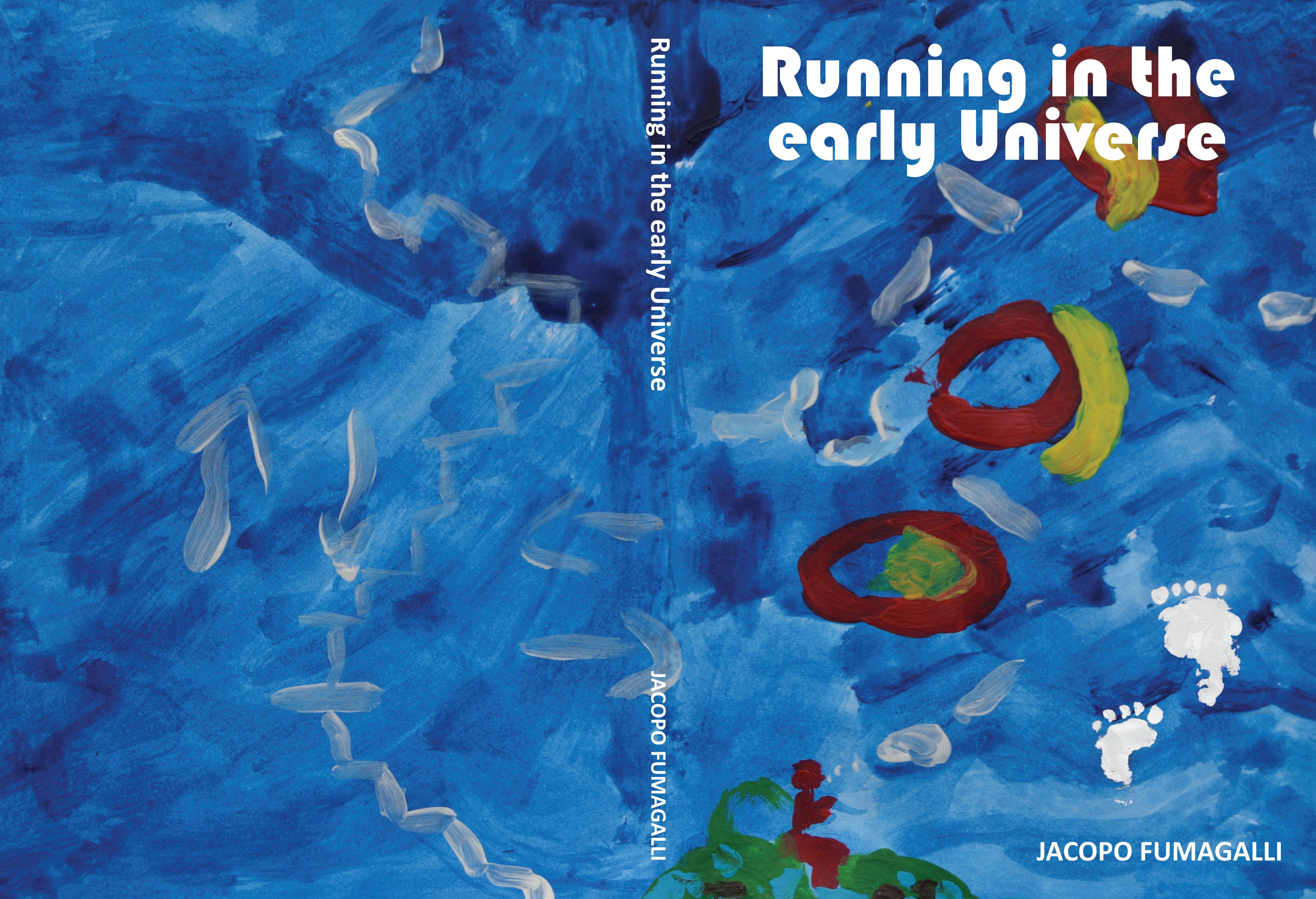
Running in the early Universe

Fumagalli, J.



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UV sensitivity of single-field inflationary models

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This work is part of the research programme of the Foundation for Fundamental Research on Matter (FOM), which is part of the Netherlands Organisation for Scientific Research (NWO). The research has been accomplished at the National Institute for Subatomic Physics (Nikhef).

Title:	Running in the early Universe
ISBN:	978-94-6380-041-9
Printed by:	ProefschriftMaken - Vianen
Cover:	Painting by Marianna Bodini

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Academisch Proefschrift

ter verkrijging van de graad van doctor

aan de Universiteit van Amsterdam

op gezag van de Rector Magnificus

prof. dr. ir. K.I.J. Maex

ten overstaan van een door het College voor Promoties

ingestelde commissie,

in het openbaar te verdedigen in de Agnietenkapel

op donderdag 25 oktober 2018, te 12.00 uur

door

Jacopo Fumagalli

geboren te Cremona, Italië

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Faculteit der Natuurwetenschappen, Wiskunde en Informatica

a Benedetta (geweldig),

Contents

Publications

This thesis is based on the following publications:

[1] Jacopo Fumagalli and Marieke Postma
"UV (in)sensitivity of Higgs inflation",
JHEP 05 (6), 049, `arXiv:1602.07234 [hep-ph]`.

Presented in chapter **3** (part in chapter **2**).

[2] Jacopo Fumagalli,
"Renormalization Group independence of Cosmological Attractors",
Phys. Lett. B **B769** (2017) 451-459, 044001, `arXiv:1611.04997 [hep-th]`.

Presented in chapter **4**.

[3] Jacopo Fumagalli, Sander Mooij and Marieke Postma
"Unitarity and predictiveness in new Higgs inflation",
JHEP 03 (2018), 038, `arXiv:1711.08761 [hep-ph]`.

Presented in chapter **5** (part in chapter **2**).

[4] Jacopo Fumagalli, Marieke Postma and Melvin van den Bout
"Running new Higgs inflation",
to appear.

Presented in chapter **5**.

Contribution of the author to the publications:

The author participated to all the conceptual discussions in all the publications. In [1] the author contributed to section 2, to part of section 3 and the two appendices A and B. In [3] the author contributed to section 3, to part of section 4 and to section 5. In the work in preparation [4] the author contributed to the conceptual aspects of the method used and to the numerical implementation, with results presented in this thesis in chapter 5.

Preface

> *"Research is a beautiful and terrible woman who reveals all of your limits and spurs you to the most absurd peaks"*
>
> G.R.

This thesis explores the intriguing possibility of connecting measurements taken at the particle accelerator at CERN (LHC) to observations of the sky, coming from satellites and telescopes, which carry information on the primordial Universe.

A beautiful story

Copernicus has taught us that we have no privileged spot in the Universe. From down here on earth, we are nevertheless able to describe the Cosmos quantitatively from a fraction of a second after the Big Bang until today, 13.8 billion years later.[1]

One of the main achievements of the hot Big Bang scenario is the prediction of an omnipresent background of photons, the afterglow of the Big Bang. This is known as the Cosmic Microwave Background (CMB) and it has been measured with incredible accuracy. Its mapping is the oldest picture we have of our Universe, a snapshot of when it was roughly 380.000 years old. At that time, the Universe, as a result of its expansion, had cooled sufficiently to allow photons to stream freely for the first time.

Inflation is the idea that the very early Universe went through a phase of exponential acceleration. The theory was initially proposed to solve puzzles of the standard Big Bang cosmology such as the unexplained homogeneity of the CMB,

[1] The question of what happened at the initial time belongs for the moment to the realm of philosophy.

the observed flatness of the Universe and the absence of magnetic monopoles. The surprising thing, and the reason why cosmologists really like this theory, is that it also opens a window into the very early Universe. If inflation indeed happened, and there are very good reasons to think so, quantum fluctuations produced during that epoch are responsible for the observed temperature inhomogeneities in the CMB. Moreover, these inhomogeneities are the seeds for all structures we see today in our Universe, such as clusters, galaxies and, ultimately, our planet. In other words, the inflationary era (about 10^{-37} s after the Big Bang) left imprints on the CMB (380.000 yr) and in large scale structure that can be measured by our satellites and telescopes today. Relating something we can observe today to what could have happened at the very beginning of the Universe is one of the most fascinating aspects of cosmology, perhaps even of science in general.

This story so far is very beautiful, but it is not all roses. After almost 40 years since its original formulation [5],[2] the physical origin of inflation still remains an open question. This is not due to a lack of candidates. On the contrary, the rather small amount of data that needs to be explained[3] has given rise to an explosion of proposals able to account for the CMB measurements. However, theoretical physics is about more than just having a model that does not contradict the experiments. If measurements do not help resolve the degeneracy between different proposals one may look for new observables. In addition, not all models are on equal theoretical footing, and one can use this as a guide to move forward.

We try to navigate through the inflationary landscape following this latter approach. We thus demand that successful inflationary models satisfy some basic requirements. First, they have to be, at the very least, *consistent* within the framework in which they are formulated, i.e. quantum field theory (QFT) and general relativity (GR). Second, our guiding principle will be *minimality*. "Entia non sunt multiplicanda praeter necessitatem".[4] In theoretical physics this translates as: use the smallest possible amount of ingredients (fields, parameters) to describe phenomena. Minimality might not be the path Nature has chosen to describe inflation but it is worth to see what can be learned by just requesting that theories have to be simple. Third, we want our models to be *predictive*, in the sense that the number of observables predicted is greater than the number of parameters required to specify the model. Pushing minimality to its extreme means, for example, trying to explain and link inflation to known physics that has

[2] An optimist may argue that this is actually nothing compared to the cosmological time scales.

[3] At the moment there are only three measured numbers that can be related to the very early Universe. These are the scalar power spectrum $P_{\mathcal{R}}$, the spectral index n_s and the number of e-folds N.

[4] Ockham's Razor: "entities must not be multiplied beyond necessity".

been measured and tested at low energy. A natural question to ask is then: can the requirements of consistency and minimality have an impact on the predictive quality of the model?

Particle accelerators such as the Large Hadron Collider (LHC) have deepened our understanding of Nature's building blocks. This has led to the Standard Model of particle physics, which provides a unified and elegant description of the (known) elementary particles and their interactions. The discovery of its last missing ingredient, the Higgs boson, was announced by the Atlas and CMS collaborations on the 4^{th} of July 2012, after a hunt that lasted more than fifty years. In the pursuit of minimality, models that use the Higgs boson to describe inflation are therefore the prime example. Specifically, they trigger the hope of connecting parameters measured at the LHC to cosmological data. However, the typical energy scales at which inflation took place (10^{12-15} GeV) are many orders of magnitude above those probed with collider physics (10^3 GeV). The questions already on the tip of the readers's tongue are those that lead directly to the work of this thesis: can we consistently extrapolate physics from one to the other regime? Can the predictions be sensitive to the unknown physics in between these two scales?

Why study models?

The inflationary paradigm is still awaiting its definitive confirmation. For instance, the primordial gravitational waves predicted by the theory have not been observed so far. Thus, confirming inflation would already be a huge achievement on its own. In this light, one might wonder to what extent studying a particular model is relevant. Probably the desire to pinpoint the "exact model" is bound to be, at present, hopeless. However, this is not what we are trying to do here.[5]

In this thesis we focus mainly on Higgs inflationary models. Looking for the consistency of such a minimal proposal is more than an academic exercise. From a particle physics perspective, if inflation cannot be simply realized within the Standard Model (SM), it would be a strong indication that physics beyond the SM is required. This is one of the main motivations for taking minimality as a guiding principle. Despite their minimal set-up, both the original Higgs inflation (HI) [6] and the alternative new Higgs inflation proposal [7] (NHI) need an extra non-minimal Higgs-gravity coupling.[6] Once quantum effects are considered, this

[5] Even if someone might think: "Excusation non petita, accusatio manifesta".

[6] In the original Higgs inflation (HI) proposal a non-minimal coupling between the Higgs and the Ricci tensor is introduced, in new Higgs inflation (NHI) the Higgs kinetic terms are non-minimally coupled to the Einstein tensor.

single non-renormalizable operator is enough to complicate the story considerably. Thus, these models become an excellent training ground to explore many non-trivial aspects of a non-renormalizable quantum field theory, that we now briefly summarize. The UV cutoff of the theory is lowered well below the Planck mass and it depends on the background field value. This raises questions on the naturalness and consistency of the theory. In both the small field SM regime and the large field inflationary regime a renormalizable effective field theory (EFT) can be defined below the cutoff [8, 9]. Even if the original motivation for Higgs inflation was minimality, the study of the renormalization group flow indicates that, when moving from one regime to the other, unknown threshold corrections become important, and this fact should be included in the analysis.

Consequently, the requirement of a consistent quantum theory introduces a UV sensitivity to these models, which is linked to their non-renormalizable character. The effects of UV physics can be parameterized by higher order operators in the EFT. The natural question to ask is then whether these new operators, as well as any effects from the UV completion entering via the renormalization group (RG) flow, will undermine the predictive power of these inflationary scenarios. Hence, one of our main goals is to analyze in detail a kind of UV sensitivity that might in principle affect any particle physics model of inflation. Note that the RG dependence of the observables can be both a curse and a blessing. A blessing because it could in principle shed light on the UV completion of the inflationary theory. On the other hand, if a model depends strongly on unknown UV corrections, it will lose any predictive power.

Thus, while awaiting for experimental developments in the CMB measurements and theoretical advancements towards defining a complete UV theory, it is highly interesting to know the possible effects that might arise from unknown physics.

Outline of the thesis

The subject of this thesis is at the interface between particle physics and cosmology. In chapter 1 we introduce basic concepts of cosmology and inflation, followed by an introduction to the kind of UV sensitivity studied in this thesis. Chapter 2 collects basic aspects of the renormalization group flow together with some more advanced notions taken from our works [1, 3], that are relevant for the understanding of the rest of the thesis. The core of the thesis is in the following three chapters (based on [1, 2, 3, 4]). In chapter 3 we consider the Standard Higgs inflation proposal as an example to illustrate the possible obstacles in connecting low energy physics to inflation. Then we compute the sensitivity of this scenario to the unknown

UV completion. In chapter 4, we step aside from minimality with the attempt to gain a more general understanding of the issues at hand. Particularly, we study the RG behavior of the wide class of Cosmological Attractors, highlighting a nice feature that makes all these models almost UV independent. Finally in chapter 5, we analyze the quantum aspects of the alternative new Higgs inflation proposal. This is the place to describe quantitative (both analytically and numerically) the difference in UV dependence between a generic model and the class of Cosmological Attractors (of which the original Higgs inflation scenario is a particular case). We compute all the ingredients needed for the analysis, such as unitarity bounds and beta-functions in the large field regime, in a way that can easily be generalized to other particle physics model of inflation. We conclude and give an outlook in chapter 6.

Jacopo Fumagalli

1 Inflation and UV sensitivity

Cosmology is the branch of physics whose modest purpose is to understand our Universe as a whole. In this chapter we briefly review why the modern approach to cosmology needs to assume a period of accelerating expansion in the early Universe, called *Inflation*. The inflationary theory provides predictions that we can test with observations of the Cosmic Microwave Background. We end with a discussion of the standard sensitivity of inflation to high energy degrees of freedom, the so-called eta problem, and we compare this to the UV dependence studied in this thesis.

As any thesis on cosmology we cannot escape from starting with the two pillars of the subject. The first is the Cosmological principle implemented via the FLRW metrics, and the second is the theory of general relativity which via the Einstein equations governs the dynamics of the space-time. The following sections do not pretend to be complete in any way.[1] The aim is just to provide in a short summary the tools to appreciate the next chapters.

1.1 Cosmological simplicity

The modern point of view begins with the fundamental assumption, known as the *Cosmological Principle*: at large scales the Universe is homogeneous and isotropic. Intuitively: there is no special point in the Universe and in any direction the Universe looks the same (on large scales). In the earliest work of Einstein this was a theoretical guess to simplify the physics. Around that time studies by Hubble [13] supported this thesis, as they showed that the known distribution of galaxies was indeed on large scale isotropic.

[1] For a real understanding of the concepts the reader should rather look here [10, 11, 12].

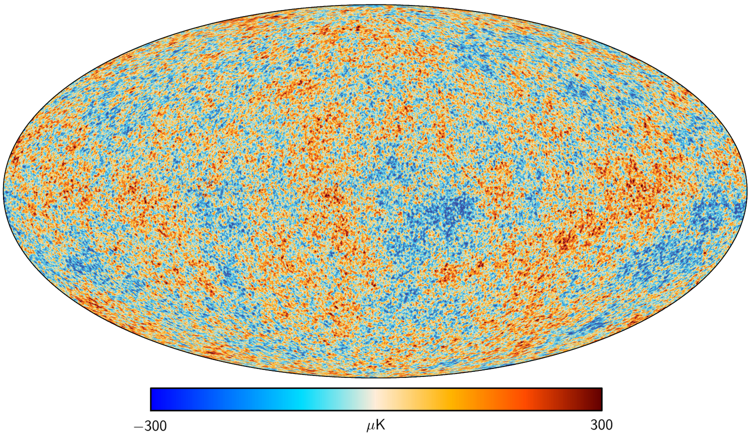


Figure 1.1: *CMB spectrum from the 2018 Planck release.*

At the end of the twentieth century the cosmological principle was empirically verified by refinements in the observation of the *Cosmic Microwave Background* (CMB). When the Universe was 380.000 years old, at the epoch of last scattering, free electrons were caught by protons and photons began to stream freely. The Universe was transparent for the first time. Some of these photons reach our telescopes today giving us the opportunity to take a look into the baby Universe. This is what we observe today as the CMB. Almost all cosmological parameters are determined by its study. As shown by the Planck data in fig. 1.1, the temperature of the radiation is the same in all directions of the sky (up to tiny anisotropies)

$$T_{\mathrm{CMB}} = 2.73 \pm 10^{-4}\ \mathrm{K}. \tag{1.1}$$

The remarkable uniformity of the CMB radiation indicates that at last scattering the universe was to a high degree of precision (order of 10^{-5}) isotropic.[2] Homogeneity cannot rigorously be tested. However, unless we are not in a special place in the Universe (accepting the Copernican Principle), the observed isotropy implies homogeneity.

The possibility to use the Cosmological principle simplifies enormously the solutions of the Einstein theory of gravity[14] which is the second ingredient on which modern cosmology is based.

[2] Nowadays we can still consider the Universe homogeneous and isotropic at scales of the order of 100 Mpc (the typical distance between two galaxies is of the order of 200 Kpc).

1.1.1 The dynamics of the Universe

The dynamics of the Universe is determined by the *Einstein equations*

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = m_{\rm P}^{-2} T_{\mu\nu}. \tag{1.2}$$

These are ten second order nonlinear partial differential equations which relate the curvature of the space time (left side) to the energy-matter content (right side). $T_{\mu\nu}$ is the energy momentum tensor.

Under some generic and reasonable assumptions on the manifold M describing our Universe, the most general metrics under the hypothesis of homogeneity and isotropy are known as the *Friedmann-Lemaitre-Robertson-Walker metrics.*[3]

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2 \right], \tag{1.3}$$

where (t, r, ϕ, θ) are referred to as comoving coordinates. The name derives from the fact that a point labeled with (r, θ, ϕ) will be labeled in the same way through the evolution of the Universe. Without loss of generality (via a rescaling of the coordinates) k can assume three values: $k = 1$ (spatial section Σ isometric to $\mathbb{S}^3$), $k = 0$ (Σ isometric to $\mathbb{R}^3$), $k = -1$ (Σ isometric to $\mathbb{H}^3$). $a(t)$ is the scale factor, it has dimensions of length and carries information on the expansion of the Universe.

The great simplification due to the Cosmological Principle is immediately visible, the only function that has to be determined to find the dynamics is $a(t)$. The simplified form of the Einstein tensor due to homogeneity and isotropy implies that T_μ^ν must be also diagonal, with time-dependent components. In covariant form

$$T_\mu^\nu = (\rho(t) + P(t)) u_\mu u^\nu + P \delta_\nu^\mu \tag{1.4}$$

which is the energy momentum tensor of a perfect fluid with 4-velocity u^μ. In comoving coordinates $u^\mu = (1, 0, 0, 0)$, which means that in this frame the fluid is at rest. ρ and P are the energy density and pressure. As usual the Bianchi identities imply the continuity equation

$$\nabla^\mu T_{\mu\nu} = 0 \implies \dot{\rho} + 3H(\rho + P) = 0, \tag{1.5}$$

[3] These are a consequence of the Killing-Hopf theorem. Let M a Riemannian manifold (we are considering the spatial sections), connected, geodesically complete and with constant sectional curvature (implied by the cosmological Principle hypothesis). Then , if M is also simply connected, it is isometric to one of the following manifolds: a) $\mathbb{S}^n$ (n-sphere) $(k > 0)$, b) $(\mathbb{R}, \delta)$ $(k = 0)$, c) $\mathbb{H}^n$ (hyperboloid) $(k < 0)$.

and the (00) component of the Einstein's equation gives[4]

$$H^2 = \frac{\rho}{3m_{\rm P}^2} - \frac{k}{a^2}, \tag{1.6}$$

which is referred to as the *Friedmann equation*. Most of the cosmic fluid can be described by a simple *equation of state* of the form[5]

$$P = w\rho. \tag{1.7}$$

For ordinary non-relativistic matter, which can be approximated as dust, the pressure is negligible and $w_M = 0$. For relativistic particles, such as radiation, $w_R = 1/3$ while for a vacuum energy/cosmological constant contribution $T_{\mu\nu} = \Lambda g_{\mu\nu}$, $w_\Lambda = -1$. Using the equation of state we can easily integrate the continuity equation

$$\rho_i \propto a^{-3(1+w_i)} \tag{1.8}$$

which gives $\rho_M \propto a^{-3}$ (Matter), $\rho_R \propto a^{-4}$ (Radiation) and $\rho_\Lambda =$ constant for the weird case in which $w = -1$. Thus, by measuring the relative densities of each component today, we can trace back their abundance at earlier times. Using (1.8) in the Friedmann equation (1.6) we obtain

$$\dot{a}^2 = \sum_i c_i a^{-(1+3w_i)} - k. \tag{1.9}$$

For small $a(t)$ it is easy to note how the main contribution on the right hand side comes from radiation ($w = 1/3$) and one finds oneself in the *radiation dominated era* (RD), $a \sim t^{1/2}$. When *matter dominates* (MD) we have $a \sim t^{2/3}$ and for $w = -1$, $a \sim e^{Ht}$. Thus in general when one component dominates over the other in the Universe we have

$$a(t) \sim \begin{cases} t^{\frac{2}{3(1+w)}} & w \neq -1 \\ e^{Ht} & w = -1. \end{cases} \tag{1.10}$$

The simple equations above show that the expansion of the Universe implies an early dense and hot state dominated by radiation. This is the so called hot big bang scenario which is able to explain, among other things, the abundance of the lightest chemical elements in our Universe. In addition, it predicts the CMB background radiation as a remnant of the time that recombination took place.

[4]This is the only information left in the Einstein's equation, all the ij components give an equation which is not independent from the other two: (1.5) and (1.6).

[5]Mathematically, in order to close the system we have to make this additional assumption.

1.1.2 Causality: Particle horizon and Hubble radius

Once the geometry of the Universe at large scale has been parametrized, it is interesting to consider the maximal region that could ever have been in causal contact with us; this is the *Particle horizon* or sometimes called Cosmological horizon.

Let us follow a signal travelling at the speed of light emitted at time $t_{\rm BB}$ arbitrary close[6] to $t = 0$ (Big Bang singularity) and reaching us today. The Particle horizon can be computed by the length of the geodesic on the spatial section, i.e.

$$R_H(t) = a(t) \int_{t_{\rm BB}\to 0}^{t} \frac{dt'}{a(t')} = a(t) \int_{t_{\rm BB}\to 0}^{t} \mathcal{H}^{-1} d\ln a, \tag{1.11}$$

where in the last expression we have rewritten the integral in terms of the comoving Hubble radius $\mathcal{H} = (aH)^{-1}$. If two points are separated by a distance larger than $R_H(t)$ at cosmic time t, it means that they could have never causally influenced each other. If we assume that from $t_{\rm BB} \to 0$ until now the Universe was always dominated by a fluid such as matter or radiation we have

$$R_H(t) = \begin{cases} H^{-1} & w = 1/3 \\ 2H^{-1} & w = 0. \end{cases} \tag{1.12}$$

In both cases the Particle horizon is, up to a factor, equal to the *Hubble Radius* H^{-1}. This is the reason why in standard cosmology the two concepts are used with the same meaning.[7] However, since they can be very different, it is important to remember that they give different notions of causality. The Hubble radius $H^{-1}(t)$ gives the greatest distance at which two points can still be in causal contact at t.[8]

Consider λ as a generic distance between two points in the Universe. Now let us assume that at a given time $t_{\rm ls}$ this scale is "super-Hubble", i.e. (in comoving coordinates) its time-independent comoving distance $\bar{\lambda} = \lambda/a$ is bigger than the comoving Hubble radius $\mathcal{H}^{-1}$. If before $t_{\rm ls}$ the Universe was always dominated either by matter or radiation two things happen. First, as discussed, the Hubble radius at that time can be used as a synonymous of the particle horizon. Second, $\mathcal{H}^{-1}$ always decreases going backwards in time. From the Friedmann equation (1.6), $H^2 \propto \rho$ and $\rho_i \propto a^{-3(1+w_i)}$. Thus, $\bar{\lambda}$ would have been always larger than

[6] In the following computations you can take $t_{\rm BB} \equiv 0$ instead of doing the limit afterwards, we keep this notation because at $t = 0$ the metric is singular and we actually do not know what happens beyond the Planck scale $t \sim 10^{-42}s$. We will briefly comment on this later.

[7] Once and for all, with standard cosmology we mean FLRW cosmology with an ordinary fluid, i.e. $\rho + 3P > 0$.

[8] The Hubble radius is the distance over which particles can travel during an expansion time ($t = H^{-1} = dt/d\ln a$), the time in which the scale factor doubles.

the Hubble radius before $t_{\rm ls}$, and two points separated by $\bar{\lambda}$ would have never been in causal contact.

1.2 Inflation: why and how

The situation just described is precisely what happens for patches of the CMB that are separated by angles bigger than roughly $\sim 2°$. These regions were super-Hubble at the time of last scattering. Thus, before taking any measurement, one would have expected to find no correlation among them. However, as already mentioned, the CMB has been measured to have almost the same temperature whatever directions of the sky we look. This is the *Horizon problem*: there are roughly 10^{60} patches in the CMB with apparently no causal relation but with the same temperature.

To solve this problem, we need to postulate a period before the time of last scattering where the particle horizon of two points arbitrary separated in the CMB increases enough to make them causally connected. This is achieved by assuming a period in the very early Universe where

$$\frac{d\mathcal{H}^{-1}}{dt} \equiv \frac{d}{dt}(aH)^{-1} < 0 \quad \Longleftrightarrow \quad \ddot{a} > 0. \tag{1.13}$$

This is the fundamental idea of Inflation, an early accelerating phase of the Universe where the comoving Hubble radius shrinks. If this period lasts long enough, the scales observed in the CMB that were outside the Hubble radius at $t_{\rm ls}$ could have been inside the Hubble radius ($\bar{\lambda} < \mathcal{H}^{-1}$) at earlier times. To parametrize how long is "enough", the amount the Universe has increased is quantified by the number of e-folds

$$N = \ln\left(\frac{a(t_{\rm E})}{a(t)_{\rm I}}\right) \tag{1.14}$$

where $t_{\rm I}$, $t_{\rm E}$ represent beginning and end of inflation. Roughly 60 e-folds of inflation[9] is enough to solve the horizon problem. There is no upper limit on how much inflation could have happened. The largest scale in the CMB becomes super-horizon approximately 60 e-folds before the end of inflation.

Inflation solves at the same time the so called *flatness problem*. This can be stated as follows. Let us rewrite the first Friedmann equation as

$$\Omega - 1 = \frac{k}{H^2a^2} \tag{1.15}$$

[9] This number depends on the history of the Universe after inflation, and how the inflaton transfers its energy to radiation (preheating and reheating). We use 60 from now on. With this estimate, our current angular resolution in the CMB can only probe modes that left the horizon between 60 and about 53-52 e-folds before the end of inflation.

where $\Omega \equiv \rho/\rho_c \equiv \rho/3H^2$. Observations teach us that today the Universe is nearly flat ($k = 0$) as $|\Omega_0 - 1| \approx 0.01 \sim O(1)$. During a matter dominated period $H^2 \propto \rho_M \propto a^{-3}$ while for radiation $H^2 \propto \rho_R \propto a^{-4}$. In both cases $\Omega - 1$ decreases going backwards in time. In order to be of order unity today we need an incredible amount of fine-tuning on its initial condition $\sim O(10^{-60})$. Inflation reverses that behavior and makes $\Omega = 1$ an attractor solution.

Both the horizon and the flatness problem can be summarized as follows: in standard cosmology there are no reasonable explanations for why the Universe is so homogeneous and why it looks so flat. It can be shown that both problems are solved if we postulate an early accelerating expansion phase where the Universe increases its size at least by a factor $\approx e^{60}$. The inflationary solution can also be understood intuitively. Imagine before inflation a Universe with some curvature and many inhomogeneous patches. After such expansion every patch will appear flat and homogeneous locally. That is why we say inflation has diluted the initial conditions.

1.2.1 Slow roll parameters

The physical mechanism behind inflation is still widely debated. As discussed, from the Friedmann equation, an accelerating evolution is achieved whenever some physics with an equation of state $P = w\rho$ and $w < -1/3$ dominates. The simplest model we can think of is a classical scalar field ϕ, the so-called *inflaton* field, moving in a potential. The action is

$$\mathcal{S} = -\frac{1}{2}\int \sqrt{-g}(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)). \tag{1.16}$$

From evaluating the energy momentum tensor we find[10]

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad P_\phi = \frac{\dot{\phi}^2}{2} - V(\phi) \tag{1.17}$$

with $\partial_0\phi \equiv \dot{\phi}$. Note that a potential energy dominating over the kinetic term $V \gg \dot{\phi}$ leads simply to a negative equation of state

$$P_\phi \approx -\rho_\phi. \tag{1.18}$$

From the continuity equation (1.5), $\rho \propto H^{-1} \approx$ constant. At the same time, the scale factor grows almost exponentially $a(t) \approx a_I e^{H(t-t_I)}$. If $\dot{\phi} = 0$ the equation of

[10] The request of homogeneity forces $T_{\mu\nu}$ to be dependent only on time. Thus the background field $\phi = \phi(t)$. Using the Friedmann equations all the way during inflation, it seems we are assuming the Universe homogeneous from the beginning. This translates in using a scalar field $\phi(t)$ constant on the spatial slices. Indeed ϕ has to be constant on one patch of the primordial Universe, after inflation that patch will be all our observable Universe.

state (1.18) is exact and we have a pure de Sitter stage. The need for a mechanism to end this epoch, and to allow for the following standard Big Bang evolution, demands a dynamical solution.

The equation of motion for the scalar field in a FLRW background is given by

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \tag{1.19}$$

where $V'(\phi) \equiv \partial V/\partial\phi$. It is easy to show that the condition $\ddot{a} > 0$ is equivalent to

$$\epsilon_H = -\frac{\dot{H}}{H^2} = \frac{\frac{1}{2}\dot{\phi}^2}{H^2} < 1. \tag{1.20}$$

The amount of e-folds generated is

$$N_\star \equiv \int_{a_\star}^{a_E} d\ln a = \int_{\phi_\star}^{\phi_E} \frac{1}{\sqrt{2\epsilon_H}} \frac{|d\phi|}{m_{\rm P}}, \tag{1.21}$$

we used $H dt = H d\phi/\dot{\phi} = |d\phi|/(\sqrt{2\epsilon_H} m_{\rm P})$, while $\phi_\star, \phi_{\rm E}$ is the field value when $N_\star$ e-folds are left before the end of inflation and at the end of inflation respectively. Since $\epsilon_H = 1$ ends inflation, in order to account for enough e-folds (to solve the horizon and flatness problem) we require another parameter to be small

$$\eta_H = \frac{d\ln\epsilon_H}{dN} = \frac{\dot{\epsilon}_H}{H\epsilon_H} = 2\frac{\ddot{\phi}}{H\dot{\phi}} - 2\frac{\dot{H}}{H^2} < 1. \tag{1.22}$$

Here ϵ_H and η_H are called the Hubble slow roll parameters. It is possible to simplify the equations of motion using the *slow roll approximation*, i.e. $\{\epsilon_H, \eta_H\} \ll 1$,

$$H^2 = \frac{\rho_\phi}{3m_{\rm P}^2} \approx \frac{V(\phi)}{3m_{\rm P}^2}, \quad 3H\dot{\phi} \approx -V'(\phi). \tag{1.23}$$

These allow one to link the scalar field potential to the feasibility of inflation. Let us define the *potential slow roll parameters*

$$\epsilon = \frac{m_{\rm P}^2}{2}\left(\frac{V'}{V}\right)^2 \;\; ; \; \eta = m_{\rm P}^2 \frac{V''}{V}, \tag{1.24}$$

which can be expressed in terms of the Hubble parameters as

$$\epsilon = \epsilon_H \left(1 - \frac{\eta_H}{2(3-\epsilon_H)}\right) \approx \eta_H, \quad \eta \approx 2\epsilon_H - \eta_H/2 + O(\epsilon_H^2). \tag{1.25}$$

The approximations above are valid only within the slow roll approximation. Thus, given a potential, $\{\epsilon, \eta\}$ have to be small in order for slow-roll inflation to occur. In this thesis we will always consider the slow roll approximation and use the potential slow roll parameters to determine the inflationary regime. Finally using $\epsilon_H \approx \epsilon$, also the number of e-folds (1.21) can be computed in terms of the potential.[11]

[11] When ϵ approaches unity we will have order one correction to the integral, however this remains a good approximation because it occurs only over a brief range in field space.

1.3 Connecting inflation to observations

Inflation is able to explain naturally the horizon and flatness problem. Actually, both issues depend on our taste of how bad fine-tuning the initial conditions is. One might legitimately question whether this is enough to "believe" in inflation.

In principle, there is nothing wrong in *assuming* given initial conditions in which the primordial Universe was simply just flat and homogeneous over super-horizon scales. The elegance of the inflationary solution avoids the need for this assumption, but at the end of the day we do not know who or what sets these initial conditions. Furthermore, the horizon problem is illustrated by showing the smallness of the particle horizon between two patches of the CMB. This is computed by sending the cosmic time in the lower bound of the integral (1.11) to zero, i.e. by going arbitrarily close to the Big Bang singularity. In truth, we know that at least at the Planck scale general relativity breaks down and there is no reason to believe in the FLRW geometry all the way till $t \to 0$. Thus, it is also legitimate to *assume* that a theory of quantum gravity will solve the horizon problem without the need for inflation.

However, even if the theory was originally postulated to answer why the CMB is so homogeneous, the strongest arguments in its favor are linked to the fact that it provides, for free, an explanation for the small inhomogeneities observed in the CMB spectrum. It turns out that the fluctuations in the temperature of the CMB have their roots in the quantum fluctuations generated during inflation.[12] Thus, the inflationary paradigm makes a connection between microscopic and macroscopic physics that can be probed by observations of the CMB.

1.3.1 Cosmological perturbations

Consider small perturbations on top of the inflaton-FLRW background

$$\phi(t,x) = \phi(t) + \delta\phi(x,t), \quad g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}. \tag{1.26}$$

A convenient way to study these perturbations is using the ADM (Arnowitt, Desner and Misner [16]) decomposition for the metric,

$$g_{\mu\nu} = -N^2 dt^2 + g_{ij}(dx_i + N_i dt)(dx^j + N^j dt) \equiv \begin{pmatrix} -N^2 + N^i N_i & N_i \\ N_i & g_{ij} \end{pmatrix} \tag{1.27}$$

[12] In particular, the analysis of the polarization spectrum in the CMB is in agreement with super-Hubble perturbations with coherent initial phases [15]. The amplitude of the perturbations is measured to be nearly scale invariant with a confidence of 6σ as evidence for the nearly exponential expansion.

and the scalar-vector-tensor decomposition of tensors under rotation on a constant time hypersurface

$$\begin{aligned} g_{ij} &= a(t)^2(\delta_{ij}(1+2\psi) + a^{-2}\partial_i\partial_j E + \partial_{(i}h^T_{j)} + h^{TT}_{ij}) \\ N &= (1+n(x)), \quad N^i = a^{-1}n^i(x), \quad n_i = a^{-1}(t)\partial_i s + n^T_i(x), \end{aligned} \tag{1.28}$$

with

$$\partial_i h^T_i = 0 \ ; \ \partial_i h^{TT\,ij} = 0 \ ; \ \delta^{ij}h^{TT}_{ij} = 0 \ ; \ \partial_i n^T_i(x) = 0. \tag{1.29}$$

In this way we have parametrized the metric perturbation in 4 scalars (s, n, ψ, E), two transverse spatial vectors (h^T_i, n^T_i) and a traceless transverse spatial tensor h^{TT}_{ij}. The beauty of this decomposition lies in the fact that in the Einstein equations (as well as the action) at first order in perturbations, scalar, vector and tensor modes decouple and they can be studied separately.

The gauge invariance (four functions) and the four constraints implied by the Lagrange multipliers (N, N_i) allows to isolate the 11-8=3 propagating degrees of freedom. The two tensor modes (spin 2) of GR plus the scalar one from the field fluctuations. These can be represented in terms of the gauge invariant quantities[13]

$$\mathcal{R} = \psi - H\delta\phi, \quad h^{TT}_{ij}, \tag{1.30}$$

$\mathcal{R}$ is called the *comoving curvature perturbation*. The Ricci scalar on a spatial hypersurfaces is at first order $R^{(3)} = -4\nabla^2\psi/a^2$. Thus $\mathcal{R}$ is the gauge invariant quantity which reduces to ψ in the comoving gauge ($\delta\phi = 0$).

Consider their Fourier expansion, i.e. $\mathcal{R}(x) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}}\mathcal{R}_{\vec{k}}(t)$ and analogously for h^{TT}_{ij}. The striking feature of the gauge invariant quantities defined above is that, in single-field inflation, their Fourier modes are time-independent once their correspondent wavelength, i.e. $k \sim \bar{\lambda}^{-1} \equiv a\lambda^{-1}$, is larger than the Hubble radius[14] $\lambda \gg H^{-1} \implies k \ll aH$,

$$\dot{\mathcal{R}}_{\vec{k}} = 0 \quad ; \quad \dot{h}^{TT}_{ij} = 0 \ , \qquad\qquad k \ll aH \ . \tag{1.31}$$

It is a good approximation to consider the Fourier modes frozen just after the "Horizon crossing", i.e. when $\lambda = H^{-1} \implies k = aH$. This is the key point that underlies our earlier statement that fluctuations observed in the CMB are ascribable to the perturbation generated during inflation. For example, the curvature perturbations are related to the late-time local densities of photon. The fact that these modes were frozen for a long period in the history of the Universe where the physics is uncertain, makes it possible to trace back the perturbations in the CMB to their primordial origin.

[13] Note that $\mathcal{R}$ is defined by partially fixing the gauge $E = 0$.
[14] Weinberg's proof of this can be found in [17].

Consider at early times during inflation a mode well inside the Horizon $\lambda \ll H^{-1}$, i.e. $k \gg aH$. At small scales the fluctuations are described by a set of harmonic oscillators that can be quantized in the usual way. We quantify the variance of the oscillators with their correlation functions (power spectrum). Any scale $\lambda \sim ak^{-1}$ is stretched during inflation and at some point it crosses the horizon $k = aH$. The conservation of $\mathcal{R}, h^{TT}$ on super-Hubble scales makes it possible to relate predictions made at the horizon crossing (high energy and during inflation) with observables that are inside the Hubble radius today. Let us take the largest scale we observe in the CMB today $k_\star = 0.002\text{Mpc}^{-1} \approx a_0 H_0$ (the one corresponding to the length of the horizon today). A fluctuation with this wavelength "begins its journey" in our direction at the time of last scattering when $k_\star \ll a(t_{\text{ls}})H^{-1}(t_{\text{ls}})$. Since it was super-horizon from a particular instant during inflation until now, its amplitude was determined as it crossed the horizon after which it remains frozen. Roughly we can think of it as related to ΔT of two photons reaching us today and separated by that distance. For all the other modes which re-enter the horizon at earlier times, the amplitudes evolved in a computable way.

1.3.2 Power spectrum

At the end, CMB measurements lead to precise values of the super-Hubble correlation functions[15] of the curvature perturbations $\mathcal{R}$ and the tensors h_{ij}^{TT}.

To quantify the properties of the perturbations consider $\mathcal{R}(\vec{x}, t)$ as a quantum operator expanded in terms of creation and annihilation operators

$$\mathcal{R}(\vec{x}, t) = \int \frac{d^3k}{(2\pi)^3} (\mathcal{R}_k(t) a_{\vec{k}} e^{i\vec{k}\vec{x}} + \mathcal{R}_k^\star(t) a_{\vec{k}}^\dagger e^{-i\vec{k}\vec{x}}). \tag{1.32}$$

The two point correlation function is

$$\langle 0|\mathcal{R}(x)\mathcal{R}(x)|0\rangle = \int \frac{d^3k}{(2\pi)^3} |\mathcal{R}_k(t)|^2 = \int d\ln k\, \mathcal{P}_\mathcal{R}(k, t), \tag{1.33}$$

where in the last equality we have used the fact that the mode function $\mathcal{R}_k(t)$, is the same for all modes with $k \equiv |\vec{k}|$. Further $|0\rangle$ is the vacuum state such that $a_{\vec{k}}|0\rangle = 0 \; \forall k$ and

$$[a_{\vec{k}}, a_{\vec{k}'}^\dagger] = (2\pi)^3 \delta(\vec{k} - \vec{k}'). \tag{1.34}$$

The dimensionless quantity

$$\mathcal{P}_\mathcal{R}(k, t) = \frac{k^3}{2\pi^2} |\mathcal{R}_k(t)|^2 \tag{1.35}$$

[15] In this thesis we focus only on two-points correlation functions, and we do not discuss non-gaussianity. The main reason is that we only focus on single-field models for which non-gaussianities are negligible [18].

defines the *power spectrum* of the curvature perturbations. This is the object directly connected with the temperature anisotropy of the CMB. Note that in order to compute the power spectrum for $\mathcal{R}(x,t)$, all we need is the expression for the mode functions $\mathcal{R}_k(t)$. This can be computed from the equations of motion. The first computations of the power spectrum can be found in [19, 20, 21, 22].

As an example, consider the action (1.16) (plus the Einstein-Hilbert term) expanded at second order in the perturbations (1.28). The scalar sector initially will depend on $S = S[n, s, E, \psi, \delta\phi]$. Fixing $E = 0$ (with a particular gauge choice) and integrating out the non dynamical degrees of freedom (s, n) leaves an action that (after some integrations by part and using the background equations of motion) is possible to rewrite in terms of the gauge invariant quantity $\mathcal{R}$. Then it is useful to introduce the Mukhanov-Sasaki variable [23, 24]

$$u = z\mathcal{R}, \qquad \text{with} \quad z = a\frac{\dot{\phi}}{H}, \tag{1.36}$$

so that the equation of motion for each Fourier mode becomes

$$u_k'' + \left(k^2 - \frac{z''}{z}\right) u_k = 0, \tag{1.37}$$

where the prime $'$ means derivative with respect to the conformal time

$$d\tau = \frac{dt}{a(t)}. \tag{1.38}$$

We can immediately appreciate the use of the new variable. The equation above is just the one dimensional harmonic oscillator with a time dependent frequency given by $\omega(\tau) = \left(k^2 - \frac{z''}{z}\right)$. Thus, we can go through the standard quantization procedure [25]: compute the conjugate field $\pi(\vec{x}, t) = \partial\mathcal{L}/\partial u' = u'$ and impose the canonical quantization relation, $[u(\vec{x},\tau), \pi(\vec{x},\tau)] = i\delta^3(\vec{x} - \vec{x}')$. Then renormalize the mode functions to get the usual commutation relation for the annihilation and creation operators as in (1.34). In order to solve equation (1.37), we rewrite z as $z = a\dot{\phi}/H = a\sqrt{2\epsilon_H}$. This leads to

$$\frac{z''}{z} = a^2H^2\left(2 - \epsilon_H + \frac{3}{2}\eta_H\right) \approx a^2H^2\left(2 + 5\epsilon - 3\eta\right) \tag{1.39}$$

where we used the slow roll approximation and the potential slow roll parameter defined in (1.24). Thus, the equation for u_k can be recast as[16]

$$u_k'' + \left(k^2 - \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right)\right) u_k = 0, \quad \nu = \frac{3}{2} - \eta + 3\epsilon. \tag{1.40}$$

[16]Using the definition of conformal time $\tau = \int \frac{dt}{a(t)} = \int \frac{da}{a^2 H} \approx -\frac{1}{aH(1-\epsilon)}$, the scale factor during a quasi de Sitter stage can be rewritten as $a(\tau) = -\frac{1}{H}\frac{1}{\tau(1-\epsilon)}$.

Equations of this form have the well known solution:

$$u_k(\tau) = \sqrt{-\tau}\left[\alpha(k)H_\nu^{(1)}(-k\tau) + \beta(k)H_\nu^{(2)}(-k\tau)\right] \tag{1.41}$$

where $H_\nu^{(1)}$, $H_\nu^{(2)}$ are the Hankel's functions of the first and second kind. A particular choice for the coefficients $\alpha(k), \beta(k)$ represents a particular choice for the set of the mode functions. This translates in a particular choice of the vacuum state. Usually in a generic curved background this represents an ambiguity, however for inflation there is a preferred and physical choice. At very early times, which means $\tau \to -\infty$,[17] all the wavelengths were deep inside the horizon $k \gg aH$. Therefore, using (1.39) the equation for the mode functions (1.37) reduces to $u_k'' + k^2 u_k = 0$. This describes a plane waves in a flat space-time. Thus, we ask that in the remote past the mode functions match the ones in Minkowski, i.e.

$$u_k(\tau) \underset{\tau\to-\infty}{\longrightarrow} \frac{1}{\sqrt{2k}}e^{-ik\tau}. \tag{1.42}$$

This can be also understood naively, when a wavelength is deep inside the horizon, $\lambda \ll H^{-1}$. The spacetime for that wave looks flat (the mode "does not see" the curvature) and the solution will be the one of Minkowski. With these initial conditions we solve the equations for $u_k(\tau)$ by matching (1.41) with the asymptotic behaviour of the Hankel functions

$$H_\nu^{(1,2)}(-k\tau) \underset{\tau\to-\infty}{\longrightarrow} \sqrt{\frac{-2}{\pi k\tau}}\exp\left[\mp i\left(k\tau + \frac{\pi}{4}(2\nu+1)\right)\right]. \tag{1.43}$$

This implies $\alpha = \frac{\sqrt{\pi}}{2}e^{i(\nu+\frac{1}{2})\frac{\pi}{2}} \equiv \bar{\alpha}$ and $\beta = 0$. The corresponding vacuum is the so-called *Bunch Davies vacuum*. The mode functions assume the form $u_k(t) = \sqrt{-\tau}\bar{\alpha}H_\nu^{(1)}(-k\tau)$. On super Hubble scale $k \ll aH \implies -k\tau \to 0$ and the Hankel function in this limit gives

$$H_\nu^{(1)}(-k\tau) \underset{\tau\to0}{\longrightarrow} \frac{i}{\pi}\Gamma(\nu)\left(\frac{-k\tau}{2}\right)^{-\nu} \tag{1.44}$$

where $\Gamma(\nu)$ is the Euler function, $\Gamma(\nu) \approx \Gamma(3/2) = \sqrt{\pi}/2$. Rewriting τ in terms of the scale factor, at first order in the slow roll parameter we get

$$|u_k(\tau)|^2 = k^{-3}\frac{(aH)^2}{2}\left(\frac{k}{aH}\right)^{2\eta-6\epsilon}. \tag{1.45}$$

The power spectrum for $\mathcal{R}$ is easily related to the modes functions by means of the redefinition (1.36) and (1.35), i.e. $\mathcal{P}_\mathcal{R}(k,\tau) = z^{-2}k^3/2\pi^2|u_k(t)|^2$. Therefore, on super-Hubble scales we finally have

$$\mathcal{P}_\mathcal{R}(k,\tau) = \left(\frac{H}{2\pi}\right)^2\frac{1}{2m_\mathrm{P}^2\epsilon}\left(\frac{k}{aH}\right)^{2\eta-6\epsilon} \qquad \text{Super Hubble} \tag{1.46}$$

[17] During inflation the conformal time is negative running from $-\infty$ to 0.

Let us point out the meaning of the expression above. This is the value of the power spectrum for a mode k at a time τ when the mode is super horizon. We us as pivot scale $k_\star = 0.002\,\mathrm{Mpc}^{-1}$, which is the largest scale we observe in the CMB today (the one that exited the horizon roughly 60 e-folds before the end of inflation). $k_\star$ exits the horizon, by definition, at the time $\tau_\star$ such that $k_\star = a(\tau_\star)H(\tau_\star) \equiv a_\star H_\star$. Even if $\mathcal{R}_{k_\star}$ remains constant just on super Hubble scales it is still a good approximation ($\dot{\phi}$ and H remain almost constant during inflation) to compute the value of the power spectrum of this mode at the horizon crossing using the expression (1.46), i.e.

$$\mathcal{P}_\mathcal{R}(k_\star, \tau_\star) = \left(\frac{H_\star}{2\pi}\right)^2 \frac{1}{2\epsilon_\star} \equiv \Delta_\mathcal{R}. \tag{1.47}$$

Therefore, the power spectrum for a generic mode at $\tau_\star$ can be written as

$$\mathcal{P}_\mathcal{R}(k) = \Delta_\mathcal{R} \left(\frac{k}{k_\star}\right)^{n_s - 1}, \quad n_s - 1 = \frac{d \ln \mathcal{P}_R(k,\tau)}{d \ln k}\Big|_{\tau_\star} = 2\eta_\star - 6\epsilon_\star \tag{1.48}$$

n_s is called the *spectral index* and it measures the deviation of the power spectrum from scale invariance.[18] The fact that $n_s \neq 1$ within 6σ is one of the major successes of the inflation paradigm.

The other dynamical degrees of freedom excited during inflation are the pure gravity tensor modes. The vector modes are not dynamical and they scale as $\sim a^{-1}$, which means they drop down during inflation when the scale factor grows exponentially. The linear tensor perturbations are represented by the traceless transverse h_{ij}^{TT}. Tensor modes do not have any source and they are free degrees of freedom propagating on the quasi de Sitter background. By means of analogous computations we get the tensor power spectrum on Super-Hubble scales

$$\mathcal{P}_T = \Delta_T \left(\frac{k}{k_\star}\right)^{n_T}, \quad n_T = \frac{d \ln \mathcal{P}_T(k)}{d \ln k}\Big|_\star = -2\epsilon_\star. \tag{1.49}$$

This is directly related to the spectrum of primordial gravitational waves.[19] Δ_T is the power spectrum for the pivot scale $k_\star$ at the horizon crossing

$$\Delta_T \equiv \mathcal{P}_T(k_\star, \tau_\star) = \frac{2}{m_\mathrm{P}^2}\left(\frac{H_\star}{\pi}\right)^2, \tag{1.50}$$

[18] It is worthy to mention that we could obtain the same results also in another way. Consider the power spectrum evaluated on the horizon crossing curve, i.e. $\tau(k)$ such that $a(\tau)H(\tau) = k$. Using for it the expression on super hubble scale in (1.46) we get $P_\mathcal{R} = (k, \tau(k)) = \frac{H^2(\tau(k))}{2\epsilon(\tau(k))}$. Which gives for the spectral index, $\left[\frac{d \ln \mathcal{P}_\mathcal{R}(k,\tau(k))}{d \ln k} = 2\frac{d \ln H}{d \ln k} - \frac{d \ln \epsilon}{d \ln k} = 2\eta - 6\epsilon\right]$

It is not obvious before the computation that we would have obtained the same result but it was expected from the reasoning we made in the text.

[19] The scalar spectral index is defined starting from $n_s - 1$ while the tensor spectral index is n_T. This is just due to some historical reasons.

most of the time considered by looking at the tensor-to-scalar ratio

$$r = \frac{\Delta_T}{\Delta_{\mathcal{R}}} = 16\epsilon_\star \tag{1.51}$$

which shows explicitly that the tensor modes are suppressed by a factor ϵ with respect to the scalar modes. That is why the main contribution in the CMB is from the scalar modes. Looking at the expression for r and the one for the tensor spectral index n_T we obtain the so called consistency relation $r = -8n_T$. This has to be valid as long as we assume single-field slow roll inflation. If any measurement in the future will falsify this formula, it would imply that something more complicated than single-field slow roll has driven inflation.

1.3.3 Observables: the recipe

From the theory of the perturbations during inflation we basically get three independent quantities. The scalar amplitude $\Delta_{\mathcal{R}}$, the spectral index n_s and the tensor to scalar ratio r. All these three parameters only depend on the value of the scalar field at the time the pivot scale left the horizon, i.e. $\phi_\star$. The power spectrum, using the FLRW equation in the slow roll approximation ($3H^2 m_{\mathrm{P}}^2 \approx V$) can be rewritten as

$$\mathcal{P}_{\mathcal{R}}\big|_{k=k\star} \equiv \Delta_{\mathcal{R}} = \frac{1}{m_{\mathrm{P}}^2}\left(\frac{H_\star}{2\pi}\right)^2 \frac{1}{2\epsilon_\star} = \frac{V(\phi_\star)}{24 m_{\mathrm{P}}^4 \pi^2 \epsilon_\star}, \tag{1.52}$$

while r, n_s are already in terms of the slow roll parameters which are functions of the potential, see (1.24). Then we can write $\phi_\star$ in terms of $N_\star$, the number of e-folds before the end of inflation at that time. Using (1.21) we have

$$N_\star = \int_{\phi_\star}^{\phi_E} \frac{1}{\sqrt{2\epsilon}} \frac{|d\phi|}{m_{\mathrm{P}}} = \frac{1}{m_{\mathrm{P}}^2} \int_{\phi_\star}^{\phi_E} \frac{V}{V'}|d\phi| \implies \phi_\star = \phi_\star(\phi_E, N_\star), \tag{1.53}$$

with ϕ_E, the value of the field at the end of inflation, computed from $\epsilon(\phi_E) = 1$. Obviously the explicit form of the inversion above will depend on the scalar field potential V. Thus, everything reduces to a function of $N_\star$ (plus some model parameters). As discussed, for the pivot scale we use $N_\star = 60$. Therefore given a potential V for a single scalar field we can compute its prediction by means of the following recipe:

- Compute the expression for the slow roll parameters $\epsilon = \frac{1}{2} m_{\mathrm{P}}^2 (V'/V)^2$ and $\eta = m_{\mathrm{P}}^2 \frac{V''}{V}$
- Find $\phi_\star$, the value of the field at the horizon crossing $N_\star$ e-folds before the end of inflation from (1.53)

– Substitute in $\Delta_{\mathcal{R}} = V(\phi_\star)/24 m_{\rm P}^4 \pi^2 \epsilon_\star$, $n_s = 1 + 2\eta_\star - 6\epsilon_\star$, $r = 16\epsilon_\star$.

The most recent Planck data [26] gives the following constraints on the CMB parameters[20]

$$\ln(10^{10}\mathcal{P}_{\mathcal{R}})|_{(k=0.05{\rm Mpc}^{-1})} = 3.045 \pm 0.016 \;\; (68\%{\rm CL}) \tag{1.54}$$

$$n_s = 0.9649 \pm 0.0044 \;\; (68\%{\rm CL}), \tag{1.55}$$

$$r < 0.10 \;\; (95\%{\rm CL}). \tag{1.56}$$

In most of the cases the value of $\Delta_{\mathcal{R}}$, that from (1.54) is roughly given by $\Delta_{\mathcal{R}} \simeq 2 \cdot 10^{-9}$, is used to fix the free parameter in a particular model. Then n_s and r are given. Fig. 1.2 shows the (n_s, r) plot and the predictions for various inflationary models. As we have already said, the tensor modes are suppressed by a factor ϵ and they have not been detected so far.[21] There is only an upper bound on r. Current experiments are looking for patterns of B-mode (parity-odd) polarization in the CMB. A very crucial point is that B-modes cannot be created by scalar perturbations but they are produced by tensor modes. This is the fundamental missing piece to confirm the inflationary scenario.

A detection of primordial B-modes would be a breakthrough also for other reasons. It would be the first time that quantized gravitational degrees of freedom provide detectable effects. Looking at expression (1.50) we notice that the tensor amplitude depends only on the Hubble parameter. This is linked through the FLRW equation to the value of the inflaton potential and thus to the energy scale $V^{1/4}$ associated to inflation. A detection of primordial gravitational waves then will give a direct measurement of the typical energy scale during inflation. In addition, a detectable signal would also imply super-planckian field excursions which are challenging to explain in a theory of quantum gravity.

1.3.4 Case study: $\lambda\phi^4$ as a no-go result

The purpose of this section is to show in one of the simplest set-up the recipe just outlined. At the same time we will see why the SM Higgs boson (without

[20] We quote 68% confidence level on measured parameters and 95% upper bound on r. We cite the results for a ΛCDM model from Planck temperature, polarization, and temperature-polarization cross-correlation, in combination with the EE measurement at low multipoles.

[21] On 17 March 2014 it was announced that the BICEP2 instrument [27] had detected for the first time B-modes consistent with inflation at the level of $r = 0.20 \pm 0.07$. At that time I just arrived as a master student in Geneva and I did not see what was all that excitement everywhere in the department. Unfortunately, later new results of the Planck experiment reported that the findings of BICEP2 can be fully attributed to cosmic dust.

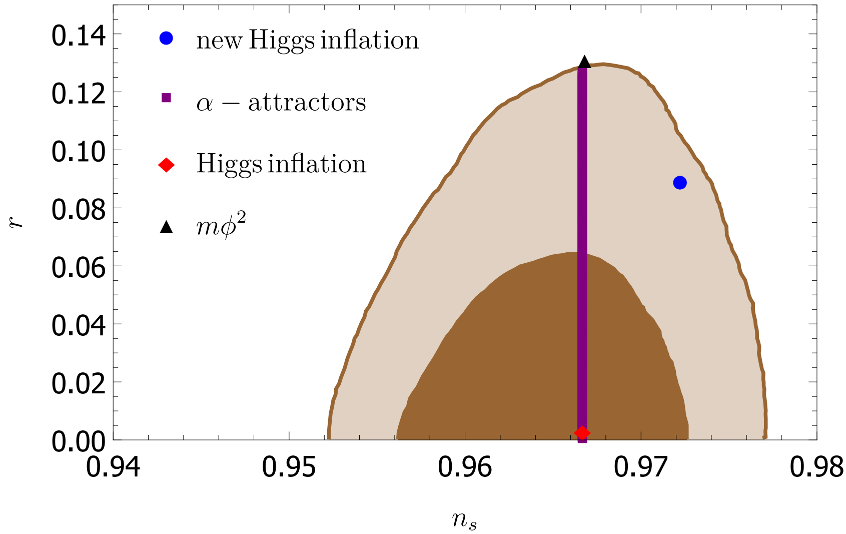


Figure 1.2: *n_s and r from the 2015 Planck release (1 and 2σ) compared to the tree-level results for some of the models studied in this thesis (60 e-folds assumed).*

additional couplings) cannot be the inflaton. Consider the action

$$S = \int \sqrt{-g}\left(\frac{m_{\rm P}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{4}\phi^4\right), \tag{1.57}$$

which can be thought to represent, for example, the action for the SM Higgs when it is well displaced from its minima $\mathcal{H}^\dagger\mathcal{H} \gg v^2$. In this case we say the Higgs is minimally coupled to gravity. To compute the CMB parameters we can apply all the machinery of the previous section:

- The slow roll parameters are

$$\epsilon = \frac{m_{\rm P}^2}{2}\left(\frac{V'}{V}\right)^2 = m_{\rm P}^2\frac{8}{\phi^2} \quad ; \quad \eta = m_{\rm P}^2\frac{V''}{V} = m_{\rm P}^2\frac{12}{\phi^2} \tag{1.58}$$

- From $\epsilon(\phi_E) = 1 \implies \phi_E = 2\sqrt{2}m_{\rm P}$, we compute

$$N_\star = \frac{1}{m_{\rm P}^2}\int_{\phi_\star}^{\phi_E}\frac{V}{V'}|d\phi| = \frac{1}{8}\frac{\phi_\star^2}{m_{\rm P}^2} - 1 \implies \phi_\star^2 = 8m_{\rm P}^2(N_\star+1) \approx 8m_{\rm P}^2 N_\star \tag{1.59}$$

  and the slow roll parameters in terms of $\phi_\star$ become $\epsilon_\star = 1/N_\star$, $\eta_\star = 3/2N_\star$.

- The amplitude for the scalar perturbation is given by

$$\mathcal{P}_R\big|_{k_\star} \equiv \Delta_{\mathcal{R}} = \frac{V(\phi_\star)}{24 m_{\rm P}^4 \pi^2 \epsilon_\star} = \frac{2}{3}\frac{\lambda}{\pi} N_\star^3. \tag{1.60}$$

In order to get $\Delta_{\mathcal{R}} \simeq 10^{-9}$ (sometimes posed also as $V_\star/\epsilon_\star = 24\pi^2 m_{\rm P}^4 \Delta_{\mathcal{R}} \simeq (0.027 m_{\rm P})^4$), λ should be of order $\lambda \sim O(10^{-13})$ which is not the case for the Higgs Boson. This is the first reason why the Higgs scalar minimally coupled to gravity is not an option to be the inflaton. We can say the reason is that the Higgs potential is too steep.

– n_s and r are given by

$$n_s = 1 + 2\eta_\star - 6\epsilon_\star \simeq 0.95, \quad r = 16\epsilon_\star \simeq 0.27 \tag{1.61}$$

These results are not dependent on λ and are general for every single scalar field with a quartic potential. The value of the tensor-to-scalar ratio r is more than 4σ above the upper limit given by the most recent Planck data ($r < 0.1$). In general, we conclude that every single-field inflationary scenario with a quartic potential (and no other couplings) is strongly disfavored by observations.

1.4 UV sensitivity of inflation

Once the toy scalar field action considered until now is upgraded to an effective field theory, inflation displays naturally a sensitivity towards high-energy degrees of freedom. It is worth clarifying that the UV sensitivity we are interested in this thesis is not the common one related to unknown Planck scale physics. This triggers the η problem and, more general, the sensitivity to Planck scale suppressed higher dimensional operators that correct the inflaton potential. We treat this kind of UV contributions as part of the definition of the model assuming these corrections are sufficiently small, either due to some symmetry principle (e.g. an approximate shift symmetry), a one-time fine-tuning, or because of the specific nature of the Planck scale physics. Let us end this chapter by making clear the difference between these two kinds of UV sensitivity.

1.4.1 Planck scale physics and eta-problem

To sustain a long enough period of inflation, the inflaton has to be light, i.e. in slow roll inflation the η parameter has to be small

$$\eta = m_{\rm P}^2 V''/V = m_\phi^2/3H^2 \ll 1. \tag{1.62}$$

The η problem is the difficulty in keeping the inflaton mass small once corrections from the UV physics are taken into account. This is a well known issue affecting

any light scalar field. In addition, the inflaton mass has to be not only below the cutoff of the theory, as in any effective field theory, but also parametrically smaller than the Hubble scale.

Consider the inflaton action as an EFT valid up to some scale Λ. Since at least at the Planck scale we expect new degrees of freedoms to help to renormalize gravity, Λ cannot be arbitrary large. Moreover, for consistency of the semiclassical approximation, Λ can also not be smaller than the Hubble scale. We conclude that $H \leq \Lambda \leq m_{\mathrm{P}}$. Integrating heavy degrees of freedom at the scale Λ shifts the parameters in the low energy EFT. In particular, the one loop matching generates radiative correction to the scalar mass of the order $m_\phi \sim \Lambda$. This sets the challenge in explaining the required smallness of the inflaton mass. Again, this is nothing else than the hierarchy problem in the context of cosmology. Furthermore, eq. (1.62), in particular $V'' \ll V/m_{\mathrm{P}}^2$, tells us that even suppressed higher order operators can change (if not spoil) the inflationary dynamics. Take, as an example, a six dimensional operator of the form $\Delta\mathcal{L} \propto \mathcal{O}^4\phi^2/\Lambda^2$. If $\mathcal{O}^4 \propto V$ during inflation, then this term will compete with the original potential by adding a correction to η, i.e. $\Delta\eta \sim O(1)m_{\mathrm{P}}^2/\Lambda^2$, which is order one even for $\Lambda \sim m_{\mathrm{P}}$.

This sensitivity to Planck scale physics affects any inflationary model. However, for models where the inflaton experiences a super-planckian excursion,[22] i.e. $\Delta\phi \gg m_{\mathrm{P}}$ (large field model) the problem is even more dramatic. Any higher order operator of the form

$$\Delta\mathcal{L} \propto \frac{\phi^{4+n}}{m_{\mathrm{P}}^n}, \tag{1.63}$$

will make things worse by adding a large contribution to η.

It is usually assumed that some property of the UV completion, such as a (quasi) shift-symmetry, will solve the problem. The challenge for any UV complete theory of inflation remains to provide a solution to this problem. As mentioned, we focus on a different kind of UV sensitivity which was first pointed out in [28].

1.4.2 UV sensitivity from the renormalization group flow

There is a type of UV sensitivity related to the RG flow and the non-renormalizabile character of an inflationary model over the whole field range.

Let us consider models where some parameters in the Lagrangian are measured (or are hoped to be measured) in experiments at low energy and then extrapolated until the inflationary scale where the CMB parameters are computed. When the

[22] Small field models are affected only by the six dimensional operators.

two regimes are defined by different renormalizable EFTs[23] (see sec. 2.1.4 and 3.5), in passing from one regime to the other unknown threshold corrections may affect the running of the couplings. As we will discuss, these can be parametrized with a tower of suppressed higher order operators in the low energy EFT. These operators have a net effect at the boundaries of the EFTs and as such provide the necessary threshold corrections (see 3.4.4 for more details). In this way the UV physics can enter the predictions through the RG flow, that is different Wilson coefficients for the higher order operators could result in different n_s and r.

For illustrative purposes consider first inflation in the SM with $V(\phi) = \lambda\phi^4$, the renormalizable quartic Higgs potential. The discussion follows almost literally part of our introduction in [3] where we tried to explain clearly this issue. The SM is renormalizable, and the running of the couplings can be computed "straightforwardly". Given the very different energy scales involved, the renormalization group (RG) improved potential (see sec. 2.2) is requested for a sensible inflationary analysis. Thus, inflationary parameters might in general be affected by the RG flow. The prediction for the scalar spectral index n_s is[24]

$$n_s = n_{s0}\left(1 + \kappa\frac{\beta_\lambda}{\lambda}\right), \tag{1.64}$$

where n_{s0} stands for the observable computed at tree level, i.e. eq. (1.61). β_λ is the beta-function for the quartic coupling λ, and $\kappa = 1$ for the quartic potential. Likewise, the tensor-to-scalar ratio will get a correction due to the running of λ. Our interest lies in the possibility that RG corrections can change the naive tree level picture. The Higgs field has superplanckian field values during inflation (see (1.59)), and consequently all SM fermions and gauge bosons are very heavy during inflation and can be integrated out [35]. With only the Higgs field itself (and the Goldstone bosons) in the spectrum, the beta-function during the inflationary epoch is $\beta_\lambda \propto \lambda^2/(8\pi^2)$ and the corrections are always small for perturbative values of the coupling. In fact, as we have already derived, for quartic inflation, $\lambda \sim O(10^{-13})$ is required to fit the CMB amplitude [36] — which is why it does not work in the SM — and the correction is completely negligible.

For the Higgs to be the inflaton new interactions beyond those already present in the SM are needed. The fact that these new interactions are non-renormalizable changes the story described above in two important ways. First, for consistency of the theory new physics at the boundary of the EFTs is needed,[25] which opens

[23]This is for example the case in Higgs inflation and New Higgs inflation models studied in chapter 3 and 5 where the beta-functions are different in the low and middle/large field regime [29, 30, 31, 32, 33, 34].

[24]This is valid as long as the RG corrections do not disrupt slow roll inflation. See for example sec. 3.7.1 for an explicit computation in Higgs inflation.

[25]In general, "new physics" can either be new fields and interactions, or that the theory becomes

up the possibility that the renormalization group equations are modified by the unknown UV completion. Second, thanks to the new coupling, during inflation the fermions and gauge bosons might be light enough to remain in the spectrum, and the beta-function will be of the form $\beta_\lambda \propto g^4$ with g a Yukawa or gauge coupling. Nothing prevents the ratio β_λ/λ from being sizeable, and the running corrections to the observables of the form (1.64) can be large. The two effects combined may introduce a UV sensitivity into the model.

Let us remark that, even if in our $\lambda\phi^4$ SM example the corrections are negligible, there is a conceptually important difference between this case and the non-renormalizable ones. When the model is renormalizable, the corrections – large or small – can be computed, in principle, by measuring the parameters in the low energy theory and assuming that no new physics between the two scales appears. In the non-renormalizable models considered, a correction of the form (1.64) would be more problematic because of our ignorance on the details of the RG flow over the whole field range.

In chapter 3 we study this problem for the Higgs inflation scenario. Using this as a benchmark, we explain in details in which way the UV physics can enter in the predictions and how to parametrize its effect in the analysis.

strongly coupled [37] and the perturbative analysis breaks down.

2 Some non-trivial aspects of the renormalization group flow

Connecting physics at low scales with the inflationary observables discussed in the previous chapter requires an understanding of the non-trivial quantum nature of the theory. Aiming to be self-contained, we provide here the basic tools for the understanding of the rest of the thesis. The material is in part a review of famous results in the literature, presented following the taste of the author, and partially coming from our works [1, 3], where renormalization group improving techniques in "non-standard set-ups" have been discussed. The first section 2.1 is an introduction to the effective action, its physical meaning, ways of computing it, and the renormalization group equations (RGEs). In the second section 2.2 standard renormalization group (RG) improving techniques are revisited, after which we discuss the RG improving in presence of a non-canonical kinetic term in 2.3. We conclude in 2.4 with a discussion on covariant quantization. By no means is this chapter meant to provide an exhaustive introduction to the renormalization group flow.

2.1 Effective action and physical interpretation

Many of the steps in this section will be obvious to the reader familiar with the formalism. Nevertheless, we include them for completeness.

In a quantum field theory, the generating functional of correlation functions is defined as

$$Z[J] \equiv e^{iW[J]} = \int \mathcal{D}\varphi \, \exp i(S[\varphi] + J\varphi) \tag{2.1}$$

where $S[\varphi]$ is the classical action and $J(x)$ an external source. We suppress space-time variables, $\int dx \, J(x)\varphi(x) \equiv J\varphi$. Any Green function can be derived by recur-

sive derivation of $Z[J]$

$$\langle\bar{0}|T\varphi(x_1)\cdot..\cdot\varphi(x_n)|\bar{0}\rangle = (-i)^n \frac{1}{Z[J]}\frac{\delta^n Z[J]}{\delta J(x_1)\cdot..\cdot\delta J(x_2)}\Big|_{J=0} \tag{2.2}$$

where $|\bar{0}\rangle$ is the ground state of the full Hamiltonian defined by $S[\varphi]$. The denominator of the right hand side expression above, i.e. $Z[0]$, is sometimes referred to as the "vacuum to vacuum transition amplitude", since it contains the exponentiation of all vacuum bubbles,

$$\lim_{T\to\infty(1-i\epsilon)}\langle 0;T|0;-T\rangle = \langle 0|T\left\{\exp\left[-i\int_{-T}^{T} dt\, H_I(t)\right]\right\}|0\rangle = \int \mathcal{D}\varphi\, e^{iS[\varphi]}, \tag{2.3}$$

with $H_I(t)$ the Hamiltonian in the interaction picture. Analogously, $Z[J]$ is the vacuum to vacuum amplitude in presence of a linear interaction with a current J. It can be shown that $W[J]$ defined in (2.1) is the generating function for connected correlation functions. These can be represented by Feynman diagrams with all legs connected.

With a Legendre transform of $W[J]$ we obtain the *effective action*

$$\Gamma[\phi] = W[J_\phi] - \phi J_\phi, \tag{2.4}$$

where

$$\frac{\delta W[J]}{\delta J} \equiv \phi, \tag{2.5}$$

and $J_\phi \equiv J(\phi)$ is the current in terms of ϕ obtained by inverting the previous expression. ϕ is called the classical field because the first functional derivative of $W[J]$ gives the expectation value of the field in presence of a source term J,

$$\frac{\delta W[J]}{\delta J(x)} = -i\frac{1}{Z[J]}\frac{\delta Z}{\delta J(x)} = \langle\varphi(x)\rangle_J \equiv \phi. \tag{2.6}$$

For $J = 0$ the expression above yields the vacuum expectation value of the theory defined by S. The first derivative of Γ with respect to ϕ gives

$$\frac{\delta\Gamma[\phi]}{\delta\phi} = \frac{\delta W[J]}{\delta J}\frac{\delta J_\phi}{\delta\phi} - \phi\frac{\delta J_\phi}{\delta\phi} - J_\phi = -J_\phi. \tag{2.7}$$

We are now in a position to outline two fundamental properties of $\Gamma[\phi]$.

First, by setting $J_\phi = 0$ in (2.7), we observe that the expectation value of the field in the vacuum (vev), i.e. $\phi = \langle\varphi\rangle_{J=0}$, is given by extremizing the effective action

$$(1) \qquad \frac{\delta\Gamma[\phi]}{\delta\phi}\Big|_{\phi=\langle\varphi\rangle_{J=0}} = 0. \tag{2.8}$$

Second, $\Gamma[\phi]$ contains all the one-particle-irreducible (1PI) Green's functions which are amputated of the external leg propagators.[1] This means that we can expand $\Gamma[\phi]$ as follows

$$(2) \qquad \Gamma[\phi] = \sum_{n=0}^{\infty} \frac{1}{n!} \int dx_1 dx_2 \cdot .. \cdot dx_n \Gamma^n(x_1, x_2, .., x_n)\phi(x_1)\phi(x_2) \cdot .. \cdot \phi(x_n) \tag{2.9}$$

where $\Gamma^n(x_1, x_2, .., x_n)$ is the 1PI n-points Green's function (we are assuming that the theory is expanded around a zero vev). In other words, the tree level vertices of Γ give the full scattering amplitudes of the theory. Eq. (2.8) is the reason why we are interested in this object, while the physical interpretation in terms of correlation functions (and the variations we are going to see) provide methods to compute it explicitly.[2]

The general form of the effective action can also be written as a derivative expansion

$$\Gamma[\phi] = -\int d^4x \left(V_{\text{eff}}[\phi] + \frac{1}{2} Z[\phi]\partial_\mu\phi\partial^\mu\phi + \mathcal{O}(\partial^4\phi) \right) \tag{2.10}$$

where the first term is called the *effective potential*. It can be determined simply by computing $\Gamma[\phi]$ setting ϕ constant, i.e.

$$\int d^4x \, V_{\text{eff}}[\phi] = -\Gamma[\phi]\big|_{\phi=\text{constant}}. \tag{2.11}$$

This object is particularly relevant in studies of symmetry breaking where the vacuum of the theory is assumed to be invariant under translation. By Fourier transforming (2.9) (see for example [40])

$$V_{\text{eff}}[\phi] = -\sum \frac{1}{n!} \Gamma^n(p^i = 0)\phi^n. \tag{2.12}$$

$\Gamma^n(p^i = 0)$ is the n-point 1PI correlation function in momentum space, with all the external momenta set to zero.

2.1.1 Sum of infinite series - an instructive example

We extend the procedure first used by S.R. Coleman and E.J. Weinberg to compute the effective potential for a $\lambda\phi^4$ theory [41], to a generic potential $V(\phi)$.

[1] The contribution to a Green's function is labeled as one-particle-reducible if it can be rewritten in a factorized form where each term is obtained by cutting one propagator from the respective diagram.

[2] All this comes with a big caveat. The 1PI generator and the effective action defined as the Legendre transform of $W[J]$ are not rigorously speaking the same thing. $W[J]$ is the Legendre transform of the 1PI generator and the effective action is the Legendre transform of $W[J]$. Thus, Γ is the convex envelop of 1PI generator (see [38, 39]) and the two coincide when the 1PI generator is convex.

Going through this calculation is instructive for different reasons. The purpose is to become acquainted with working in terms of generic functions in the Lagrangian, as it provides a good warm-up for the following chapters. Furthermore, it will make clear how much simpler it is to use the background field method, which will be introduced in the next section.[3]

As mentioned before, it is possible to compute $\Gamma[\phi]$ by matching it to an infinite series of Feynman diagrams. Consider the simple case of a theory defined by a generic potential $V(\phi)$. We can compute the effective potential in a loop expansion by summing all the diagrams with zero external momenta. At tree level, this gives nothing but the tree level potential, while at one loop we have to sum an infinite series of diagrams.[4] Let us consider the case of zero vev, i.e. the solution of (2.8) with $\phi = 0$. The mass appearing in the tree level propagator is defined as $m^2 = V''|_0 \equiv f_2$, and we label iterative derivatives of V evaluated at $\phi = 0$ with $d^nV/d\phi^n|_0 \equiv f_n$. In this way the propagator is given by

$$\Delta(p) = -\frac{i}{p^2 + f_2 - i\epsilon}, \tag{2.13}$$

while the generic n-vertex is given by

$$\mathrm{n} - \mathrm{vertex} = -if_n. \tag{2.14}$$

For each group of external legs attached to the same vertex in a diagram we add a factor $\phi^k/k!$ where k is the number of external legs in the group. The reason is simple, the products of all the ϕ^k will give you the ϕ^n in equation (2.12), and the factorial in the denominator takes into account the permutations of the external legs attached to each vertex. Thus, in general, a 1-loop diagram is

$$\mathcal{V}(n, k_1, k_2, ..) = i \int \frac{d^4p}{(2\pi)^4} \left(\frac{1}{2n}\right) (-i\Delta(p))^n n! \prod_{j=1}^{\infty} \frac{1}{k_j!} \left[\frac{f_{j+2}\phi}{j!}\right]^{k_j}, \tag{2.15}$$

with k_1 the number of 3-point interactions, k_j the number of vertices with $j+2$-points interactions and $n = \sum_{i=1}^{\infty} k_i$ the total number of vertexes in the diagram. The symmetry factor, $1/(2n)$ is for the maximally symmetric diagrams with n vertices ($1/2$ for the reflection symmetry and $1/n$ for the Z_n symmetry), which are the diagrams where all the vertices are made out of the same interaction term, for example $\mathcal{V}(n, k_1 = n, k_i = 0)$. The combinatorial takes into account the

[3] In fact, to our knowledge, the diagrammatic approach is always presented for a specific potential and the general formula (2.18) is always derived by using the functional approach. The derivation in this section in this sense is somewhat original.

[4] Coleman and Weinberg in their masterpiece [41] shows, for $V = \lambda\phi^4/4$, that although individually each diagrams looks IR divergent, they all nicely sum in a renormalizable UV divergent term.

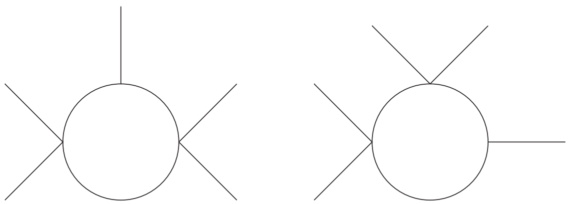


Figure 2.1: *One loop diagrams corresponding to* $\mathcal{V}(n=3, k_1=1, k_2=2, k_{i>2}=0)$.

topological different diagrams with their right symmetry factor. By summing all diagrams we obtain

$$\begin{aligned} V^{1-\text{loop}} &= \sum_{n=1}^{\infty} \sum_{\sum_{i=1}^{\infty} k_i = n} \mathcal{V}(n, k_1, k_2, ..) \\ &= i \int \frac{d^4p}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{1}{2n} (-i\Delta(p))^n \sum_{\sum_{i=1}^{\infty} k_i = n} n! \prod_{j=1}^{\infty} \frac{1}{k_j!} \left[\frac{f_{j+2}\phi}{j!} \right]^{k_j} . \end{aligned} \tag{2.16}$$

The last term is just the $n-th$ power of an infinite polynomial, i.e.

$$\begin{aligned} \sum_{\sum_{i=1}^{\infty} k_i = n} \frac{n!}{k_1! k_2! k_3! \cdot ...} \left(\frac{f_3 \phi}{1!} \right)^{k_1} \left(\frac{f_4 \phi^2}{2!} \right)^{k_2} \cdot ... &= \left(\sum_{i=1}^{\infty} f_{i+2} \phi^i / i! \right)^n \\ &= \left(\frac{d^2V}{d\phi^2}\Big|_{\phi} - \frac{d^2V}{d\phi^2}\Big|_0 \right)^n , \end{aligned} \tag{2.17}$$

where in the last step we have used $f_j \equiv d^jV/d\phi^j|_0$ and simply realized that inside the parenthesis we have the Taylor series of $d^2V/d\phi^2$ around zero minus the first term in the series. Thus eq. (2.16) becomes

$$\begin{aligned} V_1 &= -\frac{i}{2} \int \frac{d^4p}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n} \frac{1}{(p^2 + f_2 - i\epsilon)^n} \left(\frac{d^2V}{d\phi^2}\Big|_{\phi} - f_2 \right)^n \\ &= -\frac{i}{2} \int \frac{d^4p}{(2\pi)^4} \log \left(1 + \frac{\frac{d^2V}{d\phi^2}\big|_{\phi} - f_2}{(p^2 + f_2 - i\epsilon)} \right) \\ &= -\frac{i}{2} \int \frac{d^4p}{(2\pi)^4} \log \left(p^2 + \frac{d^2V(\phi)}{d\phi^2} - i\epsilon \right) , \end{aligned} \tag{2.18}$$

where in the last step we have neglected a field independent term. In the next section we will obtain the same formula in an easy to generalize way by introducing

the background field formalism. This will also make it possible to go to higher loops in the derivation by computing only a finite number of diagrams. We stress that it is still useful to keep this section in mind to understand the relation between the divergences arising from the computation of scattering amplitudes and the ones appearing at the level of the effective action.

2.1.2 Background field method

In presence of generic functions in the classical action it is much more convenient to work with the background field method. This provides also another nice physical interpretation of the effective action. The methods consists of "integrating out" quantum fluctuations around a classical background. Thus Γ can be seen as describing the backreaction of these fluctuations on the background dynamics.

Let us consider the field in the action S shifted by an arbitrary non-dynamical background ϕ_b. This will define a new generating functional as follows

$$Z_{\phi_b}[J] = e^{iW_{\phi_b}[J]} = \int \mathcal{D}\varphi \exp i(S[\phi_b + \varphi] + J\varphi). \tag{2.19}$$

Via the change of variable in the path integral $\varphi \to \varphi - \phi_b$, we have

$$W_{\phi_b}[J] = W[J] - J\phi_b. \tag{2.20}$$

We can analogously introduce its Legendre transform via

$$\Gamma_{\phi_b}[\tilde{\phi}] = W_{\phi_b}[J] - J\tilde{\phi}, \tag{2.21}$$

where $\frac{\delta W_{\phi_b}[J]}{\delta J} \equiv \tilde{\phi}$. In the same way, the current in (2.21) has to be replaced by $J_{\tilde{\phi}}$ given by inverting $\delta W_{\phi_b}[J]/\delta J$. From (2.20) and (2.6) we get $\tilde{\phi} = \frac{\delta W[J]}{\delta J} - \phi_b$, and putting everything together

$$\Gamma_{\phi_b}[\tilde{\phi}] = W[J] - J(\tilde{\phi} + \phi_b) = \Gamma[\phi_b + \tilde{\phi}] \tag{2.22}$$

which gives $\Gamma_{\phi_b}[0] = \Gamma[\phi_b]$. By just relabeling $\phi_b \equiv \phi$ we finally obtain

$$\Gamma_{\phi}[0] = \Gamma[\phi]. \tag{2.23}$$

The left hand side is the effective action of "the theory defined by $S[\phi + \varphi]$" and then evaluated in 0, i.e. it contains all the 1PI vacuum bubbles generated by $S[\phi + \varphi]$ (diagrams without external legs of the field φ in the background ϕ_b). The right hand side gives the functional expression we want to compute. $\Gamma_\phi[0]$ is equal to $W_\phi[J]$ evaluated at that value of J for which $\delta W_\phi/\delta J$ vanishes. By looking at (2.20) this happens for J such that $\delta W[J]/\delta J = \phi$, which is the same

$J(\phi)$ that gives $\delta\Gamma[\phi]/\delta\phi = -J(\phi)$. Thus, the effective action is given by the integro-differential equation

$$e^{i\Gamma[\phi]} = \int \mathcal{D}\varphi \exp i\left(S[\phi+\varphi] - \frac{\delta\Gamma[\phi]}{\delta\phi}\varphi\right) \equiv \int_{1\mathrm{PI}} \mathcal{D}\varphi e^{iS[\phi+\varphi]}. \tag{2.24}$$

The last expression is just a short-hand notation to summarize the following procedure to compute the effective action:

Expand the action around a background ϕ, i.e. $S[\phi+\varphi]$, and compute the sum of all the 1PI vacuum diagrams of the field φ from the resulting ϕ-dependent action.

At one loop, computing the 1PI vacuum bubble for $S[\phi+\varphi]$ is equivalent to only keeping the quadratic term in the expansion of $S[\phi+\varphi]$ and subtracting the tadpole $S'[\phi]\varphi$[5]

$$\begin{aligned} e^{i\Gamma[\phi]} &= e^{iS[\phi]}\int \mathcal{D}\varphi \exp\left(i\varphi\frac{\delta^2 S}{\delta\varphi^2}|_\phi \varphi\right) = e^{iS[\phi]}\mathrm{Det}^{-1/2}\left\{-\frac{\delta^2 S}{\delta\varphi^2}|_\phi\right\} \\ &= \exp\left\{iS[\phi] - \frac{1}{2}\mathrm{tr}\log\left(-\frac{\delta^2 S}{\delta\varphi^2}|_\phi\right)\right\}, \end{aligned} \tag{2.25}$$

where the definition of standard Gaussian integral has been used.[6] Thus for standard kinetic terms the effective action at one loop order is given by

$$\Gamma[\phi] = S[\phi] + \frac{i}{2}\int d^4x \langle x|\log\left(-\Box + V''[\phi]\right)|x\rangle + (\text{higher loops}). \tag{2.26}$$

At leading order the effective action is just the classical action. For ϕ constant we can compute the one loop contribution by introducing a complete set of momentum states, and we arrive at the same expression for $V^{1\text{loop}}$ as in (2.18). This can be computed by standard Wick rotation

$$V^{1-\text{loop}} = \frac{1}{64\pi^2}(V'')^2\left(\underline{-\frac{2}{\epsilon} - \log(4\pi e^{-\gamma})} + \log\left(\frac{V''}{\mu^2}\right) - \frac{3}{2}\right), \tag{2.27}$$

where $\epsilon = (4-d)$, d the space-time dimension and μ at this level is just an arbitrary scale. In a renormalizable theory the UV divergent part can be absorbed at every order in the loop expansion by the counterterms present in S. This leads to a finite effective action in agreement with its physical meaning as a generator of 1PI diagrams. For example, choosing the counterterms in order to cancel the

[5]Note that this is equivalent to solving (2.24) by expanding Γ in powers of $\hbar$, i.e. $\Gamma = S + \hbar\Gamma^{(1)} + \dots$.

[6]$\mathrm{Det}^{\mp 1/2}$ for bosons/fermions respectively. Sometimes in the literature there is a plus sign in front of the second derivative of the action inside tr log which means that the Euclidean action has been used, i.e. $S = iS_E$.

underlined part in (2.27) is referred to as the $\overline{\text{MS}}$ scheme.[7] Thus a necessary condition for the renormalizability of the theory is that $(V'')^2$ contains the same powers of the field as one of the terms presents in the tree level potential.

The results leads to the the well known Coleman-Weinberg potential [41]. This will be used extensively in next chapters (where we include fermions and gauge boson in the loops).

2.1.3 The Callan-Symanzik equation

By choosing a particular renormalization scheme we are giving a physical meaning to the parameters appearing in the effective potential (2.27) or more generally to a full effective action $\Gamma[\phi, \mu, \lambda_i]$. ϕ is the renormalized field. More precisely, for a different value of the renormalization scale μ, $\{\phi, \lambda_i\}$ has to change accordingly in such a way that $d\Gamma = 0$, i.e. the theory is invariant under the renormalization procedure. Looking at the expansion of Γ in terms of 1PI Green functions (2.9) and rewriting it in terms of the bare fields $\phi_R = Z_\phi^{-1/2}\phi_{\text{bare}}$ we see that[8] $Z_\phi^{n/2}\Gamma^n_{\text{bare}}(x_1, .., x_n, \lambda_{i\text{bare}}) = \Gamma^n(x_1, .., x_n, \lambda_i, \mu)$. Using the fact that the bare Green functions are independents of μ we derive a differential equation for the $\Gamma^{(n)}$ 1PI renormalized Green functions

$$\left(\mu\frac{\partial}{\partial\mu} + \beta_{\lambda_i}\frac{\partial}{\partial\lambda_i} - n\gamma_\phi\right)\Gamma^{(n)}(x_1, x_2, .., \mu, \lambda) = 0, \tag{2.28}$$

with

$$\beta_{\lambda_i} \equiv \frac{\partial\lambda_i}{\partial\ln\mu} \quad , \qquad \gamma_\phi \equiv \frac{\partial ln Z_\phi^{1/2}}{\partial\ln\mu}, \tag{2.29}$$

and $\beta_{\lambda_i}, \gamma_\phi$ are respectively the beta-function and the anomalous dimension. For the full effective action (2.28) implies

$$\left(\mu\frac{\partial}{\partial\mu} + \beta_{\lambda_i}\frac{\partial}{\partial\lambda_i} - \gamma_\phi\int\phi\frac{\delta}{\delta\phi}\right)\Gamma[\phi, \lambda_i, \mu] = 0. \tag{2.30}$$

This is the Callan-Symanzik or renormalization group equation (RGE) [42, 43] which describes the invariance of the theory with respect to the renormalization procedure. This provides us with a powerful tool to extract information from the effective action and to improve the validity of perturbation theory. As we are going

[7] In this way we are defining the renormalization scheme directly at the level of the effective action. It is in principle always possible to relate the effective action thus defined to its physical meaning by looking at the scattering amplitudes derived from Γ.

[8] Note that the relation between amputed Green functions scale the opposite with respect to the usual Green functions, i.e. $G^n_R = Z_\phi^{-n/2}G^n_{\text{bare}}$.

to show, knowing Γ at some loop order L, imposing the RGEs allows to include in a rigorous procedure the "leading terms" coming from higher loops, without the need to compute them explicitly.

2.1.4 Renormalizability in the EFT sense

Before discussing how to solve (2.30), it is interesting to see the relation between this equation and the renormalizability of the theory. Following Coleman and Weinberg [41], the β functions can be obtained by comparing coefficients in the RGE. At one loop $\Gamma \equiv \int(\mathcal{L}+\mathcal{L}_1)$ and we can only consider the explicit dependence on the renormalization scale μ from $\mathcal{L}_1$, i.e

$$\beta_{\lambda_i}\frac{\partial\mathcal{L}}{\partial\lambda_i} - \gamma_\phi\phi\frac{\delta\mathcal{L}}{\delta\phi} = -\mu\frac{\partial\mathcal{L}_1}{\partial\mu}. \tag{2.31}$$

For example, in the Standard Model (SM), considering the effective action of the modulus of the Higgs fields, the right hand side has contributions from all types of fields, and the equations can be solved. If the theory is not renormalizable the Lagrangian will be given by an infinite series and the system of RGEs will not be closed. In an effective field theory this series can be consistently truncated at some inverse powers of the cutoff of the theory, and the RGEs can be solved at every order.

Consider, for simplicity, the following example,

$$V = \frac{\lambda\phi^4}{4!} + \frac{\lambda_6}{6!}\frac{\phi^6}{\Lambda^2} \tag{2.32}$$

where Λ is some mass scale. The divergences in the one loop effective potential are

$$V^{(1)} = -\frac{1}{64\pi^2}\frac{2}{\epsilon}\left(\frac{\lambda^2\phi^4}{4} + \frac{\lambda\lambda_6\phi^6}{4!\Lambda^2} + \frac{\lambda_6^2\phi^8}{4!^2\Lambda^4}\right). \tag{2.33}$$

In order to reabsorb the last term we need to add a new operator to the tree level potential, i.e. $\mathcal{O} = \lambda_8\phi^8/8!$. However, the radiative corrections reevaluated by taking this new term into account will require a new addition to the Lagrangian to absorb the new divergences. Thus, the right way to look at the non-renormalizable Lagrangian is to define the potential as an infinite series,

$$V = \sum_{n=2}^{\infty}\frac{\lambda_{2n}}{2n!}\frac{\phi^{2n}}{\Lambda^{2n-4}}. \tag{2.34}$$

In theory we should fix an infinite number of coupling constants to make predictions. In practice, if we are interested in processes below the energy scale Λ, and

only up to a certain order, let's assume up to $O(E^2/\Lambda^2)$ for clarity purpose, we can truncate the series as in (2.32) and solve eq. (2.31) up to order $O(\Lambda^{-2})$. The β functions (assume for simplicity $\gamma_\phi = 0$) will be given by

$$\beta_\lambda = \frac{3\lambda^2}{16\pi^2}, \qquad \beta_{\lambda_6} = \frac{15\lambda\lambda_6}{16\pi^2}, \tag{2.35}$$

and we say that the effective field theory is perturbatively renormalizable, i.e. at every order in E/Λ only a finite number of counterterms is needed. At the level of the effective action we can obtain the same truncation by saying that we are interested in the dynamics of the background field for $\phi \ll \Lambda$ up to corrections $O(\phi^2/\Lambda^2)$. This way of thinking will be useful when the cutoff of the theory and the field value that defines the regime of validity of the EFT differ. In other words, we will consider in each field region an EFT by expanding in terms of a small parameter, in this case $\delta = \phi^2/\Lambda^2$. In the following we take the freedom to use the term *renormalizable in the EFT sense* to mean that:

It is possible to define a parameter δ in such a way that at every order in δ the theory can be renormalized with a finite number of counterterms.

Consider again (2.32) and the β-functions (2.35). It might seem that the running of the quartic coupling is not affected by the higher dimensional operators. This is indeed not correct if the scalar field acquires a non-zero vev, i.e. $\langle\phi\rangle = \bar{\phi}$. The physical, measured parameters are given by looking at the scattering of fluctuations (particles) in a given background. Take (2.34) and expand around some constant background $\phi = \bar{\phi} + \varphi$, the four point interaction will be trivially given by

$$\mathcal{L}_4 = \frac{1}{4!}\left(\lambda + \frac{\lambda_6\bar{\phi}^2}{2\Lambda^2} + \ldots\right)\varphi^4 \equiv \frac{1}{4!}\tilde{\lambda}\varphi^4, \tag{2.36}$$

and its running, using (2.35), is

$$\beta_{\tilde{\lambda}} = \beta_\lambda + \beta_{\lambda_6}\frac{\bar{\phi}^2}{2\Lambda^2} + O(\Lambda^{-4}). \tag{2.37}$$

Note that one can equivalently compute the running of the quartic coupling $\tilde{\lambda}$ expanding the action (2.32) around the background and compute the diagrams that contribute to the four point interaction.

At this level there is the ambiguity/degeneracy of absorbing the divergences in one of the two counterterms for λ and λ_6. One can, for example, define a vev dependent counterterm for λ or absorb the divergences proportional to $\bar{\phi}^2/\Lambda^2$ in λ_6. Both choices are equivalent from the prospective of the low energy theory because they give the same running for the physical coupling $\tilde{\lambda}$. Equivalently, as will be discussed in the next chapter, considering an interaction term like $(C_{\phi F}/\Lambda^2)\phi^2 F^{\mu\nu}F_{\mu\nu}$ will

give a contribution to the running of the quartic coupling. The four point interaction obtained by considering this operator attached to one 4pt vertex (see fig. (3.2) in sec. 3.4.4) can be seen as generating a vev dependent counterterm for λ or to renormalize λ_6 in (2.36), what matters is again that these operators do indeed affect the physical quartic coupling measured in an experiment at low energy.

2.2 Renormalization group improving

Let us review how to solve the Callan-Symanzik equation. We first consider the case when only one mass scale is present in the Lagrangian and then we generalize to multiple mass scales.

2.2.1 Single mass scale

Consider eq. (2.30) for the effective potential,

$$\mathcal{D}V \equiv \left(\mu\frac{\partial}{\partial\mu} + \lambda_i\frac{\partial}{\partial\lambda_i} - \gamma\phi\frac{\partial}{\partial\phi}\right)V = 0. \tag{2.38}$$

This is a first order partial differential equation and the method of characteristics provides the following formal solution

$$V(\phi, \lambda_i, \mu) = V(\phi(t), \lambda_i(t), \mu(t)) \equiv V(t) \tag{2.39}$$

where

$$\phi(t) = e^{-\int_0^t \gamma(\lambda_i(t'))dt'}\phi \equiv \rho(t)\phi, \quad \frac{d\lambda_i(t)}{dt} = \beta_i(\lambda_j(t)), \quad \mu(t) = \mu e^t \tag{2.40}$$

with initial conditions

$$\phi(t=0) = \phi, \quad \lambda_i(t=0) = \lambda_i, \quad \mu(t=0) = \mu. \tag{2.41}$$

As a result, the RGE equation reduces the number of variables on which the effective potential depends by one. The potential is constant along a characteristic curve $(\phi(t), \lambda_i(t), \mu(t))$ which is parametrized by t (sometimes referred to as the "RG time"). In order to extract information from the formal solution above we need to know the effective potential for a particular value of t, called the *boundary function*. In fact, suppose we know the exact functional form of the effective potential for a particular value $t = \tilde{t}$, then because of (2.39), $V(\phi(\tilde{t}), \lambda_i(\tilde{t}), \mu(\tilde{t}))$ will give the exact full effective potential, which, written in this way, is called the *RG improved potential.*

If, instead, we know for a given t the functional form of V only up to some log[9]

[9] We always discuss about UV divergences in this thesis, IR logs are not considered.

Leading log/loops	0-loop	1-loop	2-loops	3-loops	...	L-loops	...
λg_0	λ	$\lambda^2 \log$	$\lambda^3 \log^2$	$\lambda^4 \log^3$	...	$\lambda^{L+1} \log^L$	...
$\lambda^2 g_1$		λ^2	$\lambda^3 \log$	$\lambda^4 \log^2$	...	$\lambda^{L+1} \log^{L-1}$	...
$\lambda^3 g_2$			λ^3	$\lambda^4 \log$	...	$\lambda^{L+1} \log^{L-2}$	...
...				...	...	...	...
$\lambda^{L+1} g_L$						λ^{L+1}	...

Table 2.1: *Consider the simple $\lambda\phi^4/4!$ theory, the loop counting parameter is λ with the prescription $\hbar^{L-1} \to \lambda^{L+1}$. If you compute, for example, the 2-loop effective potential (terms in red) by RG improving you have a potential that includes also all the terms in yellow.*

corrections we can still use this as a boundary function to rewrite the RG improved potential up to these corrections. To clarify the meaning of this vague sentence let us show the generic log structure of the potential. Since the L-loop contribution will have terms which are at most Lth power of the log, we can rewrite it in the following form

$$V^{(L)} = \hbar^{-1}(v_0^{(L)} s^L + \hbar v_1^{(L)} s^{L-1} .. + \hbar^L v_L^{(L)}) \tag{2.42}$$

where

$$s = \hbar \log \left(\frac{M^2(\phi)}{\mu^2} \right). \tag{2.43}$$

We are assuming only one mass scale entering in the logs, for the theory in section 2.1.2 $M^2(\phi) = d^2V/d\phi^2$. We have introduced introduce $\hbar$ as the loop counting parameter (the L-th loop generates the term proportional to $\hbar^{L-1}$) and $v_i^{(j)}$ are functions of the field and all the other couplings/mass parameters (λ_i). The full potential can be written in general as [44]

$$V = M^4(\phi) \sum_{i=0}^{\infty} \hbar^{i-1} \left(\sum_{L=i}^{\infty} v_i^{(L)} s^{L-i} \right) \equiv M^4(\phi) \sum_{i=0}^{\infty} \hbar^{i-1} g_i. \tag{2.44}$$

The sum is referred to as the leading log expansion because g_0 is a sum that picks the leading log from each loop, g_1 the sum of the next-to-leading logs and g_i the i-th-to-leading logs. If $s = \hbar \log \sim O(1)$ we see that the first term in the sum gives the leading order contribution to the effective potential.

Notice that for $s = 0$, $g_L|_{s=0} = v_L^{(L)}$ is determined only by the L-loop potential (all the other terms in the series coming from higher loops are canceled). Thus, for $s = 0$ the full effective potential and the L-loop potential $V_L = \sum_{i=0}^{L} V^i$ are

equivalent up to (Lth)-to leading log terms, i.e. $V = V|_{s=0} = V_L|_{s=0} + O(\hbar^L)$.[10]

Let us go back to (2.39). We wish to know a particular t for which the functional form of the effective potential is known, we will refer to this in the following as the *optimal choice* for t. From what we have just seen, given the knowledge of the L loop potential, for $\tilde{t}$ such that

$$\bar{s}(\tilde{t}) = \ln \frac{\bar{M}^2(\tilde{t})}{\bar{\mu}^2(\tilde{t})} = 0 \tag{2.45}$$

we have that $V_L|_{\bar{s}=0} \equiv V_L(\tilde{t})$ gives the exact RG improved potential up to order L in this leading log expansion. See at table 2.1 for an intuitive representation. Therefore depending on which order we need the potential for doing our computations we will consider

$$V_L|_{\bar{s}=0} \equiv V_L(\tilde{t}). \tag{2.46}$$

and the RGE coefficients functions β, γ at $(L+1)$-loop order accordingly to [45, 46].

2.2.2 Multi-mass scale

The procedure described so far seems to hold only when one single mass scale is present in the potential. In general, when there is more than one field, the effective potential will be a function of the various $\log(M_i^2(\phi)/\mu^2)$, i running over all field dependent masses $M_i(\phi)$. If there is a hierarchy in mass scales, a choice of renormalization scale leaves a large log in the effective potential, that can spoil the validity of perturbation theory. Let us take the example of two mass scales. The L-loop potential will in general look like [45, 46, 47]

$$V^L = \hbar^{-1} \sum_{j,k\geq 0}^{j+k\leq L} \hbar^{L-(j+k)} v_{k,j}^{(L)}\, s_1^j s_2^k, \tag{2.47}$$

where $s_i = \hbar \ln(M_i^2(\phi)/\mu^2)$. The full potential is

$$V = \sum_{i=0}^{\infty} \hbar^{i-1} \left(\sum_{k+j\geq 0}^{\infty} v_{k,j}^{(k+j+i)} s_1^k s_2^j \right) \equiv \sum_{i=0}^{\infty} \hbar^{i-1} g_i. \tag{2.48}$$

Setting one of the two logarithms to zero, e.g. $s_1 = 0$ $(\mu = M_1(\phi))$, the L-th to leading term becomes

$$g_L|_{s_1=0} = \sum_{j=0}^{\infty} v_{0,j}^{(L+j)} u^j = v_{0,0}^{(L)} + \underline{v_{0,1}^{(L+1)} u + v_{0,2}^{(L+2)} u^2 + \ldots} \tag{2.49}$$

[10] It might seem obvious that V and V_L differ for an $O(\hbar^L)$ term. But the reader should be careful, what we mean by that are order $\hbar^L$ terms in the leading log series expansion (2.44).

where $u \equiv s_2|_{\mu=M_1(\phi)} = \hbar \log(M_2/M_1)^2$. This is not determined uniquely by the L-loop potential, in contrast to the single mass case. The difference is in the terms underlined in (2.49), i.e. a series of logarithms that can be large. Therefore using $V_L|_{s_1=0}$ as a boundary function will not give in general the exact effective potential up to L-th to leading log in this case. For example at two loops/next-to-next-to-leading order $V_2|_{s_1=0}$ will neglect all the terms highlighted in yellow in table 2.1 (with log replaced by $\log(M_2/M_1)$). These terms are higher order only if $\log(M_2/M_1) \sim O(1)$. Only in the region of field space where $s_2|_{s_1=0} = u = \hbar \log(M_2/M_1)^2 \sim \hbar O(1)$ is the difference between the L-loop potential and the full potential at $s_1 = 0$ of order $\hbar^L$ in the leading log expansion. Thus, in general, one can find a $\tilde{t}$ for which

$$s_i(\tilde{t}) = O(\hbar), \quad \forall i \tag{2.50}$$

one can use the L-loop potential V_L in $\tilde{t}$ as a boundary function to include all the L-th to leading log terms.[11]

However, this is not always possible and there are many studies in the literature to overcome this difficulty and resum large logarithms in presence of multiple mass scales. The first attempt was given in [48] and further improved in [49, 50]. The idea is to modify the bare Lagrangian in order to have two different renormalization scales in the effective potential. This leads to double the number of RGEs and each parameter comes with two beta-functions. Given the rather expensive structure of the procedure, M. Bando et al. in [46] proposed a more physical solution. In short, by using the decoupling theorem [51], the author shows[12] that in each region of the field space it is possible to define an effective field theory (EFT) where all mass scales but one decouple, and the net effect of the decoupled species for the single d.o.f. will be a shift in the definition of the parameters of the EFT ($\lambda_i \to \tilde{\lambda}_i$). Thus, region by region each loop contribution will have the same form as (2.42) in terms of the shifted parameters.

Consequently in each EFT the problem is equivalent to the single mass case shown in the previous section where only one logarithm remains relevant in the effective potential.

Schematically, they solve equations of the form $\tilde{\mathcal{D}}V = 0$ for each EFT ($\tilde{\mathcal{D}}$ being the RG operator in terms of beta-functions and shifted parameters). In a standard set-up the renormalizability of the theory ensures that in each EFT the RG operators

[11] Remember that by $\hbar$ we mean the loop counting parameter that in each specific example will be replaced by a particular coupling.

[12] The model taken as example in this works is often the Higgs-Yukawa Lagrangian

$$\mathcal{L} = -\tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 - \bar{\psi}(i\partial + g\phi)\psi - \Lambda, \tag{2.51}$$

with the two field dependent mass scales $\lambda\phi^2/2 + m^2$ and $g^2\phi^2$.

$\tilde{\mathcal{D}}$ are the same. Indeed, they define the same equation by simply using different set of parameters in different regions. Thus, the matching between the solutions is provided by the equations that relate the parameters of two adjacent EFTs evaluated at the renormalization point μ around the threshold [46].

Let us anticipate here an important caveat. In generic inflationary models, like the one studied in this thesis, that are renormalizable only in the EFT sense, the operators $\tilde{\mathcal{D}}$ are not necessarily the same in each EFT and to patch together the EFTs we would need some threshold corrections, as we will extensively explain in the following chapters.

The details, although not the main idea, of [46] relies on identifying the decoupling scale with an invariant mass parameter (field independent mass contributing to the mass eigenvalues $M_i(\phi)$). Hence, in [52] Casas et al. elaborate further by identifying the decoupling scale with the field dependent masses $M_i(\phi)$. This technique is based on running the parameters top-down from a high scale passing through the various mass thresholds of the model. Since these latter depend on the value of the field ϕ, so are the low energy parameters $\lambda_i(\mu, \phi)$. Thus, for each field value ϕ, the optimal choice $\mu(\tilde{t})$ at which the parameters have to be evaluated is simply given by $\mu(\tilde{t}) = \min\{M_i(\phi)\}$ where all the particles in the RG flow have decoupled. Although general for a full renormalizable theory, given that the procedure is top-down, this method seems to be hardly applicable when we have only partial knowledge of the RG flow.

Recently in [53] a new method has been proposed in which the RG scale $\tilde{t}$ is defined on the tree level hypersurface, i.e. the surface in the space (ϕ, λ_i, μ) where the one loop contribution vanishes. In doing so it is possible to resum the relevant logarithms (although not all the leading logs in the usual sense) in each region of the field domain.[13] Another possibility that we are currently studying is whether is possible to use factorization techniques familiar from QCD resummations to improve the RG potential when multiple mass scales are present.

For the purpose of this thesis it is worth to remember what happens in the Standard Model (SM). For any value of the Higgs field, the Higgs-Goldstone contribution is subdominant with respect to the the Gauge/fermion one in the effective potential. Furthermore, the top quark has a mass of the same order as the gauge boson W, Z. We are then in the situation where the multiple mass scales reduce immediately to the single mass scale as in (2.50). In fact, by choosing $\mu(\tilde{t}) = M_{\text{top}}(\phi(\tilde{t}))$ all the other logarithmic contributions become higher order in the leading log expansion.

[13] All these works are based on RG improving in a mass independent renormalization scheme (the $\overline{\text{MS}}$ scheme), where the decoupling has to be implemented by hand. In [47] the multi-mass scale problem has been studied in a modified mass dependent renormalization scheme where the decoupling is automatically incorporated.

This property is inherited by Higgs inflation, where all the masses rescale in the same way after the conformal transformation (see next chapter 3).

In more general sets-up when the coupling between the inflaton and the other fields is left undetermined we are using implicitly the results of [46]. In the field regime where inflation takes place we consider an EFT where only one logarithm remains relevant.

2.3 Canonical and non-canonical kinetic terms

In the previous section we have focused on improving the effective action when this is just determined by the effective potential. As already mentioned, when the aim is to find the space-time independent vacuum state of a theory this is what is needed. However, when the vev of the theory is not constant, its dynamics is determined by minimizing the full effective action. Thus, this will be the object to improve in order to be consistent in perturbation theory. In this section to avoid carrying space-time integrals we use the notation $\Gamma \equiv \int \mathcal{L}_{\text{eff}}$. Consider the first derivative correction (2.10) to the effective action

$$\mathcal{L}_{\text{eff}}[\phi] = -V_{\text{eff}}[\phi] - \frac{1}{2}Z[\phi]\partial_\mu\phi\partial^\mu\phi + \mathcal{O}((\partial\phi)^4) \tag{2.52}$$

where we have neglected higher derivatives terms $\sim Y(\phi)(\partial\phi)^4 + ...$[14]

Remember in the evaluation of eq. (2.26) it was fundamental to assume a constant field so that the operator inside the log becomes diagonal in the momentum representation. If the classical field is not constant the one loop contribution must be calculated in spacetime-dependent perturbation theory. Example computations can be found in [54, 55]. One powerful technique consists of substituting $\phi = \phi_c + \tilde{\phi}(x)$, (with ϕ_c constant) inside the log operator in an equation similar to (2.26). Then one can expand the operators inside the log in powers and derivatives of $\tilde{\phi}$ and compute each term in the series. By recollecting the coefficients in front we obtain expressions for $Z(\phi)$ as well as for the other higher derivatives terms.[15] If the theory is renormalizable the term quadratic in derivatives, i.e. $(\partial\phi)^2$, is the only one that generates divergences that can be absorbed in the counterterm $(1+\delta_Z)(\partial\phi)^2$ of the classical action. Thus in the general case Z will depend on $Z = Z(\phi, \lambda_i, \mu)$.

[14] When the effective action is used for the inflationary analysis the omission of these higher order terms is justified by the slow roll approximation.

[15] Another way it is to compute the loop corrections explicitly for $\phi = \phi(t)$ as has been done in [56].

In this section we first review the standard procedure to RG improve the action (2.52). Then we generalize to the case where a non canonical kinetic term is present.

Even if for a single scalar field it is always possible to define classically the canonical field, sometimes it is useful to work in terms of other field variables and do the improving in terms of them. In fact, it is not always possible to find an analytic expression for the canonical field over the whole field range.In addition, as we will see, in the multiple fields case, if the field space manifold is not flat, it is not possible to do a field redefinition to write the kinetic terms in canonical form. In those cases we are forced to deal with non-canonical kinetic terms.

In the rest of this section we look at how to improve an effective action in the presence of a non-canonical kinetic term. Let us first review the standard case with a canonical kinetic term.

2.3.1 Standard procedure revisited

The Callan-Symanzik equation (2.30) for the effective action is satisfied independently by each term in the expansion (2.10). Applying the RGE to the kinetic term gives, after some integrations by part, the following expression

$$\left(\mu\frac{\partial}{\partial\mu}+\beta_i\frac{\partial}{\partial\lambda_i}-\gamma_\phi\left(2+\phi\frac{\partial}{\partial\phi}\right)\right)Z(\phi,\lambda_i,\mu)=0. \tag{2.53}$$

In this case we can use straightforwardly the characteristics method to write the formal solution

$$Z(\phi,\lambda_i,\mu)=Z(\phi(t),\lambda_i(t),\mu(t))\rho^2(t)\equiv Z_{\text{eff}}(t) \tag{2.54}$$

where $\{\phi(t),\lambda_i(t),\mu(t),\rho(t)\}$ are given by (2.40) with the same initial conditions. In the leading order approximation[16]

$$Z_{\text{eff}}(t)\overset{\texttt{LO}}{\approx}\rho^2(t)=e^{-2\int_0^t\gamma_\phi dt'}. \tag{2.55}$$

The improved effective action becomes,

$$\mathcal{L}_{\text{eff}}(t)=-\frac{1}{2}Z_{\text{eff}}(t)(\partial_\mu\phi)^2+V(t). \tag{2.56}$$

As usual, it is convenient to use a *canonical field redefinition*, i.e. $Z_{\text{eff}}^{1/2}d\phi=dh$. This is useful for two reasons: the equations become simpler and in gauge theories

[16]Since also Z depends on a series of logarithms, the optimal choice of t to minimize them in the effective potential will set Z to one at the leading order (see the appendixes in [57] for the explicit expression in the SM.)

the gauge dependence in the potential is significantly reduced (see the discussion in [58]). As a first approximation we neglect the $t = t(\phi)$ dependence, then

$$h_c \approx \rho(t)\phi. \tag{2.57}$$

Later, in a specific example we will take into account first order corrections to the previous equation. In the simple example $V = \lambda\phi^4/4!$, the improved effective action, becomes

$$\Gamma = -\frac{Z_{\text{eff}}(t)}{2}(\partial_\mu\phi)^2 - \frac{\lambda(t)}{4!}\rho^4(t)\phi^4 + .. = -\frac{1}{2}(\partial_\mu h_c)^2 + \frac{\lambda(t)}{4!}h_{\text{c}}^4 + .. \tag{2.58}$$

where in the last step we used $\rho^4(t)Z_{\text{eff}}^{-2}(t) \approx 1$.

2.3.2 RG improvement with a non-canonical kinetic sector

Let us now see how the previous discussion applies to generic models with non-canonical kinetic term. We will be pedantic with the notation. The purpose is to avoid confusion, the reader should forgive us if in doing so we achieve the opposite result. Consider the following Lagrangian

$$\mathcal{L} = -\frac{1}{2}K(\phi, \lambda_i)(\partial\phi)^2 - V, \tag{2.59}$$

the kinetic sector is not canonical because the "field space metric" K is a generic function of the fields and couplings. In order to compute the improved effective action we can follow two paths.

1) We can do a field redefinition in the classical action, i.e.

$$K^{1/2}\partial\phi = \partial h \implies \phi = \phi(h, \lambda_i), \tag{2.60}$$

and compute the effective action starting from $S(\phi(h))$. Then by imposing an RGE on Γ we can go through the same procedure explained in section 2.2, to find a boundary function and an optimal choice for the renormalization scale to write

$$\mathcal{L}_{\text{eff}}(t) = -\frac{1}{2}Z_{h\,\text{eff}}(t)(\partial h)^2 - U(h, \lambda_i(t)) \tag{2.61}$$

where $U(h, \lambda_i(t)) \equiv V(\phi(\rho_h(t)h, \lambda_i(t)), \lambda_i(t))$.
As before $Z_{h\,\text{eff}} \approx \rho_h^2(t) = \exp(-2\int_0^t \gamma_h\, dt')$. By defining the canonical field

$$\rho_h(t)dh = dh_c \tag{2.62}$$

the action becomes

$$\mathcal{L}_{\text{eff}}(h_c) = -\frac{1}{2}(\partial h_c)^2 - U(\underbrace{\rho_h(t)\,h(h_c)}_{\approx h_c}, \lambda_i(t)), \tag{2.63}$$

where the approximation comes from approximating the solution of (2.62)

$$h(h_c) \approx h_c \rho_h^{-1}(t). \tag{2.64}$$

This is the action usually used if the RG improving is done for (non-canonical) kinetic terms.

2) The second option is to compute the effective action in terms of ϕ starting from (2.59). At leading order

$$\mathcal{L}_{\text{eff}}(\phi) = -\frac{1}{2} Z_\phi(\phi, \lambda_i) K(\phi, \lambda_i) \partial_\mu \phi \partial^\mu \phi + V + ... \tag{2.65}$$

The blind application of the RG techniques to the kinetic sector can in principle be problematic. However, when we rewrite our improved effective action as a formal solution of the RG equations we find that $Z' \equiv Z_\phi(\phi, \lambda_i) K(\phi, \lambda_i)$ satisfies an equation of the form (2.53). The solution, as in (2.54), is

$$Z'_{\text{eff}}(t) = Z_\phi(\phi(t), \lambda_i(t), \mu(t)) K(\phi(t), \lambda_i(t)) \rho_\phi^2(t) \overset{\text{LO}}{\approx} K(t) \rho_\phi^2(t) \tag{2.66}$$

and the improved effective action assumes the form

$$\mathcal{L}_{\text{eff}}(t) = -\tfrac{1}{2} \rho_\phi^2(t) K(\phi(t), \lambda_i(t)) (\partial_\mu \phi)^2 - V(\phi(t), \lambda_i(t), \mu(t)). \tag{2.67}$$

Now, in order to write a canonical kinetic term we can proceed in two steps.[17] First define ϕ_c in order to get rid of the anomalous dimension of the field ϕ in the kinetic term, i.e.

$$\rho_\phi(t) K^{\frac{1}{2}}(\phi \rho_\phi(t), \lambda_i(t)) d\phi = K^{\frac{1}{2}}(\phi_c, \lambda_i(t)) d\phi_c. \tag{2.68}$$

Using the approximate solution to (2.68),

$$\phi(\phi_c) \approx \rho_\phi^{-1}(t)\, \phi_c \tag{2.69}$$

we obtain

$$\mathcal{L}_{\text{eff}}(\phi_c) = -\tfrac{1}{2} K(\phi_c, \lambda_i(t)) (\partial_\mu \phi_c)^2 - V(\underbrace{\rho_\phi(t)\, \phi(\phi_c)}_{\approx \phi_c}, \lambda_i(t), \mu(t)). \tag{2.70}$$

Thus from an operative point of view one can take the action (2.59) and make every coupling running except the field ϕ. Usually this is what is used in the literature when RG improving an action with non-canonical kinetic term is considered. It is important to remember that this is valid when the approximate solution (2.69) to

[17] In principle one can combine (2.68) and (2.71) in one redefinition. We do that in two steps because the first one is what is done implicitly when one works in terms of the non canonical kinetic term.

(2.68) is a good approximation. In the same way, the approximation in (2.63) is a good approximation only if (2.64) is good as well.

From the last action, if we want to compute derivatives with respect to the canonical field we can define implicitly

$$K^{\frac{1}{2}}(\phi_c, \lambda_i(t))d\phi_c = dh_c. \tag{2.71}$$

Note that the two procedures must lead to the same action if no approximations are made. In fact, (2.61) and (2.67) are just solution of the same RG equations written in terms of different variables. Thus, the potential U in eq. (2.63) defined implicitly in terms of h_c (without using (2.64)) must be the same function as the potential V in (2.70) (without using the approximation (2.69)) once it is expressed in terms of the canonical field h_c defined in (2.71).

Thus, if one does not use the approximations (2.64) and (2.69), computing U_{h_c} $U_{h_c h_c}$, from the first method will be equivalent to computing V_{h_c},$V_{h_c h_c}$ from the second procedure. However, when the approximations above are used, is not transparent to what extend the two procedure gives the same result. Given the arbitrariness in defining a non-canonical field in the tree level action, it may be that some relevant running of the couplings are neglected by using one of the two procedures.

In the next section we will consider a specific simple example. The bottom line will be that for the quantities of interest, like the potential slow roll parameters ϵ, η, such an approximation is enough. Nonetheless, if you want to translate a result in terms of ϕ_c in a result in terms of h_c, you should consider the corrections coming from integrating (2.71), where t is a function of ϕ_c.

2.3.3 Explicit example

Let us take the following Lagrangian,

$$\mathcal{L} = -\frac{1}{2}\frac{\lambda\phi^4}{4\mathcal{M}^4}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4 - \bar{\psi}i\partial\!\!\!/\psi - \frac{y}{\sqrt{2}}\phi\,\bar{\psi}\psi \tag{2.72}$$

and consider the "large field regime", $\phi \gg 4\mathcal{M}^4$. Classically the canonical field is defined via

$$\frac{\sqrt{\lambda}\phi^2}{2\mathcal{M}^2}d\phi = dh \quad \Rightarrow \quad \phi^3 = \frac{6\mathcal{M}^2}{\sqrt{\lambda}}h \tag{2.73}$$

and the action becomes

$$\mathcal{L} = -\frac{1}{2}(\partial h)^2 - \frac{\tilde{\lambda}}{4}h^{4/3} \tag{2.74}$$

with (in Planck units)

$$\tilde{\lambda} = 6^{4/3}\mathcal{M}^{8/3}\lambda^{1/3}. \tag{2.75}$$

The field dependent mass $m_h^2 = V_{hh}$ is in the large field regime suppressed, and the only relevant loop contribution will depend on the mass of the fermion, $m_f \propto \phi(h)$. Thus, in the RG improved leading log potential we can set[18]

$$t = \ln\phi(h) = \ln ch^{\frac{1}{3}}. \tag{2.76}$$

Start with the canonical field. We work for illustrative purpose with the following assumptions

$$\frac{\beta_X}{X} \gg \frac{\beta_X^2}{X^2}, \frac{\beta'}{X}, \qquad \gamma_i \gg \gamma_i^2, \gamma_i', \tag{2.77}$$

i.e. we drop all terms quadratic and all derivatives of the beta-functions and anomalous dimensions, and only keep the leading contribution. The improved scalar action, i.e. the solution of the Callan-Symanzik is given by[19]

$$\mathcal{L}_{\text{eff}}(t) = -\frac{1}{2}Z_{h\,\text{eff}}(t)(\partial h)^2 - \frac{\tilde{\lambda}(t)}{4}h^{4/3}(t) \tag{2.78}$$

with

$$h(t) = e^{-\int_0^t \gamma_h dt'}h \equiv \rho_h(t)h, \qquad \frac{d\tilde{\lambda}(t)}{dt} = \beta_{\tilde{\lambda}}, \qquad Z_{h\,\text{eff}}(t) \approx e^{-2\int_0^t \gamma_h dt'}. \tag{2.79}$$

We can define the canonical field via $dh_c = \rho_h(h)dh$. Integrating the previous relation (approximate γ_h as a constant) by using the h dependence in t in (2.76) gives

$$h_c = \int_0^h e^{-\int_0^{t(h')}\gamma_h \mathrm{d}t'}\mathrm{d}h' = \frac{h\rho_h(h)}{(1-\gamma_h/3)} \quad \Rightarrow \quad h = \rho_h^{-1}(h)h_c\left(1 - \frac{1}{3}\gamma_h\right). \tag{2.80}$$

In parenthesis we have the first correction to (2.64) and the action becomes

$$\mathcal{L}_{\text{eff}}(h_c) = -\frac{1}{2}(\partial h_c)^2 - \frac{\tilde{\lambda}(h_c)}{4}h_c^{4/3}\left(1 - \frac{4}{9}\gamma_h\right). \tag{2.81}$$

[18] We neglect the fact that also the argument of the logarithm depends implicitly on t. However, it is not hard to show that taking that into account in the integral (2.80) will give higher order contributions in the approximation we are working.

[19] Sometimes the potential is rewritten as $V = \lambda_{\text{eff}}(t)h^{4/3}$, at LO $\lambda_{\text{eff}}(t) = \lambda(t)e^{-\frac{4}{3}\int_0^t \gamma_h dt'}$.

Likewise, for the ϕ field, the improved action is

$$\mathcal{L}_{\text{eff}}(\phi) = -\frac{1}{2}\frac{\lambda(\phi)\rho_\phi^4(\phi)\phi^4}{4\mathcal{M}^4(\phi)}Z_\phi(\phi)(\partial\phi)^2 - \frac{\lambda(\phi)}{4}\rho_\phi^4(\phi)\phi^4 \tag{2.82}$$

where now $t(\phi) = \ln\phi$, and $(\lambda, \mathcal{M})$ the couplings running with the beta-functions. We can absorb all the anomalous dimension dependence in the kinetic term by

$$d\phi_c = e^{-3\int_0^{t(\phi)}\gamma_\phi dt'}d\phi. \tag{2.83}$$

This is the analogous of (2.68) and solving as in (2.80) we find the first correction to (2.69), i.e. $\phi e^{-\int_0^{t(\phi)}\gamma_\phi dt'} = \phi_c(1 - 1/3\gamma_\phi)$. Thus, the improved action becomes

$$\mathcal{L}_{\text{eff}}(\phi_c) = -\frac{1}{2}\frac{\lambda(\phi_c)\phi_c^4}{4\mathcal{M}^4(\phi_c)}(\partial\phi_c)^2 - \frac{\lambda(\phi_c)}{4}\phi_c^4\left(1 - \frac{4}{3}\gamma_\phi\right). \tag{2.84}$$

We can define the canonical kinetic term implicitly by

$$\frac{\sqrt{\lambda(\phi_c)}\phi_c^2}{2\mathcal{M}^2(\phi_c)}d\phi_c = dh_c, \tag{2.85}$$

which in the approximation (2.77) is integrated to give explicitly

$$\phi_c^3 = \frac{6\mathcal{M}^2(\phi_c)}{\sqrt{\lambda(\phi_c)}}h_c\left(1 + \frac{1}{6}\frac{\beta_\lambda}{\lambda} - \frac{2}{3}\frac{\beta_\mathcal{M}}{\mathcal{M}}\right). \tag{2.86}$$

Using the relation between the beta functions and the anomalous dimensions in (2.90) (see below for the derivation),

$$\frac{1}{6}\frac{\beta_\lambda}{\lambda} - \frac{2}{3}\frac{\beta_\mathcal{M}}{\mathcal{M}} = \gamma_\phi - \frac{1}{3}\gamma_h, \tag{2.87}$$

we see explicitly that the two actions (2.84) and (2.81) are equivalent. By labeling, as in the previous section, with V the potential in (2.84) and with U the one in (2.81), we conclude that $V = U$.

We will be mostly interested in applying the formalism to inflation, and use it to calculate slow roll parameters. These are given by the ratio between derivatives of the potential with respect to the canonical field and the potential itself, e.g. $V_{h_c}/V = K^{-1/2}(\phi_c)V_{\phi_c}/V = U_{h_c}/U$. From a practical point of view, because of the ratios and the approximation (2.77), we are allowed to drop the anomalous dimension terms in the potentials and still obtain consistent results in both formalism.[20] Note that in order to translate a result expressed in terms of ϕ_c in a result in terms of h_c we have to integrate eq. (2.85) with the running couplings.

[20] See our appendix in [3] for the explicit check.

Relations between beta functions and anomalous dimensions

What remains is to derive eq. (2.87), which we will do now. Consider our toy model with a real scalar plus a fermion. Write the action in terms of the renormalized fields and couplings

$$\phi = Z_\phi^{1/2}\phi_b, \quad h = Z_h^{1/2}h_b, \quad \lambda = Z_\lambda \lambda_b, \quad \tilde{\lambda} = Z_\lambda \tilde{\lambda}_b, \quad \mathcal{M} = Z_\mathcal{M}\mathcal{M}_b \tag{2.88}$$

and similar for the fermionic fields and yukawa coupling. The action for the scalar field is

$$\mathcal{L} = \frac{Z_\phi^3 Z_\lambda}{Z_\mathcal{M}}\frac{\lambda\phi^4}{4\mathcal{M}^4}\frac{1}{2}(\partial\phi)^2 - Z_\phi^2 Z_\lambda \frac{\lambda}{4}\phi^4 = Z_h \frac{1}{2}(\partial h)^2 - Z_h^{2/3} Z_{\tilde{\lambda}}\frac{\tilde{\lambda}}{4}h^{4/3},. \tag{2.89}$$

This can be written in terms of either the canonical field h or the field ϕ. For the kinetic terms to be the same in either language, this gives the relation between the Z-functions:

$$Z_h = \frac{Z_\phi^3 Z_\lambda}{Z_\mathcal{M}^4} \quad \Rightarrow \quad \frac{\beta_\lambda}{\lambda} - 4\frac{\beta_\mathcal{M}}{\mathcal{M}} - 6\gamma_\phi + 2\gamma_h = 0 \tag{2.90}$$

In the second expression we took ∂_t derivative with $t = \ln\mu$ the renormalization time. The anomalous dimension is defined $\gamma_h = -\frac{1}{2}\partial_t \log Z_h$ and the beta function derived from $\partial_t \ln Z_\lambda = -\beta_\lambda/\lambda$, and likewise for the other fields and couplings. From the potential we get the relation between the Z-functions

$$Z_\phi^2 Z_\lambda = Z_h^{2/3} Z_{\tilde{\lambda}} \quad \Rightarrow \quad \frac{\beta_\lambda}{\lambda} - \frac{\beta_{\tilde{\lambda}}}{\tilde{\lambda}} - 4\gamma_\phi + \frac{4}{3}\gamma_h = 0. \tag{2.91}$$

Combining (2.90) and (2.91) gives

$$\frac{\beta_{\tilde{\lambda}}}{\tilde{\lambda}} = \frac{1}{3}\frac{\beta_\lambda}{\lambda} + \frac{8}{3}\frac{\beta_\mathcal{M}}{\mathcal{M}} \tag{2.92}$$

which is the relation that also follows from (2.75) above.

2.4 Covariant quantization

Let us conclude this chapter by introducing the concept of covariant quantization i.e. the request that the effective action is invariant under field redefinitions. If this is not the case it would mean that there is a different quantum field theory for each choice of field variables. We first show that the standard definition of the (off-shell) effective action leads, surprisingly, to a dependence on field reparameterizations.[21]

[21] Note that by on-shell we refer to fields configurations which satisfy the effective field equations. As we will show, the field reparametrization dependence goes away on-shell. However, a covariant (off-shell) effective action gives an unambiguous unique set of beta-functions [59].

Then, we review the procedure to define an unique effective action [60] by defining covariant fluctuations on the fields-space manifold. We end with a discussion of covariance in the presence of gauge fields. The main purpose of this section is to introduce the covariant formalism that will be used in Chapter 5 to compute the one loop beta-functions for New Higgs inflation.

Consider the generic multi-field (boson) action

$$\mathcal{L} = -\frac{1}{2}G_{ij}(\varphi)\partial\varphi^i\partial\varphi^j - V(\varphi^i), \tag{2.93}$$

and we use the notation $\varphi \equiv \vec{\varphi}$. There is not a universal prescription which field variables one has to use. Doing an invertible field redefinition

$$\tilde{\varphi} = f(\varphi), \qquad \tilde{S}(\tilde{\varphi}) = S(\varphi) \tag{2.94}$$

and compute the effective action should give the same result as computing the effective action and then doing a field redefinition, i.e. the following diagram should commute

$$\begin{array}{ccc} S[\varphi] & \rightarrow & \tilde{S}[\tilde{\varphi}] \\ \downarrow & & \downarrow \\ \Gamma[\phi] & \overset{\phi=f^{-1}(\tilde{\phi})}{\rightarrow} & \tilde{\Gamma}[\tilde{\phi}] \end{array} \tag{2.95}$$

where $\phi = \langle\varphi\rangle$ and $\tilde{\phi} = \langle f(\varphi)\rangle$. Equally, setting $\phi = f^{-1}(\tilde{\phi})$ in the effective action computed with $S[\varphi]$ should give the same results as the effective action computed with $\tilde{S}[\tilde{\varphi}]$.

Let us see if this is actually the case. Consider a generic Lagrangian as the one above, the $Z[J]$ functional is defined as

$$Z[J] = \int \mathcal{D}\varphi \mathscr{M}(\varphi) \exp\left[i(S[\varphi] + J_i\varphi^i)\right] \tag{2.96}$$

where $\mathscr{M}(\varphi) = (\det[G_{ij}(\varphi)])^{1/2}$ is the measure from canonical quantization. This will lead to an integro-differential equation for the effective action as in (2.24),

$$\begin{aligned} \exp\left[i\Gamma[\phi]\right] &= \int \mathcal{D}\varphi \mathscr{M}(\varphi) \exp\left[i\left(S[\varphi] - (\varphi^i - \phi^i)\partial_i\Gamma\right)\right] \\ &\overset{1-\text{loop}}{=} \exp[iS[\phi]]\mathscr{M}(\phi)(\det\left[-\partial_i\partial_j S\right])^{-1/2}, \end{aligned} \tag{2.97}$$

where we simplified the notation by using $\delta/\delta\phi^i \equiv \partial_i$ and $\delta/\delta\tilde{\phi}^i \equiv \partial_{\tilde{i}}$. If instead we first perform the field redefinition (2.94), the generating functional will be defined by coupling the new field to the current,

$$Z'[J] = \int \mathcal{D}\tilde{\varphi}\tilde{\mathscr{M}}(\tilde{\varphi}) \exp[i(\tilde{S}[\tilde{\varphi}] + J_i\tilde{\varphi}^i)]. \tag{2.98}$$

With $(K^{-1})^{\tilde{i}}_{j} \equiv \delta f^i[\varphi]/\delta\varphi^j$ the matrix describing the field redefinition, we have $\tilde{\mathscr{M}} = \det[G_{ij} \cdot K^i_{\tilde{m}} K^i_{\tilde{n}}]^{1/2} = \mathscr{M} \det[K]$. The one-loop effective action is now

$$\begin{aligned}\exp[i\tilde{\Gamma}[\tilde{\phi}]] &\overset{1-\text{loop}}{=} \exp[i\tilde{S}[\tilde{\phi}]]\tilde{\mathscr{M}}(\tilde{\phi})(\det\left[-\partial_{\tilde{i}}\partial_{\tilde{j}}\tilde{S}\right])^{-1/2} \\ &= \exp[iS[\phi]]\mathscr{M}(\phi)\left(\det\left[-\partial_i\partial_j S + (K^{-1})^{\tilde{\rho}}_{i}(K^{-1})^{\tilde{\sigma}}_{j}\partial_{\tilde{\rho}}\partial_{\tilde{\sigma}}\phi^k\partial_k S\right]\right)^{-1/2}\end{aligned} \tag{2.99}$$

where in the last line we have used $\tilde{\phi} = f(\phi)$. Eq. (2.99) is clearly different from (2.97). Thus, remarkably, we have shown explicitly that the two procedures return different results.

In order to understand the solution to this problem provided by G.A. Vilkovisky [60] we can trace the source of non-covariance in each step above. Let us first rephrase the issue. Consider the field configuration space as a manifold $\mathscr{F}$, and φ^i being a particular set of coordinates on this manifold. We would like the effective action to be a scalar on the manifold exactly as the classical action S is. At the level of the generating functional $Z[J]$ the source term $J_i\varphi^i$ depends explicitly on the choice of coordinates. This leads to the non covariant displacement $\varphi^i - \phi^i \equiv \delta\phi^i$ in Γ that at one-loop manifests itself in the non-covariant second derivative of S. Note also that the non-covariant piece in Γ is proportional to $\partial_i\Gamma$, i.e. that the two effective action computed above differ "off shell". In the one loop actions the difference is given by the term proportional to $\partial_i S$. This is closely related to the reason why in gauge theory the effective action turns out to be gauge dependent "off shell". If the effective action is non covariant under field reparametrizations, two gauge give rise to two different "off-shell" effective actions.[22]

2.4.1 The Vilkovisky-De Witt action

Thus in order to have a covariant action we should replace the displacement $\varphi^i - \phi^i$ (remember ϕ means the background field) in (2.97) with a covariant notion of the distance between two field configurations. Suppose that the field space manifold $\mathscr{F}$ is endowed with a metric $\mathcal{G}$ so that geodesics can be defined. Consider the geodesic $\varphi(\lambda)$ connecting ϕ to φ, defined such that $\varphi(\lambda = 1) = \varphi$ and $\varphi(0) = \phi$, where λ is an affine parameter (to not be confused with the quartic coupling). We define the tangent vector to the geodesic at ϕ with

$$Q^i(\varphi, \phi) \equiv \frac{d\varphi^i}{d\lambda}\Big|_0. \tag{2.100}$$

[22] In a gauge theory S is invariant under gauge transformations, but not the non-covariant term in (2.97). After adding the gauge fixing term (in order to regularize the otherwise singular $\partial_i\Gamma \to 0$ limit, see [61]) we obtain in general an off shell gauge dependent action.

This is a vector with respect to ϕ and a scalar with respect to φ. Thus by replacing in (2.97)

$$\delta\phi^i \equiv \varphi^i - \phi^i \rightarrow Q^i(\varphi,\phi) \tag{2.101}$$

we obtain a covariant equation for the effective action.[23] This is achieved by replacing in the non covariant definition of $Z[J]$, $\phi^i \rightarrow \phi^i + Q^i(\varphi,\phi)$. Thus the covariant effective action is determined by

$$\exp\left[i\Gamma[\phi]\right] = \int \mathcal{D}\varphi \mathscr{M}(\varphi) \exp\left[i\left(S[\varphi] - Q^i(\varphi,\phi)\partial_i\Gamma\right)\right]. \tag{2.102}$$

We can consider a scalar function (such as the action) $S(\varphi)$ on the manifold as function of the affine parameter λ evaluated in $\lambda = 1$. Thus we write the Taylor series of $S(\varphi(\lambda = 1))$ around $\lambda = 0$ as

$$S(\varphi) = \sum_{n=0}^{\infty} \frac{1}{n!}\frac{d^n S}{d\lambda^n}\Big|_{\lambda=0} = \sum_{n=0}^{\infty} \frac{1}{n!} Q^{i_1}\cdots Q^{i_n}[\nabla_{(i_1}\cdots\nabla_{i_n)}S][\phi], \tag{2.103}$$

where we used

$$\frac{d}{d\lambda} \equiv \frac{d\varphi^i}{d\lambda}\nabla_i, \tag{2.104}$$

with ∇ the covariant derivative on the field manifold, and the geodesic equation

$$\frac{d\varphi^i}{d\lambda}\nabla_i \frac{d\varphi^k}{d\lambda} = \frac{d^2\varphi^i}{d\lambda^2} + \Gamma^k_{ij}\frac{d\varphi^i}{d\lambda}\frac{d\varphi^j}{d\lambda} = 0. \tag{2.105}$$

In (2.103) the round brackets mean symmetrization over the indices included. What we have not specified so far is which metric $\mathcal{G}_{ij}$ defined on the field space $\mathscr{F}$ one should use. According to[24] [60] for a non-gauge theory defined by an action as (2.93), *the metric on field space is defined by the tensor contained in the highest-derivative term*, i.e. $\mathcal{G}_{ij} = G_{ij}$. Thus in the construction above all the covariant derivatives must be defined with respect to G_{ij}. By using (2.103) in (2.102) we can compute the effective action diagrammatically as before (by using the background field method for example) simply by replacing partial derivatives on S with covariant derivatives. At one loop

$$\exp[i\Gamma[\phi]] = \exp[iS[\phi]]\mathscr{M}(\phi)(\det[-\nabla_i\nabla_j S])^{-1/2} = \exp[iS[\phi]](\det\left[-\nabla^i\nabla_j S\right])^{-1/2}, \tag{2.106}$$

[23] It is important to outline that there is a subtle difference between our Q and the quantity $\sigma^i(\varphi,\phi)$ that one finds usually in the literature. This is defined as follows. Consider the Synge world function $\sigma(\varphi,\phi)$ [62, 63], which is half of the square of the geodesic distance connecting $\varphi(\lambda = 0) = \phi$ to $\varphi(\lambda)$. σ^i is defined as $\mathcal{G}^{ij}\nabla_j\sigma|_\phi \equiv \sigma^i(\varphi,\phi)$. One can show that $\sigma^i(\varphi,\phi) = -\lambda\dot{\varphi}(0)$, from which it follows that $\sigma^i\nabla_i\sigma^j = -\sigma^j$ (the minus sign difference with respect to Vilkovisky comes from the fact that he defines the geodesic in the opposite direction, meaning from φ to ϕ).

[24] This is the only tensor which a) is defined by the action itself, b) in case of a free theory gives a connection that is identically zero in the parametrization in which the action is quadratic, and c) it is local, i.e. it does not contain any derivative.

where we have used $\mathscr{M} = \det[G_{ij}]^{1/2} = \det[G^{ij}]^{-1/2}$. If the Riemann tensor of the metric G_{ij} is vanishing, it means that there exists a field reparametrization for which $G_{ij}\partial\phi^i\partial\phi^j = \delta_{ij}\partial\tilde{\phi}^i\partial\tilde{\phi}^j$ and the Christoffel symbols in these coordinates are identically zero. All the covariant derivatives become normal partial derivatives, we call the fields $\tilde{\phi}$ canonically normalized and the effective action computed diagrammatically in these coordinates gives the covariant effective action. That is why for the case that G_{ij} is flat, we usually do not introduce this formalism and are not worried about non-covariance, unless for some specific reason we want to work in terms of non-canonical fields (like the Higgs in the non-linear representations). On the other hand when the Riemann tensor associated to the metric in configuration space is non vanishing, it is not possible to find a field redefinition that makes the kinetic term canonical. It is thus indispensable to work in this formalism to obtain a covariant effective action.

In the context of inflation, the authors in [64] and later [65, 66] revisit this covariant approach to study the dynamics of the perturbations around an inflationary background. In the usual procedure to compute cosmological correlation functions the action is expanded around a time dependent background in perturbations that are not covariant. In the language of this section,

$$S(\phi + \underbrace{\varphi - \phi}_{\delta\phi}) = \sum_{n=0}^{\infty} \frac{1}{n!}\delta\phi^{i_1}\cdots\delta\phi^{i_n}\partial_{i_1}\cdots\partial_{i_n}S[\phi], \tag{2.107}$$

where again φ is the "total field" while ϕ represents the background. In order to find the relation between the non covariant displacement and Q, i.e. the tangent vector to the geodesic connecting ϕ to φ, we can expand $\delta\phi^i \equiv \varphi^i - \phi^i \equiv \varphi^i(\lambda = 1) - \varphi^i(\lambda = 0)$ in Taylor series around zero in the affine parameter and use recursively the geodesic equation (2.105); this gives

$$\delta\phi^i = \sum_{n=0}^{\infty} \frac{1}{n!}\frac{d^n\varphi^i}{d\lambda^n}\Big|_0 = Q^i - \frac{1}{2}\Gamma^i_{jk}Q^jQ^k + \frac{1}{3!}(\Gamma^i_{lm}\Gamma^m_{jk} - \partial_l\Gamma^i_{jk})Q^jQ^kQ^l + \ldots. \tag{2.108}$$

Inserting this relation in (2.107) we obtain (2.103) since both are the same Taylor expansion in the affine parameter λ.

In chapter 5 we will apply this covariant formalism to compute one loop diagrams for a specific Higgs inflationary model, where both the Higgs sector and the other fields have a non trivial metric in field space. This allows us to find the beta-functions from combining "covariant" Feynman diagrams and the one loop effective potential computed as in (2.109).

Before moving on, let us conclude by briefly mentioning the case of gauge theories.

2.4.2 Covariance in gauge theories

As already mentioned, it is well known that in gauge theories the standard gauge fixing procedure leads to a gauge dependent effective action. This dependence is just a manifestation of the reparametrization dependence arising from defining a non-covariant effective action. The basic observation to construct a metric in fields space is that the physical configuration space is given by the fields space $\mathscr{F}$ modulo the group of gauge transformations $\mathscr{G}$, i.e. $\mathcal{F} = \mathscr{F}/\mathscr{G}$ [67]. It can be shown that a change of gauge conditions is equivalent to a reparametrization in the space of gauge orbits [63, 60]. In the functional integral the volume of the gauge group is factored out by the usual Faddeev-Popov procedure [68]. Thus, by integrating over the space of gauge orbits we want to define the effective action in terms of a metric that preserves reparametrization invariance on $\mathcal{F}$ [61]. In order to define a geometry on $\mathcal{F}$ one starts from a gauge invariant metric G_{ij} defined on $\mathscr{F}$. This, as before, is usually taken from the highest derivative term in the classical action. We label covariant derivative and connection associated to G_{ij} with $(\mathcal{D}, \Gamma^k_{ij})$ while the one associated to $\mathcal{G}_{ij}$, i.e. the geometry in the field space $\mathcal{F}$, by $(\nabla, \tilde{\Gamma}^k_{ij})$. The latter is for gauge theories equivalent of the covariant derivatives appearing in (2.102) and (2.103). We did not need to make this difference before because the two coincide in the non gauge case.

Let us show the relations between this two connections following the taste of the authors in [67], to which we refer the reader for further details. Consider an infinitesimal gauge transformation $\delta\varphi^i = R^i_a \alpha^a$. Given the gauge invariance of G_{ij}, R^i_a satisfy Killing's equation for this metric, i.e. they can be seen as the tangent vector to the flow generated by the gauge group. Then one can introduce the projector Π^i_j on the gauge-fixed directions,[25] the space orthogonal to orbits generated by the Killing vectors. The distance in the orbits space $\mathcal{F}$ is then given by $G_{ij}\Pi^i_k \delta\varphi^k \Pi^j_l \delta\varphi^l \equiv \mathcal{G}_{kl}\delta\varphi^k\delta\varphi^l$. The connection $\tilde{\Gamma}$ associated to $\mathcal{G}_{ij}$ is found by asking $\nabla\mathcal{G}_{ij} = 0$. Thus one derives $\tilde{\Gamma}^k_{ij} = \Gamma^k_{ij} + T^k_{ij}$ where T^k_{ij} is given by a combination of R^i_a, see [67]. Then an effective action covariant under field reparametrization and independent of the choice of the gauge fixing functional can be defined as before by using the geometry defined by $\mathcal{G}_{ij}$. At one loop for example

$$\exp[i\Gamma[\phi]] = \exp[iS[\phi]] \det[\mathcal{Q}^a_b](\det\left[-\nabla^i\nabla_j S + \partial^i\chi^a\partial_j\chi_a\right])^{-1/2}, \tag{2.109}$$

where χ^a is the gauge fixing term contracted with an arbitrary function, and the ghost operator is given by $\mathcal{Q}^a_b = \partial_i\chi^a R^i_b$.

[25] This is the DeWitt projector, $\Pi^i_j = \delta^i_j - \gamma^{ab} R^i_a R_{bj}$ with γ^{ab} the inverse of $\gamma_{ab} = R^k_a G_{kl} R^l_b$, and indices are raised and lowered with G_{ij}.

Using the Landau De Witt gauge condition (with $\varphi = \phi + \delta\phi$), i.e. [69, 61]

$$G_{ij} R^j_a[\phi]\delta\phi^i = 0, \quad \chi^a[\phi, \delta\phi] = \lim_{\xi_g \to 0} \frac{1}{\sqrt{\xi_g}} R^i_a(\phi)\delta\phi^i \tag{2.110}$$

the contribution to the effective action coming from the difference between $\tilde{\Gamma}^k_{ij}$ and Γ_{ij} vanishes [61, 67]. By choosing this gauge fixing functional we are then allowed to compute the effective action with covariant derivatives of G_{ij}, the tensor appearing in the classical action.

Consider as an explicit example $U(1)$ massive-electrodynamics with Lagrangian

$$\mathcal{L} = -\gamma(\varphi)\mathcal{D}_\mu \Phi^\dagger \mathcal{D}^\mu \Phi - \frac{1}{4} k(\varphi) F_{\mu\nu} F^{\mu\nu}. \tag{2.111}$$

The fields are $\varphi^I \equiv \{\varphi^i, A^\mu\}$, with $D^\mu\Phi = (\partial^\mu + igA^\mu)(\varphi^1 + i\varphi^2)$ and $\mathcal{D}_\mu\Phi^\dagger\mathcal{D}^\mu\Phi = (\partial_\mu\varphi^i + g\varepsilon^{ij}\varphi^j A_\mu)^2$. The gauge transformations are given by $\delta_g\varphi^i = \alpha g \varepsilon^{ij}\varphi^j = R^i\alpha$, $\delta_g A_\mu = -\partial_\mu\alpha = R_\mu\alpha$, and

$$R^I = \{ g\varepsilon^{ij}\varphi^j, -\partial_\mu\}, \quad G_{IJ} = \mathrm{diag}\{\gamma(\varphi)\delta_{ij}, k(\varphi)g_{\mu\nu}\}. \tag{2.112}$$

We take zero background for the gauge bosons fields and $\varphi^1 = \phi$, $\varphi^2 = 0$ for the scalar. The De Witt-Landau gauge is then given by

$$G_{IJ} R^I \delta\phi^J = -k(\phi)\partial_\mu A^\mu - g\gamma(\phi)\phi\delta\phi^2 = 0 \tag{2.113}$$

so that we can take the gauge fixing functional as

$$\mathcal{L}_{\mathrm{g}} = k(\phi)\chi\cdot\chi = \frac{1}{2\xi} k(\phi) \left(\partial_\mu A^\mu + g\frac{\gamma(\phi)}{k(\phi)}\phi\delta\phi^2 \right)^2 \tag{2.114}$$

in the limit $\xi \to 0$. This is a particular type of R_ξ-gauge,

$$\frac{1}{2\xi_1} k(\phi) \left(\partial_\mu A^\mu + \xi_2 g \frac{\gamma(\phi)\phi}{k(\phi)} \delta\phi^2 \right)^2 \tag{2.115}$$

with $\xi_1 \to 0$, $\xi_2 = 1$. The ghost operator in general is given by $\partial_I\chi(\delta_g\varphi^I/\delta\alpha) = \partial_I\chi R^I(\phi + \delta\phi)$, and the ghost Lagrangian is

$$\mathcal{L}_{\mathrm{gh}} = \bar{c} \left(-\partial_\mu\partial^\mu - \xi_2 g \frac{\gamma(\phi)\phi}{k(\phi)} (\phi + \delta\phi^1) \right) c. \tag{2.116}$$

with $c, \bar{c}$ the ghost fields.

In chapter 5 we study the one-loop corrections to new Higgs inflation using the standard Landau gauge. This one is of the form above but with $\xi_1 = \xi_2 = 0$,

such that the ghosts decouple. Note that it is not the case for the Landau De Witt gauge. Thus, to rigorously extend the covariant formalism to the gauge sector, we should either consider the extra diagrams coming from setting $\xi_2 = 1$ (instead of $\xi_2 = 0$), or project the metric (2.112) to the space $\mathcal{F}$ by using the De Witt operator, as discussed above. A preliminary check on the Coleman Weinberg potential reveals that for our purpose, given the simple form of the metric, the additional effects introduced by working in this way are suppressed. However, for more complicated set-up, such as the RGEs in standard Higgs inflation, it may be that including this effects, at the moment neglected, may resolve some discrepancies between different results in the literature.

3 UV (in)sensitivity of Higgs inflation

The Higgs boson is the only scalar particle predicted by the Standard Model. Its existence has now been experimentally confirmed [70, 71]. The possibility that the Higgs scalar could have been responsible for the early inflationary period represents the Holy Grail in connecting low energy observables to the physics of the early Universe. The idea is appealing mainly for three reasons. Minimality, excellent agreement with the Planck data, and, again, the hope of connecting Standard model observables with cosmological measurements.

In this chapter, we critically discuss the interplay between these three aspects. In particular, we consider to what degree the consistent matching of the low and high energy regimes can be done in a minimal setting, and how this, in turn, influences the observational signatures of the model. We do this in the context of the standard Higgs inflation scenario, although the discussion generically apply to any non-renormalizable inflation model.

The outline of this rather long chapter is as follows. In 3.1 we warn the reader about possible problems with the proposal at the quantum level, and its implications. In 3.2 we review the role of the Higgs boson in the SM; this also allows to set the notation for the next chapters. In sec. 3.3 we introduce Higgs inflation at the classical level, identify the different field regimes and discuss the role of the conformal transformation. In the two sections 3.4 and 3.5 we critically discuss the quantum aspects of Higgs inflation. After illustrating different approaches to embed HI in an EFT we show why new UV physics is demanded. This gives us the opportunity to spell out in detail the kind of UV sensitivity considered in this thesis and ways of parameterizing its effect. The contents of these two sections is summarized in fig. 3.3. Finally, we study the UV (in)sensitivity of HI. In 3.6 we provide the details of our analysis together with an extensive discussion on the renormalization scale used. In 3.7 we present the results for the inflationary

predictions and in 3.8 we summarize with some concluding remarks.

The chapter is based on part of our work [1] augmented with considerations coming from our study in [3]. Thus, in a sense, it represents naturally the expanded and updated version of [1].

3.1 Idea and issues

Standard Model Higgs inflation [72, 73, 6, 74] has attracted much attention over the last years. This is not surprising, as the model has many appealing features — at least, at the classical level. With the Higgs field as inflaton the model is firmly embedded in the Standard Model (SM). The idea is indeed very simple. It consists of adding an extra coupling between the Higgs and the Ricci scalar to the combined Standard Model and Einstein-Hilbert Lagrangian [6]

$$\delta\mathcal{L} = \sqrt{-g}\xi\mathcal{H}^{\dagger}\mathcal{H}R, \tag{3.1}$$

with ξ a dimensionless parameter. Then, the predictions for the inflationary observables are in excellent agreement with the latest Planck data [75]. Note that the non-minimal coupling is the only dimension four operator compatible with all the symmetries of the SM which is not already included in the SM lagrangian. Since it is not suppressed by a cutoff scale $1/\Lambda$, it can be seen as the term dominating at low energy in the EFT for large Λ. Furthermore, this type of coupling is somehow inevitable, as renormalization of a scalar field on curved spacetime requires the introduction of divergent counterterms of this form, the finite part of which has to be determined by experiments. As we will discuss extensively, what seems an innocuous addition has indeed many non trivial consequences.

Although this minimal approach is attractive, it is not clear whether Higgs inflation is fully consistent as a quantum theory. First, including the running of couplings, the potential may become unstable at energy scales below the inflationary scale. In brief, the running of the Higgs quartic coupling with the SM RGEs can become negative at some intermediate scale, generating a second minimum in the Higgs potential at high scales. One of the main surprises from the measurements of the Higgs and top quark mass is the fact that their values lie exactly within the boundary region in parameter space that separates where the SM is stable or metastable up to the Planck scale,[1] see figure 3.1. Higgs inflation requires absolute stability up to the inflationary scale $m_{\rm P}/\sqrt{\xi}$. However, for the best fit values of

[1] Metastable means that the value of the potential at the second minimum is smaller than the the EW minimum, i.e. $V(\phi_{\rm min}) < V(\phi = v)$, but the lifetime of the EW vacuum is larger than the age of the Universe.

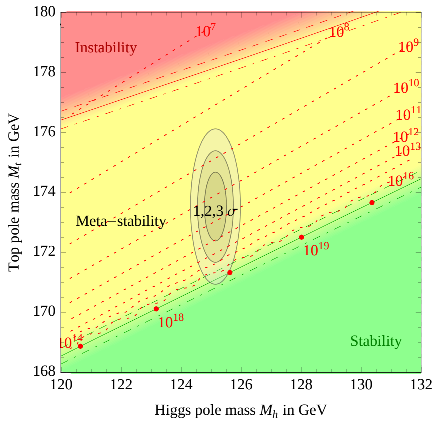


Figure 3.1: *The 1,2,3σ region of the experimental range of top quark and Higgs mass at the EW scale compared to the stability,metastability and instability regions of the EW vacuum. The couplings are run to the Planck scale assuming no new physics beyond the SM. The plot is taken from ref. [86].*

the top and Higgs mass [76] the SM potential becomes negative around 10^{11}GeV. It should be noted though that vacuum stability of the SM up to the Planck scale is only excluded at the 2σ level [77, 76, 78, 79, 80, 81, 82]. Furthermore, with extra matter, e.g. a dark matter particle, the instability bound can be evaded [83, 84, 85].

A second issue with quantum Higgs inflation is the unitarity bound [8, 87, 88, 89, 90, 91, 92]. Tree level unitarity is lost at energies well below the Planck scale, and new degrees of freedom [93, 94] or strong dynamics [37, 95] should become important at this scale. Although the energy scale of the inflationary potential is always below the field-dependent unitarity cutoff [8, 91, 96, 97], which makes the semiclassical approximation meaningful, this is not so for the field value. To forbid non-renormalizable operators that spoil the inflationary potential already at the classical level, an (approximate) shift symmetry has to be assumed. This is no different from chaotic inflation. We will further discuss this in 3.4.

Thirdly, the theory is not renormalizable over the full field range (see sec. 3.5). It has been shown that for small, mid and large field values Higgs inflation is renormalizable in the usual effective field theory (EFT) sense [34] (see also earlier work [9, 29, 30, 31, 32, 33, 98, 99, 100]). However, these EFTs need to be patched together at the boundaries of the different field regimes, and it is here that we expect non-renormalizable operators to become important. We will also refer to these higher order operators as threshold corrections, and more generically, speak

about threshold corrections to the renormalization group equations (RGEs) or to the inflationary observables, meaning the effect of the higher order operators on these quantities.

Thus for a consistent quantum field theory, new physics is needed below the Planck scale. This begs the question: how sensitive is Higgs inflation to the unknown UV physics? If the model predictions demanded a particular UV completion it would mean that the simplicity of the set-up, to which it owes much of its success, would be completely spoiled. In this chapter we will show that as long as the UV corrections do not affect the inflaton potential at tree level, but only enter at loop level via corrections to the renormalization group equations, the inflationary predictions are to a very good approximation unaffected. Indeed, whatever the exact running of the couplings, the spectral index n_s and tensor-to-scalar ratio r have at leading order in the slow roll expansion a universal value: [2]

$$n_s = 1 - \frac{2}{N_\star} + \mathcal{O}(N_\star^{-2}) \simeq 0.967, \qquad r = \frac{12}{N_\star^2} + \mathcal{O}(N_\star^{-3}, \xi^{-1}) \simeq 0.0031 \tag{3.2}$$

with $N_\star \approx 60$ the number of e-folds of observable inflation, and $\xi \gg 1$ the non-minimal coupling.

3.2 The Higgs boson in the Standard Model

Before discussing the impact of the Higgs boson in cosmology let us review the role it plays in the Standard Model. In the electroweak sector, the Standard Model of particle physics is described by the gauge group $SU_L(2) \otimes U_Y(1)$ with one gauge boson W_μ^a, B_μ associated to each generator. Experimentally, all fermions and three out of the four electroweak gauge bosons have a finite mass. This demands that the gauge symmetry is spontaneously broken, i.e. the vacuum state of the theory is not invariant under the gauge symmetry group of the lagrangian. Electroweak symmetry breaking (EWSB) can in principle be described without making any assumptions on the physics responsible for its dynamics. In the non-linear representation of the chiral lagrangian it is possible to give mass to the bosons in a gauge invariant way. However, as is well known, the resulting theory will violate unitarity at the TeV scale [101, 102, 103].[3] Here and in the following, by "violating unitarity at scale X" we mean that at the momentum scale X unitarity is violated

[2] This is for inflation on the flat plateau of the potential, as is usually meant by "Higgs inflation" (and at tree level is the only possibility). For some fine-tuned values of the couplings, inflation near a maximum or inflection point of the potential is possible; in this latter case, the predictions are sensitive to the details of the potential, and thus to the unknown UV physics.

[3] The scattering amplitude of the longitudinal gauge bosons, or, thanks to the equivalence theorem, the high energy behavior the scattering amplitude of the goldstone bosons will grow

at tree level, and thus the theory is trustworthy only at energies below this scale. It turns out that one can both dynamically describe the EWSB and unitarize the theory using only one new ingredient, namely by adding an $SU(2)$ Higgs doublet.[4] That's why sometimes the Higgs model is referred to as a parametrization of the EWSB. Thus, the role of the Higgs boson in the Standard Model is two fold: 1) its vacuum expectation value generates the masses of the Standard Model particles 2) its interactions with other SM particles ensures a theory that is unitary up to the Planck scale. As we will see, for the Higgs to be the inflaton it has to violate point 2), with a series of consequences that will be discussed in the following.

To set notation, the lagrangian for the Higgs doublet is given by

$$\begin{aligned}\mathcal{L} = &-(\mathcal{D}_\mu\mathcal{H})^\dagger\mathcal{D}^\mu\mathcal{H} + \lambda\left(\mathcal{H}^\dagger\mathcal{H} - v^2/2\right)^2 - \frac{1}{4}f^a_{\mu\nu}f^{a\,\mu\nu} - \frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &- \bar{\psi}_i i\gamma^\mu D_\mu\psi_i - \left(y_d\bar{Q}_L\mathcal{H}d_R + y_u\bar{Q}_L\mathcal{H}_c u_R + \text{h.c.}\right)\end{aligned} \tag{3.3}$$

with $\mathcal{H}_c = (i\sigma^2)\mathcal{H}^*$ and $f^a_{\mu\nu}, F^a_{\mu\nu}, B_{\mu\nu}$ the $SU(3), SU(2)$ and $U(1)$ field strenght respectively. The SM fermion fields are $\psi_i = \{Q_L, u_R, d_R, E_L, e_R\}$: the left-handed doublet, right-handed up and down quark, left-handed lepton, and the right-handed electron respectively (we suppressed family indices). $\mathcal{D}_\mu$ is the co-variant derivative of the electroweak gauge group

$$\begin{aligned}\mathcal{D}_\mu\mathcal{H} &= (\partial_\mu + ig_2\tau^a W^a_\mu + ig_1 Y_H B_\mu)\mathcal{H} \\ \mathcal{D}_\mu Q_L &= (\partial_\mu + ig_3 f^a_\mu t^a + ig_2\tau^a W^a_\mu + ig_1 Y_Q B_\mu)Q_L \\ \mathcal{D}_\mu u_R &= (\partial_\mu + ig_3 f^a_\mu t^a + ig_1 Y_u B_\mu)u_R\end{aligned} \tag{3.4}$$

where τ^a, t^a are the usual normalized generators for $SU(2)$ and $SU(3)$, $\tau^a = \frac{1}{2}\sigma^a$, while $Y_H = 1/2, Y_Q = 1/6, Y_u = 2/3, Y_d = -1/3$ are our convention for the $U(1)$ hypercharges.

3.3 Higgs inflation at tree level

The classical lagrangian for Higgs inflation, from (3.1) and (3.3), is defined as

$$\mathcal{L} = \sqrt{-g}\left[\frac{m_\text{P}^2}{2}\left(1 + 2\xi\frac{\mathcal{H}^\dagger\mathcal{H}}{m_\text{P}^2}\right)R + \mathcal{L}_\text{SM}\right], \tag{3.5}$$

with energy as $\sim E^2$, and thus violate unitarity at energies of order $4\pi M_W/g \sim 1.5$Tev. That is why running the LHC to the TeV scale was a win-win situation. Either the Higgs or a strong phase should have became manifest there.

[4] Before the discovery of the Higgs, various "Higgsless" proposal to unitarize the theory were considered [104, 105].

which is referred to as the *Jordan frame* Lagrangian. The gravitational sector can be brought in standard form, i.e. one can go to the *Einstein frame*, by a conformal transformation of the metric

$$g_{E\,\mu\nu} = \Omega^2 g_{\mu\nu}, \qquad \Omega^2 = 1 + 2\xi \frac{\mathcal{H}^\dagger \mathcal{H}}{m_{\rm P}^2}. \tag{3.6}$$

The resulting action is[5]

$$\mathcal{L}_E = \sqrt{-g_E}\left[\frac{m_{\rm P}^2}{2}R(g_E) - \frac{(\mathcal{D}_\mu\mathcal{H})^\dagger(\mathcal{D}^\mu\mathcal{H})}{\Omega^2} - \frac{3}{4}m_{\rm P}^2\frac{\partial_\mu\Omega^2\partial^\mu\Omega^2}{\Omega^4} - \frac{V}{\Omega^4} + ...\right] \tag{3.9}$$

where the indices are contracted with $g_E^{\mu\nu}$. The Lagrangian for the classical background field $\mathcal{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \phi \end{pmatrix}$ reduces to

$$\mathcal{L}_E = -\sqrt{-g_E}\left[\frac{1}{2}\gamma_{\phi\phi}\partial_\mu\phi\partial^\mu\phi + U(\phi)\right] \tag{3.10}$$

with

$$\gamma_{\phi\phi} = \frac{1}{\Omega^2}\left(1 + \frac{6\xi^2}{m_{\rm P}^2\Omega^2}\phi^2\right), \quad U(\phi) = \frac{\lambda(\phi^2 - v^2)^2}{4\Omega^4}. \tag{3.11}$$

In the large field limit $\phi^2 \gg m_{\rm P}^2/\xi$ the potential approaches a constant value developing a flat plateau that can support a period of slow roll inflation. The classical Higgs field can be canonically normalized via

$$\gamma_{\phi\phi}^{1/2}(\phi)d\phi = dh, \tag{3.12}$$

with $\Omega^2(\phi)$ evaluated on the classical background.

We conclude that, at the classical level, after the conformal transformation (3.6) all the effects of the non-minimal coupling are translated to a new shape for the scalar field potential. Thus one can apply all the formalism of slow roll single-field inflation reviewed in chapter 1 for the field h. In order to compute $N_\star, \epsilon, \eta$ we do not necessary need the analytic expression for $U(h)$, we can simply express

[5] Under a conformal transformation as in (3.6) the metric volume and Ricci scalar transforms (see for example [106])

$$\sqrt{-g} = \Omega^{-4}\sqrt{-g_E} \quad ; \quad R(g) = \Omega^2 R(g_E) - 6\Omega^3 g_E^{\mu\nu}\nabla_\mu\nabla_\nu(\Omega^{-1}), \tag{3.7}$$

where the covariant derivatives on the right-side are with respect to g_E. The Ricci part of the action becomes

$$\sqrt{-g}\Omega^2 R(g) = \sqrt{-g_E}R(g_E) - \sqrt{-g_E}\, g_E^{\mu\nu}\, \tfrac{6}{4}\Omega^{-4}\partial_\mu\Omega^2\partial_\nu\Omega^2. \tag{3.8}$$

everything in terms of ϕ using the implicit relation (3.12). Following the, by now standard, procedure we have[6]

$$
\begin{aligned}
\epsilon &= \frac{8m_{\mathrm{P}}^2}{\phi^2}\left(1+\left(\frac{\phi}{m_{\mathrm{P}}/\sqrt{\xi}}\right)^2+6\left(\frac{\phi}{m_{\mathrm{P}}/\xi}\right)^2\right)^{-1} \\
\eta &= \frac{4}{\Omega^2+\frac{6\xi^2\phi^2}{m_{\mathrm{P}}^2}}\left(3\frac{m_{\mathrm{P}}^2}{\phi^2}-2\xi-\frac{6\xi^2}{\left(\Omega^2+\frac{6\xi^2\phi^2}{m_{\mathrm{P}}^2}\right)}\right).
\end{aligned}
\tag{3.13}
$$

Now ϵ is a monotonically decreasing function of ϕ^2. This means that once we find the unique value ϕ_E for which $\epsilon \simeq 1$, i.e. $\phi_E \simeq 1.07m_{\mathrm{P}}/\sqrt{\xi}$, inflation can occur only in the region $\phi > \phi_E$. Hence taking $\phi \gg m_{\mathrm{P}}/\xi$, we have

$$
\epsilon \simeq \frac{4m_{\mathrm{P}}^4}{3\xi^2\phi^4} \quad ; \quad \eta \simeq \frac{4}{3}\frac{m_{\mathrm{P}}^2}{\xi^2\phi^4}\left(1-\frac{\xi\phi^2}{m_{\mathrm{P}}^2}\right). \tag{3.14}
$$

Using $N_\star \approx 60$ for the number of e-folds, we get $\phi_\star \simeq 9.13m_{\mathrm{P}}/\sqrt{\xi}$. Inflation takes place mostly in the regime

$$
\phi^2 \gg m_{\mathrm{P}}^2/\xi, \tag{3.15}
$$

The observational constraint on the scalar power spectrum (3.14)

$$
\Delta_{\mathcal{R}} = \frac{U(\phi_\star)}{24m_{\mathrm{P}}^4\pi^2\epsilon_\star} = \frac{3(9.13)^4\lambda}{4\cdot 4\cdot 24\pi^2\xi^2} \simeq 2.2\cdot 10^{-9}, \tag{3.16}
$$

which fixes the value of the free parameter $\xi \simeq 47000\sqrt{\lambda}$, where λ is the Higgs self coupling evaluated at the inflationary scale. Note that compared to the case where the Higgs is minimally coupled $\Delta_{\mathcal{R}} \propto \lambda$, here we have $\Delta_{\mathcal{R}} \propto \lambda/\xi^2$. This allows to fit the power Spectrum with more realistic values of the Higgs quartic coupling constant for big ξ values. Finally, to leading order in ξ^{-1} we have

$$
n_s \simeq 1-\frac{2}{N_\star} \simeq 0.967 \quad ; \quad r \simeq \frac{12}{N_\star^2} \simeq 0.0031. \tag{3.17}
$$

These results are both well in agreement with the data. The aim of this chapter is to show that these predictions are robust, and still hold once the full quantum theory is considered.

3.3.1 Different regimes and scales

There are two relevant energy scales in the problem. First, $\phi = m_{\mathrm{P}}/\xi$ above which we start to have significant deviations from the Standard Model and second,

[6] As a check, note that for $\xi = 0$ the previous expressions reduces to (1.58), as expected.

$\phi = m_{\rm P}/\sqrt{\xi}$, which marks the end of inflation. We refer to $\phi \ll m_{\rm P}$ as the small field regime (Standard Model regime), $m_{\rm P}/\xi < \phi < m_{\rm P}/\sqrt{\xi}$ as the mid-field regime and $\phi \ll m_{\rm P}/\sqrt{\xi}$ as the large field regime respectively. In the two asymptotic regimes we can simply integrate explicitly $dh = \gamma_{\phi\phi}^{1/2} d\phi$ to get the canonical field,[7]

$$\begin{aligned} \phi \ll \frac{m_{\rm P}}{\xi}, &\quad dh \simeq d\phi &&\Longrightarrow \quad h \simeq \phi \\ \phi \gg \frac{m_{\rm P}}{\xi}, &\quad dh \simeq \frac{\sqrt{6}}{m_{\rm P}} \frac{\xi\phi}{\Omega^2} d\phi &&\Longrightarrow \quad h \simeq \sqrt{\frac{3}{2}} m_{\rm P} \ln \Omega^2 . \end{aligned} \tag{3.18}$$

In the small field regime, where $\Omega \simeq 1$, we retrieve the SM potential. In the large field regime, inverting (3.18), we get

$$\phi \simeq \frac{m_{\rm P}}{\sqrt{\xi}} e^{\frac{h}{\sqrt{6} m_{\rm P}}} \quad , \quad U(h) \simeq \frac{\lambda m_{\rm P}^4}{4\xi^2} \left(1 - \exp\left(-\frac{h}{\sqrt{3/2} m_{\rm P}}\right)\right)^2 . \tag{3.19}$$

Since $\xi \sim 10^3$ and inflation takes place for $\phi \geq m_{\rm P}/\sqrt{\xi}$, the approximation $\phi \gg m_{\rm P}/\xi$ to compute the field values of h during inflation is accurate. In the limit $h \gg m_{\rm P}$ the potential goes to a constant and the theory acquires a shift symmetry. The exponential term makes the symmetry only approximate allowing slow roll inflation. The inflationary field values are translated from $\phi \in [m_{\rm P}/\sqrt{\xi}, 9m_{\rm P}/\sqrt{\xi}]$ to $h \in [0.9 m_{\rm P}, 5.4 m_{\rm P}]$. From the FRLW equations the Hubble scale and the scale set by the potential during inflation are $H^2 \simeq U/3m_{\rm P}^2 \simeq \lambda m_{\rm P}^2/12\xi^2$ and $U^{1/4} = (\lambda/4)^{1/4} m_{\rm P}/\sqrt{\xi}$ respectively.

3.3.2 Strengths/weaknesses of a conformal transformation

In the previous subsection we have studied the classical inflationary dynamics in the Einstein frame, where the analysis is more straightforward. In the rest of the chapter (and thesis) we will keep working in this frame. We now take a moment to pause and list the main advantages as well as the changes introduced by the conformal transformation (3.6). Most of the features listed below are not unique to the conformal transformation but they apply (except for the specific details) whenever an "Einstein frame" can be found.[8] The advantages of the Einstein frame are:

– As already shown, at the classical level we can apply all the standard slow roll analysis for single-field inflation reviewed in Chapter 2. The inflationary

[7] An analytic, although non illuminating, solution is indeed possible over the all field range [107]. We perform the integration using the boundary condition $\chi_0 = f(\phi_0)$ where $\int_{\chi_0}^{\chi} d\chi = \chi - \chi_0 = [f(\phi)]_{\phi_0}^{\phi} = f(\phi) - f(\phi_0)$.

[8] In chapter 5 we will consider the case of a disformal transformation.

dynamics is simply given by the canonically normalized background field rolling down the Einstein frame potential.[9]

– In the quantum analysis one can treat gravity classically to leading order in the slow roll approximation [9]. Note that treating gravity classically (or: freezing the gravitational degrees of freedom) leads to different results in the two frames (and thus cannot be done in the Jordan frame). For example, the Mukanov-Sasaki variable [109, 110, 111, 112, 113] is a different combination of the scalar metric degree of freedom and the Higgs in the two frames.

– We can neglect graviton loops since these are suppressed by inverse powers of m_{P}.

– Unitary violation can be studied by looking at scatterings of SM particles.

The conformal transformation makes the non-renormalizable nature of Higgs inflation explicit. Thus, considering the problem in the Einstein frame gains interested on its own, independently of the specifics of the set-up.

– One non-minimal coupling in the Jordan frame generates an infinite series of non-renormalizable operators in the Einstein frame. Expanded around a particular background the Higgs potential in (3.11) is given by an infinite series. The kinetic term for the gauge bosons is invariant under the conformal transformation. Thus, the effect on the Higgs-gauge bosons interaction,[10]

$$\mathcal{L}_E \supset -\frac{1}{\Omega^2}(\mathcal{D}_\mu \mathcal{H})^\dagger \mathcal{D}^\mu \mathcal{H} \tag{3.20}$$

is given by expanding the function $1/\Omega^2$. For a fermion field the kinetic term can be made canonical after the rescaling $\psi \to \psi' = \Omega^{-\frac{3}{2}}\psi$, and the Yukawa interaction terms become

$$\mathcal{L}_E \supset -\left(y_d \bar{Q}_L \cdot \frac{1}{\Omega}\mathcal{H} d_R + y_u \bar{Q}_L \frac{1}{\Omega}\tilde{\mathcal{H}} u_R + \text{h.c.}\right). \tag{3.21}$$

It follows that for the fields not appearing in the non-minimal coupling function, the masses in the Einstein frame are given by the rescaling

$$m_E = \frac{m_J}{\Omega} \propto \frac{\phi}{\Omega} \tag{3.22}$$

[9] Single field inflation is an attractor. Treating the Higgs-Goldstone sector as a multifield model, as it should be done, it has been shown that the multifield dynamics becomes negligible before perturbations on scales of observational relevance first cross the Hubble radius [108]. Thus we can safely consider, at the background level, solutions for which $\theta_i = 0$, $\dot{\theta}_i = 0$. In this way the adiabatic fluctuation evolves as in the single-field case and we can apply the formalism reviewed in the first chapter.

[10] $\mathcal{L}_E$ is defined through $\sqrt{-g}\mathcal{L} = \sqrt{-g_E}\mathcal{L}_E$.

where $m_{\rm J}$ means a generic mass term in the Jordan Frame. This rescaling of mass scales has lead to some confusion over the renormalization prescription for the RG improved effective action, as will be discussed extensively in sec. 3.6.1.

– The Higgs-Goldstone sector has a non-trivial metric in field space,

$$-\frac{1}{2}G_{ij}\partial\varphi^i\partial\varphi^j = -\frac{1}{2}\left(\frac{\delta_{ij}}{\Omega^2}+\frac{6\xi^2}{m_{\rm P}^2\Omega^4}\varphi^i\varphi^j\right)\partial\varphi^i\partial\varphi^j \tag{3.23}$$

where we used $\mathcal{H}=\frac{1}{\sqrt{2}}\begin{pmatrix}\theta^1+i\theta^2\\ \phi+\delta\phi+i\theta^3\end{pmatrix}$ and $\varphi^i=\{\phi+\delta\phi,\theta^i\}$. Since the curvature in field space $R[G_{ij}]$ does not vanish, it is not possible to find a global field reparametrization to make the kinetic term canonical. This has to be taken into account when computing loop corrections [34].

3.4 Higgs inflation as an effective field theory

Despite its simplicity and its predictive power (at the classical level) Higgs inflation has sparked, from the very beginning, a long debate about its viability as a full quantum field theory (if we want to take the theory seriously, it is unavoidable to address its quantum nature). As often happens, the dispute has also produced misunderstanding regarding the nature of the issues themselves and sometimes confusion about what has been settled and what instead remains to be solved. In the following we clarify the main aspects and how they are relevant for our analysis.

To not lose the big picture, the reader can turn to fig 3.3, which nicely summarizes the contents of the next six subsections (Sec. 3.4 and 3.5).

3.4.1 Unitarity and consistency

Since HI includes gravity, it defines a non-renormalizable theory. As such it has to be interpreted as an effective field theory (EFT) valid only for energies below its regime of validity.

The introduction of the non-minimal coupling lowers the cutoff below the Planck mass. The first question to ask is thus the range of validity of the EFT description. The model is consistent if the typical energy scales involved in the dynamics (described by the model) are always below the unitarity cutoff of the theory. For

typical energy scales we mean the typical energy of the fluctuations or more conservatively the background energy density stored in the potential.[11]

Consider the non-minimal coupling in the Jordan frame. A naive computation around a trivial background, $g_{\mu\nu} = \eta_{\mu\nu} + m_{\mathrm{P}}^{-1} h_{\mu\nu}, \mathcal{H}^\dagger = 1/\sqrt{2}(0 \quad \delta\phi)$ (in unitary gauge) gives

$$\sqrt{-g}\xi\mathcal{H}^\dagger\mathcal{H}R = \frac{1}{m_{\mathrm{P}}/\xi}(\delta\phi)^2\Box h_{\mu\nu} + \ldots \tag{3.24}$$

Simple power counting shows that the theory has a cutoff at the scale $\Lambda = m_{\mathrm{P}}/\xi$. The energy of the fluctuations during inflation are of the same order as the Hubble scale, $H \simeq \lambda m_{\mathrm{P}}/\xi \sim \Lambda$. One might conclude that the model is ill-defined, because the semiclassical approximation upon which the inflationary analysis relies is not valid, as the EFT is taken beyond its regime of validity [89, 88, 90]. In the Einstein frame the same cutoff can be naively inferred by looking at the second term in the kinetic part of the lagrangian (3.23).[12]

However, a more careful analysis (see for example [8]), shows that the scale of unitarity violation is actually *background dependent*. The argument in [8] proceeds schematically as follows. The action is expanded around a time dependent background $\phi = \phi(t) + \delta\phi$. After diagonalizing the quadratic terms in the kinetic sector one can estimate the cutoff by a dimensional analysis of the higher order operators in the expansion which are of the form $O^n(\delta\phi)/\Lambda^{(n-4)}(\phi)$. The result is that for large field values $\phi > m_{\mathrm{P}}/\xi$ the cutoff rises to $m_{\mathrm{P}}/\sqrt{\xi}$ making the semiclassical approximation valid. Even if this approach has been criticized for its non-rigorous derivation [112], subsequent and more refined analyses [28, 114] have basically confirmed its main message. *Throughout the history of the Universe the relevant energy scales are always below the field dependent cutoff.* Indeed in the Einstein frame $U^{1/4} < \Lambda(\phi)$.[13]

As mentioned, to study unitarity, gravity does not play an essential role in the Einstein frame. The unitarity bound can then be found from scattering amplitudes of SM particles. The interactions are found expanding around a non-zero Higgs field value, and will thus depend on the background. In [28] the chiral lagrangian, which gives a non-linear representation of the electroweak sector, has been used to compute the cutoff due to 2-to-2 Goldstone-Goldstone and Higgs-Goldstone scattering (in the high energy limit Goldstone scattering is equivalent to scattering of the longitudinal gauge bosons, as follows from the equivalence theorem). Only

[11]The energy scale sets by the potential can be seen as a reservoir available for random scatterings between scalar fluctuations [114].

[12]In the Palatini formulation of the model [115, 116] this term is not present after the conformal transformation, and the unitarity cutoff is lifted to $m_{\mathrm{P}}/\sqrt{\xi}$ in the small field regime.

[13]In the mid-field regime the cutoff is also parametrically larger than the typical momenta of relativistic particles produced during reheating [8, 117].

for the SM Higgs couplings is the theory unitary up to arbitrary scale, and any modification will give rise to unitarity violation at finite scale below the Planck mass.[14] The authors of [114] computed the 2-to-n Higgs self-scattering. This gives a similar result for the unitarity cutoff as follows from the 2-to-2 scattering mentioned above.[15] In Higgs inflation the lowest cutoff over the whole field regime comes from the gauge boson sector, and is given by

$$\Lambda \simeq O(1) 4\pi \frac{m_{\rm P}}{\xi} \left(\frac{1 + 6(\xi\phi/m_{\rm P})^2}{6(\xi\phi/m_{\rm P})^2/\xi + 1/\xi + 6} \right)^{1/2} \longrightarrow \left(\frac{m_{\rm P}}{\xi}, \phi, \frac{m_{\rm P}}{\sqrt{\xi}} \right), \tag{3.25}$$

where the last expression is the unitarity violation scale in the small, mid and large field regime respectively. The self-interactions of the Higgs also lead to unitarity violation around the same scale $m_{\rm P}/\xi$ in the small field regime, and to a cutoff of order $m_{\rm P}$ in the large field regime [8]. In the large field regime the only relevant cutoff comes from the scattering of the Goldstone bosons as given by (3.25).

3.4.2 UV completion: an open question

Even if the field dependent cutoff ameliorates the problem of the validity of the EFT analysis, it is still an open question whether it is meaningful or not to extrapolate SM parameters from the electroweak to the inflationary scale, one of the main selling points of Higgs inflation. There are two main reasons for this concern and both are related to the UV completion. In particular, they can be illustrated by the two philosophies of dealing with the UV physics, which is either in the context of an EFT framework, or by working with an explicit UV completion.

1. *Explicit UV completion.* The more conservative, and in a sense, traditional approach from a particle physics perspective is the following reasoning. There is a unitarity cutoff in the small field regime at the scale $\Lambda_{\rm low} = m_{\rm P}/\xi$. Therefore, one should first unitarize the theory in this regime and only after doing that study the behaviour of the model in the high field regime (where the energy scales relevant for inflation are roughly of the same order as $\Lambda_{\rm low}$). In other words, one should prove that whatever UV completes the theory at small field values does not change significantly the interactions and the potential during inflation. In this spirit two UV completions of Higgs inflation have been constructed [118, 119]. Apart from the details, the two approaches share a common feature: an extra scalar field is introduced to

[14] We will review this methods in Chapter 5 when we compute the cutoff with the same techniques for New Higgs inflation.

[15] In particular they include the phase space factor typical of multi-particles scattering. This latter lifts the cutoff for a growing number of particles involved with respect to 2-to-2 Higgs self-scattering.

unitarize the model up to the Planck scale and the inflationary dynamics becomes a two-field model. Inflation takes place mainly along the direction of the auxiliary field. One might thus wonder whether this still can be considered Higgs inflation, or whether it is simply another model without the attractive feature of HI. As just mentioned, the Higgs boson does not play the role of the inflaton anymore, and the connection between the Standard Model couplings measured at the LHC and inflationary observables now also depends on the extra coupling of the Higgs with the auxiliary field.

2. *EFTs and threshold corrections.* In the two asymptotic regimes of Higgs inflation perturbatively renormalizable EFTs can be defined [8, 34] and the β-functions can be computed [29, 30, 31, 32, 33, 99, 9, 100]. However, the theory is not renormalizable over the whole field range since radiatively generated quantum corrections are not suppressed at the boundary of the EFTs. Thus, even if the original motivation for HI was minimality, we cannot escape the fact that threshold corrections are expected and should be included to run the parameters consistently [120, 121]. These corrections can for example be implemented adding non-renormalizable operators which become important at the boundary between the two regimes. There is a big assumption underlying this approach. *The unknown UV completion that makes the model consistent is relevant only at the boundary of the EFTs without affecting the inflaton potential significantly at tree level.* If this is the case, the effects of the UV completion will enter the inflationary predictions via their effects on the running couplings. No one has proved so far that an explicit UV completion with this feature does exist.

The EFT approach introduces unavoidably uncertainties due to the unknown threshold corrections. It is then worth asking whether these can influence the predictions for the CMB observables. In the next section we illustrate different approaches to model the threshold corrections. However, as we will show (with the assumption that they only affect the running but not the inflaton potential), our conclusions are independent of the specific implementations.

3.4.3 Threshold corrections - different approaches

Consider Higgs inflation embedded in an effective field theory with a (field-dependent) cutoff given by the scale of unitarity violation (3.25).

$$\mathcal{L} = \mathcal{L}_{HI} + \sum_i C_i \frac{\mathcal{O}^{n+4}}{\Lambda^n}. \tag{3.26}$$

The usual EFT philosophy is to consider *all possible operators* of theory. This includes, for example, all higher operators of the form

$$\mathcal{O} = \frac{(\mathcal{H}^\dagger \mathcal{H})^{n+2}}{\Lambda^n}. \tag{3.27}$$

Since the Higgs field value during inflation exceeds the unitarity cutoff, the model is extremely sensitive to this kind of UV physics. All higher dimensional operators of the form above will completely spoil the flatness of the inflationary potential. Also operators of the form $(\mathcal{H}^\dagger \mathcal{H})^n \mathcal{O}^4/\Lambda^n$ should be forbidden during inflation; here $\mathcal{O}$ is a dimension four operator made up of standard model fields, e.g. $\mathcal{O} = F^{\mu\nu}F_{\mu\nu}$ and $F_{\mu\nu}$ the SU(2) field strength tensor. Indeed, during inflation these operators will give rise to effective couplings that are non-perturbatively large, and thus spoil the predictiveness of the model. In this sense the situation in Higgs inflation is not different from chaotic models of inflation. In the latter case the cutoff is the Planck scale and inflation takes place at superplanckian field values, and thus also chaotic inflation is highly sensitive to operators of the above form. We simply assume these operators to be absent, this can be motivated and implemented independently in two ways:

A. *Minimal approach* to UV corrections. The minimal approach to UV corrections is to add only the higher dimensional operators that are necessary for consistency of the theory [121, 120, 122], i.e. only the ones that are radiatively generated by the HI action itself. In the large field regime these operators are not only suppressed by inverse powers of the cutoff but also by a small parameters δ that makes the theory renormalizable in the EFT sense (see below).

B. Assume that the UV completion respects an *approximate shift symmetry*, which forbids the most dangerous operators that already change the potential at tree level [28, 1]. From an operative point of view this translates into adding all the higher order operators discussed in section 3.4.4.

The result in both cases is that the unknown UV physics is only important in the mid-field regime where it is needed for consistency of the theory, and that it can only enter the inflationary potential via its effect on the renormalization group equations. Thus, the UV physics will affect the running of the couplings (in the mid-field regime), and consequently change the inflationary potential at loop level.

Below we will motivate our choice of higher dimensional operators that we add, and that we will use in our numerical computations. We would like to stress, though, that this choice is not critical to our results, and other parameterizations can be chosen and additional corrections can be added. As we will show, our main conclusions will remain intact.

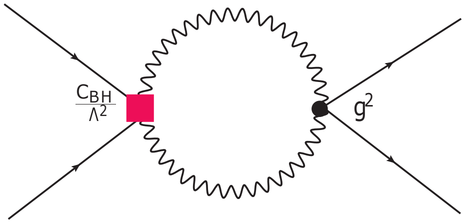


Figure 3.2: 2-*to*-2 *Higgs scattering with a gauge boson loop. The red-squared vertex is built out of the new interaction* (3.29).

3.4.4 Dimension-six operators

We assume the UV completion respects the approximate shift symmetry of the action in the inflationary regime, which is only broken by a non-zero (but small) Higgs mass. This implies that at dimension six, which are the leading corrections, only operators of the form [28]

$$\mathcal{L} \supset \sum_i C_i \frac{m_h^2}{\Lambda^2} \mathcal{O}_i^4 \tag{3.28}$$

are allowed. Here C_i are unknown Wilson coefficients, and the sum is over all dimension four operators invariant under the SM symmetries. In the small field regime $m_h^2 \propto \mathcal{H}^\dagger \mathcal{H}$ and the operators (3.28) reduce to the 59-independent SM dimension-six operators listed in [123].

These operators can affect the inflationary potential at the quantum level. The reason is that the SM couplings are run from the electroweak scale where they are measured, to the inflationary scale where they enter the inflationary observables. To connect the low and high scale parameters, one has to pass the mid-field region where the renormalization scale is $\mu \sim m_{\rm P}/\xi$ and the threshold corrections — if large enough — cannot be neglected, as they give corrections to the RGE equations [28, 123]. For the sake of clarity, consider the following operator $m_h^2 B_{\mu\nu} B^{\mu\nu}$, in the small field regime it will look like

$$\mathcal{O}_{\mathcal{HB}} = \frac{C_{HB}}{\Lambda^2} \mathcal{H}^\dagger \mathcal{H} B_{\mu\nu} B^{\mu\nu}. \tag{3.29}$$

In order to understand how this, for example, changes the running of the Higgs quartic coupling λ, consider the diagram in 3.2. One vertex is given by the usual SM Higgs-gauge boson coupling while the other is built out of the new interaction generated by the six-dimensional operator. The divergence generated by this diagram will depend on the Higgs mass (due to the momentum dependent vertex).

This will in general depend on the background field value and it will modify the beta-function for λ in the following way

$$\delta\beta_\lambda = \frac{C_{HB} m_h^2(\phi)}{16\pi^2\Lambda^2}(12 g_1^2 y_H^2). \tag{3.30}$$

In the SM regime $m_h^2 = 3\lambda\phi^2 - \lambda v^2$ and with the Higgs in the electroweak vacuum $m_h^2 = 2\lambda v^2$.[16]

Using the explicit form of the Higgs mass (see (3.32) below) and unitarity bound (3.25), it can be seen that these operators are only unsuppressed around the scale $\phi \sim m_\mathrm{P}/\xi$. As a result, parameterizing the UV completion with operators of the form (3.28) does not affect the tree level inflaton potential.

We can arrive at the same conclusion from a different perspective (following approach A. in 3.4.3). Namely, take the minimalistic approach to add new physics only when really necessary for the consistency of the theory. For this purpose we do not need higher dimensional operators that become important at the large field values during inflation, only corrections around the scale $\phi \sim m_\mathrm{P}/\xi$ are necessary. First of all, although $V^{1/4} < \Lambda$ at all field values, these scales become of the same order at $\phi \sim m_\mathrm{P}/\xi$ and corrections to the Higgs inflation action are unsuppressed. Secondly, the counterterms introduced to absorb the UV divergences of the quantum corrections make a jump at the scale $\phi \sim m_\mathrm{P}/\xi$ [34, 121], which signals that new physics should enter at this scale. We will explain this second point in detail in the next section. This will also give us the opportunity to discuss in general how to deal with renormalization in models that are not renormalizable, such as Higgs inflation.

3.5 Renormalizability of non-renormalizable models

We discuss the one-loop CW correction in the Einstein frame showing that radiatively generated corrections are suppressed in the two asymptotic regimes. This allows to define EFTs in each regime even if, again, the matching of the parameters between the two different EFTs depends on unknown threshold corrections.

[16] As explained in chapter 2 sec. 2.1.4, instead of absorbing a background field dependent divergence in the counterterm of λ we can equivalently absorb the divergence in the coefficient of the dimension-six operator $(\mathcal{H}^\dagger\mathcal{H})^3$. Once expanded around the background this will give the same result for the physical quartic coupling at low energy.

3.5.1 Coleman Weinberg potential

The one-loop effective potential for the classical field ϕ is given by the tree level potential plus the Coleman-Weinberg potential [41]. Explicitly

$$U_{\rm eff} = U + \frac{1}{64\pi^2}\sum_i (-1)^{F_i} S_i m_i^4(\phi)\left[\ln\left(\frac{m_i^2(\phi)}{\mu^2}\right) - c_i\right] \tag{3.31}$$

in the $\overline{\rm MS}$ renormalization scheme. Here μ is the normalization scale, $F_i = 0\,(1)$ for a boson (fermion) field, S_i counts the degrees of freedom of each particle with mass m_i, and $c_i = 3/2$ for fermions and scalars and $c_i = 5/6$ for gauge bosons. The gauge boson, top quark, Higgs and (three) Goldstone boson (GB) masses are given (in Landau gauge) by [34, 121]

$$m_{A^i}^2 = \frac{g_i^2\phi^2}{2\Omega^2}, \quad m_t^2 = \frac{y_t^2\phi^2}{2\Omega^2}, \quad m_h^2 = 3\lambda\phi^2\frac{1+4\xi^2\phi^2-4\xi^3\phi^4}{\Omega^4(1+6\xi^2\phi^3)^2}, \quad m_{\theta^i}^2 = \frac{\lambda\phi^2}{\Omega^4(1+6\xi^2\phi^2)}, \tag{3.32}$$

with $g_i = \{g_2, \sqrt{g_1^2+g_2^2}\}$ for the W and Z bosons with g_1, g_2 the hypercharge $U(1)_Y$ and and $SU(2)$ couplings respectively, and y_t the top Yukawa.

In the Einstein frame Lagrangian, the term that is of quadratic order in the background field is schematically of the form

$$\mathcal{L}_E \supset -U(h) - gG(h)A_\mu A^\mu + y_t Y(h)\,\bar\psi\psi, \tag{3.33}$$

with A_μ, ψ_t a gauge and fermion field respectively. The special relation between potential, gauge and fermion interactions

$$U \equiv \lambda F^4(h),\;\; G(h) = F^2(h),\;\; Y = F(h), \quad F = \frac{\phi(h)}{\sqrt{2}\Omega(h)}, \tag{3.34}$$

assures that loop contributions from the gauge bosons and fermions have the same field dependence as the tree level potential, and the divergences can be absorbed over the whole field range.[17] However, that is not the case for the Higgs and GB masses and *the theory is not renormalizable over the whole field range.*

To illustrate the non-renormalizability of the theory, consider the theory with a real Higgs field and no other degrees of freedom. Write the Higgs potential in the form $U \equiv \lambda\mathcal{U}(h)$. Working with the background field method, the one-loop divergences to the effective action are of the form

$$U^{1-\rm loop} \supset \frac{1}{2}{\rm Tr}\log(-\Box + \mathcal{U}''({\rm h})) = -\frac{1}{\epsilon}\frac{\lambda^2}{32\pi^2}(\mathcal{U}''({\rm h}))^2 \tag{3.35}$$

[17] This can be shown also at the level of Feynman diagram, working as in section 2.1.1 and Taylor expanding the function $F(\phi(h))$. The result is that we can resum and reabsorb an infinite number of diagrams by renormalizing only the quartic coupling λ.

where a prime denotes derivative with respect to the canonical background field, and with $\epsilon = (4-d)$ and d the number of space-time dimensions. For the theory to be renormalizable $(\mathcal{U}''(h))^2$ needs to have the same functional dependence on h as the tree level potential. If this is not the case, the following operator needs to be added to the original Lagrangian,

$$\mathcal{L}^{(1)} = \lambda^2 C_1 \mathcal{U}_1 \equiv \lambda^2 C_1 [(\mathcal{U}''(h))^2 - 36\mathcal{U}(h)] \tag{3.36}$$

where C_1 is a new independent parameter (the Wilson coefficient of the new operator), whose counterterm cancels the divergence generated by $\lambda\mathcal{U}$. The $36\mathcal{U}$ is added to subtract the same powers of $\mathcal{U}$ already contained in $\mathcal{U}''(h)$, such that $\mathcal{U}_1$ is negligible in the small field regime. The one loop corrections reevaluated with this new operator will need the addition of an extra operator. Iterating the procedure will generate the infinite series,

$$\mathcal{U}_{\text{eff}} = \lambda\mathcal{U} + \lambda^2 C_1 \mathcal{U}_1 + \lambda^3 C_2 \mathcal{U}_2 + ... \tag{3.37}$$

These operators are automatically ordered in an EFT series in the small and large field regime, i.e. at each iteration they are increasingly suppressed by the small parameter defining the EFT. Consider the operator (3.36) in the two regimes

$$\mathcal{L}^{(1)} = \begin{cases} \lambda^2 C_1 U(h) \sum_{n=1}^{\infty} c_n \left(\frac{\phi(h)}{\Lambda}\right)^{4n} & \frac{\phi}{\Lambda} \ll 1 \\ -36\lambda^2 C_1 U(h) + \lambda^2 C_1 \sum_{n=1}^{\infty} d_n \delta^n, & \delta = \frac{1}{\Omega^2} \ll 1 \end{cases} \tag{3.38}$$

with $\Lambda = m_{\text{P}}/\xi$ the cutoff in the SM regime. In the small field regime trivially $\phi \ll \Lambda$ and the theory is renormalizable in the usual EFT sense, as the additional operators needed to cure the divergences starts at higher order in $O(1/\Lambda)$. In the large field regime the situation is similar. The higher order operators generated radiatively are suppressed by increasing powers of the small parameter $\delta = 1/\Omega^2$. [18] This parameter plays the role of $(\phi/\Lambda)^4$ in consistently ordering the EFT in the large field regime.

3.5.2 Low energy parameters and high energy parameters

The crucial thing to notice is that once $\mathcal{L}^{(1)}$ is written in the large field regime as in (3.38) the first term can be reabsorbed in a redefinition of λ. *This breaks the connection between the parameters in the two regimes.* Considering the series (3.37), we obtain

$$\lambda_{\text{high}} = \lambda(1 + C_1'\lambda + C_2'\lambda^2 + ...), \tag{3.39}$$

where we have reabsorbed the numerical coefficient in the Wilson coefficient. Note

[18] If you expand around the background the perturbations are suppressed both by increasing powers of the cutoff (in the Higgs sector alone this is just m_{P}) and by increasing powers of δ.

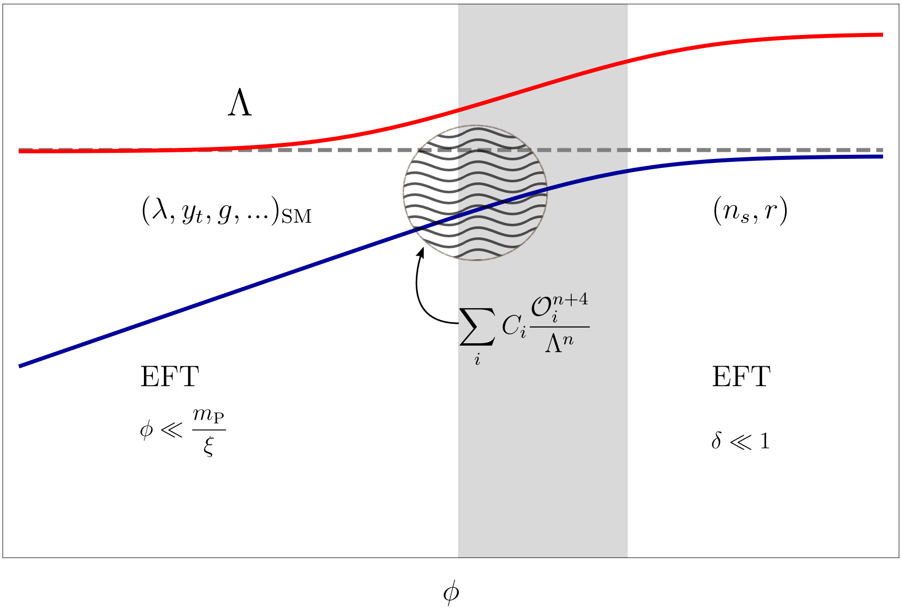


Figure 3.3: *Overview of quantum aspects in Higgs inflation. The grey area represents the mid-field regime $m_{\mathrm{P}}/\xi < \phi < m_{\mathrm{P}}/\sqrt{\xi}$. The blue line gives the typical energy of the Higgs sector $U^{1/4}$ as a function of the real Higgs field ϕ. It is always below the field-dependent unitarity cutoff given by the red line, and an EFT approach is amenable. The dashed line gives the unitarity cutoff of the small field regime Λ_{low}, extrapolated over the whole field range. It is of the same order as the inflationary potential, and it is thus by no means guaranteed that whatever UV completes the theory does not spoil the inflationary potential, as discussed in sec. 3.4.1. This observation has lead to different approaches in dealing with unknown UV physics — the construction of an UV complete theory or using an EFT approach — as explained in sec. 3.4.2. Working within an EFT framework, in the small and large field regimes renormalizable EFTs can be constructed. These are valid only within the given regime, and for energies below the unitarity cutoff. At the boundaries between the small and mid-field regime ($\phi \sim m_{\mathrm{P}}/\xi$), threshold corrections are unavoidable as it is here that the RG equations change. These threshold corrections can be implemented and motivated in different ways (case A. and B. in sec. 3.4.3). The conclusion about the inflationary observables will be independent of the specific implementation.*

that in (3.36) we have explicitly extracted a factor λ^2 from the Wilson coefficients (and then a λ at every iteration in (3.37)) to give a hierarchical order to our procedure. Thus, the new operator $\mathcal{U}_1$ will itself contribute to the one-loop corrections at the same order as the two-loop corrections coming from the original potential [121], and analogously for the rest of the series. In this way we can make all these extra operators hierarchically suppressed by increasing powers of the coupling. This is a working assumption that allows to parametrize the connection between the parameters in the two regimes. It has been used [120, 121] to truncate consistently the series in the regime where no other order parameter is present.[19] As discussed, in the two asymptotic regimes this is not needed. It simply represents another way to parametrize the unknown UV completion. We can parameterize our ignorance about the mid field regime in many different ways, as long as the UV completion is not known one is as legitimate as the other.

Summarizing, in the two asymptotic regimes the theory is renormalizable in the EFT sense[20] while in the mid field regime all these additional operators will contribute. The ignorance about the UV completion is parameterized by the arbitrary Wilson coefficients. The connection between the quartic coupling in the small and large field regime depends on this parameterization, and thus on the specifics of the UV completion. The net effect is a jump in the quartic coupling in the mid field regime, which effectively smears the boundary conditions on the running coupling at the EW scale. Both approaches to parameterizing the threshold corrections, motivated by minimality or by an approximated shift symmetry of the potential, lead to the same result: the direct connection between the low and high scale parameters is lost as it depends on the unknown UV completion.

However, as we will show analytically in section 3.7, in Higgs inflation the inflationary predictions are universal, and all dependence on the running, and thus on the threshold corrections, drops out. Thus, as long as inflation is possible at all — no large corrections to the tree level potential and UV physics only enters via the RGE equations— the inflationary predictions are robust.

[19] The coefficient of this operator are used in [120] to get a posteriori (after the numerics) inflation near the inflection point (where the predictions might depart from the tree level one and they can explore possibilities to fit the Bicep data). In [121], facing the case of metastable vacuum, the jump is used to lift the quartic coupling λ towards positive values at the inflationary scale.

[20] This has been shown for the full set of one loop SM beta-functions in [34].

3.6 Renormalization group flow

We want to compute the inflationary observables from the RG improved effective action during inflation including the effects on the running of the couplings due to higher order operators relevant in the mid-field regime.

As explained in the previous chapter (sec. 2.2), using the L-loop RG improved potential and the beta-functions computed at $L+1$ loop allows us to resum all the L-to leading logs in the effective action. In the SM sector the effective potential is known up to two-loops [76]. One can simply (and approximately) substitute the modified masses (3.32) in the SM expression to obtain the two-loop potential in the Einstein frame [29, 99]. This can be rewritten as

$$U = \frac{\lambda_{\rm eff}\phi^4}{4\Omega^4}, \qquad \lambda_{\rm eff} = \lambda(t) + \lambda^{(1)}(t) + \lambda^{(2)}(t), \tag{3.40}$$

where $\lambda^{(1)}(t) + \lambda^{(2)}(t)$[21] are the one and two loop contributions that are left after resumming the logs. Now, in order to extend the NNLO SM analysis of [76] to the case of HI, one should use the potential in (3.40), two-loop matching conditions at the EW scale (known) and three-loop beta-functions. Even if these last ones are known in the SM regime [125, 126], for the mid and large field regime the beta-functions have been computed explicitly only up to one-loop by different groups [29, 98, 30, 31, 32, 33, 9, 100], with small differences. There are approximate methods to derive the higher order beta-functions in the large field regime from the SM ones (for example, suppressing each off-shell Higgs propagator in the SM RG equation [99]). However, since there are already discrepancies at one-loop between these methods, we do our analysis by simply using the recent one-loop beta-function computed in [34] for the inflationary regime.[22] In addition, we also have at our disposal only the one-loop contribution to the beta-functions coming from the dimension-six operators [123] that we use to parametrize the threshold corrections. Thus, we restrict to the tree level RG improved potential for our numerical analysis. For the analytic discussion in sec. 3.7.1 nothing would really change at higher orders since one can simply replace $\lambda \to \lambda_{\rm eff}$ and $\beta_\lambda \to \beta_{\lambda_{\rm eff}}$ and draw the same conclusions.[23]

We do not elaborate on the small differences between explicit computation of the beta-functions in the large field regime at one-loop since they are irrelevant for our analysis. Our main conclusion does not depend on this choice, only the exact

[21] Explicit expression can be found in appendix C of [124].

[22] The discrepancies between different methods at one-loop is of the same order as going from LO to NNLO using one of the approximate methods.

[23] Moreover, absorbing the anomalous dimension in the "canonical field" as is done in sec. 2.3.2 is valid at LO. It is rather cumbersome to extend that beyond that accuracy.

numerical values of parameters differs slightly. However, since this is crucial for our study, in the next section we extensively motivate our choice for the renormalization scale.

3.6.1 Renormalization prescription

Higgs inflation can be analyzed, and loop corrections can be calculated in both the Jordan and Einstein frame. Even if the frames are merely related by a field transformation it is not universally accepted that they describe the same physics.[24]

There remains confusion in the literature on the frame dependence of the results at quantum level, and in particular on the choice of renormalization scale. We first review why in the literature two different renormalization prescriptions appear. Then we provide at least three reasons why if a consistent RG procedure is implemented there is no ambiguity in the choice of the renormalization scale.[25]

In this section, compared to sec. 3.3, in order to be more explicit we use $V \equiv V_J$ and $U \equiv V_E$ for the tree level potential in the Jordan and Einstein frame respectively (and similar notation for the quantum corrections in both frames).

To include the (one-loop) corrections one could proceed in two ways:

1. First go to the Einstein frame and then add the quantum correction to V_E.

$$V_{E_1} = V_E^{(0)}(\phi) + V_E^{(1)} = \frac{V_J^{(0)}}{\Omega^4} + V_E^{(1)}, \tag{3.41}$$

   where the superscript (0) and (1) refer to the tree level and one-loop Coleman-Weinberg potential respectively.

2. Add the CW corrections to the Jordan frame potential and only after transform to the Einstein frame

$$V_J^{(0)}(\phi) + V_J^{(1)} \xrightarrow{E} V_{E_2} = \frac{V_J^{(0)}(\phi)}{\Omega^4} + \frac{V_J^{(1)}}{\Omega^4}. \tag{3.42}$$

In both cases we want to end up in the Einstein frame where slow roll inflation is most easily studied. As can be seen in the above equations, but this is general, all

[24] The equivalence of the Jordan and Einstein frame [127, 128, 129] can be made explicit by rewriting the action in terms of dimensionless quantities which are invariant under a conformal transformation [130, 131]. The equivalence can also be checked on a case-by-case basis. For example, in [109, 110, 111, 112, 113] it was shown that both frames give the same result for the curvature perturbation during inflation, [132] uses a covariant approach to show that both frames gives the same on-shell effective action, Finally, in [34] it was shown that the Coleman-Weinberg potential and the renormalization procedure is one-to-one in both frames.

[25] Remember that even if the true effective action is independent on μ, any approximate truncation is, from where it stems the importance of choosing μ.

mass scales are rescaled by the conformal transformation

$$m_J \xrightarrow{E} m_E = \frac{m_J}{\Omega}. \tag{3.43}$$

As already mentioned, if one does not consider the back reaction from gravity, following one of the two paths leads to different results in the Higgs-gravity sector. Since the main contribution to the CW potential comes from the top quark and gauge boson loops — the Higgs and GB loops are suppressed — we do not have to worry about this.

Let's consider then the contribution of the top quark to the Coleman-Weinberg potential in the Einstein frame, following procedure 1

$$V_{E_1} = \frac{(\lambda + \delta\lambda)\phi^4}{4\Omega^4} + \frac{3}{8\pi^2} m_{t,E}^4 \ln\left(\frac{\Lambda_E^2}{m_{t,E}^2}\right), \tag{3.44}$$

where cutoff regularization has been used; $\delta\lambda$ is the counterterm, and the Einstein frame top mass has been defined in (3.32). Choosing the counterterm

$$\delta\lambda = -\frac{3y^4}{(8\pi^2)} \ln\left(\frac{\Lambda_E^2}{\mu_E^2}\right) \tag{3.45}$$

gives

$$V_{E_1} = \frac{\phi^4}{4\Omega^4}\left(\lambda(\mu_E) + \frac{y^4}{8\pi^2}\ln\left(\frac{\mu_E^2}{m_{t,E}^2}\right)\right). \tag{3.46}$$

The log will be resummed for

$$\mu_E \sim m_{t,E} \sim \frac{\phi}{\Omega(\phi,\xi)}; \tag{3.47}$$

in the RG improved action this is the optimal choice defined in section 2.2. This choice of renormalization scale is often referred to as *prescription I*. For Higgs inflation it corresponds to the usual prescription that the renormalization scale is chosen to be the typical energy scale involved in the process.

Procedure 2 gives

$$V_J = \frac{(\lambda + \delta\lambda)\phi^4}{4} + \frac{1}{8\pi^2} m_{t,J}^4 \ln\left(\frac{\Lambda_J^2}{m_{t,J}^2}\right) = \frac{\lambda\phi^4}{4} + \frac{3}{8\pi^2} m_{t,J}^4 \ln\left(\frac{\mu_J^2}{m_{t,J}^2}\right) \tag{3.48}$$

where $m_{t,J} = \Omega m_{t,E}$ is the top mass in the Jordan frame. In the second expression we set the counterterm $\delta\lambda = -\frac{y^4}{4\pi^2}\ln\left(\frac{\Lambda_J^2}{\mu_J^2}\right)$. After applying the conformal transformation we have $V_J \to V_J/\Omega^4$ and the potential is equivalent to the Einstein

frame renormalized potential with the exception that, at this stage, the log in the potential will be minimized for

$$\mu_J(t) \sim m_{t,J} \sim \phi. \tag{3.49}$$

This choice is often referred to as *prescription II*. Thus, the couplings in the RG improved potential seems to have a different field dependence following one of the two procedures. The issue can be solved by noticing that *all mass scales*, including the Planck mass, cutoff scale and renormalization scale, *are rescaled under a conformal transformation* as in (3.43).[26] Thus the physical renormalized coupling $\lambda(\mu_J)$ in the Einstein frame becomes $\lambda(\mu_J/\Omega = \mu_E)$ and the transformed Jordan frame potential coincides with the one given by the first procedure, i.e. $V_{E_2} = V_{E_1}$. More details can be found in [34, 1].

Thus, by rescaling all the mass scales, different prescriptions do not arise from frame dependence. However, one could still argue that different renormalization prescriptions encode different UV completions of the theory.[27] As discussed in sec. 3.5 no counterterms can be defined that absorb the (subdominant) Higgs and GB contributions over the whole field range. Thus, we are forced to add field dependent counterterms (or equivalently new operators) to the action. Consider, for example following procedure 1, to add a field dependent counterterm of the form

$$\delta\lambda' = -\frac{y^4}{8\pi^2}\ln\left(\frac{\Lambda^2}{\mu^2}f(\phi)\right). \tag{3.50}$$

instead of (3.45). The Einstein frame potential becomes

$$V_E = \frac{\lambda\phi^4}{4\Omega^4} + \frac{1}{8\pi^2}m_t^4\ln\left(\frac{m_t^2}{\mu^2}f(\phi)\right), \tag{3.51}$$

and the choice $f(\phi) = \Omega^2 = 1 + \xi\phi^2/m_{\rm P}^2$ will lead to prescription II since $\mu = f(\phi)m_t$ will be the optimal choice to resum the log. However, note that a field dependent counterterm implies adding new operators to the action. Indeed, the difference between the two choices for the counterterms gives the following contri-

[26] The situation is completely analogue to going from a conformal FLRW metric to a Minkowksi metric by doing a conformal transformation with $\Omega = a(t)$ the scale factor. All masses are rescaled by the scale factor, *cf.* the physical momentum (the canonical momentum in the FLRW metric) and comoving momentum (the canonical momentum in the Minkowski metric) are related by $k_{\rm com} = k_{\rm phys}/a$. Spurious factors of $a(t)$ (the scale factor is by definition unobservable) only appear when comoving scales are erroneously compared to physical mass scales [133].

[27] This fact has been first mentioned in [121], then illustrated in detail in our [1] and later re-found in the analysis worked out in [134].

bution to the lagrangian

$$\begin{aligned}\Delta V_{\text{c.t.}} &= \frac{(\delta\lambda' - \delta\lambda)\phi^4}{4\Omega^4} = -\frac{3}{8\pi^2} m_\tau^4 \ln\left(f(\phi)\right)\Big|_{f=\Omega^2} \\ &= -\frac{3y^4}{8\pi^2}\frac{\phi^4}{\Omega^4}\left(\frac{\phi^2}{m_{\text{P}}^2/\xi} - \frac{\phi^4}{2m_{\text{P}}^4/\xi^2} + ...\right)\end{aligned} \tag{3.52}$$

All these new operators are innocuous suppressed marginal operators in the small field regime. However, in the large field regime the operators in eq. (3.52) are not suppressed. In general, a non-trivial choice of $f(\phi)$ implies that the potential is already changed at the classical level! Turning the argument around, we can say that prescription II translates in a type of UV completion different from the one we considered and motivated in 3.4.3. The simple form of the action in the Jordan frame, with just a single new parameter compared to the SM Lagrangian, can no longer be used as a motivation for the model. Moreover, allowing for any UV completion possible, i.e. for any choice of $f(\phi)$, all predictivity is lost as literally any potential can be constructed. Fortunately, that is not needed. The choice $f(\phi) = 1$ is the natural choice as no new operators and counterterms beyond those present in standard Higgs inflation are needed in the large field regime.

Let us finally give a third argument in favor of the renormalization prescription I by stressing a crucial point about this way of parameterizing the renormalization scale. We have already seen that the cutoff depends on the Higgs vev. Prescription I gives $\mu(\phi) < \Lambda(\phi)$ for all field values. For prescription II, on the other hand, $\mu(\phi) > \Lambda(\phi)$ for large field values, and the RGE evolution can no longer be described in the EFT setting, the full UV completion is required. Consider the analogy with the Fermi effective theory of beta decay. At energy values below the W-boson mass the Fermi effective action can be used to compute the beta-functions etc. However, for energies above the cutoff (W mass) the knowledge of the full electroweak Lagrangian is needed. Prescription I for the renormalization scale automatically takes into account that when $\mu(\phi) > \Lambda_{\text{low field}}$ the field is no longer in the low field region and the unitary bound is still larger than μ. Thus the full form of the UV completion is not needed and one can consistently parametrize it with a series of higher order operators suppressed by the scale given by the field dependent cutoff. Instead, by using prescription II we are pretending to consider the running of the couplings at energy scales beyond the unitarity violation scale of the theory. From the physical point of view the difference between the two situations can be understood from the fact that here the increase in energy is due to a displacement of the Higgs vev.

For definiteness, we will use in the next section

$$\mu(t) = \frac{\phi}{\Omega(\phi, \xi(t))}, \tag{3.53}$$

which is proportional to the top mass in the Einstein frame.

3.6.2 Renormalization group equations

Let us enumerate the main steps of our analysis to calculate the inflationary observables including the effects of running couplings.

- *Effective Potential.* We consider the RG improved Einstein frame action (3.10) with potential and field space metric

$$U = \frac{\lambda(t)\phi^4}{4(1+\xi(t)\phi^2)^2}, \qquad \gamma_{\phi\phi} = \frac{1+\xi(t)\phi^2(1+6\xi(t))}{(1+\xi(t)\phi^2)^2}, \tag{3.54}$$

  with

$$t = \ln(\mu(t)/m_{\rm t}), \tag{3.55}$$

  m_t the EW scale top mass and $\mu(t)$ given by eq. (3.53).[28] The running of the couplings is governed by the RG equations.

- *RGEs.* In the small field regime ($\phi < m_{\rm P}/\xi$) we use the the SM two-loop beta-functions [135, 124]. From $\phi \sim m_{\rm P}/\xi$ we use the one-loop beta-functions derived in [34] for the mid and large regime, i.e.

$$\begin{aligned}
\beta_\lambda &= \frac{1}{(4\pi)^2}\left(\frac{3}{8}(2g_2^4+(g_1^2+g_2^2)^2)-6y_t^4\right)+4\lambda\gamma_\phi, \quad \beta_\xi = 2\gamma_\phi\xi, \\
\beta_{g_1} &= \frac{20}{3(4\pi)^2}g_1^3, \quad \beta_{g_2} = -\frac{10}{3(4\pi)^2}g_2^3, \quad \beta_{g_3} = -\frac{7}{(4\pi)^2}g_3^3, \\
\beta_{y_t} &= -\frac{1}{(4\pi)^2}\left(\frac{2}{3}g_1^2+8g_3^2\right)y_t+\gamma_\phi y_t, \quad \gamma_\phi = \frac{3y_t^2}{(4\pi)^2}.
\end{aligned} \tag{3.56}$$

- *Threshold corrections.* We add the corrections to the beta-functions due to the full set of 59 independent dimensional-six operators as explained in 3.4.4. These corrections were computed in [123] and have the form (3.30), i.e. $\delta\beta_\lambda = C_i m_h^2(\phi)/\Lambda^2$. They depend on unknown Wilson coefficients C_i, the Higgs mass which is given in (3.32), and for the cutoff scale we choose the natural unitary cutoff given in (3.25).

  As already discussed the operators are peaked at $m_{\rm P}/\xi$, only around this scale the corrections to the running are appreciable. For inflationary purposes the effect is that threshold corrections may give a "kick" to λ, i.e.

[28] For more details see the discussion in the previous chapter sec. 2.3.2.

change $\lambda(\mu \sim m_{\rm P}/\xi)$ by some amount compared to the SM running.[29]

– *Initial conditions.* We use the initial values of the couplings at the electroweak scale determined by the two loop matching conditions as in [124].[30] The boundary condition ξ_0 is set at the boundary of the mid-field regime, i.e.

$$\xi(m_{\rm P}/\xi) = \xi_0. \tag{3.57}$$

The Wilson coefficients are chosen randomly in an interval

$$C_i = \text{Random}[-\text{C}_{\text{max}}, \text{C}_{\text{max}}]. \tag{3.58}$$

Then all couplings are run to the large field regime.

– *Planck normalization.* Next $t_{\rm end}$ and $t_\star$ are determined, i.e. the renormalization scale (3.55) at the end and $N_\star$ e-folds before the end of inflation. From this then finally the power spectrum for the perturbations is derived. We reiterate this procedure, adjusting the value of ξ_0 till the right Planck/COBE normalization (3.75) is obtained. It may happen that for some or all ξ_0-values inflation with more than $N_\star$ e-folds is impossible. (For definiteness, we take $N_\star = 60$).

– *Inflationary observables.* (n_s, r) are computed by standard slow roll formulas. The procedure is iterated for different boundary conditions at the EW scale and/or different randomly chosen Wilson coefficients.

3.7 Inflationary observables

In this section we compute the spectral index and tensor-to-scalar ratio following the procedure just outlined. We will first show through an analytic study of the RG improved action that for inflation on a flat plateau, as it is usually assumed in Higgs inflation, all dependence on the beta-functions drops out, and the inflationary observables are the same as for the classical potential. This also allows for a simple understanding of other numerical results presented in the literature with (and without) threshold corrections (see for example [28] and [99]). In addition, it shows the amount of fine tuning required to end up in a regime where the predictions start to be sensitive to the UV completion. In fact, with running

[29] Since $\lambda \ll 1$ this kick may be appreciable for Wilson coefficients $C_i \sim O(10)$ (such that the threshold and SM contribution to the RGEs are of comparable size $\delta\beta \sim \beta_{\rm SM}$ at the scale $\mu = m_{\rm P}/\xi$). The relative kick to other SM parameters is very small.

[30] The instability of the Higgs potential (when $\lambda(\mu)$ becomes negative) is pushed to larger scales with two-loop matching conditions compared to one-loop matching.

included, it is possible for a limited range of parameters that the potential develops a maximum. As discussed in 3.7.2 for hilltop inflation, i.e. inflation near the maximum, the results depend sensitively on the running, and thus on the UV completion (entering via the beta-functions). The numerical results will illustrate this case and also confirm the analytic findings of the next section.

3.7.1 Inflation on the flat plateau

Higgs inflation takes place in the large field regime (3.15), where the action can be expanded in the small parameter (note that in this section we set $m_{\rm P} = 1$)

$$\delta = 1/(\xi\phi^2) \ll 1. \tag{3.59}$$

As follows from (3.65, 3.71) below, the δ-expansion is equivalent to an expansion in slow roll parameters, and is also equivalent to an $1/N_\star$ expansion.

In order to include the effects of running couplings on the inflationary observables we work with the renormalization group improved action. The potential and field space metric (3.54) can be rewritten as

$$V = \frac{\lambda(t)}{4\xi(t)^2}\frac{1}{(1+\delta(t))^2}, \qquad \gamma_{\phi\phi} = \frac{\delta(t)(1+\delta(t)+6\xi(t))}{(1+\delta(t))^2}, \qquad \delta(t) = \frac{1}{\xi(t)\phi^2}, \tag{3.60}$$

with

$$t = \ln(\mu(t)/m_t), \qquad \mu(t) = \frac{1}{\sqrt{\xi(t)(1+\delta(t))}}, \tag{3.61}$$

This choice minimizes the logs in the Coleman-Weinberg expansion, as already discussed in section 3.6.1. For $\delta \ll 1$ the potential reduces to a constant plus (exponentially) small corrections, and inflation takes place on a flat plateau. The running of the couplings may slightly tilt the plateau, and thus change the expressions for the observables.

To calculate the slow roll parameters the first and second derivatives of the potential with respect to the canonically normalized field h, defined in (3.12), are needed. Let's start with the slope first. Using the chain rule gives

$$V_h = \frac{1}{\sqrt{\gamma_{\phi\phi}}}\left(\frac{\partial V}{\partial\phi} + \frac{\partial V}{\partial\lambda}\lambda_\phi + \frac{\partial V}{\partial\xi}\xi_\phi\right), \tag{3.62}$$

with

$$\lambda_\phi = \beta_\lambda\frac{dt}{d\phi}, \qquad \xi_\phi = \beta_\xi\frac{dt}{d\phi}, \qquad \frac{dt}{d\phi} = \frac{\delta^{3/2}\xi^{1/2}}{1+\delta+\frac{\beta_\xi}{2\xi}}, \tag{3.63}$$

where we used the definitions $\beta_\lambda = \partial\lambda/\partial t$ and $\beta_\xi = \partial\xi/\partial t$. The last expresson follows from the explicit form of normalization scale (3.61). Putting it all together gives

$$\frac{V_h}{V} = \sqrt{\frac{8}{3}} \frac{\delta(1+\delta)}{\sqrt{1+\frac{(1+\delta)}{6\xi}}} \frac{(1+\frac{\beta_\lambda}{4\lambda})}{(1+\delta+\frac{\beta_\xi}{2\xi})}. \tag{3.64}$$

This result is still exact, no δ-expansion or other approximation has been done. In a similar way the 2nd derivative of the potential can be computed. The slow roll parameters become

$$\epsilon \equiv \frac{1}{2}\left(\frac{V_h}{V}\right)^2 = \frac{4}{3}\delta^2 F^2 \left(1+\frac{1}{6\xi}\right) + O(\delta^3), \qquad \eta \equiv \frac{V_{hh}}{V} = -\frac{4}{3}\delta F + O(\delta^2), \tag{3.65}$$

with

$$F = \frac{(1+\frac{1}{4}\frac{\beta_\lambda}{\lambda})}{(1+\frac{1}{2}\frac{\beta_\xi}{\xi})(1+\frac{1}{6\xi})}. \tag{3.66}$$

Turning off the running of the couplings $\beta_\lambda = \beta_\xi = 0$, we retrieve the standard classical results in eq. 3.14.

In the large field regime the RGE in (3.56) gives

$$\frac{\beta_\lambda}{\lambda} = \frac{1}{8\pi^2}\left(\frac{3/8 g_i^4 - 3y^4}{\lambda} - 2y^2\right), \qquad \frac{\beta_\xi}{\xi} = \frac{1}{8\pi^2} y^2. \tag{3.67}$$

Note that in the inflationary regime the contribution from threshold corrections to the beta-functions can be neglected since for our choice their contribution is important only around the scale $\mu = 1/\xi$. The main point is that $\beta_\xi/\xi < 1$ is always perturbatively small and the denominator of F is always positive. The top contribution dominates and $\beta_\lambda < 0$ at the inflationary scale. This means that F can go through zero and become large and negative in the $\lambda \to 0$ limit. When

$$F = 0 \quad \Leftrightarrow \quad \lambda_{\max} = -\frac{\beta_\lambda}{4}, \tag{3.68}$$

to the lowest order in the δ-expansion the slow roll parameters vanish. As can be seen from (3.64) this corresponds to an extremum of the potential, and $\epsilon = 0$ at all orders. For SM Higgs inflation $\lambda_{\max} \sim 5 \times 10^{-5}$. For energies well below the Planck scale the quartic coupling $\lambda(t)$ is a monotonically decreasing function, and there is at most one extremum which is a maximum as[31]

$$\eta\big|_{\lambda=\lambda_{\max}} = -\frac{8}{3}\frac{\delta^2(1+\delta)^2(1+\frac{\beta'_\lambda}{4\beta_\lambda})}{(1+\frac{1+\delta}{6\xi})(1+\delta+\frac{\beta_\xi}{2\xi})^2} < 0. \tag{3.69}$$

[31] Close to the Planck scale there is the possibility of a second extremum, a minimum, in the potential. This opens the possibility for inflation near an inflection point. We comment on this in section 3.7.2.

If the potential develops a maximum in the inflationary regime, 60 e-folds of inflation may still occur if the potential near the maximum is flat enough. We will refer to this possibility as "hilltop inflation". To end up in the electroweak vacuum of the Higgs potential, this should happen for field values $\phi < \phi_{\rm max}$ where $\phi_{\rm max}$ is the field value at the maximum (this possibility thus constrains the initial field values); this corresponds to the region where $F > 0$ is positive. Note that the δ-expansion breaks down close to the maximum, when

$$F \approx \left(1 + \frac{1}{4}\frac{\beta_\lambda}{\lambda}\right) \sim \delta \left(1 + \frac{\beta'_\lambda}{4\beta_\lambda}\right), \tag{3.70}$$

and the first order term of η in (3.65) becomes comparable to the δ^2 term in (3.69) (we took the $\xi \gg 1$ limit).

Remember our notation that the subscript $\star$ denotes the value of the parameters when observable scales leave the horizon, $N_\star$ number of e-folds before the end of inflation. We distinguish three possibilities.

1. If $F_\star \gtrsim \delta_\star$ inflation takes place on a flat plateau, and there is no maximum.[32] This is the case for coupling values $\lambda_\star \gtrsim 5 \times 10^{-5}$.
2. If $F_\star \lesssim \delta_\star$ there is a maximum in the potential. If the maximum is flat enough, hilltop inflation takes place close to the maximum at field values $\phi < \phi_{\rm max}$. This is the case for coupling values $\lambda_\star \sim 5 \times 10^{-5}$.
3. The potential near the maximum is too steep to support $N_\star = 60$ e-folds of inflation.

In this section we will discuss case 1, inflation on the flat plateau. The discussion of case 2, hilltop inflation, is postponed till the next section.[33] The value $F_\star \sim \delta_\star$ divides the two regimes, as follows from (3.70); this is in agreement with our numerical results, which are presented in 3.7.2. The slow roll parameters (3.65) are affected by the running of the couplings, and corrections may become sizeable for small λ. However, to calculate the inflationary observables, the slow roll parameters are to be evaluated at the field value $\phi_\star$ at which the observable scales leave the horizon. This field value also gets corrected by the running, and as we will show now, these corrections exactly cancel, such that the inflationary predictions are to leading order in the δ-expansion not affected by the running of the couplings.

[32] Note that in the limit $\phi \to \infty$, $\mu^2 \sim 1/\xi$ approaches a constant, and the running comes to a halt. For $F_\star > \delta_\star$ the asymptotic value of $\lim_{\phi\to\infty} \lambda(t)$ exceeds the critical value (3.68).

[33] It may happen that threshold corrections kill Higgs inflation, in that the corrections to the RGEs bring the model from case 1 to case 3. The statement we want to make is that when inflation happens, the predictions are robust and insensitive to UV corrections (except for some possible fine-tuned parameters that allow for hilltop/inflection point inflation).

Let's thus compute the number of e-folds $N_\star$ before the end of inflation, which is given by

$$N_\star \simeq \int^{h_\star} \mathrm{d}h \frac{1}{\sqrt{2\epsilon}} \simeq \frac{\sqrt{3}}{|F_\star|\sqrt{8(1+\frac{1}{6\xi_\star})}} \int^{h_\star} \mathrm{d}h\, \delta^{-1} = \frac{1}{\delta_\star |F_\star|}\frac{3}{4}. \tag{3.71}$$

On the flat plateau $F > 0$ and we can drop the absolute signs. Here we assumed that F and ξ is to first approximation field-independent and we have taken it out of the integral. This gives the leading term in the δ-expansion, as we now quickly explain. The e-folds integral can be rewritten as follows

$$N_\star = \frac{3}{2}\int^{\phi^\star} \mathrm{d}\phi \, \frac{\xi}{|F|}\phi + O(\sqrt{\delta}) = \frac{3}{2}\int^{\phi_\star} \mathrm{d}\phi \, \phi \left(D_\star + \frac{dD}{d\phi}\Big|_{\phi=\phi_\star}(\phi-\phi_\star) + ..\right). \tag{3.72}$$

In the first step we used the field space metric to express the integral in terms of the (non-canonical) field ϕ (3.12), in the second step we defined $D(\beta_i, \lambda, \xi) \equiv \xi |F|^{-1}$ and expanded the integrand around $\phi_\star$. The first term in the expansion is the only one considered in (3.71). It gives a contribution of the form

$$\int^{\star} \mathrm{d}\phi \, \phi D_\star \propto \phi_\star^2 \propto O(\delta_\star^{-1}). \tag{3.73}$$

The second term in the expansion takes the form ($f_i \equiv \{\lambda, \xi, \beta_\lambda, \beta_\xi\}$),

$$\int^{\phi_\star} \mathrm{d}\phi \, \frac{dD}{d\phi}\bigg|_\star \phi(\phi-\phi_\star) = \int^\star \mathrm{d}\phi \, \phi(\phi-\phi_\star) \left(\frac{dD}{df_i}\frac{df_i}{dt}\frac{dt}{d\phi}\right)\bigg|_\star \propto \phi_\star^3 \delta_\star^{\frac{3}{2}} \propto O(\delta_\star^0), \tag{3.74}$$

where we used $dt/d\phi|_\star \propto \delta_\star^{\frac{3}{2}}$ from (3.63). It is higher order in the δ expansion and can be neglected. We also neglected the lower bound of the integral; this correction is likewise higher order in $1/N_\star \sim \delta_\star$. Using (3.71) from the COBE normalization we get [75]

$$\left(\frac{V}{\epsilon}\right)_\star = \frac{4}{3}\frac{\lambda}{\xi^2 N_\star^2} = (0.027)^4 \qquad \Rightarrow \qquad \frac{\xi(t_\star)}{\sqrt{\lambda(t_\star)}} = 5 \times 10^4. \tag{3.75}$$

Plugging (3.71) in the expressions (3.65) gives the spectral index and tensor-to-scalar ratio

$$n_s = 1 + 2\eta + O(\delta^2) = 1 - \frac{2}{N_\star} + O(\delta^2) \tag{3.76}$$

and

$$r = 16\epsilon = \frac{12}{N_\star^2}\left(1 + \frac{1}{6\xi}\right) + O(\delta^3) \tag{3.77}$$

The COBE normalization can always be fit by choosing the non-minimal coupling appropriately. All parameters in the model are then fixed. For the large non-minimal couplings needed ($\xi_\star > 10^2$), the spectral index and tensor-to-scalar ratio

only depend on the number of e-folds. All dependence on the beta-function has cancelled in the final expression, and the results are identical to those for classical Higgs inflation. This means that *the results for plateau inflation are very robust: they are independent from the running, and thus insensitive to UV physics that change the running, and also independent of the electroweak boundary conditions on the couplings.*

At next order in the δ expansion the beta-functions do enter, see appendix A, but this is too small an effect to be measured.

3.7.2 Numerical analysis and inflation near the maximum

As discussed previously, for $F_\star > \delta_\star$ inflation takes place on the flat part of the potential and the inflationary observables are insensitive to the running of the couplings, to first order they depend only on the number of e-folds. Here we discuss the case $F_\star < \delta_\star$. When $F = 0$ the potential develops a maximum. Requiring the Higgs field to end up in the electroweak vacuum, inflation should take place at field values before the maximum, where $F > 0$. We expect Hilltop inflation to be highly sensitive to the form of the potential and thus to the details of EW boundary conditions and to threshold corrections. Unfortunately, because of this sensitivity, it is hard to obtain analytical expressions, and we will only present numerical results. We point out that even though hilltop inflation is sensitive to the UV completion, it only occurs for very fine-tuned boundary conditions. Thus the numerical results presented in this section confirm our statement that the predictions for Higgs inflation are remarkably robust, and they verify the analytical result of section 3.7.1. For more details on our numerical implementation see section 3.6 and in particular 3.6.2.

Let's start by considering just the SM running, and turn off all threshold corrections. We can tune F small by adjusting the boundary conditions at the electroweak scale. We choose to decrease the Higgs mass, while keeping the top mass and gauge couplings fixed.[34] Our results are summarized in Table 3.1. They agree with the discussion above. For large enough Higgs mass values, inflation takes place on the flat plateau, and n_s and r are independent on the running. In some fine-tuned range of Higgs mass values, inflation can happen near a maximum; in this case the inflationary results depend sensitively on the EW boundary conditions. For an even smaller Higgs mass the maximum is too steep and 60 e-folds of inflation is not possible. In Fig. 3.4 we show an example potential for inflation on

[34] We choose $m_t = 171\text{GeV}$ which is about 2σ below its central value, to avoid that the quartic coupling becomes negative before inflation.

the plateau and for inflation near the maximum,[35] the parameters corresponding to the first and last line of Table 3.1. Our numerical results agree with similar studies in the literature [98, 99].

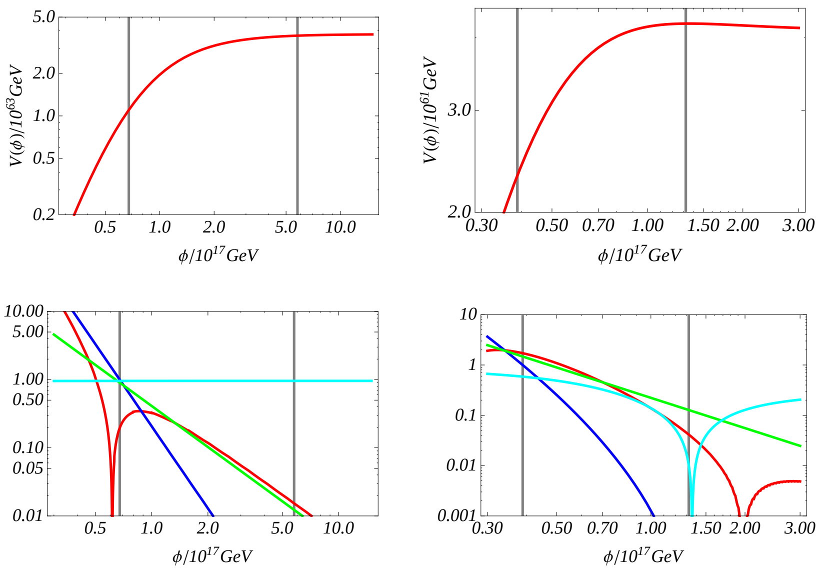


Figure 3.4: *Top left the potential $V(\phi)$ for inflation with $m_t = 171\,GeV$ and $m_h =$ 125.5 GeV; the grey vertical lines correspond to $\phi_\star$ and $\phi_{\rm end}$ respectively (i.e. the beginning and end of inflation). Bottom left shows $|\eta|$ (red), ϵ (blue), δ (green) and F (cyan) respectively for the same top mass. Right top and bottom, same plots but for $m_t = 171$ GeV and $m_h = 125.245$ GeV.*

Now turn on the threshold corrections. We choose $m_t = 171$ GeV, $m_h = 125.5$ GeV and did 500 simulations with Wilson coefficients randomly chosen between c_i = Random$[-10, 10]$. We found 382 times that inflation takes place on the flat plateau, and the other 118 times there was no inflationary solution. Hilltop inflation does not happen. The spread in spectral index, tensor-to-scalar ratio, ξ_0 and kick $\Delta\lambda$ for the successful models are shown in Fig. 3.5. The kick in λ is defined with respect to the reference set-up without threshold corrections, corresponding to the highlighted line in Table 3.1. Define $(\lambda_\star^{\rm SM}, t_\star^{\rm SM}, \xi_0^{\rm SM}) =$

[35] For these plots we numerically inverted $t(\phi)$. This inversion is not needed to calculate n_s and r, which is done with t as the clock variable.

m_h(GeV)	$h_\star$	$\lambda_\star$	$\xi_\star$	$F_\star$	$\delta_\star$	n_s	r
127	0.15	6.3×10^{-3}	3863	0.99	0.01	0.968	3.0×10^{-3}
126	0.18	2.7×10^{-3}	2505	0.98	0.01	0.968	3.0×10^{-3}
125.5	0.24	9.0×10^{-4}	1451	0.96	0.01	0.968	3.0×10^{-3}
125.3	0.33	1.9×10^{-4}	667	0.84	0.01	0.968	2.9×10^{-3}
125.26	0.34	4.2×10^{-5}	344	0.42	0.03	0.970	2.4×10^{-3}
125.255	0.20	3.1×10^{-5}	451	0.12	0.06	0.968	9.5×10^{-4}
125.253	0.13	3.3×10^{-5}	730	0.05	0.08	0.958	3.7×10^{-4}
125.25	0.09	3.7×10^{-5}	1314	0.07	0.12	0.941	1.2×10^{-4}
125.245	0.05	4.3×10^{-5}	2678	0.01	0.12	0.917	3.4×10^{-5}

Table 3.1: *Inflationary parameters for different Higgs mass while* $m_t = 171$GeV *is kept fixed, in the absence of threshold corrections. Above the double line the potential has a flat plateau, below the line the potential develops a maximum. For* $m_h < 125.245$ *no inflationary solutions with* $N_\star = 60$ *e-folds exists.*

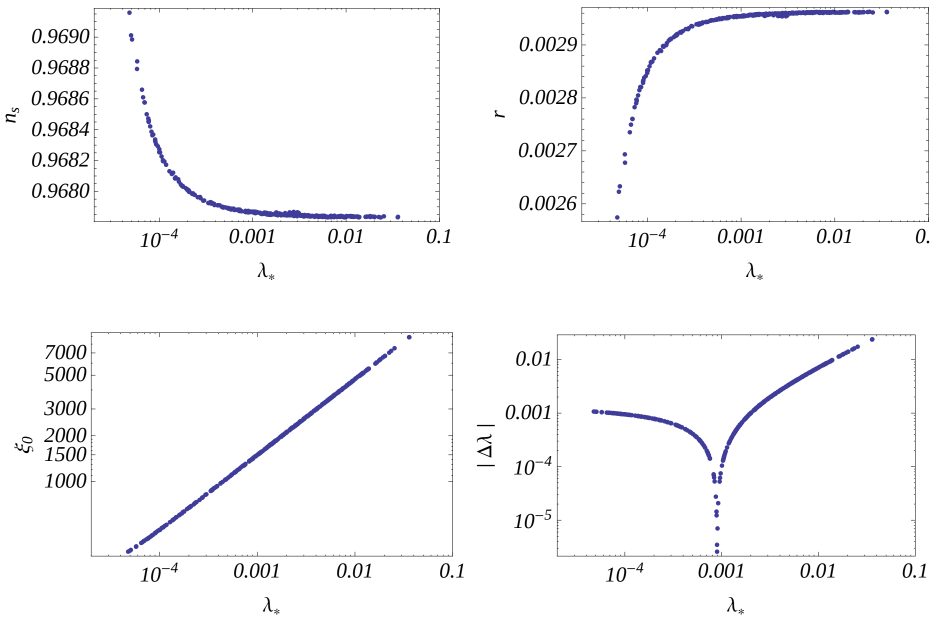


Figure 3.5: *The spread in spectral index* n_s*, tensor-to-scalar ratio* r*,* ξ_0 *and* $\Delta\lambda$ *as a function of* $\lambda_\star$ *for the 382 succesfull models with threshold corrections* $c_i =$ Random$[-10, 10]$.

$(9.0 \times 10^{-4}, 33.55, 1417)$ for this model, with $t = \ln(\mu/m_t)$ and ξ_0 the boundary condition $\xi(1/\xi) = \xi_0$. We then define the kick in λ for the models with threshold corrections as

$$\Delta\lambda = \lambda(t_\star^{\rm SM}) - \lambda_\star^{\rm SM}, \quad \text{for } \xi_0 = \xi_0^{\rm SM}. \tag{3.78}$$

For our run of 500 simulations, the average kick is upwards $\langle \Delta\lambda \rangle = 1.5 \times 10^{-3}$ with standard deviation $\sigma = 3.9 \times 10^{-3}$; the average absolute kick size is $\langle |\Delta\lambda| \rangle = 2.3 \times 10^{-3}$. If the kick is upwards, or downwards but not so large, plateau inflation is still possible. The value ξ_0, which is a free parameter, has to be adjusted with respect to the reference model, to fit the power spectrum (3.75). However for large kicks downwards this is no longer possible, and the potential is too steep for all ξ-values. The critical kick dividing the successful models from the unsuccessful ones is

$$\Delta\lambda^{\rm crit} = -1.1 \times 10^{-3} \quad \Rightarrow \quad \lambda_\star^{\rm SM} + \Delta\lambda^{\rm crit} = -1.7 \times 10^{-4}. \tag{3.79}$$

As mentioned, we do not find any examples of hilltop inflation in our 500 simulations with threshold corrections, in contrast to the pure SM running. One can either generate a kick, with respect to the reference model, by changing the boundary conditions at the electroweak scale (e.g. changing the EW top/Higgs mass) or by turning on threshold corrections. Adding thus a kick to the reference model, the CMB power spectrum constraint is no longer satisfied; we retune ξ_0 to fit the CMB data. Fig. 3.6 shows the result, it plots ξ_0 for downwards kicks $\Delta\lambda < 0$. The reference model is again $m_t = 171$ GeV, $m_h = 125.5$ GeV, that is the highlighted line in Table 3.1. In this plot the green line correspond to pure SM running and different values of the EW top mass, the red line for SM running and different values of the EW Higgs mass (corresponding to the results in Table 3.1), and the blue line for fixed top and Higgs mass but a kick generated by threshold corrections. Inflation near the maximum only happens in the first two cases for the small kick interval where ξ_0 increases again (i.e. where the red and blue line increase). It matters whether the kick is produced by EW boundary conditions or by threshold corrections. In the 2nd case, inflation is possible for larger kick values. This can be understood as follows. For SM running without threshold corrections, changing ξ_0 mainly affects the size of the power spectrum, but it has only a small effect on the running. In contrast, for the set-up with threshold corrections, changing ξ_0 will both affect the power spectrum and the running. Indeed, $\mu \sim 1/\xi_0$ is the scale where the kick is produced. For a smaller ξ_0 this happens at a higher scale, where the value of $\lambda(\mu)$ is smaller and since the size of the kick is proportional to $\lambda(\mu)$, this results in a smaller kick. Hilltop inflation is only possible if the curvature near the maximum is tuned small. This depends on the details of the potential. It is no surprise that this gives slightly different results for SM running, and SM including

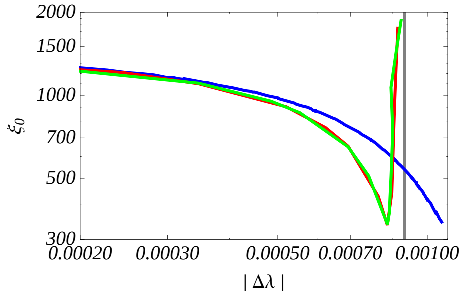


Figure 3.6: *The boundary value ξ_0 for the non-minimal coupling vs. the kick $|\Delta\lambda|$ for SM running (green/red) and SM running with threshold corrections (blue). In the former case the kick is from changing the top/Higgs mass at the electroweak scale, whereas in the latter it is due to the threshold corrections. The kick is downwards $\Delta\lambda < 0$. The vertical grey line corresponds to the value $\lambda_*^{\rm SM} + \Delta\lambda = 0$.*

threshold corrections, even for a similarly sized kick. No matter what the exact form of the threshold corrections is, if the kick is not too large ($\Delta\lambda + \lambda_*^{\rm SM} > 0$) inflation takes place on the flat plateau with universal predictions for the observables. The larger $\lambda_*^{\rm SM}$ is without threshold corrections, the larger kick is needed to disrupt inflation, which is only possible for large Wilson coefficients of the non-renormalizable operators. Consider for example the first line in Table 3.1, with $\lambda_*^{\rm SM} = 6.3 \times 10^{-3}$. Also for this case we did 500 simulations with random Wilson coefficients, choosing $c^{\rm max} = 20$ with $c_i = {\rm random}[-c^{\rm max}, c^{\rm max}]$. We found that 61 out of 500 times the downwards kick was large, and inflation no longer possible; we found no examples of hilltop inflation.

There might be additional sources of threshold corrections. If they only affect the potential via modifications of the running, our results apply: (except from some possible fine-tuned cases near a maximum) inflation takes place on the plateau and the observables have universal values. Our choice of higher order terms in the Lagrangian (3.28) with $c^{\rm max} \sim 10$ can be viewed as a (specific) parameterization of the kick in λ due to all possible threshold corrections.

Finally we would like to comment on the possibility of Higgs inflation near an inflection point, as has been discussed in the literature [120, 136]. Close to the Planck scale the potential may develop a second minimum. For fine-tuned boundary conditions the maximum and minimum merge into an inflection point with $V_h = V_{hh} = 0$, where inflation can take place.[36] Such solutions only exist for relatively small non-minimal coupling $\xi = O(10)$; this is because the renormalization

[36] This scenario has been recently questioned in [137].

scale is bounded $\mu < 1/\sqrt{\xi}$, and only for small ξ large enough scales can be reached where the Landau pole becomes important. Because of the large scales involved, inflection point inflation can give rise to a large gravitational wave signal (these models were motivated by the BICEP results). In our numerics we did not search for this possibility, and it is not included in our results.

3.8 Summary

In Higgs inflation the unitarity cutoff, signaling the breakdown of the effective theory, is well below the Planck scale and introducing an UV completion is demanded by the consistency of the theory. This breaks the connection between the parameters in the two asymptotic regimes and raises the question how sensitive the CMB predictions are to the UV completion. In this chapter we have shown that as long as the UV corrections do not affect the inflaton potential at tree level but only enter at loop level via corrections to the renormalization group equations, the inflationary predictions are (almost) unaffected. Indeed, as we proved analytically in section 3.7.1, to leading order in the slow roll expansion all dependence on the running cancels, and thus the predictions are insensitive to threshold corrections. The spectral index and tensor-to-scalar ratio are exactly the same as for the classical, tree level potential, which is in excellent agreement with data.

The inflationary predictions are universal if inflation takes place on the flat plateau of the potential. However, it may happen that due to the running of the couplings the potential develops a maximum. Inflation near the maximum will depend on the details of the RGE evolution and thus on the UV completion. The perturbative expansion used in section 3.7.1 does not capture this case, and we used a numerical analysis to also study the possibility of hilltop inflation, where we parameterized the threshold corrections by a specific set of higher order operators in the Lagrangian (3.28). Our numerical analysis confirms our analytical results for inflation on the flat plateau of the potential. We further found that hilltop inflation is a possibility, but it only happens for very fine-tuned boundary conditions (the top/Higgs mass at the electroweak scale, and the Wilson coefficients of the non-renormalizable operators). Indeed for our run with randomly chosen Wilson coefficients (taken large enough, such that the effect on the running is appreciable — see section 3.7.2 for more details) we did not find a single instance of hilltop inflation. We conclude that, apart from the very fine-tuned case of inflation near the maximum, if inflation happens, the predictions are the same as those derived from the classical potential (3.2).

A previous study of threshold correction to HI has been done in [28]. They con-

cluded that Higgs inflation is extremely sensitive to the UV completion, which was modeled by the same set of higher order operators (3.28) [123]. We expect that the difference is mainly due to the choice of the renormalization prescription. While [28] allows for both prescription I and II in their numerical analysis, we showed analytically (and confirmed numerically) that for our choice of μ, which has been discussed extensively in (3.6.1), such dependence does not arise in general. As argued in (3.6.1), prescription I is the only consistent renormalization scale parametrization to study the RG improved potential in the Einstein frame. Furthermore, there are some slight differences in the numerical implementation, for example the set of RGEs for the inflationary regime, and the parameterization of the unitarity cutoff. However, the main conclusion that inflation on the flat plateau of the potential is insensitive to UV physics, does not depend on these choices.

As mentioned above, we do find deviations from the universal predictions if inflation occurs near a maximum of the potential. We studied numerically the fine-tuned parameter space for hilltop inflation, which depends sensitively on the boundary conditions as well as on the UV completion — and thus also on the specifics of the numerical implementation. For standard model inflation, without threshold corrections, our numerical results agree with earlier work [98, 99]. Our study in [1] was later implemented (and confirmed where the analyses overlap) by other groups [122, 138, 139, 140]. In particular, the authors in [122, 138] study in this context the critical regime where inflation takes place near an inflection point, while [139, 140] analyses the loop corrections in the Palatini formulation and the hilltop scenario in this case.

We conclude with a small remark. It is well known that for the central values of the electroweak scale top and Higgs mass the Higgs potential becomes unstable at $\phi \sim 10^{11}$ GeV [77, 76, 78, 79, 80, 81, 82], well before the potential flattens in Higgs inflation. The top/Higgs mass values separating a stable from an unstable Standard Model Higgs potential are close to those separating Higgs inflation from models where inflation is not possible. There are small differences with respect to SM running (without a non-minimal coupling), because 1) we include threshold effects, 2) we run until the inflationary scale and not the Planck scale, and 3) the RGE equations get modified in the mid and large field regime. The measured Higgs and top masses [141] are surprisingly close to the border separating the regions where the Higgs boson can or cannot be the inflaton.

4 Renormalization group independence of Cosmological Attractors

In this chapter we study the effect of perturbative corrections due to the renormalization group flow for generic classes of inflationary models. The motivation comes from what we just have learned by studying Higgs inflation. The RGEs for inflation models with non-renormalizable interactions can be sensitive to the UV completion of the theory.

In particular, we will investigate this effect for the large class of models known as α- and ξ-attractors. As the authors in [142] have shown, due to their underlying mathematical structure, these models all give identical predictions at tree level for the inflationary parameters.[1] A natural question to ask then is if these predictions are robust in the full quantum theory. Are the attractor models consistent and predictive at the quantum level? Can this degeneracy be lifted?

The chapter is based on my work in [2]. The outline is as follows. We first summarize the key idea in the next section, keeping the discussion fully general. Then we show the details and its actual implementation in sec. 4.2. In sec. 4.3 we apply it to the class of Cosmological Attractor models of inflation. As we will discuss in 4.3.4, a simple and rather natural condition on the renormalization scale allows for a model independent analysis of the quantum corrections to inflation. In sec. 4.4 we summarize the results.

[1] More precisely, for $\alpha \to 1$ and $\xi \to \infty$ in the α- and ξ-attractors respectively, we have $n_s = 1 - 2/N + O(N^{-2})$ and $r = 12/N^2 + O(N^{-3})$.

4.1 Renormalization group sensitivity in single-field inflationary models

In single-field inflation, the quantum corrected dynamics of the inflaton is given by an effective action of the form

$$\frac{\mathcal{L}_{\text{eff}}}{\sqrt{-g}} = -\tfrac{1}{2}Z(\phi)K_\phi(\phi)(\partial\phi)^2 - V(\phi) + ... \tag{4.1}$$

with $Z(\phi)$ the (non-trivial) renormalization wavefunction, K_ϕ the metric in field space in presence of a non-canonical kinetic term, V the full quantum potential and the dots stand for higher derivative terms that can be safely neglected in the slow roll approximation. With the gravity sector in the standard form, i.e. for the action in the Einstein frame, the slow roll parameters as well as the inflationary indexes n_s and r are given in terms of derivatives of the effective potential with respect to the canonical inflaton field. Using standard renormalization group techniques it is possible to rewrite the effective action in a form suitable for the inflationary analysis, which takes into account the leading log expansion of the quantum potential.

We are interested in the possibility that quantum corrections enter at first order in the $1/N$-expansion. The observables can then be written in the general form

$$n_s \simeq 1 - \frac{2}{N} f_n(\beta_{\lambda_i}, \lambda_j)_\star\,, \quad r \simeq \frac{12}{N^2} f_r(\beta_{\lambda_i}, \lambda_j)_\star, \tag{4.2}$$

evaluated at the time the pivot scale ($k_\star = 0.002\,\mathrm{Mpc}^{-1}$) leaves the horizon. Here f_n and f_r are two generic functions of the beta-functions β_i and the couplings of the model λ_i. If the inflationary parameters have such a dependence,[2] this would imply that the knowledge of the details of the renormalization group (RG) flow during inflation are needed (to find out the expressions for β_{λ_i}) to draw conclusions on the model. Even more, to ever connect the low and high energy regimes of the model, one would need to know the details of the RG flow through the entire energy domain.

4.1.1 RG improving and renormalization scale

As discussed extensively in sec. 2.2, the effective potential can be rewritten as a series of leading logs. A well known result in quantum field theory [45, 46] tells us

[2] As noted before, even though β_{λ_i} might be small in general, this is not necessarily true for combinations of $(\beta_{\lambda_i}, \lambda_j)$, e.g. $\beta_{\lambda_i}/\lambda_i$.

that in each region of the field space it is possible to define an effective field theory (EFT) where only one logarithm remains relevant in the full effective action. All the other mass scales decouple and their net effect will be a shift in the definition of the parameters of the EFT. We labeled the only relevant log in the EFT as

$$s = \hbar \ln\left(\frac{M^2(\phi)}{\mu^2}\right) \tag{4.3}$$

$\hbar$ is the loop counting parameter. V satisfies in each EFT the renormalization group equation (RGE) [42, 43]:

$$\mathcal{D}V \equiv \left(\mu\frac{\partial}{\partial\mu} + \beta_{\lambda_i}\frac{\partial}{\partial\lambda_i} - \gamma\phi\frac{\partial}{\partial\phi}\right)V = 0, \tag{4.4}$$

where γ is the anomalous dimension of the scalar field. This allows us to take into account the leading log expansion of the effective potential (for more details see 2.2).

Compared to the standard renormalizable case, given a generic inflationary model renormalizable in the EFT sense in the inflationary regime,[3] the operators $\mathcal{D}$ are not necessarily the same in each EFT (defined at different energy scales).When we formally solve the RG equation in the inflationary regime, only one log remains relevant. The characteristic methods shown in 2.2 tells us that the effective potential is determined once its functional form is known for a certain value of t. The standard procedure to derive useful information from the formal solution (2.39), is to choose t in such a way that

$$s(\tilde{t}) = \ln\frac{M^2(\tilde{t})}{\mu^2(\tilde{t})} = 0 \implies \tilde{t}(\phi, \lambda_i(\tilde{t})) \tag{4.5}$$

where the t dependent quantities are the quantities on the characteristic curve (2.40),(2.41). We will consider the leading corrections, that is equivalent to use the RG improved tree level action and the 1-loop beta-functions.[4] Even if our results will not depend on the loop order of the beta-functions considered, this does not imply that it holds automatically beyond the leading order. In section 4.2.4 we comment on the generalization of our results to higher orders.

Let us make an important remark here. In the following (we used this already in (4.5)) we will consider ϕ instead of its t dependent version $\phi(t)$ in the RG improved potential (and ρ instead of $\rho(t)$ in the next sections). We are allowed to do this

[3] Remember that by this we simply mean that in each field region it is possible to define a small parameter; there should be a finite number of counterterms at every order in the expansion in this small parameter.

[4] The knowledge of the L loop potential (and the function $\tilde{t}$) provides an exact RG improved potential up to order L in this leading log expansion.

for the reason extensively discussed in sec. 2.3, but basically we can absorb the wavefunction in a field redefinition via $Z_{\text{eff}}(t)(\partial\phi)^2 = (\partial\phi_{\text{c}})^2$. Then we have, at leading order

$$\phi(t) = e^{-\int \gamma dt'}\phi = e^{-\int \gamma dt'} Z_{\text{eff}}^{-1/2}\phi_{\text{c}} \approx \phi_{\text{c}}, \tag{4.6}$$

where we simply omit the subscript "c".

4.1.2 Key idea

The key point is that since only one log remains relevant during inflation there is (up to some irrelevant numerical factors) a unique choice for the function $\tilde{t}$, i.e. the one implicitly defined by eq. (4.5). In order to compute the inflationary parameters n_s and r we take derivatives of the effective potential with respect to the scalar field. These will be a function of derivatives of $\tilde{t}$ as well as of the couplings and the beta-functions. Thus the predictions can in principle depend on the RG flow during inflation (through the value of the beta-functions in this regime) and on the full RG flow (through the value of the running couplings at the horizon). Expanding the equations in powers of the small parameter ρ defining the inflationary regime the RG flow dependence can be discussed analytically, without having to solve explicitly the RG equations.

In order to work in a model independent approach we do not specify how the inflaton couples to other fields. As already discussed, this information is usually needed to determine the field dependence of the renormalization scale $\tilde{t}$, fixed by demanding the leading log correction to vanish. Indeed, our results follow by simply asking the renormalization scale to satisfy a quite generic condition. This turns out to be ensured by requesting the theory to be renormalizable in the EFT sense in the small/large regime. Thus, this requirement is enough to draw our main conclusion: for all the Cosmological Attractors the inflationary parameters (n_s, r) are (nearly) independent on the Renormalization group flow. This also implies that any kind of UV physics whose effect enters only via the RG flow will in general have no effect on the predictions for these models.[5]

[5] The scalar power spectrum constrains one combination of couplings in the theory, which can always be satisfied fixing the free parameter of the model. For different RG evolutions, the individual couplings may have different values at horizon exit, but as long as the combination is kept fixed by adjusting the free parameter, this has no direct observable consequences.

4.2 Inflationary parameters

4.2.1 General set up: tree level

Let us start by reviewing the predictions for the Cosmological Attractors at tree level [142, 143, 144, 145, 146]. The Lagrangian of the models considered can be written as (with the Planck mass set to one)

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}K(\rho)(\partial\rho)^2 - V(\rho)\right], \tag{4.7}$$

with

$$K = \frac{a_p}{\rho^p} + \frac{a_{p-1}}{\rho^{p-1}} + ..., \quad V = V_0(1 + c\rho + c_2\rho^2 + ..) \tag{4.8}$$

where $\rho \ll 1$ is the parameter identifying the inflationary regime.[6] V is the tree level potential in the Einstein frame, while $V_0 = V|_{\rho=0}$ is the coupling dependent part of it. a_p are generic coefficients that might depend on the couplings.

To first approximation the following happens [142]: the slow roll parameter η, and consequently the spectral index, is completely determined by the order of the leading pole in the kinetic term (p); for $p = 2$ the tensor-to-scalar ratio r will depend only on the residue of this leading pole.[7] We will now show this explicitly.

The first and second slow roll parameters are

$$\epsilon = \frac{1}{2}\left(\frac{V_\rho}{V}\right)^2 K^{-1} = \frac{1}{2}\left(\frac{V_\rho}{V}\right)^2 \frac{\rho^p}{a_p}, \tag{4.9}$$

and

$$\eta = \frac{V_{\chi\chi}}{V} = -\frac{V_\rho}{V}\frac{d^2\chi}{d\rho^2}\left(\frac{d\chi}{d\rho}\right)^{-3} + \frac{V_{\rho\rho}}{V}\left(\frac{d\chi}{d\rho}\right)^{-2} = \frac{p}{2}\frac{\rho^{p-1}}{a_p}c + O(\rho^p). \tag{4.10}$$

Here χ labels the canonical field defined via $d\chi/d\rho = K^{\frac{1}{2}}$. Further we introduced the notation $V_\rho = dV/d\rho$, and likewise for higher derivatives. From the previous expression we see that in all these models $\eta \gg \epsilon$ (one order in ρ difference). The number of e-folds is given by

$$N \simeq \int^{\rho_\star}\left(\frac{V}{V_\rho}\right) K d\rho \simeq \frac{a_p}{c\rho_\star^{p-1}(1-p)}, \tag{4.11}$$

[6] $\rho \to 1$ towards the end of inflation. Thus ρ is defined in order for c to be negative in (4.8).

[7] This approach is robust under perturbation of the non-canonical kinetic term K with terms of one order higher in the leading pole, i.e. $K \subset a_{p+1}/\rho^{p+1}$ [147].

which implies

$$\rho_\star = \left[\frac{Nc(1-p)}{a_p}\right]^{\frac{1}{1-p}} ..11 \tag{4.12}$$

Evaluating the slow roll parameters at horizon exit then gives

$$\begin{aligned}\eta_\star &\simeq \frac{p}{2(1-p)}\frac{1}{N}\,,\\ \epsilon_\star &\simeq \frac{1}{2}c^{\frac{p-2}{p-1}}a_p^{\frac{1}{p-1}}\left(\frac{1}{(1-p)N}\right)^{\frac{p}{p-1}}\,.\end{aligned} \tag{4.13}$$

For $p = 2$ all dependence on the potential drops from the inflationary parameters, which become

$$n_s \simeq 1 - \frac{2}{N}\,, \qquad r \simeq \frac{8a_2}{N^2}, \tag{4.14}$$

where a_2 is the coefficient of the pole of order two in the expansion (4.8).

4.2.2 General set up: quantum corrections

As we discussed in section 4.1.1, even if the inflaton field has a non-canonical kinetic term, the net effect of considering leading order quantum corrections is captured by substituting in the tree level action each coupling by its running counterpart, i.e. $\lambda_i \to \lambda_i(t)$, modulo a proper choice of the RG time $t = \tilde{t}$ (the one solving (4.5)). Thus, we consider the RG improved version of (4.7), given by[8]

$$\frac{\mathcal{L}_{\rm eff}}{\sqrt{-g}} \simeq -\tfrac{1}{2}K(\rho, \lambda_i(\tilde{t}(\rho)))(\partial\rho)^2 - V(\rho, \lambda_i(\tilde{t}(\rho)). \tag{4.15}$$

It is worth making a remark here. The previous action captures the leading quantum correction as long as the effective action satisfies a Callan-Symanzik equation as (4.4), i.e. the theory is perturbatively renormalizable during inflation. For the case of interest ($p = 2$) this can be seen in two ways. In terms of the canonical field, $K^{\frac{1}{2}}\partial\rho = \partial\chi$, $\rho \propto e^{-\chi/M}$, the action has an approximate shift symmetry, i.e. for $\chi \to \infty$ the action is invariant under $\chi \mapsto \chi+$const. This implies that all the divergences are proportional to $e^{-\chi/M}$ and the counterterms will organize into a series of the same form as the tree level potential $V = V_0\sum_{n=1}^{\infty} c_n\rho^n = V_0\sum_{n=1}^{\infty} c_n e^{-n\chi/M}$. Thus we have at every order a finite number of counterterms. In terms of the non-canonical field ρ the approximate shift symmetry turns into a scale symmetry of the action, $\rho \mapsto$ (const)ρ, in the limit of ρ going to zero. In terms of the non-canonical field ρ this symmetry prevents that loop corrections generate higher inverse powers of ρ in the non-canonical kinetic term.

[8] In computing the effective action one usually expands around a constant background $\rho = \bar{\rho}$ where the tree level action is analytic. Even if we are in the limit of small ρ we consider an expansion around a small but finite $\bar{\rho}$ where the action is analytic.

Let us now compute the effect of the quantum corrections on the inflationary parameters. For a potential of the form (4.8), the dependence on the couplings is in V_0. The derivative of the potential with respect to ρ, denoted by V_ρ, then becomes[9]

$$V_\rho = V_0 \sum_{n=1}^{\infty} n c_n \rho^{n-1} + \frac{\beta_{V_0}}{V_0} \frac{d\tilde{t}}{d\rho} V \tag{4.16}$$

where $c_1 \equiv c$ (to match the tree level notation (4.8)). At leading order we have

$$\frac{V_\rho}{V} \simeq c + \frac{\beta_{V_0}}{V_0} \frac{d\tilde{t}}{d\rho}. \tag{4.17}$$

Now a key point of the argument kicks in. On the weak assumption that $\tilde{t}$ can be simply expanded in a Taylor series about zero,

$$\frac{d\tilde{t}}{d\rho} = \sum_{k=0}^{\infty} d_k \rho^k \ , \tag{4.18}$$

where the coefficients d_k can depend implicitly on ρ, we have

$$\frac{V_\rho}{V} = c + \frac{\beta_{V_0}}{V_0} d_0 + O(\rho). \tag{4.19}$$

Thus, the effect of the RG flow will be only a rescaling of the factor

$$c \to \mathcal{C} \equiv c \left(1 + \frac{\beta_{V_0}}{V_0} \frac{d_0}{c} \right). \tag{4.20}$$

As we show now, this will be the only relevant effect which has no consequences for the inflationary parameters.

Let us compute the number of e-folds The leading term in the integrand is the same as (4.11) with the replacement (4.20). On the other hand, the factor $a_2/\mathcal{C} \equiv D$ is not a constant anymore and it depends implicitly on ρ. Expanding it in a Taylor series about $\rho_\star$ gives

$$N \simeq \int^{\rho_\star} \frac{d\rho}{\rho^2} \frac{a_2}{\mathcal{C}} = \int_{\rho_\star} \frac{d\rho}{\rho^2} \left(D_\star + \beta_{D\star} \frac{d\tilde{t}}{d\rho} (\rho - \rho_\star) + .. \right). \tag{4.21}$$

Given the assumption (4.18), we observe that D can be considered constant over the integration domain within our approximation, i.e. $D \simeq D_\star$. In fact, all the other terms, starting from the second one within the brackets, give contributions that are at most of order $\sim \ln \rho_\star$, which gives an order higher in the $1/N$ expansion.

[9] Note that for the set-ups considered, even if the coefficients c_i had a dependence on the couplings that would only give contributions to the derivative that are higher order in ρ, of the form $\sim V_0 \left(\rho \frac{dc}{dt} \frac{d\tilde{t}}{d\rho} + .. \right)$.

These contributions enter at the same order as the corrections from the subleading poles in the kinetic term, which were already neglected at tree level. The number of e-folds then becomes[10]

$$N \simeq -\frac{a_{p\star}}{\mathcal{C}_\star \rho_\star}, \tag{4.22}$$

which is simply (4.11) with $p = 2$ and the couplings (which now are not constant anymore) evaluated at the horizon crossing $\rho_\star$. Therefore $\epsilon_\star$ will be exactly the same as the tree level expression, but with c replaced by $\mathcal{C}_\star$ (whose dependence drops out for $p = 2$). From (4.22) we note that the quantum corrections, encoded in $\mathcal{C}_\star$, generically will not prevent this model from generating a large enough number of e-folds. In fact $\mathcal{C}_\star$ can be in principle arbitrarily large, this will simply imply a shift of $\rho_\star$ towards smaller values in order to get the desired number of e-folds. In computing η, one has to be a little more careful since an extra contribution could come from the derivative of the non-canonical kinetic term in (4.15). In fact, in (4.10), we should consider

$$\frac{dK^{\frac{1}{2}}}{d\rho} \equiv \frac{d^2\chi}{d\rho^2} = \frac{1}{2}\rho^{\frac{p}{2}} a_p^{\frac{1}{2}} \left(-p\rho^{-p-1} + \frac{\beta_{a_p}}{a_p}\frac{d\tilde{t}}{d\rho}\rho^{-p} \right) \Big|_{p=2}. \tag{4.23}$$

However, as long as (4.18) is satisfied, the second term in this expression gives higher order contributions to η. Thus (4.10) becomes

$$\eta = -\frac{V_\rho}{V}\frac{d^2\chi}{d\rho^2}\left(\frac{d\chi}{d\rho}\right)^{-3} + O(\rho^2) = \rho\frac{\mathcal{C}}{a_2} + O(\rho^2), \tag{4.24}$$

which is the tree level expression at leading order with again $\mathcal{C}$ playing the role of c. It is then obvious that inverting (4.22) and substituting $\rho_\star$ in the the slow roll parameters gives the same (4.14) for n_s and r,

$$n_s \simeq 1 - \frac{2}{N} \ , \quad r \simeq \frac{8a_{2\star}}{N^2}. \tag{4.25}$$

Summarizing, the effect of the RG flow enters in three ways. First, in the e-folds dependence of $\rho_\star = \rho_\star(N)$. Second, by giving a rescaling $c \to \mathcal{C}$ in the slow roll parameters (before evaluating them at the horizon crossing) and third from the extra contribution to the derivative of K in η. Nevertheless, if the condition (4.18) is satisfied, this latter gives simply higher order contributions, while the first and second points compensate each other. Different running histories (encoded in $(\beta_{V_0}/V_0)_\star$ in $\mathcal{C}_\star$) will just imply a different value of the field at the horizon exit $\rho_\star$. This effect cancels with the shifted expressions of the slow roll parameters.[11]

[10] We will comment on the possibility of quantum corrections flipping the sing of $\mathcal{C}_\star$ in the next section.

[11] Suppose that (4.18) is not satisfied, for example $\tilde{t} \propto \frac{k}{\rho}$. In the number of folds the second term in (4.21) will now give $\int \frac{d\rho}{\rho^2}\beta_{D\star}\frac{d\tilde{t}}{d\rho}(\rho - \rho_\star) = \frac{k}{\rho_\star}\beta_{D\star} + \text{h.o.}$, which is of the same order as the leading term $D_\star/\rho_\star$, and thus (4.22) and the relation $\rho_\star(N)$ is altered. As a consequence, the leading order slow roll parameters at horizon exit will depend on the beta-functions.

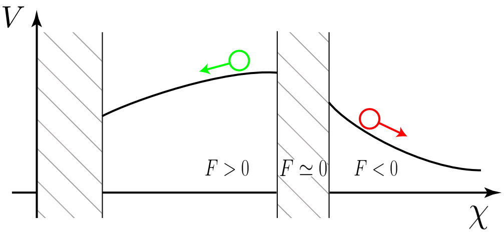


Figure 4.1: *Possible features of the effective potential to RG effects.*

Note that the arguments presented are valid as long as the quantum corrections encoded in $\mathcal{C}$ do not break the perturbative expansion in ρ. In general $|\mathcal{C}| \sim O(1)$ and (4.25) follows. Consider the term between brackets in (4.20), denoted by F, i.e.

$$\mathcal{C} = c\left(1 + \frac{d_0}{c}\frac{\beta_{V_0}}{V_0}\right) \equiv cF, \tag{4.26}$$

if F is positive during inflation, then $\mathrm{sign}[\mathcal{C}_\star] < 0$ and the predictions will be the same as for the tree level case.

4.2.3 Maximum and breakdown of perturbativity

The conclusion that the tree level predictions are not affected by the RG corrections is valid as long as the perturbative expansion in ρ holds. It may be that the potential develops an extremum because of the running; this is purely a quantum effect in that the tree level potential has no extremum in the inflationary regime. For fine-tuned parameters it is then possible to obtain inflation near the maximum or inflection point. In this case the details of the inflationary scenario will depend sensitively on the quantum corrections.

To see the appearance of an extremum in the potential, consider its slope. For $|F| \gtrsim \rho$ the perturbative expansion is valid and $V_\rho = V_0 cF + \mathcal{O}(\rho)$, see (4.19). It follows that going from a region in field space with $F \gtrsim \rho$ to a region with $F \lesssim -\rho$, the slope of the potential has changed sign. This can only happen if there is a (at least one) maximum in between. In general, one cannot calculate the location of the maximum analytically though, as the perturbative expansion breaks down exactly in the in between region where $F \sim 0$.

For the particular choice of normalization scale $\mu(\tilde{t}) \propto V^{\frac{1}{4}}$ the slope of the potential factorizes in a classical piece times a quantum correction at all orders, see (4.29) below. Then it can actually be shown analytically that the regime where perturbativity breaks down coincides with the development of a maximum in the

effective potential. This choice of normalization scale is appropriate for Higgs inflation and it also appears generically in the α and ξ-attractor models considered. It can be parametrized $\mu(\tilde{t}) = (V_0\mu_0)^{1/4}(1 + c\rho + ..)^{1/4}$, where V_0 and μ_0 depend explicitly on the couplings but do not explicitly depend on the field ρ. It is not hard to see that this satisfies (4.18). The RG time is then

$$\tilde{t} = \ln \mu(\tilde{t}) = \frac{1}{4} \ln (\mu_0 V) \,. \tag{4.27}$$

Taking the derivative with respect to ρ gives

$$\frac{d\tilde{t}}{d\rho} = \frac{1}{4}\left(\frac{V_\rho}{V} + \frac{\beta_{\mu_0}}{\mu_0}\frac{d\tilde{t}}{d\rho}\right) \implies \frac{d\tilde{t}}{d\rho} = \frac{V_\rho/4V}{1-\beta_{\mu_0}/4\mu_0}, \tag{4.28}$$

which allows us to write an exact expressions for V_ρ without the need to solve (4.16) iteratively. Inserting the previous expression in (4.16) gives

$$V_\rho = V_0 \left(\sum_{n=1}^{\infty} n c_n \rho^{n-1}\right) \frac{\left(1 - \frac{\beta_{\mu_0}}{4\mu_0}\right)}{\left(1 - \frac{\beta_{\mu_0}}{4\mu_0} - \frac{\beta_{V_0}}{4V_0}\right)}, \tag{4.29}$$

and thus

$$\mathcal{C} = c\frac{\left(1 - \frac{\beta_{\mu_0}}{4\mu_0}\right)}{\left(1 - \frac{\beta_{\mu_0}}{4\mu_0} - \frac{\beta_{V_0}}{4V_0}\right)} \equiv cF. \tag{4.30}$$

Note that (4.26) and (4.30) are in agreement. Combining (4.28) and (4.29), $d\tilde{t}/d\rho$ can be written in the form (4.18) with $d_0 = (c/4)(1 - \beta_{\mu_0}/4\mu_0 - \beta_{V_0}/4V_0)^{-1}$. Using eq. (4.26) gives for F the same expression as in (4.30). Now switching to the canonical field χ, we thus find that for $F = 0$ the slope vanishes $V_\chi = -K^{-\frac{1}{2}}F(c\rho + O(\rho^2)) = 0$.[12] The inflaton potential develops an extremum and ϵ is identically zero at all orders. Approaching this point, when $F \simeq \rho$, our perturbative analysis breaks down. To show that the potential develops a maximum consider the curvature at the extremum $V_{\chi\chi}|_{F=0} = K^{-1}F_\rho c(1 + O(\rho))$. As c is negative, it follows that $\mathrm{sign}[\mathrm{V}_{\chi\chi}]|_{\rho_{\rm ext}} = -\mathrm{sign}[\mathrm{F}_\rho]|_{\rho_{\rm ext}}$ where $\rho_{\rm ext}$ is the field value at the extremum. Now since $F(\rho) > 0$ for $\rho > \rho_{\rm ext}$, we have $F_\rho(\rho_{\rm ext}) \geq 0$. This implies that $\mathrm{sign}[\mathrm{V}_{\chi\chi}]|_{\rho_{\rm ext}} < 0$, i.e. the extremum is indeed a maximum (or an inflection point for double fine-tuned parameters [120, 136, 122]). Note that, also this result is independent on the particular beta-functions.

One can contemplate the possibility of inflation happening on the other side of the maximum. F is negative here as $\mathrm{sign}[\mathcal{C}_\star]$ is reversed. However, this describes a completely different kind of inflation with the inflaton rolling on the other side of the maximum (see Fig. (4.1)). Therefore, when a maximum develops due to

[12] Here we used that $d\rho/d\chi = -K^{-\frac{1}{2}} < 0$ since $K > 0$ and $\rho \to 0$ for $\chi \to \infty$.

quantum corrections, one has to assume initial conditions such that inflation starts with the inflaton always on the "correct side" of the maximum. In Higgs inflation this is equivalent to the observational request to end up in the electroweak vacuum after inflation. For the wider class of models considered here, the assumption is still reasonable since the vacuum at the origin is tuned to have zero (small) cosmological constant and this is where inflation is assumed to end; the minimum at large field values might not only be large (negative or positive), but also in the regime where any calculation control is lost as quantum gravity corrections may be large.

4.2.4 Higher orders

Even if the arguments shown in section 4.1.1 are general, it is still not clear if the result presented in section 4.2.2 hold beyond leading order (LO). Consider the RG improved L-loops potential, i.e. $V = V_{0,\text{eff}}(\tilde{t})(1+..)$ where $V_{0,\text{eff}} = V_0(\tilde{t})+V_0^1(\tilde{t})+..$ only depend explicitly on the couplings. Since the cancellation of the RG effects in the inflationary predictions does not depend explicitly on the particular V_0 nor on the order of β_{V_0}, everything still follows replacing $V_0 \to V_{0,\text{eff}}$ and $\beta_{V_0} \to \beta_{V_{0,\text{eff}}}$. However, including higher loops contributions we can in general no longer absorb the effect of the anomalous dimensions in the canonical field, as was discussed at the end of section 4.1.1. This may not necessarily hold beyond LO, for which further investigations would be required.

4.3 Applications

We now discuss the classes of models that can be written in the general form (4.8); these are the α- and ξ- attractors. As a particular case of this latter we first consider again Higgs inflation (HI) in view of the above. Here the condition (4.18) on the renormalization scale is determined by couplings of the Higgs to the other Standard Model particles. Nothing guarantees beforehand that this is still valid for the more general class of models considered. Nevertheless, in 4.3.4 we discuss how the weak condition (4.18) holds naturally also for the Cosmological Attractors.

It thus follows that the tree level predictions for all these models are robust against quantum corrections.

4.3.1 Higgs Inflation revisited

The argument outlined in the previous section applies to Higgs inflation. This is the reason why in the previous chapter it was found that the dependence on the beta-functions drops out of the inflationary predictions. In fact, the kinetic term and the potential for Higgs inflation (in the Einstein frame and for the background field) can be written as a Laurent series in $\rho = \Omega^{-2} = 1/(1+\xi\phi^2)$ as

$$K = \frac{3}{2\rho^2} + \frac{1}{4\xi(1-\rho)\rho^2} \simeq \frac{3}{2}\left(1+\frac{1}{6\xi}\right)\frac{1}{\rho^2} + \frac{1}{4\xi}\sum_{i=-1}^{\infty}\rho^i \tag{4.31}$$

and

$$V = V_0(1-2\rho+\rho^2) \tag{4.32}$$

with $V_0 = \lambda/(4\xi^2)$. This is exactly of the general form (4.8) with $p = 2$, $c = -2$ and $a_2 = 3/2(1+1/6\xi)$. The slow roll parameters at tree level are given by (4.13), which implies (3.17) at tree level.

Let us now turn to the quantum corrections. In Higgs inflation there is a natural choice for the RG time $\tilde{t}$ as discussed in 3.6.1. This is chosen such that it minimizes the largest logs in the Coleman-Weinberg potential in agreement with (4.5).[13] This is the already widely discussed Prescription I (see sec. 3.6.1) that can be rewritten as

$$\tilde{t} = \ln\frac{\phi}{(1+\xi(\tilde{t})\phi^2)^{\frac{1}{2}}} = \frac{1}{4}\ln\left(\frac{4V(\tilde{t})}{\lambda(\tilde{t})}\right). \tag{4.33}$$

To get the last expression it was used that the classical potential in the Einstein frame is given by $V = \lambda\phi^4/4\Omega^4/4$.[14] It follows that $\tilde{t}$ is actually of the form (4.27) with $\mu_0 = 4/\lambda$ which satisfies the generic assumption (4.18). Using (4.28), (4.29) and (4.30) with $\beta_{\mu_0}/\mu_0 = -\beta_\lambda/\lambda$ and $\beta_{V_0}/V_0 = \beta_\lambda/\lambda - 2\beta_\xi/\xi$ gives[15]

$$\frac{d\tilde{t}}{d\rho} = -\frac{1}{2}\frac{1}{\left(1+\frac{\beta_\xi}{2\xi}\right)} + O(\rho) \tag{4.34}$$

[13] As discussed the dominant quantum corrections come from the W and Z bosons and from the top quark masses and their all scale the same way with the Higgs field, namely as $M_i \sim f(\phi)/2$, with $f(\phi) = \phi/\Omega$.

[14] Remember that even if initially the loop corrections and renormalization scale is computed in the Jordan frame, and only afterwards the results are transformed to the Einstein frame, this would give the same result for the renormalization scale as shown in sec. 3.6.1.

[15] This can be matched to the notation used in 3, where the small parameter $\delta = (\xi\phi^2)^{-1}$ was used as expansion parameter. In that notation $\rho = \delta(\delta+1)^{-1} \simeq \delta + O(\delta^2)$ and $\phi = \left(\frac{1-\rho}{\xi\rho}\right)^{\frac{1}{2}}$, which gives

$$\frac{dt}{d\phi} = \frac{\xi^{\frac{1}{2}}\delta^{\frac{3}{2}}}{1+\frac{\beta_\xi}{2\xi}+\delta}, \quad \frac{d\phi}{d\rho} = -\frac{1}{2\xi^{\frac{1}{2}}}\frac{\rho^{-\frac{3}{2}}}{(1-\rho)^{\frac{1}{2}}} + \frac{\beta_\xi(1-\rho)^{\frac{1}{2}}}{\xi^{\frac{3}{2}}\rho^{\frac{1}{2}}}\frac{dt}{d\rho},$$

It then follows that at leading order $\frac{dt}{d\rho} = \frac{dt}{d\phi}\frac{d\phi}{d\rho}$ agrees with (4.34).

and

$$\mathcal{C} = c\left(1 + \frac{\beta_{V_0}/4V_0}{1 - \frac{\beta_{\mu_0}}{4\mu_0}}\right) = -2\left(\frac{1 + \frac{\beta_\lambda}{4\lambda}}{1 + \frac{\beta_\xi}{2\xi}}\right) \equiv -2F. \tag{4.35}$$

Since Higgs inflation is a specific example of the general set-up considered in section 4.2.2 the shift of C by the quantum corrections drops out of the inflationary predictions, which are equal to the tree level results. This conclusion holds as long as the perturbative expansion in ρ is valid. As discussed in 4.2.3 for $F_\star \simeq \rho_\star$ the potential develops a maximum, which for fine-tuned parameters can be used for "hilltop inflation" in agreement with the CMB data. We studied this possibility numerically in the previous chapter.

In sec. 3.6.1 we illustrate how different renormalization prescription arise in the context of Higgs inflation. We argue that Prescription I is the natural choice since it minimizes the log preserving the asymptotic shift symmetry of the potential. Prescription II can be rewritten in the language of this chapter as

$$\tilde{t} = \ln \phi = \frac{1}{2} \ln\left[\frac{1}{\xi}\left(\frac{1-\rho}{\rho}\right)\right]. \tag{4.36}$$

This corresponds to a different UV completion of the theory (where the potential is already altered at tree level in the large field regime). Using this prescription for the renormalization scale we can immediately see that the previous cancellation does not take place anymore. In fact $\frac{dt}{d\rho} = -\frac{1}{2}\frac{1}{\rho(1-\rho)}$ which is not of the form (4.18). This is the reason why in [29, 99, 28], where prescription II was considered in the numerics, features for n_s and r have been observed. In fact, going through the same steps of section 4.2.2 we would obtain an F factor of the form $\sim (1 + c\rho^{-1}\beta_\lambda/4\lambda)$. Since $\rho^{-1} \approx 10^2$ (see footnote 15 and table 3.1), for small values of $\beta_\lambda/4\lambda|_\star \sim -10^{-2}$ (occurring already for $\lambda_\star \sim 10^{-3}$) the RG dependence is enhanced and change the predictions.

4.3.2 α-attractors

The α-attractors [143, 144],which are a generalization of conformal attractors [148, 149], are described by the Lagrangian

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{1}{2}K(\partial\phi)^2 - V(\phi) \tag{4.37}$$

with

$$K = \frac{\alpha}{(1-\phi^2/6)^2}\,, \quad V = \alpha f^2(\phi/\sqrt{6}). \tag{4.38}$$

The Starobinsky model [150] also belongs to this class for a particular choice of f and $\alpha = 1$. Through the change of variable

$$\frac{\phi}{\sqrt{6}} = \frac{1-\rho}{1+\rho}, \tag{4.39}$$

the inflaton Lagrangian becomes

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{1}{2}\left(\frac{3\alpha}{2\rho^2}\right)(\partial\rho)^2 - \alpha f^2\left(\frac{1-\rho}{1+\rho}\right). \tag{4.40}$$

From (4.13) it then follows that for quite generic[16] f the tree level results are

$$n_s \simeq 1 - \frac{2}{N}\,, \quad r \simeq \frac{12\alpha}{N^2}. \tag{4.41}$$

As we will motivate in section 4.3.4 the RG time can be chosen as in (4.27). The analysis including quantum corrections is then a special case of the general discussion in section 4.2.2. As was shown there, the inflationary observables are not affected by quantum corrections as long as F does not break the perturbative expansion in powers of ρ.

4.3.3 ξ-attractors

The ξ-attractors are models in which the inflaton is non-minimally coupled to the gravity sector [145, 146, 151]. They are described by a Lagrangian of the form

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{\Omega^2}{2}R - \frac{1}{2}K_J(\partial\phi)^2 - V_J(\phi). \tag{4.42}$$

After the usual conformal transformation of the metric $g = \Omega^{-2}g_E$, the gravity sector is in the standard form and the Einstein frame field metric and potential are

$$K = \frac{K_J}{\Omega^2} + 6\frac{\Omega'^2}{\Omega^2}\,, \quad V = \frac{V_J}{\Omega^4}. \tag{4.43}$$

Let us briefly review which classes of models belong to the ξ-attractor family. In [142] it has been shown that for the special choice $K_J = \frac{1}{2\xi}\frac{\Omega'^4}{\Omega}$, $V_J = \Omega^2 U(\Omega)$ (*special attractors*) the models are completely equivalent to the α-attractors with the identification $1 + \frac{1}{6\xi} \equiv \alpha$.[17] In fact, with this choice of K_J in (4.43), the

[16] The function f should not be singular at $\phi = 1$, or equivalently at $\rho = 0$, such that it is possible to expand it as $f = V_0(1 + c\rho + O(\rho^2))$.

[17] Notational remark; the relation between our Ω and the one defined in [142] is $\Omega^2_{\text{our}} = \Omega_{\text{their}}$. We keep this notation to be consistent with the previous chapter.

Einstein frame field space metric K becomes exactly the one of (4.40). Other subclasses of ξ-attractors are the *induced inflation models* [146] described by

$$\Omega^2 = \xi f(\phi)\,, \quad K_J = 1\,, \quad V_J = V_0(\Omega^2 - 1)^2, \tag{4.44}$$

and the condition that $\Omega \to 0$ as $\phi \to 0$; the *universal attractors* [145] satisfy $\Omega \to 1$ as $\phi \to 0$, with

$$\Omega^2 = 1 + \xi f(\phi) \tag{4.45}$$

and the same V_J and K_J as the induced inflation models. Higgs inflation is a particular example of a universal attractor model. For $\xi \to \infty$ all these models give classically the same predictions at leading order in N^{-1}, which coincide with the predictions of the α-attractors for $\alpha \to 1$. Indeed, in this limit the first term in K in (4.43) can be neglected. Thus the Lagrangian in the Einstein frame, after the field redefinition $\rho = \Omega^{-2}$, becomes

$$\frac{\mathcal{L}}{\sqrt{-g}} \simeq \frac{R}{2} - \frac{1}{2}\left(\frac{3}{2\rho^2}\right)(\partial\rho)^2 - V_0(1-\rho)^2. \tag{4.46}$$

This is of the form (4.7)-(4.8) with the leading pole of order two in the kinetic term ($p = 2$). Therefore the predictions coincide at first order with (4.41) in the $\alpha \to 1$ limit and given (4.18) the conclusions on the RG flow corrections are the same as in the previous sections. In the next section we further discuss the choice of renormalization scale.

4.3.4 Renormalization scale for Cosmological Attractors

Whereas the choice of $\tilde{t}$ in Higgs inflation is determined by the known couplings of the Higgs to the Standard Model fields, it is not clear a priori whether $\tilde{t}$ for a generic α or ξ attractor satisfies (4.18). This depends on the inflaton couplings to the other fields, which is model dependent. Let us consider first the case where only the inflaton loops are relevant in the inflationary regime (which happens when the coupling to all other fields is suppressed).[18] The one loop correction in the Einstein frame is then of the form (remember that the canonical inflaton field is labeled with χ) [153, 154, 155]

$$V^{(1)} \propto m_\chi^4 \ln\left(\frac{m_\chi^2}{\mu^2}\right) \simeq m_\chi^4 \ln\left(\frac{H^2}{\mu^2}\right) \tag{4.47}$$

[18] The general argument in this context [152] states that the one loop correction is negligible since it is suppressed by an extra factor m_χ^2 as compared to the tree level potential.

with the inflaton mass given by[19]

$$
\begin{aligned}
m_\chi^2 &\simeq V_{\chi\chi} - 2H^2 \simeq V\left(\eta - \frac{2}{3}\right) \\
&\simeq -\frac{2}{3}V_0\left(1 + c\left(1 - \frac{3}{2a_p}\right)\rho + O(\rho^2)\right).
\end{aligned} \tag{4.48}
$$

Thus the RG time satisfying (4.5) is

$$
\tilde{t} \quad \simeq \quad \ln\left(m_\chi^2(\tilde{t})\right) \sim \ln(H^2) \sim \ln V, \tag{4.49}
$$

where with $\ln(..) \sim \ln V$ we mean that the arguments of the two logs contain the same powers of ρ. This is enough to ensure that (4.18) is satisfied. The reason for this is simple. If the theory is renormalizable in the EFT sense, the one loop term (as any other log term in the effective potential) can be reabsorbed order by order in the tree level part.[20]

Let us now consider the case where the loop corrections are dominated by other fields running in the loops. We focus on the inflaton ϕ coupled to a scalar field σ; the results straightforwardly generalize to fermion and gauge fields. Here ϕ is the original field appearing in the Lagrangians that the define the models, (4.37) for the α-attractors and (4.42) for the ξ-attractors. In the small field regime $K \simeq 1$ and $\phi \simeq \chi$, and the ϕ field is the canonical renormalizable field. If we demand the theory to be renormalizable in the small field regime, the coupling between σ and ϕ has to be in a standard renormalizable form. This automatically implies that a one loop term like (4.49), with $m_\sigma^2 = \partial^2 V(\phi,\sigma)/\partial\sigma^2$, can be reabsorbed in the tree level potential in the inflationary regime. Indeed, m_σ^4 and V share the same field dependence over the whole field regime. Thus, since the renormalization scale accommodates a subset of the powers of ϕ contained in the classical potential it implies that, once rewritten in terms of ρ, it will equally have a subset of the powers of ρ that are contained in V.

Let us illustrate the previous statements with a simple example. Consider a coupling $\mathcal{L} \supset g^2\phi^2\sigma^2$ in the action (4.37) [152]. Assume for simplicity that the scalar field σ has no bare mass term. In the small field regime the 1-loop contribution will be of the form $V^{(1)} \sim g^4\phi^4\ln(g^2\phi^2/\mu^2)$, which can be absorbed in a quartic tree level potential $V \sim \lambda\phi^4$. For α-attractors, in the large field regime $V^{(1)}$ and V have exactly the same functional form, which implies $\tilde{t} = 1/4\ln(g^4\phi^4(\rho))$. This written in terms of ρ using (4.39) will give a RG time $\tilde{t}$ satisfying (4.18) (in particular, it will be of the form (4.27)). A similar reasoning applies to the ξ-attractors.

[19] In all these kind of models we can neglect the backreaction of gravity since $\epsilon \ll \eta$ [9].

[20] The slow roll parameter η in terms of the canonical field is $\eta \propto e^{-\chi/a_2^{1/2}}$. The fact that we can reabsorb the η-dependent term in (4.48) is basically another way of saying that radiative corrections do not spoil the quasi shift symmetry of the classical potential.

The coupling generates loop contributions that can be reabsorbed in the Jordan frame potential V_J over the whole field range, fully analogously to the situation in the α-models. In the large field regime, once we transform to the Einstein frame, all the mass scales (including the renormalization scale, (see 3.3.2, [1, 9] and [156]) are rescaled as $m \to m/\Omega$. This still ensures that $\tilde{t}$ is of the form (4.18); the explicit example in this context is given by Higgs inflation in section 4.3.1.

The argument presented so far comes with a caveat. We demanded the low energy regime to be renormalizable, which is not necessary for a working inflationary model. On top, we can relax the constraint by only asking the theory to be renormalizable in the EFT sense in the small field regime; this opens the possibility that the theory is only defined up to some cutoff scale Λ in this regime. To be specific, consider the α/ξ-attractors actions (4.37) and (4.42) augmented with an interaction term of the form

$$\mathcal{L}_{\mathrm{I}} = \Lambda^2 g\sigma^2 \ln\left(1 + \frac{\phi^2}{\Lambda^2}\right). \tag{4.50}$$

Expanding in $\phi \ll \Lambda$ gives the first term of the previous example plus a tower of higher order operators suppressed by powers of the cutoff. The theory is clearly renormalizable in the EFT sense in the small field regime. However, things might change in the large field regime.

In the α models, using the expression of ϕ in terms of ρ given by (4.39), we get

$$\mathcal{L}_{\mathrm{I}} \sim g\sigma^2 \left(a_0 + a_1\rho + O(\rho^2)\right), \tag{4.51}$$

where a_i are just the numerical coefficients of the expansion. Therefore, even in the high field regime, the quantum corrections generated by the field σ (proportional to m_σ^4) can be absorbed order by order in the tree level potential. The RG time $\tilde{t}$ satisfies a relation like (4.49) and the main conclusions of section 4.2.2 still follow. The same argument does not apply to the ξ-attractors though. Here the interaction term in the Einstein frame becomes

$$\mathcal{L}_{\mathrm{I}} \propto \frac{g\sigma^2}{\Omega^2} \ln\left(1 + \frac{\phi^2}{\Lambda^2}\right) \simeq g\sigma^2\rho^2 \ln\left(\rho^{-1}\right), \tag{4.52}$$

where for simplicity we have taken $f(\phi) \propto \phi^2$ in (4.45). Similar results follow for generic $f(\phi)$ polynomial in ϕ and generic powers of ϕ appearing in the log in (4.50). Eq. (4.52) cannot be expanded in powers of ρ as before, leading apparently[21] to a choice for the renormalization scale which does not fulfill condition (4.18).

[21] We say "apparently" because if the theory is not renormalizable even perturbatively, all the arguments of section 4.1.1 do not apply anymore and talking about RG improved action is misleading.

The interaction renders the theory not renormalizable in the EFT sense. Hence, interaction terms like the one considered here are simply not allowed on this ground — it is important to remember that our whole analysis in section 4.1.1 is based on the assumption that inflation is described by a perturbatively normalizable EFT.

4.4 Summary

In this chapter we studied more generally how the RG flow can affect the predictivity of inflation, considering a large class of models. In particular, we analyzed the Cosmological Attractor models using the RG improved action to include the leading log quantum corrections. A consequence of this is that the slow roll parameters and the number of e-folds will depend on the beta-functions, and differ from the respective tree-level expressions. However, when calculating the spectral index and tensor-to-scalar ratio, all corrections exactly cancel. This can be shown, expanding in the small parameter defining the inflationary regime, without explicitly solving the RGEs. This is our main result, the inflationary predictions for α and ξ attractors are not affected by quantum corrections (to leading order in the $1/N$ expansion). This generalizes our previous work on Higgs inflation [1] presented in the previous chapter to the larger class of Cosmological Attractors. At the same time it extends the unified picture provided in [142] beyond the tree level.

There is one caveat which allows quantum corrections to become important. It may be that the potential develops an extremum because of the running; this is purely a quantum effect since the tree level potential has no extremum in the inflationary regime. This coincides with the breakdown of our perturbative expansion and analytical control is lost. This allows for hilltop or inflection point inflation which are sensitive to the loop corrections. However, these cases are realized for very fine-tuned values of the parameters and they can be studied numerically for specific inflationary scenarios.

Our main conclusion is valid as long as the kinetic term and the potential can be written in the form (4.8) and the leading log in the effective potential is minimized by an RG function satisfying (4.18). This is indeed satisfied provided the theory is perturbatively renormalizable in both the low/high field regime; which is the *sine qua non* for our full discussion.

In the next chapter we finally discuss a model that cannot be recast in this form and show explicitly its RG dependence.

5 Quantum corrections and predictions in new Higgs inflation

We now discuss quantum aspects and the RG sensitivity of the main alternative Higgs inflationary scenario, called new Higgs inflation (NHI). The reason for doing that is the following. We have alarmed the reader that the predictiveness of single-field inflationary models might be lost because of a specific UV effect. However, so far we have only discussed classes of models with the special property of being (almost) independent on the RG flow. Thus, as a proof of principle, it is interesting to work with a model that lies outside the set analysed in the previous chapter, and see how running effects can indeed move the tree-level predictions on the Planck plot. Hence, the model in principle brings new hopes in connecting low energy physics with CMB data. With the Higgs field as the inflaton, we can spell out this connection and compute explicitly all the ingredients necessary for the analysis. Unitarity bounds are derived in the chiral formalism extended to include non-minimally coupled gauge bosons and fermions (sec. 5.3). The beta-functions and the effecticve potential are computed with a covariant method that can easily be applied to other models (sec. 5.4). The chapter is based mostly on our work [3], and on work in preparation [4].

5.1 Déjà vu

In new Higgs inflation the Higgs kinetic terms are non-minimally coupled to the Einstein tensor, allowing the Higgs field to play the role of the inflaton. The new interaction is non-renormalizable, and the model only describes physics below some cutoff scale. One way to see the need for new physics is to consider the tree level scattering amplitudes, and to see at what energy scale unitarity is lost. This gives

the unitarity bound of the theory. EFTs can be defined in the two asymptotic regimes and unknown physics can affect the matching between the two. Thus, even if the UV physics does not affect the tree level inflaton potential significantly, it may still enter at loop level and modify the running of the Standard Model (SM) parameters. As a result the values of these couplings during inflation will depend both on the boundary conditions at the electroweak scale (the measured values), and on the UV completion via this kick.

If the reader feels a sense of déjà vu, this is quite understandable. The story so far is equally valid for both standard Higgs inflation (HI) and new Higgs inflation (NHI) (although the details, such as the explicit values of the cutoff and the form of the RGEs differ). A key difference, though, is that in NHI the inflationary predictions are sensitive to this running. In HI, the running corrections to the spectral index and the tensor-to-scalar ratio are suppressed to leading order in slow roll parameters through a cancellation of different effects, as we saw in chapter 3. This particular feature is shared by the more general class of Cosmological Attractors (chapter 1.4.2) of which HI is a particular case. Thus whatever the kick, whatever the effect of the UV completion on the running,the predictions for n_s and r are given by the tree-level results, and thus robust. In NHI inflation on the other hand, as we will show, this is not the case. Thus, the boundary conditions at the EW scale as well as the unknown UV completion[1] may leave a signature on the inflationary parameters. This conclusion can be avoided if the fermions and gauge bosons are coupled non-minimally to gravity as well. In that case, they are very light during inflation and effectively decouple. Just as for the example of the quartic potential discussed in sec 1.4.2, the inflationary predictions are not affected by the uncertainty in the running of the couplings. Non-minimally coupling the other fields gives us the opportunity to explore interesting details of the RGEs.

5.2 New Higgs inflation: a quick review

In this section we give the action for new Higgs inflation (NHI), show how its analysis can be simplified after performing a disformal transformation, identify the different field regimes, and study the inflationary dynamics at the classical level.

[1]It should also be noted that as in standard Higgs inflation, the unitarity bound in the mid field regime is lower than the typical potential energy scale during inflation. Therefore we work under the same assumptions on the UV physics discussed in sec. 3.4.3.

5.2.1 The action

The model is built out of the only gravity-Higgs kinetic interaction that does not introduce extra degrees of freedom with respect to GR and the SM [7]. We will also consider the possibility that the kinetic terms of the fermions and gauge bosons contain a non-minimal coupling to gravity [157, 158]. To assure second order equations of motion for both gravity and matter, the new couplings should be to divergenceless tensors constructed from the Riemann tensor. In four dimensions there is only the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - (R/2)g_{\mu\nu}$, and the double dual Riemann tensor[2] [159, 160] $\mathcal{G}_{\alpha\beta\gamma\delta} = *R_{\alpha\beta\gamma\delta}* = \frac{1}{4}\epsilon_{\alpha\beta\mu\nu}R^{\mu\nu\rho\sigma}\epsilon_{\rho\sigma\gamma\delta}$, which satisfy $\nabla_\mu G^{\mu\nu} = 0$ and $\nabla_\mu \mathcal{G}^{\mu\nu\rho\sigma} = 0$.

The action is

$$\mathcal{L} = \sqrt{-\bar{g}}\left[\frac{1}{2}m_{\mathrm{P}}^2\bar{R} + \mathcal{L}_{\mathrm{Higgs}} + \mathcal{L}_{\mathrm{fermion}} + \mathcal{L}_{\mathrm{gauge}}\right]. \tag{5.1}$$

For reasons that become clear in a moment we denote the metric and the quantities derived from it by an overbar $\bar{g}_{\mu\nu}$. The Higgs, fermion and gauge Lagrangian contain the SM terms (using the notation defined in and around eq. (3.3)) plus a non-minimal coupling to gravity via the kinetic term. Explicitly [158]

$$\begin{aligned}
\mathcal{L}_{\mathrm{Higgs}} &= -\left(\bar{g}^{\mu\nu} - \alpha_h\frac{\bar{G}^{\mu\nu}}{M^2}\right)D_\mu\mathcal{H}^\dagger D_\nu\mathcal{H} - V\\
\mathcal{L}_{\mathrm{gauge}} &= -\sum\frac{1}{4}\left(\bar{g}^{\alpha\mu}\bar{g}^{\beta\nu} - \alpha_A\frac{3\bar{\mathcal{G}}^{\mu\nu\alpha\beta}}{M^2}\right)F^a_{\alpha\beta}F^a_{\mu\nu}\\
\mathcal{L}_{\mathrm{fermion}} &= -\sum_i\left(\bar{g}^{\alpha\beta} - \alpha_\psi\frac{\bar{G}^{\alpha\beta}}{M^2}\right)\bar{\psi}_i i\gamma_\alpha D_\beta\psi_i - (y_d\bar{Q}_L\mathcal{H}d_R + y_u\bar{Q}_L\mathcal{H}_c u_R + \mathrm{h.c.}).
\end{aligned}$$

The couplings α_i can be Higgs field dependent: $\alpha_i = \alpha_i(\mathcal{H})$. NHI assumes a constant Higgs coupling, which can always be set to unity by redefining the mass scale M. From now on we set $\alpha_h = 1$. In the original NHI scenario this is the only non-minimal coupling and $\alpha_A = \alpha_\psi = 0$ [7]. The mass scale M is the only additional free parameter. However, one can consider the more general possibility $\alpha_i = \mathcal{O}(1)\left(\frac{\sqrt{V}}{Mm_{\mathrm{P}}}\right)^n$ with $i = \psi, A$. The basic idea of NHI is to enhance the friction of the Higgs kinetic term ($\bar{G}_{\mu\nu} \propto \bar{H}^2$) in such a way that the Higgs boson rolls very slowly down its potential, allowing inflation to occur.

[2] $\epsilon^{abcd} = \frac{1}{\sqrt{-g}}\epsilon^{abcd}$, with ϵ^{abcd} the Levi-Civita tensor and ϵ^{abcd} the completely antisymmetric symbol.

5.2.2 Disformal transformation

The Higgs-gravity sector can be brought in (approximate) standard form: an Einstein-Hilbert term plus a scalar field Lagrangian. Consider a disformal transformation of the metric [158]

$$g_{\alpha\beta} = \bar{g}_{\alpha\beta} + \varepsilon_{\alpha\beta} = \bar{g}_{\alpha\beta} - 2\frac{D_\alpha \mathcal{H}^\dagger D_\beta \mathcal{H}}{\mathcal{M}^4}, \tag{5.2}$$

where we introduced the scale

$$\mathcal{M}^2 = M m_{\mathrm{P}}. \tag{5.3}$$

To leading order in covariant derivatives, the action after the disformal transformation is

$$\begin{aligned} \sqrt{-\bar{g}}\left(\bar{R} + 2\bar{G}^{\mu\nu}\frac{D_\mu \mathcal{H}^\dagger D_\nu \mathcal{H}}{\mathcal{M}^4}\right) &= \sqrt{-g}R + \mathcal{O}(\varepsilon), \\ \sqrt{-\bar{g}} = \sqrt{-g}\left(1 + \frac{D_\alpha \mathcal{H}^\dagger D^\alpha \mathcal{H}}{\mathcal{M}^4} + \mathcal{O}(\varepsilon^2)\right)&. \end{aligned} \tag{5.4}$$

Higher powers of derivatives will be suppressed by higher powers of $\mathcal{M}^2$. The gravity-Higgs sector transforms into standard Einstein gravity plus the Higgs Lagrangian; the effect of the original Higgs-gravity coupling is now transferred to non-minimal kinetic terms

$$S_{\mathrm{EH+Higgs}} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}m_{\mathrm{P}}^2 R - \left(1 + \frac{V}{\mathcal{M}^4}\right)D^\mu \mathcal{H}^\dagger D_\mu \mathcal{H} - V + \mathcal{O}(\varepsilon^2)\right]. \tag{5.5}$$

Note that to describe the evolution of the classical field $\langle|\mathcal{H}|^2\rangle = \phi(t)^2/2$ the expansion is certainly valid. Indeed, evaluated on the background [161]

$$\varepsilon_0^0 = \frac{\dot{\phi}^2}{\mathcal{M}^4} = \frac{2(3\bar{H}^2 m_{\mathrm{P}}^2 - V)}{9\bar{H}^2 m_{\mathrm{P}}^2 + \mathcal{M}^4} < \frac{2}{3}, \tag{5.6}$$

where in the second step we have used the Friedmann equations. During inflation the potential dominates the energy density and the numerator $3\bar{H}^2 m_{\mathrm{P}}^2 - V \ll 3\bar{H}^2 m_{\mathrm{P}}^2$ is small. In this regime $\varepsilon_0^0 \sim \epsilon_H$, with $\epsilon_H = -\dot{H}/H^2$. At small field values $\bar{H}^2 m_{\mathrm{P}}^2 \ll \mathcal{M}^4$ and $\varepsilon_0^0 \ll 1$ as well. The maximum $\varepsilon_0^0 \approx \frac{2}{3}$ is reached at the end of inflation/onset of preheating.

To leading order in ε the gauge and fermion Lagrangians are invariant, and we can replace $\bar{g}_{\mu\nu} \to g_{\mu\nu}$ in $\mathcal{L}_{\mathrm{gauge}}$ and $\mathcal{L}_{\mathrm{fermion}}$ in (5.2). In the small derivative limit, by using the Einstein equations we have [162]

$$\begin{aligned} \mathcal{L}_{\mathrm{gauge}} &= -\frac{1}{4}\sum\left(g^{\alpha\mu}g^{\beta\nu} - \alpha_A\frac{3\mathcal{G}^{\mu\nu\alpha\beta}}{M^2}\right)F^a_{\mu\nu}F^a_{\alpha\beta} + \mathcal{O}(\varepsilon) \\ &\sim -\frac{1}{4}\sum\left(1 + \alpha_A\frac{V}{\mathcal{M}^4}\right)F^a_{\mu\nu}F^{a,\mu\nu}, \end{aligned} \tag{5.7}$$

and

$$\mathcal{L}_{\text{fermion}} = -\sum_i \left(g^{\alpha\beta} - \alpha_\psi \frac{G^{\alpha\beta}}{M^2}\right) \bar{\psi}_i i\gamma_\alpha D_\beta \psi_i - (y_d \bar{Q}_L \mathcal{H} d_R + \text{h.c.} + ...) + \mathcal{O}(\varepsilon)$$
$$\sim -\sum_i \left(1 + \alpha_\psi \frac{V}{\mathcal{M}^4}\right) \bar{\psi}_i i\gamma^\mu D_\mu \psi_i - (y_d \bar{Q}_L \mathcal{H} d_R + \text{h.c.} + ...), \tag{5.8}$$

where for concreteness we have focused on the top quark Lagrangian, but obviously all Yukawa interactions have the same structure.

To summarize, and to set the notation used throughout this chapter (although different parameterizations might be used to answer different question, but this will always be indicated), we work with the NHI Lagrangian in the Einstein frame given in (5.5, 5.7, 5.8)

$$\begin{aligned}\frac{\mathcal{L}}{\sqrt{-g}} =& -\gamma(\mathcal{H})|D_\mu \mathcal{H}|^2 - V(\mathcal{H}) - \sum_a \frac{1}{4} k(\mathcal{H})(F^a_{\mu\nu})^2 \\ &+ \sum_i q(\mathcal{H}) \bar{\psi}_i (i\not{D}) \psi_i - \frac{y_t}{\sqrt{2}} \bar{Q}_L \mathcal{H}_c t_R + \text{h.c.}\end{aligned} \tag{5.9}$$

where the sum is over all SM gauge groups; we only included the top quark and the sum in the fermion kinetic terms is over $\psi_i = \{Q_L, t_R\}$. The non-minimal Higgs, gauge and fermion field space metrics γ, k, q are given explicitly by

$$\gamma = (1+\delta)\,, \qquad q = (1 + \alpha_F \delta)\,, \qquad k = (1 + \alpha_A \delta)\,. \tag{5.10}$$

with

$$\delta \equiv \frac{V}{\mathcal{M}^4} = \frac{\lambda \phi^4}{4\mathcal{M}}. \tag{5.11}$$

We write in general the non-minimal gauge boson and fermion couplings as

$$\alpha_i = \alpha_{0i} \delta^{n_i/2} \tag{5.12}$$

for $i = A, F$. They are vanishing for $\alpha_{0i} = 0$ and constant for $n_i = 0$, which are arguably the most interesting cases.

5.2.3 Large field regime

As usual we parametrize the Higgs field on the classical background as

$$\mathcal{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}. \tag{5.13}$$

The action for the background field becomes

$$\mathcal{L} = \sqrt{-g}\left(\frac{1}{2}m_{\rm P}^2 R - \frac{1}{2}\gamma(\phi)(\partial_\mu\phi)^2 - \frac{\lambda}{4}\phi^4\right). \tag{5.14}$$

We neglect the Higgs-vev term at the electroweak scale which is irrelevant during inflation. The dynamics of the system is very different for small and large field values, for which the correction to the Higgs kinetic term is not important respectively dominates. We distinguish between two regimes and we use the parameter δ in (5.11) evaluated on the background to parameterize them:[3]

$$\text{small field}: \quad \delta \ll 1, \qquad \text{large field}: \quad \delta \gg 1. \tag{5.15}$$

In the small field regime limit the action reduces to the SM Lagrangian. The large field regime corresponds to the inflationary regime. Consider $\phi \gg 4\mathcal{M}^4$.[4] The canonically normalized background field h is [5]

$$dh = \sqrt{\delta}d\phi \to h = \frac{\sqrt{\lambda}\phi^3}{6\mathcal{M}^2}. \tag{5.16}$$

In terms of h the potential becomes simply

$$V = \frac{\tilde{\lambda}}{4}m_{\rm P}^4\left(\frac{h}{m_{\rm P}}\right)^{4/3}, \quad \tilde{\lambda} = 6^{4/3}\lambda^{1/3}\left(\mathcal{M}/m_{\rm P}\right)^{8/3}. \tag{5.17}$$

For large field values the theory is nothing but old-fashioned canonical chaotic inflation, albeit with a rather unconventional exponent of 4/3 in the potential. Taking $N_* = 60$ at tree level the spectral index and tensor-to-scalar ratio are $n_{s\star} \simeq 0.972$ and $r_\star \simeq 0.089$. The field value during and at the end of inflation is $\phi_\star^6 = (96N_*\mathcal{M}^4 m_{\rm P}^2/\lambda) = (3N_\star)\phi_{\rm end}^6$. The power spectrum fixes the free parameter $\mathcal{M}$ in the theory via $V/(m_{\rm P}^4\epsilon)|_\star = (0.027)^4$ [75], and we find

$$\begin{aligned} M &\simeq 1.5\times 10^{-8} m_{\rm P}\lambda^{-1/4}, & \mathcal{M} &\simeq 1.2\times 10^{-4} m_{\rm P}\lambda^{-1/8},\\ \delta_* &\simeq 1.3\times 10^{7}\sqrt{\lambda}, & \phi_* &\simeq 1.0\times 10^{-2} m_{\rm P}\lambda^{-1/4}. \end{aligned} \tag{5.18}$$

Since $\delta_{\rm end} \simeq 4\times 10^5\sqrt{\lambda}$, the end of inflation is well inside the large field regime. For $\delta \sim 1$, i.e.

$$\phi_{\rm eq} = \frac{\sqrt{2}\mathcal{M}}{\lambda^{1/4}}. \tag{5.19}$$

both the small and large field expansion breaks down, and the theory is non-renormalizable. Moreover, in the "mid field regime", i.e. $1 < \delta \ll \delta_{\rm end}$ the $\mathcal{O}(\varepsilon, \epsilon)$ corrections may become large.

[3] With slight abuse of notation, we sometimes use the same letter δ to both denote the full field-dependent quantity and the quantity denoted on the background.

[4] This corresponds to the set-up discussed in sec. 2.3.3.

[5] The integration constant C can be fixed by matching it to the small field solution at the boundary region $h(\phi_{\rm eq}) = \phi_{\rm eq}$. At large field values this constant can be neglected, and we ignore it from now on.

5.3 Unitarity bound

The semiclassical approximation is valid if typical energy scales are below the one at which tree level unitarity breaks down. Because of the non-renormalizable interactions in NHI the model becomes ill-defined at large scales. Expanding the Lagrangian around $\phi = 0$, the higher order interactions are suppressed by the cutoff scale $\sim \mathcal{M}$. Likewise, one can expand the Lagrangian around large field values ϕ, and read off the typical cutoff scale from the non-renormalizable interaction. The cutoff in NHI is field dependent, and different in the asymptotic regimes. A more systematic approach to determine the cutoff scale for the validity of the theory is to calculate the scale at which tree-level unitarity is lost, obtained from scattering amplitudes/cross sections for specific interactions. In this section we compute the 2-to-2 Higgs/Goldstone boson scattering complementing the analysis done in [114], in order to check whether the typical energy scale is close to the tree level cutoff of the theory; for non-minimally coupled fermions and (transverse) gauge bosons, it is in addition interesting to look at their scattering rates. We compare to the results for 2-to-n Higgs scattering obtained in [114] and find order one agreement.

We first recap the general approach for extracting the unitarity bounds of the Standard Model EFT in chiral representation [163], which was applied to Higgs inflation in [28]. We set $m_{\mathrm{P}} = 1$ for the rest of the chapter.

5.3.1 Chiral SM with non-minimal gauge/fermion sector

In the chiral approach/non linear representation, the Goldstone boson are grouped together by means of an $SU(2)$ valued sigma field. The SM Higgs boson can be parametrized as

$$\mathcal{H} = e^{-i\vec{\sigma}.\vec{\chi}/F_0} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi_r \end{pmatrix} = \Sigma \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi_r \end{pmatrix} \tag{5.20}$$

with $\phi_r = \phi + \delta\phi$ the Higgs field split in its background value and fluctuations, and $\vec{\chi}$ the Goldstone bosons. F_0 is a normalization constant to give the Goldstone bosons canonical kinetic terms.[6] The kinetic terms for the Higgs are non-minimal, and it is convenient to rewrite the Lagrangian in terms of the canonical Higgs denoted by $h_r(\phi_r) = h + \delta h$. If the top quark and gauge bosons are non-minimally coupled, their kinetic terms are of the form

$$\mathcal{L} \supset -\sum \frac{1}{4} k(h_r) F_{\mu\nu}^2 + q(h_r)(\bar{Q}_L i\not{D} Q_L + \bar{t}_R \not{D} t_R). \tag{5.21}$$

[6] For the purpose of this section we only "approximately" canonicalize the quadratic terms.

Since we are only interested in the order of magnitude of the unitarity bound, we neglect Lorentz violating effects and use the approximations (5.7,5.8). We assume universal α_A couplings for all gauge bosons, and universal α_F couplings for all fermions. The results are easy to generalize. We rescale

$$A_\mu = \frac{\hat{A}_\mu}{\sqrt{k_0}}, \quad Q_L = \frac{\hat{Q}_L}{\sqrt{q_0}}, \quad t_R = \frac{\hat{t}_R}{\sqrt{q_0}}, \quad g = \sqrt{k_0}\hat{g}, \quad y_t = q_0\hat{y}_t, \tag{5.22}$$

with g and y_t the gauge and top Yukawa coupling, and we have used the notation $f_0 \equiv f(h)$, to emphasize the function evaluated at its background value. With this rescaling the top and gauge boson fields have canonical quadratic kinetic terms, and the gauge and Yukawa interactions are still in standard form as $gA_\mu = \hat{g}\hat{A}_\mu$ and $y_t\bar{Q}_L d_R = \hat{y}_t\hat{\bar{Q}}_L\hat{d}_R$. This assures the Goldstone boson equivalence theorem holds as usual. The Lagrangian is then

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}(\partial h_r)^2 - \frac{1}{4}F^2(h_r)\mathrm{Tr}\left[(\hat{D}_\mu\Sigma)^\dagger(\hat{D}^\mu\Sigma)\right] - \sum\frac{1}{4}\frac{k(h_r)}{k_0}(\hat{F}_{\mu\nu})^2 \\ &+ \frac{q(h_r)}{q_0}(\hat{\bar{Q}}_L\hat{\not{D}}\hat{Q}_L + \hat{\bar{t}}_R\hat{\not{D}}\hat{t}_R) - V(h_r) - \frac{1}{\sqrt{2}}\left(y_t\hat{\bar{Q}}_L\Sigma\begin{pmatrix}\hat{b}_R\\ \hat{t}_R\end{pmatrix}\right)Y(h_r) + \text{h.c.}\end{aligned} \tag{5.23}$$

For notational convenience we will drop the hat, but in the rest of the section we will always work with the Lagrangian (5.23). Next, expand[7]

$$\begin{aligned} F^2 &= F_0^2\left(1 + 2a\frac{\delta h}{F_0} + b\frac{(\delta h)^2}{F_0^2}\right), & \frac{k}{k_0} &= \left(1 + \frac{k_1}{F_0}\delta h + ...\right), \\ Y &= Y_0\left(1 + c\frac{\delta h}{F_0} + ...\right) & \frac{q}{q_0} &= \left(1 + \frac{q_1}{F_0}\delta h + ...\right). \end{aligned} \tag{5.24}$$

The coefficients a, b, c, k_1, q_1 are dimensionless. In the SM $F_0 = Y_0 = h = v$, $a = b = c = 1$ and $k_1 = q_1 = 0$. The amplitude for $\chi^+\chi^- \to \bar{\psi}\psi$ mediated by the Yukawa interaction does not give a strong bound for NHI (where $Y = \phi_r$.), and we will not consider it explicitly. In terms of the non-canonical ϕ-field — related to the canonical field h via $\frac{1}{2}\gamma(\partial\phi)^2 = \frac{1}{2}(\partial h)^2$ — we can extract the coefficients in the expansion as follows:

$$\begin{aligned} a &= \frac{1}{2F}\frac{1}{\sqrt{\gamma}}\frac{\partial F^2}{\partial\phi_r}\bigg|_\phi, & b &= \frac{1}{2}\frac{1}{\sqrt{\gamma}}\frac{\partial}{\partial\phi_r}\left(\frac{1}{\sqrt{\gamma}}\frac{\partial F^2}{\partial\phi_r}\right)\bigg|_\phi, \\ k_1 &= \frac{F_0}{k_0}\frac{1}{\sqrt{\gamma}}\frac{\partial k}{\partial\phi_r}\bigg|_\phi, & q_1 &= \frac{F_0}{q_0}\frac{1}{\sqrt{\gamma}}\frac{\partial q}{\partial\phi_r}\bigg|_\phi, \end{aligned} \tag{5.25}$$

all evaluated on the background at $\phi_r = \phi$.

[7] In the SM one has $F = Y = h_r = \phi_r$, and the expansion is formulated in terms of δh.

The amplitude for $2 \to 2$ scattering of Goldstone bosons into Goldstone bosons and Higgs fields can then be expressed [163]

$$\begin{aligned} A_1(\chi^+\chi^- \to \chi^+\chi^-) &= \frac{s+t}{F_0^2}(1-a^2) + \mathcal{O}\left(\frac{m_h^2}{E^2}\right), \\ A_2(\chi^+\chi^- \to \delta h\,\delta h) &= \frac{s}{F_0^2}(b-a^2) + \mathcal{O}\left(\frac{m_h^2}{E^2}\right). \end{aligned} \tag{5.26}$$

In the Standard Model all amplitudes vanish up to $\mathcal{O}(m_h^2/E^2)$, but in NHI the amplitudes are non-zero because of the new interactions from the non-minimal Higgs-gravity coupling. In addition there are extra diagrams because of the non-minimal gauge- and fermion-gravity couplings, mediated by the k_1 and q_1 interactions respectively. The k_1-term in the Lagrangian gives a $h(\partial A)^2$-vertex. For the scattering of longitudinal gauge bosons this interaction provides amplitudes that do not grow with energy, but as $\mathcal{O}(M_W^4/E^4)$. The terms proportional to $\mathcal{O}(s/M_W^2)$ cancel in agreement with the Goldstone equivalence theorem (the Goldstone and thus the longitudinal gauge boson interactions do not depend on the non-minimal gauge-gravity coupling). The k_1-interaction still gives a growing contribution for the transverse gauge boson scattering (via the diagram with Higgs exchange). The q_1-term generates a $h\bar{\psi}\psi$ interaction; this provides an extra contribution to the $\chi\chi \to \psi\psi$ process. The additional amplitudes are given by

$$\begin{aligned} A_A(AA \to AA) &\sim \frac{s}{F_0^2}k_1^2, \\ A_F(\chi^+\chi^- \to \bar{\psi}\psi) &\sim \frac{\sqrt{s}}{F_0}aq_1. \end{aligned} \tag{5.27}$$

The $2 \to 2$ scattering amplitudes are of the form $\mathcal{A}(s,t,u) = \mathcal{A}(s,\theta)$; in the relativistic limit $\vec{p} \gg m$ the Mandelstam variables are $t = -(s/2)(1-\cos\theta)$ and $u = -(s/2)(1+\cos\theta)$. To determine the unitarity bound we project the amplitude into partial waves

$$a_l = \frac{1}{32\pi}\int_{-1}^{-1} \mathrm{d}\cos\theta\,\mathcal{A}(s,\theta)P_l(\cos\theta), \tag{5.28}$$

with P_l the Legendre polynomial $P_0(x) = 1$, $P_1(x) = x$. The s-wave unitarity bound is $|a_0| < \pi/2$.[8] Consider first the bound for Goldstone boson scattering, for which we find

$$a_0 = \frac{1}{32\pi}\frac{|1-a^2|}{F_0^2}s < \frac{\pi}{2} \quad \Rightarrow \quad \sqrt{s} < \Lambda_1 = 4\pi\frac{F_0}{\sqrt{|1-a^2|}}, \tag{5.29}$$

[8] Also $|a_0| < 1$ and $|a_0| < 1/2$ are used sometimes; all are acceptable as estimates, the spread can be seen as theoretical uncertainty [163]. In [114] instead the bound $\sigma < 4\pi/s$ is used. This gives a cutoff that is lower, and thus a bound that is stronger, by about a factor of 2 for NHI.

which gives the unitarity bound on the center of mass energy $\sqrt{s} = E_{\rm cm}$. The bound is a factor $\sqrt{2}$ stronger if $\chi^+\chi^- \to \chi^0\chi^0$ is included. To find the unitarity bound for the other processes, we proceed in the same way. The result is

$$\Lambda_1 = 4\pi\frac{F_0}{\sqrt{|1-a^2|}}, \quad \Lambda_2 = 2\sqrt{2}\pi\frac{F_0}{\sqrt{|b-a^2|}}, \quad \Lambda_A \sim 2\sqrt{2}\pi\frac{F_0}{|k_1|}, \quad \Lambda_F \sim 8\pi^2\frac{F_0}{|aq_1|}. \tag{5.30}$$

The SM is renormalizable, and indeed all bounds diverge for the SM values of the coefficients. The SM without the Higgs boson has $a = b = 0$, and the above expressions give the unitarity cutoff of the Fermi theory.

In [114] the unitarity bound derived from $2\,\delta h \to n\,\delta h$ scattering was derived (with $n \geq 3$) using the criterion on the cross section $\sigma < 4\pi/s$. This gives

$$\Lambda_m \sim |V_m| F_m^{\frac{1}{2(m-2)}}, \quad F_m = 2^{4m-10}\pi^{2m-6}(m-3)!(m-4)!, \quad V_m = \left(\frac{\mathrm{d}^m V}{\mathrm{d}h^m}\right)^{\frac{1}{4-m}}, \tag{5.31}$$

with $m = n + 2$.

5.3.2 New Higgs inflation

The Lagrangian of NHI (5.9) is of the form of the chiral SM with

$$\gamma = (1+\delta)\,, \quad F^2 = \phi_r^2\,(1+\delta)\,, \quad q = (1+\alpha_F\delta)\,, \quad k = (1+\alpha_A\delta)\,, \tag{5.32}$$

and $\delta = V/\mathcal{M}^4$, see (5.11), and the non-minimal gauge boson and fermion couplings α_i are given in (5.12). The coefficients in the expansion of the Lagrangian (5.25) are then

$$a = \frac{(1+3\delta)}{(1+\delta)}, \quad b = \frac{\left(1+14\delta+(3\delta)^2\right)}{(1+\delta)^2}, \quad k_1 = \frac{2(2+n_A)\alpha_A\delta}{1+\alpha_A\delta}, \quad q_1 = \frac{2(2+n_F)\alpha_F\delta}{1+\alpha_F\delta}, \tag{5.33}$$

with δ evaluated on the background. Consider first the bounds from the Higgs sector, that are always present in NHI. For $\delta \gg 1$, the coefficients a, b approach an $\mathcal{O}(1)$ constant, and the unitarity cutoff for Higgs and Goldstone scales as $\Lambda \propto F_0 \propto \sqrt{\delta}$. This estimate is actually too naive for Higgs scattering, as the leading term in the denominator in Λ_2 cancels in the large field regime, and consequently the cutoff grows faster: $\Lambda_2 \propto \delta$. In the small field regime $a, b \to 1$ approach the SM values, and the cutoff scales as $\Lambda \propto 1/\sqrt{\delta}$. Thus both in the limit of small and large field values, the bound increases, with a minimum at the midfield region

$\delta \sim 1$. Numerically, the bound from scattering into Higgses is slightly stronger, which we give here explicitly

$$\Lambda_2 = \pi\phi \frac{(1+\delta)^{3/2}}{\sqrt{\delta}} = \pi\phi \left\{ \frac{1}{\sqrt{\delta}}, \delta \right\}, \tag{5.34}$$

with the right most expression the approximation in the small and large field limit respectively. The minimum of the cutoff is in the midfield regime for $\delta = 1/5$, and is given by

$$\Lambda_{2,\mathrm{min}} \approx 6\phi_{\mathrm{eq}} \approx 6 \times 10^{-3} m_{\mathrm{P}} \left(\frac{10^{-2}}{\lambda} \right)^{3/8}, \tag{5.35}$$

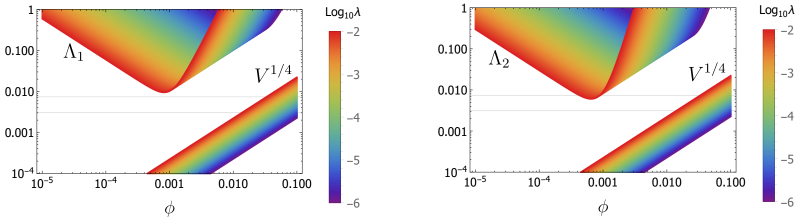
with ϕ_{eq} given in (5.19). It is interesting to compare this minimum with the potential at the same point, and with the potential during inflation

$$\frac{V^{1/4}(\phi_{\mathrm{min}})}{\Lambda_{2,\mathrm{min}}} \approx 2.4 \times 10^{-2} \left(\frac{\lambda}{10^{-2}} \right)^{1/4}, \qquad \frac{V_\star^{1/4}}{\Lambda_{2,\mathrm{min}}} \approx 1.2 \left(\frac{\lambda}{10^{-2}} \right)^{3/8}. \tag{5.36}$$

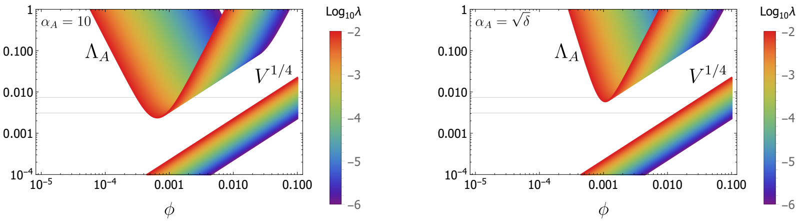
where we used (5.18). We confirm the conclusion reached in [114] that $V^{1/4} < \Lambda$ for all field values, and an EFT can be constructed for energies below the cutoff. However, it should be noted that the inflationary energy is of the same order as the minimum value of the cutoff, unless $\lambda \ll 10^{-2}$ tuned to small values during inflation. Thus, as in the original HI proposal, it is not guaranteed *a priori* that what lifts the unitarity bound to the Planck scale over the whole field range will not affect the inflationary regime already at tree level. We assume this is not the case and the UV completion only affects the running.

The results for the unitarity cutoff are shown in Fig. 5.1 for the quartic coupling λ in the range 10^{-2} to 10^{-6}. Note that $\lambda(\phi)$ is a running coupling whose value is scale dependent.[9] Figure 5.1a shows Λ_1 and Λ_2; for comparison also the energy scale set by the potential $V^{1/4}$ is shown. As can easily be seen, lower values of λ raises the cutoff and lowers the potential, with the net result that the tension between the cutoff and the inflationary scale is relieved. If the gauge bosons and/or fermions are non-minimally coupled there are additional bounds (5.30). In the small field limit $k_1 \simeq 2(1+n_A)\alpha_{0A}\delta$ and the cutoff Λ_A increases rapidly for small δ as $\Lambda_A \propto \phi/(\alpha_{0A}\delta)$. In the large field limit $k_1 \simeq 2(1+n_A)$ and the bound is $\Lambda_A \propto \phi\sqrt{\delta}/(2(1+n_A))$. The cutoff is minimal in the midfield regime $\delta \sim 1$, and scales as $\Lambda_{A,\mathrm{min}} \propto \phi_{\mathrm{eq}}/((4+2n_A)\alpha_{0A})$. That is, the larger n_A and the larger α_{0A}

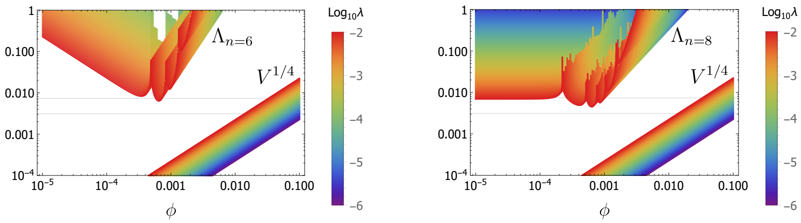
[9] To be precise, in the large field regime at first order in δ^{-1} we cannot find the running of the original Higgs quartic coupling λ but only the RGE for $\tilde{\lambda}$ and the set of parameters defined in (5.37). We can still define λ at the inflationary scale, i.e. $\lambda_\star$, as defined by inverting eq. (5.17). The value of $\mathcal{M}$ is fixed at the boundary between the two regimes in such a way that $\tilde{\lambda}_\star$ satisfies the power spectrum constraint (see sec. (5.6.2)).



(a) Cutoff from $2 \to 2$ scattering of Goldstone bosons into Goldstone bosons (Λ_1 in the left plot) and Higgs fields (Λ_2 in the right plot).



(b) Cutoff from $2 \to 2$ scattering of non-minimally coupled gauge bosons, for different non-minimal coupling $\alpha_A = 10$ (left plot) and $\alpha_A = \sqrt{\delta}$ (right plot).



(c) Cutoff from $2 \to n$ Higgs scattering, with $n = 6$ (left plot) and $n = 8$ (right plot).

Figure 5.1: *Unitarity cutoff Λ_i for different interactions as a function of the field ϕ in Planck units; also shown is $V^{1/4}$. In all figures horizontal gridlines correspond to $\{V_{\rm end}^{1/4}, V_\star^{1/4}\}$ which are independent of λ. In the plots λ varies between 10^{-2} and 10^{-6}.*

the smaller the unitarity cutoff, and in this limit the bound can become stronger than the one from the Higgs sector. The behavior for the cutoff Λ_F from fermion scattering is similar, except that it contains an extra factor of a in the denominator, which suppresses the bound for similar n_i and α_{0i} values. For example, for $n_i = 1$ and $\alpha_{0i} = 1$, we find $\Lambda_{A,\mathrm{min}} \approx 6.3\phi_{\mathrm{eq}}$ and $\Lambda_{F,\mathrm{min}} \approx 260\phi_{\mathrm{eq}}$. The results are shown in Fig. 5.1b which plots the bounds from gauge boson scattering for a constant and field dependent non-minimal coupling α_A.

Finally, there are the bounds from $\delta h \delta h \to n\delta h$ scattering (5.31). These bounds are plotted for two representative values $n = 6$ and $n = 8$ in Fig. 5.1c. The minimum cutoff is of the same order as $\Lambda_{2,\mathrm{min}}$. However, this many-body amplitude gives a lower cutoff in the small field regime than the other processes.

The power spectrum fixes $\tilde{\lambda}$ defined in (5.17). This means that for smaller Higgs coupling during inflation $\lambda_\star$, (see the discussion in footnote 9) the scale $\mathcal{M}$ becomes larger (5.18). This has the effect, as can also be seen from the plots, that the hierarchy between the cutoff scale and the scale set by the potential increases for small coupling. In fact, for $\lambda = 10^{-6}$ the cutoff is well above the inflationary scale $\Lambda_{\mathrm{min}} \gg V_\star^{1/4}$, suggesting that the effects of the unknown UV completion are small. This is, however, not necessarily the case. First of all, as we discuss in the next section, the light degrees of freedom in the EFT differ in the small field and large field regime, and consequently there is a jump in counterterms between the two regimes. This jump can be seen as the effect of threshold corrections, which therefore cannot be arbitrarily small (especially if the jump is proportional to the gauge or Yukawa coupling, and not only to the quartic coupling — which depends on the possible non-minimal couplings of the gauge and fermion fields).

Secondly, the loop corrections may grow for small coupling (this also depends on the possible non-minimal couplings of the gauge and fermion fields, as discussed in the next sections), and if they become large they may alter the shape of the potential significantly.

5.4 Covariant quantization

The purpose of next section is to compute the one-loop beta-functions in the large field regime. In this section we will set up the formalism, methodology and notation.

In the large field regime we can use (5.9) and expand in small δ^{-1} to capture the dominant effects. At leading order in the δ^{-1}-expansion the mass $\mathcal{M}$ is not an independent parameter, as it can be rescaled from the Lagrangian. Indeed, if we

define the tilde fields and couplings via[10]

$$\tilde{\mathcal{H}}^6 = \frac{\lambda}{\mathcal{M}^4}\mathcal{H}^6, \qquad \tilde{\lambda} = 6^{4/3}\lambda^{1/3}\mathcal{M}^{8/3}, \qquad \tilde{y} = y\left(\frac{\mathcal{M}^4}{\lambda}\right)^{1/6}, \quad \tilde{\alpha}_{0i} = \alpha_{0i}\left(\frac{\lambda}{\mathcal{M}^4}\right)^{\frac{1}{3}(1+\frac{n_i}{2})} \tag{5.37}$$

the lagrangian in the large field regime becomes

$$\begin{aligned}\mathcal{L} = &-\tilde{\mathcal{H}}^4|D_\mu\tilde{\mathcal{H}}|^2 - 6^{-4/3}\tilde{\lambda}|\tilde{\mathcal{H}}|^4 - \frac{\tilde{y}_t}{\sqrt{2}}\bar{Q}_L(\tilde{\mathcal{H}}_c)t_R + \text{h.c.}\\ &-\sum_a \frac{1}{4}\tilde{\alpha}_{0A}|\tilde{\mathcal{H}}|^{4(1+\frac{n_A}{2})}(F^a_{\mu\nu})^2 + \sum_i \tilde{\alpha}_{0F}|\tilde{\mathcal{H}}|^{4(1+\frac{n_F}{2})}\bar{\psi}_i(i\not{D})\psi_i + \mathcal{O}(\delta^{-1}).\end{aligned} \tag{5.38}$$

Also in the small field regime, where the model reduces to the SM, the scale $\mathcal{M}$ drops out of the Lagrangian at leading order in the δ-expansion. The scale $\mathcal{M}$ cannot be removed from the Lagrangian over the whole field range though. In fact, it still plays a fundamental role in determining the boundaries between the small and large field region and the matching conditions between the parameter of the two (see 5.6.2).

We will calculate the loop corrections in the untilde variables, as this makes the δ^{-1}-expansion more transparent. Although, as just shown, the original variables are not all independent at first order in the δ^{-1}-expansion. Thus, the resulting counterterms will form a system that is not closed. By translating these counterterms to those for the set of independent tilde-variables (5.58), we derive the beta-functions for the tilde variables.

Consider for simplicity a $U(1)$ Abelian-Higgs model with the NHI action (5.9), coupled to a chiral fermion with non-minimal couplings for the various fields. The results can be generalized to the full SM implementation of NHI. Counterterms are introduced by rescaling the bare fields and couplings by $Z_i = 1 + \delta_i$, with δ_i the counterterm, via

$$\begin{aligned}&\mathcal{H}_b = \sqrt{Z_\phi}\mathcal{H}, \qquad \psi_b = \sqrt{Z_\psi}\psi, \qquad A^\mu_b = \sqrt{Z_A}A^\mu, \qquad (\alpha_i)_b = Z_{\alpha_i}\alpha_i,\\ &\lambda_b = Z_\lambda\lambda, \qquad \mathcal{M}_b = Z_\mathcal{M}\mathcal{M}, \qquad y_b = Z_y y, \qquad g_b = Z_g g,\end{aligned} \tag{5.39}$$

We write the Higgs field as background plus perturbations

$$\mathcal{H} = \frac{1}{\sqrt{2}}(\varphi + i\theta) = \frac{1}{\sqrt{2}}(\phi + \delta\phi + i\theta), \quad \varphi^a = \{\varphi, \theta\}, \tag{5.40}$$

with $\phi(t)$ the classical background and $\delta\phi(x,t)$ and $\theta(x,t)$ the Higgs and Goldstone

[10] If one tries to find the running of $\mathcal{M}$ (and the other original parameters in the Lagrangian), there are not enough conditions to solve for all counterterms and derive all beta-functions.

fluctuations. The Lagrangian (5.9) becomes

$$\mathcal{L} = -\frac{1}{2}\gamma(\varphi^a)Z_\phi\partial_\mu\varphi^a\partial^\mu\varphi_a - \sum_a \frac{1}{4}k(\varphi^a)Z_A(F_{\mu\nu})^2 + q(\varphi^a)Z_\psi\bar{\psi}(i\partial\!\!\!/)\psi$$
$$- Z_\phi^2 Z_\lambda V(\varphi^a) - Z_\psi Z_y Z_\phi^{1/2}\frac{y}{\sqrt{2}}\bar{\psi}F_\psi(\varphi^a)\psi + Z_g Z_A^{1/2}Z_\psi\left(gq_L\bar{\psi}A\!\!\!/P_L\psi + gq_R\bar{\psi}A\!\!\!/P_R\psi\right)$$
$$- Z_g^2 Z_A Z_\phi\frac{1}{2}g^2A^2F_A(\varphi^a) + Z_g Z_A^{1/2}Z_\phi^{1/2}\gamma(\varphi^a)gA(\varphi\partial\theta - \theta\partial\varphi). \tag{5.41}$$

With the charge of the Higgs field fixed to unity, gauge invariance implies the relation $q_L - q_R = 1$ for the charges of the left and right handed fermions. Further,

$$V = \frac{\lambda}{4}(\varphi^2+\theta^2)^2, \quad F_\psi = \left(\varphi + i\gamma^5\theta\right), \qquad F_A = \gamma(\varphi^2+\theta^2),$$
$$\gamma = (1 + Z_\delta\delta), \qquad q = (1 + Z_{\alpha_F}Z_\delta\alpha_F\delta), \qquad k = (1 + Z_{\alpha_A}Z_\delta\alpha_A\delta) \tag{5.42}$$

with δ given in (5.11), and we introduced the notation $Z_\delta = Z_\phi^2 Z_\lambda/Z_{\mathcal{M}}^4$.

The gauge fixing term for the generalized R_ξ-gauge is

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi}k(\phi)\left(\partial^\mu A_\mu - g\xi\frac{\gamma(\phi)\phi}{k(\phi)}\theta\right)^2$$
$$= -\frac{1}{2\xi}k(\phi)(\partial^\mu A_\mu)^2 - g\gamma(\phi)\phi A_\mu\partial^\mu\theta - \frac{1}{2}\xi g^2(\gamma(\phi)\phi)^2\theta^2 \tag{5.43}$$

The first term combines with the quadratic term in the expansion of gauge kinetic term (the gauge boson propagator is derived from the quadratic Lagrangian), the second term cancels the quadratic $A_\mu\partial^\mu\theta$ interaction in the higgs kinetic terms, and the last term adds to the goldstone mass.

We work in Landau gauge $\xi = 0$, for which the ghosts fields decouple.

5.4.1 Covariant fields

Consider the first line in (5.41). Grouping together the Higgs-Goldstone-gauge sector, i.e. $\varphi^I = \phi^I + \delta\phi^I = \{\varphi, \theta, A\}$, the field space manifold has a non trivial geometry defined by the metric

$$G_{IJ} = \{\gamma(\varphi^I)\delta_{ij}, k(\varphi^I)\eta_{\mu\nu}\} = \{(1+\delta)\delta_{ij}, (1+\alpha_A\delta)\eta_{\mu\nu}\}. \tag{5.44}$$

As a first approximation (what we did for example in sec. 5.3) one can define approximate canonical kinetic term by rescaling the background values, $\{\varphi, \theta, A\} \to \{\gamma(\phi)^{-1/2}\{\varphi,\theta\}, k(\phi)^{-1/2}A\}$. Expanding the action and using the equation of motions gives a covariant (under fields reparameterizations) result only up to quadratic order.[11] However, the higher order vertices are not in covariant form,

[11] This can be explained a posteriori given the relation between $\delta\phi^I$ and Q^I starts at linear order.

and can only be trusted up to order one corrections. As we discussed extensively at the beginning of the thesis in sec. 2.4, the fluctuations $\delta\phi^I$ are not covariant objects on the field-space manifold. Thus, to expand the Lagrangian in a form that is fully covariant under field redefinitions we use the formalism presented in 2.4. We replace the ordinary field displacement $\varphi^I - \phi^I$ with the tangent vector to the unique geodesic connecting the background ϕ^I to the field φ^I,

$$Q^I = \frac{d\varphi^I(\lambda)}{d\lambda}\Big|_{\lambda=0}, \tag{5.45}$$

where λ is the affine parameter parameterizing the geodesic such that $\varphi^I(\lambda = 1) = \varphi^I$ and $\varphi(0) = \phi^I$. Let us summarize the notations in the following table

I	Non-covariant fields $\delta\phi^I$	Covariant fields Q^I
ϕ	$\delta\phi \quad =$	$Q^\phi + \mathcal{O}(Q^2)$
θ	$\theta \quad =$	$Q^\theta + \mathcal{O}(Q^2)$
A	$A_\mu \quad =$	$Q^{A_\mu} + \mathcal{O}(Q^2)$

$$\delta\phi^I = \{\delta\phi, \theta, A_\mu\}\,, \qquad Q^I = \left\{Q^\phi, Q^\theta, Q^{A_\mu}\right\}, \qquad I = \{\phi, \theta, A\}\,. \tag{5.46}$$

The relation between non-covariant and covariant fluctuations is given by eq. (2.108),

$$\delta\phi^I = Q^I - \frac{1}{2}\Gamma^I_{JK}Q^JQ^K + \frac{1}{3!}(\Gamma^i_{LM}\Gamma^m_{JK} - \partial_L\Gamma^I_{JK})Q^JQ^KQ^L + ..., \tag{5.47}$$

where Γ^K_{IJ} are the Christoffel symbols associated to G_{IJ} in (5.44) on the background. To rewrite the Lagrangian in covariant fields we can expand in covariant derivatives as in (2.103). The coefficients of the expansion are evaluated on the background and will determine the strength of the interactions. In particular, we expand in this way the scalar functions defined in (5.42), $f = \{V, F_\psi, F_A, \gamma, q, k\}$. Equivalently the action can be expanded as in (2.107), e.g. by normal Taylor series in the fluctuations $\delta\phi^I$, and then substitute their expression in terms of the covariant fields given in (5.47).[12] We use this second approach to expand the kinetic terms in covariant fluctuations. In this expansion we neglect $\dot\phi_0^2$-corrections, as they are slow roll suppressed during inflation.

The masses of the bosonic fields are given by the covariant expression $(m^2)_I^J = -G^{IJ}\nabla_I\nabla_J\mathcal{L}$ evaluated on the background.[13] The mass matrix is diagonal, with

[12] As discussed in 2.4.1, the two procedures give the same results since both represent the same expansion of the action in the affine parameter λ.

[13] The full mass matrix would be given by $m_{IJ} = \nabla_I\nabla_J V - R_{IMNJ}\dot\phi^M\dot\phi^N$. We neglect the $\dot\phi$ terms that are irrelevant in our case. Recently, it has been noted that in general this term can play in general an important role in premature ending inflation [164, 165].

masses

$$m_h^2 = \lambda\phi^2 \frac{(3+\delta)}{(1+\delta)^2}, \quad m_\theta^2 = \lambda\phi^2 \frac{(1+3\delta)}{(1+\delta)^2}, \quad m_A^2 = \frac{g^2\phi^2(1+\delta)}{(1+\alpha_A\delta)} + \frac{\delta(2+n)\alpha_A\lambda\phi^2}{(1+\delta)(1+\alpha_A\delta)}, \tag{5.48}$$

with δ evaluated on the background. We use the notation $(m^2)_{Q^\phi}^{Q^\phi} \equiv m_h^2$, $(m^2)_{Q^\theta}^{Q^\theta} \equiv m_\theta^2$, $(m^2)_{Q^A}^{Q^A} \equiv m_A^2$. In the small field regime the masses approach the Standard Model values. The last term in the gauge boson mass arises from mixing between the Higgs and gauge sector (specifically, because $\Gamma^\phi_{AA} \neq 0$), it is suppressed at large field values for $n < 1$. Its specific form is never important, and we will neglect this term from now on. Finally, note that, in principle we should also define covariant fermion fluctuations. It is not clear how to rigorously do that.[14] A parametrically estimate of the size of the vertices will be enough for our purposes, and we can set $q \to q(\phi)$ on the background and just rescale $q(\phi)\psi \to \psi$ to obtain approximately canonically renormalized fermions with mass

$$m_\psi \approx \frac{1}{\sqrt{2}} \frac{y}{(1+\alpha_F\delta)}\phi. \tag{5.49}$$

This is discussed further in section 5.5.1. Feynman rules are given in appendix B.

5.5 Renormalization group equations

In this section we calculate the one-loop beta-functions for NHI. In the small field regime the set-up reduces to the SM EFT with the SM RGEs to leading order in the δ-expansion. In the large field inflationary regime, such that $\mathcal{O}(\epsilon, \varepsilon)$ corrections are small as discussed in section 5.2, the EFT can be expanded in δ^{-1}. The EFT is renormalizable in the usual sense if all divergences can be absorbed in counterterms order by order in the δ^{-1}-expansion. Here we only consider the leading order result.

5.5.1 Coleman Weinberg potential

Before deriving the RGEs it is useful to consider the Coleman-Weinberg potential. The masses of the gauge and fermion fields depend on the functions k and q in their kinetic terms. For different choices, they are of the order of the inflationary scale $m \sim V^{1/4}$ and contribute to the loop corrections, they are much heavier and the fields are to be integrated out, or they are much lighter and the fields decouple.

[14] Methods as used in [166] may work.

Based on the spectrum of particles with masses of the inflationary scale, we will distinguish four different cases.

We neglect the backreaction from gravity, as well as the corrections due to the rolling of the background field, which are both slow roll suppressed during inflation. The divergent part of the one-loop effective potential is (using dimensional regularization) [41]

$$V_{\text{eff}} = Z_V \frac{1}{4}\lambda\phi^4 - \frac{1}{32\pi^2\epsilon}\left[m_h^4 + \sum m_\theta^4 + 3\sum m_A^4 - 4N_c m_\psi^4\right], \tag{5.50}$$

with $Z_V = Z_\lambda Z_\phi^2$ and $\epsilon = 4-d$. It is clear that substituting the masses (5.48, 5.49) in the effective potential (5.50) the divergences cannot be absorbed in field independent counterterms over the whole field range. Thus, reasoning the same way as in section 3.5.1, one can conclude that there is a tower of radiatively generated higher order operators which are not suppressed in the mid field regime $\delta \sim 1$. It is not surprising as $\delta \sim 1$ corresponds to field values of the order of the unitarity bound in the small field regime $\phi \sim \lambda^{-1/2}\mathcal{M} \sim \Lambda_{\text{min}}$. This breaks the connection between low and high scales in the same way as for HI, as explained in 3.5.1.

Asymptotically, in the small and large field regime, a renormalizable EFT may be constructed. In the small field regime $\delta \ll 1$, all masses reduce to their SM values values, and all divergences can be absorbed just as in the SM. Explicitly,

$$V_{\text{eff}}|_{\delta\ll 1} = \frac{\lambda\phi^4}{4}\left[Z_V - \frac{1}{8\pi^2\epsilon}\frac{1}{\lambda}\left((9+N_{GB})\lambda^2 + 3\sum g_i^4 - N_c y^4\right)\right] + \mathcal{O}(\delta), \tag{5.51}$$

We will calculate the loop-corrections first for the $U(1)$ abelian-Higgs model, introduced in the previous section. In this set-up there is one Goldstone boson $N_{GB} = 1$, one gauge boson with coupling $g_i = g$, and one colorless ($N_c = 1$) Dirac fermion with yukawa coupling y. Generalizing to the full SM, there are $N_{GB} = 3$ Goldstone bosons, the summation is over $g_i = \frac{1}{2}\{g_2^2, g_2^2, \sqrt{g_1^2 + g_2{}^2}\}$, with g_2 and g_1 the gauge coupling of the $SU(2)$ electroweak and $U(1)$ hypercharge gauge groups respectively, and the dominant fermion contribution comes from the top quark with $N_c = 3$ and $y = y_t$ the top quark yukawa.

In the large field regime $\delta \gg 1$ the effective potential during inflation becomes

$$V_{\text{eff}}|_{\text{infl}} = \frac{\lambda\phi^4}{4}\left[Z_V - \frac{1}{8\pi^2\epsilon}\frac{1}{\lambda}\left(3\sum\frac{g_i^4(1+\delta)^2}{(1+\alpha_A\delta)^2} - N_c\frac{y^4}{(1+\alpha_F\delta)^4} + \mathcal{O}(\delta^{-1})\right)\right]. \tag{5.52}$$

The Higgs field and Goldstone bosons are light during inflation, their contribution to the effective potential is $\mathcal{O}(\delta^{-2})$, and they effectively decouple. Whether the gauge field and fermions decouple or remain in the spectrum depends on their possible non-minimal couplings. We consider the various cases in turn.

– Case A: Original NHI ($\alpha_A = \alpha_f = 0$) with canonical gauge bosons and fermions as in the original NHI model [7]. The fermion has mass of the order of the energy scale set by the potential $m_t \sim V^{1/4}$ and is included in the spectrum. The gauge boson mass, on the other hand, is large, of the order of the cutoff scale (5.30) during inflation $m_A^2 = g^2\Lambda_1^2/(2\pi^2)$, and should be integrated out.

  Thus, during inflation the Higgs/Goldstone bosons and the gauge fields decouple and the only d.o.f in the spectrum is the top quark. The counterterm derived from (5.52) is

$$Z_V = 1 + \frac{1}{8\pi^2\epsilon} \cdot \left(-3\frac{y^4}{\lambda}\right). \tag{5.53}$$

  Matching to the SM effective field theory at small field values, in the midfield regime the Higgs/Goldstone bosons and the gauge fields are "integrated back in", and there are threshold corrections suppressed by the unitarity cutoff $\Lambda_{1,2}$ and m_A respectively. This new physics is needed to restore unitarity and renormalizability in the UV.[15]

– Case B: Maximal number of degrees of freedom during inflation ($\alpha_A = \alpha_{0A}$ constant and $\alpha_f = 0$).

  Introducing a non-minimal coupling for the gauge fields $\alpha = \alpha_0$ with $n = 0$ in (5.12) will lower their mass and bring them back in the spectrum as now $m_A \sim m_t \sim V^{1/4}$ during inflation. Both the gauge and fermion fields contribute and the counterterm is

$$Z_V = 1 + \frac{1}{8\pi^2\epsilon}\frac{1}{\lambda}\left(3\sum_i \frac{g_i^4}{\alpha_{0A}^2} - 3y^4\right). \tag{5.54}$$

  In the midfield regime the Higgs/Goldstone bosons are integrated back in, and there are threshold corrections suppressed by the unitarity cutoff $\Lambda_{1,2}$; in addition the gauge contribution is different in the asymptotic regime and additional threshold corrections suppressed by Λ_A are expected.

[15] Note, however, that the contribution of the gauge boson to the CW-potential is non-renormalizable. Consider the case $g \ll 1$ such that there is a hierarchy of scales $V_0^{1/4} \ll m_A \ll \Lambda$. Then at energy scales $\mu \sim m_A$ the gauge boson is in the spectrum. To absorb the loop divergence one would need to add a new operator $\Lambda_{\rm h.o} \propto \phi^4\delta^2$. It is interesting to note that in this particular case the new operator does not generate additional divergences. In terms of the canonical field in the high field regime $\phi^4\delta^2 \approx h^4$. Thus, (at least at leading order) we do not need to add an infinite tower of higher order operators. On the other hand, unless tuned to be small, this operator will completely change the inflationary dynamics already at tree level. Thus, to write (5.53) we are implicitly assuming that the physics arising at the cutoff enables us to integrate out the heavy gauge bosons in the usual EFT sense. This scenario can only work with extra assumptions on the UV physics.

– Case C: Constant non-minimal couplings for the Higgs, gauge and fermion fields ($\alpha_A = \alpha_{0f}$ and $\alpha_A = \alpha_{0f}$). Because of the non-minimal coupling the fermion is light/weakly coupled and decouples during inflation. Only the gauge field contributes to the loop corrections and the counterterm is

$$Z_V = 1 + \frac{1}{8\pi^2\epsilon}\frac{1}{\lambda}\left(3\sum_i \frac{g_i^4}{\alpha_{0A}^2}\right). \tag{5.55}$$

– Case D: All fields decouple ($\alpha_i = \alpha_{0i}\delta^{n_i/2}$ with $n_A \geq 1$ or $\alpha_{0A} = 0$ such that the gauge field is too light respectively heavy, and $n_F \geq 0$ such that the fermion is light). To leading order all d.o.f. are weakly coupled/super heavy and the running can be neglected. The counterterm is trivial $Z_V = 1$.

5.5.2 Beta-functions

We calculate the one-loop corrections to the Higgs, Goldstone boson, fermion and gauge boson 2-point functions, and expand in δ^{-1} to find the leading order contribution in the large field regime.[16] This gives the various counterterms in the theory, and consequently the beta-functions.

The momentum dependent part of the one-loop 2-point function gives the counterterm for the kinetic terms, whereas the momentum independent part the counterterm for the 2pnt vertex. We will denote these with Z_{2f} and $Z_{c\,2f}$ respectively, with $f = \{Q^\phi, Q^\theta, Q^A, \psi\}$ the (covariant) fields in question, and $c = \{\lambda, g, y\}$ if it renormalizes the Higgs, gauge or Yukawa coupling. The relevant counterterms are those of the quadratic interactions, and of the potential. These Z-factors can be expressed in terms of the basis set of counterterms introduced in (5.39). For example, from the Higgs kinetic term in the large field regime

$$\mathcal{L} \supset \frac{1}{2} Z_\delta \delta\, Z_\phi (\partial\varphi)^2 \supset \frac{1}{2} Z_\phi^2 Z_\lambda Z_{\mathcal{M}}^{-4} Z_\phi \delta\, (\partial Q^\phi)^2 \equiv \frac{1}{2} Z_{2Q^\phi}\delta\, (\partial Q^\phi)^2. \tag{5.56}$$

The full set of Z-relations in the large field at leading order is given by

$$\begin{aligned}
&Z_{2Q^\phi} = Z_{2Q^\theta} = Z_\lambda Z_\phi^3 Z_{\mathcal{M}}^{-4}, && Z_{\lambda 2Q^\phi} = Z_{\lambda 2Q^\theta} = Z_\lambda Z_\phi^2, && Z_V = Z_\lambda Z_\phi^2\\
&Z_{2Q^A} = Z_A Z_k, && Z_{g2Q^A} = Z_k Z_g^2 Z_A Z_{\alpha_A}^{-1}, && Z_k = Z_{\alpha_{0A}} \left(Z_\lambda Z_\phi^2 Z_{\mathcal{M}}^{-4}\right)^{1+\frac{n_A}{2}}\\
&Z_{2\psi} = Z_\psi Z_q, && Z_{y2\psi} = Z_\psi Z_y Z_\phi^{1/2}, && Z_q = Z_{\alpha_{0f}} \left(Z_\lambda Z_\phi^2 Z_{\mathcal{M}}^{-4}\right)^{1+\frac{n_f}{2}}
\end{aligned} \tag{5.57}$$

[16] Reminder: we will calculate the loop corrections in the untilde variables, as this makes the δ^{-1}-expansion more transparent.

Note that the calculation of the self-energy for the Goldstone bosons is redundant; its calculation serves as a consistency check.

In the large field regime the scale $\mathcal{M}$ can be rescaled from the lagrangian, and the independent set of couplings is given by the tilde couplings defined in (5.37). The resulting counterterms can be translated to those for the tilde-variables, to derive the beta-functions for the tilde variables. i.e.

$$Z_{\tilde{\phi}}^3 = Z_\lambda Z_{\mathcal{M}}^{-4} Z_\phi^3, \quad Z_{\tilde{\lambda}} = Z_\lambda^{1/3} Z_{\mathcal{M}}^{8/3}, \quad Z_{\tilde{y}} = Z_y Z_{\mathcal{M}}^{2/3} Z_\lambda^{-1/6}, \quad Z_{\tilde{\alpha}_i} = Z_{\alpha_i}(Z_\lambda Z_{\mathcal{M}}^{-4})^{\frac{1}{3}(1+\frac{n_i}{2})}. \tag{5.58}$$

The beta-functions for the couplings and anomalous dimensions for the fields are derived via

$$-\partial_t(\ln Z_\lambda) = \lim_{\epsilon\to 0}\epsilon(Z_\lambda - 1) = \frac{\beta_\lambda}{\lambda}, \qquad \frac{1}{2}\partial_t(\ln Z_\phi) = -\frac{1}{2}\lim_{\epsilon\to 0}\epsilon(Z_\phi - 1) = \gamma_\phi \tag{5.59}$$

Case A

Start with case A. The one-loop expressions for the self-energies are given in appendix B. The results are

$$Z_{2Q^\phi} = Z_{2\psi} = Z_{y2\psi} = 1 + \mathcal{O}(\delta^{-1}), \qquad Z_{\lambda 2Q^\phi} = 1 + \boldsymbol{A} + \mathcal{O}(\delta^{-1}) \tag{5.60}$$

with

$$\boldsymbol{A} = -\frac{1}{8\pi^2\epsilon}\left(\frac{y_t^4}{\lambda}\right) = -\frac{1}{8\pi^2\epsilon}\left(\frac{6^{4/3}\tilde{y}_t^4}{\tilde{\lambda}}\right). \tag{5.61}$$

The gauge bosons are integrated out (otherwise they would give a non-renormalizable $\mathcal{O}(\delta^2)$ correction, just as for the CW potential). The result is consistent with the calculation of the one-loop potential in the previous section as $Z_{\lambda 2Q^\phi} = Z_V$. The beta-functions are defined in (5.59), and depend on the logarithm of the Z-factors. We are thus interested in

$$\begin{aligned} &0 = \ln(Z_{2Q^\phi}) = \ln(Z_{\tilde{\phi}}^3), && 0 = \ln(Z_{2\psi}), \\ &0 = \ln(Z_{y2\psi}) = \ln\left(Z_\psi Z_{\tilde{y}} Z_{\tilde{\phi}}^{1/2}\right), && \ln(1+\boldsymbol{A}) = \ln(Z_{\lambda 2Q^\phi}) = \ln\left(Z_{\tilde{\lambda}} Z_{\tilde{\phi}}^2\right), \end{aligned} \tag{5.62}$$

where in the second step we used (5.57) and (5.58). Note that $Z_{\mathcal{M}}$ has dissapeared when written in terms of the tilde-variables, as it should. We can solve this system of equations to get

$$Z_{\tilde{\phi}} = Z_\psi = Z_{\tilde{y}} = 1, \quad Z_{\tilde{\lambda}} = 1 + \boldsymbol{A}. \tag{5.63}$$

The beta-functions are then

$$\beta_{\tilde{y}} = \gamma_\psi = \gamma_{\tilde{\phi}} = \mathcal{O}(\delta^{-1}), \qquad \beta_{\tilde{\lambda}} = -\frac{1}{8\pi^2}6^{4/3}\tilde{y}_t^4. \tag{5.64}$$

Case B

Consider non-minimal kinetic terms for the gauge fields $k = 1 + \alpha_0\delta$, which brings them back in the spectrum during inflation.

One can find approximate results for the gauge contribution to loop diagrams by simply setting the non-canonical kinetic function $k = k(\phi)$ constant on the background. Rescaling the field and coupling $\hat{A}^\mu = \sqrt{k(\phi)}A^\mu$ and $\hat{g} = g/\sqrt{k(\phi)}$, the gauge interactions are of standard form. The mass can be read off from the action, from the 1st term in (5.9), and is $m_A^2 = \gamma(\phi)\hat{g}^2\phi^2 = (\gamma(\phi)/k(\phi))g^2\phi^2$. The effect of the non-minimal gauge coupling k is thus that the mass is reduced by a factor δ^{-1} compared to case A, and is now of the order of the inflationary scale. Although we expect the simple rescaling of the field will give the vertex-interations only up to order one corrections, this is enough to see that all gauge coupling dependence drops from the counterterms $Z_{2\hat{A}}$, $Z_{g2\hat{A}}$, $Z_{2\psi}$, $Z_{y2\psi}$ and Z_{2Q^ϕ}, as it is suppressed in the δ^{-1}-expansion. What remains is to calculate the non-zero gauge contribution to the Higgs self-energy, which should give the same counterterm as the Coleman-Weinberg potential and thus the Z_V found in (5.54).

The calculation of the Higgs self-energy serves as a consistency check, however, it is not possible without using the covariant formalism. It is interesting to note that only by including some of the "genuinely" new interactions coming from the Q expansion of the gauge fields (and that are absent for canonical gauge fields), we are able to find exactly the same counterterm as in (5.54). Particularly important are the interactions[17]

$$\mathcal{L}_\mathrm{k} = -\frac{1}{2}G_{IJ}(\varphi^I)\partial\varphi^I\partial\varphi^J \supset -\mathcal{K}_{2Q^\phi 2\partial Q^I}(Q^\phi)^2(\partial Q^I)^2 - \mathcal{K}_{2Q^\theta 2\partial Q^I}(Q^\theta)^2(\partial Q^I)^2 \tag{5.65}$$

where $\mathcal{K}_{2Q^I\partial Q^J}$ are the background dependent coefficients given by expanding the non-canonical kinetic term in covariant fields. For example $\mathcal{K}_{2Q^\phi 2\partial Q^A} = \alpha_{A0}\delta\{0, -1/3\phi^2\}$. See appendix B. These new interactions give a contribution to the Higgs self-energy

$$\delta\Pi_{Q^\phi} = -\sum_I 2n_I\mathcal{K}_{2Q^\phi 2\partial Q^I}G^{II}m_I^4 \tag{5.66}$$

and similar for the goldstone boson self-energy. Here the sum is over $Q^I = \{Q^\phi, Q^\theta, Q^{A_\mu}\}$ with $n_I = \{1, 1, 3\}$ the d.o.f. Only the gauge boson contributes at leading order, as the Higgs/goldstone mass is suppressed during inflation. Including this correction, see appendix (B) for more details, the results are consistent

[17] The couplings $k_{Q^\phi 2\partial Q^I} = k_{Q^\theta 2\partial Q^I} = 0$ vanish.

with the CW calculation. The final result is

$$Z_{2Q^\phi} = Z_\psi = Z_{m_\psi} = Z_{2Q^A} = Z_{g2Q^A} = 1 + \mathcal{O}(\delta^{-1}), \tag{5.67}$$

$$Z_{\lambda 2Q^\phi} = Z_{\lambda 2Q^\theta} = Z_V = 1 + \boldsymbol{A} \tag{5.68}$$

with

$$\boldsymbol{A} = \frac{1}{8\pi^2\epsilon}\frac{1}{\lambda}\left(\frac{3g^4}{\alpha_0^2} - y^4\right) = \frac{1}{8\pi^2\epsilon}\frac{6^{4/3}}{\tilde{\lambda}}\left(\frac{3g^4}{\tilde{\alpha}_0^2} - \tilde{y}^4\right). \tag{5.69}$$

The Higgs and fermion Z-factors give the same results as in case A given in (5.62), except that $\boldsymbol{A}$ now includes the gauge contribution. In addition, the gauge interactions give

$$\begin{aligned} 0 = \ln(Z_{2Q^A}) &= \ln\left(\frac{Z_{\tilde{\alpha}_0} Z_{\tilde{\phi}}^2}{Z_g^2}\right), \\ 0 = \ln(Zg_{2Q^A}) &= \ln\left(Z_{\tilde{\phi}}^3\right) \end{aligned} \tag{5.70}$$

where we used the Ward identity $Z_g^2 Z_A = 1$. We can solve the system of equations to get

$$Z_{\tilde{\phi}} = Z_\psi = Z_{\tilde{y}} = \frac{Z_g^2}{Z_{\tilde{\alpha}_0}} = 1 + \mathcal{O}(\delta^{-1}), \quad Z_{\tilde{\lambda}} = 1 + \boldsymbol{A}. \tag{5.71}$$

The beta-functions are then

$$\beta_{\tilde{y}} = \beta_{g^2/\tilde{\alpha}_0} = \gamma_\psi = \gamma_{\tilde{\phi}} = \gamma_A = O(\delta^{-1}), \qquad \beta_{\tilde{\lambda}} = \frac{1}{8\pi^2}6^{4/3}\left(3\frac{g^4}{\tilde{\alpha}_0^2} - \tilde{y}^4\right), \tag{5.72}$$

Case C and D

Case C is analogous to case B with the only difference that this time the fermion decouples. Thus $\beta_{\tilde{\lambda}} = 1/8\pi^2 6^{4/3}(3g^4/\tilde{\alpha}_0^2)$. In fact, due to the constant non-minimal coupling the fermion is weakly coupled during inflation. In case D all fields are weakly coupled and nothing runs. All couplings are suppressed and $\beta_{\tilde{\lambda}} = \mathcal{O}(\delta^{-1})$.

5.5.3 Renormalization group equations

Generalizing to full SM spectrum, the RGEs during inflation are

$$\begin{aligned} \beta_{\tilde{y}} &= \beta_{g^2/\tilde{\alpha}_0} = \gamma_\psi = \gamma_{\tilde{\phi}} = \gamma_A = \mathcal{O}(\delta^{-1}), \\ \beta_{\tilde{\lambda}} &= \frac{1}{8\pi^2}6^{4/3}\left(3f_1\sum_i \frac{g_i^4}{\tilde{\alpha}_0^2} - f_2 3\tilde{y}_t^4\right), \end{aligned} \tag{5.73}$$

The summation is over $g_i = \frac{1}{2}\{g_2^2, g_2^2, \sqrt{g_1^2 + g_2{}^2}\}$, with g_1 and g_2 the gauge coupling of the SU(2) and U(1) hypercharge gauge groups respectively. Here we assumed universal non-minimal couplings for both gauge groups, but that can be generalized. The f_i take on a value of zero/one depending on the non-minimal couplings of the fermion/gauge fields, specifically

$$
\begin{aligned}
\text{Case A}: &\quad (f_1, f_2) = (0,1), \\
\text{Case B}: &\quad (f_1, f_2) = (1,1), \\
\text{Case C}: &\quad (f_1, f_2) = (1,0), \\
\text{Case D}: &\quad (f_1, f_2) = (0,0).
\end{aligned} \tag{5.74}
$$

5.6 Predictions for inflation

We are now in the conditions to calculate corrections to the inflationary observables n_s and r due to the running of the couplings. The SM small field EFT and the inflationary large field EFT are matched in the midfield regime. As extensively discussed, the corrections due to unknown UV physics and unknown running (due to $\mathcal{O}(\varepsilon, \epsilon)$-corrections) gives rise to threshold corrections. Here we will follow the approach outlined in chapter 3 sec. 3.5.2 and parameterize these by a jump in the couplings in the midfield regime.

Before moving to the actual numerical implementation, in the same way as in the previous chapters we can do an analytical estimate, this time using the support of the simple RGEs just derived.[18]

[18] In sec. 2.3.3 we considered a toy model with the same Lagrangian for the scalar field. For illustrative purpose we discussed the RG improved action in terms of h and in terms of ϕ. In both cases, to compute the inflationary parameters one has to define a canonical field obtained by absorbing the contribution from the anomalous dimension. We showed that one find the same results for the slow roll parameters if one uses the approximations $\gamma_h \approx \text{const}, \gamma_\phi \approx \text{const}$ and the relations (2.87) between beta-functions and anomalous dimensions. As already stressed, in the full model considered in this chapter, the running of the original parameters $\beta_\lambda, \mathcal{M}, \gamma_\phi$ cannot be found independently. Thus, working in terms of ϕ the just mentioned approximations cannot be checked explicitly. However, by an analogous computation one can see that improving in terms of $\tilde{\phi}$ gives the same result as improving in terms of h with the assumptions $\gamma_h = 3\gamma_{\tilde{\phi}} \approx \text{const}$. These assumptions are indeed valid, as we showed in the previous section.

5.6.1 Renormalization group dependence

We consider the RG improved effective action in terms of the canonical field h defined in 5.16 as[19]

$$h = \frac{\sqrt{\lambda}\phi^3}{6\mathcal{M}^2} = \frac{\tilde{\phi}^3}{6}. \tag{5.75}$$

We can neglect the anomalous dimension contribution since $\gamma_h = \mathcal{O}(\delta^{-1}), \gamma'_h = \mathcal{O}(\delta^{-1})$. In fact, from (5.75), $\gamma_h = 3\gamma_{\tilde{\phi}}$, and in all cases considered in the previous section $\gamma_{\tilde{\phi}} = \mathcal{O}(\delta^{-1})$. Thus the RG improved action becomes

$$\mathcal{L} = \sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{\tilde{\lambda}(t)}{4}h^{4/3}\right). \tag{5.76}$$

We choose as renormalization scale the top mass. The RG time is then

$$t = \ln\left(\frac{\mu}{m_{\mathrm{t}}^{\mathrm{EW}}}\right), \qquad \mu = y\phi = \tilde{y}\tilde{\phi} = \tilde{y}\, 6^{1/3}h^{1/3}. \tag{5.77}$$

Often the Yukawa coupling is neglected for simplicity as $y_t = \mathcal{O}(1)$. However, one should keep $\tilde{y}_t$ explicitly as it can be small. Indeed, using as example the tree level relations derived at the beginning (see eq. (5.18)), $\mathcal{M} = 10^{-8}\lambda^{-1/4}$ and $\tilde{y} = y\lambda^{-1/6}\mathcal{M}^{2/3} = 10^{-8/3}y\lambda^{-1/4}$. With $y \simeq 10^{-1}$ and $\lambda \simeq 10^{-4}$ we have $\tilde{y} \simeq 10^{-4}$.

Running effects enter the observables, because calculating slow roll parameters (derivatives of the potential) also involves taking the derivative of the $\tilde{\lambda}$-coupling in the potential:

$$\frac{d\tilde{\lambda}(\mu)}{dh} = \frac{d\tilde{\lambda}(\mu)}{dt}\frac{dt}{dh} \equiv \beta_{\tilde{\lambda}}\frac{dt}{dh}, \qquad \frac{d\beta_{\tilde{\lambda}}}{dt} \equiv \beta'_{\tilde{\lambda}} = \mathcal{O}(\delta^{-1}), \tag{5.78}$$

where the last equation is determined by the RGEs summarized in (5.73). Furthermore, the derivative of the RG time with respect to the canonical field becomes

$$\frac{dt}{dh} = \frac{1}{3h(1-\beta_{\tilde{y}_t}/\tilde{y}_t)} \simeq \frac{1}{3h}, \tag{5.79}$$

where in the last step we use $\beta_{\tilde{y}} = \mathcal{O}(\delta^{-1})$ valid in all cases. When the top quark decouples and the gauge boson remains in the spectrum such as in case C, the gauge boson mass is the appropiate scale and the same argument holds after replacing $\tilde{y}_t$ with $g/\tilde{\alpha}_0$. This saves us from working with a transcendental equation where the RG dependence of the optimal choice t is on both side of its definition

[19] This correspond to the example of section 2.3.3, eq. (2.78).

which was indeed the case in the previous chapter and in Higgs inflation. The potential slow roll parameters are

$$\begin{aligned}\epsilon &\equiv \frac{1}{2}\left(\frac{V_h}{V}\right)^2 = \frac{8}{9h^2}\left(1+\frac{\beta_{\tilde\lambda}}{4\tilde\lambda}\right)^2\\ \eta &\equiv \frac{V_{hh}}{V} \simeq \frac{4}{9h^2}\left(1+5\frac{\beta_{\tilde\lambda}}{4\tilde\lambda}\right).\end{aligned} \tag{5.80}$$

For the inflationary observables we need the slow roll parameters evaluated $N_\star$ e-folds before the end of inflation:

$$N_\star \simeq \int_{h_{\rm end}}^{h_*} \mathrm{d}h \frac{1}{\sqrt{2\epsilon}} = \frac{3}{4m_{\rm P}^2}\int_{h_{\rm end}}^{h_*}\mathrm{d}h\, h\, D, \qquad \text{with } D = \left(1+\frac{\beta_{\tilde\lambda}}{4\tilde\lambda}\right)^{-1}. \tag{5.81}$$

Note that here, in contrast to the case of the Cosmological Attractors (see sec. 4.2.2), one cannot consider the beta-functions dependent factor D constant over the integration domain. In fact, expanding the integrand as in eq. (4.21), all the terms of the series are not higher order in the small parameter, in this case δ^{-1}, anymore. Thus, we can have an analytic estimate only *assuming* $D_\star$ nearly constant, i.e.

$$\frac{\beta_{\tilde\lambda}}{\tilde\lambda} \ll \left(\frac{\beta_{\tilde\lambda}}{\tilde\lambda}\right)^2. \tag{5.82}$$

In that case $N_\star \approx 3h_\star^2 D_\star/8$. To leading order in the $1/N_\star$ expansion the observables become

$$n_s - 1 \simeq -\frac{5}{3N_\star}\left(1+\frac{\beta_{\tilde\lambda}}{4\tilde\lambda}\right)_\star, \quad r \simeq \frac{16}{3N_\star}\left(1+\frac{\beta_{\tilde\lambda}}{4\tilde\lambda}\right)_\star \tag{5.83}$$

The running corrections can become large if the ratio $\beta_{\tilde\lambda}/(4\tilde\lambda)$ is order one during inflation. We stress that the above result is markedly different from that in the original Higgs inflation scenario and all the models studied in the thesis until now. In HI for example, the RG corrections to n_s and r disappear at first order in the $1/N_\star$ expansion due to a cancellation between the running dependence of the slow roll parameters and of the number of e-folds. In NHI, in contrast, such a cancellation does not take place. Therefore, the influence of the RG flow (and consequently, the influence of the theory's UV completion) on the inflationary predictions in NHI is parametrically larger than in HI.

5.6.2 Matching and running: numerical results for case A

The linear analysis of the previous section suggests an RG dependence of the inflationary observables. We now compute explicitly the size of these corrections.

We will calculate the predictions for n_s and r given the boundary conditions on the couplings at the EW scale determined by the two loop matching as in [124] and the threshold corrections in the midfield regime. In the small field regime we run the SM parameters $\{\lambda, y, ..\}$ with the 2-loop SM beta-functions [135, 124]. At the boundary between the two regimes, i.e.

$$\delta|_{t_{\mathrm{eq}}} = 1 \implies \phi_{\mathrm{eq}} = \left(\frac{4\mathcal{M}^4}{\lambda(t_{\mathrm{eq}})}\right)^{1/4}, \tag{5.84}$$

we switch to the rescaled couplings $\{\tilde{\lambda}, \tilde{y}, ..\}$. Threshold corrections can be incorporated (and parameterized) by a jump in the coupling constants at t_{eq}. Beyond the matching point we run with the one-loop beta-functions valid in the large field regime (5.73). The power spectrum constraint fixes

$$P_{\mathcal{R}} = 2 \cdot 10^{-9} \implies \tilde{\lambda}_\star \equiv \tilde{\lambda}(t_\star) = 4 \cdot 10^{-10}, \tag{5.85}$$

where $t_\star$ is as usual the value of the RG time at horizon crossing.

The large field RGEs (5.73,5.74) for case A are

$$\beta_{\tilde{y}} \approx 0, \qquad \frac{\beta_{\tilde{\lambda}}}{\tilde{\lambda}} = -\frac{3 \times 6^{4/3}}{8\pi^2}\frac{\tilde{y}^4}{\tilde{\lambda}}. \tag{5.86}$$

The first equation trivially implies $\beta'_{\tilde{\lambda}} \approx 0$. The matching conditions at the boundary depend on the scale $\mathcal{M}$

$$\tilde{\lambda}_{\mathrm{eq}} = 6^{4/3}\lambda_{\mathrm{eq}}^{1/3}\mathcal{M}^{8/3}, \qquad \tilde{y}_{\mathrm{eq}} = y_{\mathrm{eq}}\lambda_{\mathrm{eq}}^{-1/6}\mathcal{M}^{2/3}, \tag{5.87}$$

where we used (5.37), and we introduced the notation $X(t_{\mathrm{eq}}) \equiv X_{\mathrm{eq}}$. The predictions for n_s and r depend on the ratio $(\beta_{\tilde{\lambda}}/\tilde{\lambda})_\star$ where $\lambda_\star$ is fixed. We can now understand how they depend on the running. Different boundary conditions at the EW scale will result in different values of $\mathcal{M}$ required to adjust the matching conditions at t_{eq} in such a way that $\tilde{\lambda}_\star = 4 \cdot 10^{-10}$ is obtained. Furthermore, different values of $\mathcal{M}$ (and λ) at the matching point will give a different value for $\tilde{y}_{\mathrm{eq}}$. Since $\beta_{\tilde{y}_t} \approx 0$, $\tilde{y}_{\mathrm{eq}} = \tilde{y}_\star$ and this value will determine the correction to the inflationary parameters.

Given the simple form of (5.86) we can integrate $d\tilde{\lambda}/dt = \beta_{\tilde{\lambda}}$ explicitly

$$\tilde{\lambda}(t) = \tilde{\lambda}_{\mathrm{eq}} + (t - t_{\mathrm{eq}})\beta_{\tilde{\lambda}} = \tilde{\lambda}_{\mathrm{eq}} + \ln\left(\frac{\tilde{y}\tilde{\phi}}{y_{\mathrm{eq}}\phi_{\mathrm{eq}}}\right)\beta_{\tilde{\lambda}}. \tag{5.88}$$

It is possible to express $\tilde{\lambda}$ as a function of the field h and the low energy parameters at the matching point t_{eq}. Using

$$\beta_{\tilde{\lambda}} = -\frac{3 \times 6^{4/3}}{8\pi^2}\tilde{y}_{\mathrm{eq}}^4 = -\frac{3 \times 6^{4/3}}{8\pi^2}y_{\mathrm{eq}}^4\lambda_{\mathrm{eq}}^{-2/3}\mathcal{M}^{8/3} \equiv C(t_{\mathrm{eq}}) \tag{5.89}$$

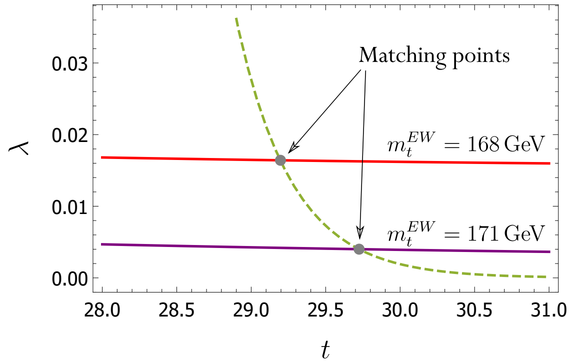


Figure 5.2: *In red and purple the running of the SM quartic coupling $\lambda(t)$ for $m_h = 125.6\,\text{GeV}$ and different values of the top mass. The dashed green line is the curve $\alpha e^{-8t/3}$ from (5.94). The intersection points give $(t_{\rm eq}, \lambda_{\rm eq})$ and define the matching point. A smaller top mass implies a larger $\lambda_{\rm eq}$, and consequently larger corrections to the observables.*

we have

$$\tilde{\lambda}(h, t_{\rm eq}) = 6^{4/3}\lambda_{\rm eq}^{1/3}\mathcal{M}^{8/3} + C(t_{\rm eq})\ln\left(\frac{h^{1/3}y_{\rm eq}\lambda_{\rm eq}^{-1/6}\mathcal{M}^{2/3}6^{1/3}}{m_t^{\rm EW}e^{t_{\rm eq}}}\right), \tag{5.90}$$

where we used $y_{\rm eq}\phi_{\rm eq} = m_t^{\rm EW}e^{t_{\rm eq}}$ and (5.77). Once we determine $t_{\rm eq}$, the corrections to n_s and r proportional to $C(t_{\rm eq})/\lambda_\star$ can be computed. The value of (5.90) at the field value $h_\star$ is fixed by (5.85), i.e. $\tilde{\lambda}(h_\star, t_{\rm eq}) = \lambda_\star$. This, together with the relation defining the boundary (5.84) forms a system of two equations with three unknowns $\{t_{\rm eq}, h_\star, \mathcal{M}\}$. In order to close the system we add the equation for the number of e-folds (as usual we use $N_\star = 60$) and $\epsilon_V(h_{\rm end}) = 1$. Summarizing, we want to solve the following system of equations[20]

$$\tilde{\lambda}(h_\star, t_{\rm eq}) = \tilde{\lambda}_\star, \qquad \mathcal{M} = \frac{\lambda_{\rm eq}^{1/4}}{\sqrt{2}y_{\rm eq}}m_t^{\rm EW}e^{t_{\rm eq}}, \qquad N_\star = \int_{h_{\rm end}}^{h_\star}\frac{dh}{\sqrt{2\epsilon}}, \qquad \epsilon(h_{\rm end}) = 1, \tag{5.91}$$

for the four unknowns[21] $\{h_\star, t_{\rm eq}, \mathcal{M}, h_{\rm end}\}$. The values for $\{\tilde{\lambda}_\star, N_\star, m_t^{\rm EW}\}$ are fixed. Further, $\lambda_{\rm eq}, y_{\rm eq}$ are the SM running couplings evaluated at $t_{\rm eq}$; they depend implicitly on the boundary conditions at the electroweak scale for λ, y, i.e. on the mass of the top and the Higgs measured at the LHC.

[20] We used $\phi_{\rm eq} = y_{\rm eq}^{-1}\, m_t^{\rm EW}e^{t_{\rm eq}}$ to rewrite (5.84) in terms of $\mathcal{M}$ in the second equation.

[21] In practice we do not solve explicitly the last equation but approximate $h_\star \gg h_{\rm end} \simeq 0$ (it turns out that $h_{\rm end}$ is always one or two orders of magnitude smaller than $h_\star$).

Boundary conditions at the electroweak scale

First we discuss the results without threshold corrections, when the observables only depend on the boundary conditions at the EW scale. Without running, $\mathcal{M}$ would be fixed by $\lambda_\star$ (5.18) and its constant value would define the matching point (5.84). Instead $\{\mathcal{M}, t_{\rm eq}\}$ are coupled by the first two equations in (5.91). Before solving it numerically, it is useful to build some idea about which results to expect. To do so we solve $\mathcal{M}$ from the first equation in (5.84) by neglecting the running of $\tilde{\lambda}$ in the large field regime, i.e the second term in (5.90); this will give some extra correction but does not change the qualitative nature of the solution. We find

$$\mathcal{M} \simeq \frac{\lambda_\star^{3/8}}{6^{1/2}\lambda_{\rm eq}^{1/8}}. \tag{5.92}$$

Substituting in the second equation, and solving for $\lambda_{\rm eq}$ gives

$$\lambda_{\rm eq} = \lambda_\star \left(\frac{y_{\rm eq}}{\sqrt{3} m_t^{\rm EW} e^{t_{\rm eq}}}\right)^{8/3} e^{-8t_{\rm eq}/3} \equiv \alpha e^{-8t_{\rm eq}/3} \,. \tag{5.93}$$

Thus $\lambda_{\rm eq}$ is given by the intersection of the two curves

$$\lambda(t) = \alpha e^{-8t/3}, \tag{5.94}$$

with α depending (weakly) on the boundary conditions at the EW scale. For example, for fixed Higgs mass a larger top mass will give a larger matching point $t_{\rm eq}$, and thus a smaller $\lambda_{\rm eq}$ as the coupling value decreases with renormalization time t. To first approximation we have that the corrections to the inflationary parameters go as $(\beta_{\tilde{\lambda}}/4\tilde{\lambda})|_\star$. From (5.89) and (5.90) we have

$$\left(\frac{\beta_{\tilde{\lambda}}}{4\tilde{\lambda}}\right)\Big|_\star = \left(\frac{\beta_{\tilde{\lambda}}}{4\tilde{\lambda}}\right)\Big|_{\rm eq} \left(1 + \left(\frac{\beta_{\tilde{\lambda}}}{4\tilde{\lambda}}\right)\Big|_{\rm eq} \ln(\ldots)\right)^{-1}, \tag{5.95}$$

with

$$\left(\frac{\beta_{\tilde{\lambda}}}{4\tilde{\lambda}}\right)\Big|_{\rm eq} = -\frac{3y_{\rm eq}^4}{32\pi^2}\frac{1}{\lambda_{\rm eq}} \propto \frac{1}{\lambda_{\rm eq}}, \tag{5.96}$$

Thus the corrections increase for smaller $\lambda_{\rm eq}$, i.e. for a larger top mass. This is shown in the left plot of fig. 5.2. ln(..) is the log appearing in eq. (5.90) and numerically is always order $\mathcal{O}(10)$ for different initial conditions. This log enhances the size of the corrections (see fig. 5.7). For example, for the range of boundary conditions for which eq. (5.96), that depends only on SM running, is of order $10^{-2} - 10^{-1}$, the correction in (5.95) is already order one. The full numerical results are shown in fig. 5.4.

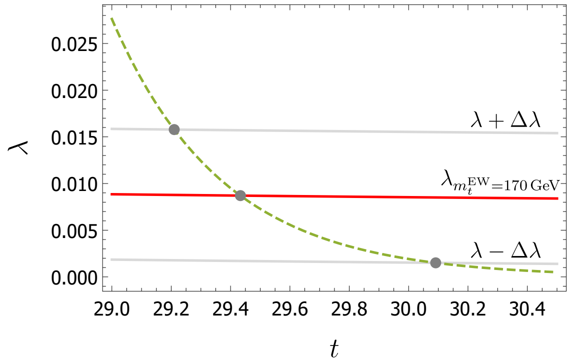


Figure 5.3: *In red the running of the SM quartic coupling for* $m_h = 125.6\,\text{GeV}$ *and* $m_t = 170\,\text{GeV}$*. The dashed green line is the curved* $\alpha e^{-8t/3}$ *from (5.94). In the presence of threshold corrections* $(t_{\rm eq}, \lambda_{\rm eq})$ *is given by the intersections between this curve and the shifted running coupling. Shown are two examples with* $\Delta\lambda = \pm 7 \cdot 10^{-3}$.

Threshold corrections

Let's now include threshold corrections, which we model by a shift in $\lambda \to \lambda + \Delta\lambda$ at the boundary between the small and large field regime where $\Delta\lambda$ has to be considered as the sum of the contribution from higher dimensional operators to the running of λ. This means that at $t_{\rm eq}$, the value of λ that is matched to the tilde parameters is shifted

$$\lambda_{\rm eq}^{\rm new} = \lambda(t_{\rm eq}) + \Delta\lambda \tag{5.97}$$

In principle there is also a jump in the yukawa coupling, which we ignore (this gives a degeneracy with the EW boundary value of the yukawa coupling). To see the effect of the threshold corrections, we solve the same system of equations (5.91), but with the substitution $\lambda_{\rm eq} \to \lambda_{\rm eq}^{\rm new}$. We fix the boundary conditions at the EW scale and let $\Delta\lambda$ vary. In order to understand the numerical results we can go through the same steps as before, with the only difference that (5.94) now becomes $\lambda(t) + \Delta\lambda = \alpha e^{-8t/3}$. For fixed boundary EW values $t_{\rm eq}$ is now given by the intersection between the shifted curve and the same $\alpha e^{-8t/3}$ as before. It follows that a positive/negative $\Delta\lambda$ will cause a smaller/bigger correction on (n_s, r) with respect to the tree level results. This is shown in fig. 5.3. The results in presence of threshold correction are shown in fig. 5.5.

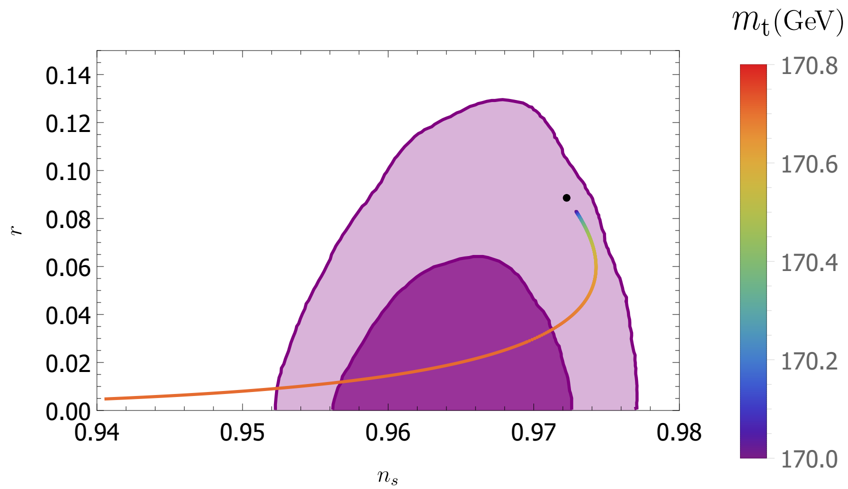


Figure 5.4: *Different predictions for the inflationary parameters with fixed Higgs mass $m_h = 125.6\,\mathrm{GeV}$, and varying top mass compared to the 2015 Planck data (1 and $2\,\sigma$). The black dot represents the tree level result which is reached for unrealistic values $m_t < 160\mathrm{GeV}$.*

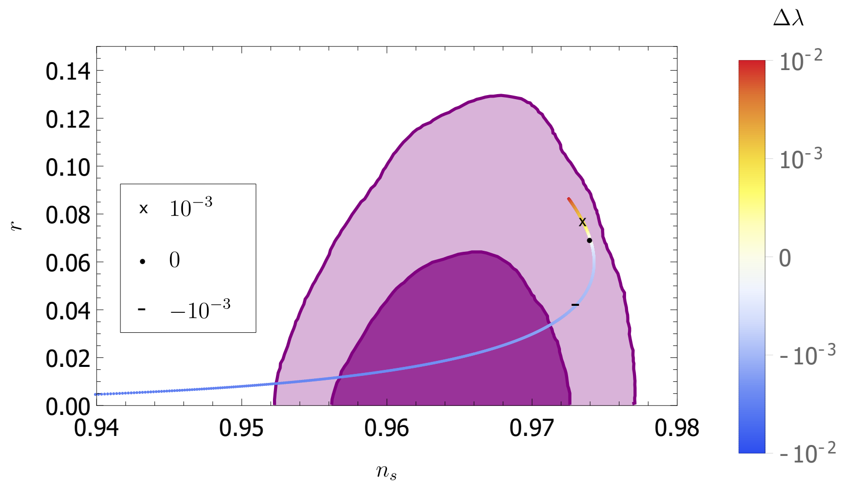


Figure 5.5: *The effect of threshold corrections is completely degenerate with changing boundary conditions at the EW scale. The black dot represents the result without threshold corrections for the top/Higgs mass values $m_t = 170.5\,\mathrm{GeV}$ and $m_h = 125.6\,\mathrm{GeV}$. Note that, in agreement with fig. 5.3 negative kicks give a bigger spread in the predictions.*

5.6.3 Matching and running: other cases

In case B the gauge bosons are back in the spectrum during inflation. We proceed exactly in the same way as in the previous section with different RGEs in the large field regime (5.73) with $(f_1, f_2) = (1, 1)$. The running of $\tilde{\lambda}$ depends on the gauge couplings only through the combinations $g_i^2/\tilde{\alpha}_0$, i.e.

$$\beta_{\tilde{\lambda}} = \frac{1}{8\pi^2} 6^{4/3} \left(3 \sum_i \frac{g_i^4}{\tilde{\alpha}_0^2} - 3\tilde{y}_t^4 \right) , \tag{5.98}$$

with $\beta_{\tilde{y}} = \mathcal{O}(\delta^{-1})$ and $\beta_{g^2/\tilde{\alpha}_0} = \mathcal{O}(\delta^{-1})$. Thus, for our purpose, we can define[22] $\tilde{g}_i^2 = g_i^2/\tilde{\alpha}_0$ and add to (5.87) the following matching condition for the gauge couplings

$$\tilde{g}_{i\mathrm{eq}}^2 = \frac{g_{i\mathrm{eq}}^2}{\tilde{\alpha}_{0\mathrm{eq}}} = g_{i\mathrm{eq}}^2 \lambda_{\mathrm{eq}}^{-1/3} \alpha_0^{-1} \mathcal{M}^{4/3}. \tag{5.99}$$

The results of the numerical implementation for case B (without threshold corrections) are given in fig. 5.6. As in case A the tree level results are never reached for realistic values of the top mass. However, for the values of the boundary conditions that allow inflation to happen, we note that there is less dependence on the RG flow. This can be explained by looking at fig. 5.7. The positive contribution from the gauge bosons to $\beta_{\tilde{\lambda}}$ leads to $(\beta_{\tilde{\lambda}}/\tilde{\lambda})_{\mathrm{eq}}$ changing rapidly for a relatively narrow range of the top mass, roughly between $171 - 171.2\,\mathrm{GeV}$. Since the size of the correction proportional to $(\beta_\lambda/\lambda)_\star$ are enhanced through (5.95), we jump quickly from having zero corrections to spoil inflation completely, i.e. from $(\beta_\lambda/\lambda)_\star \simeq 0$ to $(\beta_\lambda/\lambda)_\star \simeq -1$. On the contrary in case A, the correction grows more linearly for the range of top mass shown in the left plot in fig. 5.7.

[22] In principle g and $\tilde{\alpha}_0$ are two independent parameters, see (5.38). Knowing the running of both separately would be irrelevant for our analysis.

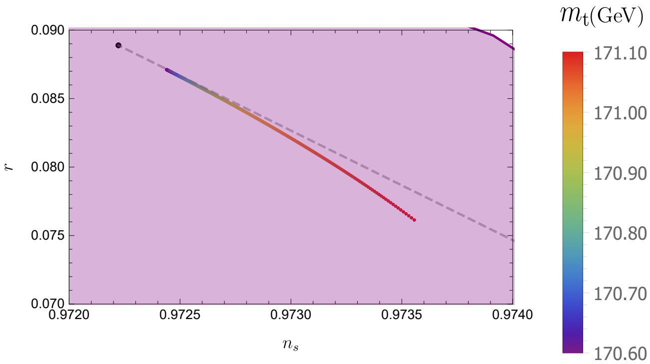


Figure 5.6: *(n_s, r) predictions for Case B without threshold corrections and for $\alpha_0 = 1$. The dashed line represents the analytic approximation of eq. (5.83). We have zoomed in on the Planck plot since the inflationary observables are less RG dependent than in case A. This outcome can be explained (see main text) by looking at fig. 5.7.*

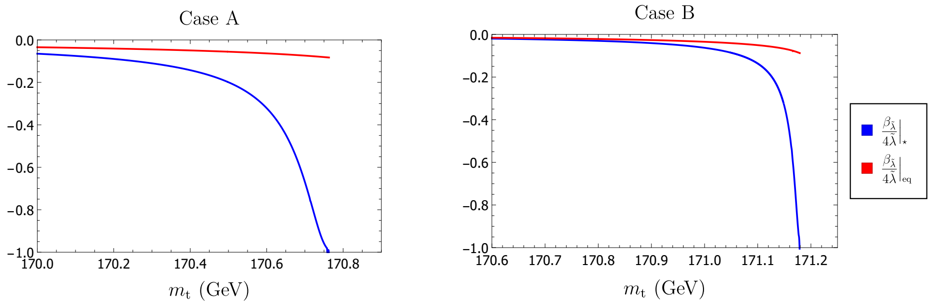


Figure 5.7: *The value of $(\beta_{\tilde\lambda}/\tilde\lambda)_{\rm eq}$ (red curves) depends only on the SM couplings at the EW scale and on the running (no threshold corrections are included). Its value translates in the actual size of the RG correction (blue curves) through the enhancement given in (5.95). For $(\beta_{\tilde\lambda}/\tilde\lambda)_\star < -1$, the potential cannot sustain 60 e-folds of inflation anymore. In case B, due to the gauge boson contributions, the absolute value of the correction takes almost a step function shape. This causes a smaller region in parameter space (given by the boundary conditions at the EW scale) with respect to case A for which the predictions are sensitive to the RG flow.*

In case C the implementation is very similar to the previous cases with the quantitative difference that now, since the top quark decouples, $\beta_{\tilde\lambda} > 0$ in the large field

regime. This implies positive corrections to n_s, r (as can be understood from the approximate solution (5.83)). Thus, by increasing the top mass (i.e. increasing the size of the corrections), the inflationary parameters“move up” in the Planck plot to higher values of the tensor-to-scalar ratio r compared to the tree-level result.

Finally, case D gives trivially the tree level results since $\beta_{\tilde{\lambda}} = \mathcal{O}(\delta^{-1})$.

5.7 Summary

We have investigated the renormalization of New Higgs inflation and how this can affect its predictions. In many respects, the situation is analogous to the original Higgs inflation (HI) proposal, where inflation takes place thanks to a non-minimal coupling of the Higgs field to the Ricci scalar. In both models, non-renormalizable interactions are added to the SM and consequently tree level unitarity is violated at energies below the Planck scale. At every stage in the Universe's history the field-dependent unitarity cutoff is above the typical scale set by the Higgs potential as we showed in sec. 5.3. In both the small field and large field regime, one can easily identify an order parameter in which a renormalizable EFT can be organized. However, the theory is non-renormalizable in the mid field regime, and new operators are needed to absorb the divergences. To make the above statements more explicit we have calculated the one-loop beta-functions in the large field regime. We have used a covariant (under field redefinitions) approach which takes into account the non-trivial geometry of the field space manifold. We consider different cases where also gauge boson/fermions are non-minimally coupled to gravity. The results for the RGEs are summarized in (5.73). Given the simple form of the beta-functions, it may seem that by introducing the formalism of sec. 5.4 we took a sledgehammer to crack a nut. However, as we saw for example in case B, the covariant formalism is necessary to have a consistency check and thus obtain reliable results. This suggests that in general a covariant formalism is desirable to compute the RGEs in presence of a non-flat metric in field space.

Although details differ, such as the value of the cutoff scale and counterterms, the above qualitative discussion is equally valid for both HI and NHI. The key difference between the two scenarios is the RG/UV sensitivity of the inflationary observables. In HI, the running corrections to the spectral index and the tensor-to-scalar ratio are suppressed to leading order in the $1/N_\star$-expansion (as shown in the previous chapters). For inflation in the universal regime, the predictions are thus robust. This is in contrast with NHI, where the inflationary predictions do depend on the running. This dependence can also be seen as a virtue since in this case the boundary conditions at the EW scale as well as the (for the moment) unknown

threshold corrections will leave a direct imprint on the inflationary parameters, which thus can be probed. The explicit dependence of n_s and r on the running coupling $\tilde{\lambda}$ can be computed via an analytical approximation with results given in (5.83). The full non-linear RG dependence for the original NHI proposal is shown in fig. 5.4 (assuming negligible threshold corrections and different boundary conditions at the EW scale), and in fig. 5.5 (fixing the EW boundary conditions and including threshold corrections). In fig 5.6 the sensitivity of the predictions when the gauge bosons are also non-minimally coupled (case B) is illustrated.

6 Outlook

The starting question of this thesis was whether it is possible to connect low energy observables to data from the Cosmic Microwave Background in a theoretically consistent way. We have seen that, even in the minimalistic and self-consistent approach, the non-renormalizability character of the Higgs inflationary models breaks the connection between low and high energy parameters. Therefore, as a general lesson, we have learned that additional physics is demanded to describe inflation.

Nevertheless, as our analysis in chapter 3 reveals,[1] in SM Higgs inflation the inflationary observables are (almost) insensitive to the details of the UV completion. Through an analytic study of the RG improved action we were able to explain the reasons for this insensitivity. This opened the way to a systematic study of a more general class of models. In particular, we have shown in chapter 4 that, due to their underlying mathematical structure, the full class of Cosmological Attractors is protected against UV corrections. Although the insensitivity to the RG flow strengthens the predictivity of a model, it also depletes our hopes of connecting inflationary observables to the low energy regime and/or to the UV physics. In this sense, it is interesting to study other models outside the previous set that are sensitive to the RG flow. This motivated the study of new Higgs inflation in chapter 5. From our findings we have learned a second lesson: in some cases the classical description is not enough to sensibly compare the model to the Planck data.

As discussed at length in this thesis, our analysis and a big part of the literature on Higgs inflation (and its variations) comes with an underlying assumption on the UV completion: the new physics that is important in the mid-field regime, and that is needed for consistency, does not prevent the Higgs to be the inflaton at

[1] We end with a brief conclusion and an outlook to the future. For a more detailed discussion of the results we refer the reader to the summary of the respective chapters.

large scales.[2] Constructing an explicit UV completion with these qualities remains a big unsolved problem. A possibility that we explored in the context of Higgs inflation is to adapt some of the beyond the SM proposals to unitarize the SM in the absence of a Higgs field [104] (these models were proposed, of course, before the discovery of the Higgs boson) to lift the tree level unitarity cutoff of the theory. Although we found that the Goldstone/gauge sector can be unitarized in this way, it is still unclear how to deal with the non-trivial interactions of the radial Higgs component. Despite the issues encountered, I think it is still worth further investigating this and other possibilities, such as the possible role of a strong phase in solving the unitarity problem without the need for extra degrees of freedom.

Regarding Cosmological attractors, our results [2] are valid for single field models and for one-loop improved effective actions. It might be interesting to see how the picture changes in the presence of a non-trivial manifold in field space (i.e. in the multifield case). Moreover, going beyond the one-loop improved order forces one to deal with some subtleties linked to RG improving the action in presence of a non-canonical kinetic term that we have only just begun to explore.

The lost connection between the parameters in the low and high field regime (and the need for threshold corrections in the middle) in the inflationary models described comes from a perturbative study. I believe that looking at the problem from a different perspective, using the framework of the exact RG flow [167], can reveal interesting surprises. Apart from the primary interest in solving the physical problem, this will also be an opportunity to apply these techniques to a cosmological set-up.

I would like to conclude with a simple thought. Particle physics data from the LHC has not found any evidence for physics beyond the Standard Model. At the same time, cosmological measurements from the Planck satellite are perfectly consistent with slow roll single-field inflation. As this thesis shows in its own little way, even from this simple scenario there is still a lot that can be learned. I think that it is thus well worth further investigating the possibilities of connecting present-day particle physics with inflation, and to keep running in the early Universe.

[2] To be precise, we assume that the UV completion only enters via the RGEs and does not (significantly) affect the tree-level inflationary potential, see sec. 3.4.2.

A CMB parameters at higher order in δ in Higgs inflation

In this appendix we compute the perturbation spectrum at second order in the slow roll expansion for SM Higgs inflation. At this order the results do depend on the running. We check that there is no accidental cancellations or terms blowing up, and that the leading order results are indeed the dominant terms.

In order to compute the CMB parameters (n_s, r) at second order in $\delta = 1/(\xi\phi^2)$ we need the slow roll parameter η at 2nd order, and ϵ at 3rd order (as $\sqrt{\epsilon}$ enters the integral for the number of efolds). Define

$$\mathcal{K} \equiv \frac{V_h}{V} = \frac{1}{h_\phi}\frac{V_\phi}{V} = \sqrt{\frac{8}{3}}\frac{\left(1+\frac{\beta_\lambda}{4\lambda}\right)\delta(\delta+1)}{(\frac{\delta+1}{6\xi}+1)^{\frac{1}{2}}\left(\delta+1+\frac{\beta_\xi}{2\xi}\right)}. \tag{A.1}$$

Then the slow roll parameters can be written as

$$\epsilon = \frac{1}{2}\mathcal{K}^2 = \frac{8}{3}\left(1+\frac{\beta_\lambda}{4\lambda}\right)\frac{\delta^2(\delta+1)^2}{A\,B^2}, \tag{A.2}$$

$$\eta = \frac{V_{hh}}{V} = \frac{1}{V}\frac{d}{dh}(\mathcal{K}V) = \mathcal{K}_h + \mathcal{K}^2 = \frac{1}{h_\phi}\mathcal{K}_\phi + 2\epsilon, \tag{A.3}$$

where we have defined

$$A = \left(\frac{\delta+1}{6\xi}+1\right), \qquad B = \left(1+\frac{\beta_\xi}{2\xi}+\delta\right). \tag{A.4}$$

Now $\mathcal{K}_h = h_\phi^{-1}\mathcal{K}_\phi$ with $h_\phi = \sqrt{\gamma_{\phi\phi}}$ given in terms of the field space metric (3.12). Explicitly

$$\mathcal{K}_h = \frac{1}{h_\phi}\left(\frac{\partial\mathcal{K}}{\partial\delta}\delta_\phi + \frac{\partial\mathcal{K}}{\partial f_j}\frac{df_j}{dt}\frac{dt}{d\phi}\right), \tag{A.5}$$

where $f_j \equiv \{\lambda, \beta_\lambda, \xi, \beta_\xi\}$ and thus $df_j/dt \equiv \{\beta_\lambda, \beta'_\lambda, \beta_\xi, \beta'_\xi\}$. The full non-expanded

slow roll parameters are given by (A.2) and[1]

$$\eta = \frac{V_{hh}}{V} = \frac{4}{3}\left(1+\frac{\beta_\lambda}{4\lambda}\right)\left(\frac{\delta^2(\delta+1)^2}{A\,B^2}\left(1+\frac{1}{12\xi}\frac{B}{A}+2\left(1+\frac{\beta_\lambda}{4\lambda}\right)\right)-\frac{\delta(\delta+1)(2\delta+1)}{A\,B^2}\right)$$
$$+\frac{2}{3}\left(\frac{\beta'_\lambda}{4\lambda}-\frac{\beta^2_\lambda}{4\lambda^2}\right)\frac{\delta^2(\delta+1)^2}{A\,B^2}+O\left(\frac{\beta_\xi}{\xi^2},\frac{\beta'_\xi}{\xi}\right) \tag{A.6}$$

For $\lambda_{\max} = -\beta_\lambda/4$ or equivalently $F = 0$, ϵ reduces to zero (extremum of the potential) while η reduces to (3.69).

In order to compute the number of efolds we expand ϵ at third order in δ, which means we need to expand $\mathcal{K}$ at second order in δ,

$$\mathcal{K} \approx k_1\delta + k_2\delta^2 + O(\delta^3). \tag{A.7}$$

Then ϵ is given by $\epsilon = \epsilon_0\delta^2 + \epsilon_1\delta^3 + O(\delta^4)$ with $\epsilon_0 = k_1^2/2$ and $\epsilon_1 = k_1k_2$. $N_\star$ becomes

$$N_\star = \int_{\phi_{\rm end}}^{\phi_\star} \frac{1}{\sqrt{2(\epsilon_0\delta^2+\epsilon_1\delta^3+..)}}h_\phi\,d\phi, \tag{A.8}$$

with

$$\epsilon \approx \frac{1}{2}k_1^2\delta_{\rm end}^2 = 1 \quad\Longrightarrow\quad \phi_{\rm end} \approx \left(\frac{4}{3}(1+\tfrac{1}{6\xi})\right)^{1/4}\left(\frac{F_{\rm end}}{\xi_{\rm end}}\right)^{1/2} \approx \left(\frac{F_{\rm end}}{\xi_{\rm end}}\right)^{1/2}. \tag{A.9}$$

To understand which terms are important we expand the integrand, i.e. $N_\star \equiv \int f$, schematically as $f \sim O(1/\sqrt{\delta}) + O(\sqrt{\delta}) + O(\delta) + ...$, where $\int O(1/\sqrt{\delta}) \propto \phi_\star^2$; $\int O(\sqrt{\delta}) \propto \ln(\phi_\star)$; $\int O(\delta) \propto 1/\xi\phi_\star$. The results can be written in term of $\delta_\star$ as[2]

$$N_\star \approx a_1\frac{1}{\delta_\star} + a_2\ln\delta_\star + \mathcal{C}, \tag{A.10}$$

with

$$a_1 = \frac{3}{4F_\star},\quad a_2 = \frac{3}{4F_\star}\left(\frac{1}{1+\frac{1}{6\xi_\star}}+\frac{\beta_{\xi\star}}{2\xi_\star(1+\frac{\beta_{\xi\star}}{2\xi_\star})}\right),\quad \mathcal{C} = -\frac{3}{4F_\star}\xi_\star\phi_{\rm end}^2 + a_2\ln(\xi_\star\phi_{\rm end}^2). \tag{A.11}$$

Now rewrite (A.10) as

$$\delta_\star = \frac{a_1}{N_\star} + \frac{a_2\delta_\star\ln\delta_\star}{N_\star} + \frac{\mathcal{C}\delta_\star}{N_\star}, \tag{A.12}$$

[1] For completeness $\frac{1}{3}\left(1+\frac{\beta_\lambda}{4\lambda}\right)\frac{\delta^2(\delta+1)^2}{A\,B^3}\left[\frac{\beta_\xi}{\xi^2}\left(\frac{B}{6A}+\beta_\xi\right)-\frac{\beta'_\xi}{\xi}\right] \equiv O\left(\frac{\beta_\xi}{\xi^2},\frac{\beta'_\xi}{\xi}\right)$.

[2] Following the arguments below (3.72) we neglect the implicit ϕ dependence of the couplings and β-functions.

which can be solved iteratively. At leading order $\delta_\star = a_1 N_\star^{-1} + O(N_\star^{-2})$ and plugging that back in (A.12) gives

$$\delta_\star = \frac{a_1}{N_\star} + \frac{a_2 a_1}{N_\star^2} \ln\left(\frac{a_1}{N_\star}\right) + \frac{\mathcal{C} a_1}{N_\star^2} + O(N_\star^{-3}). \tag{A.13}$$

Then ϵ evaluated at horizon exit becomes at second order

$$\epsilon_\star \approx \frac{4}{3}\left(1 + \frac{1}{6\xi_\star}\right) F_\star^2 \frac{a_1^2}{N_\star^2} = \frac{3}{4}\frac{1}{N_\star^2}\left(1 + \frac{1}{6\xi_\star}\right) \tag{A.14}$$

Expanding (A.6) at second order in δ and using (A.13) (we set also $\xi \gg 1$ for simplicity and we neglect $O(\beta_\xi/\xi^2, \beta'_\xi/\xi))$ we obtain

$$\begin{aligned}\eta_\star \approx & -\frac{1}{N_\star} + \frac{3}{2N_\star^2} - \frac{3}{4N_\star^2}\frac{1}{F_\star}\left[1 - \frac{1}{2F_\star}\left(\frac{\beta'_{\lambda\star}}{4\lambda_\star} - \frac{\beta_{\lambda\star}^2}{4\lambda_\star^2}\right)\right. \\ & \left. - \ln\left(\frac{\xi_{\rm end}}{\xi_\star}\frac{F_\star}{F_{\rm end}} N_\star\right) - \frac{\xi_\star}{\xi_{\rm end}} F_{\rm end}\right] + O(N_\star^{-3}).\end{aligned} \tag{A.15}$$

Therefore the CMB parameters are given by the expressions

$$\begin{aligned} n_s &= 1 + 2\eta_\star - 6\epsilon_\star \simeq 1 - \frac{2}{N_\star} - \frac{3}{2N_\star^2} - \frac{3}{2N_\star^2 F_\star}\left[1 - \frac{1}{2F_\star}\left(\frac{\beta'_{\lambda\star}}{4\lambda_\star} - \frac{\beta_{\lambda\star}^2}{4\lambda_\star^2}\right)\right. \\ & \quad \left. - \ln\left(\frac{\xi_{\rm end}}{\xi_\star}\frac{F_\star}{F_{\rm end}} N_\star\right) - \frac{\xi_\star}{\xi_{\rm end}} F_{\rm end}\right], \\ r &= 16\epsilon_\star \simeq \frac{12}{N_\star^2}. \end{aligned} \tag{A.16}$$

Turning off the running $F_\star = 1$, $\beta_i = 0$, the tree level result are recovered at second order in $N_\star^{-1}$, i.e. $n_s = 1 - \frac{2}{N_\star} - \frac{3}{N_\star^2} + \ldots$. The spectral index n_s feels the effect of the running only at second order. This dependence goes as $F_\star^{-1}$. Note, however, that for values of $F_\star$ close to zero the δ expansion breaks down, and we can no longer trust our analytical results. This is exactly the case where the potential has a maximum and we study the problem numerically.

B Covariant one-loop corrections in new Higgs inflation

B.1 Feynman rules

In this subsection we will give the Feynman rules for the action in terms of the covariant fields and set the notation. We leave the background values in terms of ϕ, as this makes the δ^{-1} expansion more transparent. The counterterms, and in particular the wave functions renormalization, are defined in the original action (5.41); they obey the usual relations from gauge invariance.[1] In the end it is useful to rewrite the counterterms and beta-functions in terms of the tilde fields and couplings defined in (5.37), as these represent the set of independent couplings (at leading order in the δ^{-1} expansion).

First, we expand the NHI action (5.41) in covariant fields as in (2.103). The covariant derivatives are defined with respect to the metric (5.44)

$$G_{IJ} = \{\gamma(\varphi^I)\delta_{ij}, k(\varphi^I)\eta_{\mu\nu}\} = \{(1+\delta)\delta_{ij}, (1+\alpha_A\delta)\eta_{\mu\nu}\}, \quad I, J = \{\phi, \theta, A\} \tag{B.1}$$

and evaluated on the background. The Q fields are defined in (5.46). From the expanded action we define the effective couplings

$$\begin{aligned}\mathcal{L}_{\text{int}} = &- \lambda_{mQ^\phi nQ^\theta}(Q^\phi)^m(Q^\theta)^n - y_{mQ^\phi nQ^\theta}(Q^\phi)^m(Q^\theta)^n\bar{\psi}(i\gamma^5)^\alpha\psi \\ &- y_{2\psi}\bar{\psi}\psi + g_L\bar{\psi}Q^A P_L\psi + g_R\bar{\psi}Q^\phi P_R\psi \\ &- (g_{Q^A\partial Q^\phi mQ^\phi nQ^\theta}\partial Q^\phi - g_{Q^A\partial Q^\theta mQ^\phi nQ^\theta}\partial Q^\theta)Q^A(Q^\phi)^m(Q^\theta)^n \\ &- g_{2Q^A mQ^\phi nQ^\theta}(Q^A)^2(Q^\phi)^m(Q^\theta)^n + \ldots\end{aligned}$$

With $\alpha = 1$ if the number $n =$ odd, and $\alpha = 0$ otherwise (signs are absorbed in

[1] Note that since the field redefinition (5.47) is non-linear, it is not a priori possible to define Z_Q.

the couplings). Equivalently we expand the kinetic term, for example

$$\mathcal{L}_{\rm k} = -\mathcal{K}_{Q^\phi 2\partial Q^I}(Q^\phi)^2(\partial Q^I)^2 - \mathcal{K}_{2Q^\theta 2\partial Q^I}(Q^\theta)^2(\partial Q^I)^2 + ... \tag{B.2}$$

All interactions are defined with a minus sign (the only exception is for one of the derivative interactions and the fermion-gauge interaction), and without numerical factors. This means that for a vertex with m Q^ϕ-fields and n Q^θ-fields and with or without fermion/gauge lines we have, respectively:

$$\begin{aligned} V^{(mQ^\phi nQ^\theta)} &= (-i)m!n!\lambda_{mQ^\phi nQ^\theta}, \\ V^{(mQ^\phi nQ^\theta 2\psi)} &= (-i)m!n!y_{mQ^\phi nQ^\theta}(i\gamma^5)^\alpha, \\ V^{(mQ^\phi nQ^\theta 2Q^A)} &= (-i)2!m!n!g_{2Q^A mQ^\phi nQ^\theta}. \end{aligned} \tag{B.3}$$

For the derivative interaction we get

$$\begin{aligned} V^{(Q^A\partial Q^j mQ^\phi nQ^\theta)} &= -ig_{Q^A\partial Q^j mQ^\phi nQ^\theta}(-ip^\mu), \quad j=\{\phi,\theta\} \\ V^{2Q^j 2\partial Q^I} &= 2!2!(-i\mathcal{K}_{2Q^j 2\partial Q^I})p^\mu p_\mu), \quad j=\{\phi,\theta\} \end{aligned} \tag{B.4}$$

with p the momentum running through the vertex.

The fermion, scalar and gauge propagators are given by:

$$\begin{aligned} -iD_\psi(p) &= q^{-1}\frac{-i(-\not{p}+m_\psi)}{p^2+m_\psi^2-i\epsilon}, \\ -iD_{Q^I}(p) &= \gamma^{-1}\frac{-i}{p^2+(m^2)_I^I-i\epsilon}, \\ -iD_{\mu\nu}(p) &\overset{\xi_G=0}{=} -ik^{-1}\frac{g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}}{p^2+m_A^2-i\epsilon}. \end{aligned} \tag{B.5}$$

where k^{-1} in the gauge boson propagator is the non-minimal gauge coupling factor, not to be confused with momentum. The scalar masses are given by the covariant expression $(m^2)_I^J = -G^{JK}\nabla_K\nabla_I\mathcal{L}$ evaluated on the background. The explicit expressions are given in (5.48) and (5.49).

In the next section we are going to calculate the 1-loop corrections to the effective potential and to the self-energies of all fields in the theory. The relevant counterterms are those of the quadratic interactions, and of the potential. The full set of Z-relations in the large field regime is given by (5.57).

B.2 Loop diagrams

B.2.1 Two point functions

Consider the 2-pnt functions of the various fields, which gives the wave functions and mass renormalizations. Start with the Higgs sector. We first give the result for both the Higgs and Goldstone, although their relation is fixed by the gauge symmetry, and they do give the same results for the counterterms. From the expansion of the kinetic term we include only terms that in NHI are not suppressed by δ^{-1}.

$$
\begin{aligned}
\Pi_{Q^\phi}\left(p^2\right) = \frac{1}{8\pi^2\epsilon}\Bigg\{ & 18\lambda_{3Q^\phi}^2\left(G^{\phi\phi}\right)^2 + 2\lambda_{Q^\phi 2Q^\theta}^2\left(G^{\theta\theta}\right)^2 + 12\lambda_{4Q^\phi}G^{\phi\phi}m_h^2 \\
& + 2\lambda_{2Q^\phi 2Q^\theta}G^{\theta\theta}m_\theta^2 - \Big[2y_{Q^\phi}^2\left(p^2+6m_\psi^2\right)q^{-2} + 8y_{2Q^\phi}m_\psi^3 q^{-1}\Big] \\
& + G^{AA}\Big[6g_{2Q^A 2Q^\phi}m_A^2 + 6g_{2Q^A Q^\phi}^2 + \frac{3}{4}G^{\theta\theta}\left(g_{Q^A\partial Q^\phi Q^\theta} + g_{Q^A\partial Q^\theta Q^\phi}\right)^2 p^2\Big] \\
& - 6\mathcal{K}_{2Q^\phi 2\partial Q^A}G^{AA}m_A^4\Bigg\} - \Big[\left(Z_{\partial Q^\phi}-1\right)p^2\mathcal{G}_{hh} + \left(Z_{\lambda 2Q^\phi}-1\right)2\lambda_{2Q^\phi}\Big], \\
\Pi_{Q^\theta}\left(p^2\right) = \frac{1}{8\pi^2\epsilon}\Bigg\{ & 12G^{\theta\theta}\lambda_{4Q^\theta}m_\theta^2 + 2G^{\theta\theta}\lambda_{2Q^\phi 2Q^\theta}m_h^2 + 4G^{\phi\phi}G^{\theta\theta}\lambda_{Q^\phi 2Q^\theta}^2 \\
& - \Big[2y_{Q^\theta}^2\left(p^2+2m_\psi^2\right)q^{-2} + 8y_{2Q^\theta}m_\psi^3 q^{-1}\Big] \\
& + 6G^{AA}g_{2Q^A 2Q^\theta}m_A^2 + \frac{3}{4}G^{\phi\phi}G^{AA}\left(g_{Q^A\partial Q^\phi Q^\theta} + g_{Q^A\partial Q^\theta Q^\phi}\right)^2 p^2 \\
& - 6\mathcal{K}_{2Q^\phi 2\partial Q^A}G^{AA}m_A^4\Bigg\} - \Big[\left(Z_{\partial Q^\theta}-1\right)p^2\mathcal{G}_{\theta\theta} + \left(Z_{\lambda 2Q^\theta}-1\right)2\lambda_{2Q^\theta}\Big].
\end{aligned}
\tag{B.6}
$$

where p is the external momentum. For standard kinetic terms for the gauge and fermion fields one can set $k \equiv G_{AA} = (G^{AA})^{-1} \to 1$ and $q \to 1$ respectively. All couplings and metric factors are evaluated on the background.

For the fermions

$$
\begin{aligned}
\Pi_\psi\left(\not{p}\right) = \frac{1}{8\pi^2\epsilon}\Bigg\{ & y_{Q^\phi}^2 G^{\phi\phi}q^{-1}\left(m_\psi - \frac{1}{2}\not{p}\right) - y_{Q^\theta}^2 G^{\theta\theta}q^{-1}\left(m_\psi + \frac{1}{2}\not{p}\right) \\
& + y_{2Q^\phi}G^{\phi\phi}m_h^2 + y_{2Q^\theta}G^{\theta\theta}m_\theta^2 - 3m_\psi G^{AA}g^2 q_L q_R\Bigg\} \\
& - \left[\left(Z_{m_\psi}-1\right)m_\psi + q\left(Z_\psi - 1\right)\not{p}\right].
\end{aligned}
\tag{B.7}
$$

For the gauge fields

$$\begin{aligned}
\Pi^A_{\mu\nu} = \frac{1}{8\pi^2\epsilon}\Bigg\{ & \left(3G^{\phi\phi}G^{AA}g^2_{Q^\phi 2Q^A} + 2G^{\phi\phi}g_{2Q^\phi 2Q^A}m_h^2 + 2G^{\theta\theta}g_{2Q^\theta 2Q^A}m_\theta^2\right)g_{\mu\nu} \\
& - G^{\phi\phi}G^{\theta\theta}\left[\frac{1}{4}\left(g_{Q^A\partial Q^\phi Q^\theta} + g_{Q^A\partial Q^\theta Q^\phi}\right)^2\left(\frac{p^2}{3} + m_h^2 + m_\theta^2\right)g_{\mu\nu}\right. \\
& \left. - \left(g^2_{Q^A\partial Q^\phi Q^\theta} - g_{Q^A\partial Q^\phi Q^\theta}g_{Q^A\partial Q^\theta Q^\phi} + g^2_{Q^A\partial Q^\theta Q^\phi}\right)\frac{1}{3}p_\mu p_\nu\right] \\
& - \frac{2}{3}q^{-2}\left(p^2 g_{\mu\nu} - p_\mu p_\nu\right)g^2\left(q_R^2 + q_L^2\right) - 2q^{-2}g^2\left(q_L - q_R\right)^2 m_\psi^2 g_{\mu\nu}\Bigg\} \\
& - \left(Z_{2Q^A} - 1\right)G^{AA}\left(p^2 g_{\mu\nu} - p_\mu p_\nu\right) - \left(Z_{m_A^2} - 1\right)g_{\mu\nu}m_A^2.
\end{aligned} \tag{B.8}$$

B.2.2 Case B

We give the couplings and results for the counterterms for Case B, where both fermions and gauge bosons are in the spectrum. Case A can be obtained from this result by setting $g \to gk(\phi)$, and then subsequently integrating out the gauge bosons; thus effectively we can set the gauge coupling to zero. Case C is obtained by rescaling the yukawa coupling $y \to y/q(\phi)$, which makes the fermion contribution subleading, and effectively we can set the yukawa to zero. Case D is obtained by rescaling both g and y with the appropiate metric factors; the net effect is that both fields decouple and we can effectively set both couplings to zero. We expand in covariant derivatives the scalar functions defined in eq. (5.42). All couplings are evaluated on the background and given for $\delta \ll 1$ and $\delta \gg 1$, with $\delta = \frac{\lambda\phi_0^4}{4\mathcal{M}^4}$.

The vertices derived from the potential are valid in all four cases, and are given by

$$\begin{aligned}
\lambda_{2Q^\phi} &= \frac{1}{2}m_h^2 = \frac{1}{2}\nabla^2_\phi V = \frac{3}{2}\lambda\phi^2\frac{(1+\frac{1}{3}\delta)}{(1+\delta)} = \lambda\phi^2\{\frac{3}{2},\frac{1}{2}\}, \\
\lambda_{2Q^\theta} &= \frac{1}{2}m_\theta^2 = \frac{1}{2}\nabla^2_\theta V = \frac{1}{2}\lambda\phi^2\frac{(1+3\delta)}{(1+\delta)} = \lambda\phi^2\{\frac{1}{2},\frac{3}{2}\}, \\
\lambda_{3Q^\phi} &= \frac{1}{3!}\nabla^3_\phi V = \lambda\phi\left\{1,-\frac{1}{3}\right\} \\
\lambda_{Q^\phi 2Q^\theta} &= \frac{1}{3!}\left[\nabla_{(\theta}\nabla_\theta\nabla_{\phi)}\right]V = \lambda\phi_0\{1,-3\} \\
\lambda_{4Q^\phi} &= \frac{1}{4!}\nabla^4_\phi V = \lambda\left\{\frac{1}{4},\frac{5}{12}\right\}, \quad \lambda_{4Q^\theta} = \frac{1}{4!}\nabla^4_\theta V = \lambda\left\{\frac{1}{4},-\frac{9}{4}\right\} \\
\lambda_{2Q^\phi 2Q^\theta} &= \frac{1}{4!}\left[\nabla_{(\theta}\nabla_\theta\nabla_\phi\nabla_{\phi)}\right]V = \lambda\left\{\frac{1}{2},\frac{15}{2}\right\}.
\end{aligned} \tag{B.9}$$

The vertices derived from the yukawa interactions are

$$\begin{aligned} y_{Q^\theta} &= -i\gamma_5 \nabla_\theta F_\psi = \frac{y}{\sqrt{2}}\{1,1\}\,, \quad y_{Q^\phi} = \nabla_\phi F_\psi = \frac{y}{\sqrt{2}}\{1,1\} \\ y_{2Q^\phi} &= \frac{1}{2!}\nabla^2_\phi F_\psi = -\frac{y}{\sqrt{2}\phi}\{0,1\}\,, \quad y_{2Q^\theta} = \frac{1}{2!}\nabla^2_\theta F_\psi = \frac{y}{\sqrt{2}\phi}\{0,1\}\,. \end{aligned} \tag{B.10}$$

The vertices involving gauge fields are

$$\begin{aligned} g_{2Q^A 2Q^\phi} &= \frac{1}{4!}\nabla_{(A}\nabla_A\nabla_\phi\nabla_{\phi)}F_A = \frac{1}{2}g^2\left\{1, -\frac{5}{3}\delta\right\} \\ g_{2Q^A 2Q^\phi} &= \frac{1}{4!}\nabla_{(A}\nabla_A\nabla_\chi\nabla_{\chi)}F_A = \frac{1}{2}g^2\{1, 5\delta\} \\ g_{2Q^A Q^\phi} &= \frac{1}{3!}\nabla_{(A}\nabla_A\nabla_{\phi)}F_A = g^2\phi\{1,\delta_0\} \\ g_{Q^A \partial Q^\phi Q^\theta} &= g_{Q^A\partial Q^\theta Q^\phi} = g\{1,\delta\} \\ g_{A\bar{\psi}_s\psi_s} &= g q_s \end{aligned} \tag{B.11}$$

with $s = L, R$, and gauge invarince sets $q_\Phi - q_L + q_R = 0$ (where we normalize $q_\phi = 1$). The relevant couplings coming from the derivative expansion are

$$\mathcal{K}_{2\partial Q^A 2Q^\phi} = \alpha_{A0}\delta\left\{0, \frac{-1}{3\phi^2}\right\}, \quad \mathcal{K}_{2\partial Q^A 2Q^\theta} = \alpha_{A0}\delta\left\{0, \frac{1}{\phi^2}\right\} \tag{B.12}$$

The masses of the fields are given in (5.48, 5.49).

With these couplings the self-energies in the large field limit become

$$\begin{aligned} \Pi_{Q^\phi} &= p^2\delta\left[-(Z_{2Q^\phi}-1) + \mathcal{O}(\delta^{-1})\right] + \phi^2\left[-(Z_{\lambda 2Q^\phi}-1)\lambda + \frac{1}{8\pi^2\epsilon}\left(\frac{3g^4}{\alpha_0} - y^4\right)\right] \\ \Pi_{Q^\theta} &= p^2\delta\left[-(Z_{2Q^\theta}-1) + \mathcal{O}(\delta^{-1})\right] + 3\phi^2\left[-(Z_{\lambda 2Q^\theta}-1)\lambda + \frac{1}{8\pi^2\epsilon}\left(\frac{3g^4}{\alpha_0} - y^4\right)\right] \\ \Pi_\psi &= p\left[-(Z_\psi - 1) + \mathcal{O}(\delta^{-1})\right] + \phi\left[-(Z_{m_\psi}-1)\frac{y}{\sqrt{2}} + \mathcal{O}(\delta^{-1})\right] \\ \Pi^A_{\mu\nu} &= (p^2 g_{\mu\nu} - p_\mu p_\nu)\delta\alpha_0\left[-(Z_{2Q^A}-1) + \mathcal{O}(\delta^{-1})\right] + g_{\mu\nu}\phi^2\delta\left[-(Z_{m_A^2}-1)g^2 + \mathcal{O}(\delta^{-1})\right]. \end{aligned} \tag{B.13}$$

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Samenvatting

In wat volgt zal ik proberen uit te leggen waarom ik bijna vier jaar lang een fenomeen heb bestudeerd dat waarschijnlijk slechts 10^{-33} (nul komma drieëndertig nullen, gevolgd door een één) seconden[1] heeft geduurd en waarvan we denken dat het maar liefst 13.8 miljard jaar voor het schrijven van deze thesis heeft plaatsgevonden.

Panku de reus

"[...] zijn zweet werd de regen en de dauw, en ondertussen werden alle sterren in de hemel geboren uit zijn haar. Op deze wijze werd de wereld geschapen door Panku de reus."

(Chinese scheppingsmythe)

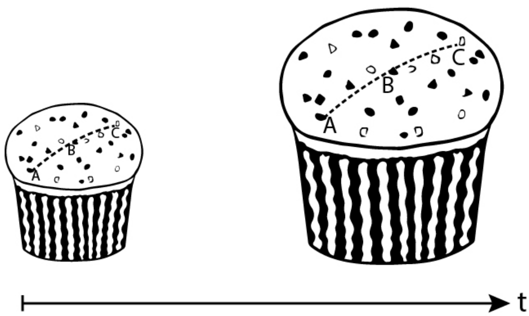
Sinds het begin van de beschaving heeft de Homo Sapiens zich afgevraagd wat de oorsprong van het universum is waarin hij zich bevindt en wat de reden is waarom het is zoals het is. Iedere cultuur had zijn eigen mythes die de antwoorden op deze vragen bevatten.

Het verschil tussen de afgelopen honderd jaar en de zeventigduizend jaar daarvoor is dat deze vragen in de afgelopen eeuw op een wetenschappelijke manier zijn aangepakt. Dit betekent dat we allereerst moeten beginnen met het toegeven van onze onwetendheid. Hierna gaan we een verhaal schrijven over ons universum met behulp van vergelijkingen en wiskundige modellen. Een dergelijk verhaal voorspelt normaal gesproken een aantal gebeurtenissen en consequenties die door telescopen en satellieten geobserveerd zouden kunnen worden. Elke keer dat een klein deel van het verhaal door een observatie wordt bevestigd, vertrouwen we het een klein beetje meer. Uiteindelijk komen we zo uit bij een theorie die de meest recente versie

[1] De precieze waarde is model-afhankelijk, maar de orde van grootte is altijd vergelijkbaar.

van onze kennis over de realiteit representeert (maar die nog steeds falsifieerbaar is).

Laten we ons bijvoorbeeld eens richten op de theorie van de Big Bang. In 1927 paste de Belgische priester en astronoom Georges Lemaître de vergelijkingen van een destijds nieuwe theorie over zwaartekracht (de algemene relativiteitstheorie) toe op het universum als geheel. Het resultaat was verrassend: Het universum leek uit te dijen.[2] Twee jaar later rondde de Amerikaanse astronoom Edwin Hubble een serie observaties af waaruit hij concludeerde dat alle sterrenstelsels van ons af bewegen. Hij deed de opmerkelijke ontdekking dat, hoe verder een sterrenstelsel van ons af staat, hoe sneller het van ons af beweegt. De observaties van Hubble leidden tot een simpele en plausibele conclusie: Het heelal dijt uit. Het is eenvoudig in te zien hoe men tot deze conclusie kwam. Neem bijvoorbeeld een 'panettone', een Italiaanse kerstcake met rozijnen. In deze analogie stelt de panettone het universum voor en de rozijnen de sterrenstelsels. We bekijken drie van de rozijnen en noemen ze voor het gemak A, B en C. Op het eerste tijdstip dat we de panettone bekijken is de afstand tussen A en C tweemaal zo groot als de afstand tussen A en B. Als de panettone over de periode van een uur zal gaan rijzen tot tweemaal zijn oorspronkelijke grootte, dan is de snelheid waarmee de rozijnen A en C van elkaar af bewegen twee keer zo groot als die waarmee rozijnen A en B van elkaar af bewegen.



Een panettone als een uitdijend universum. De drie rozijnen A, B en C stellen sterrenstelsels voor.

Een uitdijend universum heeft een logische consequentie: Er moet een moment in het verleden zijn waarop alles dicht bij elkaar is geweest en zich in een klein

[2]In zijn originele artikel (in het Frans) verzamelde Lemaître de weinige data die destijds beschikbaar was en die leek zijn ideeën te ondersteunen. In de Engelse vertaling besloot hij deze data achterwege te laten vanwege de tussentijdse publicatie van de nieuwe, completere data van Hubble.

volume met een hoge dichtheid en hoge temperatuur bevond, de zogenaamde Big Bang.

Vandaag de dag wordt de theorie van een universum dat uitdijt vanuit een klein volume met een hoge dichtheid alom voor waar aangenomen. Om te kunnen begrijpen waarom dit resultaat zo volledig onverwacht was op het moment van ontdekking, moeten we teruggaan in de geschiedenis en ons verplaatsen in de denkwijze van een astronoom aan het begin van de twintigste eeuw. In die tijd zou iedereen die het gevraagd zou worden, van priesters tot gerespecteerde wijsgeren, hebben beweerd dat het universum een statische entiteit was, je zou tenslotte iedere dag en zelfs ieder jaar dezelfde dynamica kunnen gebruiken om het universum te beschrijven.Tweeduizend jaar geleden beschreef Aristoteles het universum als eeuwig en onveranderlijk en hoewel wetenschappers in die tijd vele ideeën van Aristoteles inmiddels hadden verworpen, waren ze nog altijd content met een eeuwigdurend universum. Dit beeld weerhield hen ervan om te speculeren over het begin van het heelal. Zelfs zij die in de veronderstelling waren dat er een begin was, waren er nog steeds van overtuigd dat er vanaf de creatie tot de huidige dag niets was veranderd. Toen bleek dat zijn vergelijkingen leidden tot een uitdijend universum telde Einstein, nou niet bepaald een conservatieveling, er een extra term (de zogenaamde kosmologische constante) bij op, om te zorgen dat hij op een statisch universum uit zou komen, wat in lijn was met zijn persoonlijke geloof. Later verklaarde Einstein dat dit een van zijn grootste fouten was geweest.[3] De observaties van Hubble werden dus ook voorspeld door de theorie van Lemaître, maar is dat genoeg om te concluderen dat we vandaag de dag nog steeds een universum observeren dat zijn expansie begon vanuit een extreem hete en dichte toestand?

Nee, dat is het niet. Een wetenschappelijke theorie die één enkele observatie kan verklaren kan simpelweg worden vervangen door een alternatieve theorie die diezelfde observatie verklaart (het "steady state model" was bijvoorbeeld erg populair in die tijd). Dit is zeker het geval indien het beeld dat de theorie schetst in conflict is met de algemene overtuigingen van een tijdperk. Wetenschappers begonnen de Big Bang theorie pas serieus te nemen toen ze zich realiseerden dat die beschrijving van de werkelijkheid meerdere voorspellingen deed die te testen waren. In 1948 lieten de twee natuurkundigen Alpher en Gamow zien dat het overschot van waterstof en helium in het heelal kon worden verklaard met behulp van de Big Bang theorie. Daarnaast voorspelden ze, in samenwerking met Herman, wat tegenwoordig als het belangrijkste bewijs voor de theorie wordt gezien en wat als fundament dient

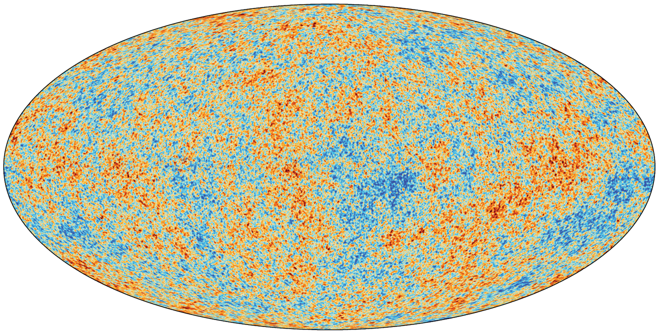
[3] Recentelijk (1998) werd er met behulp van observaties van type Ia supernovae ontdekt dat het universum een fase van versnellende uitdijing ondergaat. Dit kan worden verklaard door het toevoegen van eenzelfde kosmologische constante aan Einstein's vergelijkingen. Dus het lijkt erop dat Einstein, zelfs wanneer hij zelf dacht dat hij verkeerd zat, revolutionaire concepten introduceerde.

van deze thesis: Als het universum uit een extreem hete en dichte oertoestand is geëvolueerd dan leiden zijn expansie en de daaropvolgende afkoeling tot een onontkoombaar feit: Er moet een alomtegenwoordige straling bestaan, de zogenaamde *Cosmic Microwave Background* (CMB) of kosmische achtergrondstraling.

Toen het universum zo'n 380 duizend jaar oud was, daalde de temperatuur zo ver dat vrije elektronen in het oerplasma gevangen konden worden door protonen om zo waterstofatomen te vormen. Het gevolg hiervan was dat fotonen, de deeltjes waaruit licht bestaat, niet meer verstrooid en vastgehouden werden door een zee van elektronen, maar dat ze zich voor het eerst vrij en onverstoord konden voortbewegen door het universum. Sommige van deze fotonen komen vandaag de dag, na een reis van 13.8 miljard jaar, aan op onze aarde en zij vormen de eerder genoemde kosmische achtergrondstraling. Het is alsof het heelal in zijn eerste periode was gevuld met een dichte mist. Toen de mist optrok kon het licht zich vanaf elk punt en in alle richtingen voortbewegen. Fotonen die van gebieden dichtbij ons kwamen, bereikten ons al in een ver verleden, terwijl lichtdeeltjes van verderaf ons op dit moment pas kunnen bereiken. Deze laatstgenoemde fotonen laten ons dus zien hoe het universum er uit zag toen de mist optrok, 380 duizend jaar na de Big Bang.

In 1964, in New Jersey, waren Arno Penzias en Robert Wilson, twee fysici werkzaam voor een telecommunicatiebedrijf, aan het experimenteren met een antenne die oorspronkelijk bedoeld was om radiogolven op te vangen. Net zoals men soms ruis hoort op de achtergrond tijdens het afstellen van een analoge radio, werden ook hun metingen geplaagd door een dergelijke interferentie. Ze probeerden van alles om van de ruis af te komen, tot aan het verwijderen van uitwerpselen van duiven die zich op de antenne hadden genesteld. Na een jaar lang experimenteren, namen ze contact op met collega's in Stanford en bleek dat hun doorzettingsvermogen ze had geleid tot een ontdekking die later een Nobelprijs waard zou blijken te zijn: De ruis die ze opvingen was de kosmische achtergrondstraling.

Vanuit elke richting worden we gebombardeerd door de CMB en al deze fotonen hebben dezelfde temperatuur, onafhankelijk van waar ze vandaan komen. Vandaag de dag weten we met een hoge precisie dat de temperatuur van de CMB $2.73\,\mathrm{K}$ (ongeveer-270 $^\circ$ C) is, plus minus minuscule variaties van ongeveer $0.000001\,\mathrm{K}$, waarop we later nog terug zullen komen. Als totaal verschillende gebieden in het universum op een bepaalde manier (in dit geval hun temperatuur) toch zoveel op elkaar lijken, dan is het redelijk om aan te nemen dat er een moment in het verleden is geweest waarop deze gebieden met elkaar in contact hebben gestaan. Twee fotonen die de aarde bereiken vanuit tegenovergestelde richtingen hebben beide 13.8 miljard lichtjaar afgelegd. Als we aan zouden nemen dat het heelal niet uitdijt, dan zou dat betekenen dat deze fotonen gescheiden zijn geweest door een



De CMB is het oudste licht dat onze telescopen bereikt. Het komt van de bolvormige rand van ons zichtbare universum. In de figuur is een projectie van deze bol te zien, waar de temperatuurfluctuaties in kleur te zien zijn. Deze fluctuaties corresponderen met gebieden die een licht afwijkende dichtheid hadden toen het universum 380 duizend jaar oud was.

afstand van ongeveer 27 miljard lichtjaar toen ze hun reis begonnen. Zo ontstaat dus de volgende contradictie: Hoe is het mogelijk dat twee gebieden die zo ver van elkaar zijn verwijderd, toch in contact met elkaar lijken te hebben gestaan als het universum slechts 380 duizend jaar oud was toen de CMB werd uitgezonden?

Men zou mogelijk nog kunnen redeneren dat het universum zich aan het begin van zijn uitdijing al in een homogene toestand bevond. Dit soort aannames over de begintoestand van het universum (ook wel fine-tuning genoemd) laten natuurkundigen liever buiten beschouwing. De reden hiervoor wordt duidelijk aan de hand van de volgende metafoor: Als we aannemen dat er een examen wordt gegeven aan 10^{60} kandidaten,[4] en alle deelnemers halen exact dezelfde score, is het dan waarschijnlijker dat dit puur toeval is, of dat alle kandidaten van tevoren dezelfde hints hebben gekregen?

Een indrukwekkende uitdijing

In 1980 stelde Alan Guth, een Amerikaanse kosmoloog, een simpele oplossing voor om deze homogeniteitspuzzel op te lossen: Ver voordat de eerste seconde van het universum voorbij was, maakte het een gigantische expansie door waardoor het een factor 10^{26} groeide. Om de proporties van een dergelijke uitdijing duidelijk te maken: Het is alsof het universum van het formaat van een bacterie (10^{-6} m)

[4] Dit getal is vele malen groter dan de totale wereldpopulatie, maar het is ongeveer de hoeveelheid verschillende gebieden in het universum met dezelfde temperatuur die niet causaal verbonden zijn.

Een klein fragment van het oeruniversum dat inflateert tot ons huidige, zichtbare universum.

binnen een fractie van een seconde groeide tot 100 maal de grootte van onze huidige melkweg (de melkweg heeft een diameter van ongeveer honderdduizend lichtjaar, zo'n 10^{20} m).

Deze indrukwekkende expansie, ook wel *kosmische inflatie* genoemd, vergrootte een klein oeruniversum tot wat nu ons zichtbare heelal is, en is dus de reden dat het zo homogeen is. Toch is het gerechtvaardigd dat men zich afvraagt of dit genoeg is om erop te kunnen vertrouwen dat de eerste seconde van het universum inderdaad werd gekarakteriseerd door zo'n indrukwekkende uitdijing.

Zeer waarschijnlijk zou dit niet genoeg zijn geweest om de grote populariteit die de theorie vandaag de dag geniet te bereiken. Ondanks dat inflatie werd geïntroduceerd als oplossing voor het hierboven beschreven probleem, komt het grootste succes van de theorie uit het feit dat een periode van de inflatie een andere observatie op een simpele en elegante manier kan beschrijven, namelijk dat ons universum kleine inhomogeniteiten bevat, zoals onze melkweg of onze planeet.

Laten we een stapje terug nemen. Dankzij de bijdragen van verschillende satellieten weten we vandaag de dag dat de temperatuur van de CMB *vrijwel* homogeen is. Als we terugdenken aan de minuscule variatie van 0.000001 K, dan betekent dit dat, op het moment dat de CMB werd uitgezonden, sommige gebieden van het universum net iets kouder waren. Dit impliceert dat de zwaartekracht in deze regio's net iets sterker was dan gemiddeld en dus dat op die plekken materie beetje bij beetje iets sneller dan gemiddeld begon te accumuleren. De evolutie die daarna volgde creeërde vervolgens de structuren die we vandaag de dag observeren in ons universum, maar hoe is dit alles gerelateerd aan inflatie?

Een periode van inflatie verschaft ons een mechanisme dat op een vrij verrassende manier kan verklaren waar de minuscule temperatuurverschillen in de CMB

vandaan komen. Om te kunnen laten zien hoe dit mechanisme in zijn werk gaat, moeten we een aantal concepten introduceren die ons later nogmaals van pas zullen komen, dus het geduld van de lezer zal zich tweemaal uitbetalen. Kwantummechanica, de tak van de natuurkunde die zich richt op het bestuderen van de wereld op microscopisch kleine schaal (vergelijkbaar met afstanden ter grootte van een atoom), leert ons dat er altijd een onontkoombare onzekerheid is in de hoeveelheid energie die een systeem bevat. Uit deze onzekerheid, in combinatie met de beroemde formule van Einstein, $E = mc^2$, volgt dat twee deeltjes uit het vacuüm gecreeërd kunnen worden voor een korte periode. Hierdoor is het realistischer om het vacuüm niet te zien als lege ruimte, maar als een continuüm vol met deeltjes die constant ontstaan en weer verdwijnen. Tijdens een periode van exponentiële expansie, zoals inflatie, worden deze fluctuaties als het ware uitgerekt en versterkt tot schalen die het mogelijk maken om de zwaartekracht op macroscopische schaal te kunnen beïnvloeden. Mukhanov, Starobinsky en Hawking lieten als eerste, onafhankelijk van elkaar, zien hoe fluctuaties, die tijdens inflatie ontstaan zijn, kunnen leiden tot de kleine temperatuurverschillen in de CMB. Op deze wijze verschaft de hypothese van inflatie ons dus een mogelijkheid om wat we observeren in de hemel te kunnen relateren aan wat er een fractie van een seconde na de Big Bang zou kunnen zijn gebeurd.

Tot zo ver ziet alles er rooskleurig uit, maar we zijn nog steeds niet in staat om een bevredigend antwoord te kunnen geven op de vraag: Wie of wat was er verantwoordelijk voor zo'n indrukwekkende expansie? Sinds de introductie van inflatie zijn er honderden modellen en mogelijke scenarios geopperd. Elk van deze modellen beschrijft de dynamica van het zogenaamde inflaton, het deeltje (of meer algemeen gesteld, het fysische mechanisme) dat verantwoordelijk is voor inflatie. Dit verschaft precieze voorspellingen van de eigenschappen van de CMB (zoals de grootte en de verdeling van de temperatuurverschillen) die getoetst kunnen worden aan observaties.[5]

De methode die voor het grootste gedeelte van deze thesis wordt gehanteerd, is gebaseerd op het principe van Ockham's razor. Om als het inflaton te kunnen dienen, moet een deeltje een bepaalde intrinsieke eigenschap bezitten. Het enige deeltje met deze eigenschap waarvan men het bestaan heeft aangetoond, is het Higgs boson, dat werd ontdekt in de deeltjesversneller van CERN (de LHC) in Genève in 2012. Een minimale aanpak suggereert dat het Higgs boson het inflaton zou kunnen zijn en meerdere modellen met dit als onderliggende gedachte zijn voorgesteld. Ondanks het feit dat de energieschalen van de LHC hoog zijn (van de orde van duizenden elektronvolt), zijn ze nog steeds tien ordes van grootte kleiner dan de typische energieschalen die relevant waren tijdens de periode van inflatie.

[5] De vergelijkingen die dit mechanisme beschrijven werden kort samengevat in hoofdstuk één.

De vraag die dan opkomt is de volgende:

Als we de eigenschappen van het Higgs boson alleen bij lage energieschalen kennen, hoe kunnen we dan zijn gedrag bij de hoge energieschalen ten tijde van de inflatie beschrijven en betrouwbare voorspellingen doen over de CMB?

Running in het vroege universum

Het doel van het onderzoek in deze thesis is om de observaties van telescopen en satellieten (die de eigenschappen van de CMB waarnemen) te relateren aan de parameters van het Higgs boson die gemeten worden door CERN. Deze parameters, koppelingsconstantes genoemd, zijn analoog aan de elektrische lading voor een elektron.

Laten we recapituleren wat we zojuist hebben bestudeerd: Een gegeven inflatiemodel doet voorspellingen over de eigenschappen van de CMB, maar we kunnen de dynamica van inflatie niet simpelweg beschrijven met de parameters die gemeten zijn bij lage energieschalen vanwege de volgende, ietwat tegenstrijdig klinkende, reden: De koppelingsconstanten zijn niet constant! De waardes van deze parameters veranderen als functie van de energie.

Om dit concept te begrijpen kunnen we kijken naar de elektrische lading. Deze parameter bepaald de kracht van afstoting tussen twee elektronen. In een goede benadering kunnen we de elektrische lading van een elektron meten door er een ander elektron op af te schieten en te observeren hoe deze afkaatst. Als we terugdenken aan het feit dat het vacuüm niet leeg is, kunnen we ons realiseren dat hetzelfde geldt voor de ruimte om een elektron heen. Deze ruimte lijkt meer op een kokende pan vol deeltjes en antideeltjes (met tegengestelde lading) die constant worden gecreeërd en weer verdwijnen. Deze steeds aanwezige fluctuaties in het vacuüm vormen een soort wolk om het elektron die de elektrische lading tot op zeker hoogte afschermt. In het zojuist beschreven experiment bedoeld om de lading van een elektron te meten, zal een elektron met een hogere energie dat op het stationaire elektron af wordt geschoten verder doordringen in de wolk en dus een andere lading voelen dan een elektron dat minder energie heeft (en dus minder ver doordringt).

Om voorspellingen te kunnen doen over de CMB zijn de Higgs parameters normaal gesproken de parameters die we graag zouden hebben bepaald tijdens de periode van inflatie (bij een hoge energie), maar aangezien we ze destijds niet hebben kunnen meten, moeten we een manier vinden om ze af te leiden uit de experimenteel gevonden waarden bij lagere energieën. Een van de belangrijkste

doelen van deze thesis is geweest om te begrijpen hoe deze parameters veranderen als functie van de energie. In de natuurkunde zeggen we dat de parameters *runnen* (letterlijk: rennen) tot aan de typische energieschalen van het oeruniversum. Dus we bestuderen het mechanisme van *running in het vroege universum*.

In hoofdstuk drie hebben we het meest populaire model geanalyseerd dat het Higgs boson als inflaton bevat, het Higgs inflatie model. We bekeken daarin of de voorspellingen die dit model deed over de CMB consistent waren door op een rigoreuze manier het runnen van de parameters te bestuderen. Om te kunnen berekenen hoe de parameters over vele ordes van grootte runnen, moet er echter nog een ander aspect mee worden genomen. Ver boven de energieën die we gebruiken bij onze experimenten, zouden er nieuwe deeltjes of fenomenen kunnen bestaan die we tot op dit moment nog niet kennen (we duiden deze vaak collectief aan met de term *nieuwe fysica*). Deze nieuwe fysica zal ook meedoen aan de 'dans' van deeltjesfluctuaties in het vacuüm en daarom is het noodzakelijk om de mogelijke effecten van nieuwe fysica goed te parametriseren. Wat we hebben ontdekt is dat, verrassend genoeg, de voorspellingen onafhankelijk zijn van nieuwe fysica en perfect in overeenstemming zijn met de observaties van de Planck satelliet.[6]

In hoofdstuk vier hebben we het voorgaande resultaat uitgebreid naar een breder scala van inflatiemodellen. Door Ockham's razor even te laten voor wat het was, lieten we zien dat de wiskundige structuur van deze modellen (waarvan Higgs inflatie een specifiek geval is) de robuustheid van hun voorspellingen garandeert, zodra de running van de parameters mee wordt genomen.

Tenslotte hebben we in hoofdstuk vijf een alternatief voor Higgs inflatie, simpelweg 'nieuwe Higgs inflatie' genaamd, bekeken die niet tot de klasse van theorieën bestudeerd in hoofdstuk vier behoort. Door terug te gaan in de geschiedenis van ons universum lieten we zien dat de voorspellingen van dit model wel gevoelig zijn voor het runnen van de koppelingsconstanten. Dit verlaagt de voorspelbare waarde van dit model, maar maakt ook zijn sensitiviteit voor nieuwe fysica duidelijk. Het hoofddoel van dit laatste hoofdstuk was om aan te tonen dat de running van parameters in het algemeen van fundamenteel belang is om een model op een zinnige manier aan experimentele data te kunnen toetsen.

Het is interessant om op te merken dat de belangrijkste motivatie achter het opstellen van Higgs inflatie modellen de vraag naar minimalisme is geweest: Om te voorkomen dat we nieuwe natuurkundige fenomenen of deeltjes nodig hadden in onze theoretische modellen. Echter, zo laat onze analyse (en dat van andere onderzoeken naar ditzelfde onderwerp) zien, is de toevoeging van nieuwe fysica

[6] Nauwkeuriger gezegd, tenzij hun bijdrage zo groot is dat inflatie in zijn geheel uitgesloten is, blijven de voorspellingen van het Higgs inflatie model stabiel onder de bijdragen van nieuwe fysica. Dit is de reden van de titel "UV (on)gevoeligheid van Higgs inflatie".

op een bepaalde manier onontkoombaar als we op een consistente manier willen beschrijven hoe de moderne deeltjesfysica gerelateerd is aan de fysica van inflatie.

Voor natuurkundigen is dit niet per se slecht nieuws. Normaal gesproken zijn we blij als we ons realiseren dat we niet alles weten wat er te weten valt. Is het immers niet zo dat ons verhaal begon met het toegeven van onze onwetendheid?

Summary

In the next lines I will try to explain why I dedicated almost 4 years of studies to a phenomenon that presumably lasted 10^{-33} (zero comma thirty-two zeroes and then a one) seconds,[1] and that we think happened 13.8 billion years before this thesis was written.

Panku the giant

"[...] his sweat became rain and dew, meanwhile, all the stars in the sky were born from his hair. In this way, Panku, the giant, made the World."

(Chinese creation myth)

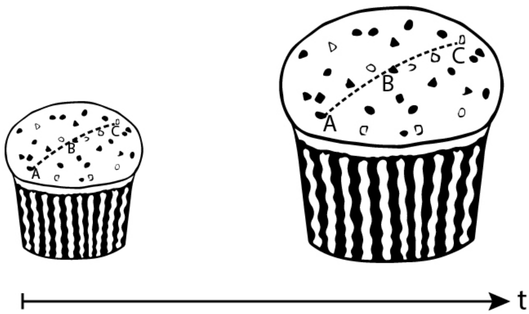
Since cultures exist, Homo Sapiens has always wondered what's behind the origin and the appearance of the Universe in which he ended up. In every civilization, myths served to this need. They contained the answers to these questions.

The difference between the last hundred years and the previous 70 thousand is that in the last century these questions can be answered scientifically. This means that we first have to admit our ignorance. Then we start building a story about our Universe based on equations and mathematical models. The story usually makes some predictions on what telescopes and satellites are going to observe. Every time an observation confirms a small piece of the story, we start to trust it a little bit more. Ultimately we have a theory which represents our most up-to-date version of reality (but which is still falsifiable).

Let us consider the theory of Big Bang. In 1927 a Belgian priest and astronomer, Georges Lemaitre, applied the equations of the new theory of gravity (general relativity) to the entire Universe. The result was surprising: the Universe might

[1] The exact values are model dependent but the orders of magnitude are always similar.

be expanding.[2] Two years later Edwin Hubble, an American astronomer, finalized a series of observations from which he concluded that all the galaxies are moving away from us. Remarkably, he noted that the further the galaxy the higher was its recession velocity. The observations made by Hubble led to a simple and possible conclusion: the Universe is expanding. It is not hard to understand why. Consider a panettone, an Italian Christmas "cake" which contains raisins. The panettone is the Universe while the raisins are the galaxies (see picture). Consider three raisins A,B and C. At a given time A and C are separated by a length which is two times the distance between A and B. If after an hour the panettone has rises doubling its dimensions, than A and C will move apart with a speed which is two times the velocity with which A and B are separated.



A panettone as an expanding Universe. The three raisins A,B and C are three galaxies.

An expanding Universe has a logical consequence: there should be a moment in the past where everything was close and packed into a small volume with high density and temperature, the so called Big Bang.

Today, the notion of a Universe that is expanding from a point with high density is taken for granted. In order to understand why the result was completely unexpected at that time we have to look back and think as an astronomer at the beginning of the twentieth century. If you were born at that time, everybody around you, from the priest to the most respected scholar, once questioned on the issue, would have replied that the Universe was something static. In the end, by contemplating every day and every year the same celestial dynamics you would have agreed. For two thousand years, Aristotle's view prevailed that the Universe was eternal and immutable, with no beginning. Even if scientists had overcome most of Aristotle's ideas, they were still happy with an eternal Universe. This

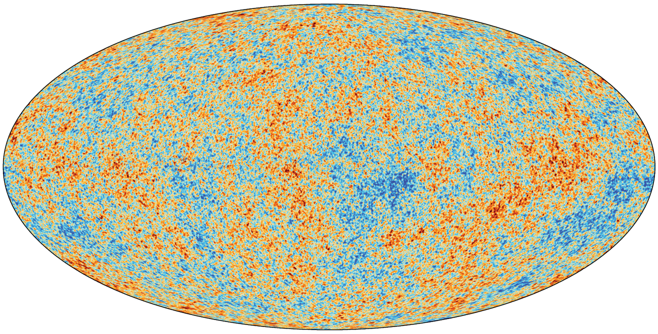
[2] In his original paper (in French), Lemaître collected the few data available at that time which roughly supported his idea. Laimatre himself chose to omit them from the translated English version after the publication of the more complete results of Hubble.

saved them from speculating about what causes its beginning. Even those who believed that the Universe had a beginning were still persuaded that from the creation onwards nothing had been changed. Einstein himself, not exactly a conservative, after realizing that his theory led to an expanding Universe decided to introduce an additional term to his equations, the so called cosmological constant, in order to obtain a static Universe aligned with his beliefs. Later on Einstein declared this was one of his worst mistakes.[3] In short, the Hubble's revolutionary observations had a counterpart in Lemaître's theoretical derivation. Is this enough to conclude that we are still observing the consequences of a Universe that has started is expansion from a very hot and dense state?

No. A scientific theory able to explain one observation can be easily replaced by an alternative description (for example the "steady state model" was popular at that time). This is true in particular if the world view offered by the theory is in contrast with all beliefs of its epoch. The moment in which scientists started to take seriously the Big Bang hypothesis is when they realized that the same description had new implications that could be tested. In 1948, physicists Alpher and Gamow showed that the abundance of hydrogen and helyum could be explained by the Big Bang hypothesis. Moreover, together with another physicist, Herman, they made a new prediction: if the Universe evolved from a primordial hot and dense state, the expansion and subsequent cooling led to an inescapable fact: it must exist a ubiquitous radiation, the so called *Cosmic Microwave Background* (CMB). The observation of the CMB is now considered the biggest evidence in favour of the Big Bang as well as fundamental for the work of this thesis.

When the Universe was about 380 thousand years old, the temperature had decreased enough such that free electrons in the primordial plasma were caught by the protons to form hydrogen atoms. As a result, photons, the light particles, no longer scattered with this sea of electrons anymore, but they were allowed to propagate freely for the first time. Some of these photons reach our Earth today after a journey 13.8 billion years long. This is the Cosmic Microwave Background. It is as if the Universe was permeated by a dense fog in the past. At a given time the fog disappeared and the light started to travel freely from every point and in all directions. Photons coming from regions close to us have reached us in the past while the ones further away reach us today. Thus, these photons show us how the Universe looked like when the "fog" disappeared 380 thousand years after the Big Bang.

[3]Recently (1998), from the observations of type Ia Supernovas, it has been discovered that our Universe is undergoing a phase of accelerating expansion. This can be explained by adding the cosmological constant to Einstein's equations. Thus, it seems that Einstein was meant to introduce futuristic concepts even when he made mistakes.



The CMB is the oldest light reaching our telescopes. It surrounds us as a sphere at the edge of our observable Universe. The figure shows the projection of this sphere where temperature fluctuations have been highlighted. These correspond to regions of slightly different densities at the time the Universe was 380 thousand years old.

In 1964 in New Jersley, two physicists working for a telecommunications company, Arno Penzias and Robert Wilson, were experimenting with an antenna to capture astronomical radio signals. In the same way as one tries to tune to a radio station but an annoying noise mumbles in the background, there was an interference bothering their apparatus. They tried everything to get rid of this noise, they even cleaned up the excrements of some pigeons who nested on the antenna. After a year of attempts they called their colleagues in Stanford. Their perseverance has led them to a Nobel prize worthy discovery: the noise captured was the Cosmic Microwave Background.

The CMB bombards us from all directions with photons of the same temperature. Today we know with high precision that the CMB temperature is 2.73 K (about -270 ° C) plus or minus tiny variation of the order of 0.000001 K.[4] We will get back to those fluctuations later. If different regions share the same information (in this case the temperature), it is reasonable to assume that there was a moment in the past where they have interacted. Two photons reaching the Earth from two opposite directions traveled 13.8 billion light years each. Without taking into account the expansion of Universe, this means that these two photons were separated by a distance greater than 27 billion light years when they start their journey. The contradiction is clear. How is it possible that regions so widely displaced could have interacted if the Universe was only 380 thousand years old at the time the CMB was emitted?

One could argue that, maybe, at the beginning of its expansion the Universe was

[4] As a consequence of the expansion of the Universe, these photons reach us after loosing most of their energy.

A tiny fragment of the primordial Universe inflated resulting in our observable Universe.

already in a homogeneous state. This is an assumption or a tweak (fine-tuning) on the initial conditions that physicists do not like to consider. I will use a metaphor to persuade the reader that this is more than a whim. Suppose that in an exam with a huge number of participants, let us say[5] 10^{60}, everybody gets the same score. Would you think that the attendants received a common hint before the exam or that the result is just a coincidence?

An impressive expansion

In 1980 Alan Guth, an American cosmologist, proposed a simple solution to solve the homogeneity puzzle: well before reaching its first second of life, our Universe underwent an exponential expansion, growing by a factor 10^{26} within a fraction of a second. In order to appreciate this number one can think that in proportion it is as if the Universe expanded, in a fraction of a second, from the dimensions of a bacteria (10^{-6} m) to 100 times the dimensions of our galaxy (the Milky Way has a diameter which is roughly 100 thousand light years $\approx 10^{20}$ m).

This impressive expansion, called *cosmic inflation*, inflated a tiny fragment of the primordial Universe to what now is our observable Universe, which is thus homogeneous. Still, it is legitimate to ask whether this is enough to be confident that the first second of our Universe has been characterized by such an impressive expansion? Most likely, this would have not been enough to explain the great popularity the theory has today. Even if inflation was introduced to solve the problem mentioned above, its big success lies in the fact that a period of inflation can explain, in a simple and elegant way, another observation: our Universe has

[5]The number is obviously bigger than the world population, but these are the number of causally disconnected regions with the same temperature in the CMB.

small inhomogeneities such as our galaxy or our planet.

Let's back up. Thanks to the contribution of different satellites today we know that the CMB temperature is *almost* homogeneous. Remember the tiny variation of the order of 0.000001 K? It means that, at the time the CMB was emitted, some regions were slightly colder than the average. This implies that in these regions gravity was a little bit stronger than the average. Over there, matter starts to accumulate growing little by little under the effect of gravity. The subsequent evolution creates the structures we observe in the Universe today.
But how is this all related to inflation?

Well, a period of inflation provides a mechanism that explains in a rather surprising fashion, how the tiny temperature variation of the CMB originates. In order to have an idea of how this mechanism works we introduce a couple of concepts that will also be helpful later on. Thus, the patience of the reader will be rewarded twice. Quantum mechanics, the branch of physics that studies the microscopic world (sizes comparable to atomic distances), teaches us that there is always an unremovable uncertainty in the amount of energy present in a system. From this uncertainty and from the famous Einstein formula $E = mc^2$, two particles can be created out of the vacuum for a short period of time. Thus, it is advantageous to picture the vacuum not just as empty space, but as if it is full of particles continuously arising and annihilating. During a period of exponential expansion like inflation, these fluctuations are stretched and amplified until scales able to influence the gravity of the macroscopic world. Mukhanov, Starobinsky and Hawking first showed independently how the fluctuations generated during inflation have a counterpart in the tiny temperature variations of the CMB. Therefore, the inflationary hypothesis provides us with the exciting possibility to relate what we observe in the sky with what could have happened a fraction of a second after the Big Bang.

Everything looks beautiful so far, however, we are still not able to bring a satisfactory answer to the following question: who or what was responsible for such an impressive expansion? Since inflation was introduced hundreds of models and possible scenarios have been proposed. Each model describes the dynamics of the inflaton, the particle (or more general, the physical mechanism) responsible for inflation. This provides precise predictions for the properties of the CMB (such as the size and the distribution of the temperature fluctuations) that can be tested by observations.[6]

The approach mainly considered in this thesis is based on Ockham's razor. In order to be the inflaton a particle needs to have a certain intrinsic property. The only

[6]The equations describing this mechanism have been briefly summarized in chapter one.

particle with this property, that has been proved to exist, is the Higgs boson which was discovered in 2012 at the particle accelerator of CERN in Geneva (LHC). A minimal approach suggests the Higgs boson as a candidate to be the inflaton and models in this direction have been proposed. Despite that the energy scales at the LHC are high (of the order of thousands of electron-volts), these are still ten orders of magnitude lower than the typical energy scale relevant during the inflationary era. The question now arises: If we know only the properties of the Higgs boson at low energy, how can we describe its behaviour at the inflationary scales and make reliable predictions about the CMB?

Running in the early Universe

The motivation behind this thesis is to relate what we observe in telescope and with satellites (the properties of the CMB) to the Higgs parameters measured at CERN. These parameters, called coupling constants, are the equivalent of the electric charge for an electron.

Let us remember what we have just learned: given an inflationary model this provides predictions for the features of the CMB. However, we cannot simply describe the dynamics of inflation with the parameters measured at low energy for a reason that might seem counterintuitive: the value of the parameters change with the energy. The coupling constant are not really constant!

In order to understand this concept consider the electric charge. This parameter measures the repulsion strength between two electrons. Approximately, we can think to measure this charge by shooting an "electron probe" towards another electron and observe the following bounce. Do you remember that the vacuum is not exactly empty? The same is true for the space surrounding the electron. It is more like a boiling pot with particles and antiparticles (with opposite charge) that are created and disappear. These restless vacuum fluctuations form a cloud around the electron that "screens" the electric charge. In our experiment, the higher the energy of the probe electron is the "more deeply" it will penetrate in the cloud. In this way it will "feel" a different charge.

To be able to make predictions about the CMB, the Higgs parameters are the ones that we would have liked to determine during the period of inflation (at high energies), but since we were not in a position to do so at that time, we have to find a way to derive them from our experimental results found at lower energies. One of the main goal of this thesis has been to understand how the parameters change by increasing the energy, in physical language they *run*, until the typical energy scales of the primordial Universe. Thus, we have studied the *running in*

the early Universe.

In particular, in chapter 3 we analyzed in detail the most popular model in which the Higgs is the inflaton, so-called Higgs inflation. We asked if the predictions of this model for the CMB are consistent with a rigorous study of the running of the parameters. In computing how the parameters run across many orders of magnitude another aspect has to be carefully taken into account. Well above the energy tested in our experiments new particles or new phenomena that we do not know at the moment (in short we refer to them with the name *new physics*) may turn up (remember $E = mc^2$). This new physics will participate as well in the "dance" of the vacuum fluctuations. As a consequence, the running of the parameters can be affected. Thus, it has been necessary to properly parameterize the effects of new physics. What we have discovered is that, surprisingly, the predictions are independent of the contribution of new physics and in perfect agreement with the observations of the Planck satellite.[7]

In chapter 4 we have extended the previous result to a broader class of inflationary models. Setting aside Ockham's razor for a moment, we show that the mathematical structure of these models (of which Higgs inflation represents a particular case) guarantees the robustness of their predictions once the running of the parameters is taken into account.

Finally, in chapter 5, we have considered an alternative to Higgs inflation, simply labeled as new Higgs inflation, which does not belong to the class studied in chapter 4. By tracing back the history of our Universe, we show that the predictions of this model are sensitive to the running of the couplings. This weakens the predictivity of this model but also shows its sensitivity to new physics which thus can be probed. The main goal of this last chapter was to show that in general the running of the parameters is fundamental to sensibly compare a model to the data.

It is interesting to note that the main motivation behind building Higgs inflationary models was the request for minimality: to avoid introducing in our theoretical model additional physics and particles. However, as our analysis shows (as well as other studies on the subject), the introduction of new physics is somehow unavoidable to consistently explain how present-day particle physics is related to the physics of inflation. For physicists this is not necessary bad news. Usually we are happy when we realize that what we know is not everything there is to know.

Is it not true that by admitting our ignorance this story has begun?

[7] More precisely, unless the effect is so strong that a period of inflation is precluded, the predictions for Higgs inflation remain stable against the contribute of new physics. That's the reason for the title "UV (in)sensitivity of higgs inflation".

Riassunto

Nelle prossime righe cercherò di spiegare perchè ho dedicato quasi 4 anni di studi ad un fenomeno che si stima sia durato 10^{-33} (zero virgola trentadue zeri uno) secondi,[1] e si pensa sia avvenuto 13.8 miliardi di anni prima che questa tesi venisse scritta.

Il gigante Panku

"[...] il suo sudore si trasformò in pioggia e rugiada, mentre dai suoi capelli nacquero tutte le stelle del cielo. Fu così che il gigante Panku creò il Mondo."

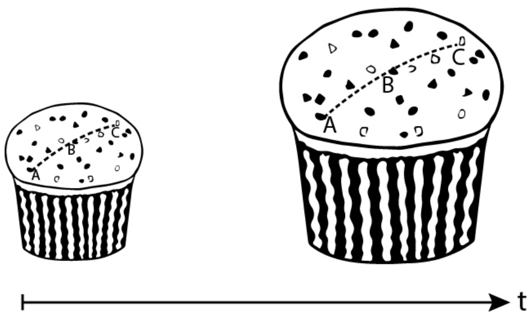
(Mito cinese della creazione)

Da quando esistono le culture, Homo Sapiens si è sempre domandato quali dinamiche abbiano determinato l'origine e l'aspetto dell'Universo in cui si è ritrovato a vivere. I miti, in tutte le civiltà, servivano proprio per soddisfare questa esigenza, essi contenevano le risposte a questi interrogativi.

La differenza tra gli ultimi cento anni e i 70mila precedenti è che da circa un secolo queste domande sono diventate una realtà scientifica. Questo significa che prima di tutto ammettiamo di non conoscere la risposta. In un secondo momento ci immaginiamo una storia riguardo al nostro Universo, formulandola attraverso equazioni e modelli matematici, che prevede una serie di conseguenze su quanto osservato da telescopi e satelliti. Ogni volta che le osservazioni confermano un altro pezzetto della storia che ci siamo immaginati tendiamo a dare loro più credito. Il risultato è una teoria che rappresenta sempre la nostra versione più aggiornata (ma non per questo non falsificabile) della realtà.

[1] Gli esatti valori dipendono dal modello ma gli ordini di grandezza in gioco sono simili.

Si consideri la teoria del Big Bang. Nel 1927 l'astronomo e sacerdote belga Georges Lemaître, applica le equazioni della nuova teoria della gravità di Einstein (la relatività generale) all'intero Universo. Il risultato è un fatto sorprendente per l'epoca: l'Universo si starebbe espandendo.[2] Due anni più tardi un astronomo statunitense, Edwin Hubble, porta a termine una serie di osservazioni con le quali conclude che quasi tutte le galassie si stanno allontanando da noi. Cosa ancora più importante, egli notò che maggiore è la distanza di una galassia da noi e maggiore è la la sua velocità di allontanamento. Le osservazioni di Hubble conducevano ad una semplice e possibile spiegazione: l'Universo si sta espandendo. Non è difficile capire perchè. Immaginate che l'Universo sia un panettone e l'uvetta al suo interno rappresenti le galassie. Prendete ora tre uvette A, B e C. Ad un certo momento A e C hanno una distanza doppia rispetto ad A e B. Se dopo un'ora il panettone è lievitato raddoppiando le sue dimensioni allora A e C si saranno allontanate ad una velocità doppia rispetto ad A e B.



Un panettone che lievita come un Universo in espansione, le tre uvette A, B e C rappresentano tre galassie.

Un Universo in espansione ha una logica conseguenza: deve esserci stato un momento nel passato in cui tutto era concentrato in un volume molto piccolo e in uno stato ad elevata densità e temperatura, il cosidetto Big Bang.

Oggi, la nozione di un Universo che si espande da un punto a densità molto elevata potrebbe sembrarci quasi scontata. Per capire però come mai il risultato fosse del tutto inaspettato bisogna guardare con uno sguardo lontano dal nostro e pensare come un astronomo di inizio novecento. Se vi fosse capitato di nascere in quell'epoca, tutti intorno a voi, dal prete, al vostro insegnante, al più erudito

[2] Nell'articolo originale (in francese) di Lemaître vi sono già raccolti i pochi dati osservativi disponibili all'epoca che confermavano in modo rudimentale la sua idea. Nella successiva pubblicazione inglese, Lemaître decise di omettere questi dati osservativi data la pubblicazione di quelli più completi da parte di Hubble.

filosofo, una volta interrogati sulla questione vi avrebbero convinto che l'Universo fosse qualcosa di statico. E alla fine, osservando ogni giorno e ogni anno il ripetersi delle stesse dinamiche celesti, ve ne sareste convinti. Da un lato per più di duemila anni la visione aristotelica ha considerato l'Universo come qualcosa di eterno e immutabile, senza inizio. Sebbene uno dopo l'altro i precetti aristotelici siano stati col tempo superati dagli scienziati, un Universo eterno tornava utile per non dover speculare circa la causa prima alla sua origine. Anche chi credeva che l'Universo avesse avuto un inizio, era comunque convinto che dalla creazione in poi esso fosse rimasto pressoché immutato. E' celebre la circostanza per cui persino Einstein, non proprio un conservatore, accortosi di come la sua teoria implicasse un Universo dinamico, introdusse un termine alle sue equazioni, la cosidetta costante cosmologica, così da ottenere un Universo statico e in linea con le sue convinzioni. Successivamente lo stesso Einstein dichiarò come l'introduzione della costante cosmologica fosse stato uno, se non il più grave, dei suoi errori.[3]

In breve; le rivoluzionarie osservazioni di Hubble trovavano conforto nella derivazione teorica di Lemaître. E' sufficiente questo per concludere che l'Universo si è espanso da uno stato iniziale a densità molto elevata i cui effetti sono visibili ancora oggi?

No. Una teoria scientifica in grado di spiegare una sola osservazione può essere facilmente rimpiazzata da una descrizione alternativa (come ad esempio la teoria dello stato stazionario in voga a quel tempo). Questo è vero in particolar modo se la visione del mondo che fornisce è in contrasto con tutte le convinzioni della sua epoca. Il momento in cui gli scienziati hanno iniziato a considerare seriamente la teoria del Big Bang è quando ci si è resi conto che questa descrizione portava con sé altre implicazioni che potevano essere verificate. I fisici Alpher e Gamow nel 1948 mostrarono come questa ipotesi fosse in grado di spiegare l'abbondanza di elementi leggeri come gli isotopi di elio e idrogeno presenti nel nostro Universo. Inoltre, con la compartecipazione di un altro fisico, Herman, ebbero l'intuizione per quella che è considerata la conferma più importante della teoria del Big Bang, nonché fondamentale per il lavoro di questa tesi. Se l'Universo si è evoluto da uno stato primordiale denso e caldo, l'espansione e successivo raffreddamento implicano un fatto ineludibile: deve esistere una radiazione onnipresente nello spazio, la cosidetta *radiazione cosmica di fondo* (CMB).

Dopo circa 380mila anni dalla sua iniziale espansione la temperatura è scesa al punto che gli elettroni liberi presenti nel plasma primordiale vennero catturati dai

[3]Recentemente (1998) dall'osservazione delle Supernovae di tipo Ia si è scoperto che l'Universo è in una fase di espansione accelerata. Questa può essere spiegata matematicamente includendo la costante cosmologica nelle equazioni di Einstein. Così, pare che Einstein fosse destinato ad introdurre concetti avveniristici anche quando sbagliava.

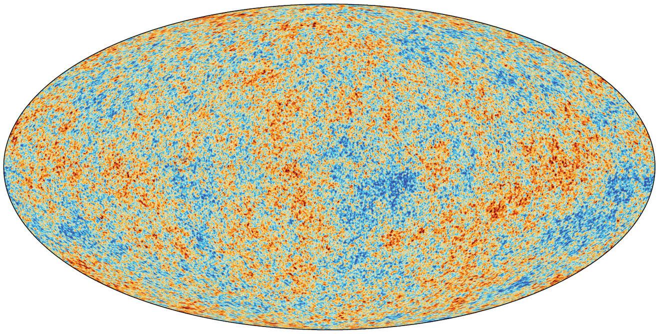
protoni così da formare atomi neutri di idrogeno. Come risultato le particelle di luce, i cosiddetti fotoni, non vennero più rimbalzati e trattenuti da questo mare di elettroni ma furono in grado di propagarsi liberamente per la prima volta. Alcuni di questi fotoni raggiungono la Terra solo oggi, dopo un viaggio di 13.8 miliardi di anni, producendo la radiazione cosmica di fondo. E' come se l'Universo in passato fosse permeato da una densissima nebbia. In un preciso momento la nebbia scomparve e la luce iniziò a viaggiare liberamente da ogni punto e in tutte le direzioni. I fotoni provenienti da regioni a noi vicine ci raggiunsero in passato, mentre i fotoni che ci raggiungono solo oggi provengono da regioni più lontane e mostrano come era l'Universo nel momento in cui la nebbia è svanita. Così, la radiazione cosmica di fondo ci consegna un'immagine dell'Universo primordiale quando aveva appena 380mila anni.

Nel 1964, nel New Jersley, Arno Penzias e Robert Wilson, due fisici che lavoravano per una compagnia di telecomunicazioni, stavano testando un'antenna che volevano utilizzare per esperimenti di radioastronomia. Esattamente come quando cerchiamo di sintonizzarci su una stazione radio ma il segnale è disturbato, essi cercarono di eliminare in tutti i modi un'interferenza dal loro apparecchio. Ripulirono persino gli escrementi di alcuni piccioni che avevano nidificato sull'antenna, ma questò non bastò per sbarazzarsi di quel fastidioso segnale che arrivava da tutte le direzioni dello spazio. Dopo un anno di tentativi, fecero una telefonata ai loro colleghi di Stanford, la loro perseveranza li aveva portati a una scoperta che gli sarebbe valsa il Nobel: il rumore captato era la radiazione cosmica di fondo.

Questa radiazione ci bombarda da tutte le direzioni con la stessa temperatura. Oggi sappiamo essere con precisione di 2.73 K (circa -270 ° C) più o meno piccolissime variazioni dell'ordine di 0.000001 K[4] sulle quali torneremo in seguito. Se le diverse regioni da cui la radiazione proviene condividono tutte la stessa informazione (la temperatura in questo caso) è ragionevole pensare che esse abbiano interagito tra di loro in passato. Tuttavia, due fotoni che arrivano sulla Terra da due direzioni diametralmente opposte hanno viaggiato rispettivamente 13.8 miliardi di anni luce. Senza contare l'espansione dell'Universo, ciò significa che essi erano separati da una distanza maggiore di 27 miliardi di anni luce all'inizio del loro viaggio. E' subito chiara la contraddizione. Com'è possibile che regioni così distanti possano aver avuto il tempo di interagire se l'età dell'Universo era di appena 380mila anni quando la CMB è stata emessa?

Si potrebbe replicare affermando che, forse, al principio della sua espansione l'Universo era semplicemente già omogeneo. Questa è una assunzione o meglio una messa a punto (fine tuning) delle condizioni iniziali che ai fisici non piace prendere

[4] Come conseguenza dell'espansione dell'Universo, questi fotoni ci raggiungono dopo aver perso gran parte della loro energia.



La CMB è la luce più antica che raggiunge i nostri telescopi. Essa ci circonda come una sfera posta ai limiti del nostro Universo osservabile. In figura la sua proiezione dove sono evidenziate le minuscole fluttuazioni di temperatura. Esse corrispondono a regioni diversa densità quando l'Universo aveva appena 380 mila anni.

in considerazione. Per capire che questa presa di posizione è qualcosa di più che un capriccio cerchiamo di rappresentare il problema con una metafora. Supponiamo che in un concorso pubblico con un enorme numero di partecipanti, diciamo pari a 10^{60} (1 seguito da 60 zeri),[5] tutti realizzino lo stesso punteggio. Pensereste che abbiano ricevuto un'informazione in comune prima del concorso o che il risultato sia frutto di una coincidenza?

Un'espansione sorprendente

Nel 1980 il fisico Alan Guth propose una semplice soluzione per risolvere il problema dell'omogeneità della radiazione: il nostro Universo, molto prima di raggiungere il suo primo secondo di vita, si espanse in maniera esponenziale per una frazione di secondo, aumentando le sue dimensioni di un fattore 10^{26}. Per apprezzare il significato di questo numero bisogna pensare che in proporzione è come se l'Universo si fosse espanso, in una frazione di secondo, dalle dimensioni di un batterio (10^{-6} m) a 100 volte le dimensioni della nostra Galassia (la via Lattea ha un diametro di circa 100mila anni luce $\approx 10^{20}$ m).

Questa espansione sorprendente, la cosidetta *inflazione cosmica*, avrebbe fatto si che un piccolo frammento dell'Universo (prima dell'inflazione) si sia gonfiato sino a diventare quello che oggi è il nostro Universo osservabile e che così risulta essere

[5]Numero che è ovviamente maggiore della popolazione mondiale ma queste sono le regioni causalmente disconnesse che condividono la stessa temperatura nella CMB.

Un piccolo frammento dell'Universo primordiale si "inflaziona" sino a diventare il nostro Universo osservabile.

omogeneo. Ancora una volta possiamo chiederci, basta questo per convincerci che ci sia stato un periodo di inflazione nel primo secondo di vita del nostro Universo?

Molto probabilmente questo non sarebbe stato sufficiente per giustificare il successo che questa teoria ha oggi tra i cosmologi. Sebbene essa sia stata introdotta per risolvere il problema menzionato sopra, il suo grande successo risiede nel fatto che un periodo di inflazione riesce a spiegare in modo semplice ed elegante un altro fatto osservabile: esistono delle disomogeneità nel nostro Universo, come ad esempio la nostra Galassia o il nostro pianeta.

Facciamo un passo indietro. Grazie ai contributi di vari satelliti oggi sappiamo che la temperatura della CMB è *quasi* completamente uniforme. Ricordate le piccolissime variazioni dell'ordine di 0.000001 K? Significa che quando la CMB è stata emessa alcune regioni erano infinitesimamente più fredde di altre. Questo implica che in queste regioni la gravità era lievemente maggiore che nelle immediate vicinanze. Queste regioni hanno iniziato ad attrarre più materia amplificandosi a poco a poco sotto il costante effetto della gravità. La loro successiva evoluzione ha creato le strutture che osserviamo oggi nell'Universo. Ma tutto questo che cosa ha a che fare con l'inflazione?

Ebbene l'inflazione è in grado di spiegare in modo sorprendente come le piccolissime variazioni di temperatura nella CMB si siano originate. Per avere un'idea di come questo sia possibile è opportuno introdurre un paio di concetti che ci torneranno utili anche sul finale. La pazienza del lettore sarà dunque premiata per ben due volte.

La meccanica quantistica, ovvero la branca della fisica adibita a studiare scale di lunghezza molto piccole (dell'ordine delle dimensioni di un atomo), ci insegna che vi è sempre un'incertezza irremovibile sul quantitativo di energia presente in un

sistema. Da questa incertezza possono nascere, anche dal vuoto, coppie di particelle per un breve periodo di tempo, secondo la celebre equivalenza tra massa ed energia $E = mc^2$. Così è utile immaginarsi il vuoto, non proprio vuoto, ma come costituito da coppie di particelle che nascono e si annichiliscono in continuazione. Durante un periodo di espansione esponenziale come l'inflazione queste fluttuazioni vengono amplificate e gonfiate fino a scale in grado di influenzare la gravità del mondo macroscopico. Per primi, Mukhanov, Starobinsky e Hawking mostrarono in modo indipendente come le fluttuazioni originatesi durante l'inflazione abbiano una controparte nelle minuscole variazioni di temperatura osservate nella radiazione cosmica di fondo. L'ipotesi dell'inflazione offre così l'entusiasmante possibilità di mettere in relazione ciò che osserviamo nel cielo con quello che potrebbe essere avvenuto una frazione di secondo dopo il Big Bang.

Nonostante sembri tutto meraviglioso, non siamo ancora in grado di fornire una spiegazione soddisfacente su che cosa abbia causato l'inflazione. Centinaia di modelli e scenari possibili sono stati ipotizzati. Ognuno di essi descrive la dinamica del cosidetto inflatone, la particella (o più in generale il meccanismo fisico) responsabile per l'inflazione. Questa a sua volta fornisce precise previsioni riguardo alle proprietà della radiazione cosmica di fondo (come l'intensità e la distribuzione nello spazio delle fluttuazioni di temperatura) che possono essere verificate dalle osservazioni.[6]

L'approccio preso maggiormente in considerazione in questa tesi è quello del Rasoio di Occam. Per giocare il ruolo dell'inflatone una particella deve possedere almeno una proprietà intrinseca. L'unica particella verificata sperimentalmente e con questa particolare proprietà è il bosone di Higgs, scoperto nel 2012 presso l'acceleratore di particelle del CERN Ginevra (LHC). Un approccio minimale vorrebbe il bosone di Higgs come motore dell'inflazione e infatti sono stati proposti modelli che considerano questa ipotesi. Nonostante le scale di energie in gioco all'LHC siano molto elevate (dell'ordine di migliaia di elettronvolt) esse sono comunque almeno dieci ordini di grandezza inferiori a quelle che si ipotizza abbiano caratterizzato il fenomeno dell'inflazione. Ma possiamo realmente descrivere il comportamento del bosone di Higgs alle scale dell'inflazione e fare previsioni realistiche riguardo i parametri della CMB se conosciamo solo le sue proprietà a energie molto più basse?

[6]Le equazioni che definiscono questo meccanismo sono state brevemente riassunte nel primo capitolo.

Running nell'Universo primordiale

La suggestione da cui parte questo lavoro di tesi è quindi quella di mettere in relazione quanto osservato nei telescopi (le proprietà della CMB) con i parametri che descrivono l'Higgs misurati al CERN di Ginevra. Questi parametri, detti costanti di accoppiamento, per il bosone di Higgs sono da intendersi, ad esempio, come l'equivalente della carica elettrica per un elettrone.

Ricordiamo quanto visto in precedenza: dato un modello per l'inflazione questo fornisce delle previsioni riguardo alle proprietà della CMB. Tuttavia, non possiamo semplicemente usare i parametri misurati a basse energie per descrivere la dinamica dell'inflazione a causa di un motivo che a prima vista potrebbe sembrare controintuitivo: il valore dei parametri cambia al variare dell'energia. Le constanti di accoppiamento non sono realmente costanti!

Per capire questo concetto si consideri la carica elettrica, parametro che misura la forza con la quale due elettroni si respingono. In modo approssimativo possiamo immaginarci di misurare questa carica sparando un "elettrone sonda" verso un altro elettrone e osservare il loro successivo rimbalzo. Ricordate che il vuoto non è esattamente vuoto? Lo stesso principio vale per lo spazio che circonda l'elettrone che, come una pentola in ebollizione, è pieno di particelle e antiparticelle (di carica opposta) che nascono e si annichiliscono. Queste continue fluttuazioni del vuoto creano una nuvola attorno all'elettrone che funge da "schermo" per la sua carica elettrica. Nel nostro esperimento, maggiore sarà l'energia posseduta dall'elettrone sonda e "più in profondità" esso riuscirà a penetrare nella nuvola, sperimentando di conseguenza una diversa carica elettrica.

Per fare previsioni riguardo alla CMB, i parametri dell'Higgs che entrano in gioco sono quelli ipoteticamente misurati alle scale dell'inflazione. Uno degli scopi principali di questa tesi è stato capire come i parametri variano variando l'energia, in termini fisici fanno *running* (corrono), sino alle scale caratteristiche dell'Universo primordiale. Così abbiamo studiato il *running nell'Universo primordiale*.

In particolare, nel capitolo 3 abbiamo analizzato nel dettaglio il modello più popolare dove l'Higgs è l'inflatone, chiamato Higgs inflation (inflazione di Higgs). Ci siamo così chiesti se le sue previsioni per le proprietà della CMB siano consistenti con uno studio rigoroso del running dei parametri. Nel calcolare come i parametri variano aumentando l'energia di diversi ordini di grandezza, è fondamentale tenere in considerazione un altro aspetto. Ad energie più elevate di quelle sperimentate nei nostri laboratori, nuove particelle o nuovi fenomeni che non conosciamo, e ai quali ci riferiremo in seguito con l'appellativo di *nuova fisica*, possono fare il loro ingresso in scena (ricordate $E = mc^2$) e partecipare alla danza delle fluttuazioni

del vuoto. Questa nuova fisica può influenzare il running dei parametri ed è stato quindi necessario parametrizzarne in modo sistematico i possibili effetti. Ciò che abbiamo scoperto è che le previsioni sono indipendenti dal contributo che potrebbe avere l'avvento di nuova fisica e in perfetto accordo con le osservazioni del satellite Planck.[7]

Nel capitolo 4, abbiamo esteso questo risultato ad una classe molto più ampia di modelli inflazionari. Mettendo da parte per un momento il Rasoio di Occam, abbiamo mostrato come la struttura matematica di questi modelli (di cui Higgs inflation è un caso particolare) garantisca la solidità delle loro previsioni una volta tenuto conto del running dei parametri.

Infine nel capitolo 5, abbiamo considerato un modello alternativo a Higgs inflation, semplicemente chiamato new (nuova) Higgs inflation e non facente parte della classe di modelli studiata nel capitolo 4. Così, ripercorrendo a ritroso la storia del nostro Universo, abbiamo mostrato come le previsioni per questo modello siano sensibili all'evoluzione dei parametri dell'Higgs. Se da un lato questo rappresenta un problema in termini predittivi, allo stesso tempo rende il modello sensibile a dinamiche di fisica che ancora non conosciamo. Lo scopo principale di quest'ultimo capitolo è stato così di mostrare come sia importante tenere in considerazione il running per determinare in modo affidabile le previsioni di un modello.

E' interessante ricordare che la *ratio* che ha portato all'introduzione dei modelli di Higgs inflazione era quella di una richiesta di minimalità: evitare di introdurre nel modello teorico per spiegare l'inflazione nuova fisica oltre quella conosciuta e sperimentata. Tuttavia, dal nostro studio (così come da altri sul tema) emerge come l'introduzione di nuova fisica sia necessaria per spiegare, attraverso un quadro teorico consistente, come l'attuale fisica delle particelle sia in relazione con la fisica dell'inflazione.

Questa per i fisici è sicuramente una buona notizia, di solito siamo contenti quando realizziamo che quanto scoperto non è tutto quello che c'era da scoprire.
Non è poi forse ammettendo la nostra ignoranza che questa storia è iniziata?

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[7] Più precisamente, a meno che l'effetto non sia tale da precludere completamente un periodo di inflazione, allora le previsioni date da Higgs inflation rimangono indipendenti dal contributo di nuova fisica. Da qui il titolo "UV (in)sensitivity of Higgs inflation".