Low-frequency performance improvement of seismic attenuation systems and vibration sensors for next generation gravitational wave detectors

Joris van Heijningen
Cover: mixed media with technical drawings of the devices described in Chapter 3 and Chapter 4 of this thesis. Reverse: several gravitational waveforms are shown. From top to bottom: LIGO waveform reconstruction of GW170104, both LIGO time series of GW150914, LIGO waveform reconstruction of GW151226, and Virgo waveform reconstruction of GW170814.

This work has been performed at the National Institute for Subatomic Physics Nikhef and is part of the research programme of the Foundation for Fundamental Research on Matter (FOM), which is part of the Netherlands Organisation for Scientific Research (NWO).
Turn up the bass!

Low-frequency performance improvement of seismic attenuation systems and vibration sensors for next generation gravitational wave detectors

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Abstract

Since September 2015, the LIGO Virgo Collaboration is detecting gravitational waves. After the first detection of GW150914, many binary black hole mergers were recorded. GW170814 was detected by both the Advanced LIGO detectors and the Advanced Virgo detector, which joined LIGO on August 1, 2017. Three days later, GW170817 marked the first ever display of the power of multi-messenger astronomy; a binary neutron star merger and several electromagnetic counterparts were coincidentally detected.

The measurement of gravitational waves has been possible only because most (optical) elements of the detectors have been decoupled from the Earth’s ever-present vibrations. In the Advanced Virgo design, not only is this decoupling necessary for the core optics of the detector, but also for the auxiliary optics. Some of these auxiliary optics are housed on optical benches, which are isolated from the Earth’s vibration using so-called vibration isolation systems. Five such systems have been developed at Nikhef and installed at the Virgo site. The systems are called Multistage Seismic Attenuation System (MultiSAS).

MultiSAS has been designed, constructed and tested at Nikhef. Finite element (FEM) and state space modeling have taught Nikhef engineers much about the (internal) mechanical modes of the system, designing and installing dampers where necessary. The overall transfer function measurement did not show surprises. Five more systems were constructed with the lessons learned from the first characterization campaign and, so far, the systems are behaving according to expectation, meeting requirements set by the Advanced Virgo design. The prototype set-up now serves as an advanced sensors and controls test-bed at Nikhef. MEMS accelerometers and a monolithic accelerometer with interferometric readout are developed on the seismically isolated bench. The most quiet optical table in Europe is now available in Amsterdam for industry to test their
Four systems are now suspending an optical table and are operated in vacuum. SIB2, an optical table of the injection system, is ready to be suspended. This was not critical for noise hunting of the 13 W input power, power recycled, Fabry-Perot Michelson interferometer operated at the end of observation run 2 with Advanced LIGO. The other four systems, SNEB, SWEB, SPRB and SDB2 suspend critical optical components for linear and angular alignment controls. SDB2 also houses the photodiodes which receive the signal containing the information about a passing gravitational wave. All systems have been installed and commissioned with a dummy mass. After this 2014 campaign, the dummy masses were removed and the MultiSASs were ready to be used. Later, the benches full of optics were attached to the lower suspension wire and in 2016 and 2017 optical tables were suspended. The MiniTowers, the vacuum vessels around the tables and suspensions, were subsequently evacuated.

Many tests have been performed to validate the expected performance of the isolation systems. The vacuum envelopes of the isolation systems at Virgo are below 1 mbar pressure and this has been tested to be enough at Nikhef not to have (measurable) acoustic coupling. The construction tolerances have been such that the so-called cradle effect has no effect on the inertial sensors on the top stage used for control. Thermal shielding has been designed and tested to protect the GAS filter blades from sagging too much. MultiSAS performance at the site shows to be compliant with set requirements and, apart from a GAS blade failure in April 2016, no significant problems have been encountered. The blade failure has been analyzed and modeling of Von Mises stress induced hydrogen migration, a possible culprit for the failure, is underway. Installation and commissioning of five MultiSASs has been reasonably smooth and already more systems are planned to suspend the squeezing optical table and filter cavity mirrors for Advanced Virgo. Further advancement in advanced control strategies will improve rms performance by better predicting system behavior and acting on it accordingly.

MultiSAS has provided suspended optical tables that are vibrationally so quiet that the best commercial sensors will only measure self-noise from 5 Hz onwards. Nikhef has set forth a combination of two proven concepts to create the world’s most sensitive vibration sensor in the 10 Hz to 100 Hz regime. This is pursued in order to monitor the suspended table motion down to MultiSAS specifications, which entails femtoradians and femtometer motions. The interferometric readout of the sensor features a tabletop interferometer and has a proven performance of $4 \text{ fm}/\sqrt{\text{Hz}}$ from 5 Hz onwards. When this readout was used to determine the position of the proof mass of a monolithic accelerometer, an unprecedented $8 \text{ fm}/\sqrt{\text{Hz}}$ from 30 Hz onwards has been achieved. This is a factor ten better at 30 Hz than the best available sensor: the LIGO/GeoTech GS-13 geophone.

The modeled shot noise limited sensitivity of the accelerometer with interferometric readout ($3 \text{ fm}/\sqrt{\text{Hz}}$) has not been achieved yet. More advanced control can improve the proof mass rms motion, keeping the subtraction of common mode noise in the interferometer more optimal. Additionally, the thermal noise could be reduced significantly by using a different material for the mechanics. Already, titanium versions are being produced and these are expected to have 36 times less electrical
susceptibility, which greatly reduces the viscous damping. These titanium monolithic accelerometers will most probably be dominated by structural damping. Looking into the future, quantum demolition techniques, such as squeezing, can possibly smash the $f_m/\sqrt{Hz}$ barrier.

A fiber-optic version of the same sensor design has been tested and achieved a sensitivity of $4\ \text{pm}/\sqrt{Hz}$. The readout is limited by thermal effects in the vicinity of the fibers, frequency noise and (non-isolated) seismic noise in the Nikhef lab. The next step is installing this sensor on the MultiSAS prototype optical bench in vacuum. Such a sensor can be installed in radiation or high magnetic field environments as the electrical components don’t have to be in the vicinity of the mechanics of the sensor. The interferometric readout in fiber could also be used as a high precision test mass motion sensor to replace e.g. the OSEM in the LIGO suspensions; this was the initial plan of the open air version of this readout.

The author has visited Japan as part of the ELiTES exchange program to help and learn while building the KAGRA gravitational wave detector. At NAOJ (Mitaka, Tokyo), control and sensor development for the room temperature payload for Type B(p) suspensions was performed. For the Type B(p) suspension payload, a simple inertial damping loop with appropriate roll-off filtering has shown to damp the translational, yaw and pitch modes of (dummy) test mass modes. The author also aided fabricating the cabling and set-up for (tests of) the intermediate mass and its recoil mass. All sensing and actuation is done using OSEMs (Optical Sensor and ElectroMagnetic actuation) and these have been calibrated and their designs improved.

Advanced sensor characterization and remote simulation on LVDT read out monolithic accelerometers for Type A or Type B pre-isolators has been performed. Low-frequency open loop operated sensors promise to be able to blend at lower frequency than the now used L4C geophones. This is important as no ground subtraction is foreseen in the KAGRA design. Simulation shows that, without tilt, 20 mHz blending gives better results. There is no proper tilt measurement available of the KAGRA site at this point, and this is almost surely impacting on the found simulation result.
The past four and a bit years have gone by so fast. What an absolute joy to be part of this research and this vast team of researchers working towards the mutual goal of the first detection of gravitational waves a hundred years after the prediction by Einstein! I am humbled by the time a lot of people have spent on explaining me all I wanted to know. When I started, I was new to the field and I have learned so much.

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About ten years ago, my promotor Jo van den Brand (quasi-)single-handedly took Nikhef towards what is now the hottest science of them all: gravitational wave astronomy. Even now, besides becoming Virgo’s spokesman, you are pushing for the next generation detector to be built in the Netherlands! I have enormous respect for your perseverance and enthusiasm. Thank you for the sharp questions when I presented new results and your patience with me. I know I have been quite some work at times, but I think I have made a contribution to Nikhef’s part in Advanced Virgo and our group’s endeavors.

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modeling, hammering and (GAS) blade studies presented in this thesis are your work. Your infectious enthusiasm, especially when giving demonstration about gravitational wave instrumentation to visitors or on fairs, have been inspirational throughout my time at Nikhef!

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Introduction

At the turn of the 19th century, after Maxwell had published the description of electrodynamics, physicists thought physics was complete. Albert A. Michelson displayed an example of that feeling when in 1894 he stated: “It seems probable that most of the grand underlying principles have been firmly established. An eminent physicist remarked that the future truths of physical science are to be looked for in the sixth place of decimals” [1].

On Earth, a formidable mass in itself, gravity has always been present. Gravity was first described by Newton, who, in the form of Kepler’s Laws, predicted all (but one) of the planets’ orbits to precision greater than measurement uncertainties in pre-Einstein times. All this was before the development of Quantum Mechanics and a very different description of gravity that was postulated by Einstein in 1915 [2]. Descriptions of gravity are asymptotically getting more precise in the last few centuries with the ones from Newton and Einstein and the work spawning from that. It is now believed that what is observed as gravity is actually a curvature of space-time and space-time in turn is curved due to the presence of mass (or energy), as shown in Fig I.1. Understanding the mechanism behind the force itself is also under (theoretical) study, e.g. more recently by Ref. [3].

For thousands of years men have gazed up into the skies, wondering where we are from, why we are here and where we are going. Almost all of mankind’s knowledge of the Universe and the laws that govern it are derived from observations of electromagnetic waves. Astronomy originated by visible-light astronomy, i.e. what could be seen with the naked eye. As technology advanced, it became possible to observe other parts of the electromagnetic spectrum, from radio to gamma rays. Each new frequency band gave a new perspective on the Universe and heralded new discoveries. Late in the 20th
Figure I.1: Space-time is 4-dimensional, but if it is imagined as 2-dimensional and the 3rd dimension is used to depict curvature, one can represent Einsteinian mathematics in the way shown here. Both Sun and Earth curve the space-time in their vicinity and the curvature of space-time by the Sun causes the gravitational pull on the Earth, much like a large ball in a valley. Credit: T. Pyle/ Caltech/ MIT/ LIGO Lab.

century, the detection of solar neutrinos founded the field of neutrino astronomy, giving new insights, such as the inner workings of the Sun.

The effort to detect gravitational waves seeks to expand human knowledge by observing the Universe in an entirely different way, and to further verify the correctness of the theory of General Relativity. Any theory of gravity that avoids instantaneous action at a distance must feature some kind of gravitational waves. Even Newtonian gravity can be modified to account for propagation delays from massive bodies that are the sources of attraction [4]. Gravitational waves travel as a ripple in space-time with the speed of light. The waves have a modest effect when arriving on Earth, i.e. the amplitude of the induced space-time deformation is small. The relative length change that they induce is called strain, denoted by $h$ in literature. The strain that was expected (before the first detection) from a typical nearby (~100 Mpc), perfectly oriented, source is only of the order of $10^{-23}$. Any experiment trying to perform a measurement over such a span in orders of magnitude is challenging.

Einstein predicted these waves in 1916, but, after fixing self-proclaimed blunders, he wrote it up more clearly in another paper in 1918 [5]. And yet, General Relativity is such a subtle theory that even its founder Einstein was not sure that what rolled out of the mathematics was physical. Einstein feared that the solutions which appeared to be gravitational waves were purely gauge effects, not physically real. On the experimental side, at the Chapel Hill Conference in 1957, talks by Pirani and Feynmann were pointing
in the direction of the belief that these waves were real. The use of a device to measure the relative accelerations between particles could give access to the Riemann tensor. By the end of the conference, Joe Weber and John A. Wheeler had informed conference sponsor Josh Goldberg that they were working on a paper to describe a possible implementation of Pirani’s thought experiment [6]. Experimental attempts for direct measurement of gravitational waves have been going on since then.

In September 2015 gravitational waves coming from a collision of two black holes were measured for the first time. This discovery paves the way for a new field in astronomy, coined gravitational wave astronomy. More detections followed after the first one, providing new insights in black hole population. More recently, also waves coming from a collision of neutron stars were detected. All these detections, which are also the most precise distance measurement ever done by mankind, are made by terrestrial detectors. The detectors are designed to have high sensitivity for gravitational radiation in the 10 Hz to 10 kHz band. Three of the world’s kilometer-scale interferometric gravitational wave detectors are presented in Fig. I.2.

![Image of gravitational wave detectors](image-url)

**Figure I.2:** Current kilometer scale Advanced era gravitational wave detectors: (a) an aerial photograph of one of the LIGO detectors near Livingston, Louisiana, United States, (b) an aerial photograph of the Virgo detector near Pisa, Italy and (c) a photo of one of the interferometer arm vacuum tubes of KAGRA, an underground gravitational wave detector in the mountains south of Toyama, Japan.

To have sufficient sensitivity for gravitational waves, seismic vibrations have to be suppressed by mechanical systems known as vibration isolation or seismic attenuation systems. To that extent, optical components are typically suspended to exploit the attenuating nature of a pendulum above its resonance frequency. In other words, all elements of the huge interferometers are decoupled from the ever-present minuscule, but typically more than ten orders of magnitude too high, motions of the Earth. At Advanced Virgo, the recently finished upgrade of the Virgo detector shown in Fig. 2(b), optical benches for auxiliary optics are suspended. In the previous so-called Virgo+ set-up, these benches were not suspended and outside the vacuum, and large noise contributions due to light scattered back into the interferometer were already identified back then. To reach the Advanced Virgo design sensitivity, the benches had to be in vacuum and are suspended by a compact seismic isolator. This compact isolator is coined MultiSAS and is designed and built at Nikhef.
The isolation performance of seismic attenuation systems cannot be measured directly. This is because the seismic attenuation is so large that available sensors do not have sufficient sensitivity to directly measure the motion of the optical components, which see their high frequency motion cut dramatically from a few Hz onwards. Most commercial vibration sensors are made for geophysical research and need to have sufficient sensitivity to monitor even the most quiet locations on Earth. The optical components of any gravitational wave detector are much more (seismically) quiet than these locations. Nikhef has started to develop sensors to be able to monitor these man-made quiet objects by developing sensors that reach displacement sensitivities of a few $10^{-15}$ m/$\sqrt{\text{Hz}}$.

The projects presented here are summarized in a gravitational wave scientist’s wish for (seismically) quiet objects. The low-frequency improvements fulfilling this wish can be granted by proper design and improvement of active control strategies. The ideal control performance provides a rms motion as low as possible, while maintaining the more high frequency ($f > 5$ Hz) passively part of seismic attenuation. This part can then be monitored by sufficiently sensitive sensors for unexpected displacement noises possibly introduced by modes of vacuum envelopes in combination with nearby machinery.

Chapter guideline

This thesis consists of 5 chapters. The first two chapters describe the theoretical aspects, context and detection principles, whereas chapters 3, 4 and 5 present the work done over the course of the author’s doctorate.

- Chapter 1 starts with a short description of General Relativity and how gravitational waves emerge from it. The first detections are presented after that before continuing to build a theoretical basis for the chapters to come. The theory behind vibration isolation, its controls and sensors as well as examples of these three topics are shown in succession in the final sections.

- Chapter 2 is an overview of how to detect gravitational waves with kilometer scale interferometers. It commences with a description of why it actually works. Then it expands the toy model of the interferometer to more complex optical set-up to increase sensitivity. The real life example of Advanced Virgo is presented and future detectors, namely the underground Einstein Telescope and the space-bound LISA detector, are briefly discussed.

- Chapter 3 is a description of the development and installation of a seismic isolation system called MultiSAS. Five of these systems were developed and built at Nikhef. They have been installed at the Advanced Virgo detector and their performance is shown and discussed. Issues that arose during the commissioning phase and their solutions or mitigation strategies end the chapter. Details of the prototype
campaign and all non-general descriptions of each of the five Virgo systems are summarized in Appendix A.

- Chapter 4 presents a novel sensor developed at Nikhef. It features an interferometric readout for a horizontal monolithic accelerometer, which will be able to perform a shot-noise limited vibration measurement of $3 \times 10^{-15} \text{ m/ } \sqrt{\text{Hz}}$ from 10 Hz onwards. This accelerometer is compact for its performance and vacuum compatible. It can perform a measurement that monitors the required displacement vibration levels of an optical bench suspended by MultiSAS. A fiber version of the same optical readout scheme was also developed and preliminary results are presented in Appendix B.

- Chapter 5 summarizes the projects in which the author participated in instrumentation for the cryogenic and underground gravitational wave detector KAGRA in Japan. The author spent a total time of four months at NOAJ, Mitaka, Tokyo and two months at the KAGRA site in three visits to Japan. After a short overview of the KAGRA detector, a description of the vibration isolation systems is given and preliminary measurements of LVDT read out monolithic accelerometers on the first vibration isolation stage for so-called Type A or Type B suspensions are presented. Finally, work on control of the final isolation stage of the so-called Type B(p) suspension is presented.

The conclusions, recommendations and future work are followed by two appendices. A summary in English, Dutch and Italian is provided at the end of this thesis.
The theory of General Relativity (GR), as put forward by Einstein in 1915 [2], has tremendous predictive power. He predicted that light would be bent by the curvature of space-time surrounding the Sun, and bent light was indeed observed in 1919 as the Sun was eclipsed by the Moon [7]. Light from stars at known positions in the sky suddenly seemed to come from a slightly different position. Most predictions from or published after GR have been tested, but one prediction had such a minuscule effect that Einstein himself thought it would never be measured: Gravitational Waves (GWs) [5].

Here, the mathematical foundations towards concepts and results found in subsequent chapters are presented. Starting with a brief introduction to GR and its linearized version, GWs from sources that are strong enough to be measured billions of light-years away are discussed. One such source, a binary black hole merger, produced a GW so strong that on September 14, 2015, scientists were able to measure its effects even after the wave spread out over the Universe for about 1.3 billion years.

In addition, theoretical aspects and descriptions of instrumentation for GW detectors are presented. An overview of used vibration isolation concepts and the parts that make up a typical seismic attenuation system are explained. The mechanical modes of these systems are damped by digital control systems, which are subsequently discussed. The chapter ends with sensor theory and the typical sensors that are used in the various GW detectors.
1.1 General relativity and gravitational waves

Where Newton states that gravity is an instantaneous force between massive objects, Einstein postulated that mass and energy curve space-time and this curvature tells mass and energy how to move. Mass, energy and space-time are related by the equations of GR known as the Einstein Field Equations (EFE). These ten, coupled, non-linear differential equations can be represented by the single tensor equation

\[ G_{\mu \nu} = R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = \frac{8\pi G}{c^4} T_{\mu \nu}, \tag{1.1} \]

where the Einstein tensor \( G_{\mu \nu} \) contains information on the curvature of space-time, and the energy momentum tensor \( T_{\mu \nu} \) captures the mass, energy and momentum. For a given \( T_{\mu \nu} \) the EFE yield the metric \( g_{\mu \nu} \). From the metric and its derivations one can derive the Riemann curvature tensor and its contractions \( R_{\mu \nu} \) and \( R \), the Ricci tensor and curvature scalar, respectively. The metric tensor \( g_{\mu \nu} \) is the central object that determines the fundamental characteristics of a space-time, describing distances, angles, volumes, and past and future directions.

1.1.1 Linearized general relativity

The Minkowski metric defines the space time interval, or world line \( ds^2 \), between two events in flat Cartesian space which is empty in terms of mass and energy

\[ ds^2 = g_{\mu \nu} dx^\mu dx^\nu = -c^2 dt^2 + dx^2 + dy^2 + dz^2. \tag{1.2} \]

The Minkowski metric is usually written as \( \eta_{\mu \nu} \) and can be represented in the form of the 4×4 matrix

\[ g_{\mu \nu} = \eta_{\mu \nu} = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{1.3} \]

According to usual GR conventions [8], the Minkowski metric is defined with respect to the Cartesian basis

\[ x_0 = t, \ x_1 = x, \ x_2 = y, \ x_3 = z. \tag{1.4} \]

The Ricci tensor \( R_{\mu \nu} \) and Ricci scalar \( R \) are calculated from derivatives of the metric. They are contractions of the Riemann tensor

\[ R^\sigma_{\mu \lambda \nu} = \partial_\lambda \Gamma^\sigma_{\mu \nu} - \partial_\nu \Gamma^\sigma_{\mu \lambda} + \Gamma^\theta_{\mu \nu} \Gamma^\sigma_{\theta \lambda} - \Gamma^\theta_{\mu \lambda} \Gamma^\sigma_{\theta \nu}, \tag{1.5} \]

where \( \partial_\nu \) denotes the partial derivative, which is for a vector field

\[ \partial_\alpha A = \frac{\partial A^\beta}{\partial x^\nu} e_\beta. \tag{1.6} \]
in which $e_\beta$ represents a basis vector. $\Gamma^\nu_{\beta\gamma}$ represents the Christoffel symbol and is given by

$$\Gamma^\nu_{\beta\gamma} = \frac{1}{2}g^{\alpha\delta}(\partial_\gamma g_{\beta\delta} + \partial_\beta g_{\gamma\delta} - \partial_\delta g_{\beta\gamma}). \quad (1.7)$$

Inspecting Eq. (1.5), it can be seen that

$$R_{\mu\nu} = R^i_{\mu\nu} \quad \text{and} \quad R = g^{\mu\nu} R_{\mu\nu} = R^i_i. \quad (1.8)$$

In cataclysmic astrophysical events involving compact objects and large curvatures the metric $g_{\mu\nu}$ can become complicated and the EFE describing such systems are highly non-linear. However, if we are interested in the metric near an observer, e.g. here on Earth, far away from all large sources of curvature, one can assume that the effect on this coordinate system is weak. The metric of such distant observer can be approximated by a small, linear perturbation to the Minkowski metric, or

$$g_{\mu\nu} = \eta_{\mu\nu} + \varepsilon h_{\mu\nu}, \quad (1.9)$$

where $|\varepsilon| \ll 1$. This is in the case of a special observer which lives in a flat space-time. This linear formalism, in combination with the EFE, predicts that the perturbation $h_{\mu\nu}$ may be interpreted as waves interacting with the observer’s coordinate system. These perturbations contain information about the original source dynamics and subsequent space-time curvature evolution.

### 1.1.2 Gravitational waves

Starting with the linearized metric given in Eq. (1.9), one can formulate all terms of the field equations of Eq. (1.1). Partial derivatives of the Minkowski metric vanish, leaving the Christoffel symbols to only depend on the perturbation (dropping terms $O(\varepsilon^2)$),

$$\Gamma^\nu_{\beta\gamma} = \frac{1}{2}\eta^{\alpha\delta}(\partial_\gamma h_{\beta\delta} + \partial_\beta h_{\gamma\delta} - \partial_\delta h_{\beta\gamma}), \quad (1.10)$$

and the Riemann tensor simplifies to

$$R^\nu_{\mu\lambda\nu} = \frac{1}{2}\eta^{\alpha\sigma}(\partial_{\lambda\sigma} h_{\mu\nu} + \partial_{\nu\sigma} h_{\mu\lambda} - \partial_{\mu\sigma} h_{\lambda\nu} - \partial_{\nu\sigma} h_{\lambda\mu}). \quad (1.11)$$

The four-space d’Alembertian operator is denoted as

$$\Box \equiv \eta^{\mu\nu}\partial_\mu\partial_\nu = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x_i \partial x_i}. \quad (1.12)$$

With $h \equiv h^\mu_{\mu}$ the trace of $h_{\mu\nu}$, the Ricci tensor $R_{\mu\nu}$ and curvature scalar $R$ can be written as

$$R^i_{\mu\nu} = R^i_{\mu\nu} = \frac{1}{2}\varepsilon(\partial_{\lambda\sigma} h^\mu_{\nu} + \partial_{\nu\sigma} h^\lambda_{\mu} - \Box h_{\mu\nu} - \partial_{\nu} \partial_{\mu} h), \quad (1.13)$$

$$R = \eta^{\mu\nu} R^i_{\mu\nu} = \varepsilon(\partial_{\mu} \partial_{\nu} h^{\mu\nu} - \Box h).$$
Substituting these results in Eq. (1.1) and again dropping terms $O(\epsilon^2)$ gives
\[
\frac{8\pi G}{c^4} T_{\mu\nu} = \frac{1}{2} \epsilon (\partial_\lambda \partial_\mu h^\lambda_\nu + \partial_\nu \partial_\mu h^\lambda_\mu - \Box h_{\mu\nu} - \partial_\mu \partial_\nu h - \eta_{\mu\nu} \partial_\rho \partial_\sigma h^{\rho\sigma} - \partial h). \tag{1.14}
\]

Without loss of generality, Eq. (1.14) may be further compressed by substituting the so-called trace reverse $\tilde{\eta}_{\alpha\beta} = h_{\alpha\beta} - \frac{1}{2} \eta_{\alpha\beta} h$ for $h_{\alpha\beta}$ to
\[
\frac{8\pi G}{c^4} T_{\mu\nu} = \frac{1}{2} \epsilon \left( \partial_\lambda \partial_\mu \tilde{h}^\lambda_\nu + \partial_\nu \partial_\mu \tilde{h}^\lambda_\mu - \Box \tilde{h}_{\mu\nu} - \eta_{\mu\nu} \partial_\rho \partial_\sigma \tilde{h}^{\rho\sigma} \right). \tag{1.15}
\]

Acknowledging one is free to transform the coordinate system defined in Eq. (1.4) as long as the Riemann tensor, and therefore the Ricci tensor and curvature scalar remain unchanged, the transverse traceless (TT) transformation is applied, which is
\[
\chi^{\alpha}_{TT} = x_\alpha + \epsilon \phi^\alpha_{TT}, \tag{1.16}
\]
where $\phi^\alpha_{TT}$ is a function of position and $\partial^i \phi^\alpha_{TT} = 0$, $\Box \phi^\alpha_{TT} = \partial_\beta \tilde{h}^\beta_\alpha = 0$ [9]. The TT transformation makes the first, second and fourth term in Eq. (1.15) vanish, resulting in
\[
\frac{8\pi G}{c^4} T_{\mu\nu} = -\frac{1}{2} \epsilon \Box h^\mu_\nu. \tag{1.17}
\]

Finally, in a space-time far away from the source, the stress-energy tensor $T_{\mu\nu}$ vanishes leaving only a three dimensional wave equation in vacuum
\[
0 = \Box h^\mu_\nu = \left( \frac{1}{-c^2} \frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^i \partial x^i} \right) h^\mu_\nu \tag{1.18}
\]
from which the existence of time dependent waves, which travel at the speed of light $c$, can be inferred. Furthermore, it can be shown in the TT gauge, after employing symmetries demanded by the Riemannian geometry, only two independent components of the original perturbation remain. The waving tidal deformations are found in $\chi^{1}_{TT}$ and $\chi^{2}_{TT}$, while the wave is traveling in $\chi^{0}_{TT}$ and $\chi^{3}_{TT}$ (in the z-direction), and are
\[
h^\mu_\nu = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & h_{11} & h_{12} & 0 \\
0 & h_{12} & -h_{11} & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \tag{1.19}
\]
where each component is transverse and time dependent, and may be expressed as plane waves
\[
h^\mu_\nu = A_{\mu\nu} e^{ik_\alpha x^\alpha}, \tag{1.20}
\]
as long as $k^1 k_1 = 0$, i.e. the wave is transverse, and $A_{\alpha\beta} k^\lambda = 0$ (from the gauge condition $\Box \phi^\alpha_{TT} = \partial_\beta \tilde{h}^\beta_\alpha = 0$). With only two independent components it is often convenient to express the plane wave solution as two independent polarizations or wave forms
\[
h^\mu_\nu = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & h_{11} & 0 & 0 \\
0 & 0 & h_{12} & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} + \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & h_{12} & 0 \\
0 & h_{12} & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} \tag{1.21}
= h^+_{TT}(t - z/c) + h^\times_{TT}(t - z/c),
reverting back from tensor notation into the Cartesian basis. It is often more convenient to write these waveforms in the Fourier domain as $h_{+,\times}^{TT}(f)$. The Fourier transform is two-sided and can be written as

$$h_{+,\times}^{TT}(t - z/c) = \int_{-\infty}^{\infty} h_{+,\times}^{TT}(f)e^{2\pi if(t - z/c)}df.$$  \hspace{1cm} (1.22)

How these small perturbations arriving at the otherwise flat metric on Earth can be measured by an interferometer, is discussed in the start of section 2.1.

1.1.3 Sources of gravitational waves

At the source of the GW, mass and energy are abundant and curvature is large. One can derive how astrophysical sources produce these GWs by using a multipolar expansion of the stress energy tensor $T_{\mu\nu}$, instead of assuming vacuum [8]. The expansion yields an expression for the spatial components of the strain in the TT gauge [10],

$$h_{jk}^{TT} = \frac{2}{r} G \frac{\ddot{Q}_{jk}}{c^4},$$  \hspace{1cm} (1.23)

where $\ddot{Q}_{jk}$ denotes the second time derivative of the quadrupole moment of the mass distribution of the source, $G$ the Newtonian constant of gravitation, and $r$ the distance between the source and the observer. There are many astrophysical sources believed strong enough to produce gravitational radiation that is measurable on Earth.

Stochastic Background Radiation

Stochastic Background Radiation of cosmological or astrophysical origin is predicted by GR. Analogous to the Cosmic Microwave Background, it is omnipresent in the Universe and could entail a snapshot of (just after) the Big Bang! The sources include the amplification of quantum vacuum fluctuations due to inflation and cosmic string cusps [11]. Coalescing binary systems, rotating neutron stars, and supernovae that are too frequent in time to resolve are astrophysical sources believed to contribute to this background as well [12, 13].

Continuous Waves

The non-axisymmetric deformations of, for example, a rapidly rotating neutron star cause disturbances in space-time [14]. In realistic neutron stars, such quadrupolar distortions may be supported by either elastic stresses or strong magnetic fields misaligned to the axis of rotation. For the latter case, if the magnetic field poles are in the line of sight, polarized electromagnetic emission causes the star to pulse, and those stars are referred to as Pulsars. Pulsars in accreting binary systems, may have in-falling matter guided by magnetic fields which can also lead to quadrupolar moments [15]. Radio observations of the spin rate decrease of pulsars place upper limits on their gravitational wave emission. However, this emission may be directly detectable, and if not, a search would at least provide the opportunity to further refine these upper limits.
Current estimates of the strain produced by these stars range from \[ h_{TT,jk,CW} \approx 1.06 \times 10^{-25} \left( \frac{\epsilon}{10^{-6}} \right) \left( \frac{I}{10^{38} \text{ kgm}^2} \right) \left( \frac{10 \text{ kpc}}{r} \right) \left( \frac{f}{1 \text{ kHz}} \right) \] (1.24) to as low as \( h_{TT,jk,CW} \approx 10^{-27} \) [16]. Here, \( \epsilon \) denotes the equatorial ellipticity, \( I \) the moment of inertia about the spin axis and \( f \) the frequency of the gravitational wave, which is in general twice the spin frequency.

### Burst

Events that also result in Long Gamma Ray Bursts (GRBs) and supernovae are believed to be promising sources of GW radiation. GRBs are flashes of gamma rays whose origins are uncertain. Presumably, they are massive energy releases (jets) of stellar mass objects in events of catastrophic proportions [17]. Such a cataclysmic event would produce gravitational waves, although it is currently impossible to predict what the waveform would look like.

Supernovae, as a result of core collapse at the end of star’s life, are believed to emit gravitational radiation if the collapse is asymmetric in any fashion, i.e. the quadupolar moment of the gravitational field is significant [18]. However, modeling of asymmetric core collapse at its current state provides minimal guidance of the strength of the signal. In addition, the explosions pose extreme technical difficulties when calculating the bulk properties of degenerate matter during collapse.

It is difficult to model these sources and to predict the waveforms of their gravitational radiation. Confidence in the detection would increase if a signal was observed electromagnetically, by e.g. neutrino telescopes or radio telescopes as well as by GW detectors. Therefore a multi-messenger approach for their detection is pursued [19].

### Compact Binary Coalescence

Compact binary coalescence has always been the candidate of first GW detection as it is expected to give the strongest, best modeled signals. A large fraction of the stellar population is found in binary systems [20, 21]. The first binary system with a pulsar was found by Russell A. Hulse and Joseph H. Taylor in 1975 [22] and the Hulse-Taylor binary was soon determined to radiate GWs according to the GR prediction [23]. This was the first indirect evidence for GWs as the binary system was shedding energy, i.e. decreasing orbital period in the form of a negative shift in periastron time. Fig. 1.1 shows that 30 years after this measurement started, data points still differed less than 0.2% from GR predictions. Gravitational radiation has since then been inferred from several other binary pulsar systems including the double-pulsar binary system PSR J07373039 [24], from which the most precise values (before GW150914 [25]) of Post-Newtonian parameters were determined.

Eventually, the binary system will lose so much energy that it collapses, and the two compact objects will coalesce, releasing massive amounts of gravitational radiation while doing so. The frequency and amplitude of the strain depends on the (chirp) mass of the
compact binary system as [10]

\[
f(\tau) \approx 135 \text{ Hz} \left( \frac{M_c}{1.2 M_\odot} \right)^{5/8} \left( \frac{\tau}{1 \text{ s}} \right)^{3/8},
\]

\[
h_{jk,\text{CBC}}^{\text{TT}}(\tau) \approx 2.1 \times 10^{-23} \left( \frac{M_c}{1.2 M_\odot} \right)^{5/3} \left( \frac{1 \text{ s}}{\tau} \right)^{1/4} \left( \frac{200 \text{ Mpc}}{r} \right),
\]

where \(\tau\) represents the time to coalescence of the binary stars, \(r\) the distance to the source and \(M_c = (M_1 M_2)^{3/5}/(M_1 + M_2)^{1/5}\) the chirp mass, a characteristic mass defined by the mass of each component of the binary. For a neutron star binary, where typically \(M_1 = M_2 = 1.4M_\odot\), at a distance of 100 Mpc, this amplitude is roughly \(h_{jk,\text{CBC}}^{\text{TT}} \approx 10^{-22}\) at inspiral frequency of 200 Hz.

The inspiral is followed by the merger phase. In the merger phase, which last only milliseconds, the two objects get close to each other and the gravitational forces become sufficiently strong that the two bodies start to plunge towards each other, colliding and forming one object. During the ring-down phase, the newly formed object calms down to a steady state by radiating gravitational waves. Computation of the inspiral, merger and ring-down phase of the binary coalescences is actively being researched. The goal of this computation is to provide templates for matched filtering. In this process, the numerical waveforms are compared to the output signals generated by gravitational wave detectors.

The waveforms of such events are fully predictable using numerical relativity [27] and the astrophysical occurrence rates of systems may be constrained by electromagnetic observations [28]. These, among other factors, make these systems the most promising sources of GWs, strong enough to be measured with current GW detector sensitivities.
1.2 Gravitational wave detections

Advanced LIGO's first observing run (O1) lasted from September 12, 2015 to January 19, 2016. In O1, the first direct detection of GWs was achieved with the discovery of two binary black hole (BBH) mergers, GW150914 [29] and GW151226 [30]. In total, 51.5 days of coincident data were recorded in O1. After data with excess noise were removed from the analysis, the total amount of coincident data was 49.8 days.

The second observing run (O2) saw the introduction of a third detector to the worldwide GW detector network: Advanced Virgo. The Advanced LIGO detectors started O2 on November 30, 2016, Advanced Virgo joined on August 1, 2017 and the run ended on August 25, 2017. In O2, three BBH mergers were detected, which were coined GW170104, GW170608 and GW170814. The latter detection was the first detection where the signal was coincidently measured in all three detectors, paving the road for multi-messenger astronomy. The ability of the grown network to better determine the position of the GW source paid off three days later. GW170817 was determined to be a binary neutron star (BNS) merger and traditional telescopes were able to detect several electromagnetic counterparts.

1.2.1 First detection of gravitational waves

On February 11, 2016, the LIGO Virgo Collaboration (LVC) announced the first confirmed observation of gravitational waves from colliding black holes. The gravitational wave was observed by the LIGO's twin observatories on September 14, 2015. Along with the publication detailing this first direct detection of GWs, several companion papers were released that provide a complete description of the O1 analyses and the state of the Advanced LIGO interferometers during the run, e.g. Ref. [25, 31–34]. A description of an interferometer like Advanced LIGO and all the difficulties involved with measuring GWs is found in Chapter 2.

Fig. 1.2 shows a calibrated and filtered time domain representation of GW150914 in the Livingston (L1) and Hanford (H1) LIGO detectors with the best estimated waveform overlaid on top. Both the signal and the waveform have been bandpass filtered to isolate the frequency range where the detectors are sensitive and the signal has power. Notch filters were used to remove noise sources with a static frequency, such as the 60 Hz power line frequency. As two compact objects revolve closer and closer and faster and faster around each other, an increasing amplitude and frequency that constitutes the characteristic chirp signal is expected.

It is rather remarkable that GW150914 is so visible in the data even with minimal filtering. Due to the high total mass of the system, 65 $M_\odot$, the two black holes of GW150914 coalesced quickly and at a low frequency, spending about 0.2 seconds in the frequency range where LIGO has the highest sensitivity. Not all CBC signals or other sources for that matter, that are loud enough to be detected, are expected to be as
Figure 1.2: Time signals of GW150914 in both Advanced LIGO detectors, band pass filtered and whitened by each instrument’s noise amplitude spectral density. The red curve is a CBC waveform generated for BBH mergers using the best estimated parameters. The overlap between the three curves is significant, demonstrating many cycles of clear coherence and the expected chirp signal. Time is relative to GPS time 112625946.22, which corresponds to September 14, 2015, 9:50:45.22 UTC. Reproduced from Ref. [35].

visible in the time series as is the case for GW150914. That is why considerable effort has been devoted to data analysis of noisy detector output signals in which a GW signal may be hiding.

1.2.2 Other LIGO detections in O1 and O2

The second GW detection, GW151226, required state-of-the-art techniques such as matched filtering to be extracted from the data, which is presented in Fig. 1.3. The binary black hole system that produced GW151226 was roughly three times less massive than that of GW150914 and merged at a similar distance. This implies that GW151226 spent about two seconds in the frequency band that LIGO is sensitive to, which is a factor of ten longer than GW150914.

The third loudest foreground event in the O1 analysis, LVT151012, stands out from the background distribution but is not statistically significant enough to be labeled as a GW detection [35]. While it is not being claimed as a detection, there is no obvious reason to believe that it is a noise artifact based on detector performance. Main parameters and signal strength with respect to detector sensitivities of the three O1 events are summarized in Table 1.1 and Fig. 1.4, respectively. By adding $M_1$ and $M_2$ and comparing this sum to $M_{\text{final}}$, one can deduce that 3.0, 0.9 and 1.0 solar masses were converted into GWs for GW150914, GW151226 and LVT151012, respectively.

Plots similar to Fig 1.4 (left panel) are very common in the GW field. Amplitude Spectral Density (ASD) plots, typically with some unit (e.g. meter or Watt) per $\sqrt{\text{Hz}}$ on the vertical
Figure 1.3: Upper panels: filtered time signals of GW151226 in both Advanced LIGO detectors. The black curve represents a CBC waveform generated for BBH using the best estimated parameters and filtered similar to the detector strain curves. Lower panels: accumulated peak SNR\(p\) of GW151226 in both Advanced LIGO detectors. Hanford, having better overall sensitivity, reached an SNR of 10, most of which was accumulated in the final phase of the CBC, i.e. during the merger and ringdown phase. Adapted from Ref. [30]

Axis and frequency on the horizontal axis, are used to present the spectral content of some measured or observable quantity, such as displacement or power. The integral of the curves in these plots results in the rms motion or power (expected) in the frequency interval over which one integrates. For example, the value between 100 Hz and 200 Hz of the strain spectrum in Fig. 1.4 (so the red and blue curve for the Hanford and Livingston LIGO detector strain sensitivity) could be approximated as a straight line with amplitude \(10^{-23} \text{1/}\sqrt{\text{Hz}}\).

<table>
<thead>
<tr>
<th>Event</th>
<th>FAR [yr(^{-1})]</th>
<th>(M_1) [(M_\odot)]</th>
<th>(M_2) [(M_\odot)]</th>
<th>(M_{\text{final}}) [(M_\odot)]</th>
<th>(R) [Mpc]</th>
</tr>
</thead>
<tbody>
<tr>
<td>GW150914</td>
<td>(&lt; 6.0 \times 10^{-7})</td>
<td>36.2(^{+5.2}_{-3.8})</td>
<td>29.1(^{+3.7}_{-4.4})</td>
<td>62.3(^{+3.7}_{-3.1})</td>
<td>420(^{+150}_{-180})</td>
</tr>
<tr>
<td>GW151226</td>
<td>(&lt; 6.0 \times 10^{-7})</td>
<td>14.2(^{+8.3}_{-3.7})</td>
<td>7.5(^{+2.3}_{-2.3})</td>
<td>20.8(^{+6.1}_{-1.7})</td>
<td>440(^{+180}_{-190})</td>
</tr>
<tr>
<td>LVT151012</td>
<td>0.37</td>
<td>23(^{+18}_{-6})</td>
<td>13(^{+4}_{-5})</td>
<td>35(^{+14}_{-4})</td>
<td>1000(^{+500}_{-500})</td>
</tr>
</tbody>
</table>

Table 1.1: Foreground events found in the first observing run O1. The quoted false alarm rates (FAR) are calculated by the data analysis search pipelines. All quoted mass measurements are in the source frame. Given are median values with 90% credible intervals. The astrophysical parameters are further explained in the parameter estimation companion paper [32].

Assuming this constant value, the area underneath (i.e. integral of) this line segment, where the horizontal axis interval should be square rooted, is equal to \(10^{-22}\) relative displacement of the test masses of a detector. Ignoring all spectral content below 100
Figure 1.4: Left panel: Amplitude Spectral Density of the total strain noise of the H1 and L1 detectors during O1 (both Hanford and Livingston had strain sensitivities resulting in a BNS inspiral range of about 80 Mpc), $\sqrt{S(f)}$, in units of strain per $\sqrt{\text{Hz}}$, and the recovered signals of GW150914, GW151226, and LVT151012 plotted so that the relative amplitudes can be related to the SNR of the signal. Right panel: Time evolution of the recovered signals from when they enter the detectors sensitive band at 30 Hz. Both figures show the 90% credible regions of the LIGO Hanford signal reconstructions from a coherent Bayesian analysis using a nonprecessing spin waveform model. Reproduced from Ref. [35].

Hz and above 200 Hz for this example, the test mass displacement expected is thus $10^{-22} \times 4 \text{ km}$ (in the LIGO detectors, arms are 4 km long) $= 4 \times 10^{-19}$ meter over $1/(100 \text{ Hz}) = 0.01$ second time scales. Integrating down to lower frequencies (and including now also the spectrum above 200 Hz) gives the rms strain of the detector. This is then up to time scales which are the inverse of the frequency down to which the integral is taken, e.g. down to 10 mHz gives expected rms over 1 minute and 40 seconds.

O2, the second observation run in the Advanced detector era started November 30, 2016. On January 4, 2017, GW170104 was detected with high statistical significance [36]. The source of GW170104 is a heavy stellar mass binary black hole system, with a total mass of about 50.7 $M_\odot$. Presented in Fig. 1.5, this signal is visible in the band filtered time signal. The component masses of this merger are determined [36] to be $31.2^{+8.4}_{-6.0} M_\odot$ and $19.4^{+5.3}_{-5.9} M_\odot$, and the radiated energy is $2.0^{+0.6}_{-0.7} M_\odot c^2$. The luminosity distance is determined to be $880^{+450}_{-390}$ Mpc which corresponds to a redshift of $0.18^{+0.08}_{-0.07}$.

On June 8, 2017, GW170608 was detected by the two LIGO detectors. The source of this GW is the lightest binary black hole system measured to date, with a total mass of about 19 $M_\odot$. In Fig. 1.6, the signal is visible in the time-frequency maps of both Hanford and Livingston LIGO detectors. The component masses of this merger are
Figure 1.5: Time signals of GW170104 in both Advanced LIGO detectors. The upper panel shows the time-series data from each detector with a 30 - 350 Hz bandpass filter and corrected for each instrument’s noise amplitude spectral density. The Livingston data have been shifted back by 3 ms, and the sign of its amplitude has been inverted to account for the detectors different responses. The maximum-likelihood binary black hole waveform model is shown in black. The bottom panel shows the residuals between each data stream and the maximum-likelihood waveform. Reproduced from Ref. [36]

The luminosity distance is determined to be $340_{-140}^{+140}$ Mpc which corresponds to a redshift of $0.07_{-0.03}^{+0.03}$. Determined [37] to be $12_{-7}^{+7} M_{\odot}$ and $7_{-2}^{+2} M_{\odot}$ and the radiated energy is $0.85_{-0.17}^{+0.07} M_{\odot} c^2$.

Figure 1.6: Time-frequency power maps of LIGO strain data at the time of GW170608. Reproduced from Ref. [37].
All BBH detections in O2 are entirely consistent with the astrophysical rates inferred from O1. Expanding the catalog of binary black holes will provide further insight into their formation and evolution, and will allow for tighter constraints on potential modifications to GR.

1.2.3 First triple-coincident detection by LIGO and Virgo

On August 1, 2017, Advanced Virgo expanded Advanced LIGO to a global GW detector network. Advanced Virgo joined with a BNS inspiral range around 28 Mpc and the three detectors measured until the end of O2, August 25, 2017. Fourteen days after Advanced Virgo joined, a GW signal coined GW170814, coming from the merger of two stellar mass black holes, was published as the first triple-coincident detection. The component masses of this merger are determined to be $31.2^{+8.4}_{-6.0} M_{\odot}$ and $25.3^{+2.8}_{-4.2} M_{\odot}$, and the radiated energy is $2.7^{+0.4}_{-0.3} M_{\odot} c^2$ [36]. The luminosity distance is determined to be $540^{+130}_{-210}$ Mpc which corresponds to a redshift of $0.11^{+0.03}_{-0.04}$. The detection strain-time series and time-frequency plots are shown in Fig. 1.7.

While the signal had a low SNR in the Advanced Virgo detector, the triple detection allowed for a significant improvement in the measurement of both the distance and sky-position of the source, as shown in Fig. 1.8. In particular, the triple detection improves the sky localization of the source, reducing the 90% of the confidence region of the double-coincident detection from 700 to ultimately 60 square degrees (deg$^2$). This enhances the opportunity of identifying an electromagnetic counterpart to the gravitational wave signal, simply because conventional telescopes have a smaller so-called box in the sky. With a network of three detectors, the LIGO Virgo Collaboration was able to probe the polarization content of the signal, and find that the data favor the pure tensor polarization of gravitational waves, over pure scalar or pure vector polarizations.

Nature has so far provided scientists with five (plus one) complementary BBH detections. GW150914 was a merger in the most sensitive part of the LIGO detectors, which is optimal for tests of GR. GW151226 provided a long in-band inspiral, which allowed for a more precise measurement of chirp mass, a particular combination of the two black hole masses, and component spins. The spin precession can be inferred best from the modulation on the monotonically rising amplitude of the signal up to the merger, as is visible in the black curves Fig. 1.3. Also the masses of the black holes prior to merger differ relatively the most with respect to the other events. LVT151012 has a similar waveform compared to the other events, but the cosmic distance of the event is larger.

GW170104 was a heavy system but at about twice the distance of the source of the first detection. Together with LVT151012 (even further away), this provides possibly more information on how GWs travel through space-time. If there would have been any dispersion, then a massive graviton could be proposed. GW170608 was the lightest system measured to date, further expanding the catalog of BBH mergers. Finally, GW170814 is the only BBH merger that was detected in all three detectors. This allows...
Figure 1.7: First triple-coincidence detection: time signals of GW170814 in both Advanced LIGO detectors and the Advanced Virgo detector, with times shown from August 14, 2017, 10:30:43 UTC. Top row: SNR time series produced in low latency and used by the low-latency localization pipeline on August 14, 2017. The single-detector SNRs in Hanford, Livingston and Virgo are 7.3, 13.7 and 4.4, respectively. Middle row: time-frequency representation of the strain data around the time of GW170814 in each detector. Bottom row: GW170814 in both Advanced LIGO detectors and the Advanced Virgo detector: the time-series data (colored), and 90% confidence intervals for reconstructed waveforms (light gray) and BBH models (dark gray), whitened by each instrument’s sensitivity. The right vertical axes are in units of noise standard deviations for each individual detector. The merger times at the Hanford and Virgo detectors were delayed by 8 ms and 14 ms with respect to the arrival time at Livingston, respectively. Reproduced from Ref. [38].

for much more precise localization of the source. Not only could the position in the sky be better inferred from timing and phase-at-arrival differences between the three detectors, also the source distance could be estimated with better precision. The latter is a result of now being able to properly measure the GW polarization, which allows breaking the degeneracy between source plane inclination and source distance. The volume in the sky, in which the source must have been, decreased by more than an order of magnitude.
1.2.4 First observation of a binary neutron star inspiral

On August 17, 2017 at 12:41:04 UTC, Advanced LIGO and Advanced Virgo made their first observation of a BNS inspiral. The signal, GW170817, was detected with a combined SNR of 32.4, which made it the loudest GW event detected to date. The source was localized within a sky region of 28 deg$^2$ (90% probability) and had a luminosity distance of $40^{+8}_{-14}$ Mpc, the closest and most precisely localized GW signal yet. The component masses were determined to be in the range $1.17 \, M_{\odot}$ and $1.60 \, M_{\odot}$ consistent with previously measured neutron star masses. The total mass of the system was measured to be $2.74^{+0.04}_{-0.01} \, M_{\odot}$. The signal was visible for about 100 seconds in the frequency band where the detectors have sufficient sensitivity. The last 30 seconds of the signal are presented as time-frequency power maps in Fig. 1.9.

This precise localization allowed for association with the gamma-ray burst (GRB) GRB 170817A, detected by Fermi-GBM and INTEGRAL shortly after the coalescence. This corroborated the hypothesis of a neutron star merger and provided the first direct evidence of the previously theorized link between these mergers and short GRBs [40]. Subsequent identification of transient counterparts across the electromagnetic spectrum in the same area in the sky further supported the interpretation of this event as a neutron star merger. A summary of this multi-messenger approach is presented in Fig. 1.10.

Several measurements can be done for the first time with GW170817 and its optical
Figure 1.9: First observation of a binary neutron star inspiral: time-frequency power maps of GW170817 in both Advanced LIGO detectors and the Advanced Virgo detector, with times shown from August 17, 2017 12:41:04 UTC. The amplitude scale in each detector is normalized to that detector’s noise amplitude spectral density. Adapted from Ref. [39].

counterparts. The GW can be used as a standard siren, which is the GW counterpart of the standard candle used in cosmology for distance measurements. Together with the recession velocity of the host galaxy arising from the electromagnetic data, the Hubble constant can be determined [41]. A value of $H_0 = 70.0^{+12.0}_{-8.0} \text{km s}^{-1} \text{Mpc}^{-1}$ is inferred, which is consistent with existing measurements, while being completely independent from those measurements.

Figure 1.10: Localization of gravitational wave, gamma-ray and optical signals associated with GW170817, showing an orthogonal projection of the 90% credible regions from LIGO (light green), LIGO-Virgo (dark green), triangulation from the time delay between Fermi and INTEGRAL (light blue) and Fermi GBM (dark blue). The inset shows the location of the apparent host galaxy NGC 4993 in the Swope optical discovery image at 10.9 hours after the merger (top) and a DLT40 image taken 20.5 days before the merger of the same area in the sky (bottom). The reticle marks the position of the transient in both images. The scale of 30 arcseconds can be viewed in comparison with the Moon, which on average subtends a diameter of about 1900 arcseconds in the sky. Reproduced from Ref. [40].
GR prescribes GWs to travel with the speed of light. The first electromagnetic counterpart to reach Earth was GRB 170817A that arrived 1.7 s after LIGO and Virgo detected the merger. With this delay, that could have occurred due to the merger process itself or an actual difference between the speed of gravity and the speed of light, a constrain on this difference was inferred to be between $-3 \times 10^{-15}$ and $+7 \times 10^{-16}$ times the speed of light [42].

The transients across the electromagnetic spectrum detected after the merger and the GRB came from a kilonova, which is caused by the synthesis of large amounts of very heavy elements via rapid neutron capture (also called r-process) and subsequent decay of this neutron rich material [43]. The abundance of heavy elements, such as gold and platinum, is now believed to have originated from GW170817-like BNS merger events [44]. The probability distribution of ejected mass for this event ranges between $10^{-3} M_\odot$ and $10^{-2} M_\odot$.

1.3 Vibration isolation

Strain ASDs for gravitational radiation typically fall below $10^{-21} 1/\sqrt{\text{Hz}}$ as shown in the bands for the different detected GWs in Fig. 1.4. It is thus necessary to do relative position measurements more precise than $10^{-18} \text{m}/\sqrt{\text{Hz}}$ for kilometer scale detectors, such as LIGO and Virgo. All test masses involved in such a measurement have to be isolated from the Earth's vibrations. This section will discuss the theoretical background of vibration isolation, starting from damped harmonic oscillators and moving on to how these concepts are applied in practice. The main components of a seismic isolator as built for existing GW detectors are then presented.

1.3.1 Damped harmonic oscillators

Examples of basic concepts that are used in suspensions for GW detectors are presented in Fig. 1.11. In both cases, the pendulum or spring decouples the motion of the mass from the ground motion by use of a so-called harmonic oscillatory system. The position of the ground is denoted $x_g$ and the position of the suspended object (e.g. a test mass) is given by $x_M$. The relative coordinate between the two is denoted as

$$x_r(t) = x_M(t) - x_g(t).$$  \hspace{1cm} (1.26)

The second derivative with respect to time of this coordinate is the relative acceleration $\ddot{x}_r(t)$. When subject to an excitation the magnitude of the subsequent oscillation is constrained by the damping forces that act on the mass. The damping forces can be categorized as viscous or structural

$$F_v = -\gamma \dot{x} \quad \text{viscous damping}$$

$$F_s = -i\phi k x \quad \text{structural damping},$$
Figure 1.11: (a) Horizontal and (b) vertical vibration isolation systems can be modeled as damped harmonic oscillators such as a pendulum and a damped mass-spring system, respectively. The tangential component of gravity and the spring constant $k$ act as the restoring force. The air resistance and the dashpot act as the viscous damping $\gamma$. Structural damping loss angle $\phi$ could be the friction in the suspension point pivot for the pendulum and the internal friction in the spring material for the mass-spring system.

where $\gamma$ represents the damping factor and $\phi$ the structural loss angle associated with the relative energy loss per oscillation cycle due to internal friction and material dissipation. If one now excites the system with a harmonic excitation $x = x_0 e^{jt}$, the damping forces become

$$F_v = -\gamma \dot{x} \omega x \quad \text{viscous damping}$$
$$F_s = -i \phi k x \quad \text{structural damping}.$$

Inspecting the pre-factors of these expressions, the structural loss angle in combination with the stiffness and frequency $\frac{k}{\omega}$ assumes a similar role as the viscous damping factor $\gamma$.

Applying Newton’s Second Law of Motion to the systems shown in Fig. 1.11 yields

$$M\ddot{x}_M(t) + \gamma \dot{x}_M(t) + kx(t) = 0.$$  \hspace{1cm} (1.27)

Taking the Fourier transform of this equation, the idealized frequency dependent transfer function of ground motion to mass motion motion is

$$X_M = \frac{\omega_0^2}{-\omega^2 + 2i\zeta \omega \omega_0 + \omega_0^2}X_g = \mathcal{H}(\omega)X_g,$$  \hspace{1cm} (1.28)

where $\omega_0^2 = k/M$ and $\zeta = \gamma/(2M\omega_0)$. Using the relation between the damping factor $\gamma$ and loss angle $\phi$ described above, the following expression for $\zeta$ can be used for structural
damping in Eq. (1.28), which is

\[ \zeta = \frac{\phi \omega_0}{2 \omega}. \]

From Eq. (1.28), the system’s viscously damped and structurally damped frequency response \( H_v(\omega) \) and \( H_s(\omega) \), respectively, can be obtained, so that

\[
H_v = \frac{\omega_0^2}{-\omega^2 + 2i\zeta\omega_0 + \omega_0^2} = \frac{\omega_0^2}{\omega_0^2 + i\frac{\gamma\omega}{M} - \omega^2},
\]

\[
H_s = \frac{\omega_0^2}{\omega_0^2(1 + i\phi) - \omega^2}.
\]

The quality factor \( Q \) is defined as \( 1/2\zeta \) and is a standard measure of the quality of the resonance. It represents the amplification factor of driven oscillations at the resonance frequency, and \( \pi \) times the amount of cycles before the amplitude of a step response decreases to a value \( 1/e \), i.e. about 37\%. For viscous damping and for purely elastic structural damping, the \( Q \) is given by

\[ Q_v = \frac{M\omega_0}{\gamma} \quad \text{and} \quad Q_s = \frac{1}{\phi}. \]  

In Fig. 1.12 the transfer function is shown for different values of \( Q \) as a function of frequency. Below the resonance frequency \( (\omega < \omega_0) \) the mass follows the ground motion. At the resonance frequency, the relative motion is amplified and above the resonance frequency the mass is isolated with respect to the ground.

**Figure 1.12:** The linear magnitude of the angular frequency response \( \mathcal{H} \) of a structurally or viscously damped simple harmonic oscillator with \( f_0 = 1 \) Hz for different values of \( Q \).

Large amplitude ground motion can lead to subsequent build up at the resonance frequency. In order to damp the system, a force \( f \) can be applied to the mass. In the case of both structural and viscous damping and giving also the spring a mass \( m \), the
The equation of motion is

\[
\left(M + \frac{m}{4}\right)\ddot{x}_M + \frac{m}{4}\ddot{x}_g + k(1 + i\phi)x_t + \gamma \dot{x}_M = f, \tag{1.31}
\]

of which the Fourier transform is

\[
-\left(M + \frac{m}{4}\right)\omega^2 X_M - \frac{m}{4}\omega^2 X_g + k(1 + i\phi)X_t + i\gamma\omega X_M = F. \tag{1.32}
\]

The undamped resonance frequency is now \(\omega_0 = \sqrt{k/(M + \frac{m}{4})}\) and the displacement and forced transfer functions can be written as

\[
\begin{align*}
\mathcal{H}_x(\omega) &= \frac{X_M}{X_g} = \frac{\omega_0^2(1 + i\phi) + \frac{m}{4M+m}\omega^2}{\omega_0^2(1 + i\phi) - \omega^2 + i\omega\frac{\gamma}{M+\frac{m}{4}}}, \\
\mathcal{H}_F(\omega) &= \frac{X_M}{F} = \frac{1}{\left(M + \frac{m}{4}\right)\left(\omega_0^2(1 + i\phi) - \omega^2 + i\omega\frac{\gamma}{M+\frac{m}{4}}\right)}. \tag{1.33}
\end{align*}
\]

Note that for the displacement transfer function we assume \(F = 0\), while \(F\) is assumed to be much larger than the forces imposed by the ground motion in the forced transfer function. In general, systems will be engineered such that \(m/(4M + m) \ll 1\) and \(\gamma/(M + m/4) \ll 1\) so that the transfer function decreases with \(\omega^{-2}\) above the resonance frequency \(\omega_0\). Well below the resonance frequency \(\mathcal{H}_x = 1\). As \(\omega \gg \omega_0\) the transfer function will level out to a value \(\beta = m/(4M + m)\). This is known as the center of percussion (CoP) effect and can be tuned by adjusting the mass distribution of the oscillator with the use of counter weights, as will be demonstrated later. Note that the CoP effect does not appear in the forced transfer function.

### 1.3.2 Seismic attenuation system components

From a combination of simple harmonic oscillators, having the isolation characteristics shown in the previous section, vibration isolating mechanical filters can be constructed. Two strategies can be pursued and combined here, which are low natural frequency filters and the chaining of filters. A low natural frequency of a single oscillator results in the isolation slope of \(f^{-2}\) starting at a lower frequency, which improves the isolation ratio in the bandwidth of current GW detectors, \textit{i.e.} from 10 Hz onwards. Lowering the resonance frequency without \textit{e.g.} increasing the size of the pendulum, can be done by using the anti-spring effect. Combining an unstable system, which can be seen as a system with a negative spring constant, with a stable system results in a system with tunable stiffness or resonance frequency.

The isolation ratio can also be improved by cascading mechanical filters, essentially multiplying the transfer function of each stage. Typical GW detector mirror suspensions have several cascaded isolation stages each performing as a pendulum or spring providing vibration attenuation. For some suspension designs, an additional horizontal
Figure 1.13: (a) A schematic of a mass $M$ supported by an inverted pendulum. A leg of mass $m$ carries the load and is attached to the floor via a flexible joint with a spring constant $k_\theta$. The horizontal ground and mass displacement are given by $x_g$ and $x_M$, respectively, and the angle of the legs with respect to their vertical position by $\theta$. Below the point where the bottom flexure is attached, counter weights are installed. (b) Artist impression cross-section of the inverted pendulum installed in MultiSAS, the isolator described in Chapter 3: 1) Top plate support. 2) Top flexure. 3) Inverted pendulum leg. 4) Bottom flexure. 5) Counter weights.

For some GW detector suspensions, three inverted pendulum legs support a top plate from which further filters can be suspended. As shown in Fig. 1.13, each inverted pendulum is comprised of a rigid rod of mass $m$ and length $l$, supporting a mass $M$. It is well-known that such a system is unstable, just like a broom balancing on the palm of your hand. As the system moves away from equilibrium it will continue to do so. By fixing the inverted pendula to the ground with a flexure that has a positive angular spring constant $k_\theta$, a stable system is obtained. In a two dimensional approach and with the small angle approximation, the top plate is able to move horizontally. By adjusting the amount of supported mass it is possible to tune the resonance frequency. This frequency is given by [45]

$$\omega_0 \approx \sqrt{\frac{k_\theta}{l^2} \frac{(M + \frac{m}{2})g}{M + \frac{m}{3}}}.$$  \hspace{1cm} (1.34)

The inverted pendulum system is described by replacing the factor $m/(4M + m)$ with $\beta = m/(6(M + m/3))$ in Eq. (1.33). When the base of the leg is displaced by translational ground motion, the leg rotates around a point below the top flexure, according to the
CoP effect described earlier. Above resonance, where the mass $M$ is isolated from the ground motion, the top of the leg now moves out of phase with the ground motion. For frequencies far greater than the resonance frequency ($\omega \gg \omega_0$), the top of the leg translates with amplitude and direction $x_M \approx -\beta x_g$. Lowering $\beta$, such that the frequency $\omega_0 / \sqrt{\beta}$ is as high as possible and the saturation level $\beta$ is as low as possible, is desired for the inverted pendulum. This is equivalent to setting the leg CoP rotation point as close to the top flexure as possible.

To mitigate the CoP effect, $\beta$ can be lowered by reducing the mass of the legs. For this reason the legs are designed as thin walled pipes. Additionally, the inertia of the leg may be altered by adding counterweights on an extension of the pendulum rod that reaches below the lower flexure. This is done in order to shift the CoP point as close as possible to the top flexure. This design is used in both the superattenuator [46] and the Nikhef MultiSAS (see Fig. 1.13(b) and Ref. [47]).

GW detectors measure displacement in the horizontal direction. While light travels in a straight line, the vertical axes of the suspension chains ending in the mirrors point to the center of the Earth, as explained in Fig 1.14. Vertical isolation of the mirrors is thus necessary. In addition, tolerances in the construction process of all the different components may lead to horizontal-to-vertical couplings. Typically, the vertical-to-horizontal coupling for suspensions in GW detectors is of the order of 1%.

![Figure 1.14: Mirror suspensions feature vertical-to-horizontal coupling in kilometer scale GW detectors, because of the curvature of the Earth. Over a 3 km length the vertical axis changes by an angle $\alpha = 3 \cdot 10^{-4}$ rad. A vertical displacement of the mirror $\delta y$ turns into a differential displacement $\delta z = \alpha \cdot \delta y$ in the beam direction. Reproduced from Ref. [48].](image)

For the Virgo superattenuator, vertical isolation is achieved by means of the so-called Magnetic Anti-Spring (MAS) [49]. Fig. 1.15(a) shows a standard Virgo filter from below. Twelve blade springs are bolted to the filter body in a circular configuration and they carry the load hanging from the filter. The higher up the chain, the more load must be suspended and more and/or thicker blades are used. These blades are pre-bent and are straight when loaded. A single blade suspension would have a typical resonance frequency of 1.5 Hz.

The resonance frequency can be lowered by means of the magnetic anti-spring effect.
At the top of the red part visible in Fig. 1.15(b), arrays of magnets face other arrays of magnets which are attached to the filter body. The arrays of magnets have their poles in the opposite direction, resulting in a repulsive force with a vertical component. Note that this repulsive force nulls in the vertical direction at the filter equilibrium point. The moving parts can only move in the vertical direction, so when the magnets move away from their equilibrium position, the magnetic repulsive force increases. This acts as an anti-spring as its force acts in the opposite direction as that from the blade springs thus lowering the resonance frequency of the system.

Another vertical isolation technique is the Geometric Anti-Spring (GAS) [50], which is used in the seismic isolation of the suspended benches that house the auxiliary optics for Advanced Virgo and in KAGRA suspensions. As shown in Fig. 1.16(a), the blades are positioned radially, each supporting a load $F_y$. They are compressed such that the horizontal force they exert on each other results in a tunable anti-spring effect along the vertical axis. The static properties of a GAS filter can be solved analytically for a simplified model, as was done by Cella et al. [51] where a Poisson ratio of 0 is assumed. Finite element models of the GAS blades were used at Nikhef to accurately describe stress levels along the blade, including realistic values for the Poisson ratio. In addition, various blade geometries were studied to optimize the loading capability [47].

A GAS filter can be tuned by changing the compression distance $x_L$ using the adjustment screws at the blade clamps (see Fig. 1.16(b)). The red curve is an example of a well tuned GAS filter; the tuned load remains around 320 kg for over 10 mm keystone movement and the resonance frequency is as low as 250 mHz. For increasing compression it is possible to tune a GAS filter to increasingly lower frequencies down to a critical point after which the system becomes bistable, as shown in the black curve.

**Figure 1.15:** Overview of a MAS suspension as used in the Virgo superattenuator: (a) drawing of a MAS filter and (b) a cross section of such a filter, where the red part can move with respect to the filter body. Reproduced from Ref. [49].
Figure 1.16: Overview of the an GAS suspension as used in Nikhef’s EIB-SAS [47] and MultiSAS. Panel (a) Left: Example of a GAS filter: maraging steel blade springs are radially distributed around a base plate. Right: A model of a single blade clamped at a set angle $\theta_0$ at a horizontal distance $x_L$, which can be adjusted to tune the anti-spring effect. The tips of the blades are connected to a single central keystone at an angle $\theta_{L}$. The suspended object load force is given by $F_y$. The distance along the blade is denoted by $s$ and $\theta(s)$ describes the tangential angle. Panel (b) Left: Finite element results of applied filter load versus the vertical position of the blade tip, for various compression distances $x_L$. The solid red curve corresponds to a well-tuned GAS filter. Right: Resonance frequency versus vertical position of the blade tip for various compression distances. Tuning the GAS filter essentially is finding a compression distance $x_L$ that corresponds to the desired resonance frequency at the required load. Reproduced from Ref. [52]
The blades of both the MAS and the GAS filters are made of maraging steel to accommodate the high loads and stress levels. This type of steel is a precipitation hardened alloy with high tensile strength and very low creep [53]. GAS filters also suffer from the CoP effect. For a typical GAS filter $\beta = 10^{-3}$. The value of $\beta$ can be lowered by adding so-called magic wands [54]. A magic wand is a light-weight rigid tube attached to the filter base near the blade clamp via a flexible pivot. A counterweight is fixed to one end and the other end is attached to the keystone via a thin flexure. In this way the counterweight will follow the vertical movement of the keystone but in opposite phase. More details are found in section A.3. By tuning the mass or position of the counterweight, it is possible to reduce the $\beta$ value to below $10^{-4}$.

![Torque balance to explain the workings of a GAS filter: (a) half of a 2-dimensional GAS filter (only showing one blade attached to thekeystone) in equilibrium position and (b) the same model, but now displaced by $\Delta y$ to determine the stiffness of the system.](image)

**Figure 1.17:** Torque balance to explain the workings of a GAS filter: (a) half of a 2-dimensional GAS filter (only showing one blade attached to the keystone) in equilibrium position and (b) the same model, but now displaced by $\Delta y$ to determine the stiffness of the system.

Using a torque balance rather than a force balance [47], a simple model to explain the workings of a GAS filter is shown in Fig. 1.17. In the equilibrium position, shown in Fig. 1.17(a), the torque balance at the clamp side is

$$0 = \tau_0 - mgx_L,$$

where $x_L$ represents the horizontal distance of the blade, $m$ the mass of the suspended load and $\tau_0$ the torque after the GAS blade is bent to the equilibrium position. In the figure, $F_c$ represents the compression force, $k_\phi$ the angular stiffness at the clamp and $F_y = mg$ the load of the suspended object. Once the system is brought a distance $\Delta y$ out of equilibrium by a vertical force $\Delta F_y$, as shown in Fig. 1.17(b), the torque balance reads

$$0 = \tau_0 + \Delta F_y x_L - mgx_L + F_c \Delta y - k_\phi \phi,$$

where $\phi$ represents the angle of the displacement and is equal to $\phi = \Delta y/x_L$. In the imaginary case in which there is no compression of the GAS blades, $k_y$ can be defined as the vertical stiffness without the anti-spring effect. Substituting $k_\phi = k_y x_L^2$ and $\tau_0 = mgx_L$ from Eq. (1.35) in Eq. (1.36), one can obtain the effective stiffness of the anti-spring system as

$$k_y, eff = \frac{\Delta F_y}{\Delta y} = k_y - \frac{F_c}{x_L}.$$
Real GAS blades are not straight and the pivotal point that is assumed in the discussion above is actually fixed. When the keystone moves, the shape of the GAS blades change and numerical analysis of a differential equation is necessary [51]. Further FEM modeling of GAS blades is necessary when taking into account non-triangular blade shapes as so-called corrugation effects (bending of the blades along the width of a blade) arise [47].

The vibration isolation systems described above are used in suspensions of Advanced Virgo. In particular, the inverted pendulum stage and GAS filters are cascaded to suspend optical tables for auxiliary optics. MultiSAS is the compact vibration isolation system that consists of such cascaded stages and it is described in Chapter 3.

1.4 Controls

This section starts with a description of basic properties of an analog and linear system with feedback control. The purpose of such a system is to keep a physical parameter as close as possible to a predefined value in the presence of fluctuations due to various origins. In order to do so, this parameter is monitored continuously by some sensor. When the parameter deviates from its preset value (the so-called working or lock point), a signal proportional to the deviation, the error signal, is generated, filtered and fed back to the system with opposite sign to compensate for the deviation.

Standard terminology of control systems is extensive, but there are four terms that make up the classical control loop block scheme. The plant is the physical system to be controlled. Usually, the purpose of the feedback loop is to control only one parameter of the plant, which in itself might be a complex system. An example of a plant is (part of) a suspension system. The sensor is a detector for the parameter to be controlled. The output of the sensor is called the error signal, measured at the error point. The servo or control filter is any kind of system that transforms the signal from the sensor into a feedback signal that can be fed back to the actuator. Typically, it is an electric analog board or digital block in the loop that has a frequency dependent gain. The servo output is called the feedback signal, which is sent to an actuator, which acts on the plant. It is used to change the controlled parameter under command of the servo output.

Any linear system can be characterized by its transfer function [55]. Consider a plant or control filter that has an input $x_{\text{in}}(f)$ with a certain frequency content and output $x_{\text{out}}(f)$. The linear transfer function or gain is then $G = x_{\text{out}}(f)/x_{\text{in}}(f)$ and is a complex function having both amplitude and phase information. If such systems are in series, then gains of the subsystems can be multiplied; if systems are in parallel having the same input signals, then their outputs may be summed. Multiplying all the (parallel added) blocks of a loop gives the open loop (OL) response of the total control loop.
1.4.1 Feedback

A control loop is closed when all subsystems are connected sequentially and form a loop, i.e. the sensor measures some parameter of the plant, its signal is connected to the filter, and the filter generates the feedback signal that is connected to the actuator, which, in turn, changes the previously measured property of the plant. The most simple block scheme of a feedback loop is shown in Fig.1.18, where the sensor is part of the plant $G$ at the error point and the actuator brings the feedback signal back to the plant. The OL response of such a system is $\text{OL} = G \cdot H$ which is a frequency dependent complex function.

Figure 1.18: A simple block representation of a feedback loop. Here $x_N$ represents the noise, e.g. seismic noise, that flows into the plant $G$ and the feedback loop is trying to suppress the effect of that noise. Sensors produce error signal $x_{EP}$ by measuring e.g. the position of the plant at the error point. The control filter $H$ produces signal $x_{FB}$, which is fed to the actuators producing a negative feedback on the plant by counteracting on $x_N$.

The signals in the loop can be calculated by using $x_O = x_N - x_{FB}$ and $x_{FB} = x_O \cdot GH$. This yields for the signal to be sent to the actuator

$$x_{FB} = \frac{GH}{1 + GH} x_N = \text{CL} \cdot x_N. \quad (1.38)$$

The feedback signal is proportional to the closed loop (CL) response as it shows the response of the system to noises from outside the loop. If the OL gain is high, i.e. $GH \gg 1$, the CL approaches 1. At the error point, $x_{EP}$ responds to closing the loop as

$$x_{EP} = \frac{G}{1 + GH} x_N. \quad (1.39)$$

When the loop is open $x_{EP,OL} = G x_N$ and substituting this in Eq. (1.39) yields $x_{EP} = 1/(1 + GH) x_{EP,OL}$. The term $1/(1 + GH)$ in the bandwidth of the feedback loop and suppresses $x_{EP,OL}$. It seems from this that the limit to suppression of noise in a GW detector is determined purely by the gain of the feedback filter. However, this simple model assumes noiseless sensors and actuator drivers. Sensor noise typically defines the ultimate limit to loop performance and stability. An overview of the variety of noises GW detectors have to cope with will be given in section 2.2.1.

Feedback loops can become unstable. Careful consideration of the definition of CL in Eq. (1.38) shows that it can blow up when $\text{OL} = GH$ goes to -1, i.e. for a magnitude of 1
Figure 1.19: Example simulated loop design of a conditionally stable loop. The unity gain frequency (UGF) is the frequency where the OL gain is unity or 0 dB. For stable loops, the phase must not be (close to)180° at the UGF. The difference between the phase at the unity-gain point and 180° is called phase margin. The gain margin is the difference between the maximum and minimum gain around unity gain in the interval where the phase is not 180°. Adapted from Ref. [55].

and a phase of ±180°. A fictive example of an OL response of a feedback loop is shown in Fig. 1.19. The loop is conditionally stable as changes in phase and magnitude can change the conditions such that the system becomes unstable. Non-linearities over the range of operation can change the overall OL gain. Examples are changes in the sensor or actuator calibration or changes in the plant. In order to cope with sudden changes in these conditions, sufficient phase and gain margins must be taken into account when designing the loop.

1.4.2 Control of a single degree of freedom

A popular control filter used in GW detector loops is a proportional - integral - derivative (PID) controller and it has vast applications in GW instrumentation. Such a controller continuously calculates the difference between a desired setpoint and a measured process variable and applies a correction based on proportional, integral, and derivative terms which give their name to the controller type. The three terms are summed in parallel to calculate the output of the PID controller

\[ x_{FB} = H_{PID} \cdot x_{EP} = K_p x_{EP} + K_i \int_0^\tau x_{EP} d\tau + K_d \frac{dx_{EP}}{dt}, \]  

(1.40)

where \( K_p, K_i \) and \( K_d \) are the proportional, integral and derivative gain, respectively, which can be varied to tune the PID controller to the desired functionality. The contribution from the integral term is proportional to both the magnitude of the error and the duration of the error. Here \( \tau \) is the variable of integration and takes on values from time 0 to the
The derivative of the process error is calculated by determining the slope of the error over time and multiplying this rate of change by the derivative gain. Equivalently, the transfer function in the Laplace domain of is

\[ H_{\text{PID}} = K_p + K_i/s + K_d s, \]  

(1.41)

here \( s \) is the Laplace variable, which is usually substituted by \( s = i\omega \). This equation shows that the proportional term is flat in frequency space, the integral term is proportional to \( 1/s \) and the derivative term is proportional to \( s \). Alternatively, when the proportional gain \( K_p \) is set, integrator \( (f_i) \) and differentiator \( (f_d) \) frequencies determine where the slope changes to \( 1/s \) and \( s \), respectively.

**Figure 1.20:** Example of a simple harmonic oscillator damped by PID controller. The plant is the \( Q=50, 1 \text{ Hz} \) harmonic oscillator from Fig. 1.12 and the PID controller has a linear gain of 3, \( f_i = 0.1 \text{ Hz} \) and \( f_d = 1 \text{ Hz} \). A 10 Hz second order cut-off is also applied to further lower the gain for frequencies higher than the UGF.

As can be seen in Fig. 1.20, the open loop response is a multiplication of the plant and the controller. The UGF is around 3 Hz with a phase margin of about 30° and the 1 Hz resonance peak is effectively damped out at the expense of a small so-called gain peaking around 3 Hz. This small bump is due the phase being close to -180° around UGF which makes the denominator in the CL of Eq. (1.38) smaller than unity. The bump phenomenon is called *gain peaking*. If the phase margin around the UGF becomes smaller, then this bump will gradually turn into a resonant peak.

The cut-off applied here from about 10 Hz onwards also has an effect on the phase, as is visible in the lower panel, so it can not be applied at a much lower frequency as it will reduce the phase margin. Cut-off filters can be more complex than shown in this simple example, for example filters with notches at the resonance frequencies of the actuator support structures. This is done to have the least gain possible at those frequencies, not to excite the modes of e.g. the vacuum vessel in which the controlled suspension is housed. When the mode is (sufficiently) damped, the open loop gain should diminish for
higher frequencies than the UGF, such that the passive vibration isolation of the harmonic oscillator, e.g. a chain of pendulums, is preserved.

1.4.3 Digital and modern control

Typically, digital control systems are used in GW detectors. Signals from sensors can be read out by a real-time computer with Analog-to-Digital Converters (ADCs). These signals can be fed into digitally designed filters and the output of the control filters can be sent to the actuators via Digital-to-Analog Converters (DACs). In general, the advantage of digital control filters is the increased programmability and flexibility of the filter design. The disadvantage is the reduced bandwidth as a result of the transition from continuous to discrete signals. Anti-aliasing and anti-imaging filters (essentially low-pass filters) cause phase loss in these transitions. A digital system also plays a role in storing data of signals, creating excitation signals for system characterization, and automatizing control and (interferometer) locking procedures.

The control described in the previous section is a classical approach. It uses the signals from sensors as the only resource of information about the state of the plant. Typically virtual sensors in a Cartesian coordinate system are geometrically reconstructed by using sensor signals from various physical locations. These locations are chosen such that all degrees of freedom one is interested in can be reconstructed in this way. Putting three (horizontal) sensors in a so-called pinwheel configuration, \(120^\circ\) apart in a circular configuration, allows for both horizontal degrees of freedom as well as the yaw degree of freedom of some platform to be reconstructed. These virtual sensors provide error signals that are fed to single-input-single-output (SISO) control filters, which each send correction signals to virtual actuators in the same Cartesian base. The virtual actuator signals are then properly distributed to the physical actuators. This distribution, typically determined by an iterative process coined diagonalization [52], takes into account the geometry of the system as well as the individual actuator strengths.

Despite diagonalization efforts to decouple the cross-coupling terms between the different degrees of freedom of the virtual actuators, several percent magnitude couplings typically still exist in classically controlled plants [56]. Additionally, the use of only sensor measurement data to control a plant misses the opportunity to exploit the knowledge that can be extracted from the (modeled) reaction of the plant, i.e. its resulting motion as a result of signals being fed to the actuators. Modern or advanced control strategies, such as control filters that use a Kalman state observer, use (among other ingredients) a combination of both these data streams [52]. Modeling of the plant is usually performed using so-called state space models. These models are used to create state observers. Finding a suitable observer, i.e. an observer gain matrix, is the subject of various state estimation techniques, for example, Ackermann’s formula [57] or the Luenberger observer [58, 59].

Naively, one would think that designing an observer to respond rapidly to differences
between measured and estimated outputs would be most effective. However, this strategy makes the state estimates more sensitive to uncorrelated noise in the system, such as measurement noise at the system output or the uncertainties in the input response. Given the statistical properties of the various noise sources and knowledge of the internal dynamics of the system a so-called optimal observer can be devised that minimizes the mean square difference between the measured and estimated states. Such a state observer is known as a Kalman filter. Other gravitational wave detector applications of observer based control have also been proposed for different types of suspension systems, e.g. for the LIGO quadruple suspensions [60].

Standard Kalman filter methods assume that the noise terms are zero mean, white Gaussian distributed and mutually uncorrelated. In reality, it is often the case that the relevant noise sources are non-white. A widely used method [61] typically extends the state space models to include the colored properties of the disturbance and measurement noises. The method utilizes shaping filters that produce colored noise from a white Gaussian noise input. The extended state space model can then be manipulated to generate a Kalman state observer and subsequent feedback controllers, e.g. the so-called Linear Quadratic Regulator (LQR) or Linear Quadratic Gaussian (LQG) controller (coined LQ filters below).

Examples of other control strategies are so-called $H_2$ (or $H_{\infty}$) filters [56] (coined $H$ filters below). Compared to LQ controllers, there are some differences that mainly concern the type of problem one faces. LQ filters are used when all the states are measurable or updateable. In LQ control, if the states are not measurable directly, they are first estimated by an observer and by the so-called separation principle a state feedback controller can be designed. In $H$ filter problems one can select special performance channels and use partial state information. There is no need to measure or estimate all the states. $H$ synthesis can be used for output feedback control. $H$ controllers try to find a filter by using the minimization of some plant transfer function as a figure of merit, whereas LQ filters try to find a Kalman gain to minimize the steady-state covariance of the error. Compared to well designed (agressive) PID filtering, the advantages of modern control are mostly found in the stability and robustness in case of plant variations. Examples of plant variations are shifted mode frequencies or quality factor by e.g. temperature effects on mechanical parts or, in the extreme case, vanishing of a mode by mechanical bypassing or blocking by e.g. cabling shifting to an unintended position. To understand better why LQ controllers seem more robust to plant variations compared to (agressive) PID controllers is under study [62].

1.5 Sensing

In order to apply all control filters described in the previous section or to measure the performance of the control loop, sensors are used to measure the parameter which has to be controlled or monitored, e.g. position, angle, velocity or acceleration, but also power, phase or frequency. Focusing on motion sensors here, a distinction between
two types of sensors can be made. A position sensor measures the difference between two parameters, e.g. a position sensor measuring the position of a suspended object with respect to the ground. Here, the sensor does not distinguish between displacement of either the object or the ground. For this reason, these types of sensors are typically used for DC positioning with respect to ground or for low frequency control.

The other type of sensor is an inertial or absolute sensor, which measures a parameter at the location where the sensor is placed, e.g. the absolute velocity of a suspended object. This type of sensor is used for the derivative part of the control. Typically, they consist of a relatively small mass suspended in a frame or housing. The frame is attached to or placed on the object of which the motion is measured. The mass will lag behind the motion of the frame above the resonance frequency as it behaves as an harmonic oscillator. By determining the position of the mass with respect to the frame, an inertial sensor can be constructed. An inertial sensor will in principle measure the absolute velocity or acceleration of the sensor frame itself.

### 1.5.1 Vibration sensors

Returning to Eq. (1.27), a harmonic oscillator can be used as a sensor. With the definition of the relative coordinate \( x_r \), the ground motion \( x_g \) can be extracted as

\[
-\ddot{x}_g(t) = \ddot{x}_r(t) + \frac{\gamma}{M} \dot{x}_r(t) + \frac{k}{m} x_c(t).
\]  

(1.42)

The measured motion in frequency space is \( X_r = \mathcal{H} X_g \), with the transfer function from ground to relative motion for a structurally or viscously damped sensor as

\[
\mathcal{H}_s(\omega) = \frac{\omega^2}{\omega_0^2(1 + i\phi) - \omega^2}, \quad \text{and} \quad \mathcal{H}_v(\omega) = \frac{\omega^2}{\omega_0^2 + i\frac{2\gamma\omega}{M} - \omega^2}.
\]  

(1.43)

Above the resonance frequency the response goes to 1. This allows for a one-to-one measurement of the motion for frequencies above the resonance frequency. To obtain a measurement at and below the resonance frequency, the output signal must be corrected by dividing out the transfer function of the mass suspension. Examples of such transfer functions are shown in Fig. 1.21 for differently damped sensors.

The mechanical response of the suspended mass and the precision of the measurement between proof mass and frame determine the resolution or sensitivity of a sensor. Low natural frequency or soft suspensions result in relatively large proof mass motion compared to a stiffer suspension subject to the same frame motion. This has however implications on the dynamic range of the sensor. Large proof mass motion in the frame can cause the mass to touch the frame or result in other non-linear effects. Some sensor designs solve this by incorporating an actuator on the proof mass and by operating the sensor in closed loop. The measurement of proof mass motion is then used as an error signal and the signal sent to the proof mass to keep it in place is then the sensor output. Lowering the resonance frequency to increase mechanical response can only increase the overall sensitivity of a sensor until other noise sources become
Figure 1.21: (a) The linear relative response between proof mass or sensor frame of the angular frequency response $H(\omega)$ of a structurally and viscously damped simple harmonic oscillator working as a sensor for different values of $Q$. Panel (b) shows responses for absolute proof mass motion to applied force (or ground accelerations coupled via the suspension) for different suspension natural frequencies. Mechanical response is inversely squared proportional to this natural frequency.

dominant. A typical example of such a contribution is the thermal noise of the proof mass suspension itself.

1.5.2 Sensors for gravitational wave detectors

A position sensor widely used in Virgo and KAGRA is the LVDT, which is an acronym for Linear Variable Differential Transformer. An LVDT, as used in the GW community [63], consists of three coaxial coils, i.e. one primary coil usually attached to the suspended object and two secondary coils usually attached to the ground or reference frame, as shown in Fig. 1.22. The two secondary coils are identical, counter-wound and connected in series. A signal, typically 10 or 20 kHz, drives the primary coil and is picked up by the secondary coils.

The two secondary coils are connected in series at ends of opposite polarity. By symmetry, when the primary coil is centered with the respect to the secondary coils, the current induced into the outer coils cancels out. Within the linear range of the LVDT, the amplitude of the signal increases linearly with the distance away from the center. The sign or phase depends on the direction as the signals of the two secondary coils are in practice subtracted. Demodulating the signal coming back from the secondary coils with the reference signal, i.e. the primary coil signal, results in a DC signal which is proportional to displacement. The displacement noise of a typical LVDT used in a GW detector suspension system with a linear range of about 8 mm is about $1 \text{ nm/}\sqrt{\text{Hz}}$. 
Figure 1.22: Schematic bisection of a magnetic voice coil actuator (left) and an LVDT (right) unit as used in MultiSAS, the seismic attenuation system described in Chapter 3. Reproduced from Ref. [52].

A geophone is an inertial sensor that has an output voltage proportional to velocity. Voltage is generated in a pickup coil wound on the proof mass $M$ when it passes permanent magnet attached to the housing, as shown in the electromechanical schematic in Fig. 1.23(a). The suspension system consists of a proof mass, typically a few 100 g to a few kg, suspended on soft springs with resonant frequency of typically several 100 mHz to a few Hz. Damping is represented by the dashpot of strength $b$. The system acts as a damped mechanical oscillator electromagnetically coupled to the output by the coil moving through the field of a permanent magnet attached to the housing. An electromotive force

$$\mathcal{E} = -\frac{\delta \Phi}{\delta t} = -\frac{\delta \Phi}{\delta x} \dot{x}_r$$

is induced in the coil, where $\Phi$ denotes the magnetic flux through the coil and $\dot{x}_r = \dot{x}_M - \dot{x}_g$ the velocity of the proof mass relative to its housing. The geophone sensitivity is defined as $G = \mathcal{E}/\dot{x}_r$ and has units V/(m/s).

The motion sensing coil is mounted above proof mass $M$ and is represented by inductance $L_c$ in series with coil resistance $R_c$. The geophone signal is read over a loading resistor $R_d$ and, in the Laplace domain, has a transfer function given by

$$H_{geo}(\omega) = G \frac{R_d}{R_d + R_c} \frac{\omega^2}{\omega^2 - 2i\omega_0\zeta \omega - \omega_0^2},$$

where $\omega_0 = \sqrt{k/m}$ represents the resonance frequency of the proof mass suspension and $\zeta$ the damping coefficient. The latter is given by

$$\zeta = \frac{1}{2M\omega_0} \left( b + \frac{G^2}{R_d + R_c} \right).$$

Damping the resonance to a desired response is typically done by choosing an appropriate value for the loading resistor. For ground motions above mechanical
Figure 1.23: (a) An electromechanical overview of a vertical geophone and (b) an artist impression of a (commercial) horizontal geophone (in blue) as used on an in-vacuum suspension. The geophone is fixed in a vacuum pod which also houses its preamplifier.

resonance, the output signal is proportional to the ground velocity $\dot{x}_g$. For frequencies below resonance, output response falls as the proof mass motion starts to follow ground motion. The signal has to be corrected by using Eq. (1.45). The signal-to-noise ratio of the geophone is dominated by the Johnson noise of the electronic circuit and the suspension thermal noise [64]. To preserve the signal-to-noise ratio in the digitization process, a geophone is typically equipped with a low-noise preamplifier.

The Optical Sensor and Electro-Magnetic actuator, for which OSEM is the acronym, is a collocated position sensor and coil-magnet actuator [65] and is shown in Fig. 1.24. These sensors are typically used in the final stages of mirror suspensions of GW detectors as an alternative to LVDTs. The displacement of the to-be-sensed test body with respect to a recoil body in the reaction chain can be determined to below $\text{nm}/\sqrt{\text{Hz}}$ precision over a large bandwidth. The flag in the OSEM determines the relative position between recoil and test body with a shadow sensor, but in the flag a magnet is also present to be used in a coil magnet actuator. The magnet position, which has a nominal position in the middle of the coil, is optimized to reduce effects by the field gradient to keep the actuator as linear as possible. The sensitivity curve of the three translation sensors described above are summarized in Fig. 1.25.

Additionally, a widely used optical sensor is the so-called optical lever. It is mostly used to perform angular measurements of suspended optics. Typically, a laser diode and a so-called Position Sensitive Detector (PSD) are placed at a certain distance from each other on a support structure attached to the suspension vacuum tank. The laser shines a beam on to the suspended object and the reflected beam is detected by the PSD. The change in position on the position sensitive active area of the PSD is a measure for the angle when calibrated by the geometrical knowledge of the system. A typical sensitivity for an optical lever is $1 \, \text{mrad}/\sqrt{\text{Hz}}$. 
Figure 1.24: (a) The OSEM sensor and actuator and (b) a cutview of the same device, where the flag cuts the light beam, thus making it possible to sense the relative position of the test body with respect to the recoil body. The actuator coil acts on the magnet in the flag.

Figure 1.25: Sensitivity curves for various widely used sensors in gravitational wave detectors. The geophone curve is a specification of the widely used Sercel L4C, the LVDT curve is a modeled curve for the devices used in MultiSAS and the OSEM curve is a modeled curve for the devices used in LIGO and KAGRA.
The search for GWs started in the 1960s with Joseph Weber [66]. In 1966 he constructed the first resonant-mass detector - a two meters long, half a meter in diameter aluminum bar - isolated from vibrations, held at room temperature in vacuum. If a GW, with energy content at or near the longitudinal mechanical mode of the bar, were to travel perpendicular to such bar, it would stretch and squeeze it. This excitation of the natural frequency of the bar was to be measured. Weber used piezoelectric crystals to detect the changes in length. Weber’s original idea was to actually make the entire bar out of piezoelectric material [67]!

In 1969, Weber had built a network of one detector at Argonne National Laboratory in Illinois, and five other resonant-mass detectors located at the University of Maryland. One bar in Maryland, whose properties matched closely the Argonne bar, was used to look for coincident excitation by a passing GW. Weber claimed [68] that he had observed coincidences between them. Other groups, such as the group led by Richard Garwin at IBM [69] or by Heinz Billing [70] at Max Planck Institute in Munich, could not confirm Weber’s claim. The strain sensitivity of Weber’s detectors was about $10^{-16}$ and, as the devices were at room temperature, was limited by thermal noise. Moreover, the GW would need to have enough power in the narrow band in which such a bar detector had high enough sensitivity, i.e. around the resonance frequency, typically between 1 and 4 kHz. Up to until 2009, further improvements to this kind of detector were made such as cryogenic cooling of the bar to reduce thermal noise, better suspensions for seismic isolation or with resonant transducers, in which a smaller mass is tuned to the excitation of the bar increasing the signal. Strain sensitivities of about $10^{-21}$ [71] were achieved. No
detections were made and this technique was abandoned.

Here, the successful method for detecting GWs with kilometer scale laser interferometers is described. First, the effect of GWs on such an interferometer is explained. Next, several technological improvements are discussed that have allowed these detectors to achieve performances near fundamental quantum limits. Furthermore, the present status and future outlook of the large land-based and space-based interferometers is presented.

### 2.1 Interferometric detectors

The idea of detecting GWs by means of optical interferometers was mentioned already in the 1960s [72], but practical designs for actual detectors were only conceived from the 1970s onwards [73–75]. The response of an interferometer to a GW can be examined by discussing the behavior of space-time between the two perpendicular arms as shown in Fig. 2.1.

**Figure 2.1:** A simple Michelson interferometer: a laser with power $P_L$ directs light onto a beamsplitter (BS) with reflection and transmission coefficients $r_{BS}$ and $t_{BS}$. Light reflects off two test masses, which are mirrors with reflection coefficients $r_1$ and $r_2$. A signal is read out at the photodiode (PD).
2.1.1 Measuring a space-time perturbation

Each mirror is used as a test mass (a mass free from all forces) and has a time-like world line $x^a(\tau)$, whose tangent vector $\partial_\tau x^a \equiv u^a$ remains constant. TT coordinates are employed (see section 1.1.2) and, for example, the beamsplitter can be chosen as the origin of the local coordinate system.

Let $\xi^a(\tau)$ denote the separation of any two test masses at proper time $\tau$, so that

$$\xi^a(\tau) = x^a_2(\tau) - x^a_1(\tau), \quad (2.1)$$

where the subscripts reference to one of the two test masses. The Taylor expansion of the Christoffel symbol at $x_1$ in terms of $\xi$ can be written as [76]

$$\Gamma^\alpha_{\beta\gamma}(x_2) = \Gamma^\alpha_{\beta\gamma}(x_1 + \xi) = \Gamma^\alpha_{\beta\gamma}(x_1) + \partial_\mu \Gamma^\alpha_{\beta\gamma}\xi^\mu, \quad (2.2)$$

where $\Gamma^\alpha_{\beta\gamma}$ denotes the Christoffel symbol as defined by Eq. (1.7). Each test mass follows a geodesic in the space-time evolution and this can be expressed with the geodesic equation as

$$\frac{\partial^2 x^\alpha}{\partial \tau^2} + \Gamma^\alpha_{\beta\gamma} \left( \frac{\partial x^\beta}{\partial \tau} \right) \left( \frac{\partial x^\gamma}{\partial \tau} \right) = 0, \quad (2.3)$$

where $\tau$ represents the affine parameter for which proper time is used. If the world line of each particle is defined by its geodesic equation, subtracting the geodesic equation for $x_1$ from the geodesic equation for $x_2$, substituting from Eq. (2.2), keeping only terms up to first order in $\xi$ yields, and dropping the subscripts yields

$$\frac{\partial^2 \xi^\alpha}{\partial \tau^2} + \Gamma^\alpha_{\beta\gamma} u^\beta \left( \frac{\partial \xi^\gamma}{\partial \tau} \right) + \Gamma^\gamma_{\beta\gamma} u^\gamma \left( \frac{\partial \xi^\beta}{\partial \tau} \right) + \partial_\mu \Gamma^\gamma_{\beta\gamma} \xi^\mu u^\beta = 0, \quad (2.4)$$

where, again, $u^\beta \equiv \partial_\tau x^\beta$. The covariant derivative of the separation vector is defined as

$$\frac{D \xi^a}{D \tau} \equiv \frac{\partial \xi^a}{\partial \tau} + \Gamma^a_{\beta\gamma} \xi^\beta u^\gamma, \quad (2.5)$$

and can be used to obtain an expression for $D^2 \xi^a / D\tau^2$, which yields

$$\frac{D^2 \xi^a}{D \tau^2} = \frac{\partial^2 \xi^a}{\partial \tau^2} + \partial_\mu \Gamma^a_{\beta\gamma} u^\mu \xi^\gamma + \Gamma^a_{\beta\gamma} \left( \frac{\partial \xi^\gamma}{\partial \tau} \right) u^\beta + \Gamma^a_{\beta\gamma} \left( \frac{\partial \xi^\mu}{\partial \tau} \right) u^\beta + \left( \Gamma^\nu_{\gamma\nu} \Gamma^\gamma_{\beta\nu} - \Gamma^\gamma_{\beta\nu} \Gamma^\gamma_{\nu\mu} \right) u^\nu \xi^\beta. \quad (2.6)$$

The result of Eq. (2.4) can be substituted in this equation and, with the definition of the Riemann tensor of Eq. (1.5), a compact expression is obtained

$$\frac{D^2 \xi^a}{D \tau^2} = R^a_{\beta\gamma\mu} u^\beta u^\gamma \xi^\mu. \quad (2.7)$$

When a GW passes by in the TT gauge, the Christoffel symbols only depend on the perturbations of the metric and, remembering that $h^{TT}_{00} = h^{TT}_{\mu0} = h^{TT}_{0\mu} = 0$, they become

$$\Gamma^a_{00} = \frac{1}{2} \eta^{a\mu} (\partial_0 h^{TT}_{\mu0} + \partial_0 h^{TT}_{0\mu} - \partial_\mu h^{TT}_{00}) = 0. \quad (2.8)$$
Consider two test masses initially at rest and separated by distance \( L_x \) along the \( x \)-axis, i.e. \( \xi^a = (0, L_x, 0, 0)^T \). For a weak gravitational field, taking proper time approximately equal to coordinate time, Eq. (2.7) then simplifies to

\[
\frac{\partial^2 \xi^a}{\partial t^2} = L_a R^a_{001} = -L_a R^a_{010}.
\]  

From Eq. (1.11) one can see that \( R_{010}^1 = -(1/2) \frac{\partial^2 h_{11}^{TT}}{\partial t^2} \). Hence, two test masses initially separated by \( L_x \) in the \( x \)-direction, have a geodesic deviation vector which obeys the differential equation

\[
\frac{\partial^2 \xi^a}{\partial t^2} = \frac{1}{2} L_x \frac{\partial^2 h_{TT}^a}{\partial t^2}.
\]  

Integration of this equation gives the change of the physical distance to be measured between the test masses, which is

\[
\Delta L_x = \frac{1}{2} h_{TT}^+ L_x. \tag{2.11}
\]

As depicted in Fig. 2.1, a third test mass can be placed at a distance \( L_y \) along the \( y \)-axis of the local coordinate system to form a Michelson interferometer. From the common test mass, the beamsplitter, photons traveling back and forth to the test mass along the \( y \)-axis experience

\[
\Delta L_y = -\frac{1}{2} h_{TT}^+ L_y. \tag{2.12}
\]

In such an interferometer, a beam of many coherent, monochromatic photons is split at the beamsplitter and injected in the two arms. Upon return, these beams have accumulated a round-trip travel time difference of

\[
\Delta t = \Delta t_x - \Delta t_y = h_{TT}^+ \frac{L_x + L_y}{2c} = h_{TT}^+ \frac{L_{\text{avg}}}{c}, \tag{2.13}
\]

while the differential arm length change there is

\[
\Delta L = \Delta L_x - \Delta L_y = h_{TT}^+ L_{\text{avg}}. \tag{2.14}
\]

This corresponds to a round-trip optical phase shift of

\[
\Delta \phi = \frac{4\pi}{\lambda} (\Delta L_x - \Delta L_y) = \frac{4\pi}{\lambda} h_{TT}^+ L_{\text{avg}}, \tag{2.15}
\]

where \( \lambda \) represents the photon wavelength.

In general, a GW can be incident at arbitrary polar and azimuthal angles with respect to the detector plane and contains contributions from both \( h_{TT}^1 \) and \( h_{TT}^x \), as defined by Eq. (1.22). In this case, the strain on the interferometer \( h \) is a linear combination of both polarizations. In the optimally oriented case, where \( \Delta L = h_{TT}^+ L_{\text{avg}} = h L_{\text{avg}} \), the interferometer, measuring the differential arm length \( \Delta L \), will produce a larger signal with increasing arm length \( L_{\text{avg}} \). In Fig. 2.2 it can be seen that a GW coming in optimally oriented for one orientation will result in the detector having no sensitivity for the other polarization.
2.1.2 Interferometry in practice

Revisiting Fig. 2.1, consider now a laser producing planar monochromatic light waves having a total power $P_L$. A 50-50% beamsplitter can be modeled by multiplying the incoming electric field component amplitude of the light wave by a factor $r_{BS} = 1/\sqrt{2}$ for reflection from one side and $r_{BS} = -1/\sqrt{2}$ from the other side, while a transmission always gives a factor $t_{BS} = 1/\sqrt{2}$ [77]. For mirrors the amplitudes are multiplied by $r_1$ and $r_2$ for mirror 1 and mirror 2, respectively. At the photodiode, the two beams (one with factor 1/2 and the other with factor -1/2 from the beamsplitter) that return give rise to

$$E_{\text{out}} = \frac{1}{2}E_{\text{in}} \left( r_1 e^{2ikL_x} - r_2 e^{2ikL_y} \right). \tag{2.16}$$

The power at the photodiode can be obtained by multiplication of the electric field amplitude by its complex conjugate, which yields

$$P_{\text{out,static}} = \frac{1}{4}E_{\text{in}}^2 \left( r_1 e^{2ikL_x} - r_2 e^{2ikL_y} \right) \left( r_1 e^{-2ikL_x} - r_2 e^{-2ikL_y} \right) = \frac{1}{4}P_L \left( r_1^2 + r_2^2 - 2r_1 r_2 \cos \left[ 2kL_{\text{static}} \right] \right). \tag{2.17}$$

A passing GW with strain $h(t)$ changes the proper distance between the mirrors by an amount $h_+/2$. This introduces a phase difference $\Delta \phi_{GW}(t) = 2kL_{\text{avg}}h(t) = 4\pi L_{\text{avg}}h(t)/\lambda$. Substituting this in Eq. (2.17) gives

$$P_{\text{out}} = \frac{1}{4}P_L \left( r_1^2 + r_2^2 - 2r_1 r_2 \cos \left[ \phi_0 + \Delta \phi_{GW}(t) \right] \right), \tag{2.18}$$

where $\phi_0$ represents the phase difference at the working point of the interferometer. As a GW has a similar effect as changing the arm lengths by moving the mirrors, it is natural to introduce the GW induced phase in the cosine term in Eq. (2.18). Using the trigonometric identity $\cos (a + b) = \cos a \cos b - \sin a \sin b$ for small $\Delta \phi_{GW}(t)$ gives

$$P_{\text{out}} \approx \frac{1}{4}P_L \left( r_1^2 + r_2^2 - 2r_1 r_2 \cos \phi_0 \cos \Delta \phi_{GW}(t) + 2r_1 r_2 \sin \phi_0 \sin \Delta \phi_{GW}(t) \right), \tag{2.19}$$
This can be separated into a static and a time varying part as

\[ P_{\text{out,static}} = \frac{1}{4} P_L \left( r_1^2 + r_2^2 - 2r_1 r_2 \cos \phi_0 \right) \]  
and

\[ \Delta P_{\text{out,GW}}(t) = \frac{1}{2} P_L r_1 r_2 \Delta \phi_{\text{GW}}(t) \sin \phi_0. \]  

The largest response to a passing GW is obtained by maximizing \( \Delta P_{\text{out,GW}}(t) / \Delta \phi_{\text{GW}} \), and thus when \( \phi_0 = (2n + 1) \frac{\pi}{2} \), i.e. halfway up the fringe. Substitution of Eq. (2.15) into the expression for \( \Delta P_{\text{out,GW}}(t) \) of Eq. (2.20) results in

\[ \Delta P_{\text{out,GW}}(t) = \frac{2\pi L_{\text{avg}}}{\lambda} P_L \sin \phi_0 h(t). \]  

The lasers used in GW detector interferometers are the best available and typically have laser power fluctuations of about \( \delta P / P = 10^{-8} / \sqrt{Hz} \) in the detection band \( (f > 10 \text{ Hz}) \). This means the minimal strain measurement that can be made when operating the detector halfway up the fringe is

\[ h_{\text{min}} = 10^{-8} \frac{\lambda}{2\pi L_{\text{avg}}}. \]  

In the absence of noise, halfway up the fringe would be the most sensitive condition of the interferometer. However, the quantum nature of light results in statistical fluctuations in the arrival times of the photons at the photodiode. This is called shot noise and brings an uncertainty to the intensity measurement in which a GW signal could be hiding. The counting statistics of photons is given by a Poisson distribution. The average number of photons per unit time reaching the detector is

\[ \dot{N}_y = P_{\text{out,static}} / (\hbar \omega_L), \]  

where \( \hbar \) represents the reduced Planck constant and \( \omega_L \) the angular frequency of the laser light. In the frequency domain the power fluctuations associated with shot noise have a flat spectrum with

\[ P_{\text{shot}} = \sqrt{\dot{N}_y \hbar \omega_L} = \sqrt{P_{\text{out,static}} \hbar \omega_L}, \]  

where \( \omega_L \) denotes the angular frequency of the laser light. To maximize the sensitivity of the detector, the signal-to-noise ratio (SNR) needs to be maximized. The SNR is calculated by dividing \( P_{\text{out,GW}}(t) \) and \( P_{\text{shot}} \). This yields

\[ \text{SNR} = \sqrt{\frac{P_L r_1^2 r_2^2}{2 \hbar \omega_L \Delta f} b(\phi) \Delta \phi_{\text{GW}}(t)}, \]  

where \( b(\phi) = \sin^2 \phi / (r_1^2 + r_2^2 - 2r_1 r_2 \cos \phi) \). In order to see where the maximum occurs, the derivative of \( b(\phi_0) = (1 - \cos^2 \phi_0) / (r_1^2 + r_2^2 - 2r_1 r_2 \cos \phi) \) to \( \cos(\phi_0) \) is taken, set to zero, and by using the quadratic formula, one can show that

\[ \cos \phi_0 = \frac{2(r_1^2 + r_2^2) \pm 2(r_1^2 - r_2^2)}{4r_1 r_2} = \left\{ \begin{array}{cc} \frac{r_1}{r_2} & \frac{r_2}{r_1} \end{array} \right\}. \]
Now, inserting $\phi_0$ in the equation for $b(\phi)$ and using $r_1 \approx r_2 \approx 1$ and $r_1^2 \approx r_2^2 \approx r_1 r_2 \approx r^2$, it can be shown that

$$b(\phi_0) = \frac{1 - \cos^2 \phi_0}{2r^2(1 - \cos \phi_0)} = \frac{1 + \cos \phi_0}{2r^2}.$$  \hspace{1cm} (2.27)

Since $\cos(\phi_0) \leq 1$, this becomes $b(\phi_0) = 1/r^2 \approx 1$ and the SNR is

$$\text{SNR} = \sqrt{\frac{P_L}{2\hbar \omega_L}} \cdot \Delta \phi_{GW}(t).$$ \hspace{1cm} (2.28)

At the dark fringe, the dark fringe condition is employed with $r_1 = r_2 \approx 1$. Given Eq. (2.19), this yields

$$P_{\text{out}} = \frac{1}{4} P_L \left( r_1^2 + r_2^2 - 2r_1 r_2 \cos \phi_0 \cos \Delta \phi_{GW}(t) + 2r_1 r_2 \sin \phi_0 \sin \Delta \phi_{GW}(t) \right)$$

$$= \frac{1}{4} P_L (2 - 2 \cos \Delta \phi_{GW}(t)) \approx \frac{1}{4} P_L \Delta \phi_{GW}^2(t).$$ \hspace{1cm} (2.29)

This implies that $P_{\text{out}} \propto h^2$ exactly at the dark fringe, which will not give output of the detector that is linear in the GW signal. A method to extract the GW signal has been used to circumvent this non-linearity and is called heterodyne or RF readout and has been used in the initial GW detector configurations. All advanced detectors use DC readout (a type of homodyne readout), where a signal linear in $h$ is obtained by locking the interferometer with a slight offset in the differential arm length. These readout methods will be discussed in section 2.2.2. Detectors typically have a working point a few picometer from the dark fringe. Working close to the dark fringe strongly suppresses the laser intensity noise.

The sensitivity of an interferometer to GWs, $S_{GW}$, is defined as the phase measurement per unit bandwidth for which SNR = 1. Inspecting Eq. (2.28) and replacing $\Delta \phi_{GW}(t)$ by $S_{GW}$ in the frequency domain, it can be seen that

$$S_{GW} = \sqrt{\frac{2\hbar \omega_L}{P_L}}.$$ \hspace{1cm} (2.30)

In case of a simple Michelson interferometer, it was shown in the previous section that $\Delta \phi_{GW}(t) = 4\pi h(t) L_{\text{avg}} / \lambda$, where $\phi(t)$ and $h(t)$ vary on time scales much longer than the round trip time of the light. The strain sensitivity can thus be defined as

$$S_h = \frac{\lambda}{4\pi L_{\text{avg}}} \cdot S_{GW} = \frac{\lambda}{4\pi L_{\text{avg}}} \sqrt{\frac{2\hbar \omega_L}{P_L}}.$$ \hspace{1cm} (2.31)

For the most basic interferometer consisting of one beamsplitter and two end mirrors, having an arm length of 3 km, an input laser power of 25 W, and a laser light wavelength of 1064 nm, this results in a strain sensitivity of $S_h = 1.4 \cdot 10^{-21} 1/\sqrt{\text{Hz}}$. Using these values for the expression for operation halfway up the fringe of Eq. (2.22) gives a minimum possible strain measurement of $5.3 \cdot 10^{-19} 1/\sqrt{\text{Hz}}$, which is more than 2 orders of magnitude higher than operating slightly off the dark fringe.


2.2 Sensitivity improvements

The sensitivity reached by a simple Michelson interferometer is insufficient to measure GWs. Moreover, the maximum strain induced by a GW happens when the light spends half a gravitational wave period in the arms. This means to optimally measure a GW of, for example, about 150 Hz ($\lambda_{GW} = c/f_{GW} \approx 2000$ km), the arms would need a length of about $L = \lambda_{GW}/4 = 500$ km.

Over the years, several technological improvements have been implemented. The most important components are shown in Fig 2.3. The core optics in a GW detector are suspended with techniques described in section 1.3 to decouple them from the Earth’s ever-present minuscule vibrations. This ensures that these components act as freely falling masses that can be perturbed by the gravitational field.

![Dual Recycled Fabry-Perot Michelson interferometer diagram](image)

**Figure 2.3**: A Dual Recycled Fabry-Perot Michelson interferometer: laser light is filtered by an input mode cleaner (IMC) and passes through the Power Recycling (PR) mirror. The beam is split by the beamsplitter (BS) and injected through the Input Test Masses (ITMs) into the Fabry-Perot cavities. Light in the cavity bounces hundreds of times between End Test Mass (ETM) and ITM before returning to the beamsplitter. Most of the light reflects from the PR mirror and the power is recycled in the interferometer. The small portion that goes to the Signal Recycling (SR) mirror is partially reflected as well for even more signal build up. The signal that comes out of the SR side is filtered one more time by the Output Mode Cleaner (OMC) before it is measured by the photodiode.

Ideally, the laser source for an advanced interferometer has a spatial profile given by a Gaussian distribution. In reality, spatial modes other than the laser fundamental or so-called carrier mode TEM$_{00}$ exist. These can be detrimental to the operation of the interferometer. Optical cavities in the interferometer are designed to keep the TEM$_{00}$
mode resonant; the radius of curvature of the end mirrors matches the spatial profile of the carrier mode. The other spatial modes cause unwanted intensity fluctuations at the interferometer output and are therefore rejected as much as possible. The triangular ring cavity prior to the main interferometer, known as the input mode cleaner, strips the injected beam of non-fundamental transverse spatial modes. In addition, this cavity suppresses laser beam jitter, which is the translational and angular deviations of the beam from the nominal optical axis of the system. Lastly, being a suspended cavity with low length noise, it is used as a first-stage reference for the stabilization of the laser frequency.

A Power Recycling (PR) mirror is placed between the laser and the beamsplitter to reflect light coming back out of the interferometer, effectively boosting the stored laser power $P_L$. The position and reflectivity of this mirror are chosen such that it forms a resonant cavity with the rest of the interferometer. In Advanced Virgo, the recycling power gain is a factor of 36 [78], which from inspection of Eq. (2.31), increases the sensitivity by a factor of 6 to $S_h = 5.7 \cdot 10^{-22} \, 1/\sqrt{\text{Hz}}$.

Fabry-Perot cavities are made by placing Input Test Masses (ITMs) or input mirrors at the beginning of each arm. These partially transmissive mirrors form a high gain optical cavity together with the highly reflective End Test Masses (ETMs) or end mirrors. Light entering the cavities will interfere with light already circulating inside it. If, after a round trip, the light returns in phase with the impinging light beam, then constructive interference will occur and the light will resonate in the cavity. Optical power inside a Fabry-Perot cavity, for a typical GW detector, can be as high as almost a megawatt! This increases the amount of time photons spend in the arms of the interferometer, increasing the round-trip time and phase response.

Keeping the length of the cavity constant, the resonant frequencies of the cavity are given by $\nu_n = (n + 1/2)c/2L$. The difference between two adjacent frequencies is called the Free Spectral Range (FSR) and is given by $\nu_{\text{FSR}} = c/2L$. For a 3 kilometer cavity such as in Advanced Virgo, the FSR is close to 50 kHz, whereas the optical frequency is about 280 THz. These resonant lines in frequency space have a full width half maximum (FWHM) related to the cavity finesse $\mathcal{F} = \nu_{\text{FSR}}/\nu_{\text{FWHM}}$ as shown in Fig. 2.4.

In terms of reflectivity of the cavity mirrors, the finesse is given by [80]

$$\mathcal{F} = \frac{\pi \sqrt{r_1 r_2}}{1 - r_1 r_2}, \quad (2.32)$$

where $r_1$ and $r_2$ represent the reflectivity of the input and end mirror of the cavity, respectively. The finesse is related to the average photon bounce number $\mathcal{N} = 2\mathcal{F}/\pi$ and this number multiplied by the cavity length $L$ yields the effective length $L_{\text{eff}}$. Substituting the effective length in Eq. (2.31) gives

$$S_h = \frac{\lambda}{8 \mathcal{F} L} \sqrt{\frac{2\hbar \omega_L}{P_{\text{BS}}}}. \quad (2.33)$$

A typical GW detector has Fabry-Perot cavities with $\mathcal{F} \approx 400$ and this corresponds to
Figure 2.4: Transmitted cavity power as a function of the frequency of the light that is traveling in the cavity. The FSR and FWHM are shown for three different values for $\mathcal{F}$. Adapted from Ref. [79]

$\mathcal{N} \approx 250$ and $L_{\text{eff}} \approx 800$ km. The strain sensitivity increases to about $S_h = 2.3 \cdot 10^{-24} \text{ 1/}\sqrt{\text{Hz}}$.

After building up phase difference, i.e. due to a GW signal, in the two Fabry-Perot cavities, the photons interfere at the beamsplitter and a tiny portion of them travels on towards the photo detector to be measured. In a similar way as is done by the PR mirror, the signal light can also be recycled at the interferometer output. By suspending a Signal Recycling (SR) mirror in between the beamsplitter and the photo detector, photons can be sent back into the interferometer, where they continue to interact with the GW to build up even more signal. The SR mirror effectively forms an optical cavity with the rest of the interferometer.

The interferometer response can be tuned in two ways. When the position of the SR mirror is chosen such that the carrier frequency, i.e. the frequency of the main laser light, is on resonance, the overall signal can be enhanced within a certain bandwidth by changing the transmittance of the SR mirror. An design choice is to be made here since the greater the transmission, the larger the bandwidth, but this reduces the enhancement within that bandwidth as less photons are reflected. This procedure is referred to as tuned signal recycling. Another way of tuning the cavity made with the SR mirror is to change the position of the SR mirror with respect to the beamsplitter. The position can be chosen such that particular GW sideband frequencies are resonant, thus enhancing those sidebands within a chosen frequency range. In other words, the cavity is tuned to the laser frequency plus the GW frequency band for which you want to enhance the signal. The GW sidebands are created by the modulation of the carrier frequency with the GW frequency, when photons bounce through the interferometer in cavities that are just not resonant anymore as the wave passes. This method is known as detuned signal recycling and allows the sensitivity of the detector to be optimized at certain frequencies at the expense of some bandwidth. It can be convenient to enhance the sensitivity within a certain bandwidth where certain sources are expected to have most of their power, e.g. in the case of colliding neutron stars. In section 2.3.2,
examples of signal recycling configurations are presented, together with their effect on the projected sensitivity of Advanced Virgo.

The discussion of the dual recycled Fabry-Perot Michelson interferometer so far has been taking place in an ideal world. Apart from shot noise, no other types of (fundamental) noise have been considered. Also, when using all the recycling cavities, photon travel times in the interferometer can easily exceed one millisecond, causing the assumption, that the period of the GW is much larger than the photon storage time, to break down. As established earlier, the maximum transfer of strain induced by a GW to the measured signal happens when the light spends half a gravitational wave period in the arms. That means, that if, for example, a GW with $f_{GW} = 100$ Hz is passing, light spending 5 ms in a Fabry-Perot cavity will produce the largest signal. On the other hand, if the photons spend longer in the stretched arm, this arm starts to squeeze again and the phase delay is partly canceled. If the photons leave the arm earlier, the arm is still being stretched after the photon is already gone and the maximum possible phase delay is not obtained. In the extreme case that the photons spend one GW period in the cavity, 10 ms in this example, this cavity has stretched and squeezed by the same amount, and no signal will be observed. By choosing a fixed value for the finesse (and thus for the bounce number and average photon storage time), the cavities are optimized for a particular GW frequency. This optimal frequency is called the cavity pole

$$f_p = \frac{1}{2\tau} = \frac{c}{4\mathcal{F}L},$$

where $\tau$ represents the average photon storage time. The cavity pole makes the shot noise limited sensitivity frequency dependent. The strain induced by the GW can no longer be assumed constant during the journey of the photon. Consequently, the induced phase in Eq. (2.25) is no longer $\Delta \phi(t) \propto h_{\text{max}}$, where $h_{\text{max}}$ is the maximum strain during the passing of the GW. This expression will need to be replaced by $\Delta \phi(t) \propto \int h(t)dt$, so that

$$\Delta \phi(\tau) = \int_0^\tau \frac{4\pi L}{\lambda} h(t)dt = \frac{4\pi L}{\lambda} \int_0^\tau h_{\text{max}} \sin(\omega t)dt = \frac{4\pi Lh_{\text{max}}}{\lambda\omega_{GW}} [1 - \cos(\omega \tau)].$$

Using the same method as in section 2.1.2 and taking into account that during the measurement the shot noise is integrated $P_{\text{shot}} = \tau \Delta P_{\text{shot}}$, the strain sensitivity can be written as

$$S_h = \frac{\lambda \pi}{8\mathcal{F}L} \sqrt{\frac{2\hbar \omega_c}{P_{\text{BS}}} \frac{1}{f_p} \frac{1 - \cos\left(\frac{\pi f}{f_p}\right)}}.$$

The cosine in the last term of the equation can in principle cause the sensitivity to blow up. This is artificial since the expression was derived for fixed photon storage time $\tau$. In reality, some photons will stay shorter and some longer in the cavity and the actual strain sensitivity is obtained from a phasor sum of all these contributions [81]. The result is given by

$$S_h = \frac{\lambda}{8\mathcal{F}L} \sqrt{\frac{2\hbar \omega_c}{P_{\text{BS}}} \left[1 + \left(\frac{f}{f_p}\right)^2\right]}.$$
Fig. 2.5 displays the strain sensitivity improvements for a simple Michelson interferometer with and without power recycling. The figure also shows the sensitivities calculated according to Eq. (2.36) and Eq. (2.37).

2.2.1 Sensitivity limitations

Here, only fundamental noise contributions are discussed. Technical noises, such as (angular) control noise and noise arising from scattered light, are not discussed. As the overall measurement is done with light, and light consists of quanta, i.e. photons, quantum noise can be dominant in some frequency regions. The shot noise, discussed in section 2.1.2, is an example of this. As can be seen in Eq. (2.31), increasing the amount of laser power will increase the sensitivity by lowering the shot noise. However, a different noise associated with the quantum nature of light must be considered. Photons reflected by a mirror change direction of motion and transfer momentum to the mirror equal to twice the individual photon momentum $|p_\gamma| = \hbar k$. If an equal amount of photons per second bounce off the mirror, this would not pose a problem. However, just as with shot noise, the Poisson distribution of arrival times makes radiation pressure noise a fundamental noise source to be reckoned with. The displacement noise caused by this effect for a single mirror [82] is given by

$$S_{\text{rp,sm}} = \frac{2}{M\omega^2} \sqrt{\frac{2\pi\hbar P_{\text{circ}}}{\lambda c}},$$

(2.38)
where \( M \) represents the mass of the mirror and \( P_{\text{circ}} \) the power circulating in the cavity. The power in the two arms of the interferometer are anti-correlated as every photon entering one arm does not enter the other. The total effect of radiation pressure noise on the detector’s sensitivity is therefore doubled. Furthermore, the induced phase shift is proportional to the effective arm length \( \mathcal{F}L \), so the total radiation pressure noise for \( f < f_p \) can be written as

\[
S_{\text{rp}} = \frac{2}{\mathcal{F}L} S_{\text{rp,sm}} = \frac{4}{M\mathcal{F}L\omega^2} \sqrt{\frac{2\pi\hbar P_{\text{circ}}}{\alpha c}}.
\] (2.39)

Because of their shared origin, radiation pressure noise and shot noise are combined into \( S_{\text{qn}} \) as

\[
S_{\text{qn}} = \sqrt{S_{\text{rp}}^2 + S_{\text{shot}}^2},
\] (2.40)

where \( S_{\text{rp}} \) is given by Eq. (2.39) and \( S_{\text{shot}} \) is given by Eq. (2.37). An example of radiation pressure noise and shot noise combining into quantum noise is shown in Fig. 2.6(a). The radiation pressure noise is proportional to \( 1/f^2 \) and is dominant at low frequencies while shot noise is dominant at higher frequencies.

\[\text{Figure 2.6: (a) Quantum noise of a power recycled Fabry-Perot Michelson interferometer, with arm cavities of } L = 3 \text{ km, a laser power } P_L = 175 \text{ W, a power recycling gain of } 36, \mathcal{F} = 443 \text{ and mirrors of } M = 42 \text{ kg (Advanced Virgo parameters). (b) Quantum noise curves for several values of } P_{\text{BS}} \text{ and the Standard Quantum Limit}\]

As shot noise \( S_{\text{rp}} \propto 1/\sqrt{P} \) and \( S_{\text{shot}} \propto \sqrt{P} \) and both are frequency dependent, for each frequency an optimal power can minimize the overall quantum noise. This optimal power \( P_{\text{opt}} \) at the beamsplitter at which \( S_{\text{rp}} = S_{\text{shot}} \) is given by

\[
P_{\text{opt}} = \frac{\alpha c M \omega^2}{32} \sqrt{\frac{2}{\pi N}}.
\] (2.41)

Substituting this expression in Eq. (2.40) obtains the expression for the minimum amount
of quantum noise at each frequency, also known as the Standard Quantum Limit (SQL),

\[
S_{\text{SQL}} = \frac{1}{F \omega} \sqrt{\frac{\hbar \sqrt{2\pi N}}{M}}.
\]  

(2.42)

The SQL is a manifestation of Heisenberg’s uncertainty principle [83]. With the photons in the interferometer, a position measurement of all test masses is made, but, by momentum transfer of the photons to the mirrors the uncertainty in the momentum of the mirror increases. More power would increase the sensitivity of the position measurement at high frequencies as the shot noise decreases, but will decrease the sensitivity at low frequencies as the radiation pressure increases.

Squeezing is a technique which has proven to break through the SQL by injecting so-called squeezed vacuum states in the interferometer. First proposed in the 1980s [84,85], it took some 20 years to implement this technique in km-scale GW detectors. By injecting squeezed light in the otherwise open dark port of the interferometer, the phase noise can be reduced at the expense of increasing amplitude noise. Phase and amplitude vacuum fluctuations are equal for normal laser light, but squeezing the phase noise increases the overall sensitivity as noise on the phase measurement reduces. Continuous operation of a so-called squeezer on a large interferometer has first been developed at GEO600 [86], established subsequently Advanced LIGO and is starting to be implemented in Advanced Virgo.

**Seismic noise** limits the sensitivity of Earth-based GW detectors at frequencies below 10 Hz. The ground vibrations exceed the mirror displacement to be measured in some frequency intervals by more than 10 orders of magnitude. Ground vibrations due to ambient noise were modeled by Peterson et al. [87]. In Fig. 2.7, the displacement ground spectra for four GW detector sites are presented. The LIGO seismic spectra were taken with a Streckeisen STS-2 seismometer from August 2009 until July 2010. From January 2011 to January 2012, a G"uralp 40T took seismic data at the central building of the Virgo site. Analysis of the 1.5-year (September 2009 - February 2011) data of ambient seismic motion, taken by a G"uralp CMG-3T seismometer located at the CLIO site obtained the KAGRA spectrum. The spectra above 1 Hz are fit curves proportional to $1/f^2$ [88].

In all ground vibration spectra, as well as the Peterson models, a large bump is observed between 100 and 500 mHz. This so-called microseismic peak are due to the oceanic activity. Vibrations caused by waves bashing on the coast shake entire continents. For the large oceans, the frequency of wave arrival is around 200 mHz, while for smaller seas, such as the Mediterranean Sea, the frequency is about 400 mHz. This is also apparent in the displacement spectrum measured at the Virgo site (near Pisa, Italy), where the 400 mHz peak is larger than the 200 mHz one (see green curve). Due to rough weather at sea these peaks can increase, sometimes by an order of magnitude. Wind also plays a role as it induces vibrations in trees and buildings, but this is typically at higher (> 1 Hz) frequencies.

Above 1 Hz, most of the noise is cultural, as day-to-day operations by humans cause
Figure 2.7: Ground motion displacement spectra of GW detector sites. For LIGO [89] and Virgo [90] spectra 1% and 99% percentiles are shown. For Kagra [91] the noise spectra shown coincide with 10% and 90% percentiles. Clearly visible is the different location in frequency of the microseismic peak for different sites. The lines observed in the measured spectra of LIGO and Virgo are anthropogenic due to nearby equipment, such as vacuum pumps. The Kamioka mine provides a significantly more quiet site to host a detector.

The ground to vibrate, e.g. trucks driving over bumps in the road nearby. Its level changes dramatically from day to night, from weekdays to weekends and even during lunch breaks [52]. At higher frequencies the typical spectrum falls off with a $1/f^2$ slope, but also the levels vary greatly from place to place as is clearly visible in Fig. 2.7. In the LIGO and Virgo spectra, several lines are visible above 10 Hz. These are attributed to machinery nearby the measurement location, such as air-conditioning or vacuum pumps.

Selection of a site for a new GW detector is influenced by such differences in seismic noise; it makes the requirements for seismic isolation systems less stringent. Measuring GWs can mean making measurements of mirror displacement around $10^{-10} \text{m/}\sqrt{\text{Hz}}$, so at 10 Hz a isolation ratio in excess of 10 billion is needed at the Virgo site. This isolation from the Earth’s motion is provided by superattenuators [92, 93] for the most critical elements of the Virgo detector, such as the beamsplitter and the input and end test masses. An isolation ratio of $10^{-14}$ is obtained at 10 Hz in both horizontal and vertical directions. This reduces the motion of the critical elements to less than $10^{-25} \text{m/}\sqrt{\text{Hz}}$ above 10 Hz, such that the measurement band (10 Hz to 10 kHz) is free from seismic motion.

**Newtonian Noise** [94] is noise associated with Newtonian attraction of a suspended object by a time varying local mass distribution. As a seismic wave travels in the
subsurface near the suspended object, it changes the local mass distribution and so the direction of gravity changes locally ever so slightly. For current detectors, this is not expected to be a limiting noise source; in their final configuration, Advanced Virgo will only see it in extreme weather conditions and, at Advanced LIGO, it will be the dominant noise source between 1 - 2 Hz [95]. However it is expected to be a dominant noise source for future detectors, which wish stretch the detection band down to 2 Hz. No type of isolation can shield an object from this noise, but subtraction solutions are an active field of research at present. The fact that it may be possible to observe Newtonian Noise in present detectors gives ample opportunity to test suggested mitigation strategies.

**Thermal Noise** arises as the thermal bath exchanges energy with the mechanical degrees of freedom of the test masses. All the elements of a GW detector are in thermal equilibrium with their surroundings, which in general are at room temperature. The energy exchange is provided by the same stochastic forces that at a microscopic level cause the structural damping in the mechanical system. According to the fluctuation and dissipation theorem [96], the same forces determine fluctuations of the position of each test mass. Most dominant are the thermal suspension noise and the mirror coating thermal noise. The magnitude of the thermal noise is determined by applying the fluctuation and dissipation theorem. This theorem postulates a relation between the response of a driven dissipative system and the spontaneous fluctuations of a generalized variable, e.g. the position of the system in equilibrium. The position dependency is similar to the Johnson-Nyquist noise and its power spectral density can be written as [97]

$$x^2_{th} = \frac{4k_B T}{\omega^2} \text{Re} \left[ Z(\omega)^{-1} \right], \quad (2.43)$$

where $k_B$ represents Boltzmann’s constant, $T$ the absolute temperature of the system, $\omega$ the angular frequency of the fluctuation and $Z(\omega)^{-1}$ denotes the mechanical admittance.

The position fluctuation of the mirrors caused by the dissipation in the suspension fibers is called **Suspension Thermal Noise** and, for structurally damped objects, can be written as [98]

$$S_{stn} = \sqrt{\frac{4k_B T \omega_0^2 \phi(\omega)}{m \omega \left[ (\omega_0^2 - \omega^2)^2 + \omega_0^2 \phi^2(\omega) \right]}} \quad (2.44)$$

which falls off with $\omega^{-5/2}$ for frequencies $\omega > \omega_0$. Here, $\phi(\omega)$ represents the loss angle, which is the phase lag of the response of the system to a sinusoidal force at a frequency $\omega < \omega_0$. It is typical for structural damping forces. In Advanced LIGO and at a later stage in Advanced Virgo the suspension thermal noise is lowered below the radiation pressure noise level by using ultra-low loss (effective $\phi < 10^{-9}$) fused silica fibers to support the test masses [99].

In the frequency range from 50 to 300 Hz the limiting factor of the sensitivity of present detectors is the thermal motion of the mirror faces or **Mirror Thermal Noise**, associated mainly with the mechanical losses of the coating. The mechanical admittance of a mirror is derived for frequencies below the lowest internal mode frequency of the substrate,
which usually has a frequency of several kHz. The mirror thermal noise is given by [100]

\[ S_{\text{mtn}} = \sqrt{\frac{4k_B T(1 - \nu^2)}{\omega \sqrt{\pi r_0 E}}} \phi_{\text{eff}}, \]  

(2.45)

where \( \nu \) represents the Poisson ratio of the coating, \( E \) its Youngs modulus, \( r_0 \) the radius of the Gaussian laser beam, and \( \phi_{\text{eff}} \) the total loss angle of the first mirror internal mode, to which the substrate and the coating contribute with different weight factors. The coating thermal noise falls off with \( \omega^{1/2} \) for frequencies below this first mode.

Several mitigation strategies exist to lower the coating noise contribution, such as lowering the temperature or increasing the beam size. Moreover, there is a large research effort ongoing to design materials for very low loss coatings [101]. Another proposal is to use Laguerre-Gauss modes as main laser carrier light, e.g. \( \text{LG}_{33} \) instead of \( \text{TEM}_{00} \), which are spatially broader. Owing to their more homogeneous light intensity distribution these beams average more effectively over the thermally driven fluctuations of the mirror surface, which in turn reduces the uncertainty in the mirror position sensed by the laser light.

All the noise sources presented above need to be modeled when designing a GW detector. It is difficult to suppress contributions from the coating thermal noise, noise due to residual pressure in the kilometer long pipes, noise induced by stray magnetic fields, scattered light and noise in the alignment and length control of the cavities. Angular (control) noise is an example of such a noise contribution and is suspected to have been one of the dominant noise source between about 20 and 100 Hz during the first detection(s) in the O1 observation run. The measured noise was found to be about a factor of 2 higher than the modeled noise sources [102]. As the angle of a mirror in a Fabry-Perot cavity of a GW detector has a lever arm of several kilometers, e.g. 1 \( \mu \)rad variation in angle moves the beam by several millimeters on the other mirror. This causes all kinds of instabilities and asks for robust, low-noise control. Nikhef is responsible for sensors providing error signals for angular control. The angular control system of Advanced Virgo is described in section 3.1.2.

### 2.2.2 Readout methods

Three main methods for readout of a GW signal are used in GW detectors: RF (or heterodyne) readout, homodyne readout or DC readout (a type of homodyne readout). RF readout was used for first generation detectors and DC readout is used in the Advanced detector era. The signals ending up at the GW signal readout photodiode are shown in Fig. 2.8. For laser light of 1064 nm, the frequency is about 280 THz. Differential phase modulation in the arms by a GW becomes amplitude modulation of the (suppressed) carrier at \( f_1 \pm f_{\text{GW}} \). This frequency is also several hundred THz, so the photodiode cannot directly detect the gravitational wave signal unless an optical local oscillator is provided. Heterodyne, homodyne and DC-readout use different concepts [103] to ensure the presence of a low-noise optical local oscillator at the output port photodiode.
Figure 2.8: Depiction of the fields in RF and homodyne/DC readouts of an interferometer. The height of the arrows indicate field amplitudes. In (a) RF readout, the laser carrier, at frequency $f_L$, is suppressed by operating on the dark fringe for the carrier. The GW signal reveals itself as an amplitude modulation of the carrier and is depicted as the audio-frequency sidebands at $\pm f_{GW}$. RF sidebands are generated to end up at the photodiode. This allows to extract the beat signal between the GW signal and the RF sidebands. In (b) homodyne readout, a carrier-frequency local oscillator is introduced by a pick-off mirror before the laser light enters the interferometer. In DC readout this is done by introducing a microscopic asymmetry between the two arms. The GW induced sidebands appear as amplitude modulation on the carrier, which is sensed directly by the photodiodes. Adopted from Ref. [104].

In RF readout, the main laser light or carrier is suppressed by operating the Michelson on a dark fringe for the carrier frequency $f_L$. RF sidebands at frequency $f_L \pm f_{RF}$ are generated on the laser bench by an Electro-Optical Modulator (EOM). An EOM is an optical device in which a signal-controlled element uses the electro-optic effect to modulate a beam of light. The sidebands are then transferred through the interferometer to the output port by introducing a macroscopic arm length difference of several centimeter (so-called Schnupp asymmetry), as shown in Fig. 2.9(a). While this asymmetry results in the output of the interferometer to be dark for the carrier, it is not dark for the sidebands. At the output port they serve as an optical local oscillator for the gravitational wave signal. The photo-current produced by the beat between the different optical field components (optical demodulation) contains a radio frequency component at $f_{RF} \pm f_{GW}$. In a second demodulation process the photo-current is then electronically demodulated at $f_{RF}$ (using a mixer) in order to finally arrive at a signal stream at $f_{GW}$.

In the homodyne readout scheme a small fraction of the carrier light is split off before entering the interferometer and guided directly to the output photo detector, as shown in Fig. 2.9(b). This optical path also passes through a phase shifter, which has the advantage that the optical demodulation phase can be changed easily. The homodyne readout scheme has the disadvantage that the length and alignment of the local oscillator path needs to be extremely stable. Moreover, the spatial overlap of the local oscillator beam and the GW sidebands on the photodiodes must be perfect. In practice, this implies that the optical components, guiding the local oscillator to the output port, need to be actively stabilized by a low-noise control system, and that all components of the path must be seismically isolated in vacuum. Due to these demanding noise and hardware requirements, so far there have been no serious plans to use this form of homodyne readout.
Figure 2.9: Simplified optical layouts for different readout methods for gravitational wave detectors (a) heterodyne, (b) homodyne and (c) DC-readout. The colors used for the light wavelength/frequency are the same as in Fig. 2.8.

The other form of homodyne readout is called DC readout. The difference to the former description of homodyne readout is that the carrier light needed at the output port is not delivered there via a separate path, but also comes out of the interferometer. This is achieved by introducing a small offset from the dark fringe condition. The advantage of this method is that it exploits the filtering action of the compound interferometer to produce the local oscillator. Any fluctuations in the amplitude or frequency of the input laser light are attenuated by the so-called coupled cavity pole before reaching the output port [104].
Because the two arm cavities are made to be as nearly identical as possible, and because the interferometer is operated close to the dark fringe, the two arm cavities can be condensed into a single cavity for the analysis purposes. In considering the effect of amplitude or frequency fluctuations of the input light, the power-recycled Fabry-Perot Michelson can be considered as a three mirror cavity. There is no solution for the exact value of the coupled-cavity pole [105], but a good approximation is achieved when the reflectivity of the shorter cavity on resonance is computed first, and then substituted into the expression for the cavity pole of the larger cavity. This yields

\[ f_{ccp} \approx \frac{1}{2\pi} \nu_{FSR} \log \left( \frac{r_3 - r_1}{1 - r_1 r_2} r_2 \right), \]  

(2.46)

where \( \nu_{FSR} \) represents the free spectral range of the Fabry-Perot cavities and \( r_1, r_2 \) and \( r_3 \) the reflectivity of the input, end and power recycling mirrors of the coupled-cavity, respectively. For frequencies above the \( f_{ccp} \) amplitude or phase noise is attenuated with a \( 1/f \) slope.

Typically \( f_{ccp} \) has a value around 1 Hz for the Advanced detectors. As the carrier itself oscillates at 280 THz, its amplitude or phase fluctuations are attenuated by more than 14 orders of magnitude. This is, in effect, one of the key motivations for DC readout. Furthermore, there is better spatial overlap with respect to the two other methods. In DC readout, the local oscillator and the GW signal are resonant in the same cavities, so spatial overlap comes naturally. Lastly, squeezed vacuum injection is more easily realized in conjunction with homodyne detection than with RF readout, since it requires squeezing in only the audio band rather than at both audio and RF frequencies [106].

Homodyne detection results in a fundamental improvement in SNR compared to RF readout at shot-noise-limited frequencies by a factor \( \sqrt{3/2} \) (for the same power circulating in the interferometer). The extra noise in heterodyne detection is the result of cyclostationary shot noise [107] due to the beat between the upper and lower RF sidebands. This results in the power and thus shot noise on the photodiode not being constant. RF readout was chosen for the first generation detectors over homodyne readout because the performance of the state-of-the-art lasers back then was insufficient for DC readout. A first demonstration of DC readout of a suspended prototype interferometer without signal recycling was shown in 2008 [108]. A further description of how DC Readout is implemented in detectors such as Advanced Virgo is given at the end of section 2.3.1.

### 2.3 Advanced Virgo

Initial Virgo was first proposed in 1989 and funded in 1994 by the French and Italian agencies, CNRS and INFN, respectively. Construction started in 1997 and the main interferometer’s infrastructure was finished in 2003. The first science run was undertaken between May and October 2007 in coincidence with LIGO and is labeled Virgo Science Run 1 (VSR1) [109]. Smaller upgrades, mostly in the laser system, the input mode
cleaner and the final suspension stages, resulted in Virgo+, which reached a NS-NS horizon of 13 Mpc. A long shutdown started in March 2012 for a major upgrade called Advanced Virgo. The Advanced Virgo project has been finalized in February 2017. The optical configuration of Advanced Virgo is shown in Fig. 2.10.

**Figure 2.10:** Optical layout of Advanced Virgo. The suspended optical benches that house (quadrant) photodiodes are depicted as pink squares. Some of these photodiodes generate signals that are used for the automatic alignment of the interferometer mirrors. Different modulated beams (so-called RF sidebands) are used to lock the various optical cavities. Reproduced from Ref. [52].

### 2.3.1 From Virgo+ to Advanced Virgo

The two yellow squares on the left in Fig. 2.10 are the laser bench and external injection bench (EIB). For Advanced Virgo, this optical bench which is the last bench before the laser beam enters the vacuum is seismically isolated. Its isolator is coined EIB Seismic Attenuation System (EIB-SAS). EIB-SAS has been designed, fabricated and tested at Nikhef and at the Virgo site. Specific details on this isolation system are found in Ref. [47].

The most important improvements from Virgo+ to Advanced Virgo start at the source,
an increase of laser power. Advanced Virgo will operate with an input power of 125 W arriving at the PR mirror. The laser will need to produce 200 W, because of losses and filtering in the input mode cleaner and rest of the injection system. Initially, the Virgo laser will be used, which produces up to 60 W. After completing the laser R&D, the higher power laser will be installed [110].

While Virgo+ was operated as a Power Recycled Fabry-Perot Michelson, Advanced Virgo will be operated as a Dual Recycled Fabry-Perot Michelson interferometer. Hence, a signal recycling mirror will be installed. The mirrors of the Fabry-Perot cavities have been replaced by mirrors twice the weight. Cylinders of fused silica weighing 42 kg are suspended from superattenuators. This is done to accommodate for the ten times higher power that will be circulating in the cavities. The finesse increases to $F = 443$, making the effective length about 850 km and the storage time 8.8 ms, which optimizes the cavity response to $f_p = 56$ Hz. To reduce the impact of the coating thermal noise, the spot size is enlarged to 48.7 mm and 58 mm for input and end mirrors, respectively.

The mirrors are suspended by superattenuators, which only required small adaptations from their Virgo+ design. The superattenuator is a seismic isolator consisting of chained mechanical filters, a combination of the ones described in section 1.3.2. Chaining the filters results in a steep cut-off of seismic noise from a few Hz onwards. The transfer function features a slope of $1/f^{14}$ resulting in an attenuation factor of $10^{12}$ at 10 Hz. The chain of filters is shown in Fig. 2.11.

Optical path length distortions are present in every real world interferometer, despite the ultra high vacuum, the extremely well polished mirror surfaces and homogeneous absorption of laser light by the ultra-pure optics. Thanks to a novel corrective coating process step the mirrors feature a $<0.3$ nm rms deviation from the nominal radius of curvature over a 150-mm diameter area [111].

To cope with these distortions a Thermal Compensation System (TCS) has been designed, consisting of ring heaters around the mirrors and compensation plates thermally actuated by high power CO$_2$ lasers. Error signals for the TCS are provided by a set of two sensors. A Hartmann sensor is used as primary sensor for measuring all thermal effects at a test mass. It can precisely measure relative distortions independently on all test masses. Phase Cameras, which have been developed at Nikhef [112], are capable of measuring the so-called cold defects in the recycling cavities [78].

An overview of the main parameters of Advanced Virgo and its predecessor Virgo+ is given in Table 2.1. As the overall sensitivity of Advanced Virgo will be an order of magnitude better than Virgo+, the requirements on both sensing and control are much stricter. The longitudinal and angular degrees of freedom have to be controlled with less noise and more precision. Initially, signal recycling will not be employed, but when the SR mirror is installed there will be five interferometer longitudinal control degrees of freedom that have to be controlled.
Figure 2.11: The Advanced Virgo superattenuator suspending a test mass. Panel (a) shows the chain from Earth to optical element: the three legs of the inverted pendulum, Filter 0 surrounded by the top ring, the passive Standard Filters 1 to 4, the last vertical filter coined Filter 7 and the so-called marionetta. Panel (b) shows the Advanced Virgo payload: 1) Optic or test mass. 2) Baffling plates to mitigate scattered light. 3) Marionetta with attached LVDT/ coil magnet actuators. 4) Cage support structure, attached to Filter 7. 5) Cage around mirror with coil magnet actuators to act on the mirror.

One combination of the mirror motions indeed corresponds to a global translation of the entire interferometer and it is clearly irrelevant, since only the relative distance between mirrors is important [113]. The resonance conditions of the two long arm cavities are independent from the rest of the interferometer and therefore their lengths can be considered as two of the degrees of freedom to be controlled. The common and
### Table 2.1: Comparison between the main parameters for Virgo+ and Advanced Virgo.
The symbol † means that this value is calculated for detuned signal recycling operation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Virgo+</th>
<th>Advanced Virgo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optical set-up (Michelson)</td>
<td>Power recycled</td>
<td>Dual recycled</td>
</tr>
<tr>
<td>Arm length</td>
<td>3 km</td>
<td>3 km</td>
</tr>
<tr>
<td>Laser wavelength</td>
<td>1064 nm</td>
<td>1064 nm</td>
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<tr>
<td>Optical power after IMC</td>
<td>8 W</td>
<td>125 W</td>
</tr>
<tr>
<td>Optical power at the BS</td>
<td>0.3 kW</td>
<td>4.9 kW</td>
</tr>
<tr>
<td>Optical power in Fabry-Perot cavities</td>
<td>6 kW</td>
<td>650 kW</td>
</tr>
<tr>
<td>Fabry-Perot cavity finesse</td>
<td>50</td>
<td>443</td>
</tr>
<tr>
<td>Beam radius at ITM/ETM</td>
<td>21 mm / 52.5 mm</td>
<td>48.7 mm / 58 mm</td>
</tr>
<tr>
<td>Test mass material</td>
<td>Fused silica</td>
<td>Fused silica</td>
</tr>
<tr>
<td>Test mass suspension</td>
<td>Steel wires</td>
<td>Fused silica fibres</td>
</tr>
<tr>
<td>Test mass weight</td>
<td>21 kg</td>
<td>42 kg</td>
</tr>
<tr>
<td>Test mass flatness</td>
<td>&lt; 8 nm rms</td>
<td>&lt; 0.3 nm rms</td>
</tr>
<tr>
<td>Vacuum pressure</td>
<td>$10^{-7}$ mbar</td>
<td>$10^{-9}$ mbar</td>
</tr>
<tr>
<td>In-vacuum suspended benches</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>Binary neutron star inspiral range</td>
<td>13 Mpc</td>
<td>146 Mpc †</td>
</tr>
<tr>
<td>Minimum strain sensitivity</td>
<td>$4 \cdot 10^{-23} / \sqrt{\text{Hz}}$</td>
<td>$3.4 \cdot 10^{-24} / \sqrt{\text{Hz}}$ †</td>
</tr>
</tbody>
</table>

To control the distance from the beamsplitter to the input mirrors, a measurement of the differential of these distances is used. To keep the position the power and signal recycling mirror with respect to the interferometer in resonant condition, the length of the recycling compound cavities are used as control signal. In line with standard control naming of which the lengths are defined by Fig. 2.12, the five longitudinal degrees of freedom are called

- **CARM** = $\frac{L_x + L_y}{2}$, the common arm length.
- **DARM** = $\frac{L_x - L_y}{2}$, the differential arm length. This signal also holds the GW signal!
- **MICH** = $l_x - l_y$, where $l_x$ and $l_y$ are the distances from beamsplitter to each input mirror. It constitutes the differential arm length of the small interferometer in the central building.
- **PRCL** = $l_{PR} + \frac{l_x + l_y}{2}$, where $l_{PR}$ is the distance from power recycling mirror to the beamsplitter.
- **SRCL** = $l_{SR} + \frac{l_x + l_y}{2}$, where $l_{SR}$ is the distance from signal recycling mirror to the beamsplitter.

These longitudinal degrees of freedom have to be controlled down to 0.1 picometer accuracy level for all but DARM, which has to be controlled down to the femtometer level. Except for DARM, all are controlled by a Pound-Drever-Hall locking method [114] by using the modulation RF sidebands that are reflected by the cavity at resonance to be controlled. For Advanced Virgo, the modulation frequencies are 6.270777 MHz,
56.436993 MHz and 8.361036 MHz and in Fig. 2.10 one can see in which cavities they are used for locking. The DARM degree of freedom is controlled with DC signals. Studies on the control of the longitudinal degrees of freedom are reported in Ref. [115]. In section 3.1, the case of angular alignment will be presented.

In Advanced Virgo, a DC readout scheme is employed. The DC readout scheme uses an offset to a differential degree of freedom, in order to have a static carrier field reaching the output port. In principle, it is possible to generate such a static field both with an offset in DARM or MICH. The use of a DARM offset introduces a detuning of the arm cavities from resonance, which increases the influence of radiation pressure. This is because the slope of transmitted cavity power is higher when the cavity is off-resonance (see Fig 2.4). A MICH offset instead does not move the arm cavities out of resonance. In the presence of a signal recycling cavity the distinction is however not so trivial.

Simulations [116] predict that a MICH offset results in lower noise couplings in the power recycled Fabry-Perot interferometer, while a DARM offset results in lower noise couplings in the dual recycled case. In both simulations MICH and DARM offsets have been added separately, resulting in the same amount of carrier power reaching the output port. For the different stages of Advanced Virgo, the offsets used are summarized in Table 2.2.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Degree of freedom</th>
<th>Offset</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power recycled 25 W</td>
<td>MICH</td>
<td>1.6 nm</td>
</tr>
<tr>
<td>Dual recycled 25 W</td>
<td>DARM</td>
<td>23 pm</td>
</tr>
<tr>
<td>Dual recycled 125 W</td>
<td>DARM</td>
<td>1.0 pm</td>
</tr>
</tbody>
</table>

Table 2.2: Offsets needed for 80 mW of power to reach the output port for Advanced Virgo [78].
2.3.2 Strain sensitivity of Advanced Virgo

The design sensitivity of the detuned signal recycled Advanced Virgo configuration is shown in Fig. 2.13(a). At high frequencies, shot noise will be dominant and in the 50 Hz to 300 Hz region, the coating Brownian noise is expected to dominate. Below 50 Hz, the sensitivity differs from the Advanced Virgo Technical Design Report reference curve. Initially, suspension thermal noise was expected to dominate in that region. This was due to an overestimation of the loss angle of the silica wires of the payload suspension [117]. Instead radiation pressure noise and Newtonian noise are expected to dominate and, as stated before, this could constitute an excellent test bed for Newtonian Noise subtraction techniques.

The effect of signal recycling on the sensitivity is shown in Fig. 2.13(b). The design sensitivity for the initial configuration is represented by the dashed red curve. In this configuration, no signal recycling mirror is installed. An increase in laser power to 125 W (dashed blue curve) shows improved high frequency sensitivity, due to shot noise reduction and signal recycling. As mentioned earlier, it is possible to adjust the signal recycling mirror position and transmittance to achieve broad or narrow-band enhancement of the gravitational wave signal. As baseline, Advanced Virgo features a broadband signal recycling configuration with a transmittance of 20%. Represented by the black solid curve, the narrow-band operation will involve a transmittance of 20% and a detuning by 0.35 radians (about 60 nm off carrier resonance position) to optimize the sensitivity to signals from BNS mergers and BBH mergers, with an emphasis on the former sources [118].

Virgo joined Observation Run 2 (O2) on August 1 2017 at 11:00 CET. During the commissioning period prior to joining, there were severe problems with the monolithic silica suspensions (see Fig. 2.11(b), silica fibers suspending the test mass (1) from the marionetta (3)). These loaded silica fibers were shattering when the test mass towers were evacuated or vented.

It was decided to equip the suspension with an optimized C70 steel suspension, with wires that are 0.3 mm thick [119], while the cause of the suspension failures was not found. The cause was established to be a surplus of dust in the vacuum chambers during venting and/or pumping that hit the loaded fibers resulting in the shattering of these fibers. This problem will be fixed and, in the period between O2 and O3, the monolithic suspensions will be installed.

In Fig. 2.14 the Advanced Virgo sensitivity during O2 is presented. The reconstructed strain sensitivity is measured with 13 W input power (carrier power after the mode cleaner) and steel Fabry-Perot payload suspensions. The typical shape of GW detector spectra is clearly visible, but there structures and lines can be found over the entire frequency range. The structures below 30 Hz are believed to be control noise stemming from timing issues in the digital system [120]. The lines around 10.5 Hz, 12.5 Hz, 16.3 Hz, 35 Hz, 45 Hz, 60 Hz, 90 Hz and 350 Hz are calibration lines. The line at 18.6 Hz and 24.6 Hz is associated with air conditioners in the central building clean room and the DAQ room, respectively.
Figure 2.13: Advanced Virgo sensitivity evolution [117]. Panel (a): the total noise (black dashed curve) shows the sensitivity curve. This curve is updated from the Reference AdV curve [78] which is dominated by suspension thermal noise between 10 and 50 Hz. In that interval, the total noise is dominated by quantum (radiation pressure) noise and Newtonian Noise (i.e. Gravity Gradients). Panel (b): the best sensitivity obtained by Virgo+ and the design sensitivity for Advanced Virgo. The black curve corresponds to 134 Mpc BNS inspiral range.
Figure 2.14: Advanced Virgo sensitivity during O2: a BNS range of about 27 Mpc was achieved. The data displayed are 300 s FFTs (60 averages) of the strain sensitivity taken between 20:00 UTC and 23:00 UTC on August 9, 2017. The sensitivity is compared to a simulated expected sensitivity of Advanced Virgo with 13 W and 25 W input power with steel wires and silica fibers Fabry-Perot payload suspensions, respectively. Also plotted are the expected sensitivity ranges for O2 (Early) and O3 (Mid) calculated before commissioning [121].

The broad line at 20 Hz is a mechanical mode of the locked EIB-SAS suspension. The mains power line is at 50 Hz. The broad line at 100 Hz is a mechanical mode of the West input test mass suspension. Current understanding is that the structures and lines between 100 Hz and 1 kHz are associated with SDB1, e.g. ringing optic mounts or (violin) modes of the suspension [122]. The broad line at 207 Hz is a mechanical mode of M6 mirror mount on the external injection bench, which is not yet isolated by EIB-SAS. The line at 315 Hz is the violin mode of the (steel) test mass suspensions (also visible in the solid black curve). Harmonics of this mode are seen at multiples of this frequency value.

The line around 1.9 kHz is the beamsplitter drum mode, a mechanical mode of the fused silica cylinder. Lastly, the structure around 4 kHz and the lines between 4 kHz and 5 kHz is associated with the demodulation system of the OMCs [120]. It is a beating of the SDB1 local control LVDT lines and OMC demodulation line. The location of these lines are available and regularly updated at the known lines database [123].

The Early and Mid expected ranges are strain sensitivity optical simulations with silica fiber suspensions and 25 W input power. The steeper slope of the simulation below 100 Hz is again due to the overestimation of the losses in the marionetta suspension. As discussed earlier, more recent simulations [119] (see dashed line in Fig. 2.14) take into account the updated losses.
2.3.3 Global network

Fig. 2.15 shows the site locations of the (planned) GW detectors around the globe. All instruments are on the Northern Hemisphere, which is suboptimal for source locating capabilities of the network. Having more than one GW detector is required for at least two reasons. Interferometric GW detectors are highly complex instruments and the data often contain a large number of noise transients that are not easily distinguishable from possible GW signals. The output stream of each instrument is subject to certain veto mechanisms. These vetos are based on empirical models of the coupling of instrumental noise to the output strain channel. However, not all noise transients can be vetoed and two or more detectors are required to distinguish between noise and signal. A true GW signal must be spectrally identical in both detectors and can not be further apart than the amount of time that it takes light to travel between the detectors. For the first detection, the LIGO-Virgo collaboration had two detectors operational and sufficiently sensitive to observe the event: LIGO Hanford and LIGO Livingston, both in the USA.

![Figure 2.15: Global network of GW detectors showing Advanced Virgo in Italy, the two Advanced LIGO detectors in Hanford, Washington and Livingston, Louisiana, USA, a third LIGO-India detector to be built west of Pune in Western India, GEO600 in Germany and KAGRA in Japan. Numbers between kilometer-scale GW detectors are lightmillisecond distances or maximum arrival time differences between the sites.](image)

To identify the sky position of the source, triangulation techniques are used, that are based on the arrival times at the various detectors. Event GW150914 was localized with 95% confidence in an area of about 600 square degrees, i.e. 3000 times the area of the Moon. For GW170814, also Advanced Virgo detected a signal and localization and distance measurements were much more precise, as presented in section 1.2.3.

The LIGO collaboration operates two gravitational wave detectors in the US. One 4 km detector is situated near Livingston, Louisiana, and another 4 km detector is located near Hanford, Washington. The first joint science run of the LIGO detectors started in 2002. After several years of intermittent science runs, upgrades and commissioning, a strain sensitivity of $2 \times 10^{-23}/\sqrt{\text{Hz}}$ around 200 Hz was reached. LIGO met its initial design sensitivity and the upgrade project Advanced LIGO was started. Advanced LIGO is a dual recycled Fabry-Perot Michelson interferometer, but differs in a few aspects from the Advanced Virgo design. The LIGO power and signal recycling cavities are folded to increase stability, and the test mass suspension systems are more compact. LIGO
employs a more active approach to seismic attenuation, which involves an extensive set-up of sensors and advanced control strategies. One extra kilometer in Fabry-Perot cavity length and an operational squeezer are examples of other differences between LIGO and Virgo. In addition to the US sites, a project known as LIGO-India will realize a third Advanced LIGO detector in India. An obvious scientific gain is an increase in the distance between the detectors, and hence an improved sky resolution for sources. LIGO-India is planned to become operational in 2022.

GEO600 is a British/German detector with arm lengths of 600 m which has been constructed near Hannover, Germany. It is unique in that instead of Fabry-Perot cavities it uses folded arm cavities to create a 4-pass delay-line interferometer. It achieved first lock in 2001 and several advanced detector technologies were developed there, such as signal recycling, fused-silica suspensions and squeezed light implementation. These technologies all were successfully implemented in GEO600 for the first time. Despite its relatively small size it has achieved sensitivities comparable to the first stages of LIGO and Virgo operation. The next phase of this detector is known as GEO-HF and will concentrate on improved high frequency sensitivity. The cryogenic and underground detector KAGRA in Japan will be discussed in more detail in Chapter 5.

Although localization has been limited for the first detections, triggers containing sky localization maps were sent to users of conventional telescopes. The telescopes searched in the direction of the sources, but found no evidence of electromagnetic counterparts. Since the sources of the first events were binary black hole coalescence, they were not expected to show any electromagnetic radiative counterparts. Other GW sources, such as supernovae or colliding neutron stars, are expected to send out electromagnetic signals. As the global network of GW detectors grows, sky localization will become more precise. As both GWs and their possible electromagnetic counterparts travel at the speed of light, faster pointing of electromagnetic telescopes in the correct direction increases the chance of multi-messenger detection, such as was observed with GW170817.

### 2.4 Future detectors

The future for GW physics and astronomy is bright as the first detections opened a new field. GWs can be used to study the Universe in new ways and will surely allow us to study objects like black holes and neutron stars with high precision. When the telescope was invented, stars were discovered to be fiery balls of gas instead of dots in the sky, and Saturn’s rings were seen for the first time. Future detectors have the potential to map the entire visible Universe in GWs, from frequencies in the range of 1 Hz to 10 kHz with Earth-based instruments and from $10^{-4}$ Hz to $10^{-1}$ Hz with interferometers in space.

Other techniques that would indirectly measure the effect of GWs are Pulsar Timing Array (PTA) and GW induced polarization effects in the Cosmic Microwave Background (CMB). A PTA is a set of pulsars which is analyzed to search for correlated signatures
between arrival times of pulses emitted by the millisecond pulsars (MSPs) as a function of the pulsar angular separations. The most interesting influence on these propagation properties is low-frequency GWs, with a frequency of $10^{-5}$ to $10^{-6}$ Hz. The expected astrophysical sources of such gravitational waves are massive black hole binaries in the centers of merging galaxies, where tens of millions of solar masses are in orbit with a period between months and a few years.

The basic experiment exploits the predictability of the pulse arrival time from MSPs and uses them as a system of Galactic clocks. A disturbance from a passing GW will have a particular signature across the ensemble of pulsars, and can be detected. Several astronomy groups have joined their data output to form the international PTA (IPTA) collaboration [124]. IPTA is comprised of the European Pulsar Timing Array (EPTA), the North American Nanohertz Observatory for Gravitational Waves (NANOGrav), and the Parkes Pulsar Timing Array (PPTA). The principal goal of the IPTA is to detect GWs using an array of approximately 30 pulsars.

One as-yet-unobserved prediction of inflation is that it would produce a background of GWs. These GWs are too faint to be directly detected today, but they can be observed through their signature on the CMB polarization. The dominant contribution to CMB polarization anisotropies is from density perturbations in the early Universe, but these density perturbations only create polarization patterns of a particular type, known as E-modes. GWs from inflation can source B-mode polarization, so a B-mode search targets the signal of inflation without being swamped by the larger E-modes.

Models of inflation predict that gravitational waves will source B-modes at angular scales of a degree or larger. In March 2014, a discovery of just such a signal with BICEP2 was reported [125], but subsequent results from the Planck satellite have shown that Galactic foregrounds, specifically dust, accounts for most if not all of the B-mode signal.

### 2.4.1 Einstein Telescope

The next generation detector Einstein Telescope (ET) is well under way in terms of conceptualization [126]. Site selection research has been performed by Nikhef [52] and others in Europe and research into vital R&D is starting. Apart from the site selection, Nikhef is performing R&D into Newtonian noise subtraction techniques and sensor development for future detectors. Newtonian noise is expected to be dominant for frequencies from 2 Hz to 20 Hz. Sensor grids of thousands of high sensitivity seismic sensors have to be developed to measure passing seismic waves to correctly subtract the changing mass distribution’s Newtonian attraction from the test mass motion this attraction produces.
Figure 2.16: Einstein Telescope [127]: (a) sensitivity curve compared to other GW detectors, (b) sketch of the 10 km triangle below the surface, (c) an ET corner station with ET-LF and ET-HF components and suspensions and (d) a beam tube with 6 beams, 4 for the interferometers and 2 for squeezing filter cavities for ET-LF.

The projected sensitivity and conceptual design of ET is shown in Fig. 2.16 and a more detailed schematic outline is shown in Fig. 2.17. Table 2.3 presents values for the most important parameters. A triangular tunnel of 10 km long interferometers is envisaged. The 60° arm angle will give an 18% loss of strain sensitivity compared to a 90° interferometer, although multiple interferometers can be accommodated. This complex is supposed to be (several) hundred meters underground and will be a GW observatory for generations to come.
Figure 2.17: Layout of Einstein Telescope, with an insert for further clarification of the design. A zoom-in is available at Ref. [128]. Einstein Telescope will consist of six nested interferometers, which will be three (each in two legs of an equilateral triangle) times two (optimized for different frequency intervals) different interferometers. The tunnel cross section, depicting the different beams and their function, is shown within the triangle.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>ET-LF</th>
<th>ET-HF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arm length</td>
<td>10 km</td>
<td>10 km</td>
</tr>
<tr>
<td>Laser wavelength</td>
<td>1550 nm</td>
<td>1064 nm</td>
</tr>
<tr>
<td>Optical power after IMC</td>
<td>3 W</td>
<td>550 W</td>
</tr>
<tr>
<td>Optical power in cavity</td>
<td>18 kW</td>
<td>3 MW</td>
</tr>
<tr>
<td>Fabry-Perot cavity finesse</td>
<td>880</td>
<td>880</td>
</tr>
<tr>
<td>Beam radius at ITM/ ETM</td>
<td>90 mm</td>
<td>72 mm</td>
</tr>
<tr>
<td>Test mass material</td>
<td>Silicon</td>
<td>Fused silica</td>
</tr>
<tr>
<td>Test mass suspension</td>
<td>Silicon</td>
<td>Fused silica</td>
</tr>
<tr>
<td>Test mass weight</td>
<td>211 kg</td>
<td>200 kg</td>
</tr>
<tr>
<td>Test mass temperature</td>
<td>10 K</td>
<td>290 K</td>
</tr>
<tr>
<td>Vacuum pressure</td>
<td>$10^{-10}$ mbar</td>
<td>$10^{-10}$ mbar</td>
</tr>
<tr>
<td>Binary neutron star inspiral range</td>
<td>about 10 Gpc</td>
<td>(combined sensitivity)</td>
</tr>
<tr>
<td>Minimum strain sensitivity</td>
<td>$3 \cdot 10^{-25} / \sqrt{\text{Hz}}$</td>
<td>$9 \cdot 10^{-25} / \sqrt{\text{Hz}}$</td>
</tr>
</tbody>
</table>

Table 2.3: The xylophone design of Einstein Telescope combines a cryogenic low frequency interferometer (coined ET-LF) and a high power high frequency interferometer (coined ET-HF)

2.4.2 LISA

Electromagnetic radiation is measured in different frequency intervals: from radio-waves to visible light and X-rays. These different measurements provide complimentary information to our understanding of the Universe. Terrestrial GW detectors now have a bandwidth roughly between 10 Hz and 10 kHz and this may be extended down to 1 Hz in the future. Lower frequencies will be very difficult to achieve on Earth as the effects of seismic waves become a dominant noise source in that region. In addition, it will be difficult to realize sufficiently long arms for efficient low frequency measurements.

To solve both issues, space is the next frontier of GW detection. Space based detectors, of which LISA, the Laser Interferometric Space Antenna, is the most mature project, will operate in the $10^{-4}$ Hz to $10^{-1}$ Hz range. In 2013, the LISA project was selected as the L3 mission within ESA’s Cosmic Vision Program, with a tentative launch date in 2034. Potential sources for signals are merging massive black holes at the centre of galaxies, massive black holes orbited by small compact objects, known as extreme mass ratio inspirals (EMRI), binaries of compact stars in our Galaxy, and possibly other sources of cosmological origin, such as the early phase of the Big Bang and speculative astrophysical objects like cosmic strings and domain boundaries [129].

The interferometer arms will be a few million kilometer long and the optical set-up will involve digital interferometry. Bouncing a beam back from mirrors at such large distances does not result in enough photons returning. Therefore, the phase information will be digitized at each of the satellites. A forerunner mission, LISA Pathfinder, was launched by ESA on 3 December 2015 with the goal of testing several new technologies planned for eLISA. An overview of LISA Pathfinder and its main results are shown in Fig. 2.18.
Figure 2.18: LISA Pathfinder. (a) An overview of the main components inside the spacecraft: a measurement of relative acceleration between two gold platinum alloy cubes by a heterodyne interferometer. Panel (b) shows the LISA Pathfinder results. Grey: ASD of the relative differential acceleration of the two cubes. Red: ASD after correction for the centrifugal force (visible at the lowest frequencies). Light blue: ASD after correction for the pickup of spacecraft motion by the interferometer, visible in the 20 to 200 mHz range. Reproduced from Ref. [130].
After subtraction of both the centrifugal force and the pick up of spacecraft motion by the interferometer, the light blue line shows the main result of LISA Pathfinder. The obtained noise floor is dominated by two noise sources. Brownian noise associated with viscous gas damping dominates the flat low frequency region of the acceleration spectrum. It is attributed to residual water molecules in the volumes surrounding the test mass cubes. At the low frequencies (< 0.5 mHz), there are several effects [130] that contribute. Charge fluctuations of the test mass cubes, thermal gradient effects by onboard electronics, laser radiation pressure fluctuations, self-gravity change from propellant depletion and the interplanetary magnetic force effect (or a combination of these) are suspected to cause the observed excess. This is currently under investigation. The higher frequency (> 50 mHz) noise floor is flat in displacement and represents the interferometer sensing noise of about 35 fm/√Hz. If transferred to the LISA observatory configuration, the acceleration noise performance already achieved on LISA Pathfinder would allow for an observatory performance near the original LISA mission goals [131].
With the detection of gravitational waves, mankind has made its most precise distance measurement to date. This would not have been achievable without the vibration isolation of all components of the gravitational wave detectors, as described in section 1.3.2. In line with the earlier strategy of Initial Virgo, all new seismic attenuation systems for Advanced Virgo achieve their (in-band) vibration isolation performance through passive isolation. Another important feature of these system designs is the active feedback control of low frequency ($f < 2$ Hz) modes of the isolation chain.

Here, the compact vibration isolator designed, built and tested at Nikhef is presented. This multi-stage seismic attenuation system is coined MultiSAS and in total five systems are part of Advanced Virgo. First, the need for these devices is discussed by presenting the case of the angular alignment requirements for Advanced Virgo. After that, an overview of the MultiSAS design, characterization modeling and measurements is presented. Then the design of the control system for the Advanced Virgo systems is discussed. Lastly, an account of the commissioning of the five systems at the Virgo site is presented.
3.1 Auxiliary optics on suspended benches

Radiation pressure plays an important role in high power interferometers with suspended optics. The laser beam acts on each mirror with a force proportional to the power. Torques are also generated in presence of misalignments. The effect is largest in the arm cavities, since this is where the highest amount of laser power is stored.

3.1.1 Angular modes of core optics

It has been shown [132] that in a Fabry-Perot cavity with suspended mirrors, the angular motion of the optics is no longer independent when the radiation pressure is sufficiently large. The cavity dynamics is then better described by means of a set of coupled angular normal modes shown in Fig. 3.1.

![Angular modes in the Fabry-Perot cavities that can be excited by radiation pressure. (a) The soft (-) and stiff (+) mode of angular misalignment in a single Fabry-Perot cavity and (b) common and differential (-) and (+) modes in a Fabry-Perot Michelson interferometer.](image)

For (-) modes the destabilizing torque from radiation pressure increases with increasing rotation of the mirrors. As a result, the frequency of each (-) mode is lower or softer than the corresponding one of the uncoupled mirrors. For the (+) mode, the reasoning is the opposite. Radiation pressure provides an additional restoring torque that increases the mode's natural frequency with respect to the uncoupled situation. During Advanced Virgo full power (125 W input power) operation the (-) and (+) modes are expected to have resonance frequencies of 1.1 Hz and 3.5 Hz, respectively.

In Fig. 3.1(b), the effect on alignment at the beamsplitter is shown. The two Fabry-Perot cavities are coupled via the beamsplitter resulting in two soft and two stiff coupled angular modes. All soft (-) modes have to be controlled down to a precision 110 nrad...
Alignment mode | Photodiode signal to be used
---|---
Differential (-) | Diff. of B7 and B8 DC signals (on SNEB and SWEB)
Common (-) | Sum of B7 and B8 DC signals
Differential (+) | B1p (on SDB2, demodulated at $f_2 = 56.436993$ MHz)
Common (+) | B2 (on SIB2, demodulated at $f_3 = 8.361036$ MHz)
Power recycling mirror | B4 (on SPRB demodulated at $f_1 = 6.270777$ MHz)
Signal recycling mirror | B1p DC signal
Beamsplitter | B4 demodulated at $f_2$

Table 3.1: Interferometer angular alignment modes and corresponding photodiode sensing signals for alignment control scheme. More details are found in Ref. [110].

and all stiff (+) modes down to 2 nrad [78]. Similar modes exist in combination with the power and signal recycling mirror cavities and these have to be controlled to 25 nrad and 280 nrad, respectively. These values are all angular deviation rms requirements from the ideal optical axis in the cavity and not individual angular requirements for the optics involved.

The rms accuracy requirements of the alignment of the cavity mirrors are set in order to have acceptable fluctuations (smaller than 0.1%) of the carrier and the sidebands power in the cavities. The choice of 0.1% for the maximum tolerable power fluctuations is an estimate based on the consideration that 1% of stability of the carrier field was reached in Virgo+. Since Advanced Virgo is aiming at a ten times better sensitivity, the goal for the angular stability was also tightened by the same factor. The overall angular control scheme has seven degrees of freedom for Advanced Virgo. An overview of the signals used for the angular alignment system is given in Table 3.1.

### 3.1.2 Sensing of the suspended end benches

In the Advanced Virgo configuration, shown in Fig. 3.2, five optical benches are shown as pink squares, of which three (SIB2, SPRB and SDB2) are housed in the Central Building and two (SWEB and SNEB) at the West and North end station, respectively. All of them are in vacuum and seismically isolated by MultiSAS units. Optical systems on these benches use reflected - picked off - or transmitted light out of the interferometer to provide control signals for the detector longitudinal and angular degrees of freedom. The decision of suspending the benches in vacuum was made for two reasons:

- to limit the vibrations of the optical sensors themselves that can couple to the alignment control signals [133];
- to mitigate the noise caused by the light scattered back towards the interferometer by the control photodiodes and their telescopes optics. This was a serious limitation on the achievable sensitivity of first generation detectors [134].
Figure 3.2: The Advanced Virgo configuration with the five optical benches which are suspended by MultiSAS (pink squares). The optical scheme shows all beams used for control, diagnostics and readout. Reproduced from Ref. [78].

The signals produced on suspended benches SNEB and SWEB have the most stringent requirements. The DC output of the B7 and B8 quadrant photodiodes (QPD) is a measure for the relative displacement between the laser beam transmitted by the end mirror and the QPD itself. Any bench motion will be indistinguishable from a beam displacement due to a misalignment of the cavity mirrors. Moreover, the bench motion must be small enough that its displacement noise does not exceed the shot noise limit of the photodiodes.

<table>
<thead>
<tr>
<th></th>
<th>translational ($z$)</th>
<th>angular ($\theta_z$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrated rms (down to 10 mHz)</td>
<td>$1 \times 10^{-6}$ m</td>
<td>$3.1 \times 10^{-8}$ rad</td>
</tr>
<tr>
<td>ASD from 10 Hz onwards</td>
<td>$2.1 \times 10^{-12}$ m/ $\sqrt{Hz}$</td>
<td>$3.3 \times 10^{-15}$ rad/ $\sqrt{Hz}$</td>
</tr>
</tbody>
</table>

Table 3.2: Requirements for translational and angular motion of the optical benches suspended by MultiSAS.

Light that is scattered or diffused on the optical components can reenter the interferometer. This light is modulated by the residual motion of the optics. Any non-linear behavior of this coupling can lead to the upconversion of low-frequency seismic excitations ($< 10$ Hz) into the detection band ($> 10$ Hz) [134]. Typically the microseismic peak provides the largest contribution to the integrated motion of the
As discussed before in section 2.2.1, this peak is roughly between 200 and 500 mHz at the Virgo site. A summary of the requirements is given in Table 3.2.

### 3.2 MultiSAS characterization

MultiSAS is designed to comply with the ASD requirements set for the optical benches at 10 Hz by using chains of low frequency \( f_0 < 1 \text{ Hz} \) mechanical filters. Around their natural frequencies the ground motion is amplified and the rms requirement is endangered. A set of sensors and actuators is used to effectively damp these modes, without spoiling the passive isolation performance by keeping the control unity gain frequency below about 5 Hz.

![Mechanical overview of the MultiSAS isolation system](image)

**Figure 3.3:** Mechanical overview of the MultiSAS isolation system. The location of sensors and actuators is visible in the structure. Bench translational control uses sensors and actuators at the top stage. Vertical control is done only with the top filter LVDT/voice coil pair. The top and intermediate GAS filters have stepper motors with springs for vertical positioning. The Virgo coordinate system is displayed, which has the z-direction along the beam and y as vertical.

Fig. 3.3 shows the complete mechanical design of MultiSAS, a compact, vacuum compatible and high performance vibration isolation system. From the base ring, an inverted pendulum stage supports the top filter structure. From the first GAS filter stage, a steel wire suspends the intermediate filter. The intermediate filter body holds a second set of GAS blade springs, which suspends the optical bench from a second steel wire.
The system design has to be compliant with the limited space available in the existing Virgo infrastructure. Hence, it is relatively small for the performance it achieves; the structure is 1200 mm high and has a 1070 mm diameter. Each system is placed in a vacuum vessel coined MiniTower and the base ring rests on an appropriate support inside this vacuum vessel, as is shown in Fig. 3.4.

Figure 3.4: An impression of MultiSAS suspending an Advanced Virgo optical (end) bench in a MiniTower vacuum chamber. 1) Optical bench. 2) MultiSAS. 3) Transmission beam from end mirror. 4) MiniTower vacuum chamber. 5) Removable cupola. 6) Observant physicist.
3.2.1 Requirements on the transfer function

In order to reach the requirements stated in Table 3.2, the mechanical transfer function of the MultiSAS suspension (see a simplified block scheme in Fig. 3.5) should reach a factor $10^{-7}$ of horizontal vibration isolation at 10 Hz. The soil at the Virgo site is typically moving with $10^{-9}$ m/$\sqrt{\text{Hz}}$ at that frequency and this isolation requirement will get displacement ASD levels down to the $10^{-15}$ m/$\sqrt{\text{Hz}}$ level. Those figures in the horizontal bench motion are in fact mandatory to achieve the required femtoradians level of residual angular motion because of horizontal-to-angular coupling.

![Figure 3.5: MultiSAS model showing the three horizontal and two vertical isolation stages. The ground is actually a support ring on top of the Minitower vacuum chamber, while the bench is simply sketched here as a thin plate. Reproduced from Ref. [52].](image)

The MultiSAS residual translational motion (in $x$ and $z$) couples to angular motion of the suspended bench. For tilt and roll, this is because the center of mass of the bench is physically located below the suspension point, for overall stability reasons. A horizontal motion will couple to a tilt and roll motion of the bench. Yaw motion of the bench is passed more directly by the angular shear of the wires. This last coupling is difficult to model and, because for the optical benches the mode is expected to be about 20 mHz, the residual angular motion at 10 Hz is not deemed a problem.

The equation of motion for the tilt or roll degree of freedom (denoted $\theta_x$ in the following discussion) for the bench is

$$I_{cm,x} \ddot{\theta}_x + k_{\theta_x} \theta_x = d_{\text{susp}} \cdot m_b \ddot{x}, \quad (3.1)$$

where $I_{cm,x}$ represents the moment of inertia around the $x$ axis, $k_{\theta_x}$ the angular stiffness at the suspension point, $m_b$ the mass of the bench and $d_{\text{susp}}$ the distance between the suspension point and the bench center of mass. Modeling the bench as a homogeneous, rectangular box yields

$$I_{cm,x} = \frac{1}{12} m_b (h_b^2 + l_b^2), \quad (3.2)$$
where $h_b$ and $l_b$ represent the height and length of the bench, respectively. A more precise $I_{cm}$ can be obtained from a CAD model of the suspended object.

Taking the Laplace transform of Eq. (3.1) with $I_{cm,x}$ from Eq. (3.2) results in

$$\bar{\theta}_x = \frac{12d_{susp}}{(h_b^2 + l_b^2) \left(1 - \frac{12}{h_b^2 + l_b^2} \frac{\omega_{0,\theta}}{\omega}\right)} \bar{X}, \quad (3.3)$$

where $\omega_{0,\theta_i}$ is the tilt or roll resonance frequency. For high frequencies Eq. (3.3) reduces to

$$\bar{\theta}_x \approx \frac{12d_{susp}}{(h_b^2 + l_b^2)} \bar{X}. \quad (3.4)$$

The vertical requirements are less strict as they do not couple to angular motion as directly as the horizontal motion. Assuming a 1% vertical to horizontal spurious coupling along the suspension, a target $10^{-5}$ vibration isolation factor was considered. A measurement of the vertical-to-horizontal coupling from the intermediate filter to the bench was performed on the MultiSAS prototype. The result between 2 Hz and 6 Hz was a 2% vertical-to-horizontal coupling [135]. Measuring at lower or higher frequencies was not possible because of the resonance modes of the chain and the isolation of the filter in combination with the low coupling ratio, respectively.

### 3.2.2 Transfer function measurements

In the earliest stages of the prototype tests [52], the inverted pendulum and the GAS filter stages were characterized individually. The total frequency response of the MultiSAS was constructed by multiplying these individual transfer functions. Several detailed aspects of the prototype tests performed at Nikhef are presented in Appendix A. After MultiSAS was installed into a MiniTower further characterization was performed. Unwanted in-band (> 10 Hz) resonances were identified by hammering tests and transfer function measurements and modeled by FEM. Examples of such resonances are presented in Fig. 3.6, Fig. 3.7 and Fig. 3.8.

In a GAS filter the blades are held in place by clamps on one side, while their tips are connected all together to a keystone from which the load is suspended. This keystone has certain modes, e.g. angular tilt and roll resonances. By placing a damper (essentially a circular slab of stainless steel on three pieces of an O-ring) on the keystone, its resonances are successfully damped, as shown in Fig. 3.6(b). This particular transfer function is measured from top filter to intermediate filter.

An example of such a keystone resonance is presented in Fig. 3.7. The rotation and translation of the keystone has its effect of the wire hanging from it. The base of the wire is bent as it is clamped in the keystone and this alters the shape of the wire profile in that region. This has subsequent effects on the modal behavior of the wire, e.g. the violin modes are expected to slightly differ from simulations without this effect. More detail
Figure 3.6: Effect of a damper on the MultiSAS Topstage GAS filter keystone: (a) a photograph of the damper on the keystone and (b) a comparison in horizontal isolation ratio from top stage to intermediate filter without and with such a damper. The keystone resonances are the peaks at 55 Hz and 125 Hz. Other structures found at 80 Hz, between 75 Hz and 100 Hz and between 140 Hz and 150 Hz are the resonance of the filter frame, coupling with the actuator support structure and the top filter GAS blades resonances, respectively. More details found in Ref. [135].

on this so-called parasitic resonance modeling and measurement during the MultiSAS prototyping campaign can be found in section A.2.

Figure 3.7: Example of a keystone mode simulated from a FEM model of the GAS filter on the MultiSAS top stage. This mode is modeled to occur around 55 Hz. The $y$ displacement is highly exaggerated, but provides insight in the combination of translation and rotation of the keystone.

The wire linking the intermediate filter and bench is split in two halves joined by a connector. This connector is visible in the right zoom-in inset in Fig. 3.8(a). It is used to trim the vertical position of the bench. Due to the mass of the connector another parasitic resonance is introduced. However, the cabling needed to deliver the power to the electronics on the bench and routed around the wire has proven to effectively damp this mode. The junction resonance at 75 Hz is damped successfully and is believed to shift to lower frequency, become broader in shape, and add to the structure between 45 Hz and 65 Hz, as it is shown in the intermediate filter to bench measured transfer.
function displayed in Fig. 3.8(b).

Below 20 Hz both transfer functions shown in Fig. 3.6 and Fig. 3.8 show the characteristic $1/f^2$ slope. The difference in the level of these slopes, despite both being a similar transfer function, is caused by the fact both measurements are performed in composite pendulum set-up. This results in the upper pendulum having a parasitic resonance from the lower pendulum and this causes the overall level of the transfer function from top stage to intermediate filter to have a larger magnitude.

![Image of cabling and transfer function graphs](image)

**Figure 3.8:** Effect of cabling from the MultiSAS intermediate GAS filter to bench: (a) a photograph of the cabling from intermediate filter to the bench and (b) a comparison in horizontal isolation ratio from intermediate filter to bench without and with such cabling. The 75 Hz mode is damped and shifted around 55 Hz because of the cabling. More details are found in Ref. [135].

The transfer function of the inverted pendulum stage was determined by a measurement where the stage was loaded by a single pendulum dummy mass. After tuning the counterweights on the legs, the transfer function follows the ideal $1/f^2$ slope up to about 20 Hz with an achieved isolation plateau better than $2\times10^{-4}$. More details on this measurement are found in section A.1.

The experimental overall transfer function of the system is subsequently constructed and is shown in Fig. 3.9. The vertical transfer function is taken from measurements described in Ref. [52]. The vertical and horizontal transfer function have a slope of $1/f^4$ and $1/f^6$ up to about 30 Hz, respectively. The vibration isolation ratio achieved at 10 Hz, compliant with the requirements, is $10^{-5}$ in vertical and $10^{-7}$ in horizontal.
Figure 3.9: Vertical [52] and horizontal [135] transfer functions of MultiSAS. Both results are obtained by multiplying intermediate transfer functions, e.g. from actuator structure to top stage to characterize the inverted pendulum stage. The structures from 65 Hz onwards in vertical are resonances of modes of the system, where the first higher order vertical mode at 135 Hz is associated with an intermediate filter keystone bouncing mode. The structure from 40 Hz onwards in horizontal are the (damped) keystone modes described above.

3.2.3 Inverted pendulum leg parallelism

Because of tolerances in machining and assembly, MultiSAS inverted pendulum legs might not be perfectly parallel. Additionally, there might be a mismatch in leg length, causing similar effects as described in this section. In Fig. 3.10(b) a perfect stage is shown in the left picture, where lateral displacements do not result in the introduction of tilt to the top plate. Two other possibilities, where the legs are not parallel, are shown to the right of that. The effect is highly exaggerated, but the two distinct cases can be distinguished when looking at the phase of the signals with respect to each other. The middle picture shows in-phase transfer from the $x$ degree of freedom to $\theta_z$, whereas the right picture shows a $180^\circ$ out-of-phase transfer.

The parasitic coupling from displacement to tilt in the top stage cannot exceed a certain value. This is because geophones cannot distinguish between translation and tilt. Tests were performed both on the prototype and on the SWEB MultiSAS to measure the coupling coefficient, i.e. the misalignment between the inverted pendulum legs. Fig. 3.10(a) shows the tilt meter in the top stage of SWEB when it was still suspending a dummy mass. The measurement entails large 2 mm peak-peak, 5 mHz sine injections in both $x$ and $z$ degrees of freedom. The injection is done at such a low frequency to be sure not to excite any mode down the chain.

Fig. 3.11 shows the results of the two, several hour long injections, where immediately it can be seen that all couplings are below the $3 \times 10^{-4}$ level. Similar results were obtained
Figure 3.10: (a) A photograph of the capacitive readout bubble level tilt meter on the SWEB MultiSAS. The tilt meter is an Applied Geomechanics 755-1129 Miniature Tilt Sensor. The sensor has a 100 nrad / √Hz resolution [136]. Panel (b) gives an overview of different possible cradle effects in the case of real world misalignments in leg-to-top-plate connections due to construction tolerances.

at Nikhef with the MultiSAS prototype. The geophone measurement of the top stage displacement can be trusted down to \( f_{\text{trust}} = 1/(2\pi) \sqrt{g c_{x\rightarrow \theta_z}} \), where \( c_{x\rightarrow \theta_z} \) is the coupling between \( x \) and \( \theta_z \). This means, in order to have the geophone correctly measuring the displacement down to e.g. 100 mHz, the coupling should not exceed the 4 percent level. The results of the measurements show that the typical leg misalignment is on the safe side by two orders of magnitude.

Figure 3.11: Measured coupling factor \( c_{x\rightarrow \theta_z} \) from horizontal to angular displacement for the top stage of the SWEB MultiSAS. The injection frequency of 5 mHz is indicated by the vertical dashed black line. All couplings are 2 orders of magnitude smaller than the maximum allowed value in order to have the geophones measure displacement instead of tilt down to 100 mHz.

3.2.4 Thermal shields

All suspended benches feature the electronics for acquisition and digital processing of the signals from all installed photodetectors. The electronics are housed in an air tight container which is a structural part of the suspended bench as shown in Fig. 3.12. The suspended in-vacuum electronics are expected to dissipate relatively high power with of 280 W for SDB2 as the extreme case. An integration test at LAPP has been performed and concluded cooling can be successfully achieved by radiative heat transfer [137].
The most temperature sensitive elements of MultiSAS are the two GAS filters. The thermo-elasticity of maraging steel causes a change of -250 ppm/K in the loading capability corresponding to -1.05 N/K for the top stage GAS filter (nominal load 430 kg) and -0.78 N/K for the intermediate stage GAS filter (nominal load 320 kg). The result is a position drift

$$\Delta y = \frac{g}{\omega_0^2} \frac{\Delta E}{E},$$  \hspace{1cm} (3.5)

where $E$ represents the Young's modulus and $\omega_0$ the filter's resonance frequency. Less important but also present is the detuning of the filter caused by the differential thermal expansion coefficient between maraging steel and the filter body material, which alters the blade compression rate slightly. The filter body material is aluminum for the top stage, and stainless steel for the intermediate filter. This detunes the filter, i.e. causes a change in the resonance frequency.

Continuous compensation for the position drift $\Delta y$ during operation of the sensing optics on the bench is provided by the top stage GAS filter built-in voice coil actuator, which has a dynamic range of $\pm$ 1.5 N. Long term drift compensation can be made by using the fishing rod actuators installed on both the filters and with a dynamic range from 3.3 N to 9.3 N (always pulling). Since a $\Delta T$ larger than 7.5 K would exceed the compensation capability and the benches are expected to operate in vacuum at a temperature around 40° Celsius, a thermal shield was designed and its efficiency was tested [138]. The thermal shield consists of a stack of two closely spaced (about 10 cm) non-anodized reflective aluminum sheets attached to the MultiSAS base ring.

A dedicated set-up was installed in the MultiSAS prototype at Nikhef. An anodized black aluminum plate to simulate the radiating surface of the warm optical bench equipped with resistive heaters was secured on top of the suspended bench. This set-up with the position of two thermal shields below the intermediate filter is shown in Fig. 3.13(a).
Figure 3.13: (a) Thermal shield test set-up, where two thermal shields are installed below the intermediate filter. The hot plate can be adjusted in height $h$ and (b) temperature at various sensing points during venting of the MiniTower and opening the doors for about 5 hours. Afterwards the doors were closed but the system was in air. Clearly, the filters recovered or stayed within the range of fishing rod or actuator. Adapted from Ref. [138].

In Fig. 3.13(b) results of a test to simulate recovery of MultiSAS after an intervention are presented. The hot plate is kept on as this simulates the electronics being on as well during venting and working with open doors for about 5 hours. The filters keep the achieved temperature for many hours even if the MiniTower doors are completely open. In this way operation can commence again with no delay due to thermal settling of the GAS blades after closing the doors and pumping down.

In Fig. 3.14, the heat shields are tested when both shields are reflective. Prior to that, a test was performed with the bottom shield anodized black. Especially the shields and also the filter blades were warmer when the temperature settled. The tests are
Figure 3.14: Thermal shield equilibrium performance of (a) GAS filter keystone and bench position. The small jump down around 18h into the test is attributed to hysteresis in the top stage blades. (b) MiniTower and MultiSAS temperatures. Adapted from Ref. [138].

performed under vacuum, of which one can see the effect in Fig 3.14(a) in the first hour. The buoyancy effect of the bench not floating in a bath of air anymore increases the load slightly for the MultiSAS and causes a sag. After that, the increasing temperature of the filter blades makes the keystones sag even more. In a day, the bench has gone down by almost 2 mm and longer tests have shown that this process continues. The bench can end up 4 mm below starting point, but this is within the range of the 2 fishing rods on the top stage filter and intermediate filter.

The filters in this test are, after one day, at 24.8°C and 23.1°C for the intermediate filter and top stage filter, respectively. The hotplate has a temperature of about 45°C, which is about 5°C warmer than expected for the hottest bench at Virgo, SDB2. Thus these results are conservative and show that MultiSAS can be used with no loss of performance once the operating temperature is established.

The power dissipation in the cabling in the tank that run via the filters to the bench was also simulated and determined to be about 15 W. Most of this heat is expected to be transferred to the outside by the MiniTower cupola walls. The fishing rods can cope with about 7.5°C deviation from room temperature and recover the system. Shielding has proven to keep the filters within this range. All five MultiSASs at Virgo are fitted with two reflective thermal shields below the intermediate filter.

3.2.5 Acoustic coupling

Operating a suspension in air has certain limitations at in-band ($f > 10$ Hz) frequencies. Acoustic pressure waves push on the otherwise isolated suspended object, such as an
optical table. Removing the air eliminates the medium these pressure waves use to travel and reduces the so-called acoustic coupling. The optical benches that are suspended by MultiSAS for Advanced Virgo are all in their MiniTower, where a vacuum of below $10^{-4}$ mbar can be achieved. A scroll pump can bring the pressure below 1 mbar and a turbo pump can bring the pressure down even further. A turbo pump is installed in the MultiSAS test facility at Nikhef, but this is not the case for the five MiniTowers at the Virgo site, where an operating pressure of about 0.5 mbar is achieved.

![Figure 3.15: Pressure in the MultiSAS test facility vacuum vessel during the acoustic coupling test. At $t = 50$ min after start of the test, the turbo pump takes over to proceed to pressures well below 1 mbar](image)

To test if this is sufficient to reduce the acoustic coupling, a test was performed at the Nikhef MultiSAS test facility. This set-up has out-of-loop sensors, i.e. Sercel L22 geophones on the bench. With these six geophones (three horizontal and three vertical), a proper measurement of the bench motion and acoustic coupling can be made until the effect of the coupling is lower than the self noise of the geophones. The self noise of the L22 geophone at 10 Hz is about $2 \times 10^{-12}$ m/√Hz and falls off with $1/f$ from that point. This implies that, if the (residual) coupling is lower than the L22 self noise, the pressure at which this coupling is observed is enough to not spoil the translational ASD requirement. For the angular motion, for which a horizontal motion at the $10^{-15}$ m/√Hz is desirable, this measurement gives an upper limit.

For the test the vacuum vessel is vented to a pressure of 10 mbar. The scroll pump continues to pump and the pressure is monitored. When the pressure is below 1 mbar, the turbo pump takes over and reduces to pressure to below $10^{-4}$ mbar an hour after the test was started. The evolution of the pressure in this test is presented in Fig. 3.15. In Fig. 3.16, the main result of the test is displayed. Acoustic coupling falls below the L22 self noise below 5 mbar, thus achieving low enough coupling not to spoil the bench translational ASD requirements.

The MultiSAS test facility at Nikhef has a different vacuum vessel as the MiniTowers used at Advanced Virgo. The vacuum vessel at Nikhef is about a factor 1.5 larger in volume. It features a cylinder shape main door, which can be lifted by three motor-turned threaded bars. This feature allows for a researcher to be able to physically reach the suspended bench from all sides. It makes for a more ideal test-bed for (commercial) sensor development.
Figure 3.16: Acoustic coupling at a translationally and angularly controlled MultiSAS prototype suspended bench at different pressures for a (a) horizontal and (b) vertical L22 (out-of-loop) geophone on the bench. Clearly, the acoustic coupling falls below the L22 self noise below 5 mbar.

3.3 Design of the control system

The control scheme adopted for the MultiSAS systems in Virgo is relatively traditional. The control of the translational degrees of freedom (x, z and y) for the bench are done from the top stage, whereas the angular degrees of freedom (θy, θx and θz) are done at the bench level. As shown in Fig. 3.3, the sensors used in the MultiSAS controls are the in-house produced LVDTs and commercial Sercel L4C geophones. PID filters with appropriate time constants and roll-off are used to meet rms requirements, without spoiling the passively achieved spectral requirements. Despite these traditional approaches, significant effort is needed to install, commission and optimize the control performance.
3.3.1 Top stage control

The LVDT is a differential sensor, as explained in section 1.5.2, measuring the MultiSAS position with respect to the reference frame which is connected to the MiniTower. The L4C geophone is instead an inertial sensor. The signals from LVDT and geophone are combined in the frequency domain (blended) to construct an inertial broadband so-called super sensor with a DC positioning capability. Blending is done preferably below the microseismic peak, located between 200 to 500 mHz, so that the L4C purely inertial signal is dominant in the blended signal at those frequencies. The microseismic peak can then be suppressed and the bench rms motion can be reduced.

![Diagram showing horizontal digital control strategy for MultiSAS.](image)

**Figure 3.17:** Horizontal digital control strategy for MultiSAS. A multitude of sensor signals is geometrically added (using matrix $S$) and blended to reconstruct virtual supersensors ($\tilde{x}$, $\tilde{z}$, $\tilde{\phi}_y$) and implement single-input-single-output (SISO) control in those degrees of freedom. Control signals for virtual actuators are subsequently sent to each actuator by multiplying by matrix $D$. Adapted from Ref. [52].

L4C geophones in three axes, *i.e.* the $x$-, $y$- and $z$-direction, are also installed on the ground next to each MiniTower. The signals from the ground geophones can be used to correct the LVDTs for the fact that they sense differentially with respect to the ground. This process is called ground subtraction and is done by adding the ground geophone signals to the LVDT signals as the ground is in the LVDT signal with opposite sign. For the vertical control, where only an LVDT is used to sense the top GAS filter keystone, this ground subtraction is vital. The use of sub-microseism blending makes the ground subtraction in horizontal less crucial.

Fig 3.17 shows the horizontal control strategy for MultiSAS in Virgo. The control
scheme features a fully digital control, using eight sensor inputs to be blended resulting in an inertial super sensor including DC position information for three horizontal degrees of freedom. Before blending, the signals are geometrically added using the sensing matrix $S$. This matrix is determined by extraction of the different degrees of freedom ($x$, $z$ and $\theta_y$) from the geometric content of each sensor signal. Using the SNEB map presented in Fig. A.6 as an example, the sensor signal content is

$$d_{\text{Hor } 0} = -\sin(30^\circ)x - \cos(30^\circ)z + r\theta_y,$$
$$d_{\text{Hor } 1} = -\sin(30^\circ)x + \cos(30^\circ)z + r\theta_y,$$
$$d_{\text{Hor } 2} = x + r\theta_y,$$

where $d_{\text{Hor } 0,1,2}$ represents the measured displacement at position 0, 1 or 2 for each sensor and $r$ the radial distance of the sensor position to the center of the top plate. The matrix $S$ for the SNEB LVDTs that can be extracted using this method is

$$S = \begin{bmatrix}
x_{\text{SNEB}} \
z_{\text{SNEB}} \\
\theta_y,_{\text{SNEB}}
\end{bmatrix} = \begin{bmatrix}
-0.3333 & -0.3333 & 0.6667 \\
-0.5777 & 0.5777 & 0 \\
0.725 & 0.725 & 0.725
\end{bmatrix} \begin{bmatrix}
d_{\text{Hor } 0} \\
d_{\text{Hor } 1} \\
d_{\text{Hor } 2}
\end{bmatrix}. \quad (3.7)
$$

For the horizontal translational degrees of freedom the loop design is shown in Fig. 3.18. The zeros and poles of the control filter are presented in Table 3.3.

<table>
<thead>
<tr>
<th>Zeros</th>
<th>Poles</th>
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</thead>
<tbody>
<tr>
<td>$f$ [Hz]</td>
<td>$Q$</td>
</tr>
<tr>
<td>0.07</td>
<td>0.5</td>
</tr>
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<td>25</td>
<td>50</td>
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<table>
<thead>
<tr>
<th>$f$ [Hz]</th>
<th>$Q$</th>
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</thead>
<tbody>
<tr>
<td>4.58</td>
<td>0.88</td>
</tr>
<tr>
<td>30</td>
<td>0.707</td>
</tr>
</tbody>
</table>

Table 3.3: $f$ and $Q$ values for the zeros and poles that make up the horizontal PID controller with elliptic roll-off filter. The real pole at 0 Hz represents the integrator. The gain is 35 at 1 Hz. The other top stage filters differ by the position of the $Q = 0.5$ zero, which is $f = 0.1$ Hz for $\theta_y$ and $y$ filters. The other difference is that the gain is 1.5 and 4 at 1 Hz for $\theta_y$ and $y$ filters, respectively.

Visible in the plant transfer function are the main translational modes of MultiSAS. The inverted pendulum stage has a resonance frequency of about 100 mHz. From the top stage the intermediate filter and suspended bench act as a double pendulum. The common pendulum mode is located around 0.7 Hz. The differential pendulum mode - the intermediate filter and bench move out of phase - is located around 1.8 Hz.
Figure 3.18: Loop design for the top stage MultiSAS horizontal degrees of freedom (x, z). A forced transfer function measurement of the SDB2 MultiSAS loaded with a dummy mass is compared to a MultiSAS modelled transfer function. The control filter is a conventional PID filter with a steep roll off provided by a 1st order elliptic filter with a notch at the MiniTower mode frequencies (here as an example at 25 Hz). Similar PID filters are used for the SISO controllers for $\theta_y$ and the $y$ degree of freedom. All filters have a phase margin of more than 30 degrees.

The roll-off filter is a 1st order elliptic filter in combination with a 2nd order Butterworth filter at 30 Hz. The elliptic low pass filter features a steep roll-off with minimal phase loss around unity gain. An additional benefit of the elliptic filter is that it features a notch, which can be aligned in frequency with the first rigid body mode of the MiniTower. The frequency of this mode differs in each system depending on the vacuum chamber installation. The measured mode for each MiniTower is given in Table A.4.

### 3.3.2 Signal blending

In the MultiSAS top stage control, the error signal that is fed to the horizontal feedback filter is a blended signal between the LVDT and the L4C geophone. The top stage sensor blending is done below the microseismic peak, typically at $s_0 = 1$ ($f = 0.16$ Hz). The blending filters are 5th order polynomials such as

\[
\mathcal{H}_\text{blend LP} = \frac{s_0^5 + 5s_0^4s + 10s_0^3s^2}{(s + s_0)^5} \quad \text{low pass blending filter,}
\]

\[
\mathcal{H}_\text{blend HP} = \frac{10s_0^2s^3 + 5s_0s^4 + s^5}{(s + s_0)^5} \quad \text{high pass blending filter,}
\]

(3.8)
Figure 3.19: Blending filters for MultiSAS horizontal control. Blending is done around $s_0 = 1$ and $H_{\text{geo,proto}}$ also contains extra Butterworth high pass filters. The yellow curve $\sum_{\text{proto}}$ is the sum of the two prototype filters. The amplitude and phase at and around the blending frequency is not flat when summing the prototype filters. The filters that are ultimately used are the purple and green curves and summing these show a flat $\sum$ blue curve in amplitude and frequency.

the sum of which is always 1 and flat in phase by definition; the sum of the numerators of the filters is equal to $(s + s_0)^5$.

To cut further the geophone noise below 100 mHz, more Butterworth high pass filters are added to compensate for the gain that is given by the geophone calibration filters to the low frequency geophone noise. As discussed in section 1.5.2, the geophone signal has to be corrected for the mechanical transfer function of its proof mass suspension. Moreover, in order to blend the geophone velocity signal with the LVDT displacement signal, the signal has to be integrated with $1/s$. The mechanical correction and this integration result in a $1/s^3$ integration of the geophone signal below the natural frequency of the geophone suspension.

Because the added high pass filter has an effect on the phase of the geophone signals around the blending frequency, the LVDT low-pass filters have to be adjusted as well. The effect on the summed blended signal if this is not done is shown in Fig. 3.19. To adjust the LVDT filter, prototype filters are constructed. For the LVDT signal the low pass filter described in Eq. (3.8) is used as $H_{\text{LVDT,proto}}$. The L4C uses the high pass filter of Eq. (3.8) with the addition of a two second order Butterworth high pass filters. This combination of filters results in $H_{\text{geo,proto}}$ in this example. The final filters are then constructed by using

$$H_{\text{LVDT}} = \frac{H_{\text{LVDT,proto}}}{H_{\text{LVDT,proto}} + H_{\text{geo,proto}}}, \quad H_{\text{geo}} = \frac{H_{\text{geo,proto}}}{H_{\text{LVDT,proto}} + H_{\text{geo,proto}}},$$

(3.9)

the sum of which again by definition is 1.
Figure 3.20: Typical geophone filters for SNEB MultiSAS geophones for the different positions. The L4C geophone is specified to have a proof mass suspension with $f_0 = 1 \pm 5\%$ Hz, but transport and installation could cause this value to change, as is visible in the position 0 geophone. A shunt resistance across the coil of the readout is specified to lower the Q to about 0.75 $\pm$ 5%. This filter is applied to raw voltages coming out the geophone pre-amplifiers and prepares the signal for blending with the LVDT.

<table>
<thead>
<tr>
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</tr>
<tr>
<td>4 $\times$ 0</td>
<td>-</td>
</tr>
<tr>
<td>1.45</td>
<td>0.77</td>
</tr>
<tr>
<td>0.50</td>
<td>0.63</td>
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<tr>
<td></td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table 3.4: $f$ and $Q$ values for the zeros and poles that make up the SNEB position 0 geophone filter to prepare the raw geophone voltage for blending with the LVDT. The real zeros at 0 Hz result in the $f^4$ slope at low frequencies. The complex zero at 1.45 Hz represents the geophone calibration. The complex poles at 0.032 Hz ($s = 0.2$ rad/s) represent the two Butterworth filters. The complex zero at 0.50 Hz, the real pole at 0.059 Hz and the complex poles at 0.14 Hz and 0.30 Hz are the blending part of the filter.

Blending filter $H_{geo}$ in combination with the geophone calibration and the conversion from velocity to displacement, results in the filters displayed in Fig. 3.20. The filter for the SNEB position 0 geophone is summarized in Table 3.4. At high frequencies ($f > 5$ Hz), the filter has a $1/s$ slope, accounting for the conversion from velocity to displacement.
3.3.3 Vertical ground subtraction

The vertical loop relies primarily on one top stage LVDT as an error signal. This error signal has a large coupling to ground noise, which can be reduced significantly by ground subtraction. The subtraction is achieved by summing an inertial measurement of the ground motion to the LVDT signal. As for the horizontal geophones on the MultiSAS top stage, steep high-pass filtering is needed to suppress the ground geophone noise below 100 mHz.

The high pass filter that is designed for this purpose is plotted in Fig. 3.21(a). It is a steep elliptic high-pass filter, with a nearly flat (maximum 20 degrees off) response down to 250 mHz. The phase advance part of the filter is

\[ H_{pa} = \frac{0.6s + s^2}{0.02 + 0.2s + s^2}, \]

resulting in a bump of a factor 2 that is needed to keep the phase more flat. The elliptic part is constructed by the Matlab `Ellip(4, 0.01, 80, 0.25, 'high', 's')` elliptical filter design command. This command results in a fourth order elliptical filter at \( s = 0.25 \text{ rad/s} \) (\( f = 40 \text{ mHz} \)) with 80 dB noise reduction and 0.01 dB bandpass ripple. On top of that, there are two second order Butterworth filters at 70 mHz.

This filter was designed to minimize the velocity rms motion, as decreased velocity at low frequency is believed to decrease upconversion of scattered light to in-band frequencies [139]. Fig. 3.21(b) shows the velocity rms at the top stage level integrated down to below 100 mHz, which is also expected to be equal to the bench velocity rms. The ground model used for this loop design is the 50 percentile trace presented in Fig. 2.7, where most of the contribution is expected between 200 mHz and 500 mHz.

By simultaneously performing a measurement of the local control (LC) vertical signal, which are constructed by the four LVDTs below the suspended benches, of SNEB and SWEB with one sensor corrected, the geophone filter performance was validated. The result of this measurement is shown in Fig. 3.21(c). Note that the ground motion around 110 mHz is around the 90 percentile level of the Virgo spectrum of Fig. 2.7, i.e. it was an uncommonly stormy day at the Atlantic Ocean during this measurement.

Despite these conditions, the sensor correction performs well, improving the rms velocity by about a factor of two comparing the yellow dashed line (SNEB, no sensor correction applied) with the purple dashed line (SWEB, sensor correction applied). Compared to the simulated results of Fig. 3.21(b), a performance better than the ground rms is not achieved in this measurement. This is most probably because of the different spectral shapes of the ground, i.e. the 50% percentile solid green line of Fig. 2.7 compared to the blue and red solid line. Additionally, to approximate proper out-of-loop inertial sensor for this measurement at the bench level, the vertical LVDT signals were also ground corrected by the nearby L4C geophone. This causes an overestimation of the signal between 35 mHz and 150 mHz.
Figure 3.21: (a) Geophone high pass filter shape, which consists of elliptic and Butterworth high pass filters and a so-called phase advance, which results in the bump between 35 mHz and 150 mHz, but improves the phase around the general microseismic peak frequencies at the Virgo site (200 - 500 mHz). (b) The effect on ground subtraction performance showing the closed loop performance with and without the use of ground subtraction with the (filtered) L4C geophone. The high pass filter decreases rms velocity with more than a factor 3. Panel (c) shows a measurement result where SWEB control is with and SNEB control is without sensor correction by the ground geophone. The ground measurements are performed by a nearby Guralp 40T.

3.3.4 Suspended bench control

Bench local controls were first tested at the MultiSAS test facility at Nikhef, at which sensor/actuator design was validated, as well as feedback filters and strategies. The bench is sensed and controlled by eight (four horizontal and four vertical) co-located LVDT/ coil-magnet actuator pairs. It is important to carefully set the strength of the
actuators in such a way that the Digital-to-Analog-Converter (DAC) noise does not spoil the angular ASD requirements at 10 Hz. Setting of this actuator strength is done by changing the (sampling) resistance of the coil driver which determines the voltage-to-current conversion of signals that are sent to the actuator coils.

Figure 3.22: Schematic of the coil driver for MultiSAS. The voltage from the DAC is converted to current by using sampling resistor $R_s$. The transconductance gain given is equal to $1/R_s$.

The coil driver (see Fig. 3.22) converts the input voltage from the DAC to current with transconductance gain $1/R_s$, where $R_s$ is the sampling resistor. Since every angular degree of freedom is controlled by using four actuators, the angular noise caused by the DAC voltage noise $V_{n,DAC}$ is

$$
n_\theta = \frac{2\beta V_{n,DAC}}{d_{cma} R_s I_{cm,x}} \frac{1}{-\omega^2 + \omega_0^2(1 + i\phi)},
$$

(3.11)

where $d_{cma}$ represents the distance of each actuator from the rotation axis, $\beta$ the coil response in [N/A] and $I_{cm,x}$ the moment of inertia of the bench around the rotation axis. The system with natural angular frequency $\omega_0$ is assumed to be in vacuum so only structural damping is assumed with loss angle $\phi$. The calculated effect of the DAC noise on angular motion of the bench is shown in Fig. 3.23 for the prototype setup at Nikhef. The projections show that the designed performance meeting ASD requirement for the angular degrees of freedom is not expected to be spoiled when $1 \mu V/\sqrt{\text{Hz}}$ DAC noise is assumed.

At the bench level, the LVDTs that generate the error signals for the control loops of the angular degrees of freedom all have a displacement noise better than $10^{-8} \text{ m/}\sqrt{\text{Hz}}$ at 10 Hz. At the MultiSAS prototype, the bench angular modes are about 15 mHz for yaw ($\theta_y$) and about 310 mHz for pitch ($\theta_x$) and roll ($\theta_z$). The angular control loops have to be designed such that this noise is not injected in band, so steep roll-off filters are required. Especially for tilt and roll, where the gain around 300 mHz should be as high as possible, the frequency interval over which this roll-off must occur is limited.
Figure 3.23: Projected effect of DAC noise on angular displacement for the MultiSAS test facility bench. A flat DAC voltage noise of $1 \mu V/\sqrt{Hz}$ is assumed. The coil drivers employ $R_s = 2.2 \, k\Omega$ and the four coil magnet actuators, each with $\beta = 1 \, N/A$, are located such that $d_{cma}$ is 0.92 m for the yaw ($\theta_y$) and 0.64 m for pitch and roll ($\theta_x, \theta_z$). The moments of inertia are about 90 kg.m$^2$ for $\theta_y$ and 30 kg.m$^2$ for $\theta_x$ and $\theta_z$.

Tilt and roll are measured by means of four LVDTs, which are each located near one corner of the suspended bench. Since the measuring arm is about 0.5 m the readout noise for these degree of freedom is about 20 nrad/ $\sqrt{Hz}$. For this reason the open loop gain at 10 Hz should not exceed a magnitude of $5 \times 10^{-8}$ in order not to spoil the angular
ASD requirements. An example of such a loop design is shown in Fig. 3.24 and its zero and pole values are presented in Table 3.5.

<table>
<thead>
<tr>
<th>Zeros</th>
<th>Poles</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$ [Hz]</td>
<td>$Q$</td>
</tr>
<tr>
<td>0.04</td>
<td>0.5</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>12</td>
<td>50</td>
</tr>
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<td>14</td>
<td>50</td>
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<td>16</td>
<td>50</td>
</tr>
<tr>
<td>18</td>
<td>50</td>
</tr>
</tbody>
</table>

*Table 3.5:* Values for $f$ and $Q$ of the zeros and poles that make up the angular PID controller with elliptic roll-off filters. The real pole at 0 Hz represents the integrator. The other zero-pole pairs are five $Q = 50$ elliptic filters.

## 3.4 Commissioning for Advanced Virgo

With the installation of the EIB-SAS, the first physical change from Virgo+ to Advanced Virgo was achieved. From Nikhef’s side, this was the first of in total six seismic isolation systems to be constructed, shipped and installed at the site. The five other systems are MultiSASs to suspend the in-vacuum benches. After the prototyping campaign, small adjustments were made and five systems were designed, built at Nikhef and shipped to the Virgo site. Subsequently, all systems were installed in their MiniTowers (SIB2, SPRB, SDB2, SWEB and SNEB) and pre-commissioned with a dummy mass, just as they were all individually tested at Nikhef. Several detailed aspects of the (individual) Advanced Virgo MultiSASs are summarized in the second half of Appendix A.

### 3.4.1 MultiSAS performance at Advanced Virgo

Over the course of 2014 and 2015, all five MultiSAS were installed in their MiniTowers. After mechanical installation, they were fitted with a 320 kg dummy mass hanging from the wire that would eventually suspend the bench. For each system the actuation matrix $D$ was determined first, by using the iterative procedure described in Ref. [52], and then the operation in closed-loop was tested. The results for SDB2 are shown in Fig. 3.25. Looking at the ground spectrum, the microseismic peak is clearly visible between 200 and 500 mHz. At 10 Hz, the motion is below $10^{-8}$ m/$\sqrt{\text{Hz}}$ and at higher frequencies the usual $1/f^2$ slope is observed.

The top stage was instrumented with an additional out-of-loop geophone in order to get a out-of-loop measurement of the performance of the control system. The signal of the witness geophone was also used to estimate the residual motion of the suspended bench. The suspended bench motion is here estimated by multiplying the witness sensor
Figure 3.25: Open and closed loop results in the z-direction from the SDB2 MultiSAS loaded with a dummy mass. The measurement is performed by a witness out-of-loop L4C geophone placed on the suspension top stage. The plot shows the expected bench motion reconstructed by multiplying the witness sensor signal by the measured top stage to bench transfer function. Also plotted is the expected closed loop performance using the modelled MultiSAS transfer function and the PID filter corresponding to Fig. 3.18. The translational ASD requirement from 10 Hz is shown as a black dashed line. Displacement levels better than $10^{-14}$ m/√Hz are achieved above 10 Hz. At 12 and 22 Hz, the main modes of the tank are observed.

signal by the measured top stage to bench transfer function. It must be noticed that this projection gives a more realistic figure of the residual bench motion since, unlike the MultiSAS transfer function in Fig. 3.9, it also includes the mechanical response of the Minitower to the ground motion. This is visible when comparing the closed loop projection, which shows the tank modes, with the closed loop expectation, which does not take into account the Minitower transfer function.

In open loop, the MultiSAS modes at around 0.1 Hz (inverted pendulum stage), 0.7 Hz (common composite pendulum mode) and 1.8 Hz (differential composite pendulum mode) are observed. The attenuation slope increases to $1/f^6$ above to last mode. The official translational ASD requirement of $2.1 \times 10^{-12}$ m/√Hz is surpassed by more than two orders of magnitude. When the rigid body modes of the suspension are effectively damped by the controls, the integrated rms displacement reduces to a fraction of a micron.

The MiniTower vacuum vessel modes should be monitored as this motion couples to angular motion. The tank modes are clearly visible in the top stage motion and can end up as horizontal motion at the suspended bench level. This motion couples directly to pitch and roll and could spoil the angular performance. Looking at Fig. 3.25 MiniTower modes
above 10 Hz are observed. As discussed in section 3.2.1, the coupling from $X$ to $\theta_x$ can easily be around 10% or even higher for certain $d_{\text{susp}}$ (distance between suspension point and bench center of mass) in combination with certain bench mass distributions.

![Figure 3.26: SNEB (a) top stage (z) and (b) bench angular ($\theta_z$) performance in high and low wind/waves conditions. The respective rms requirements are indicated in the plots by the black arrow. Microseismic peak attenuation is apparent in both cases for translational top stage motion. Angular rms requirements are not achieved in both cases, despite good damping of the modes.](image)

The SNEB top stage and bench motion a few weeks prior to detector science mode operation in O2 (August 1 to August 25, 2017) is presented in Fig. 3.26. During low or high wind conditions, the controls damp out the modes in translational (top stage) and angular (bench level) degrees of freedom. The discrepancy between the ground and the top stage motion at frequencies below 100 mHz is explained by the fact that the (blended) top stage signal is almost purely LVDT there. Below the modes of MultiSAS the top stage follows the ground motion. The LVDT measures the differential motion between top stage and ground motion and is not corrected by a geophone in horizontal, so this in-loop sensor is expected to show a suppressed signal. The increased signal magnitude in the top stage motion from 10 Hz onwards is attributed to MiniTower modes, which are summarized in Table A.4 for different Advanced Virgo systems. The translational rms requirements are met, even in high environmental noise conditions. However, this is measured with an in-loop signal, so actual motion is expected to be higher. Comparing the ground motion traces to the ground displacement noise of Fig. 3.25, it could be characterized as quiet conditions. The obtained rms results below MultiSAS modes with an out-of-loop sensor is a factor 2 higher in closed loop. Extrapolating this to high environmental noise conditions would mean the rms requirement is not met by a factor 2 for these conditions.

With a similar control strategy as described in section 3.3.4, the rms requirements are not met in the angular degrees of freedom both in high and low environmental noise conditions. Ground motion is also present in these angular LVDT data, so, even though these are measurements done with in-loop sensors, these plots should be considered as
a worse case. At this point in time (during O2) the global angular loops are not closed, so the quadrants for which this angular rms requirement is set are not used. Only in the final Advanced Virgo set-up, i.e. with signal recycling and high input power, not meeting these requirements could result in control noise injections and upconversion. The angular rms requirements are set such that the quadrants are shot noise limited and this shot noise limit is designed to be a safety factor of 10 below design sensitivity.

The optical bench content was not finalized during the design of MultiSAS by Nikhef and the optical benches by Laboratoire d’Annecy-le-Vieux de Physique des Particules (LAPP). This resulted in the bench tilt and roll modes (about 300 mHz) now coinciding in frequency with the microseismic peak interval - 200 mHz to 500 mHz for the Virgo site. Proper a priori mass distribution control would have been necessary in order to avoid this alignment in frequency. This is however practically impossible in a design stage of an optical system. Once all the optical components are placed and they would be left untouched, the bench moment of inertia could be increased to lower the resonance frequency. This could be done by changing the mass distribution using dummy masses, but opportunities for that are slim as the MultiSAS GAS filters have been tuned to a specific load. Another option is lowering the suspension point. This could be considered in the future.

3.4.2 GAS blade failure

A broken blade spring was discovered in the intermediate GAS filter (see Fig 3.27) during an attempt to get the SPRB bench floating. The entire MultiSAS was removed from the SPRB MiniTower and all 18 GAS blades (10 top and 8 intermediate) were replaced with newly produced blades to mitigate a possible environmental exposure issue. The failure of the blade occurred somewhere in the 10 months after the SPRB pre-commissioning tests with the dummy payload, which ended in October 2014. GAS blades were under nominal stress at that time as the blades are bent and kept into this curvature by the keystone end-stops.

No defect was found in the nickel plating. Moreover, there was no sign of corrosion, i.e. no evidence of environmental contamination, in any of the blades. Several blades
were cut up in test pieces and the fractured blade underwent fractographic analysis by R. Valentini et al. at the University of Pisa. Several snapshots of this analysis are shown in Fig. 3.28.

![Fractographic analysis of the SPRB GAS blade fracture](image)

**Figure 3.28:** Fractographic analysis of the SPRB GAS blade fracture: (a) overview snapshot of the fracture over several millimeters, (b) a zoom-in of several tens of µm of the brittle fracture area, (c) a zoom-in of several tens of µm of the ductile fracture area and (d) a zoom-in of about 100 µm of a transition from brittle (top part) to ductile (bottom part) fracture area. Images made by R. Valentini et al. of University of Pisa.

Prior to this GAS blade failure, after finding many Virgo superattenuator MAS blades in poor condition or broken after 15 years of service, metallurgic research [140] on similar maraging steel blade failures had already been performed. Fig. 3.29 shows a result of that research where several test pieces went through a test process. The Ultimate Tensile Stress (UTS), a measure of the maximum ability to cope with applied stress, was first determined by pulling the sample apart in an appropriate test set-up. Subsequently, the sample was heated to 1000° C to determine the diffusible hydrogen concentration. Clearly visible is a 45% reduction of the UTS when the hydrogen concentration exceeds
about 2.5 parts-per-million (ppm) and a process coined hydrogen embrittlement [141] commences.

![Graph showing ultimate tensile strength as a function of diffusible hydrogen concentration of tested samples. Reproduced from Ref. [140].]

**Figure 3.29:** Ultimate tensile strength as a function of diffusible hydrogen concentration of tested samples. Reproduced from Ref. [140].

All samples taken out of batches of blades used for MultiSAS blades are well below this value, typically below 1 ppm. Samples from several spare blades, stored in the Nikhef clean room in a humidity controlled environment, were also analyzed, and the typical diffusible H content found was below 1 ppm in all cases. However, possible environmental factors could have deteriorated this value.

Additionally, hydrogen diffusion or migration in non-homogeneously stressed metals occurs [142]. Hydrogen diffuses towards regions of high tensile stress, e.g. initiated cracks but also highly stressed areas. FEM models of the stress distribution in a GAS blades are shown in Fig. 3.30.

![FEM models of stress distribution in GAS blades.](image)

**(a)**

**Figure 3.30:** GAS blade stress profile of the (a) top part and (b) the bottom part of the blade. The peak stress level is 1.7 GPa in the FEM model.

Comparing the peak stress level to the nominal UTS of about 2 GPa of this type of maraging steel, it can be seen in Fig. 3.29 that deterioration by hydrogen embrittlement is to be avoided. In a GAS blade the stress induced hydrogen diffusion effect is expected to make the hydrogen concentration nonuniform over the blade volume. It is therefore important to model this since all hydrogen content measurements are done in
unstressed samples, making the hydrogen distribution uniform again. In presence of stress, it is thermodynamically favored for hydrogen to occupy regions where the metal lattice is tensed instead of compressed. Diffusion and rearrangement of hydrogen concentration $C$ (dimensionless, typically in ppm) are governed by the equation [142]

$$\frac{\partial C}{\partial t} = D_H \nabla^2 C - \frac{D_H V_H \nabla(C \nabla \sigma_h)}{RT},$$

(3.12)

where $\sigma_h$ represents the hydrostatic stress in the material, $V_H$ the molar volume, $D_H$ the diffusivity of hydrogen in the material, $R$ the gas constant and $T$ the temperature.

**Figure 3.31:** Hydrogen migration in (a) MAS and (b) GAS blades due to hydrostatic stress gradients. After a month, the equilibrium of the concentration distribution is established.

Eq. (3.12) can be solved analytically for symmetric problems. For the simple case of a triangular superattenuator MAS blade in the $x$-direction of the blade thickness, this gives

$$C(x) = C_0 - \frac{\beta}{\sinh \beta} e^{(2\beta x)/t},$$

(3.13)

where $C_0$ represents the hydrogen concentration for the unstressed material, $\beta = (\sigma_h V_H)/(RT)$, $x$ the distance from the middle of the blade and $t$ the time during which the material is under stress $\sigma_h$. For a MAS blade, $C$ is 30% higher or lower than $C_0$ on the top and bottom of the blade, respectively.

A GAS blade has to be FEM modeled, as the shape and stress distribution are non-linear (see Fig. 3.30). A so-called D-type blade, which is used in MultiSAS, has a 70% higher $C$ at the regions of highest stress. The time evolution of $C/C_0$ on MAS and GAS blades is summarized in Fig. 3.31. The time it takes for the $C/C_0$ to reach equilibrium is about a month and differs from experience with the SPRB blade. Moreover, these types of blades are operated at AEI Hannover optical table suspensions in and out of vacuum conditions for more than 7 years now [143].
A short term mitigation strategy is the replacement of blades by so-called E-blades. The shape of these blades are designed to have a lower peak stress. This will lower the peak $C/C_0$ value from about 1.7 for D-blades to about 1.5 for E-blades [144]. More long term research will be performed at Nikhef on diffusible hydrogen content in samples of maraging steel during the fabrication process. This hydrogen content is determined in between the various stages of the process, which are subsequently before the process (raw material), after hardening, after plating and after baking. The baking step is performed to lower the hydrogen content, which is expected to increase during the fabrication process. Half of the samples will skip this baking phase. Additionally, there are future plans to validate the simulations of Fig. 3.31. Blades with known hydrogen content will be subjected to stress for a varying number of weeks and, once the stress is relieved, they will be immediately cut up in slices and pieces. This way the hydrogen has no time to diffuse back resulting in a homogeneous distribution again and a (coarse) three dimensional map of the hydrogen content of a GAS blade under stress can be made.

### 3.4.3 Scattered light

During the commissioning and engineering run phases prior to Advanced Virgo joining O2, measurements were performed with all the suspended optical benches (including also SDB1) by the Detection (DET) team. An example of such a measurement is presented in Fig. 3.32. White noise injections on the coils of the local control actuators can excite modes of optical mount (structures) and give insight in the couplings that exist to the DARM degree of freedom. Similar injections were also performed on SDB2, SPRB, SWEB and SNEB, but no notable effect on DARM was observed [122]. Couplings are expected but not yet visible at the sensitivity obtained in O2, so repeating these measurements could be considered before O3.

Scattered light is modeled as phase noise that couples back in to the interferometer as [146]

$$h(f) = \text{FFT} \left[ \frac{\lambda}{4\pi} \delta \sin \left( \frac{4\pi\alpha}{\lambda} z(t) \right) \right],$$

where $\delta$ represents the coupling factor and $z(t)$ the time series of the motion in the beam axis of the source of the scattering, i.e. the so-called scatterer. This is typically an optical component on an optical bench or a part of the vacuum infrastructure. Coefficient $\alpha$ was added to the model to fit the (non-linear) spread of the noise injection at the suspended bench [147]. Such scattered light models are used to understand direct couplings from scattering surfaces ultimately to the detector sensitivity.
Figure 3.32: Results of an investigation on scattered light impact on the strain sensitivity from (optical components on) SDB1 between 100 Hz and 500 Hz. Peaks and structures in the so-called LSC_DARM channel increase in amplitude when injecting white noise in SDB1 actuator coils on the bench level. The red curve corresponds to about $10^{-14} \text{m}/\sqrt{\text{Hz}}$ at 200 Hz at the bench level [122]. The strain sensitivity curve is reconstructed by correction for the Fabry-Perot cavity pole and DARM control loops and other calibration lines (which are checked by the so-called photon calibrator (PCal) [145]) on the LSC_DARM channel. Adapted from Ref. [146].

Figure 3.33: Results of an investigation on scattered light unconversion on SWEB. A 0.3 Hz line (about 35 $\mu$m peak-peak bench motion) is injected in the z-direction (beam axis). From $t = 65$ s onwards, a control loop to attenuate the microseismic motion is closed. The black line is the predicted (upconverted) noise in the B8 photodiode signal. Adapted from Ref. [148].

Understanding of upconversion of low frequency scatterer motion to higher frequency
in-band noise is also important. In Fig. 3.33 a result is presented, where a measurement was done to check upconversion models. Visible from the graph is that motion, even though it is low frequency (0.3 Hz), is upconverted into in-band (> 10 Hz) noise. Before closing a feedback loop, which decreases the low frequency motion, the upconversion effect results in noise in the end test mass monitoring photodiode up to about 100 Hz.
As the most precise commercial vibration sensors are sufficiently sensitive to measure even the most seismically quiet places on Earth, i.e. have sensor sensitivities below Peterson’s low noise model [87], commercial development of more precise sensors has stalled. The sensitivity curves of currently used vibration sensors in GW detectors are presented in Fig. 4.1.

As discussed in section 3.4.1, there are stringent requirements on the residual motion of the optical benches suspended by MultiSAS in Advanced Virgo. An inertial sensor with a broadband sensitivity in the vicinity of the \( \frac{\text{fm}}{\sqrt{\text{Hz}}} \) regime can be used to characterize the residual motion at the bench level to monitor femtometer and coupled femtoradian motion. Therefore, a novel vibration sensor was proposed at the start of this work.

Additionally, in order to achieve a lower frequency cut-off (< 2 Hz) for seismic noise, that is envisaged for next-generation gravitational wave detectors such as Einstein Telescope or Cosmic Explorer [149], an active isolation platform on a pre-isolator stage could be combined with cascaded pendulums - essentially combining the suspension systems of LIGO and Virgo. The performance of (the last stage) of such active platform is mostly dependent on the performance of the inertial sensor providing error signals for the feedback loops.

Accelerometers essentially all work in the same manner. The acceleration of a frame or object is measured by comparing its motion to the motion of a inertial so-called proof mass. Inertia can be approximated by suspending the proof mass. Nowadays,
Accelerometers are used in various well-known modern appliances, such as car airbags and, together with gyroscopes, in smartphones to determine its orientation and act accordingly, as shown in Fig. 4.2(a).

Figure 4.1: Measured or specified displacement sensitivity for inertial sensors used in geophysical and gravitational wave experiments.

Figure 4.2: (a) A modern application of accelerometers: the smartphone. Panel (b) shows a schematic picture of an accelerometer, showing a suspended mass able to engage in harmonic motion constrained by a spring with stiffness $k$. The proof mass motion is viscously damped by the dashpot with damping coefficient $c$. 
In Fig. 4.2(b), a schematic picture of an elementary accelerometer is shown. The set-up is a simple damped harmonic oscillator in one degree of freedom, where \( x_g(t) \) is the coordinate of the ground (or frame) on which the mass is suspended and \( x_m(t) \) is the position of the proof mass. When the system is accelerated instantaneously with a constant acceleration of 1\( g \), the (crude) readout will show a constant value of 1\( g \).

### 4.1 Monolithic accelerometer design

A monolithic folded pendulum (FP) design is well suitable to realise a compact low frequency accelerometer [150]. A schematic view of a monolithic FP, in which the proof mass is suspended by both a regular pendulum and a folded inverted one, is shown in Fig. 4.3. By re-distributing the load on the two pendula the natural frequency can be lowered arbitrarily down to instability while keeping the pendulum length within a few centimeters.

![Figure 4.3: The folded inverted pendulum monolithic accelerometer: (a) schematic design of this accelerometer and (b) a model of this design in order to deduce e.g. the equations of motion.](image)

### 4.1.1 Mechanical modeling

By modeling Fig. 4.3(a) in a way depicted in Fig. 4.3(b), all parts of the system Lagrangian can be identified. The proof mass is represented by means of two masses \( m_{p1}^0 \) and \( m_{p2}^0 \) located at the hinge points \( P \) and \( P' \), respectively. The pendulum and inverted pendulum legs have length \( l_1 \) and \( l_2 \), moment of inertia \( J_1 \) and \( J_2 \) and mass \( m_{a1} \) and \( m_{a2} \), respectively. The distances between upper and lower hinge points of both pendulums are coined \( l_p \),
and $M_l$ represents the tuning mass. The resulting load on each pendulum is

$$m_{p1} = m_{p1}^0 + M_l \left(1 - \frac{D}{S}\right),$$

$$m_{p2} = m_{p2}^0 + M_l \left(\frac{D}{S}\right).$$

(4.1)

The Lagrangian of the system can be written as

$$L = \frac{1}{2} \left( J_1 + J_2 \right) \dot{\theta}^2 + \frac{1}{2} \left( m_{a1} + m_{a2} \right) \dot{x}_c^2 + \frac{1}{2} \left( m_{p1} + m_{p2} \right) \dot{x}_p^2 - \frac{1}{2} \left[ \frac{1}{2} (m_{a1} + m_{a2}) g l + (m_{p1} + m_{p2}) g l_p + \kappa \right] \theta^2,$$

(4.2)

with

$$x_c = x_g - \frac{1}{2} l \theta,$$

$$\theta = \frac{x_g - x_p}{l_p},$$

(4.3)

where $x_g$, $x_p$, and $x_c$ represent the horizontal coordinates of the frame, the proof mass, and the pendulum center of gravity, respectively. The lengths of the pendula are equal, i.e. $l_1 = l_2 = l$ and $l_{p1} = l_{p2} = l_p$. $\theta$ and $\dot{\theta}$ represent the angular displacement and its time derivative and $\kappa$ the cumulative angular spring constant from all the flexures in the pendula. In order to obtain the equation of motion of the system, the Euler-Lagrange equation

$$\frac{d}{dt} \frac{\delta L}{\delta \dot{x}_p} - \frac{\delta L}{\delta x_p} = 0$$

(4.4)

is used. Using the Lagrangian of Eq. (4.2), the equation of motion from $x_g$ to $x_p$ in the time domain is

$$\left[ \frac{J_1 + J_2}{l_p^2} + (m_{a1} + m_{a2}) \frac{l}{2 l_p} + (m_{p1} + m_{p2}) \right] \ddot{x}_p -$$

$$\left[ \frac{J_1 + J_2}{l_p^2} - (m_{a1} + m_{a2}) \left(1 - \frac{l}{2 l_p}\right) \right] \ddot{x}_g +$$

$$\left[ \frac{1}{2} (m_{a1} - m_{a2}) g l + (m_{p1} - m_{p2}) g l_p + \kappa \right] \frac{x_g - x_p}{l_p^2} = 0.$$

(4.5)

Solving the equation of motion gives a similar angular frequency response as derived in section 1.3.1, a transfer function between $x_g$, the ground (frame) motion, and $x_p$, the mass movement. Each pendulum leg moment of inertia can be approximated by $J = m l^2 / 12$ and

$$\mathcal{H}_{\text{disp}}(\omega) = \frac{X_p(\omega)}{X_g(\omega)} = \frac{\omega_0^2 - A_c \omega^2}{\omega_0^2 - \omega^2}$$

(4.6)

is the angular frequency response of the monolithic accelerometer for displacement of which simulated results are shown in Fig. 4.4. The square of the resonant frequency,
which can be lowered arbitrarily by changing the mass distribution, is given by

\[
\omega_0^2 = \frac{(m_{a1} - m_{a2}) \frac{g}{l_p} + (m_{p1} - m_{p2}) \frac{g}{l_p} + \frac{K}{l_p}}{(m_{a1} + m_{a2}) \frac{g}{3l_p} + (m_{p1} + m_{p2})}
\]  

(4.7)

and

\[
A_c = \frac{\left( \frac{l}{3l_p} - \frac{1}{2} \right)(m_{a1} - m_{a2})}{(m_{a1} + m_{a2}) \frac{g}{3l_p} + (m_{p1} + m_{p2})}
\]  

(4.8)

is a parameter associated with the center of percussion effect discussed in section 1.3.1.

Figure 4.4: Simulated folded pendulum accelerometer displacement transfer functions with \( f_0 = 0.1 \) Hz for different values of \( A_c \). Adapted from Ref. [150].

Eq. (4.6) represents the response of the accelerometer suspension to ground or frame displacement. The response to a force applied to the proof mass is instead

\[
\mathcal{H}_{\text{forced}}(\omega) = \frac{X_p(\omega)}{F(\omega)} = \frac{1 - A_c}{m(\omega_0^2 - \omega^2)}.
\]  

(4.9)

In practice, \( A_c \) is much smaller than 1, so can be ignored when the sensor is used in closed loop, i.e. the proof mass is not allowed to move freely when subject to the (to be measured) vibrations. The (actuator) signal can then be taken as sensor output.

### 4.1.2 Proof mass suspension thermal noise

The accelerometers used at Nikhef were made at NWO institute AMOLF. The frame, proof mass, pendulum legs and flexures are all carved out of the same block of material with spark erosion techniques. The material used for the accelerometers presented here is an aluminum alloy, i.e. type 7076-T6. The proof mass is 0.85 kg and is connected to the pendulum legs via thin flexures (nominally 100 µm thick).

The design of the FP is such that the flexures can all be loaded in tension. This allows for much thinner flexures, decreasing the amount of energy that can be stored in the
flexures. The design minimizes suspension thermal dissipation noise, governed by the fluctuation-dissipation theorem [96]. The basic statement of this theorem is that a given oscillator only experiences thermal noise if there is a loss channel present. In other words, every dissipation mechanism causes a fluctuating force on the system, resulting in a displacement fluctuation. In the general case, the thermal noise displacement power spectral density of any suspended object for different types of damping is [98, 152]

\[
x_{\text{th,v}}^2 = \frac{4k_B T \gamma}{m^2(\omega_0^2 - \omega^2)^2 + \omega^2 \gamma^2} \quad \text{viscous damping},
\]

\[
x_{\text{th,s}}^2 = \frac{4k_B T \omega_0^2 \phi}{m \omega(\omega_0^2 - \omega^2)^2 + \omega_0^4 \phi^2} \quad \text{structural damping},
\]

where symbols from section 1.3.1 are used for the damping terms, i.e. $\gamma$ the viscous damping factor and $\phi$ structural loss angle. For a detailed derivation of thermal noise spectral densities, see Ref. [81]. It is easily seen that $x_{\text{th,v}} \propto \omega^{-2}$, whereas $x_{\text{th,s}} \propto \omega^{-2.5}$.

Using LVDT read out monolithic accelerometers (see Fig. 4.5), the mechanics of the FP accelerometer have been characterized [151]. Several accelerometers were put in vacuum to determine the effect of pressure on damping. It was determined that the $Q$ is no longer dependent on pressures below $10^{-3}$ mbar. Below those pressures, the dominant damping contribution was determined to be the eddy current damping [153] from the voice coil. In this coil-magnet actuator, there is an interaction between the actuator magnet and the conductive surfaces of the moving metal material. This creates Joule heating due to residual resistance of the material. The heating represents a power loss and results in a damping term. This type of damping is viscous and the $Q$ for an FP accelerometer with a natural frequency of $f_0 = 0.45$ Hz is determined to be about 150. A high quality factor lowers the suspension thermal noise of the proof mass.
4.2 Interferometric readout

In Fig. 4.6, an optical scheme for the readout of the movement of the proof mass is presented, based on a small interferometer. Both light transmitted to PD2 and reflected to PD1 by the interferometer are read out, matched in magnitude and subtracted in order to suppress common mode noise, such as intensity fluctuations in the laser power. The interferometer output is used as the error signal in a feedback loop in which the piezo actuated mirror tracks the displacement \( (x_p) \) of the mirror attached to the accelerometer proof mass. In this way a linear sensing range corresponding to several micron can be achieved. Such a position readout scheme was first proposed by Gray et al. [154].

The lower beamsplitter together with the two mirrors make up a classic Michelson interferometer. The upper beamsplitter is used to make it possible to readout also the light reflected back to the laser. Assuming both beamsplitters having an ideal 50/50% splitting ratio, the signal amplitude on PD1 is expected to be half of the one on PD2. For the following derivations of the amplitudes of the signal, the beamsplitter convention of

\[
\begin{pmatrix}
E_{a,\text{out}} \\
E_{b,\text{out}}
\end{pmatrix} =
\begin{pmatrix}
r_{\text{BS}} & i\tau_{\text{BS}} \\
i\tau_{\text{BS}} & r_{\text{BS}}
\end{pmatrix}\begin{pmatrix}
E_{a,\text{in}} \\
E_{b,\text{in}}
\end{pmatrix}
\]

(4.11)

is used. The incident fields \( E_{a,\text{in}} \) and \( E_{b,\text{in}} \) are partially reflected and transmitted, as shown in Fig. 4.7(a). The amplitude reflection coefficient is a purely real quantity while the amplitude transmission coefficient is purely imaginary. For a 50/50% beamsplitter \( r_{\text{BS}} = \tau_{\text{BS}} = \sqrt{2} \).

Figure 4.6: Schematic overview of interferometric position sensing by using a piezoelectric actuated tracking mirror. The mirror attached to the piezo is made to follow the mirror attached to the mechanical device by means of a feedback loop. The error signal of the feedback loop is constructed by subtraction of both photodiode (PD1 and PD2) signals.
Figure 4.7: (a) Beamsplitter conventions and (b) the optical scheme of the used interferometer, with the ability to read out both arms. The $E_{\text{in}}$ reflected beam at the top beamsplitter is dumped on an inclined black anodized aluminum surface.

Referring to Fig. 4.7(b), if $E_{\text{in}}$, $E_1$ and $E_2$ are the complex amplitudes of the electric fields of the light entering the interferometric readout and ending up at PD1 and PD2, respectively, then

$$E_1 = E_{\text{in}} t_{\text{BS1}} r_{\text{BS1}} (t_{\text{BS2}}^2 e^{i\phi_1} - r_{\text{BS2}}^2 e^{i\phi_2})$$
$$E_2 = -E_{\text{in}} t_{\text{BS1}} (r_{\text{BS2}} t_{\text{BS2}} e^{i\phi_1} + t_{\text{BS2}} r_{\text{BS2}} e^{i\phi_2}),$$

where $t_{\text{BS1}}$, $r_{\text{BS1}}$, $t_{\text{BS2}}$ and $r_{\text{BS2}}$ are the amplitude transmission and reflection coefficients for BS1 and BS2, respectively. The picked up phases in each arm are

$$\phi_1 = \frac{4\pi (L_1 + \Delta L_1)}{\lambda} \quad \phi_2 = \frac{4\pi L_2}{\lambda},$$

where $L_2$ is the length of the reference arm and is assumed fixed. Here, $\Delta L_1$ can be seen as the motion of the proof mass. In further discussion below, $\Delta L = \Delta L_2 - \Delta L_1$ will be used. Without loss of generality, the reference arm can also be assumed not fixed.

4.2.1 Readout circuit

The photocurrent from each photodiode is converted to voltage by means of standard transimpedance amplifiers (TIAs) with feedback resistor $R_{\text{TIA}} = 20 \text{ k}\Omega$ and a OPA827 op-amp. These two voltages can be read out separately, but are also fed to a balanced differential amplifier, where a AD8597 op-amp is used. The electrical scheme of the
The balanced differential amplifier has a potentiometer in order to match the amplitudes of the input signals. A fixed resistance is not desirable here as, in practice, the output of the TIAs do not differ exactly by a factor of two. The reason is that 50/50% beamsplitters have 50% reflection and transmission coefficients only at a particular wavelength and polarization of light. The gain factors given by this differential amplifier set-up before subtraction, for PD1 and PD2 outputs, are

\[
G_1 = \frac{R_f}{R_1},
\]

\[
G_2 = \frac{R_g(R_f + R_1)}{R_1(R_g + R_2 + R_{pot})}.
\]  

(4.14)

All used resistances have the same value of \( R_1 = R_2 = R_g = R_f = 1 \, \text{k}\Omega \) (all ± 1\% tolerance) and the potentiometer has a range between \( R_{pot} = 0 - 5 \, \text{k}\Omega \). This results in \( G_1 = 1 \) and \( G_2 = 1 \) for \( R_{pot} = 0 \, \text{k}\Omega \) and \( G_2 = 0.29 \) for \( R_{pot} = 5 \, \text{k}\Omega \). The parallel capacitances, visible in Fig. 4.8(b) and Fig. 4.8(c), increase the stability of the electrical circuit.
Knowing the responsivity $\rho$ (in Ampere per Watt) of the used photodiodes allows for a determination of the expected voltages out of the readout circuit. The optical power on each photodiode is first determined by squaring the amplitude of the electrical fields in Eq. (4.12). Then multiplying the power by $\rho$ and $R_{TIA}$ results into the expected voltage levels

$$V_1 = R_{TIA}\rho P_{in}T_{BS1}R_{BS1}\left[T_{BS1}^2 + R_{BS1}^2 + 2T_{BS2}R_{BS2}\cos\left(\frac{4\pi\Delta L}{\lambda}\right)\right],$$

$$V_2 = R_{TIA}\rho P_{in}T_{BS1}T_{BS2}R_{BS2}\left[2 - 2\cos\left(\frac{4\pi\Delta L}{\lambda}\right)\right]$$

out of the TIAs. Here, $T_{BS1} = t_{BS1}^2$, $R_{BS1} = r_{BS1}^2$, $T_{BS2} = t_{BS2}^2$, and $R_{BS2} = r_{BS2}^2$ represent the intensity transmission and reflection coefficients of BS1 and BS2, respectively, and $P_{in}$ the input power in Watt.

For perfect 50/50% beamsplitters, $T_{BS1} = R_{BS1} = T_{BS2} = R_{BS2} = 0.5$. The sign differences at the two cosines shows that, if $\Delta L$ is changing, the two signals will be out of phase by $180^\circ$. The output of the differential amplifier is

$$V_{\text{diff}} = G_1 V_1 - G_2 V_2$$

$$= R_{TIA}\rho P_{in}T_{BS1}\left[G_1 R_{BS1}\left[T_{BS2}^2 + R_{BS2}^2 + 2T_{BS2}R_{BS2}\cos\left(\frac{4\pi\Delta L}{\lambda}\right)\right]\right]$$

$$- G_2 T_{BS2}R_{BS2}\left[2 - 2\cos\left(\frac{4\pi\Delta L}{\lambda}\right)\right].$$

By choosing $G_2$ such that

$$G_2 = G_1 \frac{R_{BS1}(T_{BS2}^2 + R_{BS2}^2)}{2T_{BS2}R_{BS2}},$$

Eq. (4.16), in the practical case in which $G_1 = 1$ and $T_{BS2} = R_{BS2} = 1/2$, reduces to

$$V_{\text{diff}} = R_{TIA} \frac{\rho P_{in}}{4} \cos\left(\frac{4\pi\Delta L}{\lambda}\right).$$

The null condition in this equation, i.e. $\Delta L_0 = (n + 1/2)\lambda/4$, around which the system is locked by the feedback loop, also corresponds to the maximum response to a displacement of the proof mass, according to

$$\left.\frac{\partial V_{\text{diff}}}{\partial \Delta L}\right|_{\Delta L_0} = -R_{TIA} \frac{\rho P_{in}}{2} \frac{\pi}{\lambda} \sin\left(\frac{4\pi\Delta L}{\lambda}\right)\right|_{(n+1/2)\lambda/4} = (-1)^n R_{TIA} \frac{\rho \pi P_{in}}{2\lambda}.$$ 

The other advantage of this readout configuration is that the correlated noise, mostly due to the laser power fluctuations, between the two photodetectors is suppressed at first order.

## 4.2.2 Noise budget

A noise budget of the prototype accelerometer was made based on the opto-mechanical parameters listed in Table 4.1. Besides the suspension Brownian noise, that was
discussed in section 4.1.2, the accelerometer performance is expected to be limited by
the resolution of the interferometric position sensor, the noise of which will show two
dominant contributions: the shot noise and the residual intensity noise (RIN) of the light
source. Fig. 4.9 shows that the noise contribution from the readout electronics is
negligible.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
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<tr>
<td>Proof mass</td>
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<td>kg</td>
</tr>
<tr>
<td>Leg mass</td>
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<td>g</td>
</tr>
<tr>
<td>Leg length</td>
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<td>cm</td>
</tr>
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<td>Natural frequency</td>
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<td>Hz</td>
</tr>
<tr>
<td>Quality factor</td>
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<td>-</td>
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<tr>
<td>Frequency noise [155]</td>
<td>500 \cdot f^{-1/2}</td>
<td>Hz / \sqrt{Hz}</td>
</tr>
<tr>
<td>Static differential arm length</td>
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<td>mm</td>
</tr>
<tr>
<td>Injected power</td>
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<td>mW</td>
</tr>
<tr>
<td>Wavelength</td>
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<td>nm</td>
</tr>
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<td>nV/\sqrt{Hz}</td>
</tr>
<tr>
<td>Opamp current noise</td>
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<td>fA/\sqrt{Hz}</td>
</tr>
<tr>
<td>Feedback resistor</td>
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</tr>
<tr>
<td>Diode responsivity</td>
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<td>A/W</td>
</tr>
<tr>
<td>Diode dark current</td>
<td>50</td>
<td>nA</td>
</tr>
</tbody>
</table>

Table 4.1: Optomechanical and readout electronics parameters for the prototype
accelerometer. The modeled laser source is The Rock\textsuperscript{TM} from NP Photonics, the opamp
used in the transimpedance amplifier is the OPA827 and the photodiodes used are
Thorlabs FGA21.

The shot noise for this small interferometer is similar to what is presented in
section 2.1.2. Knowing the power of the light falling on a photodiode, the amount of
photons can be deduced and the shot current noise is

\[ i_{sn} = \sqrt{2eI_{PD}} = \sqrt{2e\rho P_{PD}}, \]

(4.20)

where \( e \) represents the elementary charge \( 1.602 \times 10^{-19} \) C.

RIN is excess noise on the shot noise caused by the light source used. It is usually
expressed in dB/Hz and specified by the laser manufacturer. Typically, for solid state
lasers the RIN spectrum can be roughly expressed as

\[ i_{RIN} = i_{sn} \sqrt{\frac{f_c}{f}} + 1, \]

(4.21)

in which \( f_c \) represents the corner frequency above which the light source intensity
fluctuations are just shot noise limited (corresponding to the RIN suppression ratio
quoted by the laser manufacturer, in the frequency interval of interest). Thanks to the
differential configuration of the interferometer the effective $f_c$ can be pushed to low frequency, possibly resulting in a shot noise limited position measurement in the frequency band of interest. The effective value of $f_c$ can be determined experimentally.

![Figure 4.9: Minimum detectable inertial motion for a viscously damped accelerometer with interferometric readout as in Fig. 4.6. In this noise budget the suspension natural frequency of the accelerometer was assumed to be 0.45 Hz.](image)

Laser frequency noise can impact the total noise budget. The frequency noise in Hz/√Hz is given by the laser manufacturer and typically has a $1/\sqrt{f}$ behavior. It translates to displacement as

$$x_f = \frac{\nu_L}{\nu_0} \Delta L_0,$$

where $\nu_L$ represents the laser frequency noise, $\nu_0 = c/\lambda$ the central laser frequency and $\Delta L_0$ the static arm length difference.

### 4.2.3 Sensor characterisation

A first characterization of the interferometric readout system was made by using a red laser diode Thorlabs LPS-675-FC with 1 mW output power. The fiber laser output was coupled to the free-space interferometer setup by means of a collimator lens. The beam diameter for this particular collimator is about 5 mm. The used beamsplitters, Thorlabs BS004, are held in place by screwed down PEEK holders which also serve as photodiode (Hamamatsu S1223-1) holders to ensure their reproducible position. The choice was made to replace the flat mirrors with corner reflectors in order to simplify the (manual) alignment process.

To determine important characteristics of the interferometer and calibrate the sensor, the proof mass is mechanically blocked. In this way, one of the mirrors is in a fixed position, while the piezo allows for known, controlled movement of the other mirror. By
applying a linear driving signal to the piezo, fringes appear in the signals of the photodiodes. By electrically balancing the output of the two photodiodes (see section 4.2.1), the differential signal appears as a sinusoid crossing zero at the point where its first derivative is maximal, ensuring the highest possible linearity and sensitivity in the response. This is the operation point of the sensor when the feedback loop is closed.

In Fig. 4.10 the interference fringes on the outputs of the interferometric readout are shown. The difference in amplitude of both signals is not exactly two as discussed earlier, but by means of the circuit with potentiometer the differential signal is made to be symmetric around zero. This is important as the lock point of a controller is preferably set at 0 V. The fringe visibility or contrast, defined as $100\% \times \frac{(\text{max} - \text{min})}{(\text{max} + \text{min})}$, for PD1 and PD2 is 79% and 80%, respectively.

Zooming in on the differential signal and the ramp signal sent to the piezo actuator in Fig. 4.11, a Volt-to-meter conversion factor can be determined for this actuator. The horizontal axis is determined by taking the laser wavelength as the reference, e.g. the distance between a minimum and a maximum in the fringe pattern corresponds to a displacement of $\lambda/4$ of the actuated mirror. Reading off from Fig. 4.11,

$$c_{\text{piezo}} = \frac{\lambda/4}{\Delta V_{\text{ramp}}} = \frac{168.75 \times 10^{-9}}{27.2} = 6.20 \text{ nm/V}.$$ \hspace{1cm} (4.23)

The point on the fringe at which this working point of the control is operated is the maximum slope determined earlier in Eq. (4.19). Using Eq. (4.23) and Fig. 4.12, a similar conversion factor can be obtained for the differential signal. The Volt-per-Volt conversion
Figure 4.11: Calibration of the piezo actuator: the actuator is calibrated by measuring the driving voltage needed to displace the mirror by a quarter wavelength. Horizontal axis units and ramp signal are off set and scaled, respectively, for visibility reasons.

factor of a signal applied to the piezo to the signal coming out of the differential port can be determined. Using this gain and the previously obtained $c_{\text{piezo}}$, the $c_{\text{diff}}$ coefficient can be obtained.

Figure 4.12: Determination of the interferometer gain: by zooming in on the linear part of the sinusoid, the change in differential signal one expects when applying some voltage to the piezo can be determined. A Volt-per-meter conversion factor for the differential channel can subsequently be calculated. Horizontal axis unit is off set and ramp signal is scaled for visibility reasons.
The noise levels of the different outputs can be determined and converted to m/√Hz. To get a first grasp on what is possible with this sensor, the interference and thus control is eliminated by blocking one arm by placing a black anodized aluminum surface in front of the proof mass corner reflector.

Firstly, the PDs and readout circuit are placed in a light-tight bag to measure the electronic noise of the system. The light out of one interferometer arm is distributed over the two PDs and the subtraction is tuned by the potentiometer in the differential amplifier such that its output is zero. The spectra are measured by using an Agilent 35670A signal analyzer of which the self noise, when terminating the input BNC port with a 50 Ω resistor, was measured to be 30 nV/√Hz. The traces shown in Fig. 4.13 are fused traces of two measurements. One measurement is from 62.5 mHz to 100 Hz and the other measurement is from 1 Hz to 1.6 kHz and they are fused at 100 Hz. Using $c_{\text{diff}}$ from Eq. (4.24), the measured V/√Hz trace is converted into Fig. 4.13.

**Figure 4.13:** Displacement amplitude spectral density in m/√Hz in the different channels of the interferometric readout. As one interferometer arm was blocked, input laser power was doubled to ensure typical voltages out of the channels.

Fig. 4.13 shows that the fm/√Hz regime is in reach if the interferometric readout can be made shot noise limited. Subtraction of intensity noise improves noise levels by two orders of magnitude. The LPS-675-FC Thorlabs laser diode diode has a multi-mode spectrum with an overall emission bandwidth of a large fraction of one nanometer, corresponding to a coherence length of a few hundreds of micrometers. For this reason the fringe contrast, and therefore the sensor response, is expected to be dependent on

\[ G_{\text{IFO}} = \frac{\Delta V_{\text{diff}}}{\Delta V_{\text{ramp}}} = \frac{2.65}{7.64} = 0.347 \text{ V/V,} \]
\[ c_{\text{diff}} = \frac{c_{\text{piezo}}}{G_{\text{IFO}}} = \frac{6.20 \times 10^{-9}}{0.347} = 17.87 \text{ nm/V.} \]
the length difference between the two arms of the interferometer. This was confirmed by the test that was made by scanning the interferometer output as a function of the accelerometer proof mass position. In Fig. 4.14(a) the side lobes, observable around the main one, are due to the multi-mode laser emission and correspond to the beats between the different active modes.

Figure 4.14: Fringe visibility development when changing differential arm lengths by hundreds of wavelengths: (a) putting the accelerometer on an incline and applying a ramp signal to the voice coil to make the mass slide through the fringes and (b) the center envelope of the PD2 signal normalized to maximum fringe visibility to determine the emission bandwidth of the used light source.

Figure 4.15: Readout signals of the interferometer when using the 1550 nm light source and applying a linear driving signal to the piezo.

In Fig. 4.14(b), by measuring the differential arm length change that causes the fringe
visibility in the main lobe to drop to half of its maximum level, i.e. 0.5 of the normalized value, the emission bandwidth can be estimated. In this case the differential arm length span is $\Lambda_c \approx 640 \mu m$ which corresponds to

$$\Delta \lambda \approx 0.44 \frac{\lambda^2}{\Lambda_c} \approx 0.3 \text{ nm.}$$  (4.25)

This wide linewidth, which corresponds to about 500 GHz, is expected to swamp the readout noise budget with frequency noise, as described in the previous section. Due to all the described limitations, the LPS-675 source was replaced with single mode, narrow linewidth fiber laser The Rock™ from NP Photonics operating at $\lambda = 1550$ nm. This laser delivers a 0.5 mm diameter collimated beam with a coherence length of a few hundreds of kilometers thanks to the very narrow emission linewidth (about 700 Hz). After replacing the laser, the calibration as described above was repeated. Fig. 4.15 is an example of a result of a similar calibration campaign.

The conversion factor for the piezo determined by this measurement is $c_{\text{piezo}} = 8.20 \text{ nm/V}$, which is almost similar to the previously obtained $6.20 \text{ nm/V}$. Such discrepancies are usual when using piezoelectric ceramics, as they are notorious for creep, hysteresis and other (aging) phenomena. The interferometer gain around the lock point is also determined to be $G_{\text{IFO}} = 0.381 \text{ V/V}$, making the differential signal conversion factor $c_{\text{diff}} = 21.52 \text{ nm/V}$. The same measurement as described by Fig. 4.13 was performed and similar results were obtained for intensity noise subtraction without interference.

### 4.3 Increasing the dynamic range

The sensor with readout as described above cannot be used in open loop as the proof mass motion excited by the ambient seismic noise is usually far larger than the linear range of the interferometer, i.e. about $\lambda/4$. For this reason a feedback system, acting on the reference mirror or on the proof mass itself, is implemented to enhance the dynamic range of the sensor.

#### 4.3.1 Using a piezo as actuator

Fig. 4.16 shows two different feedback strategies, one of which is the loop that uses a piezo actuated mirror to lock. The interferometer and readout circuit monitor the differential arm length change. The servo/controller monitors deviations from the lock point and sends a correction signal to the HV amplifier. The piezo is of type HPCh 150/12-6/2 by Piezomechanik and has an input range of -30 V to +150 V. The low pass filter (LPF) is used to lower the gain for higher frequencies, where typically the piezo resonance frequency is located (usually in the tens of kHz regime for piezos of this size), and keep the loop stable.
Figure 4.16: Accelerometer with interferometric optical readout. The position of the proof mass is probed by a differentially read out interferometer. The difference between the two output signals is kept null by a feedback loop. A piezo actuated mirror is part of the interferometric readout and an auxiliary voice coil actuator is located on the other side of the sensor. Both can be used for calibration or as an actuator. The piezo actuated mirror of the reference arm can be made to follow the proof mass mirror. The feedback voltage driving the piezo actuator can be taken as the position sensor output. An alternative feedback loop (V) uses the voice coil as an actuator. It keeps the mass at a fixed position with respect to the frame and the signal it needs to do that can be used as sensor output. Adapted from Ref. [156].

The block scheme in Fig. 4.17(a) shows the model of gain and frequency response of the different elements of the feedback loop. In Fig. 4.17(b), these responses are shown for the four different elements, where unconventional units are chosen to have all traces roughly at the same level for plotting reasons. The amplifier and LPF trace starts at a linear response of 20, because of the factor 20 amplification of the HVA, and rolls off after the cut-off frequency of the LPF, which is at 25 Hz. The response of the piezo is at the level determined by the two prior measurements; it is set for this model at 7 nm/V and becomes only larger near the piezo resonance, which is at 40 kHz here. The magnitude of the interferometer trace is $1/\text{c_{diff}}$. The controller has a gain of 10 and an integrator with a cut-off frequency around 1 Hz, which gives the higher gain for lower frequencies.

In the model, the unity gain frequency of the control is around 2 kHz while the closed loop response is flat to frequencies up to 500 Hz. The measured open and closed loop response of the piezo locked interferometer readout are shown in Fig. 4.18, where the traces are acquired by swept sine injection. The unity gain frequency is around 350 Hz with a phase margin of more than 80°. The piezo resonance is, unlike the example loop model of Fig. 4.17, clearly visible at 16 kHz. A peak-notch feature is visible between 1 kHz and 2 kHz, which may be caused by an internal mode of the frame or pendulum legs.
As the final check of the readout performance, a test was made, in vacuum on the MultiSAS suspended platform, with the accelerometer mechanics blocked and the piezo loop closed. The result of $4 \text{ fm/} \sqrt{\text{Hz}}$ from 5 Hz onwards is shown in Fig. 4.19. The peaks at 0.7 Hz and 1.8 Hz correspond to the open-loop bench horizontal modes, which make the proof mass move ever so slightly. The expected shot noise level is calculated from the power deduced from the measured voltages on the two photodiode channels.

During the measurement, monitoring the differential channel showed second-long 25 mV deviations from the 0 V level. This results in intensity noise not optimally subtracted anymore. Inspecting $c_{\text{diff}}$ determined at the end of section 4.2.3, 50 mV deviations correspond to a 1 nm displacement from the lock point. Simulating the intensity noise on PD2 with a flat $10 \mu \text{V/} \sqrt{\text{Hz}}$ level, introducing such a shift from lock point results in a $0.25 \mu \text{V/} \sqrt{\text{Hz}}$ level. This is higher than the calculated shot noise limited value.
Figure 4.18: Measured open and closed loop responses of the piezo locked interferometer readout with an unity gain frequency or bandwidth upper limit of about 350 Hz.

Figure 4.19: Displacement ASD traces of piezo and interferometer readout channels when the piezo loop is locked with the accelerometer mechanics blocked. This measurement was performed on the uncontrolled MultiSAS test facility bench and, despite the blocked mechanics, the pendulum modes are visible at 0.7 Hz and 1.8 Hz.

This noise contribution is cyclostationary, i.e. the $0.25 \mu V/\sqrt{Hz}$ is a maximum value when the signal is 1 nm away from the lock point, but most of the time it is less. These simulated results can be compared with the observed readout performance which is about a factor 2 worse than shot noise limit in this case. This corroborates the hypothesis that the suboptimal subtraction of the intensity noise due to the differential
signal deviation from 0 V results in the readout not achieving the shot noise limit.

### 4.3.2 Using a voice coil as actuator

The previously determined $c_{\text{piezo}}$, together with the input range of the piezoelectric actuator of -30 V to +150V, results in a sensor range of about 1.5 µm. Such a range is substantially smaller than the expected displacement of the freely swinging proof mass of the accelerometer. For this reason, the configuration with the tracking mirror is useful only for the characterization of the optical readout and for diagnostics. A more traditional approach, with the accelerometer mechanics controlled by means of the voice coil was used in the final implementation of the sensor.

![Flowchart of the loop when using the voice coil with all the noise sources that could influence the performance and the linear responses of all the elements in that loop.](image)

**Figure 4.20:** (a) Flowchart of the loop when using the voice coil with all the noise sources that could influence the performance and (b) the linear responses of all the elements in that loop.

The loop flowchart of voice coil (VC) locked sensor is shown in Fig. 4.20(a). The three different elements of the loop have responses shown in Fig. 4.20(b). The interferometer has again a flat response of $1/c_{\text{diff}}$ in V/m. The shown accelerometer force response
is the mass displacement per unit input voltage across the VC. The servo filter is a PID controller. The integrator in this model has a cut-off frequency of about 160 mHz, whereas the differentiator rolls on from about 70 Hz.

**Figure 4.21**: Voice coil locked loop design for the interferometrically read out monolithic accelerometer. The low frequency level of the transfer function is determined by the high gain of the interferometric readout in combination with the relatively weak actuator. The unity gain function of the loop is 200 Hz, but the structures around 100 Hz associated with modes of the pendulum legs determine the actual usable upper bandwidth limit.

### 4.3.3 Noise measurement in the MultiSAS test facility

The sensor is put on the seismically isolated optical table suspended by the MultiSAS in the test facility at Nikhef, as shown in Fig. 4.22. The laser, because of its heat production and vacuum non-compatibility, is outside the vacuum tank and a polarization maintaining (PM) fiber runs through a feedthrough and through the suspension to provide the beam to the interferometer.

A variable optical attenuator and fiber polarization controller are used to adjust the light intensity and polarization to maximize the fringe visibility. Fringe visibilities of more than 95% are achieved with this set-up on the suspended bench. The measurement on the MultiSAS bench motion is performed in vacuum. The pressure during the measurement was below $10^{-4}$ mbar and translational and angular loops were closed. The sensor is expected to hit its $f_m/\sqrt{Hz}$ self-noise floor above 20 Hz. Results of this measurement are shown in Fig. 4.23. From 30 Hz onwards, a noise floor of $8 \, f_m/\sqrt{Hz}$ is observed. Compared to the state-of-the-art commercial sensor sensitivity curves of Fig 4.1, this result is more sensitive by a factor of ten at 30 Hz.
The results lie above the noise budget of Fig. 4.9 below 30 Hz, which could be due to a lower $Q$ than expected. Viscously damped suspension thermal noise due to eddy currents in the actuator is the only noise source in the budget that could have a $1/f^2$ slope as the one observed between 10 and 30 Hz. From 4 Hz to 10 Hz a $1/f^3$ slope is observed; the source for this slope is unknown. A possible frequency noise that is higher than expected (the quoted laser manufacturer noise is done in an idealized set-up) or the differential arm length being more than 1 mm could also increase the noise around 20 Hz.

The structures observed around 25 Hz and 40 Hz and the line at 30 Hz are not associated with any MultiSAS mode. The line at 30 Hz is the scroll pump of the vacuum system. The structures are not expected to be actual bench motion because MultiSAS isolation performance is expected to result in the grey dashed line. Inspecting the red line of Fig. 3.9, this line is expected to follow this slope up until about 20 Hz. This expected performance is possibly slightly deteriorated by MiniTower modes, but not enough to explain displacements levels observed in the structures. Additionally, as some of the structures are also visible in the L4C with different displacement ASD magnitude, some unknown electromagnetic coupling to the L4C geophone coil and Nikhef accelerometer actuator coil seems more likely.

To measure the $Q$ of the proof mass suspension in vacuum, the interferometric readout was replaced by a simple flag. This flag is used as part of a shadow sensor set-up, similar to the OSEM sensor shown in Fig. 1.24(b). The $Q$ measurement was done by injecting a burst of current in the voice coil actuator and observing the ring-down of the proof mass motion as presented in Fig. 4.24. A quality factor of about 40 is estimated from the measurement. Although the mechanics and mass of the proof mass system was altered slightly to accommodate the flag instead of the corner reflector (part of the interferometric
Figure 4.23: A measurement on the MultiSAS bench by the Nikhef accelerometer and an L4C installed next to it. These measurements are compared to the specifications of the L4C, the LIGO/GeoTech GS13 and the goal or total noise budget projection of the Nikhef accelerometer. The L4C and accelerometer both measure the (damped) 1.8 Hz differential pendulum mode of the suspension. The L4C hits its self-noise at about 4 Hz. An 8 fm/√Hz noise level is observed from 30 Hz onwards for the Nikhef accelerometer. Added in text are the modeled dominant noise sources from the noise budget. The dashed black line is the suspension thermal noise for Q=40 and the dashed grey line is the approximate ground spectrum multiplied by the expected MultiSAS isolation performance.

From 30 Hz onwards, the observed 8 fm/√Hz is a factor two higher than the performance observed in Fig. 4.19 for the piezo locked interferometric readout. The differential signal is suspected to venture off more from the 0 V value resulting in less optimal subtraction of the intensity noise in the two photodiode signals. A measurement of the raw signal sent to the voice coil was performed, where the feedback loop setpoint was varied as shown in Fig. 4.25. This was done such that the differential signal was not at 0 V (more details in the caption).

The difference between this result of 8 fm/√Hz, the piezo-locked result of 4 fm/√Hz and the modeled level of only a few fm/√Hz is most probably related to this effect of deterioration of intensity noise subtraction performance due to residual proof mass motion. This residual motion also results in the the deviation of the differential signal
Figure 4.24: $Q$ measurement of monolithic accelerometer proof mass suspension. A shadow sensor measures the position of the proof mass, where its output voltage is proportional to displacement. Non-linear effects due to the rectangular shape of the flag cutting a circular beam are visible in the 0 V to 1.5 V and 8 V to 10.5 V region.

from the nominal value of 0 V. In other words, an increased sensed motion decreases sensitivity for this type of interferometric readout, where the subtraction of intensity noise is dependent on the sensor feedback loop ability to keep the differential signal as close to 0 V as possible.

Figure 4.25: Intentionally spoiled intensity noise subtraction for the interferometric readout. The setpoint for the differential signal to lock on is changed by 1V, 2V or 3 V (resulting in a proof mass displacement from the nominal of 21.5 nm, 43 nm or 64.5 nm, i.e. multiplied by $c_{diff}$) resulting in deteriorated intensity noise subtraction. The raw signal sent to the voice coil is proportional to acceleration.

Another possible unmodeled noise source is polarization noise by stress-induced birefringence due to fiber vibrations. The laser light passes through a feedthrough which feels Earth’s vibration. The PM fiber in the vacuum vessel picks up vibrations during its path through to suspension to the optical bench. Coherence measurements have been
performed between Wilcoxon 731A piezo accelerometers and the differential interferometer output. No significant coherence levels were observed from 4 Hz onwards. However, injection of a vibration line at high frequency (> 50 Hz) by a piezo on the flange or near the laser did result in a line in the sensor output, where direct mechanical coupling of this small vibration is expected to be filtered by MultiSAS.

Despite several unresolved noise contributions in the measured Nikhef accelerometer performance, the novel sensor shows unprecedented displacement measurement performance between about 8 Hz and 100 Hz. A similar optical interferometric readout was fabricated at Nikhef in optical fiber for performance in high magnetic field, high radiation environments such as particle colliders. Preliminary results of this project are summarized in Appendix B.
Control of KAGRA suspension prototypes

Apart from the Advanced LIGO and Virgo detectors, a fourth detector with comparable size and design performance is being built in Japan. Fig. 5.1 shows an artist impression of the mountain, which houses the caverns where KAGRA is being realized. In other caverns of this underground complex a number of other physics experiments are
housed, such as the neutrino observatory Super-Kamiokande, the dark matter liquid Xenon experiment XMASS and a predecessor of KAGRA named CLIO, which pioneered in cryogenics research for GW detectors.

The low seismic noise environment as well as the existing science infrastructure made the choice to build another experiment in the Kamioka mines a logical one. KAGRA, previously coined Large-scale Cryogenic Gravitational wave Telescope (LCGT), was approved on June 22, 2010. KAGRA is designed, built and will be operated by scientists from the Institute for Cosmic Ray Research (ICRR, Kashiwa, Tokyo), the National Astronomical Observatory of Japan (NAOJ, Mitaka, Tokyo) and the High Energy Accelerator Research Organization (KEK, Tsukuba, Ibaraki). In terms of size, the detector will feature 3 km long interferometer arms, which is comparable to the LIGO and Virgo detectors. KAGRA is built in an underground facility and will make use of cryogenics to lower the temperature of the four test masses to about 20 K to reduce suspension and mirror coating thermal noise.

**Figure 5.2:** Simplified optical scheme employed during the iKAGRA run that took place in March and April 2016. Most optics use existing suspensions from CLIO (beamsplitter, BS) and TAMA300 (end test masses, ETMX and ETMY), except for the power recycling 3 (PR3) suspension and the input mode cleaner stack suspensions. The input mode cleaner is the triangular optical cavity featuring the Mode Cleaner input (MCi), end (MCo) and output (MCe) mirrors. The GW signal is read out by using a Faraday Isolator (IFI) at the laser side to divert the beam to the photodiode (PD) and this signal is also used for end mirror control to lock the interferometer. The beam going to the (usual) detection port is discarded by a beam dump. Reproduced from Ref. [157].

The initial phase iKAGRA was operational in 2016 from March 25 9:00 to March 31 17:00 (JST) and, after a quick commissioning break, April 11 9:00 to April 25 17:00 (JST). The iKAGRA experiment, featuring the optical set-up depicted in Fig. 5.2, was performed to confirm the layout of the vacuum tanks, test the digital control system, data acquisition,
data transfer and data management, get environmental data and obtain experience in managing and operating a kilometer class interferometer [157]. In both iKAGRA runs, a duty cycle of about 90% was achieved.

Figure 5.3: Strain sensitivity curve (blue) obtained in the second iKAGRA run and the noise sources that are (expected to be) dominant. The peaks at 80 Hz and 113 Hz in the measured spectrum are from the calibration lines. Acoustic noise plotted here only shows the acoustic noise coupled via the beam splitter chamber, but it is likely that the acoustic noise is the sensitivity limiting noise source also in the neighboring frequency regions. Actuator noise is the sum of the displacement of the mirrors from DAC noise. Seismic noise is the ground displacement attenuated by the mirror suspensions. Sensor noise is the sum of the ADC noise, the dark noise of the photodiode, and the shot noise. The best sensitivity curve of iKAGRA corresponds to a BNS range of 3.21 pc. Adapted from Ref. [157].

Displayed in Fig. 5.3 is the best iKAGRA strain sensitivity curve featuring a level of about $5 \times 10^{-17} \, 1/\sqrt{\text{Hz}}$ at 4 kHz. The intended goals of this run were achieved and valuable experience for the Japanese GW community was gained. After the iKAGRA run, further installation towards the bKAGRA phase, i.e. baseline KAGRA, commenced the full dual recycled cryogenic Fabry-Perot interferometer.

5.1 KAGRA vibration isolation

KAGRA shares with Virgo, the final part of the LIGO suspension, and the earlier Japanese TAMA300 the philosophy that passive isolation chains, each suspending one
of the main optical components or auxiliary (optical) sensors, leads to the required performance. Seismic attenuation systems of different parts of the detector have different requirements in terms of attenuation. This results in various designs for the different components to be isolated from seismic noise. The KAGRA design features four different suspensions: Type A, Type B, Type Bp and Type C.

5.1.1 Suspensions overview

![Figure 5.4](image)

Figure 5.4: (a) Overview of the locations and function of the different vibration isolation systems for KAGRA and (b) in-band requirements for the different suspended optics. Adapted from Ref. [88].

Fig. 5.4 shows spectral requirements for suspending the main optical components of KAGRA. The velocity rms requirements are $0.5 \mu m/s$ for Type A and Type B suspensions and $2 \mu m/s$ for Type Bp suspensions [88].

The test masses are suspended by a Type A suspension, which is a 13 m long suspension chain with the final payload in a cryogenic environment. Type B and Type Bp suspensions will suspend the signal and power recycling folded cavity mirrors, respectively. Type B suspension is also used for the beam splitter. Type Bp suspension is a reduced version of Type B suspension. Type C suspensions, with a design similar to the TAMA300 stacks [158], are used, for instance, for the input mode cleaner optics, i.e. MCi, MCe and MCo in Fig. 5.2.

After iKAGRA ended in April 2016, installation and commissioning of bKAGRA systems started. The road to bKAGRA will follow a phased approach. Starting from 2018 a power recycled Michelson interferometer will be operated, in which the two end test masses will be suspended with Type A vibration isolators and cooled down to 20 K. Then, after a short science run, installation and commissioning of the full dual recycled cryogenic
Fabry-Perot Michelson interferometer will take place. The goal is reaching the design sensitivity in 2021 [157].

### 5.1.2 Type A and type B(p) suspensions

The suspensions for KAGRA share some similarities with the Virgo superattenuator and MultiSAS. All relevant specifications for a comparison of the different suspensions are summarized in Table 5.1. The suspension specifications do exclude the payload, that is marionetta and test mass for the superattenuator, the optical bench for the MultiSAS, and the so-called intermediate mass and the mirror for the KAGRA systems.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Superattenuator</th>
<th>MultiSAS</th>
<th>Type A</th>
<th>Type B</th>
<th>Type Bp</th>
</tr>
</thead>
<tbody>
<tr>
<td>IP (length in m)</td>
<td>6 m</td>
<td>1 m</td>
<td>1 m</td>
<td>1 m</td>
<td>no IP</td>
</tr>
<tr>
<td>Vertical filters (§)</td>
<td>MAS (6)</td>
<td>GAS (2)</td>
<td>GAS (6)</td>
<td>GAS (3)</td>
<td>GAS (2)</td>
</tr>
<tr>
<td>Position sensor</td>
<td>LVDT</td>
<td>LVDT</td>
<td>LVDT</td>
<td>LVDT</td>
<td>LVDT</td>
</tr>
<tr>
<td>Inertial sensor</td>
<td>Accelerometer</td>
<td>L4C</td>
<td>L4C</td>
<td>L4C</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.1: Relevant specifications for the different Virgo and KAGRA suspensions. MAS represents Magnetic Anti-Spring used in the Virgo suspension and GAS the Geometric Anti-Spring similar to those used in MultiSAS.

The conceptual designs of the three main KAGRA suspensions are shown in Fig. 5.5. Type A suspensions provide 8 stages of horizontal attenuation and 6 stages of vertical attenuation for the test masses. Taking advantage of the underground environment, the top stage (inverted pendulum and top vertical filter) of each Type A suspension is hosted inside an auxiliary tunnel located above the caverns where the large cryostat for the test mass is installed. The short inverted pendulum legs mitigate the issues Virgo has with low frequency internal leg modes and tilt coupling. The attenuation chain resides in a 1.2-m diameter shaft connecting the tunnel with the cavern.

Type B suspension has a smaller number of attenuation stages and a room temperature payload. The base of the suspension system is elevated from the floor and sitting on top of a support structure external to the vacuum chamber. Ideally the support frame should simply transfer the ground motion to the suspension system. In practice the support frame is not perfectly rigid and its resonances amplify the seismic noise thus enhancing the vibrations injected into the suspension, also in the interferometer detection band. This is similar to the MiniTower modes discussed in section 3.4.1. As the Type A system, the Type B suspension features an inverted pendulum stage for pre-isolation and static positioning of the suspension chain. Three stages of GAS filters provide vertical seismic attenuation. The optical element is suspended from the stage called intermediate mass by means of two wire loops. The function of the intermediate mass is similar to the superattenuator marionette, allowing to steer the suspended optical element in yaw, pitch and roll.
Although the power recycling mirrors were meant to employ the Type B suspension in the preliminary design of the KAGRA detector, their suspension systems were reduced due to budgetary constrains. The Type Bp system does not contain the pre-isolation stage with an inverted pendulum and has two stages of GAS filters for vertical isolation. In order to horizontally position the suspended optic, a motorized stage called the traverser is implemented on the top of the chain. The traverser is a frame with a GAS filter that can be moved with micrometer precision in any position in the horizontal plane by stepper motors. This system however does not provide any isolation for the microseismic peak and thus the suspended optics are expected to suffer from larger rms motion.

**Figure 5.5:** Conceptual designs of vibration isolation systems for the KAGRA detector. The vacuum envelopes are not shown for visibility reason. IP represent the inverted pendulum stages for Type A and Type B. F0-F5 denote the GAS filters for vertical seismic attenuation. MD represents the magnetic damper which is placed just above F1 and aims for damping the torsion modes of the attenuation chain. IM represents the intermediate mass, from which the optic is hung with suspension wires. Reproduced from Ref. [88].

### 5.2 Inverted pendulum stage control

In Type A and Type B suspension systems, the pre-isolator stage is responsible for seismic attenuation starting from below the microseismic peak, static positioning of the suspension point and yaw orientation of the chain. The stage consists of a GAS filter (Filter 0) supported by three inverted pendulum legs. Filter 0 has a diameter nearly
twice as large compared to the standard GAS filters downward in the chain and it can be tuned to a natural frequency lower than 100 mHz. All inverted pendulum stages for KAGRA were designed, assembled and tested at Nikhef.

First tests on pre-isolator stage control for the Type B prototype were done at NAOJ [88]. For error signals on the inverted pendulum stage, blending of LVDT signals for absolute positioning and Sercel L4C geophones for inertial damping is applied. This is similar to the MultiSAS control strategy, but the residual noise spectral requirements for the KAGRA suspensions are about five to seven orders of magnitude more strict. L4C geophones typically start to be noisier than LVDTs below 100 mHz, so blending as far as possible below the microseismic peak around 200 mHz can result in geophone noise injection or tilt issues. For further testing, another pre-isolator test stage was placed next to the beam splitter vacuum tower at the KAGRA site in January 2016, as shown in Fig. 5.6.

![Figure 5.6: (a) Photograph of the inverted pendulum test stage in the beam splitter clean area and (b) a photograph of one of the three accelerometers on the top plate. These monolithic accelerometers were assembled at Nikhef and were also used for the research described in section 4.1.2.](image)

### 5.2.1 LVDT read out monolithic accelerometers

A monolithic accelerometer with an LVDT readout was used to alleviate possible issues regarding low frequency blending with L4C geophones. The accelerometer is shown in Fig. 5.6(b), and was also presented in Fig. 4.5. These accelerometers have better low-frequency performance than the L4Cs, which make them more suitable to blend below 100 mHz and to measure inertially the microseismic peak. The accelerometers that were used employ the same mechanics as the sensor described in Chapter 4. However instead of an interferometric readout, the sensing of the proof mass position is performed by an LVDT. As the LVDT has a much larger linear range than the interferometer, the accelerometer can be operated in open loop.
The mechanical response of each sensor was characterized by injecting a white noise current into the voice coil actuator built in each accelerometer. Results of such a measurement are shown in Fig. 5.7. By fitting the data with the theoretical response of a damped harmonic oscillator, the natural frequency $f_0$ and $Q$ of each sensor have been determined.

![Graph showing transfer functions](image)

**Figure 5.7:** Three accelerometer (ACC) transfer functions obtained by white noise injection at the coil magnet actuator. The relative signs of ACC2 and ACC3 are opposite to ACC1 as their phase is not zero below the resonance. The structures visible just below 9 Hz were associated with the optical table on which the calibration measurement was performed.

The L VDT displacement-to-volt conversion factor can be determined by reading out the effect of tilt on the DC voltage of the LVDTs. Applying a certain tilt will put a component $(mg \sin(\theta) \approx mgh\theta)$ of the gravitational pull on the mass along the axis of the accelerometer. This will ensure that the mass will move by

$$d_m = \frac{g\theta}{\omega_0^2}.$$  \hfill (5.1)

In this setup, the ultimate sensitivity of the LVDT read out monolithic accelerometer is expected to be limited by the ADC noise. A comparison between L4C geophone and the expected accelerometer performance is shown in Fig 5.8.

### 5.2.2 Pre-isolator stage simulation results

The calibrated accelerometers were placed on the beam splitter pre-isolator stage. Fig. 5.9 shows the positions of the different sensors on the top plate of the test set-up.
For the LVDTS, using the same method as given by Eq. (3.6), the geometric sensing matrix $S$ is

$$
\begin{bmatrix}
    x_{\text{IP}} \\
    y_{\text{IP}} \\
    \theta_{z,\text{IP}}
\end{bmatrix} =
\begin{bmatrix}
    -0.3333 & 0.6667 & -0.3333 \\
    0.5773 & 0 & -0.5773 \\
    0.6323 & 0.6323 & 0.6323
\end{bmatrix}
\begin{bmatrix}
    x_{\text{IP LDVT},0} \\
    x_{\text{IP LDVT},1} \\
    x_{\text{IP LDVT},2}
\end{bmatrix}. \tag{5.2}
$$

For the monolithic accelerometers, the positions and angles with respect to the Cartesian coordinates result in sensing matrix $S$

$$
\begin{bmatrix}
    x_{\text{IP}} \\
    y_{\text{IP}} \\
    \theta_{z,\text{IP}}
\end{bmatrix} =
\begin{bmatrix}
    -0.1725 & 0.6439 & -0.4717 \\
    0.5444 & 0.1992 & -0.7436 \\
    0.6944 & 0.6944 & 0.6944
\end{bmatrix}
\begin{bmatrix}
    x_{\text{IP ACC},0} \\
    x_{\text{IP ACC},1} \\
    x_{\text{IP ACC},2}
\end{bmatrix}. \tag{5.3}
$$

A type B suspension is used for the beam splitter. The simulated transfer function of a Type B suspension [159] is multiplied by the KAGRA high noise model to project the expected open loop motion of the top stage at the site, as shown in Fig. 5.10(a). To compare to the velocity rms requirements discussed earlier, all curves from Fig. 5.8 are converted to velocity.

Fig. 5.10(b) shows the simulated rms velocity of the inverted pendulum stage in closed-loop for different blending frequencies. No additional effect from the ground tilt is considered, and a simple PID control with 2 Hz unity gain frequency is assumed. The rms velocity requirement, i.e. 0.5 $\mu$m/s, is met for blending frequencies lower than 80 mHz.
Without tilt effects on the horizontal accelerometers, blending at the lowest frequency practically possible would result in better performance. The effect of tilt on an accelerometer, where the suspended mass feels (a component of) gravity in its degree of freedom when the top stage plate undergoes tilt, can occur for two reasons. First, the cradle effect described in section 3.2.3 can result in horizontal-to-tilt coupling. Inverted pendulum leg parallelism measurements, such as displayed in Fig. 3.11, should be made for Type A and B inverted pendulum stages to assess that $f_{\text{trust}}$ is low enough to allow low frequency blending.

Assuming that this is the case, ground tilt is not filtered out by an inverted pendulum stage and its spectrum couples to apparent horizontal motion in the inertial sensors. Sekiguchi showed [88] that the L4C geophone blended with the LVDT at 10 mHz resulted in worse velocity rms results than 50 mHz blending, most probably because of a combination of tilt induced apparent horizontal motion and the relatively high self-noise of the geophones.

So far, a tilt measurement at the KAGRA site has not been performed. From experience with suspensions in other GW detectors, tilt is a major issue when weather conditions are bad and the microseismic peak is high [110]. The strategy at LIGO to mitigate these effects is deploying tilt sensors near suspensions of core optics. Sensors that can measure the ground rotations about the horizontal axes are needed so that this signal can be removed in real time from the horizontal seismometers. If the tilt sensor is
Figure 5.10: Results of a velocity simulation for sensor blending of a KAGRA Type B inverted pendulum stage: (a) Top stage velocity compared to sensor noise. The open-loop top stage motion has been estimated by multiplying the ground spectrum by the Type B suspension modeled transfer function. The inverted pendulum fundamental mode is at 80 mHz, while the horizontal modes of the chain are visible between 0.3 Hz and 1.2 Hz. Panel (b) shows the rms velocity spectrum calculated in closed loop for different blending frequencies. Lower blending essentially reduces ground motion coupling. These results are in the absence of tilt.

sufficiently sensitive, only the true ground translation is used to control the isolation system [160]. This requirement resulted in the development of a high-precision mechanical absolute-rotation sensor [161]. In Virgo, alternative strategies to reduce the impact of microseismic peak noise and associated ground tilt effects based on a global control of the core optics are implemented [162].

5.3 Room temperature payload prototype

Type B and Bp suspensions are used for all room temperature payloads suspending the beam splitter and all signal and power recycling mirrors. Each payload includes a GAS filter (Bottom Filter), an intermediate mass and the suspended optical element, both of them with their own recoil masses. In this section methods and results of the characterization of the room temperature payload prototypes carried out at NAOJ are presented.

The payload (bottom filter not shown here) is shown in Fig. 5.11. All parts of the payload have a different function. The bottom filter has the function of providing [163] independent suspension of the intermediate mass and the intermediate-mass recoil.
mass. Additionally it provides fine pitch, yaw and vertical relative positioning of the (rigidly fixed) intermediate mass recoil mass with respect to the intermediate mass and additional attenuation in the GW detection frequency band.

Figure 5.11: Payload design below the bottom filter. The intermediate recoil mass (IRM) is attached to the bottom filter body by three wires, while the intermediate mass (IM) is suspended from the bottom filter with a single wire. Suspended from the IM are the optic (TM) and its recoil mass (RM). Reproduced from Ref. [164]

The intermediate mass functions are to provide independent suspension of the mirror and the mirror recoil mass and static pitch and roll positioning of the mirror. The intermediate-mass recoil-mass functions are to carry the dynamic actuators and position sensor acting on the intermediate mass, to provide dynamic control forces on the intermediate mass in all six degrees of freedom by means of strong collocated sensor/actuator pairs (OSEMs), and to reduce the control authority on the mirror actuators. The mirror recoil mass primary function is to carry the dynamic position sensors and actuators acting on the optic. The positions of the OSEMs on the test mass and intermediate mass levels is shown in Fig. 5.12.

For the payload prototype tests, a dummy load was used as a test mass. The (dummy) optic and recoil mass are each suspended by two loops of metal wires. The wires for the test mass are 0.2 mm in diameter and made of high-carbon steel (piano-wire). The wires for the recoil mass are 0.6 mm in diameter and made of tungsten. The length of the wires is about 580 mm. The two loops of wires are separated by a distance of 10 mm in the optic suspension and 20 mm in the recoil mass suspension. This determines the pitch
The locations of the OSEMs on the optic (TM) and intermediate mass (IM) subsystems used to obtain the sensing matrix $S$ for the geometrically reconstructed Cartesian coordinate signals. The OSEM body is attached to the recoil masses of both the optic and intermediate mass, while the flag is attached to the optic or intermediate mass itself.

stiffness of both bodies. The suspension wires are clamped onto the intermediate mass at the same height as its nominal center of gravity.

The sensor and actuator unit used in the payload prototype for type B(p) suspensions is the OSEM as described in section 1.5.2. The OSEM was first made for initial LIGO and upgraded for Advanced LIGO. The Advanced LIGO design has been altered to tailor the needs of the KAGRA payloads. The OSEM bodies are attached to the recoil masses of both the optic and intermediate mass, while the flag is attached to optic or intermediate mass. In current KAGRA design, OSEMs were omitted in the final stage of Type A and Type B suspensions and only coil magnet actuators are present at the optic level [165].

### 5.3.1 OSEM characterization

The controls of a Type B(p) payload final suspension stage have been tested first. The four OSEMs involved in this test first had to be characterized and calibrated. The sensing part consists of a LED (Optek OP232), a photodiode (Hamamatsu S1223-01), a collimator lens, and an aluminum flag which shadows the light emitted from the LED. Calibration is needed in order to convert the signal coming out of the device to displacement. The current coming out of the photodiode is converted to a voltage by using a TIA, similar to the one shown in Fig. 4.8(b).

The right-handed axis system for the OSEM is defined as shown in Fig. 5.13. Before calibration, the coupling to output signal from translations of the flag in Y and Z is checked by setting the flag halfway and translating in Y and Z with a 3 degree-of-freedom translational stage. If possible, the coupling is minimized by repositioning the OSEM at a different angle with respect to the flag. The couplings can be due to this non-orthogonality, but also due to reflections inside the OSEM. Which of these two effects is dominant is unknown. The measurements of these couplings for
Figure 5.13: OSEM coordinate axis definition. The flag will point through the middle hole in the positive X direction. The Y direction is pointing from photo diode to light source and the Z direction is pointing upwards.

Figure 5.14: Calibration results by moving the flag across the light beam are displayed. In Fig. 5.14(a), the rise from about -12.5 V to 0 V starts slowly as the edges of the beam are cut off by the flag. Cutting through the middle of the beam ideally shows a constant gradient, as is visible for OSEM 1 between 2.2 mm and 3.3 mm. This is the (linear) range of this OSEM at a sensitivity of about 5 kV/m. OSEM 2 most probably has its LED glued under an angle, so that the beam intensity is less well distributed. In the final OSEM design a small lens has been introduced in front of the LED to have a more homogeneously distributed light beam. The linear range of the current OSEMs is about 2 mm and they have a calibration factor between 4 and 5 kV/m [165].

From Fig. 5.14(b) the flag position for each OSEM, where the sensitivity is highest, i.e. the operating point, can be inferred. The sensitivity at that point is also the conversion factor used to convert from V/√Hz to m/√Hz. Spectral noise measurements in m/√Hz of a batch of LEDs is shown in Fig. 5.15. The LEDs from this batch were also used in OSEM 1 to 4 in the tests. Due to the relatively high noise of the shadow sensor, the sensing part is not used during science mode, as this would spoil the suspension performance in the detection band.

### Table 5.2: Measured coupling factors for each of the four OSEMs being tested.

<table>
<thead>
<tr>
<th>OSEM #</th>
<th>Y [kV/m] (%)</th>
<th>Z [kV/m] (%)</th>
<th>X [kV/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.21 (3.6%)</td>
<td>0.20 (3.5%)</td>
<td>5.70</td>
</tr>
<tr>
<td>2</td>
<td>0.29 (4.3%)</td>
<td>0.28 (4.1%)</td>
<td>6.80</td>
</tr>
<tr>
<td>3</td>
<td>0.10 (1.8%)</td>
<td>0.19 (3.3%)</td>
<td>5.11</td>
</tr>
<tr>
<td>4</td>
<td>0.29 (4.8%)</td>
<td>0.13 (2.2%)</td>
<td>6.05</td>
</tr>
</tbody>
</table>

In Fig. 5.14 the results of the calibration by moving the flag across the light beam is displayed. In Fig. 5.14(a), the rise from about -12.5 V to 0 V starts slowly as the edges of the beam are cut off by the flag. Cutting through the middle of the beam ideally shows a constant gradient, as is visible for OSEM 1 between 2.2 mm and 3.3 mm. This is the (linear) range of this OSEM at a sensitivity of about 5 kV/m. OSEM 2 most probably has its LED glued under an angle, so that the beam intensity is less well distributed. In the final OSEM design a small lens has been introduced in front of the LED to have a more homogeneously distributed light beam. The linear range of the current OSEMs is about 2 mm and they have a calibration factor between 4 and 5 kV/m [165].

From Fig. 5.14(b) the flag position for each OSEM, where the sensitivity is highest, i.e. the operating point, can be inferred. The sensitivity at that point is also the conversion factor used to convert from V/√Hz to m/√Hz. Spectral noise measurements in m/√Hz of a batch of LEDs is shown in Fig. 5.15. The LEDs from this batch were also used in OSEM 1 to 4 in the tests. Due to the relatively high noise of the shadow sensor, the sensing part is not used during science mode, as this would spoil the suspension performance in the detection band.

#### 5.3.2 Inertial damping of the optic stage

After determining the performance of all OSEMs, the (dummy) optic and recoil mass are suspended and the OSEMs are attached. This is done in such a way that their outputs are reading voltages that corresponded to the (middle of the) flag position interval where
Figure 5.14: (a) Photodiode current output transformed into voltage. When the LED shines, not obstructed by the flag, on the photodiode, this particular set-up results in an output of about -12.5 V. Panel (b) shows the gradients of the TIA output upon flag translation to determine the OSEM sensitivity.

The OSEM sensitivity is highest. With these horizontal OSEMs, the translational, pitch and yaw degree of freedom of the optic - recoil mass system can be measured. These three degrees of freedom can be measured by using the geometrical sensing matrix $S$

$$
\begin{bmatrix}
L_{TM} \\
P_{TM} \\
Y_{TM}
\end{bmatrix}
= 
\begin{bmatrix}
0.25 & 0.25 & 0.25 & 0.25 \\
0.5 & 0 & -0.5 & 0 \\
0 & 0.5 & 0 & -0.5
\end{bmatrix}
\begin{bmatrix}
x_{TM \text{ OSEM,H1}} \\
x_{TM \text{ OSEM,H2}} \\
x_{TM \text{ OSEM,H3}} \\
x_{TM \text{ OSEM,H4}}
\end{bmatrix}.
$$

(5.4)

Before controlling the system, diagonalization of the driving matrix, responsible for giving weights to signals in order to cope with actuator strength, is performed. After diagonalization, couplings between the three degrees of freedom are reduced to less than 5% [88]. Damping the simple pendulum modes is provided by simple derivative
Figure 5.15: Noise measurements of OSEMs with the flag in the middle of the device, converted to displacement using a typical 5 kV/m displacement sensitivity. A discrepancy in LED noise characteristics from a single batch is observed. Especially LED 1224.06 has a larger noise than the other LEDs.

control. Fig. 5.16 shows the control performance in the beam axis degree of freedom. The resonance frequency is about 0.65 Hz, which is easily calculated as $f_r = \frac{1}{2\pi} \sqrt{g/L}$.

The damping is successful at the resonance and even the spurious vertical bounce mode at about 14 Hz is less visible, presumably as there is coupling between the main mode and this mode. The peaks in the higher frequency region are attributed to the power line frequency and the plateaus, visible from 50 Hz onwards, are attributed to electromagnetic coupling between the currents running to the coils and coming back from the photodiodes. Similar transfer functions and successful damping are observed for the other two degrees of freedom, i.e. pitch $\theta_y$ and yaw $\theta_z$. The resonance frequencies in $\theta_y$ are modeled and observed to be around 0.85 Hz and 4.5 Hz. In $\theta_z$ they are modeled and observed to be around 1 Hz and 1.3 Hz. All these resonances are successfully damped using a simple derivative control.

Transfer function measurements of the other degrees of freedom are in agreement with simulated transfer functions, as shown in Fig. 5.17. With similar damping loops, the modes in the pitch and yaw degrees of freedom are also successfully damped. Similar electromagnetic coupling plateaus as described before are also visible in these measurements.
5.3.3 Inertial damping of the intermediate stage

In a second experiment the actuation matrix for the six degrees of freedom control of the intermediate mass stage was diagonalized and the transfer functions measured. The intermediate mass is suspended from and its recoil mass is attached to the bottom GAS filter. This system contains six OSEMs, as shown in Fig. 5.12. The position of the intermediate mass along the six degrees of freedom is geometrically reconstructed from the OSEM signals by using sensing matrix $S$

$$
\begin{bmatrix}
L_{IM} \\
T_{IM} \\
V_{IM} \\
P_{IM} \\
R_{IM} \\
Y_{IM}
\end{bmatrix} =
\begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.5 & -0.5 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.333 & -0.333 & -0.333 \\
0 & 0 & 0 & 0 & 0.577 & 0.577 \\
0 & 0 & 0 & 0.667 & -0.333 & -0.333 \\
0 & -0.5 & 0.5 & 0 & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
X_{IM, OSEM,H1} \\
X_{IM, OSEM,H2} \\
X_{IM, OSEM,H3} \\
X_{IM, OSEM,V1} \\
X_{IM, OSEM,V2} \\
X_{IM, OSEM,V3}
\end{bmatrix}
$$

After performing the actuator diagonalisation, the transfer functions in those degrees of freedom can be measured and compared to simulation results, as shown in Fig. 5.18. By applying a simple damping filter with an appropriate roll off filter, all the modes which can disturb the interferometer lock-acquisition phase are successfully damped. More details can be found in Ref. [164].

**Figure 5.16:** Damping of the optic - recoil mass system pendulum mode along the beam axis. The 14 Hz line is the vertical bounce mode and the plateaus visible from 50 Hz onwards are associated with electronic couplings. More details in text.
Figure 5.17: Transfer function measurements and comparison to simulation results of the longitudinal (beam axis), pitch and yaw degrees of freedom for the optic suspension. Reproduced from Ref. [88].

Figure 5.18: Transfer function measurements and comparison to simulation results of the six degrees of freedom for the intermediate mass suspension. Reproduced from Ref. [88].
The first detection of gravitational waves opens up a whole new window on the Universe. Man has always gazed up to the sky to learn about the extra-terrestrial, but now man will be able to listen as well. This opens a new field in astronomy: gravitational wave astronomy. It is impossible to predict, but exciting to imagine, what astronomical surprises are on their way to Earth to be detected by the global gravitational wave detector network.

The status or sensitivity achievements of the LIGO Virgo Collaboration at the end of observation run O2 is summarized in Fig. C.1. Considerable progress has been made in the gravitational wave field, but the community continues to push forward towards the design sensitivities of the Advanced detectors, new cryogenic underground detectors and an entirely new (third) generation of detectors in the coming decades. Work on several aspects that are critical to the operation of gravitational wave detectors have been presented in this thesis.

Conclusions

Five MultiSASs have been installed at Advanced Virgo and four of them were operational in O2. SNEB, SWEB, SPRB and SDB2 (in this chronological order) have been brought into operation with only the SPRB GAS blade failure as a significant issue. These four seismic attenuation systems are suspending optical tables in vacuum and are operating according to expectation. SIB2 will only be suspended at a later stage as it is not critical at the sensitivity that was projected \textit{a priori} for O2.

MultiSAS fulfills the requirements set forward at the start of the project. The rms
The magnitude of the so-called cradle effect in the inverted pendulum stage of MultiSAS was investigated by low frequency (5 mHz) large magnitude (2 mm) line injections in the two horizontal degrees of freedom and monitoring the coupling to tilt and roll of the top stage platform. This coupling has been determined for two systems (the prototype and SWEB) to be more than 2 orders of magnitude lower than levels that would be problematic for the top stage inertial sensors in the blended error signals used for control. In Advanced Virgo, the high pass filter for the L4C ground geophones has been designed by simulation, improving the residual rms motion for the vertical degree of freedom. Lastly, if the vacuum vessel containing MultiSAS is pumped down to below a millibar, the L22 geophones that are on the prototype bench measure only self noise - about $2 \times 10^{-12}$ m/√Hz · 1/f from 10 Hz onwards - which is equivalent to the translational ASD requirements.

Femtometer and femtoradians level residual motion was specified in the Advanced Virgo MultiSAS design for the suspended benches, and therefore an inertial sensor that could measure in the $10^{-15}$ m/√Hz regime would be needed to characterize or monitor...
system performance. Such a sensor was developed at Nikhef by combining two proven concepts into a monolithic accelerometer with an interferometric readout. The readout reaches a $4 \times 10^{-15} \text{ m/} \sqrt{\text{Hz}}$ noise level from 5 Hz onwards. This readout is used to determine the proof mass position of a monolithic accelerometer and is the error signal for a feedback loop by using a voice coil as an actuator on the proof mass. The sensor reaches an unprecedented $8 \times 10^{-15} \text{ m/} \sqrt{\text{Hz}}$ self noise level from 30 Hz onwards. A fiber optic version of this sensor for high radiation/ high magnetic field operation (for example in particle colliders, such as the proposed Compact Linear Collider, CLiC at CERN) was also fabricated at Nikhef. The fiber interferometric readout performs a factor 4 above (CLiC specified) sensitivity requirements when thermally isolated, reaching a displacement sensitivity of $4 \text{ pm/} \sqrt{\text{Hz}}$ from 50 Hz onwards. There are no obvious reasons why this fiber version would not reach the femtometer regime as well, and could be used as a test mass displacement sensor - as was intended by Gray et al. when first published - in the future. Current design and results of this readout are summarized in Appendix B.

The author has been active in the Japanese effort towards bKAGRA by three visits to NAOJ, Mitaka, Tokyo and the KAGRA site, Toyama, Gifu. Work has been done on parts of Type B(p) suspension, i.e. the payload structure with OSEMs. OSEM calibrations were performed and improvements to the shadow sensor light source were investigated. A more homogeneous light beam was necessary and this was solved by introducing a collimator lens at the LED side. The OSEMs were used to successfully damp the modes of the final suspension of the Type B(p) payload. Additionally, simulations in order to quantify a possible change of inertial sensor (from L4C to an LVDT read out monolithic accelerometer) for the inverted pendulum stage for Type A and Type B suspensions. Calibration and simulation on the effect of substitution of L4Cs for LVDT read out monolithic accelerometers as inertial sensor was performed. Simulation results show that, in the absence of tilt, velocity rms requirements of 0.5 $\mu$m/s will be met when blending the inertial sensor signal with the LVDT signal at (or below) 80 mHz.

**Recommendations and future work**

Statistics on binary black hole mergers will continue to build up with every new detection. Advanced Virgo joined at the end of O2 and better localization of sources is now possible. The scientific community is looking forward to more events like GW170817 that have an electromagnetic counterpart, i.e. multi-messenger astronomy. Apart from the obvious astronomical gains, physicists aim at understanding more the validity of General Relativity. Gravitational waves may bring more hints of where to possibly adapt Einstein's description of gravity in order to possibly merge it with Quantum Mechanics. The future observation run participation for LIGO, Virgo and KAGRA is summarized in Fig. C.2.

Plans for more MultiSASs for Advanced Virgo to house parts of a squeezed light source are already in the design stage. MultiSAS is a well designed mechanical filter
Figure C.2: The planned sensitivity evolution and observing runs of the aLIGO, Advanced Virgo and KAGRA detectors over the coming years. The colored bars show the observing runs, with the expected sensitivities for future runs, and the achieved sensitivities in O1 and in O2. There is significant uncertainty in the start and end times of planned the observing runs. Reproduced from Ref. [166].

that makes the most out of the small space it occupies. Further control development towards more advanced control schemes will improve its performance and stability. Monitoring the MiniTower modes is critical to ensure that spectral requirements are met for the angular degrees of freedom. If the angular rms motion would be limiting for detector sensitivity, one solution may be lowering the frequency of the suspended bench angular modes (now around 300 mHz) to below 100 mHz by lowering the suspension point more towards the bench center of mass. Just as was done with the superattenuator marionetta angular modes, lowering these modes will move them away from aligning with the microseismic peak frequencies.

Further improvements in geophone high pass filtering (for both blending and ground geophone filters) can be achieved by using a blend of IIR and FIR filtering, such as used in LIGO [167]. For this to be possible in the future, also hardware improvements in Virgo’s data acquisition infrastructure will be necessary. Vertical performance will improve and possibly horizontal LVDT ground correction could be considered as well. For the Advanced Virgo MultiSASs, the horizontal ground geophones are now only used for monitoring. As the blending of LVDT and top stage geophones is around 160 mHz ($s = 1$ rad/s), top stage inertial motion is already measured across most of the microseismic frequency interval where the ground geophone filters are sufficiently flat in phase.

In section A.5, low frequency blending of the top stage error signals with suspended bench local control is described. Since the suspended benches contain optical set-ups that are designed to determine the angular alignment of a nearby core optic, the suspended benches would ideally follow the long time-scale (> 2 s) motion of that
nearby optic. To achieve this, blending strategies involving local top stage geophone signals with different LVDT signals can be pursued. The LVDT signals could be, inspired by the global inverted pendulum control (GIPC) strategies employed at Virgo, a subtraction of the nearby superattenuator LVDT signal from the MultiSAS LVDT signal. This would eliminate the coherent microseismic motion and produce a signal with no ground motion in it for frequencies below 1 Hz. This approach is currently under investigation and preliminary results can be found in Ref. [168].

Regarding the monolithic accelerometer with an interferometric readout, the control loop design, which locks the small interferometer by keeping the proof mass in its place, can be improved. Advanced control strategies would keep the differential interferometer error signal closer to 0 V, which improves the common mode noise subtraction. Also, a local light source like a LED could simplify the sensor. Obviously, an LED does not have a sharp emission spectrum, so equal arm length (i.e. white light interferometer) operation is necessary. A VCSEL single mode laser would be another option, but its frequency noise would only be suppressed sufficiently if the differential arm length is well matched.

When measuring on the seismically isolated bench, the sensor self noise is not measurable below 4 Hz. Both L4C and novel sensor measure the damped bench motion below those frequencies. This can be solved by fabricating two more identical versions of the sensor and performing a so-called huddle test. Such a three channel correlation analysis [169] can distinguish between sensor self noise and cross correlated common motion and subtract the latter, allowing for convergence to the self noise of the sensors at lower frequencies as well.

The quality factor of the proof mass suspension may have been lower than expected, which increases the thermal proof mass suspension noise. The reason for the $Q$ being 40 instead of the expected 150 is not resolved, but even $Q = 40$ does not explain the measured noise levels between 10 Hz and 30 Hz in Fig. 4.23. There are plans at Nikhef to produce new monolithic accelerometers made of titanium (Grade 5 Titanium alloy, Ti-6Al-4V). The main advantage of titanium in light of eddy current damping is its 36 times lower electrical susceptibility. Therefore the eddy currents induced by the stray magnetic field from the voice coil magnet will be 36 times lower and $Q$ is expected to improve by a factor 36. This would mean that suspension thermal noise would reduce by a factor 6, e.g. to a displacement equivalent value of $8 \times 10^{-14}$ m/√Hz at 1 Hz in the noise budget.

Many systems have to be brought into operation and commissioned in KAGRA before this detector can join the global network, but progress is fast. iKAGRA showed the world that Japan has what it takes to operate a kilometer class interferometer. Now cryogenic experience will prove vital to finish bKAGRA to unprecedented low-frequency sensitivity for gravitational radiation. Regarding the work described in this thesis, OSEMs have been improved further since the author’s work, similar loops have been employed in the power recycling 3 (PR3) mirror operated in iKAGRA and already meet baseline (b)KAGRA displacement requirements. The pre-isolators, which are designed, assembled and tested at Nikhef, could perform better with the LVDT readout monolithic accelerometers, but, as a tilt spectrum of the site is still to be measured, the results presented here are not conclusive.
Several measurements done during the MultiSAS prototype campaign and commissioning phase for Advanced Virgo are summarized below. First, the inverted pendulum transfer function measurement performed in 2013 is presented. Then, results from both the modeling and subsequent validation by measurement of internal (structural) modes of all components of MultiSAS are presented. Finally, several definitions and tuning methods specific to the MultiSASs installed at (Advanced) Virgo are summarized.

Other than the standard specifics presented in Chapter 3, there also have been non-standard events during the commissioning phase of all the MultiSAS systems. An example of this is the cabling of SWEB. Already soon after SWEB MultiSAS was installed, the top stage geo0 L4C geophone seemed to change damping coefficient and gain. It was identified to be a faulty connection somewhere in the cable. This essentially changed the loading resistance on the geophone coil and thus the amount of damping and the overall gain from time to time. In the end it was solved by having a parallel cable running through the J6 DSUB-39 connector (temperature sensor feedthrough) and connected to the extra external ‘EXT geophone’ DSUB-9.

During O2, the SDB2 MultiSAS had only one GAS filter operational [170]. When trying to float the SDB2 optical bench, the bench was found to be too heavy by more than 5 kg. Dead weight, necessary to balance the suspended bench in height and the pitch and roll angular degrees of freedom, was transferred from the intermediate filter to the bench to keep the top GAS filter at nominal load. The amount of mass now suspended
by the intermediate filter was too high and thus its keystone was mechanically blocked. Additionally, the SWEB MultiSAS inverted pendulum stage was not working properly, probably due to cabling that is touching one of the legs [171]. SWEB showed more prominent scattering arches (see section 3.4.3) than SNEB in the engineering run prior to O2. Both issues are scheduled to be solved before the start of O3.

A.1 Inverted pendulum transfer function

A transfer function measurement of the inverted pendulum stage was performed at Nikhef [172]. The primary reason for the measurement was to tune the CoP effect by adjusting the counterweights at the bottom of the inverted pendulum legs, as described in section 1.3.2.

![Set-up for the inverted pendulum stage transfer function measurement](image1)
![CAD drawing of a piezo shaker](image2)

**Figure A.1:** (a) A photograph of the set-up for the inverted pendulum stage transfer function measurement. To avoid exciting the reference frame modes, the frame was detached and lifted from the base ring. (b) A CAD drawing of a piezo shaker. The base ring rests on three custom made horizontal flexure stages driven by piezo actuators. The green arrow indicates the direction of motion.

The inverted pendulum stage was put on three piezo shaker stages indicated by red arrows in Fig. A.1(a). In order to measure a transfer function, piezoelectric accelerometers were placed on the base ring and the top stage. The horizontal shaker stage also introduced tilt at the base ring stage and this transfers directly to the top stage. Tilt coupling to the accelerometer on the top stage proved to be a problem for a correct measurement. The accelerometer position was tuned as much as possible to coincide with the rotation axis of the tilt motion. Much effort was put in tuning the CoP
effect by adding or removing mass from the counterweight holding bell located at the bottom of the inverted pendulum legs. It was determined that five blocks of 140 g, totaling at 700 g per inverted pendulum leg, gave the best results. The final result is shown in Fig. A.2.

Figure A.2: Inverted pendulum transfer function measurement result. The CoP effect is modeled to be slightly overcompensated with a Q=5 resonant zero at 25 Hz. The saturation level is almost 80 dB. Residual tilt coupling of the measurement set-up is visible from 25 Hz onwards. Several internal modes of the shaker stage are visible above 50 Hz.

A.2 Resonance modeling and measurements

An overview of the FEM model of MultiSAS is presented in Fig. A.3. The finite element analysis is used to identify the suspension rigid body modes, to validate the state space model [52], and to identify the system’s internal modes. An example of such mode, showing the translational and rotational oscillations of the top filter keystone, was already given in Fig. 3.7.

The model was validated by hammering tests on the MultiSAS prototype. The results of the measurements on the top stage are summarized in Table A.1 [173, 174].

<table>
<thead>
<tr>
<th>Top stage part</th>
<th>Frequency [Hz]</th>
<th>Mode identification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keystone</td>
<td>50, 54</td>
<td>pair of keystone modes (lateral and tilt)</td>
</tr>
<tr>
<td></td>
<td>124</td>
<td>keystone modes (lateral and tilt)</td>
</tr>
<tr>
<td></td>
<td>137</td>
<td>lateral keystone mode or blade modes</td>
</tr>
<tr>
<td></td>
<td>183</td>
<td>keystone/ motor bouncing on the wire</td>
</tr>
<tr>
<td>Blades</td>
<td>299, 300, 313, 317</td>
<td>blade modes</td>
</tr>
<tr>
<td>Wire</td>
<td>108, 260, 370, 530, 808</td>
<td>violin spectrum, with cableguide</td>
</tr>
<tr>
<td></td>
<td>170, 325, 525, 725, 925</td>
<td>violin spectrum, without cableguide</td>
</tr>
</tbody>
</table>

Table A.1: Internal mode characterization by hammering measurements of MultiSAS top filter. The measurement is done by placing a magnet on the top stage part listed here and measuring the induced current in a coil nearby.

In Fig. A.4 the results of the top wire hammering test are compared with the FEM
model. In particular, the effect on the violin modes from attaching a cable guide to the wire was investigated. The cable guide (shown in the left inset of Fig. 3.8(a)) is a light plate used to route the cabling through the suspension. Due to the added mass, the violin modes are lower in frequency and not evenly spaced when the cable-guide is installed.

All these measurements were performed on the MultiSAS test facility. The main difference between the prototype and the systems installed at Virgo is the diameter of the two pendulum wires. In the test facility the top and the bottom wire have diameters of 3.5 mm and 2.5 mm, respectively. In the Virgo systems thinner wires, i.e. 2.5 mm and 2 mm diameter respectively, were chosen. A hammering measurement done at SDB2 and SNEB of the keystone and motor bouncing on the wire mode gave a result of about 122 Hz. Compared to the 183 Hz mode measured at the test facility, this is a factor 1.5 lower. This is expected when decreasing the wire thickness as the frequency of the violin mode $f_v$ is dependent on the wave propagation velocity

$$c_v = \sqrt{\frac{T}{\rho A}} = \frac{f_v}{\lambda_v},$$

where $T$ denotes the tension in the wire, $\rho$ the wire mass density per length, $A$ the wire cross-sectional area and $\lambda_v$ the wave length of the violin mode. Since the numerator of the square root is proportional to the wire diameter squared, the violin mode frequency is inversely proportional to the thickness of the wires. The mode in the top wire in the Advanced Virgo systems is expected to be a factor 1.5 higher in frequency. This was the main argument to decrease the wire thicknesses for the MultiSASs at Virgo. Modeled values of several modes, compared with several MultiSASs are presented in Table A.2. Again, the main difference between the Virgo systems and the Nikhef system is the wire thickness of 2.5 mm and 3.5 mm for the top wire and 2 mm and 2.5 for the lower wire.
respectively.

Figure A.4: (a) Spectrum of hammering measurements of top stage wire including a cable guide at 190 mm below the wire top. Several peaks can be individually identified. Panel (b) shows a comparison between these measured frequencies and the predictions from the FEM modeling of the system including a cable guide at different positions on the wire. The FEM model of the cable guide at 190 mm from wire top shows good agreement.
Table A.2: Keystone mode hammering test mode frequencies in Hz compared to modeled values for Virgo and Nikhef MultiSASs [175]. Only the $x$-$z$/$z$-$x$ diff. values for the intermediate filter keystone are ill understood. Common (comm.) and differential (diff.) translation-and-angular modes refer to in phase or out-of-phase modes.

### A.3 Tuning methods

The resonance frequency of the inverted pendulum stage can be tuned by changing the mass supported by the legs, as shown in Eq. (1.34). In practice, the tuning is done by adding weights onto the top stage plate. All MultiSASs produced for Advanced Virgo are pre-tuned such that the nominal load plus about 20 kg of mass on the top stage results in a resonance frequency of about 100 mHz. In order to lower it for example to 70 mHz, roughly 8 kg of mass needs to be added in the form of ballast weight. Below, a summary of mathematical considerations when tuning the horizontal and vertical mechanical filters is given. More details are found in Ref. [176].

Adding the effect of the tuning of the counter weights below the lower flexure of the leg, Eq. (1.34) transforms to

$$\omega_0 \approx \sqrt{\frac{k_\theta - Mgl - m_{\text{leg+cw}} g h_{\text{leg+cw}}}{Ml^2 + I_{\text{leg+cw}}}} ,$$  \hspace{1cm} (A.2)

where $m_{\text{leg+cw}}$ and $I_{\text{leg+cw}}$ represent the mass and the position of the center of mass of the combination of leg and counterweight, respectively. The total moment of inertia of the leg and counterweight around the bottom flexure is denoted as $I_{\text{leg+cw}}$. The displacement transfer function saturation level $\beta_{\text{IP}}$, due to the CoP effect, can be adjusted by tuning the counterweights according to

$$\beta_{\text{IP}}(\omega \rightarrow \infty) = \frac{\frac{h_{\text{leg+cw}}}{l} - m_{\text{leg+cw}} l_{\text{leg+cw}}}{Ml + \frac{I_{\text{leg+cw}}}{l}} .$$  \hspace{1cm} (A.3)

Advanced Virgo MultiSASs have five counterweights of 140 g each (700 g total) installed at each inverted pendulum leg. This provides a saturation level $\beta_{\text{IP}}$ around $10^{-4}$ at high frequencies.
The magic wands, shown in Fig. A.5, are used to tune the CoP effect in the GAS filters. The resulting saturation level is

$$\beta_{\text{GAS}}(\omega \to \infty) = \frac{(m_{\text{blade}}x_{\text{blade}} + m_{\text{mw}}x_{\text{mw}}) - \left(A + \frac{m_{\text{mw}}x_{\text{mw}}^2 + I_{\text{mw}}}{x_L}\right)}{Mx_L + A + \frac{m_{\text{mw}}x_{\text{mw}}^2 + I_{\text{mw}}}{x_L}},$$

(A.4)

where $m_{\text{blade}}$ and $x_{\text{blade}}$ represent the mass and the position of the center of mass, respectively, of the blade(s). The mass and the position of the center of mass of the magic wand are denoted by $m_{\text{mw}}$ and $x_{\text{mw}}$, respectively. The total moment of inertia around the pivot point connecting the magic wand to the GAS filter plate or body, between the clamps of the blades, is denoted by $I_{\text{mw}}$. A geometric correction factor taking into account the blade curvature [176] is denoted by $A$. As in Eq. (1.37) and Fig. 1.16(a), $x_L$ represents the compression distance of the blades. In Advanced Virgo’s MultiSAS, two magic wands are installed in each intermediate filter. The counterweight has a 0.5 kg mass and it is mounted at $x_{\text{mw}} = 49$ mm. The resulting measured transfer function has a notch at 150 Hz [177] and measured $\beta_{\text{GAS}}$ of $3 \times 10^{-4}$. The parasitic mode due to the magic wand has been modeled to be around 350 Hz.

### A.4 Local coordinate systems at Virgo

Each MultiSAS has three sensor positions as depicted in Fig. A.6 for SNEB, which are the positions of LVDT/voicecoil combinations, stepper motors and (one level higher in the structure) the L4C geophones.

The local coordinates for each MultiSAS are shown in Fig. A.7. The local signals in Cartesian coordinates have to be calculated by geometrically adding, i.e. cosine and
Figure A.6: Local MiniTower coordinate system for the SNEB MultiSAS with sensor/actuator positions with respect to this coordinate system. The arrows in the rounded rectangles define the positive (calibrated) readout of the sensor, i.e. positive in $\theta_y$. Similar information for the other Advanced Virgo systems can be found in Table A.3.

sine multiplication and superposition, the different sensor positions. How the different sensor positions for each system relate to the local coordinates is presented in Table A.3.

<table>
<thead>
<tr>
<th>Bench</th>
<th>Position 0 direction</th>
<th>Position 1 direction</th>
<th>Position 2 direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNEB</td>
<td>-x &amp; 30° -z direction</td>
<td>-x &amp; 30° +z direction</td>
<td>+x-direction</td>
</tr>
<tr>
<td>SWEB</td>
<td>-x &amp; 30° -z direction</td>
<td>-x &amp; 30° +z direction</td>
<td>+x-direction</td>
</tr>
<tr>
<td>SIB2</td>
<td>-z &amp; 30° +x direction</td>
<td>-z &amp; 30° -x direction</td>
<td>+z-direction</td>
</tr>
<tr>
<td>SPRB</td>
<td>+x &amp; 30° +z direction</td>
<td>+x &amp; 30° -z direction</td>
<td>-x-direction</td>
</tr>
<tr>
<td>SDB2</td>
<td>-x &amp; 30° +z direction</td>
<td>+x-direction</td>
<td>-x &amp; 30° -z direction</td>
</tr>
</tbody>
</table>

Table A.3: MultiSAS sensor/actuator position direction with respect to the local coordinate system as defined by Fig. A.7.

The MiniTower vacuum vessels have been bolted to the ground, but how this is specifically done per system has an effect on the MiniTower modes. Typically they range from 20 Hz to 50 Hz. The elliptic roll-off filter of the PID controller (see Fig. 3.18) places a notch at the first mode and provides further roll-off after that. The frequencies of the first (notable) MiniTower mode for each system are presented in Table A.4.
Figure A.7: Local reference system of towers, minitowers and external benches. External PR benches (EPRB1, EPRB2) are not drawn for readability. They adopt the same coordinate system as the SPRB minitower. Reproduced from Ref. [178].

<table>
<thead>
<tr>
<th>Bench</th>
<th>Main tank mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNEB</td>
<td>21 Hz</td>
</tr>
<tr>
<td>SWEB</td>
<td>25 Hz</td>
</tr>
<tr>
<td>SIB2</td>
<td>22 Hz</td>
</tr>
<tr>
<td>SPRB</td>
<td>26 Hz</td>
</tr>
<tr>
<td>SDB2</td>
<td>24 Hz</td>
</tr>
</tbody>
</table>

Table A.4: Frequency values of the MiniTower modes for the different suspended bench systems.

A.5 Control in Advanced Virgo systems

The control filters discussed in section 3.3 were designed and tested with the MultiSAS test facility. The filters used at the Advanced Virgo systems are similar, except for the origin of (DC) position information. The error signals for the MultiSAS test facility are a blend of the top stage LVDTs and L4C geophones.
At Advanced Virgo, however, MultiSAS has to be able to position the suspended bench with micrometer precision. Naively one would think this is possible using top stage signals only, but e.g. temperature induced tilt effects can cause the reference frame of the top stage LVDTs to assume a slightly different position in the horizontal plane. Top stage loops follow this position and unintentionally change the position of the suspended bench, even though the top stage error (time) signals remain at the same value.

During O2, for the reasons described above, MultiSAS positioning loops used position information from the Local Control (LC) error signals. These LC signals are provided by eight LVDTs (four horizontal and four vertical) at the corners of the suspended bench. The extremely low frequency blending filters (see Fig. A.8) allow only the LC position information over long time scales to be used. The filters are constructed by using 5th polynomial filters from Eq. (3.8), but with $s_0 = 0.1 \text{ rad/s}$.
Interferometric readout in fiber for CLiC

The interferometric readout described in Chapter 4 has all the readout electronics attached to the accelerometer mechanics. In applications where this is unwanted, such as in high magnetic field or high radiation environments, a different solution is proposed here. Aimed to be used at the CLiC linear collider proposed by CERN, an adaptation of the optical scheme suffices by going from open air environment to a complete fiber set-up. Apart from using two fiber circulators instead of the upper beamsplitter, the optical set-up can practically remain the same.

The sensor can be used to generate error signals for feedback loops to stabilize the quadrupole magnets that are used in the two linear accelerators that make up the CLiC e⁺e⁻ collider. In the linear accelerators, the size of the electron and positron beams is 500 nm in the horizontal direction and 5 nm in the vertical direction. About 4000 quadrupole magnets are used to keep the beams this size. After the accelerating stage, the beams are focused by a quadrupole magnet with a much stronger gradient. This quadrupole magnet, coined QD0, will focus the particle beam to 40 nm horizontally and 1 nm vertically [179]. The sensor requirements for CLiC are less stringent than the requirements for the MultiSAS measurement. Sensors with pm/√Hz displacement sensitivity are necessary for the control of the isolation elements and the sensors have to be radiation hard and able to operate in (stray) high magnetic fields.
The test set-up, shown in Fig B.1(a), needed in-house fabrication for certain components. The fiber stretchers (see Fig. B.1(b)) and silver plated fiber ends to act as end mirrors for the interferometer were developed at Nikhef. All other components, \textit{e.g.} circulators and a polarization controller (see Fig B.2), are standard off-the-shelf.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure_b1.png}
\caption{Fiber interferometric readout (a) test set-up (using testing arm) and the accelerometer readout set-up (using sensor arm). Adapted from Ref. [180]. (b) In-house fabricated piezo stretcher used in the reference arm and testing arm.}
\end{figure}

The feedback loop uses a fiber stretcher as actuator, which mimics the moving of the reference mirror, \textit{i.e.} causing a optical path length change and, is similar to the loop described in section 4.3.1. The noise budget is similar to Fig. 4.9 and, when using
Figure B.2: Fiber interferometric readout components: (a) A fiber circulator, Thorlabs 6015-3-APC. Each fiber connector serves as an input and output. When injecting light in one port, one of the other ports has low optical output loss (< 0.1 dB), whereas the third port has a high optical isolation loss (> 50 dB). (b) Fiber polarization controller or flapper. Long stretches of single mode (SM) fiber are rolled in so-called paddles which can move to use stress-induced birefringence to create independent wave plates to alter the polarization of the transmitted light in SM fiber.

the same photodiode readout as the so-called open-air sensor, is expected to reach below 10 fm/√Hz at high frequencies. The frequency noise is expect to dominate as the current fiber set-up has large centimeter scale static differential arm length. Judging from Eq. (4.22), this couples linearly to the level of the frequency noise. This optical set-up can also be used as a test mass displacement sensor as Gray et al. first intended.

B.2 Results of the prototype set-up

The set-up presented in Fig. B.1(a) (with testing arm) was characterized and calibrated in a similar way as described in section 4.2.3. More details our found in Ref. [180]. Fringe visibilities obtained in this set-up were about 98%. The feedback loop (using piezo stretcher 2 as actuator) was subjected to a linearity test by injecting a signal in piezo stretcher 1, as shown in Fig. B.3. The green curve has an offset with respect to 0 V, because that was the DC position necessary for that piezo stretcher to lock the interferometer prior to this injection test. When taking the spectrum prior to injection, i.e. obtaining the performance of the fiber interferometric readout, the cyan curve in Fig. B.4 is obtained. The readout shows a performance of 4 pm/√Hz from 50 Hz onwards.

The set-up was placed under a box of thermally insulating material as small fluctuations in temperature are believed to cause the refractive index of the fiber to change slightly, resulting in birefringence. This will cause the polarization of the light in the fiber to change its orientation. When the temperature changes are different for the two arms of the interferometer, the refractive indices will vary differentially, generating a difference in the orientation of the polarization planes for the two arms. This is believed to cause the higher noise levels below 10 Hz [181]. The structures visible between 15 Hz and 55 Hz are associated with the direct coupling of vibrations of the optical table.
to stress-induced birefringence.

**Figure B.3:** Linearity test on the fiber interferometric readout test set-up. Injecting in piezo stretcher 1 (test piezo) a 0.25 Hz, 13 V<sub>pp</sub> (resulting in about 1.64 µm<sub>pp</sub>) saw-tooth results in the differential signal kept around 0 V while the feedback loop controlled piezo stretcher 2 (feedback piezo) corrects for the injection by following it. Adapted from Ref. [180].

**Figure B.4:** Characterization and prototype measurements on an optical table on the 3<sup>rd</sup> floor of Nikhef, Amsterdam with the fiber interferometric readout and the monolithic accelerometer fitted with this readout and an L4C installed next to it. The readout reaches a 4 pm/√Hz and 100 pm/√Hz sensitivity level with and without thermal insulation, respectively. Adapted from Ref. [180].
Adapting the set-up to Fig. B.1(a) (with sensor arm) allows to test the readout for displacement sensing (of a proof mass) of a vibration sensor. The thermal insulation box was not large enough to accommodate the FP accelerometer mechanics and was not used. The red curve in Fig. B.4 was obtained with the proof mass mechanically blocked. The larger thermal fluctuations is suspected to result in the overall level of displacement sensitivity to be about one order of magnitude worse. Also the pointing set-up of the collimator is believed to introduce vibrations, which could cause the structures around 25 Hz, 40 Hz and 50 Hz. The blue and green curve show the fiber interferometrically read out FP accelerometer and L4C measuring the Amsterdam vibration spectrum below 50 Hz, above which the Nikhef sensor hits its self-noise.

Reducing vibrations and thermal fluctuations in the fiber set-up environment is expected to improve performance. An obvious experiment would be to install this set-up on the MultiSAS test facility's suspended bench in vacuum. Additionally, fabrication of a better collimator pointer would eliminate the possible noise injection of the pointing set-up used up until now. A final improvement would be to equalize the lengths of the two fiber interferometer arms, taking into account the added open-air distance from collimator to accelerometer mirror. This will reduce the effect of the laser frequency noise on the overall sensitivity.
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Summary

Turn up the bass! Low-frequency performance improvement of seismic attenuation systems and vibration sensors for next generation gravitational wave detectors.

I think it is a great privilege to write a PhD thesis during this period in the field of gravitational waves. On these few pages I would like to summarize what has been going on in these past few years. First, I describe and discuss the grand results of the LIGO-Virgo Collaboration and how these results have been measured. Then I zoom in on the research done during my PhD.

Gravitational waves, listening to our Universe

The first detections of gravitational radiation have been a breakthrough in physics and astronomy. Just as the invention of the telescope ushered in a new era of discovery and understanding of the Universe, gravitational wave astronomy is expected to do the same. GW150914, GW151222, GW170104, GW170608, GW170814 and GW170817 (GW is the acronym for gravitational waves and the six numbers point to the detection day in year - month - day format) have shown us that (binary) black holes and neutron stars can be studied for the first time using gravitational waves. What is truly monumental about these first detections we can see the Universe in an entirely new way. Mankind’s novel ability to directly detect gravitational waves is comparable to being deaf and suddenly gaining the ability to hear. An entirely new realm of information is now available.

The measured binary systems have collided hundreds of millions to several billion years ago after a very long dance around each other. The waves the systems emitted
have traveled to up until now to arrive here on Earth. When some of the measured waves began their journey, multi-cellular life began developing here on Earth. This life evolved to the human species. Within humanity, a genius emerged and predicted a hundred years ago that gravitational waves exist. This genius, called Albert Einstein, thought that the minuscule effect of these waves on Earth would never be measured. Even if they are real - he doubted several times whether they were not just a mathematical artifact - he deemed it technically impossible to measure the tiny effects. About 50 years ago, scientists have began trying to measure it anyway and ultimately built detectors that measured these waves at the end of their cosmic journey.

Figure S.1: A new look on the Universe’s black hole and neutron star population. X-ray studies have accumulated a family of black holes (purple spheres) below 20 solar masses. Our gravitational wave detections prove there are also heavier families out there (blue spheres). It is unknown what the product of the first neutron star merger we measured is: a light black hole or a heavy neutron star? Credit: LIGO/Virgo/Northwestern/Frank Elavsky.

Implications of the first detections

The first discoveries have shed light on our understanding of the dark Universe. Black holes had never been measured directly before, let alone a merger of two of these pure
space-time objects. The measurements, over more than 23 orders of magnitude, are the most precise distance measurements ever performed. This accuracy is comparable to measuring the distance from here to the nearest star outside the solar system (Alpha Centauri, 4.32 light years away) with the precision of the width of a human hair.

The first GW150914 discovery showed us that 3 solar masses of energy could vanish into space-time perturbations traveling at the speed of light. This was the single most powerful event ever measured, clocking in at 50 times the radiative power of the entire visible Universe at peak luminosity. The black holes involved, weighing about 30 and 35 solar mass each, are the heaviest stellar mass black holes measured to date. That such heavy black hole binary systems existed was new to astronomy as well, as shown in Fig. S.1.

The study of General Relativity, i.e. gravity, can now be brought up to a whole new level, the so-called strong field regime. With these new measurements data analysts can further constrain certain parameters in which they hope to find a hint where the theory of General Relativity - the description of macroscopic things - can be unified with the theory of Quantum Mechanics - the description of microscopic things. This envisioned merger of theories is one of the modern holy grails of physics.

The first few detections have been done with two LIGO detectors. With two detectors, you can get a ring of potential source positions in the sky from the timing differences. Out of the signal strength differences between the two detectors because they are on a different plane - the LIGO detectors are on opposite sides of the United States and the Earth is round - that ring can break a bit, but an area hundreds of times larger than the moon in the night sky is typical. With the addition of the European Virgo detector you see this area shrink dramatically, especially if it is a strong signal. This opens up possibilities for so-called multi-messenger astronomy, i.e. a complementary measurement of electromagnetic and gravitational waves. For GW170814 we already saw the area shrink and, three days later, GW170817 was detected and determined to be coming from a fusion of two neutron stars. The precise localization by the three gravitational wave detectors, shown in Fig. S.2, helped conventional astronomers to find an afterglow of this massive collision. In the afterglow, we were able to see for the first time the theorized process now known to be responsible for the abundance of elements heavier than iron in
the Universe, such as gold, platinum and uranium!

How do you measure gravitational waves?

Because there is a correlation between space-time and gravitational curvature, a gravitational wave will change the way objects fall with respect to each other. When a gravitational wave passes two objects, a measurable effect will occur. The physical distance between the two objects will stretch and contract as long as the gravitational wave is passing by. Gravitational waves are measured by accurately monitoring the position and movement of so-called test masses. To do exactly this, interferometers are used; they are kilometer-long laser set-ups with all kinds of optical elements such as a semi-reflective mirror (the so-called beamsplitter), a very powerful laser (typically hundreds of Watt) and highly reflective mirrors, which in this case are silicon cylinders weighing tens of kilograms. The beamsplitter and mirrors act as test masses and the distance between them is monitored to dazzling precision.

The basic principle is relatively easy to explain. A laser shoots a beam of light to for example the east through a beam splitter and 50% of the light continues its path due east into one of the interferometer arms. The other 50% reflects to a path an angle of 90° with the other beam to the north into the other interferometer arm. Both beams meet a highly reflective mirror at the end of their respective arm and reflect back to the beam splitter. The beams return at the beamsplitter, meet and, if both arms are equally long, they extinguish each other in the southern direction where there is a photo-detector; all the light goes back west to the laser. This is the result of (destructive) interference; the light waves are in anti-phase with each other and the result is no light at the photo-detector. When a gravitational wave passes, both arms will stretch and contract in anti-phase. This causes the interference effect to have a different outcome because the waves do not extinguish each other anymore completely. Little flashes of light are now detected at the photo-detector and tell us the distances between test masses are changing.

All gravitational wave detectors would measure nothing but seismic noise without all the elements of a detector being isolated from the ever-present minuscule vibrations of the Earth. These vibrations are typically one hundred billion (!) times larger than the effects of gravitational waves. To suppress the vibrations, we work with harmonic oscillators. To quickly understand what a harmonic oscillator is in this context, imagine an unrolled yo-yo (or take it out of your drawer!). Hold the yo-yo at the end of the string. Your hand is the vibration you want to suppress - for example the Earth’s vibrations - and the yo-yo is the mirror. If you slowly move your hand back and forth, there is no suppression of the vibrations; the yo-yo moves as much as your hand. There is a speed of back-and-forth hand motion (or frequency) where you get lots of yo-yo motion; this is called the resonance frequency and there is vibration amplification at that frequency. This is the price we have to pay for the behavior of the system above the resonance frequency. Now move your hand quickly back and forth and you see that the yo-yo is not following; above the resonant frequency there is vibration suppression.
We mitigate the price we have to pay - the amplification at the resonance frequency - with control technology; sensors measure the movement and a computer tells actuators - typically a magnet and an electrical coil - when a (small) force needs to be sent to the system - the yo-yo in our example - to damp the amplification at the resonance frequency. You can imagine that these control systems are running continuously to keep those huge interferometers, in practice a complex optical arrangement with dozens of suspended mirrors, aligned during the measurements. In a typical gravitational wave detector, there are hundreds of so-called feedback loops that keep an eye on everything.

Since the nineties, a lot of work has been done to set up a global network. The two LIGO detectors in Hanford, Washington and Livingston, Louisiana in the United States of America and the Virgo detector near Pisa, Italy are already operational. KAGRA, an underground detector in Japan (from 2020 onwards) and LIGO India (from 2022 onwards) will strengthen the network. So will new future detectors designed to exceed the sensitivity of the present detectors. The so-called Einstein Telescope in Europe and the Cosmic Explorer in the United States are expected to be realized in the third decade of this century. With more sensitive detectors we can not only crank up the amount of detections per year, but also look (back) further into the Universe.

In Advanced Virgo, there are now tables on which optics perform measurements to better align the main mirrors. These optical tables have to be isolated from the Earth’s vibrations. To this end, compact seismic isolation systems have been developed at the National Institute of Subatomic Physics Nikhef. This dissertation describes that seismic isolation system for the optical tables (chapter 3), a new vibration sensor to better monitor the performance of seismic isolators (chapter 4) and the author’s work on similar aspects of the KAGRA gravitational wave detector done during three visits to Japan (chapter 5).

**Compact seismic attenuation system**

The system that suspends the optical tables in Advanced Virgo is called MultiSAS. In the prototype phase (2011 to 2014), Nikhef engineers learned a lot about the mechanical modes of the system with so-called finite element (FEM) and state space models. This resulted in minimal design adjustments and the installation of several different damping strategies. The performance measurements of MultiSAS have not shown any surprises. Following this prototype campaign, five systems were constructed to be installed in Advanced Virgo. The systems behave according to expectations and meet the requirements set by the Advanced Virgo design.

All systems were installed and tested with a dummy mass. After this 2014 campaign, the dummy masses were removed and the MultiSASs were ready to suspend the tables. In the run-up to observation run 2 (O2), all control filters were designed and there were also some other tests done. Examples of these tests are determining if construction tolerances were not detrimental to MultiSAS performance, tests on thermal shielding of certain delicate parts of the mechanics, and determining the maximum pressure allowed.
in the vacuum envelope around MultiSAS in which acoustic effects are not yet visible. Four out of five systems suspended an optical table in O2. SIB2, the injection system suspended bench, is also ready for suspending its optical table, but this was not yet necessary in O2. The remaining four systems, called SNEB, SWEB, SPRB and SDB2, isolate critical optical components for linear and angular alignment. SDB2 - suspended detection bench 2 - also houses the photodiode that captured the GW170814 and GW170817 signals at the end of O2!

The prototype MultiSAS is now used in an advanced sensor and control test bed at Nikhef. MEMS accelerometers and our vibration sensor with interferometric readout are developed on the seismically isolated table. A vibrationally quiet optical table is now also available in Amsterdam for companies outside academia to test sensors.

![Figure S.3: Femtometer precision achieved with Nikhef’s new vibration sensor. Compared to the Sercel L4C geophone (both measurement and specification) and the GeoTech GS13 (specification of the world’s best commercially available vibration sensor), this sensor gives access to vibration measurements of a few millionths of billionths of meters (femtometers, new area is shaded green). The purpose of the sensor is to measure even more accurately the vibrationally quiet locations we create with our seismic isolators.](image)

**Figure S.3:** Femtometer precision achieved with Nikhef’s new vibration sensor. Compared to the Sercel L4C geophone (both measurement and specification) and the GeoTech GS13 (specification of the world’s best commercially available vibration sensor), this sensor gives access to vibration measurements of a few millionths of billionths of meters (femtometers, new area is shaded green). The purpose of the sensor is to measure even more accurately the vibrationally quiet locations we create with our seismic isolators.

**Interferometric readout of a vibration sensor**

Optical tables suspended by MultiSAS are so quiet that the best commercial sensors only measure self-noise from about 5 Hz onwards. Nikhef has proposed a combination of two proven ideas into a vibration sensor with unprecedented sensitivity. Such a sensor is needed to better monitor the motion of a vibrationally isolated object. This vibration sensor has been tested in the MultiSAS test facility and has a self-noise level of $8 \text{ fm/}\sqrt{\text{Hz}}$ from 30 Hz onwards, which as shown in Fig. S.3 is a factor ten more sensitive than the world’s best commercial sensor at 30 Hz.
Additionally, the same interferometric readout has been realized using fiber optic. This readout achieved a preliminary sensitivity of $4 \text{ pm/} \sqrt{\text{Hz}}$ from 5 Hz onwards. The next step is to install this sensor on the MultiSAS optical table in vacuum. The advantage of using fiber optic is that electrical components do not have to be near the sensor mechanics. Such a sensor can thus be installed in radiation or high magnetic field environments, such as in next generation particle accelerators. The readout could even be used as an independent displacement sensor, e.g. for the main mirrors of our detectors.

**Controls for KAGRA’s suspension systems**

The author has visited Japan as part of the ELiTES exchange program during the development and construction phase of the KAGRA gravitational wave detector. At NAOJ (Mitaka, Tokyo) development and testing of controls has been performed on the last stage of so-called Type B(p) vibration suppression systems. The performance of that subsystem is within the requirements of the KAGRA design. Work has been done to improve the sensor part of the OSEM, the combined sensor and actuator used in KAGRA. The sensing part consists of a so-called shadow sensor and requires a light beam that is as homogeneous as possible. Testing all sorts of different LEDs and collimator lenses has improved the design which will ultimately be used in KAGRA.

At the KAGRA site, the first stage of the seismic isolation systems for the main detector elements is a so-called inverted pendulum stage. These systems - designed, assembled and tested at Nikhef - have a very low resonance frequency ($< 0.1 \text{ Hz}$), with the goal of suppressing the microseismic motions of the Earth caused by oceanic activity. Simulations to determine which sensor can be used best to achieve this goal have been performed. The next step is to physically measure the motion predicted by the simulations and decide which sensor to use.

The future of gravitational wave astronomy is bright, or should one say loud?! Hearing the sounds of the dark Universe, after centuries of being deaf, is a blessing for astronomy. New discoveries and further tests of Einstein’s theory of General Relativity are expected. Stay tuned!
Draai die bas omhoog! Prestatieverbetering in het laagfrequente gebied van seismische
verzwakkingssystemen en trillingssensoren voor de zwaartekrachtsgolfdetectoren van de
toekomst.

Ik vind het een groot voorrecht om in deze periode een proefschrift te mogen schrijven in
de zwaartekrachtsgolfphysica. Op deze paar pagina's licht ik graag toe wat er zich in deze
tijd heeft afgespeeld, zowel om mij heen als betreffende mijn eigen onderzoek. Eerst geef ik
een beschrijving en discussie van de grootste resultaten van de LIGO-Virgo Collaboration
en de gebruikte meetmethode. Daarna zoom ik in op het onderzoek bedreven tijdens mijn
promotie.

Zwaartekrachtsgolven, luisteren naar ons universum

De eerste detecties van zwaartekrachtsgolven zijn een doorbraak in de natuurkunde en
astronomie. Net zoals de uitvinding van de telescoop een nieuw tijdperk van ontdekkings
en begrip van het heelal heeft ingeleid, is de verwachting dat de bestudering van
zwaartekrachtsgolven dat ook zal doen. GW150914, GW151222, GW170104, GW170608, GW170814 en
GW170817 (GW staat voor Gravitational Wave en de zes cijfers geven de datum van de detectie aan
in jaar-maand-dag format) hebben ons laten zien dat bestudering van (paren van) zwarte gaten en
neutronensterren nu voor het eerst mogelijk is. Wat echt ongekend is aan deze eerste detecties is
dat het ons de mogelijkheid geeft om het heelal op een totaal nieuwe manier te bestuderen. Het
vermogen van de mensheid om zwaartekrachtsgolven direct te detecteren is te vergelijken met
doof zijn en nu plotseling kunnen horen. Een volledig nieuwe
De gemeten binaire systemen zijn honderden miljoenen tot enkele miljarden jaren geleden tot botsing met elkaar gekomen na een zeer lange dans rond elkaar. Deze golven hebben tot nu gereist om hier op aarde aan te komen. Toen sommige van de gemeten golven begonnen aan hun reis, begon zich hier op aarde net meer-cellig leven te ontwikkelen. Dat is doorgévaluaard naar de mensheid. Binnen die mensheid heeft een genie iets meer dan honderd jaar geleden voorspeld dat er zwaartekrachtsgolven zouden zijn. Dit genie, genaamd Albert Einstein, dacht dat het miniscule effect van die golven, als deze op aarde aankomen, nooit gemeten zou worden. Ook al zijn die golven echt - hij twijfelde of het niet gewoon een wiskundig artifact was - hij dacht dat het technisch onmogelijk was om ze te meten. Ongeveer 50 jaar geleden zijn wetenschappers toch begonnen met het proberen van deze meting en hebben zij uiteindelijk detectoren gebouwd die deze golven konden meten aan het eind van hun kosmische reis.

Figuur S.1: Een nieuwe blik op de zwarte gaten en neutronenster populatie van het universum. Studies gebruikmakend van röntgenstraling hebben een familie van zwarte gaten verzameld (paarse bollen) die bijna allemaal onder de 20 zonnemassa’s zwaar zijn. Onze zwaartekracht detecties bewijzen dat er ook zwaardere zwarte gaten zijn (blauwe bollen). Het is onbekend of het product van de eerste gemeten samensmelting van neutronensterren een zware neutronenster of een licht zwart gat is. Credit: LIGO/Virgo/Northwestern/Frank Elavsky.
Wat leren we van de eerste detecties?

De vijf detecties van binaire zwarte gaten werpen licht op ons begrip van het donkere universum. Zwarte gaten werden daarvoor nooit eerder direct gemeten, laat staan een samensmelting van twee van deze pure ruimte-tijdobjecten. De eerste detectie was de nauwkeurigste afstandsmeting ooit uitgevoerd, namelijk over meer dan 23 orden van grootte. De meting is 50.000 maal accurater dan het verschil in het waterniveau van het IJsselmeer (± 1000 km²) als er één druppel water in gegooid wordt!

De eerste ontdekking van GW150914 liet ons zien dat drie zonnemassa’s in ruimte-tijdsgolfenergie kunnen verdwijnen in 0.2 seconde. Dit was het meest krachtige evenement dat ooit werd gemeten, namelijk 50 keer het uitstralend vermogen van het zichtbare universum (op het piekvermogen van het event). Het evenement werd veroorzaakt door twee zwarte gaten van elk ongeveer 30 zonsmassa’s zwaar. Dat dergelijk zware binaire systemen van zwarte gaten bestonden, was ook nieuw in de sterrenkunde, zoals getoond in Fig. S.1.

De studie van de algemene relativiteitstheorie (lees: de zwaartekracht) kan nu tot een heel nieuw niveau worden getild, namelijk het sterke veldsregime. Met deze nieuwe metingen kunnen data-analisten nu bepaalde parameters meten die in eerder tests niet bereikbaar waren. Hierin hopen zij hints te vinden naar waar de algemene relativiteitstheorie - de beschrijving van al het grote - kan passen in de grotere context van een vereniging met de kwantummechanica - de beschrijving van al het kleine.


De eerste paar detecties zijn met de twee LIGO detectoren gedaan. Met twee detectoren kun je uit de timingsverschillen een ring in de hemel van potentiële source posities halen en deze zijn verstuurd naar (conventionele) astronomen. Uit de signaalsterkteverschillen tussen de twee detectoren - de LIGO detectoren liggen in een verschillend vlak omdat de aarde rond is en de detectoren aan weerszijden van de Verenigde Staten zijn gebouwd - kan die ring iets doorbroken worden, maar het blijft typisch een gebied honderden malen groter dan de maan in de nachthemel. Dit is te zien in Fig. S.2. Met Virgo zie je dit gebied enorm krimpen, zeker als het een sterk
signaal is, en dit geeft meer kans op zogenaamde multi-messenger astronomie, i.e. een complementaire meting met elektromagnetische- en zwaartekrachtsgolven. Voor GW170814 zagen we dit al en drie dagen later, met GW170817, werden voor het eerst zwaartekrachtsgolven van samensmeltenende neutronensterren gemeten. Na deze botsing gloeide er weken lang elektromagnetische straling vanuit een punt in het gebied dat, met behulp van de lokalisering van de LIGO en Virgo detectoren, snel was gevonden door conventionele telescopen. In deze nagloei zagen we voor het eerst het lang getheoriseerde process dat verantwoordelijk is voor de aanwezigheid van zware metalen in ons universum, zoals goud, platinum en uranium!

**Hoe meet je zwaartekrachtsgolven?**

Omdat er een verband bestaat tussen kromming van ruimtetijd en zwaartekracht, zal een zwaartekrachtsgolf de manier veranderen, waarop objecten ten opzichte van elkaar vallen. Wanneer er een zwaartekrachtsgolf twee objecten passeert, zal de er een meetbaar effect zijn. De fysieke afstand tussen de twee objecten zal uitrekken en samentrekken zolang de zwaartekrachtsgolf passeert. Zwaartekrachtsgolven worden gemeten door het zeer nauwkeurig monitoren van de positie en beweging van zogenaamde testmassa's. Hiervoor gebruikt men interferometers; dit zijn kilometers grote laseropstellingen met allerlei optische elementen zoals half-reflecterende spiegels (de zogenaamde bundel-splitter), een zeer krachtige laser (typisch honderden Watt) en zeer hoogreflectieve spiegels, welke in dit geval cilinders van silicium zijn van tientallen kilograms zwaar. De bundel-splitser en spiegels fungeren als testmassa's en de afstand tussen hen wordt constant met een duizelingwekkende precisie gemeten.

Het basis-principe is redelijk gemakkelijk uit te leggen. Een laser schiet een bundel licht, bijvoorbeeld richting het oosten, door een bundel-splitter en 50% van het licht vervolgd zijn pad oostwaarts in de ene arm van de interferometer. De andere 50% kaatst in een hoek van 90° met de andere bundel naar het noorden in de andere arm van de interferometer. Beide bundels komen aan het eind van de arm een hoogreflectieve spiegel tegen en kaatsen terug naar de bundel-splitser. Daar komen de bundels elkaar weer tegen en, als beide armen even lang zijn, doen zij elkaar uit in de zuidelijke richting waar een photo-detector staat; al het licht gaat weer terug in westelijke richting naar de laser. Dit is het resultaat van (destructieve) interferentie; de lichtgolven zijn in anti-fase met elkaar en doven elkaar uit. Als een zwaartekrachtsgolf passeert, zullen beide arm in anti-fase uitrekken en samentrekken. Dit zorgt voor een ander restulaat van de interferentie, omdat de golven elkaar niet meer precies uitdoven. Er arriveren nu kleine lichtflitsjes bij de photo-detector.

Al deze zwaartekrachtsgolfdetectoren zouden niets anders meten dan seismische ruis zonder dat alle elementen van een de detector zijn geïsoleerd van de altijd aanwezige minuscule trillingen van de aarde. Deze trillingen zijn namelijk typisch honderd miljard (!) maal groter dan het te meten effect. Toch voelt u niets van deze altijd aanwezige trillingen die voor zwaartekrachtsgolfjagers vele orden van grootte te veel zijn. De trillingen zijn
honderderden nanometers groot en men probeert nano-nanometer - ofwel attometer, $10^{-18}$ m - afstandsveranderingen te meten in de twee kilometers lange armen van de eerder genoemde interferometers.

Het onderdrukken van trillingen doen wij met harmonische oscillatoren. Om snel te begrijpen wat dat zijn kunt u zich een uitgerolde jojo voorstellen (of deze even snel uit de la pakken!). Pak de jojo aan het uiteinde van het touwtje vast. Uw hand stelt in dit geval de aarde voor - dit zijn de trillingen die u wilt onderdrukken - de jojo is de spiegel (of een ander optisch object) en het gehele systeem gedraagt zich als een harmonische oscillator. Als u uw hand langzaam heen en weer laat gaan is er geen onderdrukking van de trillingen; de jojo beweegt even veel als uw hand. Er is een tempo van bewegen (ofwel frequentie) waarbij u de beweging van de jojo opzwiept; dit heet de resonantiefrequentie en er is sprake van trillingsversterking. Dit is de prijs die we moeten betalen voor het gedrag van het systeem boven de resonantiefrequentie. Beweeg nu uw hand snel heen en weer en u ziet dat de jojo niet volgt; boven de resonantiefrequentie is er sprake van trillingsonderdrukking (en wel met één gedeeld door de frequentie in het kwadraat).

De prijs die we moeten betalen (de resonantiefrequentie) dempen we eruit met regeltechniek; sensoren meten de beweging en een computer vertelt actuatoren (typisch een magnetetje en een elektrische spoel) wanneer er een kracht(je) naar het systeem gestuurd moet worden om de resonanties te dempen. U kunt zich misschien voorstellen dat die regeltechniek continue moet draaien om die enorme interferometers, in de praktijk een complexe optische opstelling met tientallen opgehangen spiegels, uitgelijnd te houden gedurende de metingen. Er zijn honderden zogenaamde feedbackloops die alles in de gaten houden.

Sinds de negentiger jaren is er veel werk verricht om een wereldwijd netwerk op te tuigen. De twee LIGO detectoren in Hanford, Washington en Livingston, Louisiana in de Verenigde Staten van Amerika en de Virgo detector dichtbij Pisa, Italië zijn al operationeel. KAGRA, een ondergrondse detector in Japan (vanaf 2020) en LIGO-India (vanaf 2022) zullen het netwerk gaan versterken. Daarnaast worden er al nieuwe detectoren ontworpen die de gevoeligheid van de detectoren van nu zullen overtreffen. De zogenaamde Einstein Telescope in Europa en de Cosmic Explorer in de Verenigde Staten staan op het programma voor het derde decennium van deze eeuw. Met gevoeligere detectoren kunnen we niet alleen meer detecties per jaar verrichten, maar ook verder (terug) in het Universum kijken!

In Advanced Virgo zijn nu ook de tafels, waarop optica staat die metingen doen om de spiegels beter uit te lijnen, geïsoleerd van de aardse trillingen door ze aan een isolatiesysteem te hangen. De compacte seismische isolatiesystemen daarvoor zijn ontwikkeld op het Nationaal instituut voor subatomaire fysica Nikhef. Dit proefschrift beschrijft aan eigen onderzoek de seismische isolator voor de optische tafels (hoofdstuk 3), een nieuwe trillingssensor om o.a. de prestaties van seismische isolatoren beter te kunnen monitoren (hoofdstuk 4) en het werk van de auteur aan vergelijkbare aspecten van de KAGRA zwaartekrachtsgolfdetector gedaan in drie bezoeken aan Japan (hoofdstuk 5).
Compact seismisch isolatiesysteem


Alle systemen zijn getest met een dummy massa. Na deze campagne in 2014 werden de dummy massa's verwijderd en de MultiSASs waren klaar om de optische tafels op te hangen. In de aanloop naar O2, een observatie run samen met LIGO, is alle regeltechniek ontworpen en er zijn ook een aantal andere tests gedaan. Voorbeelden van deze tests zijn het kijken of constructietoleranties de prestatie van MultiSAS kunnen verminderen, het testen van thermische schilden voor delen van de mechanica en het bepalen van de maximale druk van het vacuüm waarbij akoestische effecten nog niet zichtbaar zijn. Aan vier van de vijf systemen heeft tijdens O2 een optische tafel in vacuüm gehangen. SIB2, de optische tafel van het injectiesysteem, is ook klaar om opgehangen en afgepompt te worden. Dit was nog niet noodzakelijk om aan O2 deel te nemen. De overige vier systemen, SNEB, SWEB, SPRB en SDB2 isoleren kritische optische componenten voor de lineaire- en hoekuitlijning. SDB2 - opgehangen detectietafel 2 - herbergt ook de fotodiodes die de GW170814 en GW170817 signalen aan het einde van O2 hebben opvangen!

De prototype opstelling fungeert nu als geavanceerd sensor- en regeltechniek-testopstelling bij Nikhef. De ontwikkeling van de MEMS accelerometer en de monolithische accelerometer met interferometrische uitlezing wordt gedaan op de seismisch geïsoleerde tafel. De stilste optische tafel in Europa is nu in Amsterdam ook beschikbaar voor commerciële partijen om sensoren te testen.

Interferometrische uitlezing van een trillingssensor

MultiSAS heeft opgehangen optische tafels gerealiseerd, die zo stil zijn dat de beste commerciële sensoren alleen zelfruis meten vanaf 5 Hz. Nikhef heeft een combinatie van twee ideeën voorgesteld om een trillingssensor met ongeëvenaarde gevoeligheid te realiseren. Zo een sensor is nodig om de beweging van het seismisch geïsoleerde object beter te kunnen monitoren. Deze zogenaamde monolitische accelerometer met een interferometrische uitlezing is getest in de MultiSAS testopstelling en heeft een zelfruisvloer van $8 \text{ fm/} \sqrt{\text{Hz}}$ vanaf 30 Hz, hetgeen bij die frequentie een factor 10 gevoeliger is dan de beste commerciële trillingssensor ter wereld, zoals te zien is in Fig. S.3.
Figuur S.3: Femtometer-precisie behaald met Nikhef’s nieuwe trillingssensor. vergeleken met de Sercel L4C geofoon (zowel meting als specificatie) en de GeoT ech GS13 (specificatie van ‘s werelds beste commerciële trillingssensor) geeft deze sensor toegang tot trillsmetingen van enkelen miljoenen van miljardsten van meters (ofwel femtometers, nieuw gebied is groen aangegeven). Het doel van de sensor is de overgebleven trillingen in onze seismisch geïsoleerde objecten nog preciezer te kunnen meten.

Ook is diezelfde sensoruitlezing gerealiseerd, maar dan gebruikmakend van glasvezel. Deze uitlezing behaald een gevoeligheid van 4 pm/√Hz vanaf 10 Hz. De volgende stap is het instalen van deze sensor op de door MultiSAS opgehangen optische tafel in vacuüm. Een dergelijke sensor kan geïnstalleerd worden in stralings- of hoge magnetische veld omgevingen, zoals in de volgende generatie deeltjesversnellers. Het voordeel van het gebruik van glasvezel is dat elektrische componenten niet in de buurt van de mechanica van de sensor hoeven te worden geïnstalleerd. Ook kan de uitlezing als onafhankelijke bewegingssensor gebruikt worden om bijvoorbeeld de spiegels van onze detectoren in de gaten te houden.

Controls voor KAGRA’s ophangingssystemen

De auteur heeft Japan bezocht als onderdeel van het ELTIES uitwisselingsprogramma tijdens de ontwikkeling en constructie fase van de KAGRA zwaartekrachtsgolfdetector. Bij NAOJ (Mitaka, Tokio) is meegeholpen aan ontwikkeling en testen van regeltechniek op de laatste trap van de zogenaamde Type B(p) trillingsonderdrukkende systemen. Hierbij is er werk verricht om het sensor-deel van de OSEM, de combinatie van sensor en actuator gebruikt in KAGRA, te verbeteren. Dit deel bestaat uit een zogenaamde schaduwssensor en hiervoor is een zo homogeen mogelijke lichtbundel nodig. Het testen van allerlei opstellingen met verschillende LEDs en (collimator) lensen heeft het ontwerp, dat uiteindelijk in KAGRA gebruikt zal worden, verbeterd. De prestaties van de control loops vallen binnen de door het KAGRA ontwerp gestelde eisen.
In de berg waar KAGRA wordt gebouwd is de eerste trap van de seismische isolatiesystemen voor de belangrijkste elementen van de detector een zogenaamde inverted pendulum stage. Dit is een systeem met een zeer lage resonantiefrequentie (< 0.1 Hz) met als doel de microseismische bewegingen van de aarde, veroorzaakt door golvenactiviteit van oceanen, te onderdrukken. Simulaties om te bepalen welke sensoren men het beste kan gebruiken om dit doel te behalen zijn uitgevoerd. De volgende stap is het daadwerkelijk meten van de door simulaties voorspelde beweging en te beslissen welke sensor gebruikt moet worden.

De toekomst van zwaartekrachtgolfastronomie is schitterend, of zou men luid moeten zeggen?! Het universum horen, na eeuwen van geluidsloze astronomie, is een zegen voor de sterrenkunde. Nieuwe ontdekkingen en verdere tests van Einstein’s algemene relativiteitstheorie worden verwacht. Houdt u dus het nieuws in de gaten!
Alza quel basso! Miglioramento delle prestazioni nel campo delle basse frequenze dei sistemi di attenuazione sismica e dei sensori di vibrazione per i rilevatori di onde gravitazionali di prossima generazione.

Penso che sia un gran privilegio poter scrivere una tesi sulla fisica delle onde gravitazionali in questo periodo. In queste poche pagine vorrei spiegare cos'è successo in questi ultimi anni. Per prima cosa illustrerò i grandi risultati dalla collaborazione LIGO-Virgo ed il metodo di misurazione. Successivamente approfondirò la ricerca conseguita durante il mio dottorato.

Onde gravitazionali, ascoltando il nostro universo

I primi rilevamenti delle onde gravitazionali sono una svolta nella fisica e nell'astronomia. Proprio come l'invenzione del telescopio ha aperto una nuova era di scoperte e comprensione dell'Universo, l'ulteriore studio delle onde gravitazionali farà altrettanto. GW150914, GW151222, GW170104, GW170608, GW170814 e GW170817 (GW sta per Gravitational Wave e le sei cifre indicano la data del rilevamento in formato anno-mese-giorno) hanno dimostrato che lo studio di coppie di buchi neri e di stelle di neutroni è ora possibile grazie alle onde gravitazionali. Ciò che è storico in queste prime rilevazioni è l'opportunità di studiare l'Universo in un modo completamente nuovo ora possibile per l'umanità. La capacità di rilevare direttamente le onde gravitazionali può essere paragonata a quella di essere sordi ed improvvisamente essere in grado di udire. Uno strumento completamente nuovo per misure astrofisiche è ora disponibile! I progetti per una prossima generazione di rilevatori di onde gravitazionali sono in fase avanzata.
I sistemi binari osservati sono entrati in collisione da centinaia di milioni a diversi miliardi di anni fa dopo una lunga danza l’uno intorno all’altro. Queste onde hanno viaggiato fino ad ora per arrivare qui sulla Terra. Quando le onde gravitazionali che abbiamo osservato sono state emesse, qui sulla Terra la vita multicellulare aveva appena iniziato a svilupparsi. Questa si è poi evoluta nel genere umano. Tra questi individui, un genio ha predetto poco più di cento anni fa l’esistenza delle onde gravitazionali. Questo genio, chiamato Albert Einstein, pensava che l’effetto minuscolo di quelle onde sulla Terra non sarebbe mai stato misurato. Anche se quelle onde fossero state reali (Einstein spesso dubitò che fossero semplicemente un artefatto matematico), sarebbe stato troppo difficile misurarle. Circa 50 anni fa, gli scienziati hanno iniziato a provare a misurare questi segnali e ad un certo punto hanno costruito dei rilevatori in grado di osservare queste onde alla fine del loro viaggio cosmico.

Cosa impariamo dalle prime rilevazioni?

Le cinque rilevazioni di buchi neri binari fanno luce sulla nostra comprensione dell’SUniverso oscuro. I buchi neri non erano mai stati osservati direttamente prima, per non parlare della fusione di due di questi puri oggetti spazio-temporali. La prima rilevazione è stata la misurazione di distanza più accurata mai effettuata, vale a dire una parte su $10^{23}$. L'accuratezza di questa misurazione è simile al determinare la distanza da qui alla stella più vicina fuori dal sistema solare (Alpha Centauri, 4.32 anni luce di distanza) con la precisione pari alla larghezza di un capello umano.

L’evento GW150914 ci ha mostrato come l’energia corrispondente a tre masse solari possa scomparire in 0.2 secondi e trasformarsi in perturbazioni dello spazio-tempo che attraversano l’SUniverso alla velocità della luce. Questo è stato l’evento più potente mai osservato, corrispondente al suo picco a 50 volte la potenza radiante dell’SUniverso visibile. L’esistenza di sistemi binari di buchi neri così pesanti è anch’essa una novità per gli astronomi, come mostrato in Fig. R.1.

Lo studio della relatività generale (ovvero della gravitazione) può ora essere esteso ad un livello completamente nuovo, il cosiddetto regime di campo forte. Tuttavia, grazie a questo nuovo tipo di osservazioni, gli analisti di dati confidano di poter trovare indicazioni di una teoria che unifichi relatività generale - la descrizione di cose macroscopiche - e meccanica quantistica - la descrizione di cose microscopiche.

Figura R.2: Localizzazione del cielo notturno delle rilevazioni di onde gravitazionali di Advanced LIGO (GW150914, LVT151012, GW151226 en GW170104) e in combinazione con Advanced Virgo (GW170814 e GW170817) realizzato dalla collaborazione LIGO-Virgo. Credit: LIGO / Virgo / NASA / Leo Singer / Alex Mellenger.

Con due rilevatori, in base alla differenza di tempo di arrivo del segnale, è possibile delimitare una regione nel cielo di posizioni potenziali di origine. Grazie a Virgo l’area di tale regione può essere sensibilmente ridotta rendendo possibile anche la cosiddetta astronomia multi-messaggera, ovvero una misura complementare con onde elettromagnetiche e gravitazionali. Questo è stato riscontrato sia nel caso di GW170814, sia tre giorni dopo, con GW170817, quando le onde gravitazionali prodotte dalla fusione di due stelle di neutroni sono state osservate per la prima volta. Grazie alla precisa localizzazione fornita dai due rilevatori di onde gravitazionali, i telescopi convenzionali hanno potuto rapidamente individuare il bagliore prodotto dalla collisione. Questo si può
anche vedere nella Fig. R.2. Dall’analisi spettroscopica del bagliore che ha seguito la collisione siamo stati in grado di osservare il processo, fino ad ora soltanto teorizzato, responsabile dell’abbondanza di elementi più pesanti del ferro nell’Universo, come per esempio oro, platino e uranio.

Come vengono misurate le onde gravitazionali?

Poichè esiste una connessione tra la curvatura dello spazio-tempo e la gravită, un’onda gravitazionale cambierà il modo in cui due oggetti cadono l’uno rispetto all’altro. Quando un’onda gravitazionale incontra due oggetti produce un effetto misurabile. La distanza fisica tra i due oggetti si allungherà e si contrarrà finché passerà l’onda gravitazionale. Le onde gravitazionali sono osservate misurando in modo molto accurato la posizione ed il movimento di cosiddette masse di prova. A tale scopo vengono utilizzati interferometri; questi sono sistemi laser su scala chilometrica con componenti ottici di ogni genere come specchi semi-riflettenti (il cosiddetto beam splitter), un potente laser (tipicamente capace di emettere centinaia di watt), e specchi altamente riflettenti che consistono in cilindri di quarzo di alcune decine di chilogrammi di peso. Il divisore di fascio e gli specchi funzionano come masse di prova e la distanza tra loro viene costantemente misurata con precisione incredibile.

Il principio di base è abbastanza facile da spiegare. Un laser emette un raggio di luce, ad esempio verso est, attraverso un beam splitter cosicché il 50% della luce continua il suo percorso all’interno di uno dei bracci dell’interferometro. L’altro 50% rimbalza e viene deviato di un angolo di 90° in direzione nord all’interno dell’altro braccio dell’interferometro. Entrambi i fasci di luce incontrano uno specchio altamente riflettente e tornano indietro verso il beam splitter dove i fasci si incontrano di nuovo. Quando gli specchi si trovano alla stessa distanza dal beam splitter i fasci di luce si estinguono a vicenda nella direzione sud dove si trova un fotorilevatore; tutta la luce ritorna al laser in direzione ovest. Questo è il risultato dell’interferenza (distruttiva); le onde luminose che escono dal beam splitter in direzione sud sono in controparte l’una rispetto all’altra e si estinguono a vicenda. Quando passa un’onda gravitazionale, entrambi i bracci dell’interferometro si allungano e si contraggono in controparte. Le onde luminose non si estinguono più completamente e piccoli flash di luce raggiungono il fotorilevatore.

I rilevatori gravitazionali non misurerebbero altro che rumore sismico ambientale, se tutti i loro elementi non fossero isolati dalle incessanti microscopiche vibrazioni della crosta terrestre. L’ampiezza di queste vibrazioni è infatti in genere un centinaio di miliardi (!) di volte superiore al movimento degli specchi causato da un segnale gravitazionale. La soppressione delle vibrazioni sismiche è quindi essenziale e, nella sua realizzazione pratica, sfrutta la risposta di un oscillatore armonico. Per capire rapidamente di cosa si tratta, potete immaginare uno yo-yo srotolato (o tiratelo fuori dal vostro cassetto!). Afferrate lo yo-yo all’estremità del filo. La vostra mano è la sorgente delle vibrazioni che volete sopprimere, lo yo-yo è lo specchio (o un altro componente ottico) e l’intero sistema si comporta come un oscillatore armonico. Se muovete
lentamente la mano avanti e indietro, non c’è soppressione delle vibrazioni; lo yo-yo si muove tanto quanto la vostra mano. Tuttavia c’è una specifica velocità di movimento (o frequenza) della vostra mano per cui lo yo-yo si muove moltissimo; questa è chiamata la frequenza di risonanza, in corrispondenza della quale le vibrazioni della mano vengono amplificate. Questo è il prezzo che dobbiamo pagare per beneficiare del comportamento del sistema al di sopra della frequenza di risonanza. Ora muovete ancora più rapidamente la mano avanti e indietro e vedrete che lo yo-yo non vi segue più; al di sopra della frequenza di risonanza le vibrazioni vengono attenuate.

L’effetto del prezzo da pagare (frequenza di risonanza) viene mitigato da un sistema di controllo: sensori misurano il movimento ed un computer comanda degli attuatori (tipicamente un magnet e una bobina) quando una (piccola) forza deve essere inviata al sistema per smorzare le vibrazioni dell’oscillatore alla sua frequenza di risonanza. Potete facilmente immaginare che i sistemi di controllo debbano funzionare continuamente per mantenere quegli enormi interferometri, in pratica complessi sistemi ottici con numerosi specchi sospesi, allineati durante le misurazioni. Tipicamente in un interferometro ci sono centinaia di cosiddetti loop di feedback che tengono d’occhio tutto.

Fin dagli anni ’90, si è lavorato molto per costruire una rete mondiale di osservatori gravitazionali. I due rilevatori LIGO di Hanford, Washington e Livingston, Louisiana negli Stati Uniti e il rilevatore Virgo vicino a Pisa, sono già operativi. KAGRA, un rilevatore sotterraneo in Giappone (dal 2020) e LIGO-India (dal 2022) rafforzeranno la rete, mentre verranno sviluppati nuovi rilevatori che supereranno di gran lunga la sensibilità dei dispositivi odierni. Il cosiddetto Telescopio Einstein in Europa ed il Cosmic Explorer negli Stati Uniti dovrebbero entrare in funzione alla fine della terza o all’inizio della quarta decade di questo secolo. Con rilevatori sempre più sensibili non solo potremo aumentare la frequenza degli eventi osservati, ma anche guardare oltre (o indietro) nell’Universo!

In Advanced Virgo ci sono ora banchi sui quali si trovano sensori ottici per l’allineamento degli specchi principali che devono essere isolati dalle vibrazioni del terreno. Per questo scopo sistemi compatti di isolamento sismico sono stati sviluppati dall’Istituto Nazionale di Fisica Subatomica Nikhef. Questa tesi descrive l’isolatore sismico per i banchi ottici (capitolo 3), un nuovo sensore di vibrazione per monitorare meglio le prestazioni degli isolatori sismici (capitolo 4) ed il lavoro svolto dall’autore su aspetti simili del rilevatore di onde gravitazionali KAGRA durante le sue tre visite in Giappone (capitolo 5).

**Sistema di isolamento sismico compatto**

Il sistema di sospensione dei banchi ottici di Advanced Virgo si chiama MultiSAS. Nella fase di prototipo (dal 2011 fino al 2014 incluso), gli ingegneri presso Nikhef hanno caratterizzato in dettaglio i modi meccanici del sistema con modelli ad elementi finiti (FEM) e rappresentazioni in spazio di stato. Lo studio è risultato in lievi modifiche al
progetto iniziale nell'introduzione di vari dispositivi di smorzamento. Le misurazioni della
funzione di trasferimento di tutti gli stadi di attenuazioni del MultiSAS non hanno
mostrato sorprese. Successivamente al prototipo, cinque isolatori sismici sono stati
costritti. I sistemi si comportano in base alle aspettative e ai requisiti stabiliti dal design
di Advanced Virgo.

Tutti i sistemi sono stati poi installati e testati con un carico di prova. Dopo questa fase
del 2014, i carichi di prova sono stati rimossi e i MultiSASs erano pronti per sospendere
i banchi. Durante il periodo precedente la fase di osservazione 2 (O2), sono stati
progettati tutti i filtri di controllo e svolti numerosi altri test. Per esempio è stato misurato
il cosiddetto effetto culla, è stato studiato il comportamento degli schermi termici, ed è
stato determinato il livello minimo di vuoto al di sotto del quale gli effetti dei disturbi
acustici non sono più riscontrabili. Quattro dei cinque sistemi hanno sospeso un banco
ottico nel corso di O2. SIB2, il banco ottico del sistema di iniezione, era anch'esso
pronto per essere sospeso e messo sotto vuoto, ma questo non è stato necessario dato
il ridotto livello di sensibilità di Virgo durante O2. I rimanenti quattro sistemi, SNEB,
SWEB, SPRB e SDB2 hanno isolato componenti ottici critici per l'allineamento
longitudinale ed angolare dell'interferometro. SDB2 - banco sospeso di rilevazione 2 -
ospita anche i fotorilevatori che hanno catturato i segnali GW170814 e GW170817 alla
fine di O2!

Il prototipo di MultiSAS presso Nikhef è ora impiegato come banco di prova per sensori
sismici avanzati e per lo sviluppo di tecniche di controllo più efficienti. Gli accelerometri
MEMS e l'accelerometro monolitico con lettura interferometrica vengono sviluppati sul
banco sospeso dal MultiSAS. Tale banco di prova sospeso silenzioso è ora disponibile
ad Amsterdam anche per l'industria per lo sviluppo di sensori.

**Lettura interferometrica di un sensore di vibrazione**

I banchi ottici sospesi dal MultiSAS sono così silenziosi che i migliori sismometri esistenti
non sono in grado di misurarne le vibrazioni. Nikhef ha proposto una combinazione di due
idee per creare un sensore di vibrazione che potesse raggiungere una sensibilità senza
precedenti. Tale sensore era necessario per caratterizzare le prestazioni degli attenuatori
sismici. Questo sensore di vibrazione è stato testato sul banco sospeso dal prototipo di
MultiSAS ed ha mostrato un rumore di fondo di \(8 \text{ fm} / \sqrt{\text{Hz}}\) da 30 Hz, dieci volte più
basso alla stessa frequenza rispetto al miglior sismometro commerciale esistente, come
mostrato in Fig. R.3.

Inoltre è stata realizzata la stessa lettura del sensore utilizzando la fibra di vetro.
Questa lettura ha raggiunto una sensibilità di \(4 \text{ pm} / \sqrt{\text{Hz}}\) da 10 Hz. Il prossimo passo è
quello di installare questo sensore sul tavolo ottico sospeso nel MultiSAS. Tale sensore
può essere installato in radianti o in ambienti di campo magnetico, come ad esempio
nella prossima generazione di acceleratori di particelle. Il vantaggio dell'uso di fibre
ottiche è che i componenti elettrici non devono trovarsi in prossimità delle meccaniche

Controlli per i sistemi di sospensione KAGRA

L’autore ha visitato il Giappone nell’ambito del programma di scambio ELITES durante la fase di sviluppo e costruzione del rilevatore di onde gravitazionali KAGRA. Al NAOJ (Mitaka, Tokyo) ha contribuito a sviluppare e testare i controlli dell’ultimo stadio dei sistemi di soppressione delle vibrazioni di tipo B(p). Le prestazioni dei loop di controllo sono risultate conformi alle specificazioni di progetto. Inoltre l’autore ha lavorato al miglioramento della parte di lettura dell’OSEM, il sensore/attuatore combinato utilizzato per i controlli. La parte di lettura dell’OSEM consiste in un cosiddetto sensore d’ombra e richiede un fascio di luce il più omogeneo possibile. Il test di tutti i tipi di configurazione con diversi LED e lenti a collimatore ha migliorato il design, che è stato poi adottato in tutti i sistemi meccanici di KAGRA.

Il primo stadio dei sistemi di isolamento sismico per gli elementi principali del rilevatore presso KAGRA è un cosiddetto stadio di pendolo invertito. Questi sistemi, progettati, assemblati e collaudati presso Nikhef, hanno una frequenza di risonanza molto bassa (< 0.1 Hz), che ha lo scopo di sopprimere i movimenti microsismici della terra causati dalle onde degli oceani. L’autore ha eseguito simulazioni per determinare quali sensori fossero più idonei per il controllo dei pendoli invertiti.
Il futuro dell’astronomia gravitazionale è luminoso, o meglio dire rumoroso?! Sentire il suono dell’Universo oscuro, dopo decenni di sordità, è una benedizione per l’astronomia. Nuove scoperte e verifiche sempre più importanti della teoria delle relatività generale di Einstein sono ora alla nostra portata!