Measurement of Invisible Z Decays

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Meting van Onzichtbare Z vervallen

(met een samenvatting in het Nederlands)

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Introduction

One of the fundamental questions, already pondered by the Greek philosophers Aristotle, Leukippus and Democritus around 400 BC, is what the fundamental constituents of the universe are. Democritus introduced the concept of the “atom”, the smallest indivisible building block. Aristotle’s universe consisted of the four elements fire, water, earth and air. Everything else could be built from these four elements. Another question is how these elements, or atoms interact. The concept of forces acting at a distance was posed by Newton in 1687 in his theory of gravity. The interaction of electrically charged particles was studied by Coulomb, Ampere and Faraday. It was recognized by Maxwell in 1864 that the two different forces acting upon these charged particles, could be unified into one single force, the electromagnetic force.

Five years later, in 1869, the periodic table of elements was constructed by Mendeleev. On the basis of the regularity of periodic table, the existence of three new elements was predicted. Based on their expected chemical properties, the elements Gallium, Scandium and Germanium were discovered in 1875, 1879 and 1886. The regularity of the periodic table suggested that the elements themselves are not elementary, but are made of a smaller number of elementary particles. These particles are the electron, the proton and the neutron, which together form the atoms which give the periodic table its structure. To explain why the atomic nucleus, made of protons and neutrons, is stable, a third force was introduced, the so-called strong force.

Until the 1930’s, although the underlying theories had changed radically, these particles seemed to be all that was needed. However, the study of $\beta$ decay and the observations of cosmic rays led to the discovery of new particles that did not fit into the picture. Moreover, $\beta$ decay prompted the introduction of yet another force, the weak interaction.

Over the years, more and more particles were discovered, leading to a vast “zoo” of supposedly fundamental particles. Again, the regularity of the particles which interact through the strong force hinted at the existence of yet another layer of more fundamental constituents, the quarks. Nowadays the quarks, together with the particles that are insensitive to the strong force, the leptons, are considered to be “elementary” particles.

The interactions between quarks and leptons are described by the Standard Model of Electroweak Interactions. This theory, which was developed at the end of the sixties and at the beginning of the seventies, successfully describes the measurements. It also unifies the Weak and Electromagnetic forces into a single Electroweak force. This unification led to the prediction of the existence of the massive intermediate vector bosons $W^\pm$ and $Z$, which, together with the photon, are the carriers of this Electroweak interaction. The discovery of the $W^\pm$ and $Z$ in 1983 by the UA1 and UA2 collaborations [1, 2] at CERN was a breakthrough for the Standard Model. The LEP accelerator was designed and build to produce large amounts of $Z$ bosons in an experimentally “clean” environment. The precision measurements of the properties of the $Z$ enable rigorous tests of the Standard Model.
Although the Standard Model predicts that leptons and quarks can be grouped in “families”, it does not predict the total number of these “families”. With the recent discovery of the sixth quark by the CDF and D0 collaborations at Fermilab [3, 4], all members (with the exception of the $\tau$-neutrino) of the first three generations have been observed experimentally. This opens the question whether there is a “fourth generation” of particles waiting to be discovered. The measurements at LEP allow, albeit assuming the validity of some hypotheses, the determination of the number of “families” of leptons and quarks. One of these measurements, the measurement of the production cross section for events containing a single photon is the subject of this thesis. These events can be produced by the reaction $e^+e^- \rightarrow \nu\bar{\nu}\gamma$, and the cross section for this process depends almost linearly on the number of different neutrino types that can be produced.

The outline of this thesis is as follows: in Chapter 1 the Standard Model of Electroweak Interactions is introduced, and some of the general predictions of the model are presented. Then, in Chapter 2, the different methods available to determine the number of families are discussed. Chapter 3 describes the LEP collider and the $L_3$ detector, which were used to collect the data for the measurements. The Fourth Chapter presents the analysis of the data: the determination of the various efficiencies and the estimation of the remaining backgrounds. Finally, the extraction of the results from the measurements and their interpretation is presented in Chapter 5.

Unless otherwise specified, all quantities are expressed in so-called natural units corresponding to: $\hbar = c = 1$. 
Chapter 1

The Standard Model of Electroweak Interactions

1.1 The Neutrino

Neutrinos first emerged to explain the spectrum of electrons produced in nuclear β decay, such as $^{137}\text{Cs} \rightarrow ^{137}\text{Ba} + e^- + \bar{\nu}_e$. If this decay had been a two-body decay, the electron would have had a fixed energy, equal to the difference in mass between the initial and final nuclei. However, observations ruled out this simple model: energy (and spin) seemed to be lost. To save the law of conservation of energy and momentum, a third particle, the neutrino $\nu$ (and its corresponding anti-particle $\bar{\nu}$), was introduced by Pauli. This particle, which hardly interacts with matter and therefore escapes undetected, carries the unobserved (or “missing”) energy in β decay. The observation of the inverse β decay [5], initiated by the neutrino, confirmed this hypothesis.

After the discovery of the neutron [6, 7], a nucleus could be described as consisting of protons and neutrons, and all basic ingredients for a model of β decay were present. Fermi introduced a model which described β decay through the point-like interaction between four particles, resulting in the transformation of a neutron into a proton, emitting an electron and an anti-neutrino in the process. Inspired by the formulation of electromagnetic scattering, this interaction was written by Fermi as the interaction between two vector currents.

Before the discovery of muons, pions and kaons in cosmic rays, nuclear β decay was the only known “weak” process. The weak decays of these particles and other hadrons were originally incorporated into the Fermi model by the introduction of additional currents.

From the measurements [8, 9] of the polarization of the electrons produced in β decay, it became clear that the simple picture of interacting vector currents is not quite correct as only left-handed neutrinos are produced. This observation led to the formulation [10, 11] of the “Universal V – A Theory” where the Lorentz structure of the weak currents is given by the maximal parity violating vector minus axial-vector structure. Another observation [12], i.e. that neutrinos produced by the decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$ subsequently only produce muons via $\nu_\mu + n \rightarrow p + \mu^-$, and not electrons, prompted the introduction of two distinct types of neutrinos, one accompanying the electron, and one the muon. After the formulation of the quark model [13, 14, 15, 16], which describes hadrons in terms of their constituent quarks, the weak interactions of hadrons could be absorbed into the $V – A$ Theory by the addition of quark currents.
The Universal \( V - A \) Theory can be described by the following Lagrangian density:

\[
\mathcal{L}_{\text{eff}} = -\sqrt{2}G_F j_\mu^+ j^-\mu
\]  

(1.1)

where \( G_F = 1.16630(2) \times 10^{-5} \text{ GeV}^{-2} \) and the charged current \( j_\mu^\pm \) is given by:

\[
j_\mu^\pm = \sum_i \bar{\Psi}_i^\dagger \gamma_\mu (1 - \gamma_5) T^\pm \Psi_i
\]  

(1.2)

with \( T^\pm = \frac{1}{2}(\tau^1 \pm i\tau^2) \), where \( \tau^i \) are the Pauli matrices, and \( \Psi^i \) the (weak isospin) doublets of lepton fields:

\[
\Psi^i = \left\{ \left( \begin{array}{c} \nu_e \\ e^- \end{array} \right), \left( \begin{array}{c} \nu_\mu \\ \mu^- \end{array} \right), \left( \begin{array}{c} \nu_\tau \\ \tau^- \end{array} \right) \right\}
\]  

(1.3)

and quark fields:

\[
\Psi^i = \left\{ \left( \begin{array}{c} u \\ d' \end{array} \right), \left( \begin{array}{c} c \\ s' \end{array} \right), \left( \begin{array}{c} t \\ b' \end{array} \right) \right\}
\]  

(1.4)

As the weak eigenstates of the quarks are not necessarily the same as their mass eigenstates a mixing matrix, \( V_{\text{CKM}} \), is introduced [17, 18]:

\[
\begin{pmatrix}
\frac{d'}{s'} \\
\frac{b'}{s'}
\end{pmatrix} = V_{\text{CKM}} \times
\begin{pmatrix}
d \\ s \\ b
\end{pmatrix}
\]  

(1.5)

The strong interactions of quarks can be described by Quantum Chromodynamics (QCD) and will not be considered in this thesis.

### 1.2 Weak Gauge Bosons

However successful it is in describing low energy experiments, the universal \( V - A \) model has a fundamental flaw: it predicts that the cross section of processes such as \( \nu_\mu p \rightarrow \mu^+ n \) increases as \( G_F^2 s \), where \( s \) is the square of the center-of-mass energy. At a certain point this will break the upper limit which is set by the requirement of unitarity, i.e. the conservation of probability. This problem originates in the fact that \( G_F \) has the dimensions of \( \text{ GeV}^{-2} \), caused by the point-like structure of the current-current interactions.

The high-energy behavior can be improved by replacing the point-like interaction of two fermion currents with a model in which the interaction between the currents is mediated by an intermediate boson [19], similar to Quantum Electrodynamics (QED):

\[
\mathcal{L}_{\text{vector}} = ig \left( W^- j^+ + W^+ j^- \right)
\]  

(1.6)

In this case it is no longer the coupling constant which determines the typical energy scale (and thus the range) of the weak interactions, but the mass of the \( W^\pm \) boson. As a result the coupling constant \( g \) has become dimensionless. The exchange of the intermediate boson requires the insertion of a boson propagator into the matrix element, which, as is the case in QED, improves the high-energy behavior. At low energies, where the transferred momentum \( q^2 \) is small compared to the mass of the
W$^\pm$, the propagator can be approximated by $1/m_W^2$, so the universal V – A model can be regained if the coupling constant $g$ and the mass $m_W$ of the W$^\pm$ bosons satisfy the following constraint:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

(1.7)

Although the introduction of a boson which mediates the weak interaction has removed the point-like interaction, this model still breaks unitarity, albeit at higher energies. An example is the cross section for the pair production of W$^\pm$, i.e. $e^+e^- \rightarrow W^+W^-$ or even the reaction $\nu_e\bar{\nu}_e \rightarrow W^+W^-$ which is mediated by the weak force only$^1$. A large part of the problem can be removed by introducing an additional neutral gauge boson [20]. The additional contribution of this neutral boson to the cross section ensures that the unitarity limit is not violated as long as the couplings of this neutral boson are chosen such that the resulting model remains invariant under local $SU(2)_L$ transformations. Such a non-Abelian gauge theory was first considered by Yang and Mills [21]. The requirement of local gauge invariance determines both the coupling constant and the current with which the third gauge boson interacts. However, it also seems to prohibit the bosons from acquiring any mass, as explicit mass terms break this invariance. This seems to be in conflict with Equation 1.7. Ignoring the problem of gauge boson masses, the part of the Lagrangian density that describes the interaction of this neutral gauge boson with the particle fields can be written as:

$$\mathcal{L}_{\text{interaction}} = ig W^0 j^0$$

(1.8)

As mentioned, the additional current $j^0$ is completely determined by the requirement that the three currents form a gauge algebra:

$$j^0_\mu = \sum_i \bar{\Psi}^i \gamma_\mu (1 - \gamma_5) T^0 \Psi^i$$

(1.9)

As the current $j^0$ to which the W$^0$ couples is not a vector current, it cannot be interpreted as the photon.

In 1973, neutral current interactions were discovered by the Gargamelle collaboration [22] at CERN through the observation of the reaction $\bar{\nu}_e e^- \rightarrow \nu_e e^-$. However, measurements [23] of the ratio $R = \sigma_{\nu_e e^-} / \sigma_{\bar{\nu}_e e^-}$ indicate that the neutral current interactions do not have the V – A structure as would be expected from Equation 1.9.

### 1.3 Unification of Weak and Electromagnetic Interactions

The deviation from Equation 1.9 of the neutral current interactions can be incorporated into a model that combines both the weak interactions and electromagnetism in one single gauge theory. The minimal model, which describes the interactions of both the charged chiral weak current, mixed vector-axial neutral weak current and the vector QED current can be obtained by enlarging the $SU(2)_L$ group to the group $SU(2)_L \otimes U(1)_Y$. This implies the introduction of a new quantum number, the weak hypercharge $Y$, which is defined as:

$$\frac{1}{2} Y = Q - T^0$$

(1.10)

$^1$but unlikely to be measured experimentally.
<table>
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<td>ν_μ</td>
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<td>d_R</td>
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<td>s'</td>
<td>s</td>
<td>b</td>
<td>b</td>
<td>s_R</td>
<td>1</td>
</tr>
</tbody>
</table>

| 2 | -1/2 | 1 | -1 |

Table 1.1: The quantum numbers of the fermion fields.

where $Q$ is the electromagnetic charge operator. The demand that the model be invariant under local $SU(2)_L \otimes U(1)_Y$ gauge transformation prescribes, apart from the already present $W^\pm$ and $W^0$, the introduction of a fourth vector boson $B$ and an additional coupling constant $g'$. The interaction between the gauge-fields and the currents can be described through the following Lagrangian density:

$$\mathcal{L}_{\text{interaction}} = ig \left( W^- j^+ + W^+ j^- + j^0 W^0 \right) + ig' j^- Y B$$

(1.11)

To account for the vector properties of QED interactions the additional current $j^Y$ contains both the left-handed and right-handed fermion fields:

$$j^Y_\mu = \bar{\nu}_L \gamma_\mu \frac{1}{2} (1 - \gamma_5) Y \nu^\mu + \bar{L}_R \gamma_\mu Y L^\mu_R + \bar{L}_L \gamma_\mu Y L^\mu_L + \bar{Q}_R \gamma_\mu Y Q^\mu_R + \bar{Q}_L \gamma_\mu Y Q^\mu_L$$

(1.12)

where $l_{R/L} = \frac{1}{2} (1 \pm \gamma_5) l$ and $q_{R/L} = \frac{1}{2} (1 \pm \gamma_5) q$ are the right- and left-handed lepton and quark fields, and a summation over the fermion families is implied. If the eigen-values of the different fields are assigned as in Table 1.1, a current is obtained which, by construction, has the right charge and the correct vector structure to couple to the photon:

$$j^{EM} = \frac{1}{2} j^Y + j^0$$

(1.13)

It is clear that to ensure that this current couples to the photon, it has to be a combination of the $W^0$ and the $B$.

1.4 Spontaneous Symmetry Breaking

The requirement that the weak gauge bosons acquire a mass to regain the Fermi model in the low energy limit seems to be in contradiction with the requirement of gauge invariance: introducing explicit mass terms for the intermediate vector bosons breaks the gauge invariance.

It is however possible to generate mass terms for the gauge bosons by spontaneously breaking the gauge symmetry. This can be accomplished through the Higgs [24, 25, 26] Mechanism, which
introduces a ground state (the vacuum) which does not exhibit the full symmetry of the Lagrangian. To generate this non-trivial vacuum, a complex iso-doublet $\Phi$ with a non-zero vacuum expectation value $v$ is introduced. The potential for this field $\Phi$ is chosen such that the ground state is no longer invariant under weak isospin transformations. However, it is constructed in such a way that it remains invariant under $Q$, i.e. the choice of a ground state breaks the $SU(2)_L \otimes U(1)_Y$ gauge symmetry down to a $U(1)_{EM}$ symmetry.

The interactions between the field $\Phi$ and the gauge bosons (which again are completely determined by the requirement of invariance under local gauge transformations) generates, amongst others, mass terms for the bosons when the field $\Phi$ is expanded around one of the possible ground state configurations. The location of this minimum is determined by the vacuum expectation value $v$ of the Higgs field. In terms of $v$, the mass terms of the gauge bosons can written as:

$$\mathcal{L}_{\text{vector mass}} = \frac{g^2 v^2}{8} (W^+ W^- + W^- W^+) + \frac{v^2}{8} (g W^0 - g' B)^2$$

(1.14)

The mass of the $W^\pm$ can be read off directly, but to determine the mass of the other two bosons their mass matrix must be diagonalized. This leads to the eigenvectors $A$ and $Z$:

$$\begin{pmatrix} A \\ Z \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B \\ W^0 \end{pmatrix}$$

(1.15)

where $\theta_W$, the Weak Mixing Angle, is defined as $\tan \theta_W = g'/g$. The first one, $A$, remains massless, and, by construction, couples to a current proportional to $Q$ (which leaves the vacuum invariant), i.e. $j^{EM}$, and can be identified as the photon. The second one, $Z$, together with the $W^\pm$, obtains a mass:

$$m_W = \frac{1}{2} g v,$$

$$m_Z = \frac{1}{2} \sqrt{g'^2 + g^2} v = m_W / \cos \theta_W$$

(1.16)

(1.17)

As the ratio between the $W^\pm$ mass and the coupling constant $g$ is fixed by Equation 1.7, the ground state energy of the Higgs field is determined by Equation 1.17: $v \approx 246$ GeV. This process leaves one of the degrees of freedom of the original iso-doublet $\Phi$, which should manifest itself as the much sought after Higgs-particle, denoted by $H$. The mass of this Higgs particle is not predicted: it is a free parameter of the model.

The original interaction Lagrangian density, Equation 1.11, can now be rewritten in terms of the $W^\pm$, $Z$ and $A$:

$$\mathcal{L}_{\text{interaction}} = ig \left( W^+ j^- + W^- j^+ \right) + ig_Z Z j^{NC} + ie A j^{EM}$$

(1.18)

where the photon field $A$ couples to the electromagnetic current with strength $e = gg' / \sqrt{g^2 + g'^2}$. The $Z$ couples, with the coupling constant $g_Z = e / 2 \sin \theta_W \cos \theta_W$, to the following neutral weak current:

$$j^{NC}_{\mu} = \left( j^0_{\mu} - \sin^2 \theta_W j^{EM}_{\mu} \right)$$

$$= \sum_i \bar{\psi} \gamma_\mu (g' \gamma^5 - g \gamma^5) \psi$$

(1.19)

(1.20)
where $g_V^f$ and $g_A^f$ are the vector and axial-vector coupling constants:

$$g_V^f = T_3^f - 2 \sin^2 \theta_W Q_f^f, \quad g_A^f = T_3^f$$  \hspace{1cm} (1.21)

As can be seen from this equation, the deviation of the neutral current from the pure $V - A$ coupling is determined by the weak mixing angle $\theta_W$. Using Feynman diagrams the interactions between the gauge bosons and fermions can be summarized in the following diagrams:

$$\gamma \quad \overline{\ell} \quad f \quad \rightarrow \quad -ieQ_f \gamma_\mu$$

$$Z \quad \overline{\ell} \quad f \quad \rightarrow \quad -ie \frac{e}{2 \sin \theta_W \cos \theta_W} \gamma_\mu (g_V^f - g_A^f \gamma_5)$$

$$W^\pm \quad \overline{\ell} \quad \rightarrow \quad -ie \frac{e}{2 \sqrt{2} \sin \theta_W} \gamma_\mu (1 - \gamma_5)$$

$$W^\pm \quad \overline{q_i} \quad q_j \quad \rightarrow \quad -ie \frac{e}{2 \sqrt{2} \sin \theta_W} \gamma_\mu (1 - \gamma_5) V_{ij}^{\text{CKM}}$$  \hspace{1cm} (1.22)

The fermion masses can be generated by postulating, for each type of fermion, a Yukawa interaction between the Higgs field and the fermion. Again, expanding $\Phi$ around the minimum of its potential leads to mass terms for the fermions and an interaction between the fermions and the Higgs boson which is proportional to the fermion mass. The model does not predict the values of the coupling constants between each fermion species and the Higgs field. The fermion masses are thus free parameters in this model.

The above model of weak and electromagnetic interactions in terms of a spontaneously broken gauge theory based on the group $SU(2)_L \otimes U(1)_Y$ is known as the Glashow-Salam-Weinberg Model [27, 28, 29], or simply the Standard Model.

### 1.5 Renormalization and Anomaly Cancellation

At the lowest order in perturbation theory, the Standard Model has no infinities. However, when trying to calculate higher order corrections (which include loop diagrams) to observables, one finds divergences. By redefining the original (or “bare”) parameters of the model in terms of the observable quantities themselves, these divergences can be “renormalized”, and higher order calculations yield a finite correction with respect to the lower order calculations. In 1971, it was shown [30, 31] that the Standard Model is a renormalizable theory.

However, the requirement of renormalization does lead to some conditions, so called “anomaly cancelation” conditions. For instance, the coupling of a $Z$ and two photons to a virtual fermion loop poses a potential problem, because of the combination of $V - A$ and $V$ couplings (the so-called chiral anomaly). In order to cancel the divergence caused by this diagram, one has to require that the sum of the electric charge of all contributing fermions is zero. In the Standard Model this only occurs [32] when the number of lepton doublets is equal to the number of quark doublets$^2$.

---

$^2$This also requires that the quarks themselves come in three so-called color degrees of freedom, as is the case in QCD.
1.6 Predictions from Electroweak Unification

Assuming three families of fermions and massless neutrinos, the Standard Model describes the electroweak and strong interactions between twenty-five different particles in terms of seventeen parameters. The twenty-five particles consist of twelve fermions: six quarks, three charged leptons and three massless neutrinos; twelve gauge bosons: $W^\pm$, $Z$, $\gamma$; eight gluons and one scalar boson: the Higgs. To do so, it requires as input the following parameters: one parameter which determines the ground state energy of the Higgs field; nine constants which describe the interaction between the Higgs field and the fermions; the three coupling constants of the gauge bosons and four parameters to describe $V_{CKM}$. Of these parameters, three are specific to the electroweak interactions: the two coupling constants $g$ and $g'$ and the vacuum expectation value $v$.

However, it is more convenient to parameterize the electroweak sector of the model in terms of constants which can be measured with high precision. Two convenient quantities are the electromagnetic coupling constant $\alpha_{EM}$ and the Fermi constant $G_F$. Before the measurements at LEP, the third parameter was usually taken to be $\sin^2 \theta_W$, determined from the ratio of charged current to neutral current interactions, as measured from $R = \sigma(\nu_e e^-)/\sigma(\bar{\nu}_e e^-)$: $\sin^2 \theta_W = 0.230 \pm 0.007$ [23]. Using these values, the other parameters can be determined. As an example, the masses of the $W^\pm$ and $Z$ bosons can be calculated as a function of $\alpha_{EM}$, $G_F$ and $\sin^2 \theta_W$ using Equation 1.17:

$$m_W = \left(\frac{\pi \alpha_{EM}}{\sqrt{2} G_F}\right)^{1/2} \frac{1}{\sin \theta_W} \approx 80 \text{ GeV}$$

(1.23)

$$m_Z = \left(\frac{\pi \alpha_{EM}}{\sqrt{2} G_F}\right)^{1/2} \frac{1}{\cos \theta_W \sin \theta_W} \approx 90 \text{ GeV}$$

(1.24)

Vector bosons with these masses where found by the UA1 and UA2 collaborations [1, 2] at the SppS proton antiproton collider at CERN in 1983 through their leptonic decay signatures.

With the advent of the measurements performed at LEP, the $Z$ mass has replaced $\sin^2 \theta_W$ as the third parameter. The numerical values of the three parameters are summarized in Table 1.2.

<table>
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<tr>
<th>Parameter</th>
<th>Numerical Value</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
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<td>$\alpha_{EM}^{-1}$</td>
<td>137.035990 ± 0.000006</td>
<td>anomalous magnetic moment of the electron [33].</td>
</tr>
<tr>
<td>$G_F \times 10^{-5} \text{ GeV}^{-2}$</td>
<td>1.6639 ± 0.0002</td>
<td>muon lifetime [33].</td>
</tr>
<tr>
<td>$m_Z$ (GeV)</td>
<td>91.195 ± 0.009</td>
<td>Z lineshape [34].</td>
</tr>
</tbody>
</table>

Table 1.2: Numerical values of the electroweak parameters.

This simple requirement implies that a measurement of the number of distinct neutrino species is sufficient to determine the number of fermion generations.

---

9To calculate higher order corrections, the other parameters of the Standard Model are of course also required.
<table>
<thead>
<tr>
<th></th>
<th>$\Gamma_{Z-\ell\ell}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_\nu$</td>
<td>166.2 (167.8)</td>
</tr>
<tr>
<td>$\Gamma_e$</td>
<td>83.5 (84.1)</td>
</tr>
<tr>
<td>$\Gamma_u$</td>
<td>295.3 (300.0)</td>
</tr>
<tr>
<td>$\Gamma_d$</td>
<td>381.4 (386.9)</td>
</tr>
</tbody>
</table>

Table 1.3: The predicted partial widths of the $Z$, assuming $m_t = 100(200)$ GeV and $m_H = 250$ GeV.

widths of the $Z$ and the $W^\pm$ can be predicted. For instance, the $Z$ boson will decay into any fermion-antifermion pair, as long as their masses are less than $\frac{1}{2}m_Z$. The partial widths for these decays, neglecting the fermion masses and higher order corrections, can be written as:

$$\Gamma_{Z-\ell\ell} = \frac{G_F m_Z^3}{6\sqrt{2}\pi}((g_A^f)^2 + (g_Y^f)^2) = \begin{cases} 
\frac{G_F m_Z^3}{12\sqrt{2}\pi} & \text{if } f = \{\nu_e, \nu_\mu, \nu_\tau\} \\
\frac{G_F m_Z^3}{12\sqrt{2}\pi} \left(1 - 4\sin^2\theta_W + 8\sin^4\theta_W\right) & \text{if } f = \{e, \mu, \tau\} \\
\frac{G_F m_Z^3}{12\sqrt{2}\pi} \left(3 - 8\sin^2\theta_W + \frac{32}{3}\sin^4\theta_W\right) & \text{if } f = \{u, c\} \\
\frac{G_F m_Z^3}{12\sqrt{2}\pi} \left(3 - 4\sin^2\theta_W + \frac{8}{3}\sin^4\theta_W\right) & \text{if } f = \{d, s, b\} 
\end{cases}$$

(1.25)

The numerical values, including higher order calculations which require specific values for the mass of the top quark and the Higgs boson, are summarized in Table 1.3. Assuming that only the known three charged leptons, five quarks, and three massless neutrinos contribute, the total $Z$ width, $\Gamma_Z$, is predicted (to lowest order) to be:

$$\Gamma_Z = \sum_{f=e,\mu,\tau}(\Gamma_\ell + \Gamma_\nu) + \sum_{q=u,d,s,c,b} \Gamma_q = \frac{G_F m_Z^3}{\sqrt{2}\pi} \left(\frac{1}{4} - \frac{10}{3}\sin^2\theta_W + \frac{40}{9}\sin^4\theta_W\right)$$

(1.26)

If additional fermions with masses less than $\frac{1}{2}m_Z$ exist and interact with the $Z$, the total width will increase accordingly.
Chapter 2

Determination of the Number of Families

Although the fermions are grouped into families in the Standard Model as mentioned in the previous chapter, the Standard Model does not predict the number of such families. However, there are measurements which try to determine the number of distinct families. All these measurements rely on the fact that each family contains a distinct neutrino (Section 1.5) and try to determine the number of neutrino species, \( N_\nu \). Two measurements exploit astrophysical observations:

- The measurement of the \( \bar{\nu}_e \) flux produced by supernova SN1987A. Sanduleak-69 202, the blue giant star whose supernova collapse was observed in 1987, had a mass of \( 20 \pm 5 M_\odot \). The collapse released the binding energy of the remaining neutron star (with a mass of \( 1.4 - 2 M_\odot \)), of \( 2 - 4 \times 10^{46} \) joule. According to the models [35] of type II supernovae, \( \approx 98\% \) of this energy is released in the form of neutrinos, almost independently of the flavour. This implies that the measurement of the \( \bar{\nu}_e \) flux by the Kamiokande and IMB experiments (8 resp. 11 events observed) can be used to obtain a determination of the number of neutrinos [36, 37, 38]:

\[
N_\nu = 2.5^{+4.1}_{-0.8} \text{ or } N_\nu < 8 \text{ at 95\% CL.} \tag{2.1}
\]

- The measurement of the relative abundance of \(^4\text{He}\) in the universe.

The Big-Bang model of cosmology predicts the following relation between the temperature \( T \) and the age \( t \) of the universe [39]:

\[
t^2 = \frac{3}{128\pi \sigma_{\text{SB}} G_N \kappa T^4} \tag{2.2}
\]

where \( \sigma_{\text{SB}} = \frac{1}{60} \pi^2 \), \( G_N \approx 6 \times 10^{-39} M_p^{-2} \) and \( \kappa = \kappa(T) \) is the number of massless degrees of freedom at temperature \( T \), i.e. the number of particle types satisfying \( m \ll T \):

\[
\kappa(T) = 1 + \frac{7}{8} N_\nu + \frac{7}{4} N_\ell(T) + \cdots \tag{2.3}
\]

When the temperature \( T \) is low enough protons and neutrons will be formed. The initial ratio of protons to neutrons is given by the thermal equilibrium value \( e^{-\Delta M/T} \), where \( \Delta M = M_n - M_p \). At some point the temperature will drop below the value at which this equilibrium
can be maintained. This happens when $G_F^2 T^5 \approx \sqrt{\kappa G_N T^4}$. At this time the ratio of neutrons to protons is given by:

$$\frac{N_N}{N_P} \approx e^{-\frac{\Delta M}{\kappa G_N G_F^{1/2} c_p^{3/2}}}$$  (2.4)

Starting from this point, the number of neutrons decreases exponentially as they decay. The neutrons which survive until the point when nucleosynthesis starts combine with protons to form (mainly) $^4$He. Equations 2.4 and 2.2 determine the relative abundance of $^4$He as function of $\kappa$: each additional neutrino species increases this abundance by approximately 1.5 % From the measurement of the $^4$He abundance the following limit can be derived [40]:

$$N_\nu = 2.3 \pm 0.8 \text{ or } N_\nu < 3.6 \text{ at 95\% CL.}$$  (2.5)

The measurement of the properties of the Z boson offers another opportunity to determine the number of neutrino species. As shown in Equation 1.26, the decay width of the Z boson depends on the number of fermion species satisfying $m_f < \frac{1}{2} m_Z$. As a result the measurement of Z width can be used to determine this number. But already amongst the known three families there exists a fermion with a mass larger than $\frac{1}{2} m_Z$, the top quark. Instead, as in the astrophysical measurements, the total width is used to constrain the number of neutrino species, which are assumed to be massless. However, Equation 1.26 might be modified for several reasons. For instance, a family containing a neutrino with non-zero mass contributes $\frac{1}{2} \beta \left( 1 + \beta^2 / 3 \right) \Gamma_\nu$ [41]. Another example could be a contribution from the supersymmetric partner of the neutrino, the sneutrino. Each species of sneutrino adds $\frac{1}{2} \beta^3 \Gamma_\nu$ to Equation 1.26. Additionally, if left-handed neutrinos are allowed to mix with right-handed neutrinos then measurements of the Z decay widths can only be used to set an upper limit on the number of neutrino species [42].

Taking into account these assumptions, a determination of the the Z decay width into “invisible” particles, $\Gamma_{\text{inv}}$, assuming only left-handed, massless neutrinos contribute to this width, can be used to constrain the number of distinct neutrino species. The invisible width can be determined from a measurement of the total Z width in combination with either measurements of the visible decay widths or the predictions of Table 1.3, or it can be measured by tagging the invisible decay channels.

Before the LEP and SLC colliders became operational, the former measurement was performed at p$\bar{p}$ colliders, whereas the latter was done at the PEP, PETRA and TRISTAN $e^+e^-$ colliders.

- Limit on $\Gamma_Z$ from p$\bar{p}$ collider experiments [43].

From the measurement of the ratio of the production cross sections of p$\bar{p} \rightarrow ZX \rightarrow e^+ e^- X$ and p$\bar{p} \rightarrow W^\pm X \rightarrow e^\pm \nu X$, combined with the predicted values of $\Gamma_W, \sigma_Z/\sigma_{W^\pm}$ and $\Gamma(Z \rightarrow e^+ e^-)/\Gamma(W^\pm \rightarrow e^\pm \nu)$, the total Z width can be determined using:

$$\Gamma_Z = \Gamma_W \cdot \frac{\Gamma(Z \rightarrow e^+ e^-)}{\Gamma(W^\pm \rightarrow e^\pm \nu)} \cdot \frac{\sigma_Z}{\sigma_{W^\pm}} \cdot \frac{\sigma_{W^\pm} B(W^\pm \rightarrow e^\pm \nu)}{\sigma_Z B(Z \rightarrow e^+ e^-)}$$  (2.6)

From the comparison of the so derived total Z width with the Standard Model prediction, Equation 1.25, the following limit is derived:

$$N_\nu < 4.8 \text{ at 90\% CL.}$$  (2.7)
2.1 Fermion Pair Production in $e^+e^-$ Annihilation

- Limit on $\Gamma_{\text{inv}}$ from the measurement of $\sigma_{e^+e^--\nu\bar{\nu}\gamma}$.

The cross section of the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ is almost proportional to the invisible width of the $Z$. The measurement of the cross section for this process at center-of-mass energies close to the $Z$ resonance is presented in this thesis, and the dependence of the cross section on $N_\nu$ will be given in Section 2.2.2.

Previously, the cross section for this process has been measured at center-of-mass energies below $m_Z$ by the ASP, CELLO, MAC and VENUS collaborations at the PEP, PETRA and TRISTAN $e^+e^-$ colliders. As this process is mediated by the $Z$, the production rate at the PEP, PETRA and TRISTAN center-of-mass energies is small compared to LEP. Therefore the five collaborations combined only observed (after correcting for the expected background) 3.9 events, whereas 6.8 events were expected for $N_\nu = 3$. From this the following upper limit can be derived [44, 45, 46, 47, 48, 49]:

$$N_\nu < 4.8 \text{ at 95\% CL.} \quad (2.8)$$

Both the determination of the $Z$ decay parameters and the measurements of the $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ cross section can be substantially improved upon at an $e^+e^-$ collider with a center-of-mass energy close to the $Z$ mass.

2.1 Fermion Pair Production in $e^+e^-$ Annihilation

![Diagram of $e^+e^- \rightarrow \nu\bar{\nu}\gamma$]

**Figure 2.1:** Lowest order processes for $e^+e^- \rightarrow f\bar{f}$.

Using the couplings derived in the previous chapter, the (lowest order) cross section for fermion production in (unpolarized) $e^+e^-$ collisions can be calculated. The diagrams which contribute\(^1\) are shown in Figure 2.1. In terms of the partial widths of the $Z$, the cross section can be written as:

$$\sigma^{0}_{e^+e^-\rightarrow f\bar{f}}(s) = \frac{4\pi\alpha^2}{3s} Q^2_{e} Q^2_{f} \frac{N^f_{f}}{\sigma^0_{\nu}(s)}$$

(2.9)

$$+ \frac{12\pi \Gamma_e \Gamma_f}{m^2_Z \Gamma_Z^2} \frac{s \Gamma_Z^2}{(s - m^2_Z)^2 + m^2_Z \Gamma_Z^2} \frac{1}{\sigma^0_{\nu}(s)}$$

(2.10)

\(^1\)The exchange of a Higgs boson is neglected, as its contribution is small compared to the $Z$ and $\gamma$ exchange diagrams.
by making the following substitutions in the Born approximation:

\[
\begin{align*}
\alpha_{EM} & \to \alpha_{EM}(s) = \frac{\alpha_{EM}}{1 - \Delta_{EM}(s)} \\
\Gamma_Z & \to \Gamma_Z(s) = s\Gamma_Z/m_Z^2 \\
g_V^f & \to g_V^f(s) = \sqrt{\rho_f(s)} T_f^3 \\
g_A^f & \to g_A^f(s) = \sqrt{\rho_f(s)} \left( T_f^3 - 2\kappa_f(s) Q_f \sin^2 \theta_W \right)
\end{align*}
\] (2.14)

The running of \(\alpha_{EM}\) is caused by the self-energy insertions in the photon propagator, whereas the self-energy insertions in the Z propagator are absorbed by the redefinition of the Z width. The mixing of the photon and the Z through a virtual fermion loop is taken into account by \(\kappa\). In principle, \(\kappa\) and \(\rho\) depend on the external fermion lines, but, except for the b quark, these effects are small and can be neglected. As the dependence on \(s\) of \(\rho(s)\) and \(\kappa(s)\) is small for center-of-mass energies near the Z pole, \(\rho(s)\) and \(\kappa(s)\) can be replaced by \(\rho_{eff} = \rho(m_Z^2)\) and \(\kappa_{eff} = \kappa(m_Z^2)\).

### 2.2 Measurements of the Invisible Z Width at LEP

The measurement of the invisible width of the Z, \(\Gamma_{inv}\), can be used to determine the number of families. At LEP, this invisible width can be measured in two different ways:

1) Through the measurement of both the total Z width and the visible width.

2) By tagging invisible Z decays through the initial state photon in the reaction \(e^+e^- \to \nu\bar{\nu}\gamma\), i.e. the measurement of so called “single photon” events.

#### 2.2.1 Z Lineshape

By measuring the cross section of hadrons and leptons in \(e^+e^-\) annihilation as a function of the center-of-mass energy, close to the mass of the Z, the total width, \(\Gamma_Z\), can be determined. As the cross section of \(e^+e^- \to \) hadrons is the largest, this is the dominant measurement. A fourth generation of neutrinos would increase the total width by \(\approx 6\%\). However, using just the measurement of the total width to extract the number of neutrinos increases the dependence on the top and Higgs mass. This dependence can be reduced by using the measurements of the hadronic and the three leptonic cross sections to determine both the hadronic and leptonic width of the Z. This can then be used to determine the invisible width through:

\[
\Gamma_{inv} = \Gamma_Z - (\Gamma_{had} + (3 + \delta_{\tau})\Gamma_{\ell})
\] (2.15)
Figure 2.5: The cross section for $e^+e^- \rightarrow$ hadrons for the cases of two, three and four neutrino species.

where $\delta_\tau$ is a small correction to take into account the mass of the $\tau$. To reduce the correlations between the fitted parameters, a four parameter fit (assuming lepton universality) to the measured cross sections is performed to determine $m_Z, \Gamma_Z, \sigma_{\text{had}}^0$ and $R_{\text{had}}$, where

$$\sigma_{\text{had}}^0 = \frac{12\pi \Gamma_e \Gamma_{\text{had}}}{m_Z^2 \Gamma_Z^2}, \quad R_{\text{had}} = \frac{\Gamma_{\text{had}}}{\Gamma_\ell}$$  \hspace{1cm} (2.16)

The prediction of $\Gamma_\nu$ in the framework of the Standard Model has an uncertainty of 1% due to the uncertainty in the masses of the top and Higgs particles (see Table 1.3). To minimize the dependence on $m_t$ and $m_H$ the relation

$$N_\nu = \frac{\Gamma_{\text{inv}}}{\Gamma_\ell} \left( \frac{\Gamma_\ell}{\Gamma_\nu} \right)_{\text{SM}} = \left( \frac{12\pi R_{\text{had}}}{m_Z^2 \sigma_{\text{had}}^0} - R_{\text{had}} - (3 + \delta_\tau) \right) \left( \frac{\Gamma_\ell}{\Gamma_\nu} \right)_{\text{SM}}$$  \hspace{1cm} (2.17)

is used to extract the number of neutrino generations from the cross section measurements.
2.2.2 \( e^+e^- \rightarrow \nu\bar{\nu}\gamma \)

The previous lineshape method depends on the ability to measure the total Z width and to subtract the visible part of the width to obtain the invisible width. A second method to determine \( N_\nu \) is the measurement of the cross section of the reaction \( e^+e^- \rightarrow \gamma + \text{missing energy} \). This process is dominated by the radiative production of the Z and its subsequent decay into neutrino pairs with a small contribution from the \( t \) channel \( W^\pm \) exchange (Figure 2.6). In contrast to the lineshape method, the invisible width of the Z is (almost) proportional to the measured cross section. The differential cross section for this process can be written as:

\[
\frac{d^2}{dx\,dy}\sigma_{e^+e^-\rightarrow \nu\bar{\nu}\gamma}(x,y,s) = H(x,y,s)\sigma_{e^+e^-\rightarrow \nu\bar{\nu}}(s')
\]

(2.18)

where the radiator function \( H(x,y,s) \) describes the probability of radiating a photon, with a fraction \( x = 2E_\gamma/\sqrt{s} \) of the center-of-mass energy at an angle \( y = \cos \theta_\gamma \) with respect to the beam axis; \( \sigma_{e^+e^-\rightarrow \nu\bar{\nu}}(s') \) denotes the cross section for the process \( e^+e^- \rightarrow \nu\bar{\nu} \) at the reduced center-of-mass energy \( s' = (1-x)s \). Neglecting the diagram with the double \( W^\pm \) boson propagator and terms of \( \mathcal{O}(t/m_Z^2) \), \( \sigma_{e^+e^-\rightarrow \nu\bar{\nu}}(s) \) is given by [55, 56, 57, 58]:

\[
\sigma_{e^+e^-\rightarrow \nu\bar{\nu}}(s) = \frac{G_F^2s}{12\pi} \left( N_\nu \left( g_V^2 + g_A^2 \right) + 2(g_V^2 + g_A^2) \left( 1 - s/m_Z^2 \right)^2 + 2 \right)
\]

(2.19)

The first term, proportional to \( N_\nu \), is due to the Z exchange. The second one is due to the \( W^\pm-Z \) interference and the last term, which only contributes to the production of electron-neutrinos, is due to the \( W^\pm \) exchange. At center-of-mass energies close to the Z mass (i.e. \( \sqrt{s} \approx m_Z \pm 5 \text{ GeV} \)), the \( W^\pm \) contribution to the cross section is less than 3%. Its precise value depends on the center-of-mass energy and the lower limit imposed on the photon energy.

As can be seen from Equation 2.19, if \( \Gamma_Z \) is constrained to the value measured with the lineshape method, the cross section depends linearly on \( N_\nu \), and an additional generation of neutrinos increases the cross section by \( \approx 30\% \).
Figure 2.7: The cross section for $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ as function of the center-of-mass energy, for two, three and four light neutrino species.

The angular distribution and spectrum of the visible photon is determined by $H(x,y,s)$. The lowest, $O(\alpha)$, radiator function $H^1(x,y;s)$ is given by [59]:

$$H^1(x,y,s) = \frac{\alpha}{\pi} \frac{1 + (1 - x)^2}{x} \frac{1}{1 - y^2} \Theta \left(1 - \frac{4m^2}{s} - |y|\right)$$  \hspace{1cm} (2.20)

Because the radiator function $H(x,y,s)$ is a rapidly decreasing function of the photon energy, the measurement relies on the ability to trigger on events containing only a single photon with an energy that is small compared to the center-of-mass energy and the absolute scale of the energy measurement must be well known. The maximum sensitivity to $N_\nu$ is not reached at $\sqrt{s} = m_Z$, but at higher center-of-mass energies such that $\sqrt{s'} = m_Z$. As for practical purposes it is not possible to trigger on photon energies below one GeV, this implies that the maximum sensitivity is achieved at $\sqrt{s}$ equal to $m_Z$ plus one to two GeV, as shown in Figure 2.7.

The $O(\alpha^2)$ radiative corrections to this process have been calculated in [60, 61, 62]. They consist of three different contributions: soft Bremsstrahlung (combined with photonic vertex corrections), hard Bremsstrahlung where a photon is lost at small angles with respect to the beamline and hard Bremsstrahlung where the two photons are collinear and not resolved as such. The contribution of these corrections to the total cross section, and the comparison with the lowest order
Figure 2.8: The cross section $\sigma_{e^+e^+\to\nu\nu\gamma}$ calculated using the lowest order radiator function and the second order radiator function of [60]. The three separate contributions in the second order radiator function (Equation 26 of [60]) are shown, but the contribution due to collinear photons is too small to be visible in this plot.

cross section is shown in Figure 2.8. For the small angle hard photon correction, the smallest angle at which particles can be reconstructed, $\theta_{\text{veto}}$, is taken to be 25 mrad. In case of the collinear photons, it is assumed the minimum opening angle where they can be resolved, $\theta_{\text{res}}$, is 15 mrad. Figure 2.7 shows for the dependence of the cross section on the number of neutrino species in the case for $E_\gamma > 1$ GeV, $|\cos\theta_\gamma| < 0.72$, $\theta_{\text{veto}} = 25$ mrad and $\theta_{\text{res}} = 15$ mrad. Figure 2.9 shows the differential cross section as function of the photon energy for the same parameters, for several $\sqrt{s}$ values close to $m_\gamma$.

The full $O(\alpha^2)$ cross section, including exponentiation of the infrared leading logs has been implemented in the Monte Carlo event generator NNGSTR, described in [62]. Starting from the full tree level expression this generator implements the second order corrections to the $Z$ exchange diagrams (since the $W^\pm$-diagrams contribute approximately 3% to the cross section the higher order corrections to these diagrams have been neglected). In addition, the soft photon, leading logarithmic contributions are re-summed to all orders and the non-photonic corrections include the re-summed leading log terms. The theoretical uncertainty in the event-generator is estimated
Figure 2.9: The energy spectrum for photons produced in $e^+e^- \rightarrow \nu \bar{\nu}\gamma$ for several center-of-mass energies close to $m_Z$.

(conservatively) to be less than one percent.
Chapter 3

LEP and the $L_3$ Detector

3.1 The LEP Collider

Figure 3.1: The LEP collider.

The LEP storage ring at CERN has been designed to perform precision measurements of the carriers of the electroweak interaction. This is achieved in two phases. In the first phase, electrons and positrons are collided at center-of-mass energies close to the $Z$ resonance to study in detail the
properties of the Z boson. In a second phase the LEP collider will be upgraded, by the addition of superconducting RF cavities, to reach center-of-mass energies of up to 190 GeV, allowing the production of $W^+ W^-$-pairs. The ring, with a circumference of 26.7 km, is located in a tunnel underneath the Swiss-French border near Geneva at a depth which varies between 50 and 150 m. It consists of eight circle segments and eight straight sections, as shown in Figure 3.1. Bunches of electrons and positrons circulate, in opposite directions, in the same vacuum vessel. To keep the electrons and positrons in their orbit 3304 dipole magnets, which produce a field of 0.048 T each, are installed in the curved sections. The energy lost by synchrotron radiation (0.12 GeV per turn at a beam energy of 45 GeV) is replenished by RF cavities placed on two of the straight sections. These cavities are also used to accelerate the particles from their injection energy of 20 GeV up to the final beam energy. The cavities are capable of providing up to 16 MW of power. The electron and positron bunches collide at four interaction points (IP's), where four large detectors are installed: ALEPH, DELPHI, $L_3$ and OPAL.

For precision measurements large amounts of Z bosons have to be produced. Since the event rate is equal to the product of the cross section and the luminosity $L$, the LEP performance in this respect can be shown in Figure 3.2 by the delivered, time integrated, luminosity as a function of time. In terms of the machine parameters, the luminosity is determined by the number of bunches ($n_b$), the current per bunch ($I_b$), the single turn frequency ($f$), the vertical beam-beam strength parameter ($\xi_y$), the beam energy ($E_{\text{beam}}$) and the betatron amplitude ($\beta_y^*$) [63]:

$$L \propto n_b \cdot I_b \cdot f \cdot \xi_y \cdot E_{\text{beam}} / \beta_y^*$$  \hspace{1cm} (3.1)

The single turn frequency, which is fixed by the circumference of the LEP ring, is 11.24550 kHz. Typical values for the other parameters are $I_b = 0.32$ mA, $\xi_y = 0.020$, $\beta_y^* = 5$ cm and $n_b = 4$. The limiting factor for the luminosity produced by the LEP collider is the maximum current per bunch, which, in turn, is determined by the beam-beam interactions [63]. Therefore, to increase the luminosity, the number of bunches was doubled from 4 to 8 (using a so called Pretzel mode) in 1992.

The choice of the center-of-mass energy is a compromise between the need to collect a large number of events (i.e. running at or very close to the peak of the Z resonance, where the cross sections are largest, as shown in Figure 2.3) and the need to scan around the Z resonance. From Equation 2.12, it is clear that the determination of the branching ratios is best done at the peak. An energy scan is necessary for the measurement of the total width of the Z boson. The determination of the number of neutrinos from the measurement of the cross section of single photon events reaches its highest sensitivity, as shown in the previous chapter, at a center-of-mass energy slightly above the mass of the Z. As a compromise, LEP was operated at energies very close to the Z peak in 1992, and energy scans were performed in 1991 and 1993. As a result, by the end of 1993, in total 1.8 million hadronic Z decays were recorded by the $L_3$ experiment. During the same period 1400 candidate single photon events were collected.

Detailed information on the LEP machine and its performance can be found in [63, 64, 65].

### 3.2 The $L_3$ Detector

The $L_3$ detector is located at IP 2 of the LEP ring, approximately 50 m underground. A perspective view of the detector is shown in Figure 3.3. The design has been optimized towards the accurate
3.2. The $L_3$ Detector

Figure 3.2: The integrated luminosity delivered to $L_3$ as function of time.

energy measurement of muons, electrons, photons and jets. To achieve this, $L_3$ is divided into several sub-detectors. From the inside out, they are:

- a central tracking detector, the TEC, to measure the momentum and production vertex of charged particles.
- an electromagnetic calorimeter, the BGO, to determine the energy of electrons and photons.
- a luminosity monitor, the LUMI, to determine the luminosity through the counting of small-angle Bhabha events.
- an instrumented lead ring, the ALR, to protect the TEC from beam backgrounds.
- an array of scintillators, the SCIN, to reject cosmic rays.
- a hadron calorimeter, the HCAL, to measure, in conjunction with the BGO, jet energies.
- a muon spectrometer, the MUCH, to measure the momenta of muons.

Except for the muon system, all subdetectors are mounted inside a support tube, which is 32 m long and has a diameter of 4.45 m. The muon system is mounted on the outside of this support tube. A large magnet providing a 0.5 T field along the beam axis surrounds all detectors. The
Figure 3.3: A perspective view of the L3 detector, showing the magnet and the various subdetectors.

The coordinate system used in L3 has its origin in the center of the detector, with the positive z-axis pointing along the magnetic field.

A candidate $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ event as seen by the L3 detector is shown in Figure 3.4. The signature of such an event is the presence of a single photon (which has a typical energy in the range 1–5 GeV), and no other activity.

In the following sections, the subdetectors and their role in the analysis presented in this thesis will be discussed.

### 3.2.1 Electromagnetic Calorimeter

The energy of electrons and photons is measured through total energy absorption in the BGO calorimeter. It consists of an array of 10734 Bismuth Germanate, $\text{Bi}_4\text{Ge}_3\text{O}_{12}$ or BGO, crystals which are used both as the showering and as the detection medium. This technique minimizes the error in the measurement due to the intrinsic shower fluctuations. The small radiation length of BGO ($X_0 = 12.2\text{mm}$) makes it possible to construct a compact detector. The individual crystals
3.2. The $L_3$ Detector

![Diagram](image)

**Figure 3.4:** An $e^+ e^- \rightarrow \nu \nu' \gamma$ candidate as detected by the $L_3$ detector in the $(r,z)$ plane, shown on the left and in the $(r,\phi)$ plane on the right. The enlarged view shows the photon as an energy deposit in the BGO calorimeter. Each cuboid represents an individual crystal; the height of the cuboids is proportional to the measured energy. More information on the $L_3$ event visualization program can be found in [66].

are trapezoidal shaped, 240 mm long, and measure 20x20 mm$^2$ at the front, and 30x30 mm$^2$ at the back. To minimize the mechanical stresses on the crystals, each crystal is individually mounted in a carbon-fiber support structure. In front of each crystal there is 1 cm of carbon fiber, and the walls between the crystals are 0.25 mm thick. The total amount of material, expressed as a fraction of a radiation length, between the crystals in the central part of the detector and the IP varies as a function of the polar angle between 20% and 30%.

The support structure itself is divided in four parts, 2 cylindrical half-barrels with an inner radius of 52 cm and a length of 117 cm, each holding 24 rings of 160 crystals, and 2 endcaps located 64 cm (along the beamaxis) from the interaction point. A schematic view of the crystal layout is shown in Figure 3.5. The barrel part of the detector covers the polar angle range from 42° to 138°, whereas the two endcaps cover the angular regions from 10° to 35° and from 145° to 170°. From a comparison of the BGO data (using Bhabha events) with data from the tracking chamber, the hadron calorimeter and survey measurements, the uncertainty in the angular coverage of the BGO barrel is found to be less than 0.05° [67].

The crystals are positioned in such a way that in $\theta$ they are pointing towards the IP whereas in $\phi$ they are tilted by 0.6°. This reduces the possibility of particles escaping undetected through the inactive material between the crystals. From the analysis of Bhabha events [68] the probability to escape detection this way is determined to be less than 0.35%.
Figure 3.5: The crystal layout of the BGO calorimeter, in the \((r, \phi)\) plane and in the \((r, z)\) plane.

Figure 3.6: The timing of the readout chain of the BGO calorimeter.

Readout

Because the calorimeter is positioned inside the magnetic field and little space is available, the scintillation light of each crystal is collected by two photo-diodes of 150 mm\(^2\) each, mounted on the back of each individual crystal. The resulting signal has a rise time of a few nanoseconds, followed by a (temperature dependent) decay of \(\approx 350\) ns. After preamplification this signal is
3.2. The $L_3$ Detector

fed into a shaping circuit. After shaping the signal has a time constant of 1.2 $\mu$s and it is integrated during a 5.0 $\mu$s gate (Figure 3.6). After subtraction of the pedestal value, the pulse height of the resulting signal is $\approx 50 \mu$V times the deposited energy in MeV; it is digitized by ADC cards mounted close (3 m) to the detector. To achieve a good resolution for both large, $\mathcal{O}(10 \text{ GeV})$ and small, $\mathcal{O}(100 \text{ MeV})$, energies, the smaller signals are amplified $32 \times$ before they are integrated. After integration the voltage is compared to a preset threshold and multiplied by either 4 or 16, before it is finally digitized by a 12 bit ADC.

Full digitization of the signal takes $\approx 220 \mu$s. This is too slow for triggering purposes, so the output of the preamplifier (after shaping) is made available to the energy trigger logic. To guarantee a uniform response from crystal to crystal, the relative gains of these trigger signals have been adjusted for each individual crystal during a test beam run before installation in LEP.

Energy Resolution and Absolute Energy Scale

Before an accurate energy measurement can be performed, several factors have to be taken into account. First of all, the BGO light yield depends on the temperature: it has a gradient of $-1.55\% \text{K}^{-1}$. To correct for variations in temperature, sensors are mounted on every twelfth crystal and a measurement of the temperature is performed every 1000 events. Secondly, the light yield depends on the distance between the photodiodes and the location of the energy deposit. This variation was determined from measurements of the deposited energy of cosmic rays as function of the location along the major axis of the crystal. These measurements are used to correct the measured light yield. Two additional reasons why the light yield might change are aging of the crystals and radiation damage. These effects are monitored on a daily basis by determining the response of the crystals to the light produced by Xenon flash lamps. More information on these various calibrations can be found in [69].

Once all these effects are taken into account, a first approximation for the energy of an electromagnetic shower is the sum of the observed energies in all the crystals of a reconstructed cluster. However, there is some energy loss due to both the finite length of the crystals and to the cracks between the crystals. The latter depends on the location of the impact point on the crystal: it is larger when the impact point is closer to the edge of a crystal. The fraction of the energy in the most energetic crystal over the energy in a $3 \times 3$ matrix of crystals centered around this crystal ($\Sigma_1 / \Sigma_9$) can be used as an estimator of the distance to the edge (Figure 3.7). This quantity is therefore a candidate for a parametrization of this effect. The ratio of the generated energy and the reconstructed energy before and after this correction is shown in Figure 3.8 for simulated photons. After all corrections, the energy resolution can be parameterized by [70]:

$$\frac{\sigma(E)}{E} = \sqrt{\left[ \left( \frac{2.37}{E} + 0.38 \right)^2 + 1.18^2 + \left( \frac{0.25}{E} \right)^2 \right]^2} \% \text{ with } E \text{ in GeV} \tag{3.2}$$

The first two terms describe the resolution due to the intrinsic shower fluctuations, and the inhomogeneities in both the material and the light collection. The third term is due to the fluctuations in the energy leakage and the uncertainty in the calibration constants, whereas the last term includes the contributions due to correlated noise. This curve is shown in Figure 3.9.

The absolute energy scale of the BGO is determined from the measurement of beam-energy electrons in Bhabha scattering. The resolution and absolute energy scale at lower energies are
checked by reconstructing the invariant masses of photon pairs in hadronic Z decays. In this way one observes the mass peaks of the $\pi^0$ and $\eta$. Another cross check utilizes the decay $\Sigma^0 \rightarrow \Lambda^0 \gamma$ by reconstructing the invariant mass difference $\Delta M = M_{\Lambda^0\gamma} - M_{\Lambda^0}$. This process generates photons (in the $L_3$ frame) with an average energy of 120 MeV. The position and width of the peaks in these distributions confirm that the absolute energy scale is understood at the 0.8% level [71, 72].

Identification of Electromagnetic Showers

The small Moliere radius (2.20 cm) of BGO implies that electromagnetic showers are almost completely contained in a $3 \times 3$ matrix of crystals around the impact point. As this is not the case for hadronic showers nor for showers initiated by cosmic rays, the transverse shape of the shower can be used to distinguish electromagnetic from non-electromagnetic showers. To estimate this, the ratio $\rho_{EM}$ of the energy (corrected for losses) in a $3 \times 3$ matrix and the energy in a $5 \times 5$ matrix around the impact point is used:

$$\rho_{EM} = \frac{\sum_{3 \times 3} E_i}{\sum_{5 \times 5} E_i}$$
The event shown in Figure 3.4 has $\rho_{\text{EM}} = 0.99$. As the lateral spread of showers depends on the energy, only the six most energetic crystals in the $3 \times 3$ matrix are used for clusters with less than 3 GeV.

Another estimator uses the results obtained in a test-beam setup, where the response to electrons of 2, 5 and 50 GeV of each half-barrel was determined. The comparison of the shower-shape of the showers as measured during this test-beam and the data taken at LEP is one of the variables used in the analysis:

$$\chi^2_{\text{EM}} = \sum_{3 \times 3} \left( \frac{F_i - F_i^{\text{test beam}}}{\sigma_i^{\text{test beam}}} \right)^2 \tag{3.3}$$

where $F_i$ is the fraction of the energy in the $i^{th}$ crystal. The event shown in Figure 3.4 has $\chi^2_{\text{EM}} = 1.22$.

Another quantity used to determine whether a given cluster is electromagnetic or not is the so-called skewness, $sk$. The skewness is defined as the ratio of the smallest and largest eigenvalues of the following tensor:

$$S^{\mu\nu} = \sum_i F_i x_i^\mu x_i^\nu \tag{3.4}$$
where $x_i^p$ is the position of the $i^{th}$ crystal in the cluster, and the summation is performed over all crystals in the cluster.

The skewness can be interpreted as the ratio of the major and the minor axis of an ellipse describing the transverse profile of the shower. Values close to one correspond to "round" showers, i.e. those approximately pointing towards the IP, whereas values close to zero are caused by a long, narrow row of crystals. The latter is the signature of a (cosmic) muon traveling through the calorimeter, where the path of the muon is not perpendicular to the front face of the crystals. The event in Figure 3.10 is an example of such a shower. The cluster in this event has $sk = 0.014$ and $\chi^2_{EM} = 346$, and there are matching MUCH Z-chamber hits, making this a candidate for a cosmic ray. In contrast, the event shown in Figure 3.4 has $sk = 0.68$, indicating that, based on this variable, it is probably either a photon or an electron.

Further details of the BGO detector, its data-acquisition system and calibration are described in [69, 73].
3.2.2 Luminosity Monitor

To determine the luminosity delivered by the LEP collider to $L_3$ two forward calorimeters of 304 BGO crystals each are installed at 280 cm from the interaction point. The crystal layout of the calorimeters is shown in Figure 3.11. To avoid radiation damage to the crystals during LEP injection and acceleration, each of the calorimeters is split along the vertical plane such that the calorimeter can be opened.

In case of the single photon analysis, these two detectors are of course needed for the measurement of the integrated luminosity, but they are also crucial in the rejection of the large radiative Bhabha background: they define the smallest angle at which particles can be detected. The inner radius of the detector is at 6.8 cm, so particles which scatter more than 24 mrad can be rejected. Apart from their use in background rejection, they are used to select a sample of low $q^2$ radiative Bhabha events with an electron (or positron) detected at large polar angles, which is used to cross check the single photon analysis.

To improve the definition of the fiducial volume (the limiting factor on the luminosity mea-
3.2.3 Active Lead Ring

During the 1991 running period a lead shielding ring was installed at 104 cm from the interaction point to protect the TEC from beam backgrounds. However, this caused a gap in the detection in the regions $4.6^\circ < \theta < 9.3^\circ$ and $170.7^\circ < \theta < 175.4^\circ$. This increased the background due to Bhabha scattering in the single photon sample. To close this detection gap this lead ring was replaced by a sandwich of lead and scintillator detectors (ALR) during the 1991-1992 LEP shutdown. However, to make certain that the high precision measurement of the luminosity was not compromised, a gap between the coverage of the LUMI and the ALR was left. This gap ensures that there is no possibility that the ALR casts a "shadow" on the LUMI monitors or deflects electrons into the LUMI.
3.2.4 Central Tracking Detector

To distinguish between electrons and photons, to measure both the charge and momentum of charged particles, to determine the primary interaction point, to reconstruct possible secondary vertices and to determine the impact point of charged particles on the BGO calorimeter, a proportional wire chamber (TEC), operated in time expansion mode is installed inside the BGO calorimeter.

As charged particles cross the volume of the TEC, they ionize the gas along their path. The resulting ionization clusters drift under the influence of the electric field towards a grid of sense wires. The drift speed is low ($\approx 5.92 \mu m/\mu s$), which makes it possible to determine the “center of gravity” of the time of arrival of each cluster, yielding a better resolution than a conventional drift chamber which measures the time-of-arrival of the first few electrons. To achieve a sufficiently large signal on the sense wires, a grid of field shaping wires is located close to the anode wires. In the region between this grid and the anode wires the electrical field is large enough to achieve the necessary gas amplification. The inner of the two coaxial cylindrical drift volumes is divided into 12 sectors with 8 sense wires each, and the outer volume into 24 sectors with 54 sense-wires each. The different number of sectors in the two volumes enables the pattern recognition software to resolve the left-right ambiguities by matching track segments between the inner and outer sectors. In this task it is assisted by the so-called LR wires, which consist of groups of 5 individual sense wires capable of giving unambiguous hits.

The BGO cluster shown in Figure 3.4 is identified as a photon since no matching track is reconstructed in the TEC. The cluster shown in Figure 3.12 is associated with a track, which contains hits
Figure 3.13: The difference of the reconstructed \(\phi\) angles, \(\phi_{\text{TEC}} - \phi_{\text{BGO}}\) in mrad for electrons with transverse momenta of \(1-3\) GeV.

on 59 wires, out of the 62 possible, and the hits span all 62 wires. The distance of closest approach of the track in the \((r, \phi)\) plane to the fill vertex\(^1\) is 0.2 mm, indicating that the particle which caused the track probably originated from the luminous beam spot. The azimuthal difference between the impact point as reconstructed in the BGO and the extrapolated TEC track is 5.24 mrad. Figure 3.13 shows, for isolated electrons with transverse momenta between one and three GeV, the distribution of this variable. Furthermore, the measured transverse momentum of the track matches the transverse energy of the cluster: \(p_\perp = 2.61\) GeV whereas \(E_\perp = 2.57\) GeV or \(E_\perp/p_\perp = 0.99\). The shower in the BGO calorimeter is classified as electromagnetic (\(s_k = 0.98, \chi_{\text{EM}} = 0.30, \rho = 0.99\)), so in combination with the matching track, it is identified as an electron.

As the TEC reconstruction code is optimized to detect in-time tracks originating approximately from the center of the chamber, a separate reconstruction, specific to the single photon analysis, is performed to look for tracks originating from cosmic rays. An example of such a cosmic event can be seen in Figure 3.14. Because the cosmic ray did not cross the detector at the time of the bunchcrossing, the measured drift time (which assumes ionization to occur at the time of the bunchcrossing) is not the actual drift time. Therefore the drift distance is reconstructed incor-

\(^1\)The fill vertex is the reconstructed average position of the luminous LEP beamspot during a fill.
3.2. The $L_3$ Detector

Figure 3.14: Example of a cosmic ray as detected by the $L_3$ detector.

rectly. This causes the track segments to be systematically displaced, which in turn implies that the segments no longer match within the expected error. As a result, no track is reconstructed.

To solve this, a more robust (and more simple, as it does not try to determine the momentum of the tracks) pattern recognition was introduced especially for the single photon analysis, which aims at identifying patterns of hits which lie approximately in a straight line. This is done by performing a straight line fit to the hits found in the TEC, and requiring that no more than 50 TEC hits are found which satisfy the following requirements:

- The distance between the first and last hit is at least 90% of the maximum possible distance.

- If the distance between hit $i$ and the fitted line is denoted by $y_i$, then $\sqrt{\sum y_i^2}$ is less than 5 mm.

- The points must be distributed uniformly along the fitted line, i.e. if the distance along the fitted line between the first and last hit is given by $l$, then $\sqrt{\frac{1}{12} \sum (x_i/l)^2} > 0.9$, where $x_i$ is the position along the line of hit $i$.

In the analysis described in this thesis, this fit is used to reject background events, such as Bhabhas, taus, di-muons and cosmic rays. More information on the TEC can be found in [76, 77].

3.2.5 Scintillator Counters

Between the BGO and the hadron calorimeter there is an array of 30 scintillator counters. They cover the angular range from $34^\circ$ to $146^\circ$ and, since 2 are missing because of space restrictions, 93% of $2\pi$.

The aim of these scintillators is to provide timing information on the passage of muons. If a di-muon pair is created at the center of the detector, the time-difference between opposite scintillators
hits is zero. In the case of a cosmic muon, it is 6 ns. To measure this time difference the signals from the counters are fed into 12 bit TDCs with 50 ps resolution. However, the gate during which the signals from BGO are integrated is longer than the dynamic range of these TDCs. To cover the integration time of the BGO, the signals from the counters, averaged over both sides of each counter, are fed into coarse TDCs, with 10 ns resolution. This increases the dynamic range to 8 μs, sufficient to cover the entire BGO integration gate. The information from these TDCs is used to determine the contamination in the single photon sample of out-of-time cosmic rays.

3.2.6 Hadron Calorimeter

The hadron calorimeter is a sampling calorimeter, consisting of uranium absorbers interspersed with proportional chambers, which act as the sampling medium. It consists of a barrel calorimeter covering the angular region 35° < θ < 145°, and two endcaps covering 6° < θ < 35° and 145° < θ < 174°. The barrel consists of nine rings, each containing sixteen modules which in turn each contain 60 layers of wire chambers. To reduce the complexity of the readout system, the wires are grouped into towers, which results in a segmentation of Δφ = Δθ = 2°. This fine granularity makes it possible to recognize minimum ionizing particles (MIPs).

A cluster with a MIP signature is defined as follows:

- It is reconstructed from more than 4 hits.
- The average energy per hit is less than 400 MeV.
- If the distance of hit i perpendicular to the fitted line is denoted by $y_i$, then $\sqrt{\sum_i y_i^2}$ is less than 40 mm.
- The distance along the fit between the first and the last hit is larger than 750 mm for clusters in the barrel and larger 400 mm for clusters in the endcap.

An example of a cluster which satisfying these criteria is shown in Figure 3.15. Although this signature can be used to reject muons, due to the timing constraints imposed on the readout system, it can only be used in case the muon passes through the HCAL within a gate of 400 ns with respect to the beam crossing.

3.2.7 Muon Detector

The muon detector consists of three layers of drift chambers, labeled from the inside out as MI, MM and MO. In the bending plane, track segments are measured in each layer, and are then combined into a muon track. To measure the track in the non-bending direction, so-called Z-chambers are mounted on both the inside and the outside of each MI and MO chamber, each of which again yields a track segment. The reconstruction of muon tracks is described in detail in [78, 79].

The readout is done using multihit TDCs which are operated in COMMON STOP mode, i.e. the TDCs are started when a charge is deposited on the wires and stopped by a signal which is generated such that it arrives 1.5 μs after the bunch crossing for the P-chambers and 2.0 μs for the Z-chambers. Depending on the drift distance, this limits the possibility of rejecting out-of-time cosmic rays.
3.3 The Trigger System of the $L_3$ Detector

At a luminosity of $10^{31} \text{ cm}^{-2} \text{s}^{-1}$, and at the peak of the $Z$ resonance, the visible $Z$ production rate is $\approx 0.35 \text{ Hz}$, and the rate of small-angle Bhabha events (in the angular region covered by the $L_3$ luminosity monitor) is $\approx 1 \text{ Hz}$. Thus only a small fraction of the beam crossings results in an $e^+e^-$ interaction.

The readout sequence of the $L_3$ detector is started at each bunch crossing and, unless it is aborted, takes 500 $\mu$s or 45 bunch crossings to complete. While the readout sequence is active, it will not accept any new input. To minimise this "dead" time a trigger system is designed such that beam crossings without any $e^+e^-$ interactions can be recognized and, in case of a negative trigger decision, the readout can be reset in time for the next bunch crossing. This implies that, during eight bunch operation of the LEP collider, the trigger must reach a decision within 9 $\mu$s. To achieve this, the trigger system consists of three (in the case of four-bunch operation) or four (in the case of eight-bunch operation) levels, allowing each subsequent level more time to process the data. Under normal running conditions the dead time is determined by the acceptance rate of the first level trigger, which is approximately 10 Hz. On average, the dead time is less than 8%.

3.3.1 Level One Trigger

The logical layout of the level one trigger is given in Figure 3.16. The trigger decision is based on the logical OR of five (almost) independent sub-triggers, each corresponding to a major sub-detector. These sub-triggers are organized in such a way that most $Z$ decays are recognized by more than one of them, minimizing inefficiencies:

- **ENERGY trigger**: it is based on both the BGO and the HCAL. This trigger is described in detail in Section 3.3.1.
• **TEC trigger:** it is based the signals from the $24 \times 14$ LR wires of TEC. It recognizes tracks which have a transverse momentum larger than 150 MeV and triggers on events which contain tracks which have an acoplanarity (i.e. deviation from $180^\circ$ of the angle between the tracks in their projection onto the $(r, \phi)$ plane) of less than $60^\circ$.

• **MUON trigger:** it is based on the muon system and it requires, in combination with at least one scintillator hit, either a single track with at least 1.0 GeV transverse momentum, or two tracks on opposite sides of the detector.

• **SCIN trigger:** this trigger requires at least 5 scintillators hits within a 30 ns gate around the beam crossing time. At least two of the hits have to be separated by more than 90° in $\phi$.

• **BEAMGATE trigger:** it is based solely on the beamcrossing coincidence. To obtain a sample of unbiased events, one trigger is generated every 10 seconds.

The single particle which can be observed in an $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ event is a photon detected by the BGO calorimeter. As readout of events of this type is only initiated by the energy trigger, this trigger is described in more detail.

### ENERGY Trigger

As described in Section 3.2.1, the output of the preamplifier of each BGO crystal is available to the trigger. To reduce the number of channels which have to be read out, the signals from the barrel BGO calorimeter are added together in groups of 30 crystals each, resulting in 32 azimuthal ($\phi$) segments and 8 polar ($\theta$) segments: in total 256 channels. For the endcap BGO a similar procedure results in another 256 channels. In the case of the luminosity monitor, the signals in each of the 16 azimuthal sectors are added together, resulting in 32 channels. The hadron calorimeter is split into “front” and “back” parts, corresponding to the first interaction length and beyond, and these parts are segmented in 16 channels in $\phi$. The “front” part has 11 and the “back” part 13 channels in $\theta$, resulting in 384 HCAL trigger channels. This segmentation of the detector is shown in Figure 3.17.

### FERA System

In total 928 analog signals form the input to the energy trigger. Each of these signals is digitized by a system of Fast Encoding and Readout ADC (or FERA) modules [80]. The correspondence between the energy measured by the FERA system and the “full” data-acquisition system is shown, for four randomly chosen channels from the barrel-BGO detector, in Figure 3.18. The FERA system digitizes the charge collected during a 0.5 $\mu$s gate, in contrast with the “complete” readout, which uses a 5.0 $\mu$s gate. As a result the energy measured by the ENERGY trigger only coincides with the “complete” readout for showers in coincidence with a LEP bunch crossing. The ratio of the two measurements as a function of the time difference with the bunch crossing is shown in Figure 3.19.

If the resolution of FERA channel $j$ is denoted by $\sigma_j$ and its gain by $G_j$, then the probability $P_j(N|E,\sigma_j,G_j)$ to find $N$ counts in channel $j$, given that an amount of energy $E$ has been deposited, can be calculated as follows:

$$P(N|E,\sigma_j,G_j) = \frac{1}{\sqrt{2\pi}\sigma_j} \int_N^{N+1} \exp \left[ -\frac{1}{2} \left( \frac{x - E/G_j}{\sigma_j} \right)^2 \right] dx$$  \hspace{1cm} (3.5)
3.3. The Trigger System of the $L_3$ Detector

Figure 3.16: The logical layout of the trigger and data acquisition system.

The values of $\sigma_j$ and $G_j$ are determined from the data by comparing the number of ADC counts and the measured energy as shown in Figure 3.18 using unbinned likelihood fits. For each channel $j$ the following likelihood function is maximized as function of $\sigma_j$ and $G_j$:

$$L_j = \prod_{i=1}^{N} P(N_i|E_i, G_j, \sigma_j)$$  \hspace{1cm} (3.6)

where $P$ is given by Equation 3.5. The results from fits to each of the 256 FERA channels corresponding to the barrel part of the BGO are shown in Figure 3.20. They pertain to events in which the deposited energy in the channel under consideration was required to be between 0.8 GeV and 5 GeV, the typical energy of a single photon event.

Once the digitization is performed each channel is compared to a pre-set threshold and to suppress coherent noise, channels below this threshold are discarded in the trigger calculation. This threshold, which can be set for each channel individually, has three different settings: low, high and "killed". If the threshold is set to "killed" the channel is discarded regardless of the measured
energy. Assuming a nominal gain of 10 counts per GeV, the “low” setting for both 1992 and 1993 was 0.8 GeV, whereas the “high” setting was 2.0 GeV in 1992 and 3.0 GeV in 1993. The status of all barrel-BGO channels for a typical event (during the 1992 data-taking period) is shown in Figure 3.21.

On the surviving channels several algorithms are applied. During eight bunch operation the “full” level one energy trigger is not capable of reaching a decision before the next bunch crossing. To minimize the subsequent dead time, a simple, fast, level zero algorithm is applied. This algorithm performs a simple hit count, and requires at least one channel with more than 9 ADC counts for a positive decision. If the result of the level zero algorithm is negative, the energy trigger aborts further calculation and sends an abort signal to the central level one processor in time for the next bunch crossing.

The level one energy trigger starts to process the data in parallel to the level zero logic. It applies five different algorithms, and a decision is reached after 19 μs, i.e. in the case of a level zero accept, the detector is “dead” for at least one bunch-crossing. Typically, level zero runs at a rate of less than 1 kHz, so that the dead time due to this is less than 1%.

The following algorithms are used:

Single Photon:
This algorithm is designed to trigger on events containing a single, isolated electromagnetic energy deposit, with a threshold as low as feasible (i.e. without introducing a large trigger rate which would result in a large dead time).

If the number of ADC counts of channel \((\theta_i, \phi_j)\) is denoted by \(N_{\theta_i\phi_j}\) then the following 41 quantities can be constructed from the FERA data of the BGO barrel:

\[
\Sigma_{\phi}^{i} = \sum_{j=1}^{32} N_{\theta_i\phi_j}^{\text{barrel}}; \quad \Sigma_{\theta}^{j} = \sum_{i=1}^{8} N_{\theta_i\phi_j}^{\text{barrel}}; \quad \Sigma = \sum_{i=1}^{8} \sum_{j=1}^{32} N_{\theta_i\phi_j}^{\text{barrel}}
\] (3.7)
Figure 3.18: Comparision between the FERA response and the reconstructed energies for 4 random channels in the BGO barrel.

For a positive decision the following requirements must be met:

\[ \max(\Sigma^i) > 0.8 \Sigma; \quad \max(\Sigma^i) > 0.8 \Sigma; \]

During normal data-taking conditions the rate of this trigger is approximately 1 Hz, dominated by electronic noise.

Single Electron:

To measure the efficiency of the single-photon trigger a dedicated trigger algorithm was introduced in 1991. This algorithm uses information from both the TEC trigger and the ENERGY trigger.
**Figure 3.19:** The ratio of the energy measured by the “full” data acquisition system and the FERA system. As the time at which the shower starts shifts with respect to the beam crossing time, the FERA system only integrates part of the signal, whereas the “full” readout system still integrates most of the charge.

It requires that the TEC-trigger track-finder hardware has reconstructed at least one track, in coincidence with a single-tag LUMI trigger.

**LUMI:**

The luminosity algorithm consists of the logical OR of the following three conditions:

- **Back-to-Back:**
  
  Energy deposits of at least 15 GeV are required on each side. The clusters have to be approximately back-to-back, i.e. the azimuthal difference has to be less than 2 sectors.

- **Double Tag:**
  
  More than 25 GeV on one side and 5 GeV on the other side is required.

- **Single Tag:**
  
  At least 30 GeV on one side of the detector is required.

To reduce the level one acceptance rate and to avoid the subsequent dead time, the back-to-back and double-tag triggers were prescaled by a factor of two in 1990, 1991 and 1992, i.e. in the case of a positive trigger decision only every second event is accepted. At the same time the single tag trigger was prescaled by a factor of 20. In 1993, with the introduction of the SLUM, it was decided to remove the prescaling of the back-to-back and double-tag triggers, so that the improved measurement of the luminosity made pos-
Figure 3.20: The gain and resolution of the 256 FERA channels of the BGO barrel. The killed channels are entered at zero gain and resolution.

sible by the installation of the SLUM) would not be limited by the number of triggered events.

Total Energy:
This algorithm uses as input the measured energy in both the central and forward BGO calorimeters and the HCAL. It is the main trigger for most visible Z decays. For a positive decision, at least one of the following three criteria has to be met:

1) The total calorimetric energy, i.e. both BGO and HCAL, is larger than 25 GeV.
2) The total energy measured in the BGO calorimeter is larger than 15 GeV.
3) The measured energy in the central part of the BGO calorimeter is in excess of 8 GeV.

Cluster: This algorithm performs a search for clusters. For a positive decision, is at least one cluster above a threshold of 6 GeV. When the cluster matches (in $\phi$) with a track reconstructed by the TEC trigger, this threshold is lowered to 2.5 GeV.
Figure 3.21: The status of the FERA channels for the BGO barrel during run 459001. The dots indicate the channels at their nominal settings, whereas channels flagged as “H” have a high bias setting and channels flagged “K” are killed.

More details on the ENERGY trigger can be found in [81, 82, 83, 84, 85].

3.3.2 Level Two Trigger

The level two trigger works in parallel to the level one trigger, and it has access to the same information. However, as it has more time available, it can use more complex algorithms.

Its aim is to reject the most obvious background events, such as cosmic events, beam-gas or beam-wall interactions and detector noise [86]. The algorithms used to reject these types of events depend on the level one trigger pattern, and only events triggered by a single level one subtrigger are subjected to possible level two rejection. An exception to this rule are the events which are triggered by the either a luminosity trigger or the single photon trigger: these events will always be accepted by the level two trigger. As the second level trigger keeps one out of every ten events which would otherwise be rejected, it is possible to cross check its efficiency.

A secondary aim of the level two trigger is to monitor the proper synchronization of all subdetectors. The event rate after the level two trigger is typically 6 Hz.

3.3.3 Level Three Trigger

The level three trigger has access to the results of the “full” data-acquisition system. This enables a complete reconstruction of the event. In the case of a single photon level one decision, it requires at least one energy deposit in the barrel BGO calorimeter which has an energy larger than 500 MeV and which contains at least 3 and at most 80 crystals, where the most energetic crystal has at least 50 MeV, and only crystals with more than 10 MeV are taken into account. This requirement reduces the rate (of the single photon trigger) to 0.01 Hz. More details on the implementation and the operation of the Level Three Trigger can be found in [87].
3.3.4 Data Acquisition

The final rate at which events are written to tape varies typically between 2 and 3 Hz. As the average event size is 50 Kb, and the tapes have a capacity of 200 Mb, each tape contains the data taken during a 30 to 40 minute period. This “quantum” of data, of the order of 4500 triggers, is called a “run”. The detector status (with the exception of the TEC high voltage, which is monitored every few minutes) is assumed constant during each run. If at some point the detector status changes, a new run is started to guarantee this assumption. Only good runs, for which the single-photon trigger is active, the TEC high voltage is up for every single TEC sector during the entire run, and for which there are no known problems for any subdetector, are analyzed.

3.4 Detector Simulation

To compare the detector response to the expectations, events are generated with Monte Carlo generators for specific physics processes; subsequently they are passed through the $L_3$ detector simulation program. This program, based on GEANT [88], is responsible for the tracking of particles through a geometrical description of the detector, taking into account the interactions of the particles with detector material. It calculates the response of the various sensitive elements of the detector, and uses this to generate a data structure for each event which mimics, as closely as possible, the data that is collected from the “real” detector. In this way these Monte Carlo events can be passed through the same chain of event reconstruction as the “real” data. During the reconstruction phase, the detector status, such as dead BGO crystals, is taken into account. To do this, every Monte Carlo event is assigned to a run, according to the fraction of luminosity of a given experiment run, and the known detector defects during this run are applied. In a final step, the reconstructed Monte Carlo event is combined with a “real” data event, triggered by the BEAMGATE trigger, from the same run. In this way a realistic level of noise is introduced into the simulated events.

These events offer the possibility to calculate the acceptance and efficiency for a given set of selection cuts and for a given physics process, as their origin is well known.
Chapter 4

Data Analysis

The cross section of the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ where the photon has an energy larger than some minimum energy $E_{\gamma}^{\text{min}}$ shows a strong dependence on $E_{\gamma}^{\text{min}}$ (as shown in Figure 4.2). Unless there is a reliable trigger, capable of triggering events with a low energetic photon, the measurement of $\sigma_{e^+e^-\rightarrow\nu\bar{\nu}\gamma}$ will be severely limited by the small number of observable events. In the case of the $L_3$ experiment, the trigger system is capable of triggering events containing a single, isolated photon with an energy as small as 1 GeV. The strong dependence of the cross section on $E_{\gamma}^{\text{min}}$ also necessitates a good knowledge of both the absolute energy scale and the energy resolution of the detector.

The requirement to use low energy photons introduces potentially large backgrounds. The main background, which is indistinguishable from the signal, consists of radiative Bhabha events. In these events both the scattered electron and positron escape detection at very small angles along the beam axis, and the photon is detected in the central detector. A schematic representation of such an event is shown in Figure 4.1. The size of this background can be substantially reduced by either decreasing the smallest angle at which particles can be detected, or, at the cost of reducing the signal, by increasing the minimal required transverse momentum of the photon. As the conservation of energy and momentum constrains these radiative Bhabha events through their three-body kinematics, requiring a sufficient transverse momentum of the photon will ensure that either the electron or the positron is scattered at an angle larger than the veto angle $\theta_{\text{veto}}$. The expected Bhabha background, for several different veto angles, is shown in Figure 4.2, as a function of the minimal required photon energy at $\sqrt{s} = 91.250$ GeV and for $44^\circ < \theta < 136^\circ$.

Other, smaller backgrounds include the QED process $e^+e^- \rightarrow \gamma\gamma\gamma$, which has the same experimental signature as the radiative Bhabha events; the radiative decays of the $Z$ into leptons; final states consisting of four fermions; hadronic resonances produced by photon-photon interactions and cosmic rays.

The strategy of the analysis is to first select samples of so-called single electron events. These events are used to study not only the hermeticity, i.e. $\theta_{\text{veto}}$, and the trigger efficiency, but also the quality of the detector simulation and the size of the expected Bhabha background. Once this has been done, candidate $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ events are selected and the selection efficiency and the size of the remaining backgrounds in the selected sample are estimated.
Figure 4.1: An example of a radiative Bhabha event. Only the photon is detected, making this event indistinguishable from the $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ signal.

4.1 Analysis of Single Electron Events

Single Electron events are events where one single electron (or positron) is detected in the barrel region of the detector ($44^\circ < \theta < 136^\circ$). These electrons are produced either by the radiative Bhabha process $e^+e^- \rightarrow e^+e^-\gamma$ or by the process $e^+e^- \rightarrow e^+e^-e^+e^-$. In both processes the remaining final state particles tend to scatter at very small angles with respect to the beam line. If one of these particles scatters sufficiently to be detected, the event is called a "Tagged Single Electron Event", otherwise it is classified as an "Untagged Single Electron Event".

As in addition to an energy deposit in the BGO calorimeter also a charged track is detected in these events, they can be triggered utilizing the charged track trigger. In that case the events are obtained using a trigger which is independent of the single photon trigger, and thus the efficiency of the single photon trigger can be measured. However, the requirement of just one single track at the level of the first-level trigger leads to an unacceptable high trigger rate due to beam backgrounds. To avoid this problem, these events are triggered only in case there is a coincidence between a charged track and a large energy deposit in the luminosity monitor (i.e. only the "Tagged Single Electron" events are triggered by this trigger).

Since both the energy range and the calorimetric signature of the single, large-angle electrons (or positrons) are approximately the same as for the observed photons in $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ events, the single electron events can be used to compare the simulation of the BGO calorimeter which the actual response. The large collinear peak of the differential cross section in the forward direction of these events can be used to check the hermeticity of the detector at small angles: assuming that
Figure 4.2: The cross sections for the $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ signal and Bhabha background cross sections and their ratios as function of the minimum required photon energy $E_{\gamma}^{\text{min}}$. The curves in the lower plot show the signal to background ratio for several values of $\theta_{\text{veto}}$, the smallest angle at which particles can be detected.

one of the particles escapes along the beam line, the measurement of the transverse momentum of the electron (positron) scattered in the barrel part of the detector can be used to predict where the third particle should be observed. By comparing this prediction with the actual measurement of this third particle, the detection efficiency in the forward direction can be determined.

Finally, the Monte Carlo generator [89] which is used to calculate the expected Bhabha event rate for the single electrons sample is also used to predict the size of the Bhabha background in the selected single photon sample. A quantitative comparison between the measured single electron sample and the predictions of this generator can be performed.

4.1.1 Tagged Single Electron Events

The signature of these events is one single track with a corresponding electromagnetic shower and a tag in the LUMI. As this sample is used to check the Monte Carlo description of the variables asso-
associated with an electromagnetic shower and the single-photon trigger efficiency, the event selection avoids using these variables.

The events are triggered by the single electron trigger, described in the previous chapter. The efficiency of this trigger has been determined to be 98.5%, using events triggered by the prescaled single tag trigger. Due to timing problems in the communication between the TEC trigger and the ENERGY trigger during the runs where LEP was operating with eight instead of four bunches, the requirement of a positive single-electron trigger decision limits the amount of data available for this analysis. The periods during which the single-electron trigger was operating reliably were during the initial period of four-bunch operation in 1992 and the latter part of 1993, when the timing problem was solved.

The events are selected by imposing the following criteria:

- The event must be triggered by the level one single-electron trigger.
- The event has to contain one single track, reconstructed in TEC, and this track has to satisfy the following four criteria:
  1) $p^\perp > 0.75$ GeV,
  2) at least 40 hits which span at least 45 wires,
  3) the distance of closest approach (in the $r - \phi$ plane) to the nominal beam spot must be less than 3 mm,
  4) $|\phi_{\text{BGO}} - \phi_{\text{track}}| < 0.85^\circ$.
- The event must be tagged by a reconstructed cluster in the LUMI. To guarantee that this cluster is correctly measured, it must be at least one crystal away from the edge of the detector:
  $$1.776^\circ < \theta_{\text{LUMI}}(180^\circ - \theta_{\text{LUMI}}) < 3.724^\circ$$
  and the deposited energy in the LUMI must be large enough to satisfy the trigger requirement:
  $$E_{\text{LUMI}} > 32 \text{ GeV}.$$
- The track and the LUMI cluster must be approximately back-to-back in azimuth:
  $$165^\circ < \text{mod}(\phi_{\text{LUMI}} - \phi_{\text{track}}, 360^\circ) < 195^\circ.$$

A summary of the selected events and the expected number of events from the Monte Carlo simulation is given in Table 4.1, whereas the distribution of the two “tag” variables is shown in Figure 4.3. The events selected are Bhabha events, with a large background from $e^+e^- \rightarrow e^+e^-e^+e^-$. As can be seen in Figure 4.4, the electrons (positrons) produced by Bhabha scattering tend to continue along the direction of the initial electron (positron) whereas the $e^+e^-e^+e^-$ events show no such behaviour. This sample will be used to check the simulation of the single-photon trigger and to estimate the systematic error introduced by this simulation. The number of events measured is in good agreement with the expected number from the Monte Carlo prediction (Table 4.1). In total 10106 events are observed, compared to the 10040 events which are expected by the Monte Carlo simulation.

---

1 The efficiency of the prescaled single tag trigger itself can be determined by a comparison with the double tag trigger and is larger than 99.999% at 95% CL.
4.1. Analysis of Single Electron Events

Figure 4.3: The two variables used to tag the “Tagged Single Electron” events. Left: the θ of LUMI tag is shown and on the right the observed LUMI energy.

<table>
<thead>
<tr>
<th>period</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\int L dt$ (pb$^{-1}$)</th>
<th>$N_{\text{obs}}$</th>
<th>$N_{e^+e^-}$</th>
<th>$N_{e^+e^-e^-}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>91.294</td>
<td>15.56</td>
<td>6672</td>
<td>4929.0</td>
<td>1594.9</td>
</tr>
<tr>
<td>1993</td>
<td>91.238</td>
<td>8.15</td>
<td>3434</td>
<td>2637.9</td>
<td>878.0</td>
</tr>
</tbody>
</table>

Table 4.1: Selected “Tagged Single Electron” events.

4.1.2 Untagged Single Electron Events

To check the rejection efficiency of the LUMI and the ALR, the tagged single-electron selection is loosened by dropping any reference to either the LUMI and ALR. But, to ensure that the correct trigger efficiency can be applied to the Monte Carlo events, the requirement that the events are triggered by the single-photon trigger is added. In the case of the Monte Carlo events, the trigger efficiency is calculated using the simulation of the single-photon trigger, which is explained in detail in Section 4.3. As the single-electron trigger is not required in this selection, a larger fraction of the collected data can be used for this study. The total integrated luminosity used is 31.7 pb$^{-1}$. 
Figure 4.4: The transverse momentum and $Q_e \cdot \cos \theta_e$ distributions of the selected "Tagged Single Electron" events.

The selection criteria for these events are as follows:

- They must be triggered by the single photon trigger.
- They must contain one reconstructed charged track. This track should match the following requirements:
  1) $p^\perp > 0.75$ GeV,
  2) at least 40 hits which span at least 45 wires,
  3) the distance of closest approach (in the $r - \phi$ plane) to the nominal beam spot must be less than 3 mm,
  4) $|\phi_{BGO} - \phi_{\text{track}}| < 0.85^\circ$.

The energy spectrum of the selected events is shown in Figure 4.5. The number of events observed is compared with the Monte Carlo expectation in Table 4.2. The agreement between the data and the Monte Carlo expectation as a function of the energy shows that the simulation of the single photon trigger is correct for energies between one and five GeV. In Section 4.3 this agreement will be quantified.
4.2 Small Angle Tagging Efficiency

The sample of “Untagged Single Electron” events can be used to study the efficiency of the tagging efficiency at small angles. Using the constraints of three body kinematics\(^2\) and the assumption that one particle escapes along the beam line, it is possible to predict the expected polar angle $\theta_{\text{tag}}$.

\(^2\)There is a large probability that at least one of the $e^+e^-$ pairs in an $e^+e^-e^+e^-$ event has a small invariant mass. These events can effectively be considered to be a three particle final state.

| year | $\sqrt{s}$ | $\int L dt (\text{pb}^{-1})$ | $N^{\text{obs}}$ | $N^{\exp}_{e^+e^-} | N^{\exp}_{e^+e^-}$ |
|------|------------|-------------------------------|-----------------|-------------------|
| 1992 | 91.294     | 19.29                         | 18347           | 11927.9           | 6297.5 |
| 1993 | 91.238     | 12.38                         | 11774           | 8007.6            | 4360.1 |

**Table 4.2:** The number of selected “Untagged Single Electron” events.

---

**Figure 4.5:** Transverse momentum spectrum and the angular distribution of the selected “Untagged Single Electron” events.
of the tag:

\[
\cos \theta_{\text{tag}} = \frac{a^2 - 1}{a^2 + 1}; \quad a = \frac{2E_{\text{beam}}}{E_{\text{ta}} - \frac{1 + \cos \theta_{\text{ta}}}{\sin \theta_{\text{ta}}}} \tag{4.1}
\]

where \( E_{\text{ta}} \) and \( \theta_{\text{ta}} \) are the energy and polar angle of the electron (positron) which is scattered at a large angle.

The efficiency to tag these events, as function of \( \theta_{\text{tag}} \), in either the LUMI or the ALR is compared with the Monte Carlo expectation in Figure 4.6. The difference between the two curves is due to the fact that the location of the LUMI monitor in the detector simulation program is not correct. According to a survey, the detectors are located at 273 cm and -272 cm from the interaction point, whereas in the simulation program they are assumed to be at \( \pm 267 \) cm from the interaction point.

In the data, the tagging efficiency for single electron events with a predicted tag which satisfies \( 1.4^\circ < \theta_{\text{tag}} < 178.6^\circ \), is \( 84.8 \pm 0.2\% \), compared with the Monte Carlo expectation of \( 83.0 \pm 0.2\% \). This large difference is mainly due to the difference at the low-angle limit, i.e. the value of \( \theta_{\text{veto}} \): in the range\(^3 1.8^\circ < \theta_{\text{tag}} < 178.2^\circ \) the tagging efficiency is \( 91.9 \pm 0.2\% \) for the data and \( 92.3 \pm 0.2\% \) for the Monte Carlo.

Given these differences, only events which contain tags that are reconstructed in the polar angle region \( 1.5^\circ < \theta < 178.5^\circ \) are used in the rejection of background events in the \( e^+ e^- \rightarrow \nu \bar{\nu} \gamma \) selection. Nonetheless, the uncertainty in the veto efficiency introduces a 5\% uncertainty in the background subtraction.

### 4.3 Single Photon Trigger Efficiency

A good knowledge of the single photon trigger and its efficiency is required for a measurement of \( \sigma_{e^+ e^- \rightarrow \nu \bar{\nu} \gamma} \). Not only the efficiency as function of energy and the location of the energy deposit must be known, but also how it changes during the time the data is gathered. It is therefore not only necessary to take into account all possible sources of inefficiency, such as dead or noisy channels and the limited energy resolution of the trigger system, but also the correlation between all these parameters and their evolution in time.

As described in Section 3.3.1 the single photon trigger uses the 256 FERAm signals from the barrel part of the BGO calorimeter. The height of each signal is proportional to the energy deposited, and is represented by a nine bit value. Two bits are used to flag which threshold should be applied for this channel. This information can be used to describe the trigger as a finite state machine which can be in one of \( 2048^{256} \approx 5 \cdot 10^{846} \) states and each state corresponds to either a positive or negative trigger decision. Given as input the resolution and the energy deposited in each channel the probability for an event to be in given single state can be calculated. As it is known which states correspond to a positive trigger decision and which to a negative decision, the cumulative probability for a positive trigger decision, i.e. the trigger efficiency, can be calculated. Given the large number of possible states it is not feasible to compute this for each simulated event. It is however possible to implement the above using a Monte Carlo algorithm.

To incorporate the channel status, which might change as a function of time, each simulated event is assigned to a corresponding data-run. The probability for an event to be assigned to a given run depends on the the fraction of the integrated luminosity of each run. Once the run-number

\(^3\)This is the same value as used for the selection of the “Tagged Single Electron” events.
is known, the values of the 2 status bits of each trigger channel are taken from a database. This database contains the status information on a run-by-run basis, derived from the actual data.

Next the amount of energy deposited in each FERA channel is taken from the GEANT simulation of the detector. Given the gain and resolution of the channel, the number of ADC counts $N_i$ in this channel is determined by generating a number according to the following probability density

![Graph showing Veto efficiency as function of $\theta_{tag}$](image)

**Figure 4.6:** Veto efficiency as function of $\theta_{tag}$. Events with predicted tags values $\theta_{tag} > \pi/2$ have been folded over using $\theta_{tag} \rightarrow \pi - \theta_{tag}$. The upper plot shows all events and those events tagged, whereas the lower plot shows the comparison of the veto efficiency between data and the Monte Carlo expectation. The sudden drop at $1.4^\circ$ is due both to the implicit cut on the angle imposed by the lower limit on the $p^\perp$ of the tag caused by the requirement that there be at least 1.0 GeV in the barrel part of the BGO and the edge of the detector.
Table 4.3: Comparison between the results of the trigger hardware and the software implementation of the algorithm used in the simulation. The seven events for which the results do not agree are all taken during LEP fill 1553, and the thresholds for this fill are probably slightly different from those recorded in the database.

<table>
<thead>
<tr>
<th>trigger hardware</th>
<th>software implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>161653</td>
</tr>
<tr>
<td>no</td>
<td>132428</td>
</tr>
</tbody>
</table>

function:

\[ P(N_i|E_i, G_i, \sigma_i) = \mathbb{E} \left[ \exp \left( -\frac{1}{2} \left( \frac{N_i - G_i^{-1}E_i}{\sigma_i} \right)^2 \right) \right] \]

(4.2)

where \( G \) and \( \sigma \) are the gain and resolution as determined in Section 3.3.1, and \( \mathbb{E}[x] \) is the Entier function which truncates its argument to the largest integer less or equal to the argument. Using the probability density defined in Equation 4.2, the deposited energy is converted into ADC counts. To describe the noise level, these ADC counts are added to the ADC counts as found in a “real” event, in the same run, triggered by the BEAMGATE trigger.

Once this has been done for all 256 trigger channels in the BGO barrel, the state of the system is determined and the single-photon trigger algorithm is applied. The trigger algorithm itself (and the parameters it uses, such as the thresholds) has been validated on real data, using a sample of 294088 events triggered by independent triggers, yielding the result shown in Table 4.3. This entire process, which in each step results in either zero (rejected) or one (accepted), is repeated several times for each event. Finally, the average of all the steps is taken, yielding a weight for each Monte Carlo event which is an estimate for the probability that this event is triggered by the single photon trigger.

The result of this simulation is then compared to a measurement of the trigger efficiency, using the previously selected sample of “Tagged Single Electron” events. This is shown in Figure 4.7, where the trigger efficiency as a function of the BGO energy is plotted. Unfortunately, the measured curve itself cannot be applied directly to the single photon Monte Carlo events: the angular distribution and energy spectrum of the \( e^+e^- \rightarrow \nu\bar{\nu}\gamma \) events differs from the single electron sample. This would result in a different weighting of the various channels. Also, the data from which the single photon events are selected corresponds to a longer period of time, and thus could have different efficiencies due to a different number of dead and high-bias channels.

After convoluting the trigger curves of Figure 4.7 with the spectrum of the \( e^+e^- \rightarrow \nu\bar{\nu}\gamma \) Monte Carlo events, the difference in trigger efficiency is 0.3% for the data taken in 1992, and 1.9%, 1.5% and 1.0% for the data taken at center-of-mass energies of 89.453 GeV, 91.238 GeV and 93.036 GeV in 1993. Taking the luminosity weighted average of the differences, an overall systematic error of 1.5% is assigned to the knowledge of the trigger efficiency. The statistical error on the measured curve is 0.9% for the 1992 data and 1.2% for the data collected in 1993.

The simulated trigger efficiencies for the \( \nu\bar{\nu}\gamma \) sample during the different data taking periods
4.3. Single Photon Trigger Efficiency

Figure 4.7: A comparison of the simulation of the trigger efficiency and the measurement, as function of energy.

are summarized in Table 4.4. The main contributions to the inefficiency are:

- Killed channels.

Averaged over the luminosity used, 4.31% of the channels are killed during the 1993 period. In 1992 this fraction was 9.84%.
<table>
<thead>
<tr>
<th>year</th>
<th>$\sqrt{s}$</th>
<th>trigger efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>91.294</td>
<td>0.668112 ± 0.000016</td>
</tr>
<tr>
<td>1993</td>
<td>89.453</td>
<td>0.712940 ± 0.000014</td>
</tr>
<tr>
<td></td>
<td>91.238</td>
<td>0.692665 ± 0.000015</td>
</tr>
<tr>
<td></td>
<td>93.036</td>
<td>0.723539 ± 0.000013</td>
</tr>
</tbody>
</table>

*Table 4.4:* The simulated trigger efficiency for $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ events which satisfy $E_\gamma > 1.0$ GeV and $|\cos \theta_\gamma| < 0.72$. The quoted error is due only to the limited Monte Carlo statistics, and does not contain any systematic effect.

- Channels with a raised threshold setting.
  
  In 1993, 12.45% of the channels had a high bias setting. In 1992, this was 11.60%.

- ADC resolution.

- Noise.

**Level two and three trigger**

In principle the level two trigger should pass on every single event which has been triggered by the single photon trigger. This has been checked off-line, as every tenth event rejected by the level two trigger is kept and written to tape. For this purpose, a sample of 293190 events, containing 161656 single photon triggers, of which 6540 events were triggered by the single photon algorithm only and 108327 triggered by the combination of the level zero energy trigger and the single photon algorithm were checked. In this sample no event was found which was flagged as rejected by the level two trigger. It is therefore assumed that the level two trigger is fully efficient for the events which are accepted by the level one single photon trigger.

The level three trigger has access to the same data as the final event selection, but without the final calibration. As all the selection cuts which are applied by the level three trigger are a subset of the final selection cuts, it is assumed that the level three trigger does not introduce any additional inefficiency.

**4.4 Selection of Single Photon Events**

**4.4.1 Event Selection**

The selection of single photon candidates is split in two parts. The aim of the first set of selection cuts is to select, with high efficiency, the events which satisfy the signature of a single photon event. The second set of cuts aims at purifying the event sample by rejecting as many background events as possible, without reducing the selection efficiency of the signal too much.

First of all though, to ensure that the correct correction for the trigger efficiency can be applied to this sample, only events that are triggered by the single-photon trigger are considered. This criterium removes 3 events from the final sample of selected events.
4.4. Selection of Single Photon Events

The next step is to require events to contain one single cluster in the BGO calorimeter. To reject clusters which may be caused by correlated noise, only clusters which contain at least one crystal with a deposited energy in excess of 100 MeV are considered. Furthermore, to avoid the situation where one single noisy crystal causes events to be rejected, the clusters are required to contain more than 3 crystals.

The next step is to require that the shape of the single remaining cluster is consistent with an electromagnetic shower. This is done using the three shower shape variables introduced in Chapter Three:

- \( \rho_{EM} > 0.95 \),
- \( \chi^2_{EM} < 5 \),
- \( sk > 0.25 \).

The distribution of these three variables, and a comparison with the single electron reference sample is shown in Figure 4.8.

To guarantee that the cluster is caused by a neutral particle, there should be no track reconstructed in the TEC which contains more than ten hits. Again, requiring a track to contain more than ten hits reduces the probability that a genuine event is rejected because of noise.

In combination with the above cuts, the requirement of positive single-photon trigger decision implies that the candidate photon is contained in the barrel part of the BGO and that it has an energy larger than the trigger threshold shown in Figure 4.7. To define unambiguously the phase space in which the events are selected, both these cuts are made explicit and tightened by the following requirements on the photon candidate:

- \( |\cos \theta_\gamma| < 0.72 \),
- \( E_\gamma > 1.0 \text{ GeV} \).

The angular cut ensures that the impact point of the photon is at least one crystal away from the edge of the barrel calorimeter to ensure a proper measurement of its energy. These two cuts define the phase-space for which the cross section will be determined.

4.4.2 Selection Efficiency

To determine the selection efficiency samples of 50,000 fully simulated Monte Carlo \( e^+e^- \rightarrow \nu\bar{\nu} \gamma(\gamma) \) events were used for each value of \( \sqrt{s} \). The events were generated within \( E_\gamma > 0.8 \text{ GeV} \) and \( |\cos \theta_\gamma| < 0.75 \), to take into account the possibility of feed through, i.e. events which are generated outside the phase space under consideration, but which, due to the finite detector resolution, are reconstructed within the final phase space.

The selection efficiencies at the four different center-of-mass energies are quoted in Table 4.5. The values in this table assume a fully efficient trigger, and must be combined with the trigger efficiencies of Table 4.4. This combination is performed by weighting each Monte Carlo event with its calculated trigger efficiency as described in Section 4.3. This procedure, by construction, takes into account all correlations between the trigger and selection efficiencies.

Similar to the calculation of the trigger efficiency, known detector defects are taken into account before the event selection, during the event reconstruction. For example, during part of the 1992
Figure 4.8: Comparison of the data and the Monte Carlo expectation of the shower shape variables for both the single photon sample on the left-hand side and the single electron reference sample on the right-hand side.

data taking period one single BGO readout ring – which contains sixty crystals – was inoperative. This is taken into account by selectively disabling these sixty crystals in both the data and the proper fraction of the Monte Carlo events. Similarly, a realistic level of noise is introduced into the Monte Carlo events. This procedure, and its effects, will be described in Section 4.4.3.
4.4. Selection of Single Photon Events

<table>
<thead>
<tr>
<th>$\sqrt{s}$</th>
<th>selection efficiency</th>
<th>due to feed through</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992 91.294</td>
<td>0.9163 ± 0.0015</td>
<td>0.0264 ± 0.0009</td>
</tr>
<tr>
<td>1993 89.453</td>
<td>0.9123 ± 0.0015</td>
<td>0.0223 ± 0.0008</td>
</tr>
<tr>
<td>1993 91.238</td>
<td>0.9203 ± 0.0015</td>
<td>0.0267 ± 0.0009</td>
</tr>
<tr>
<td>1993 93.036</td>
<td>0.9176 ± 0.0014</td>
<td>0.0174 ± 0.0007</td>
</tr>
</tbody>
</table>

Table 4.5: The selection efficiencies for $e^+e^- \rightarrow \nu \bar{\nu} \gamma$ events which satisfy $E_\gamma > 1.0$ GeV and $|\cos \theta_\gamma| < 0.72$ as determined from the NNGSTR Monte Carlo.

Inefficiency due to multiple radiation

According to the $e^+e^- \rightarrow \nu \bar{\nu} \gamma(\gamma)$ Monte Carlo, 2.4% of the events are rejected because they fail the requirement of one single cluster in the BGO calorimeter. In 95% of these rejected events the second cluster is caused by the radiation of a second photon. In the remaining 5% the second cluster is caused either by noise or detector occupancy.

The theoretical uncertainty on the expected rate of $e^+e^- \rightarrow \nu \bar{\nu} \gamma \gamma$ events compared with $e^+e^- \rightarrow \nu \bar{\nu} \gamma$ is determined to be 0.3%. This number is taken from a comparison [90, 91] between the NNGSTR [62] and KORALZ [92, 93] event generators for center-of-mass energies close to the $Z$ resonance.

Photo conversion and backscatter inefficiency

There are two reasons why a genuine $e^+e^- \rightarrow \nu \bar{\nu} \gamma$ event can be rejected by the charged track veto. The most important one is the conversion of the photon into an electron-positron pair in the material between the production vertex and the tracking chamber, the other one is the backscattering of charged particles, which are generated as the photon is stopped in the BGO calorimeter.

In the 1992 sample, 1.6% of the $e^+e^- \rightarrow \nu \bar{\nu} \gamma$ Monte Carlo events is rejected because it contains charged tracks. In 1993, due to the presence of the micro vertex detector, this number increases to 2.6%.

EM-selection efficiency

To study the Monte Carlo description of the BGO shower shape variables used, the data sample of single electrons is used, and the fraction of events satisfying the cuts $\rho_{EM} > 0.95$, $\chi^2_{EM} < 5$ and $sk > 0.25$ in this sample is compared with the Monte Carlo prediction (Figure 4.8). The difference between the two is taken into account by applying a correction factor on the efficiencies quoted in Table 4.5. The efficiencies listed in Table 4.5 are therefore corrected by multiplying them with $0.9760 \pm 0.0035$ (1992) and $0.9701 \pm 0.0031$ (1993).
| $\rho > 0.95$ | 0.9831 $\pm$ 0.0015 | 0.9817 $\pm$ 0.0013 | 0.9919 $\pm$ 0.0008 | 0.9931 $\pm$ 0.0007 |
| $\chi^2_{\text{EM}} < 5$ | 0.9728 $\pm$ 0.0019 | 0.9682 $\pm$ 0.0017 | 0.9834 $\pm$ 0.0012 | 0.9848 $\pm$ 0.0011 |
| $sk > 0.25$ | 0.9663 $\pm$ 0.0021 | 0.9694 $\pm$ 0.0017 | 0.9768 $\pm$ 0.0014 | 0.9779 $\pm$ 0.0011 |
| combined: | 0.9388 $\pm$ 0.0028 | 0.9354 $\pm$ 0.0024 | 0.9619 $\pm$ 0.0018 | 0.9642 $\pm$ 0.0017 |

**Table 4.6**: A comparison between the single electron data and Monte Carlo expectation of the fraction of events which satisfy the shape criteria imposed on BGO clusters.

### 4.4.3 Background Rejection

$e^+e^- \rightarrow e^+e^-\gamma$

The main background of the single-photon measurement is radiative Bhabha scattering. The cross section of this process is sharply peaked in the forward direction. Therefore, in most cases the transverse momentum of the photon which is produced by these events is balanced by either the electron or the positron. Depending on the transverse momentum of the photon, either the electron or the positron is forced to scatter into either the luminosity monitor, the ALR or the BGO endcaps. To reduce this background the following requirements are therefore imposed:

- less than 5.0 GeV total energy must be deposited in either of the luminosity monitors if there is a corresponding cluster with a reconstructed impact point which is more than 1.5° from the beam line.
- less than 100 MeV energy observed in the ALR.

The first cut removes the Bhabha events where the photon has a transverse momentum between 1.1 GeV and 2.9 GeV, the second cut where it has a transverse momentum between 3.3 GeV and 6.5 GeV. Bhabha events where the photon has a transverse momentum larger than 6.5 GeV are already rejected by the requirement that there is only one reconstructed cluster in the BGO calorimeter. Finally, to ensure that no particle escapes through the gap between the BGO barrel and the BGO endcap, the following cut is imposed:

- no more than 5 GeV deposited in the HCAL

To determine the remaining background, events were generated with the TEEG Monte Carlo generator [89]. As in the case of the signal events, these events were passed through the detector simulation program twice: once for the 1992 detector setup and once for the 1993 setup. The estimated background due to this process is listed in Table 4.7.

Figure 4.9 shows the distribution of the variables used in the above rejection criteria. The transverse energy of the photon, for events which are rejected solely by either the cut on the LUMI or the ALR is shown in Figure 4.10.
4.4. Selection of Single Photon Events

\[ e^+e^- \rightarrow \gamma\gamma\gamma \]

The events produced by the QED process $e^+e^- \rightarrow \gamma\gamma\gamma$ are constrained by the same kinematics as the Bhabha process, and are rejected by the same set of cuts. However, as this process does not receive a contribution from $t$-channel photon exchange, its cross section is two orders of magnitude smaller than the Bhabha cross section. The Monte Carlo events used to determine the remaining background due to this reaction events were generated using the $\gamma\gamma(\gamma)$ Monte Carlo generator from F.A. Berends and R.Kleiss [94]. As the expected background from the $\gamma\gamma$ final state is small compared to the $\gamma\gamma\gamma$ final state, only the $\gamma\gamma\gamma$ events were generated. The result is shown in Table 4.8.

\[ e^+e^- \rightarrow \mu^+\mu^-(\gamma) \]

At this point most $e^+e^- \rightarrow \mu^+\mu^- (\gamma)$ events are already rejected by the requirement that there is no reconstructed TEC track. However, to increase the rejection power, it is required that

---

**Figure 4.9:** The distribution of the deposited energy in LUMI, ALR and HCAL before the veto on each individual subdetector.
<table>
<thead>
<tr>
<th>year</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\sigma_{\text{gen}}$(nb)</th>
<th>$\epsilon_{\text{sel}}$</th>
<th>$\epsilon_{\text{sel}} \cdot \epsilon_{\text{trig}}$</th>
<th>$\sigma_{\text{acc}}$(pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>91.25</td>
<td>73.92</td>
<td>448/5000</td>
<td>279.0/5000</td>
<td>4.12 ± 0.24</td>
</tr>
<tr>
<td></td>
<td>89.50</td>
<td>76.84</td>
<td>441/4998</td>
<td>282.2/4998</td>
<td>4.34 ± 0.25</td>
</tr>
<tr>
<td>1993</td>
<td>91.25</td>
<td>73.92</td>
<td>441/4998</td>
<td>274.2/4998</td>
<td>4.06 ± 0.24</td>
</tr>
<tr>
<td></td>
<td>93.10</td>
<td>71.01</td>
<td>441/4998</td>
<td>286.4/4998</td>
<td>4.07 ± 0.23</td>
</tr>
</tbody>
</table>

**Table 4.7:** Estimated background due to the process $e^+e^- \rightarrow e^+e^-\gamma$. Events were generated requiring a photon in the region $40^\circ < \theta < 140^\circ$ with at least 700 MeV. The results for the off-peak points where obtained by rescaling the on-peak points according to $s'/s$.

<table>
<thead>
<tr>
<th>year</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\sigma_{\text{gen}}$(nb)</th>
<th>$\epsilon_{\text{sel}}$</th>
<th>$\epsilon_{\text{sel}} \cdot \epsilon_{\text{trig}}$</th>
<th>$\sigma_{\text{acc}}$(pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>91.25</td>
<td>0.0132</td>
<td>90/2000</td>
<td>55.0/2000</td>
<td>0.36 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>89.50</td>
<td>0.0137</td>
<td>100/2000</td>
<td>68.3/2000</td>
<td>0.47 ± 0.06</td>
</tr>
<tr>
<td>1993</td>
<td>91.25</td>
<td>0.0132</td>
<td>100/2000</td>
<td>66.4/2000</td>
<td>0.44 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>93.10</td>
<td>0.0127</td>
<td>100/2000</td>
<td>69.3/2000</td>
<td>0.44 ± 0.05</td>
</tr>
</tbody>
</table>

**Table 4.8:** Estimated background due to the process $e^+e^- \rightarrow \gamma\gamma\gamma$. The events were generated requiring that at least one photon was generated in the region $40^\circ < \theta < 140^\circ$ with an energy larger than 800 MeV. The results for the off-peak points were obtained by rescaling the on-peak points according to $s'/s$. 
Figure 4.10: The transverse momentum of the photon for events which satisfy all selection criteria except the LUMI veto (shown at the top) or the ALR veto (shown at the bottom).

- No MUCH track is reconstructed.

Unfortunately, neither the TEC nor the MUCH cover the entire space: the region $0.05 < |\cos \theta| < 0.91$ is covered by at least one layer of MUCH chambers, but due to gaps between the chambers, the azimuthal coverage in this region is approximately 95% of $2\pi$ [79]. Similarly, the requirement that a "good" TEC track contains hits on at least ten wires implicitly rules out the detection of charged particles by TEC in the region outside $20^\circ < \theta < 160^\circ$. 
<table>
<thead>
<tr>
<th>year</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\sigma_{\text{gen}}$ (nb)</th>
<th>$\epsilon_{\text{sel}}$</th>
<th>$\epsilon_{\text{sel}} \cdot \epsilon_{\text{trig}}$</th>
<th>$\sigma_{\text{acc}}$ (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>91.25</td>
<td>1.4826</td>
<td>43/249897</td>
<td>29.0/249897</td>
<td>0.17 ± 0.03</td>
</tr>
<tr>
<td>1993</td>
<td>89.50</td>
<td>0.5167</td>
<td>10/50000</td>
<td>9.1/50000</td>
<td>0.09 ± 0.03</td>
</tr>
<tr>
<td></td>
<td>91.25</td>
<td>1.4826</td>
<td>36/249897</td>
<td>28.6/249897</td>
<td>0.17 ± 0.03</td>
</tr>
<tr>
<td></td>
<td>93.10</td>
<td>0.6631</td>
<td>23/79965</td>
<td>17.4/79965</td>
<td>0.14 ± 0.03</td>
</tr>
</tbody>
</table>

Table 4.9: Estimated background due to the process $e^+e^- \rightarrow \mu^+\mu^- (\gamma)$. 

<table>
<thead>
<tr>
<th>year</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\sigma_{\text{gen}}$ (nb)</th>
<th>$\epsilon_{\text{sel}}$</th>
<th>$\epsilon_{\text{sel}} \cdot \epsilon_{\text{trig}}$</th>
<th>$\sigma_{\text{acc}}$ (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>91.25</td>
<td>1.4768</td>
<td>8/157496</td>
<td>6.8/157496</td>
<td>0.06 ± 0.02</td>
</tr>
<tr>
<td>1993</td>
<td>89.50</td>
<td>0.5267</td>
<td>0/58998</td>
<td>0/58998</td>
<td>&lt; 0.03</td>
</tr>
<tr>
<td></td>
<td>91.25</td>
<td>1.4768</td>
<td>7/157496</td>
<td>5.0/157496</td>
<td>0.05 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>93.10</td>
<td>0.6524</td>
<td>0/46444</td>
<td>0/46444</td>
<td>&lt; 0.04</td>
</tr>
</tbody>
</table>

Table 4.10: Estimated background due to the process $e^+e^- \rightarrow \tau^+\tau^- (\gamma)$. The quoted upper limits are 95% CL limits.

To reduce the background due to muons which escape in the forward direction, and to strengthen the rejection of events where the muons are emitted at large angles, the following requirement is imposed:

- No cluster with a MIP signature is found in the hadron calorimeter.

The definition of such a cluster is given in Chapter 3.

To estimate the remaining background due to radiative dimuon production, the KORALZ event generator was used to generate dimuon events over the full solid angle. The results are summarized in Table 4.9.

$e^+e^- \rightarrow \tau^+\tau^- (\gamma)$

As expected, almost all $e^+e^- \rightarrow \tau^+\tau^- (\gamma)$ events are rejected by the requirement that no charged tracks are reconstructed. In addition, the requirement that only one, good electromagnetic cluster of more than 1.0 GeV is reconstructed in the barrel reduces the final selection efficiency even further. Again the KORALZ event generator was used to generate events over the full solid angle. The results are summarized in Table 4.10.

$e^+e^- \rightarrow e^+e^- X$

The remaining source of background due to $e^+e^-$ interactions is the production of additional lepton pairs, $e^+e^- \rightarrow e^+e^- e^+e^-$, $e^+e^- \rightarrow e^+e^- \mu^+\mu^-$, $e^+e^- \rightarrow e^+e^- \tau^+\tau^-$ and the production of hadronic resonances through photon-photon collisions, $e^+e^- \rightarrow e^+e^- R, R \rightarrow \gamma X$. These events are typically boosted along the beam line, which again may cause some decay products to escape
### 4.4. Selection of Single Photon Events

<table>
<thead>
<tr>
<th>year</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\sigma_{\text{gen}}$ (nb)</th>
<th>$\epsilon_{\text{sel}}$</th>
<th>$\epsilon_{\text{sel}} \cdot \epsilon_{\text{trig}}$</th>
<th>$\sigma_{\text{acc}}$ (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>91.25</td>
<td>4.8965</td>
<td>30/93698</td>
<td>18.3/93698</td>
<td>0.95 ± 0.22</td>
</tr>
<tr>
<td></td>
<td>89.50</td>
<td>5.0890</td>
<td>11/96346</td>
<td>4.6/96346</td>
<td>0.24 ± 0.11</td>
</tr>
<tr>
<td>1993</td>
<td>91.25</td>
<td>4.8965</td>
<td>11/96346</td>
<td>4.5/96346</td>
<td>0.23 ± 0.11</td>
</tr>
<tr>
<td></td>
<td>93.10</td>
<td>4.7038</td>
<td>11/96346</td>
<td>4.7/96346</td>
<td>0.23 ± 0.11</td>
</tr>
</tbody>
</table>

**Table 4.11:** Estimated background due to the process $e^+e^- \rightarrow e^+e^-e^+e^-$. Events were generated requiring that the total energy of the particles scattered in the region $40^\circ < \theta < 120^\circ$ is larger than 700 MeV. The off-peak point results are obtained from the on-peak points by rescaling the cross section as $s'/s$.

<table>
<thead>
<tr>
<th>year</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\sigma_{\text{gen}}$ (nb)</th>
<th>$\epsilon_{\text{sel}}$</th>
<th>$\epsilon_{\text{sel}} \cdot \epsilon_{\text{trig}}$</th>
<th>$\sigma_{\text{acc}}$ (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>91.25</td>
<td>42.51</td>
<td>0/234000</td>
<td>0/234000</td>
<td>&lt; 0.54</td>
</tr>
<tr>
<td></td>
<td>89.50</td>
<td>44.19</td>
<td>0/234000</td>
<td>0/234000</td>
<td>&lt; 0.57</td>
</tr>
<tr>
<td>1993</td>
<td>91.25</td>
<td>42.51</td>
<td>0/234000</td>
<td>0/234000</td>
<td>&lt; 0.54</td>
</tr>
<tr>
<td></td>
<td>93.10</td>
<td>40.84</td>
<td>0/234000</td>
<td>0/234000</td>
<td>&lt; 0.52</td>
</tr>
</tbody>
</table>

**Table 4.12:** Estimated background due to the process $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$. The off-peak point results are obtained from the on-peak points by rescaling the cross section as $s'/s$. The quoted upper limits are at 95% CL.

<table>
<thead>
<tr>
<th>year</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\sigma_{\text{gen}}$ (nb)</th>
<th>$\epsilon_{\text{sel}}$</th>
<th>$\epsilon_{\text{sel}} \cdot \epsilon_{\text{trig}}$</th>
<th>$\sigma_{\text{acc}}$ (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>91.25</td>
<td>0.2757</td>
<td>0/9516</td>
<td>0/9516</td>
<td>&lt; 0.087</td>
</tr>
<tr>
<td></td>
<td>89.50</td>
<td>0.2865</td>
<td>0/9516</td>
<td>0/9516</td>
<td>&lt; 0.090</td>
</tr>
<tr>
<td>1993</td>
<td>91.25</td>
<td>0.2757</td>
<td>0/9516</td>
<td>0/9516</td>
<td>&lt; 0.087</td>
</tr>
<tr>
<td></td>
<td>93.10</td>
<td>0.2649</td>
<td>0/9516</td>
<td>0/9516</td>
<td>&lt; 0.084</td>
</tr>
</tbody>
</table>

**Table 4.13:** Estimated background due to the process $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$. The off-peak point results are obtained from the on-peak points by rescaling the cross section as $s'/s$. 

<table>
<thead>
<tr>
<th>$R$</th>
<th>$\frac{\sigma_{\text{gen}}}{\text{BR}} , (\text{pb}) \cdot \text{BR}$</th>
<th>$L_{\text{gen}}$</th>
<th>$\epsilon_{\text{sel}}$</th>
<th>$\sigma_{\text{acc}}$ (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0 \rightarrow \gamma\gamma$</td>
<td>2.20 \cdot (0.9880 \pm 0.0003)</td>
<td>69.0</td>
<td>4/151</td>
<td>0.058 \pm 0.028</td>
</tr>
<tr>
<td>$\eta \rightarrow \gamma\gamma$</td>
<td>15.71 \cdot (0.388 \pm 0.005)</td>
<td>110.0</td>
<td>9/1728</td>
<td>0.031 \pm 0.011</td>
</tr>
<tr>
<td>$\eta' \rightarrow \pi^0\pi^-\gamma$</td>
<td>133.20 \cdot (0.279 \pm 0.023)</td>
<td>77.0</td>
<td>0/10258</td>
<td>&lt; 0.012</td>
</tr>
<tr>
<td>($\text{including } \rho^0\gamma$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta' \rightarrow \eta\pi^0\pi^0$</td>
<td>28.66 \cdot (0.208 \pm 0.013)</td>
<td>76.9</td>
<td>0/2197</td>
<td>&lt; 0.009</td>
</tr>
<tr>
<td>$f_2 \rightarrow \pi^0\pi^0$</td>
<td>427.61 \cdot (0.849 \pm 0.013)</td>
<td>64.7</td>
<td>8/27369</td>
<td>0.106 \pm 0.037</td>
</tr>
<tr>
<td>$a_2 \rightarrow \eta\pi^0$</td>
<td>178.92 \cdot (0.145 \pm 0.012)</td>
<td>181.9</td>
<td>13/32510</td>
<td>0.010 \pm 0.003</td>
</tr>
<tr>
<td>$a_2 \rightarrow \pi^+\pi^-\pi^0$</td>
<td>171.58 \cdot (0.701 \pm 0.027)</td>
<td>189.7</td>
<td>0/32545</td>
<td>&lt; 0.012</td>
</tr>
<tr>
<td>($\text{including } \rho^0\pi^0$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.14:** The estimated background due to resonances produced by photon-photon interactions. The upper limits are set at 95% CL.

detection at small angles. The leptonic final states were generated using the DiAG36 Monte Carlo generator [95]. The results are shown in Tables 4.11, 4.12 and 4.13.

The cross section for the production of resonances is proportional to

$$\sigma(e^+e^- \rightarrow e^+e^-R, R \rightarrow \gamma X) \propto (2J + 1) \frac{\Gamma_{\gamma R}}{M_R^2} \text{Br}(R \rightarrow \gamma X)$$

The resonances considered are listed in Table 4.14. Using the Monte Carlo generator described in [96], 200K events were generated for each of these processes (corresponding to integrated luminosities in excess of 60 pb$^{-1}$), and all events where at least one particle with an energy larger than 500 MeV was generated within $|\cos \theta| < 0.75$ were simulated.

Since the experimental errors on the branching ratios are quite large, and these branching ratios are used as input for the Monte Carlo calculations, it is crucial to ensure that these processes do not constitute a large background.

**Cosmic Rays**

The problem of the contamination of the single-photon sample by out-of-time cosmic rays is peculiar to the $L_3$ detector. It is caused by two constraints coming from the BGO calorimeter: the long integration time, 5 $\mu$s (starting 2 $\mu$s before the beam-crossing) and the absence of any timing information from the calorimeter itself. This implies that an out-of-time cosmic ray, passing through the BGO and emitting a photon can fake a signal event during the time no other subdetector is fully sensitive. However, in the case of TEC some information can be recovered, using the method described in the previous chapter. This method enables to reject cosmic rays which pass through the fiducial volume of TEC by requiring that:

- no out-of-time track is found in TEC.

The remaining cosmic ray events are mainly rejected by the shower-shape cuts.
4.4. Selection of Single Photon Events

<table>
<thead>
<tr>
<th>year</th>
<th>1992</th>
<th>1993</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of events used</td>
<td>406519</td>
<td>326387</td>
</tr>
<tr>
<td>probability (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P(N_{\text{TEC tracks}} &gt; 0)$</td>
<td>0.623 ± 0.012</td>
<td>0.465 ± 0.012</td>
</tr>
<tr>
<td>$P(N_{\text{BGO clusters}} &gt; 0)$</td>
<td>0.112 ± 0.005</td>
<td>0.073 ± 0.005</td>
</tr>
<tr>
<td>$P(N_{\text{MUCH segments}} &gt; 0)$</td>
<td>1.868 ± 0.021</td>
<td>1.929 ± 0.024</td>
</tr>
<tr>
<td>$P(E_{\text{LUMI}} &gt; 5)$</td>
<td>0.566 ± 0.012</td>
<td>0.604 ± 0.014</td>
</tr>
<tr>
<td>$P(E_{\text{ALR}} &gt; 0.1)$</td>
<td>0.257 ± 0.008</td>
<td>0.177 ± 0.007</td>
</tr>
<tr>
<td>$P(E_{\text{HCAL}} &gt; 5)$</td>
<td>0.504 ± 0.011</td>
<td>0.023 ± 0.003</td>
</tr>
<tr>
<td>total</td>
<td>3.658 ± 0.029</td>
<td>3.062 ± 0.030</td>
</tr>
</tbody>
</table>

Table 4.15: The detector occupancy probability as measured from events triggered by the BEAMGATE trigger.

The only subdetector that is sensitive during the entire BGO integration period is the array of the scintillators. Using the scintillators, events where selected which are at least 150 ns out-of-time with respect to the beam-crossing. Next the fact that the FERA uses a different integration time than the “full” readout is used. If an event occurs in coincidence with a bunch crossing, the FERA system measures the same energy as the “full” readout. However, the difference in timing between the two systems causes the ratio between the energy measured to change as a function of the difference of the event time and the moment of the bunch crossing, as shown in Figure 3.19. Selecting those events where the ratio of the energy measured by the FERA system and the “full” readout is “as expected” and which are more than 150 ns out-of-time results in a sample of 3740 events. None of these events survives the rejection cuts. This can be converted into an upper limit of 1.3 (95% CL) on the number of surviving out-of-time events in the selected $\nu\nu\gamma$ sample. In the case of events within 150 ns. of the beam-crossing, most sub detectors are active, and thus such events are easier to reject.

Detector Occupancy

Due to instrumentation effects (i.e. noise) it is possible that the background rejection cuts reject genuine single-photon events. In order to study this probability, and to correct for it, events which are triggered by the BEAMGATE trigger are studied. As these events are triggered by an unbiased trigger, they should form a representative sample of “empty” events, containing the typical level of noise in the detector.

The probability distributions of the variables used in background rejection cuts for the 1992 beam-gate sample is shown in Figure 4.11. Since the number of beam-gate events triggered is proportional to time and not, as is the case with the signal, to the instantaneous luminosity which is delivered by the LEP accelerator, the contribution of the individual beam-gate events to Figure 4.11, has been re-weighted. The weight of a beam-gate event is proportional to the ratio of the total number of beam-gate events in a given run and the corresponding integrated luminosity. This procedure is valid under the approximation that the luminosity does not vary strongly during
a run. Given that the typical duration of a run is short compared to the average LEP beam lifetime, this is a valid assumption.

The noise is incorporated into the Monte Carlo by adding the reconstructed objects in an event triggered by the BEAMGATE trigger to a simulated event, where again the run number assigned to the Monte Carlo event is used to select the run from which to selected a BEAMGATE event. The resulting inefficiencies are shown in Table 4.15.

### 4.5 Summary

Combining the estimated trigger efficiency, the selection efficiency, the background cross sections and the luminosity used, the expected number of events for both the signal and background can be computed. This is shown in Table 4.16. The energy spectra for all events are shown in Figure 4.13, while the spectra for the four individual samples is shown in Figure 4.14. The angular distribution is shown in Figure 4.12.

<table>
<thead>
<tr>
<th>year</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\int L dt(\text{pb}^{-1})$</th>
<th>$N_{\text{obs}}$</th>
<th>$N_{\nu\gamma}^{\text{exp}}(N_\nu = 3)$</th>
<th>$N_{\text{background}}^{\text{exp}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>91.294</td>
<td>19.13</td>
<td>467</td>
<td>346.3</td>
<td>109.2 ± 6.4</td>
</tr>
<tr>
<td>1993</td>
<td>89.453</td>
<td>5.33</td>
<td>60</td>
<td>34.0</td>
<td>27.4 ± 1.5</td>
</tr>
<tr>
<td></td>
<td>91.238</td>
<td>12.38</td>
<td>280</td>
<td>233.2</td>
<td>61.3 ± 3.3</td>
</tr>
<tr>
<td></td>
<td>93.036</td>
<td>7.88</td>
<td>393</td>
<td>390.4</td>
<td>38.5 ± 2.0</td>
</tr>
<tr>
<td>total</td>
<td></td>
<td></td>
<td>1200</td>
<td>1003.9</td>
<td>236.4 ± 7.6</td>
</tr>
</tbody>
</table>

**Table 4.16:** The number of events observed and the expectations from the signal for $N_\nu = 3$ and background.

<table>
<thead>
<tr>
<th>Error source</th>
<th>Estimated Uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trigger efficiency</td>
<td>1.5</td>
</tr>
<tr>
<td>Event selection</td>
<td>1.2</td>
</tr>
<tr>
<td>Background subtraction</td>
<td>1.0</td>
</tr>
<tr>
<td>Cosmic contamination</td>
<td>0.5</td>
</tr>
<tr>
<td>Detector simulation</td>
<td>0.5</td>
</tr>
<tr>
<td>Absolute energy scale</td>
<td>0.3</td>
</tr>
<tr>
<td>$L$ measurement</td>
<td>0.16 (0.5)</td>
</tr>
<tr>
<td>$L$ theory</td>
<td>0.25</td>
</tr>
<tr>
<td>total</td>
<td>2.3</td>
</tr>
</tbody>
</table>

**Table 4.17:** The estimated systematic errors in percent.
Figure 4.11: The probability distributions of the variables used in background rejection for the 1992 beam-gate sample. The sharp drop in probability for energy deposits of more than 30 GeV in the LUMI is due to the fact that off-momentum beam-electrons, which are over-focused by the final-focus-superconducting-quadrupoles are bent into the fiducial volume of luminosity monitor if their energy is 30 GeV or less. In the case of the MUCH track segments, it is not always possible to resolve the left-right ambiguities. In those cases, both ambiguities are counted as a track segment.
Figure 4.12: The angular distribution of all selected candidates.
Figure 4.13: Energy spectrum of all selected events.
Figure 4.14: Energy spectra of the four different center-of-mass energies.
Chapter 5

Results

In this chapter the various quantities of interest are extracted from the event sample selected in the previous chapter. The most important is the invisible width of the Z boson and the number of neutrino generations.

5.1 Fitting Procedure

To extract the values of the underlying parameters, \(p_1, \ldots, p_n\), a binned likelihood fit is performed to the measured energy spectrum of the selected single photon candidates. The likelihood function used in this fit is given by:

\[
\mathcal{L}(p_1, \ldots, p_n) = \prod_i P_{\text{poisson}}\left(N_{i}^{\text{obs}}, M_{i}^{\text{exp}}(p_1, \ldots, p_n)\right)
\]

(5.1)

where \(P_{\text{poisson}}\) is the Poisson probability to observe \(N_{i}^{\text{obs}}\) events in bin \(i\) when, as a function of the parameters \(p_1, \ldots, p_n\), the number of events that is expected in bin \(i\) is given by \(M_{i}^{\text{exp}}(p_1, \ldots, p_n)\). Maximizing the logarithm of the likelihood function defined in Equation 5.1 gives the values of the different parameters \(p_1, \ldots, p_n\). The errors on these parameters are obtained by varying them such that the logarithm of the likelihood function changes by 1/2 from the maximum. The maximization is done using the MINUIT package [97].

The number of events expected in a given bin \(i\) can be expressed as follows:

\[
M_{i}^{\text{exp}}(p_1, \ldots, p_n) = \mathcal{L}e^{\text{cor}}e^{\text{trig}} \times \left[ \sum_{j=\pm e, \pm \gamma, \gamma \gamma} N_{i}^{j, \text{sel}} \frac{\sigma_{j}^{\text{gen}}}{N_{j}^{\text{gen}}} + N_{i}^{\nu \bar{\nu} \gamma, \text{sel}} \frac{\sigma_{\nu \bar{\nu} \gamma}^{\text{gen}}(\bar{p}_1, \ldots, \bar{p}_n)}{N_{\nu \bar{\nu} \gamma}^{\text{gen}}} \sigma_{\nu \bar{\nu} \gamma}^{\text{ana}}(p_1, \ldots, p_n) \right]
\]

where \(\mathcal{L}\) is the integrated luminosity, \(e_i^{\text{trig}}\) the trigger efficiency of the events in bin \(i\), \(e_i^{\text{cor}}\) the corrections to the Monte Carlo selection efficiency described in the previous chapter, \(N_{i}^{j, \text{sel}}\) the number of background events of type \(j\) selected in bin \(i\) and \(\sigma_{j}^{\text{gen}}\) the cross section corresponding to the number \(N_{j}^{\text{gen}}\) of generated events for background type \(j\). In the case of the \(\nu \bar{\nu} \gamma\) Monte Carlo, events where generated for a given set of numerical values \(\bar{p}_1, \ldots, \bar{p}_n\). To determine the number of \(\nu \bar{\nu} \gamma\) events for different values of \(p_1, \ldots, p_n\), the ratio of \(\sigma_{\nu \bar{\nu} \gamma}^{\text{ana}}(p_1, \ldots, p_n)/\sigma_{\nu \bar{\nu} \gamma}^{\text{ana}}(\bar{p}_1, \ldots, \bar{p}_n)\) is
introduced. In this case $\sigma_{e^+e^- \rightarrow \nu\bar{\nu}}^{\text{gen}}$ is the cross section obtained from the NNG010 Monte Carlo generator, and $\sigma_{e^+e^- \rightarrow \nu\bar{\nu}}^{\text{ana}}$ is a parameterization of the single-photon cross section which starts from the Born cross section for the process $e^+e^- \rightarrow \nu\bar{\nu}$. The various weak corrections to the Born cross section are calculated with the ZFITTER package [50, 51, 52, 53, 54]. The cross section thus obtained is convoluted with the resummed radiator function of Nicrosini and Trentadue (Equations 19,21 and 25 of [60]) to obtain an analytical expression for the $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ cross section as a function of $p_1, \ldots, p_n$.

The fit only depends on the parameters through $\sigma_{e^+e^- \rightarrow \nu\bar{\nu}}$, which is a function of $m_Z, \Gamma_Z, g_V, g_A, N_\nu$. These parameters are either constrained to the results from other measurements, or calculated in the framework of the Standard Model (using ZFITTER) as functions of the top, Higgs and Z mass, $\alpha_S$ and, in the case of $\Gamma_Z$, also of the number of neutrino species. External constraints on the parameters $p_1, \ldots, p_n$ are incorporated into the likelihood function by multiplying it by:

$$f(z; n) = \frac{1}{(2\pi)^{n/2}} |V|^{-1/2} e^{-\frac{1}{2} z_i V_{ij}^{-1} z_j}. \tag{5.2}$$

where $V_{ij}$ is the covariance matrix of the external constraints and $z_i = p_i - \bar{p}_i$ the deviation of a constrained parameter $p_i$ from its constrained value $\bar{p}_i$. In this way the measurement of the Z mass and width using the lineshape measurement can be used to increase the sensitivity of the single-photon cross section to the invisible Z width. Similarly, the systematic errors in the measurements are taken into account by introducing several scale factors, one for each dataset. The covariance matrix corresponding to these scale vectors is calculated by performing a transformation of the (diagonal) covariance matrix of the individual sources of systematic errors, taking into account the various correlations between the different samples.

Finally, the trigger efficiency $\varepsilon_i^{\text{trig}}$ for the events selected in bin $i$ is obtained by calculating the average trigger efficiency of all events $k$ selected in bin $i$:

$$\varepsilon_i^{\text{trig}} = \frac{1}{N_i} \sum_{k=1}^{N_i} \varepsilon_k^{\text{trig}}$$

where $\varepsilon_k^{\text{trig}}$ is the trigger efficiency calculated for a given event $k$ using the simulation of the trigger and $N_i$ is the total number of events selected in bin $i$.

### 5.2 Determination of $N_\nu$

Two fits are performed to extract $N_\nu$. The first fit uses the standard model framework to calculate $\Gamma_Z$ as a function of $N_\nu$. The other parameters which are needed for this fit, are $m_Z$, which is taken from the $L_3$ lineshape measurement [34], $m_t$, from the CDF and D0 measurements [3, 4] and the coupling constants $\alpha_{\text{EM}}$ and $\alpha_S$[33]. The dependence of the result of the fit on the assumed values and their errors is shown in Table 5.1. The result of the fit is:

$$N_\nu = 3.05 \pm 0.13 \pm 0.07 \tag{5.3}$$

where the first error is statistical and the second one systematic.

In the second fit, to increase the sensitivity of the single photon cross section to the number of neutrino species, the total width of the Z is fixed to the value of the $L_3$ lineshape measurement [34],
5.2. Determination of $N_\nu$

<table>
<thead>
<tr>
<th>parameter</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$\Delta N_\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_Z$</td>
<td>91.195</td>
<td>$\pm 0.009$</td>
<td>0.015</td>
</tr>
<tr>
<td>$\alpha_S$</td>
<td>0.126</td>
<td>$\pm 0.006$</td>
<td>0.006</td>
</tr>
<tr>
<td>$m_H$</td>
<td>300</td>
<td>$\pm 700$</td>
<td>0.007</td>
</tr>
<tr>
<td>$m_t$</td>
<td>180</td>
<td>$\pm 13$</td>
<td>0.005</td>
</tr>
<tr>
<td>total</td>
<td></td>
<td></td>
<td>0.018</td>
</tr>
</tbody>
</table>

**Table 5.1:** Dependence of $N_\nu$ on the constraints used in the Standard Model fit.

$\Gamma_Z = 2.494 \pm 0.010$ GeV. With this constraint the measurement of $\sigma_{\nu\nu}$ becomes a determination of the branching ratio of $Z \rightarrow \nu\bar{\nu}$. The dependence of the result on the assumed values of $m_Z$, $\Gamma_Z$, $\alpha_S$, $m_H$ and $m_t$ is shown in Table 5.2. The result of this fit yields:

$$N_\nu = 3.01 \pm 0.10 \pm 0.07$$

which corresponds to

$$\Gamma_{\text{inv}} = 502 \pm 17 \pm 12 \text{ MeV}$$

Again, the first error is statistical and the second error systematic. The cross sections as determined from the fit are listed in Table 5.3. They are shown, together with the expectations for two, three and four generations of neutrinos is in Figure 5.1. The difference of the cross sections from the Standard Model prediction assuming three generations is shown in Figure 5.2.

<table>
<thead>
<tr>
<th>parameter</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$\Delta N_\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_Z$ (GeV)</td>
<td>91.195</td>
<td>$\pm 0.009$</td>
<td>0.011</td>
</tr>
<tr>
<td>$\Gamma_Z$ (GeV)</td>
<td>2.494</td>
<td>$\pm 0.010$</td>
<td>0.012</td>
</tr>
<tr>
<td>$\alpha_S$</td>
<td>0.126</td>
<td>$\pm 0.006$</td>
<td>0.006</td>
</tr>
<tr>
<td>$m_H$ (GeV)</td>
<td>300</td>
<td>$\pm 700$</td>
<td>0.007</td>
</tr>
<tr>
<td>$m_t$ (GeV)</td>
<td>180</td>
<td>$\pm 13$</td>
<td>0.004</td>
</tr>
<tr>
<td>total</td>
<td></td>
<td></td>
<td>0.017</td>
</tr>
</tbody>
</table>

**Table 5.2:** Dependence of $N_\nu$ on the constraints used in the fit.

This should be compared with the measurement from the lineshape method which gives $N_\nu = 2.97 \pm 0.03$ or $\Gamma_{\text{inv}} = 499.5 \pm 5.3$ MeV

**The $W^\pm$ Contribution**

The $W^\pm$ contribution to the cross section can be determined from the fact that it has a different functional dependence on $\sqrt{s}$. This implies that by comparing the measurements at the various center-of-mass values the $W^\pm$ contribution can be determined. However, the $W^\pm$ contribution is
Table 5.3: The cross section for the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ at the four different center-of-mass energies.

<table>
<thead>
<tr>
<th>year</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\sigma_{\nu\bar{\nu}\gamma}^{N_\nu=3}$ (pb)</th>
<th>$\sigma^{\text{fit}}$ (pb)</th>
<th>$N_\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>91.294</td>
<td>30.4</td>
<td>$33.5 \pm 1.9$</td>
<td>$3.28 \pm 0.18$</td>
</tr>
<tr>
<td></td>
<td>89.453</td>
<td>9.9</td>
<td>$11.3 \pm 2.2$</td>
<td>$3.39 \pm 0.66$</td>
</tr>
<tr>
<td>1993</td>
<td>91.238</td>
<td>29.4</td>
<td>$28.9 \pm 2.2$</td>
<td>$2.96 \pm 0.21$</td>
</tr>
<tr>
<td></td>
<td>93.036</td>
<td>77.4</td>
<td>$71.7 \pm 3.9$</td>
<td>$2.79 \pm 0.15$</td>
</tr>
</tbody>
</table>

Figure 5.1: Fit to the number of neutrinos.
strongly correlated to $N_{\nu}$. To avoid this correlation $N_{\nu}$ is fixed to three, and the $W^\pm$ contribution is allowed to vary by multiplying the $Z$-$W^\pm$ interference term with a factor $f_W$ and the $W^\pm$ contribution with $f_W^R$. The following result for $f_W$ is obtained:

$$f_W = 0.2 \pm 0.6 \pm 0.4$$

(5.6)

which is compatible with the expected value of 1.

### 5.2.1 Comparison with other measurements

The analysis of the single photon signal at LEP has been performed by the OPAL [98, 99], $L_3$ [100, 101] and ALEPH [102] collaborations. Their published results, together with the measurements described in Chapter 2 and the one described in this thesis are summarized in Table 5.4.
<table>
<thead>
<tr>
<th>experiment</th>
<th>data</th>
<th>$N_\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Photon Counting at PEP, PETRA and TRISTAN</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MARK J</td>
<td>$&lt; 26$ (--)</td>
<td></td>
</tr>
<tr>
<td>MAC</td>
<td>$&lt; 17$ (--)</td>
<td></td>
</tr>
<tr>
<td>VENUS</td>
<td>$&lt; 11$ (15)</td>
<td></td>
</tr>
<tr>
<td>CELLO</td>
<td>$&lt; 8.7$ (11.3)</td>
<td></td>
</tr>
<tr>
<td>ASP</td>
<td>$&lt; 7.9$ (10.4)</td>
<td></td>
</tr>
<tr>
<td>Combined</td>
<td>$&lt; 3.9$ (4.8)</td>
<td></td>
</tr>
<tr>
<td>Single Photon Counting at LEP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OPAL 1990</td>
<td>$3.0 \pm 0.4 \pm 0.2$</td>
<td></td>
</tr>
<tr>
<td>$L_3$ 1990</td>
<td>$3.24 \pm 0.46 \pm 0.22$</td>
<td></td>
</tr>
<tr>
<td>$L_3$ 1990–1991</td>
<td>$3.14 \pm 0.24 \pm 0.12$</td>
<td></td>
</tr>
<tr>
<td>ALEPH 1990–1991</td>
<td>$2.68 \pm 0.2 \pm 0.2$</td>
<td></td>
</tr>
<tr>
<td>OPAL 1990–1992</td>
<td>$3.23 \pm 0.16 \pm 0.10$</td>
<td></td>
</tr>
<tr>
<td>$L_3$ (this thesis) 1992–1993</td>
<td>$3.01 \pm 0.10 \pm 0.07$</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.4: Comparison with other measurements. Upper limits are set at 90% (95%) CL.

### 5.3 Other Results

The measurement of the production rate of single photon events can not only be used to determine the number of neutrino flavours, but also for several other measurements. Whereas the average energy of the photons produced in $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ is small compared to the beam energy, this is not the case for several other production mechanisms. For instance, a non-zero magnetic moment of the neutrino or a possible $ZZ\gamma$ interaction would lead to an enhancement of the photon spectrum at energies close to the beam energy. The limits obtained on these possible sources of energetic single photon events are discussed in [103, 104]. Furthermore, events containing a single photon are a possible experimental signature for the production of the supersymmetric partners of the $\gamma$, $Z$ and $H$, the so-called neutralinos. The limits obtained on the branching ratio of $Z$ into neutralinos and on the excluded parameter space are reported in [105].

**Determinaton of the Weak Isospin of the Neutrino**

Given as input the number of neutrino species, any measurement of the invisible $Z$ width can be converted into a measurement of the weak isospin of the neutrino:

$$T_3^\nu = \pm \sqrt{\frac{\Gamma_{\text{inv}} \sqrt{2\pi}}{G_F m_Z^2 N_\nu}} = \pm 0.502 \pm 0.021.$$  \hspace{1cm} (5.7)

using Equation 5.5 and assuming that $N_\nu = 3$. This is comparable to the result of the CHARM experiment [106] which has determined $T_3^{\nu_e} = 0.502 \pm 0.017$. Combining their data with the LAMPF data [107], the following can be derived: $T_3^{\nu_e} = 0.51 \pm 0.07$. 
5.3. Other Results

Additional Types of Neutrinos

The determination of $N_\nu$ (Equation 5.4) can be converted into an upper limit, $N_\nu < 3.22$ at 95% CL. Assuming three generations of massless neutrinos, this upper limit can be translated into limits on the parameters of additional processes which would contribute to the production of single photon events. Two possible processes are the production of an additional heavy neutrino and the production of the supersymmetric partner of the neutrino, the sneutrino.

An additional heavy neutrino with standard couplings to the Z contributes $\frac{1}{2} \beta \left( 1 + \beta^2 / 3 \right)$, where $\beta$ is the velocity of the neutrino, to $N_\nu$ [41]. Using the upper limit on $N_\nu$, combined with the average recoil mass of 89.34 GeV, this can be ruled out (at 95% CL) for neutrino masses less than 42.8 GeV.

Similarly, light sneutrinos would contribute $\frac{1}{2} \beta^3$ to $N_\nu$. The above limit rules out three generations of mass degenerate sneutrinos with masses less than 44.1 GeV (at 95% CL) or one generation of sneutrino with masses less than 40.1 GeV (at 95% CL).

Exotic Visible Decays

By combining the above measurement of the invisible width with the measurement of the leptonic, hadronic and the total width Z from the $L_3$ lineshape measurement, it is possible to determine the possibility for the Z to decay into final states which are neither accounted for in the analysis of the visible decay channels nor in the present measurement. Denoting this width by $\Gamma_{\text{exotic}}$, we have:

$$\Gamma_{\text{exotic}} = \Gamma_Z - (\Gamma_{\text{had}} + (3 + \delta_\tau)\Gamma_\ell + \Gamma_{\text{inv}}) < 40.5 \text{ MeV (95\% CL)}$$

(5.8)

where $\delta_\tau$ is a small correction to take into account the mass of the $\tau$.

Non-resonant Sources of Single Photon events

The production of single photons by a source which is independent of the center-of-mass energy can be determined by fixing the number of neutrino flavours to three, $N_\nu = 3$ and adding a term $\sigma_C$ which is independent of $\sqrt{s}$ to the cross section $\sigma_{e^+e^-\rightarrow\nu\bar{\nu}}$.

$$\sigma_C = 1.0 \pm 1.1 \pm 0.7 \text{ pb}$$

(5.9)

or $\sigma_C < 3.3 \text{ pb at 95\% CL}$.

Monochromatic Photon Production

A narrow resonance X, which could be produced for instance through the reaction $e^+e^- \rightarrow Z \rightarrow X\gamma$, would show up as a monochromatic source of photons. An example of this is the decay $Z \rightarrow H^0\gamma$, which is expected in the Standard Model at the one loop level if $m_{H^0} < m_Z$. If in addition the resonance X would escape undetected, or decay into neutrinos, it would contribute to the production of single photon events. Several models allow for this possibility [108]. As the produced photons are monochromatic, such a resonance can be identified by scanning the mass of the system recoiling against the photon. In this case the recoil mass is given by the reduced center-of-mass energy, $m_{\text{recoil}} = \sqrt{s(1 - 2E_\gamma/\sqrt{s})}$.
The recoil mass distribution for the selected single photon sample is shown in Figure 5.3. Limits on the production cross section times the branching ratio are calculated by scanning the recoil mass distribution, and gathering all events which are within three times the detector resolution (as given by Equation 3.2) of a given recoil mass. The upper limit for this recoil mass is then determined from the excess of events found in the data with respect to the expected number of events from \(e^+e^- \rightarrow \nu\bar{\nu}\gamma\), \(e^+e^- \rightarrow e^+e^-\gamma\) and \(e^+e^- \rightarrow 4\gamma\). For a given confidence level \(1 - \epsilon\), the upper limit \(N\) on the signal, when \(n_0\) events are observed in the presence of an expected background of \(\mu_B\) events, is obtained by solving:

\[
\epsilon = \frac{e^{-(\mu_B+N)\sum_{n=0}^{n_0} \frac{(\mu_B+N)^n}{n!}}}{e^{-\mu_B \sum_{n=0}^{n_0} \frac{n^n}{n!}}} \quad (5.10)
\]

After determining the upper limit on the number of events, \(N\) is transformed into an upper limit on the cross section. Next, assuming an isotropic angular distribution, this limit is extrapolated to the full solid angle. This upper limit on the production cross section of \(X\gamma\) times the branching ratio of \(X\) into undetected particles is plotted as a function of the mass in Figure 5.3.
Figure 5.3: The recoil mass spectrum and the 95% CL limit on the production cross section of Xγ. No events are observed nor expected below a recoil mass of 30 GeV; the limit extends to zero mass.
Appendix A

Measurement of Luminosity

To convert the measured number of events into a cross section measurement, the integrated luminosity $\int L \, dt$ delivered to the experiment has to be determined. Although the luminosity is determined by the LEP machine parameters (according to Equation 3.1), these parameters are not known to the required accuracy. Furthermore, they are difficult to correlate with detector specific inefficiencies, such as the dead-time introduced by the readout.

Instead, to determine the luminosity, the event rate of a physics process with a well known cross section is measured. For this purpose, a reaction is needed which satisfies the following criteria:

- It does not depend on the theory under study.
- It has a high cross section.
- It is calculable with a small (theoretical) uncertainty.
- It has a clear experimental signature.

In the case of $e^+e^-$ colliders the small angle elastic scattering of the beam electrons and positrons fulfills all the above criteria. This process is dominated (for small scattering angles) by the $t$-channel photon exchange and thus shows virtually no dependence on the $Z$ parameters: the largest $Z$ contribution is due to the $\gamma t - Z t$ interference. This term contributes less than 0.3% to the total cross section. In lowest order the differential cross section for this process can be written as [109]:

$$\frac{d\sigma^0}{d\theta}(s) = \frac{32\pi\alpha^2}{s} \frac{1}{\theta^3}$$  \hspace{1cm} (A.1)

Integrating Equation A.1 from $\theta_{\text{min}}$ to $\theta_{\text{max}}$ gives the accepted cross section in the angular region $\theta_{\text{min}} < \theta < \theta_{\text{max}}$:

$$\sigma^0(s) = \frac{16\pi\alpha^2}{s} \left( \frac{1}{\theta_{\text{min}}^2} - \frac{1}{\theta_{\text{max}}^2} \right)$$  \hspace{1cm} (A.2)

Substituting the values of $\theta_{\text{min}}$ and $\theta_{\text{max}}$ of the $L_3$ luminosity monitor, Equation A.2 yields an accepted cross section (at $\sqrt{s} = m_Z$) on the order of 90 nb$^{-1}$. This is three times larger than the largest weak cross section under consideration, $e^+e^- \rightarrow$ hadrons. The steep dependence of the accepted cross section on the inner fiducial volume cut $\theta_{\text{min}}$ does imply an experimental challenge.
A.1 Trigger Efficiency

Starting from a sample of selected Bhabha events, the trigger efficiency for these events can be determined by comparing the single-tag (ST) and double-tag (DT) triggers. Assuming the efficiency $\epsilon$ to generate a trigger signal is the same for both calorimeters and it is not correlated, the probability to observe an event triggered by ST trigger and not by the DT trigger can be written as:

$$P(\text{ST and NOT DT}) = \epsilon (1 - \epsilon) + (1 - \epsilon) \epsilon$$ \hspace{1cm} (A.3)

From this the probability to observe a DT trigger can be calculated:

$$P(\text{DT}) = \epsilon^2$$
$$= \frac{1}{2} \left( 1 - P(\text{ST and NOT DT}) + \sqrt{1 - 2P(\text{ST and NOT DT})} \right)$$
$$\approx 1 - P(\text{ST and NOT DT})$$

In a sample of 949758 selected Bhabha events, 49199 events where found which where triggered by the ST trigger. Of these events, 49198 events where also triggered by the DT trigger. From this the trigger efficiency for Bhabha events is determined to be larger than 99.990% at 95% CL.

A.2 Event Selection

The event selection is based on the reconstructed energy and impact coordinates of the incident particles in the luminosity monitor. Both the energy and the location are reconstructed by fitting the observed energy sharing amongst the crystals to the known average shape of electromagnetic showers. If more than one shower is reconstructed in either of the two detectors, the clusters are ordered by energy and the vectorial sum of the cluster is taken until the difference between the summed energies and the beam energy is minimal.

To select the events, the following criteria are imposed:

- Energy:
  $$\max(E_{+z}, E_{-z}) > 0.8E_{\text{beam}}$$
  $$\min(E_{+z}, E_{-z}) > 0.4E_{\text{beam}}$$

  The asymmetric energy cut retains most of the radiative Bhabha events and also reduces the sensitivity to dead crystals. The systematic uncertainty due to this cut is estimated to be 0.2%.

- Acoplanarity:
  $$170^\circ < \text{mod}(\phi_{+z} - \phi_{-z}, 360) < 190^\circ$$

  This cut reduces the background due to random coincidences of off-momentum electrons. The estimated systematic uncertainty due to this cut is 0.1%.

To correct for the remaining background, the sidebands (as indicated in Figure A.1) of the acoplanarity distribution (after requiring the energy on neither side is within 5% of the beam...
energy) are used. The size of this background is \( O(0.1\%) \). Given the small size of the background, the systematic error introduced by the background subtraction is negligible.

- Fiducial volume:

\[
84.4 \text{ mm} < R_{+z}(R_{-z}) < 176.2 \text{ mm}
\]

\[
|\phi - 90^\circ| > 11.25^\circ
\]

\[
|\phi - 270^\circ| > 11.25^\circ
\]

The reconstructed impact points must be at least one crystal away from the edge of the detector. The boundary of fiducial volume is indicated in Figure 3.11. To reduce the sensitivity to detector misalignments and beam offsets, two samples are selected: for one sample the radial cut is performed on the \(+z\) side and for the other on the \(-z\) side. The average of the two samples is used to calculate the final luminosity. Assuming perfect knowledge of the absolute value of the inner edge, this cut introduces a systematic error of 0.2%. The error due to the uncertainty in the absolute value of the inner edge and other geometrical uncertainties is discussed in Section A.3.

An example of a selected event is shown in Figure 3.11. The result of the event selection is summarized in Table A.1. The total systematic error introduced by the above event selection is estimated to be 0.3%.

### Table A.1: The result of the event selection.

<table>
<thead>
<tr>
<th>sample</th>
<th>#triggers</th>
<th>events selected</th>
<th>( \mathcal{L} (\text{pb}^{-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>1.8M</td>
<td>490K</td>
<td>5.8</td>
</tr>
<tr>
<td>1991</td>
<td>2.0M</td>
<td>626K</td>
<td>13.2</td>
</tr>
<tr>
<td>1992</td>
<td>2.9M</td>
<td>1039K</td>
<td>22.8</td>
</tr>
<tr>
<td>BABAMC</td>
<td>490K</td>
<td>138K</td>
<td>–</td>
</tr>
<tr>
<td>BHLUMI</td>
<td>790K</td>
<td>220K</td>
<td>–</td>
</tr>
</tbody>
</table>

### A.3 Determination of the Accepted Cross Section

The largest problem in the determination of the accepted cross section is the determination of the boundaries of the fiducial volume. The main problem are the known differences between the ideal detector geometry as implemented in the detector simulation program and the actual detector:

1. The knowledge of the lateral shower depth. As the crystals do not point towards the interaction point but are aligned along the beamline, the average showerdepth is needed to convert the radial position into a polar angle. The depth of the showers is determined from the Monte Carlo simulation of the detector, and it introduces an uncertainty of 2 mm on the \( z \) coordinate.

2. The gap between the detector half modules. This gap is known with an accuracy of 0.3 mm.
Figure A.1: The distributions of the variables used in the event selection.
\[
\begin{array}{l|c|c|c}
\text{year} & 1990 & 1991 & 1992 \\
\hline
\sigma_{\text{acc}}^{\text{real}} / \sigma_{\text{acc}}^{\text{ideal}} & 0.9334 & 0.9947 & 0.9987 \\
\end{array}
\]

Table A.2: Ratio of the accepted cross sections for both the ideal geometry and the geometry as determined from survey. The large difference between 1990 and the ideal geometry is due to a shift in the position of the detector along the beamline.

3. The inner radius of the sensitive detector volume. This radius is known within 0.2 mm.

To evaluate the effect of these uncertainties, 50,000 Monte Carlo events are generated. Using a fast simulation program, the difference in acceptance due to changes in the detector geometry are determined. The result of this comparison is shown in Table A.2. By repeating this procedure whilst varying the geometry according to the uncertainty in the parameters describing the geometry, the systematic uncertainty is derived. In total a systematic error of 0.4% was assigned to the accepted cross section due to the uncertainty in the knowledge of the detector geometry.

To determine the accepted cross section for the ideal detector geometry, 1.3M events are generated with both BHLUMI v2.01 [110, 111, 112] and BABAMC [113, 114]. The events were generated in the angular region 20 < \theta < 200 mrad, to allow for feedthrough. The total cross section in this region as estimated by BHLUMI v2.01 is 321.6 nb. After the generation of the events, the detector response is simulated and the same cuts are applied as on the actual data. The accepted cross section is then given by:

\[
\sigma_{\text{acc}}^{\text{real}} = \frac{\sigma_{\text{acc}}^{\text{real}}}{\sigma_{\text{acc}}^{\text{ideal}}} \frac{N_{\text{acc}}}{N_{\text{gen}}} \sigma_{\text{Bhlumi}}^{\text{gen}}
\]

(A.4)

The resulting cross sections are quoted in Table A.3. As there is no charge determination in this polar region of the detector, a small (0.02\%) contribution from the process \(e^+e^- \rightarrow \gamma\gamma(\gamma)\) is added. Furthermore, the cross section is corrected for the small \(\gamma-Z\) interference term which is not included in the BHLUMI v2.01 generator. This correction is calculated using the BABAMC generator.

The theoretical uncertainty on the cross section calculated by BHLUMI v2.01 is estimated to be less than 0.25\%.

A.4 Conclusion

Adding all sources of systematic uncertainties in quadrature, the final systematic uncertainty in the determination of the luminosity delivered to \(L_3\) is 0.6\%, well below the design goal of 1\%. However, this uncertainty was one of the limiting factors in the measurement of the hadronic cross section and thus the determination of the number of neutrino species. The main source, the uncertainty in the geometry of the detector, is significantly reduced with the introduction of the SLUM in 1993. A detailed description of the SLUM and the improved determination of the luminosity with the SLUM are the subject of [74, 75].
<table>
<thead>
<tr>
<th>year</th>
<th>1990</th>
<th>1991</th>
<th>1992</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma^{\text{real}}_{\text{acc}}) (nb.)</td>
<td>84.73</td>
<td>90.29</td>
<td>90.66</td>
</tr>
<tr>
<td>After prescaling</td>
<td>84.73</td>
<td>45.15</td>
<td>45.33</td>
</tr>
</tbody>
</table>

**Table A.3:** The estimated accepted cross section of the observed Bhabha events for the L3 luminosity monitor. During part of the 1991 and 1992 running periods, the luminosity trigger was prescaled by a factor two, i.e. only every second event satisfying the trigger conditions was read out and written to tape.

<table>
<thead>
<tr>
<th></th>
<th>Systematic uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event Selection</td>
<td>0.3</td>
</tr>
<tr>
<td>Geometry</td>
<td>0.4</td>
</tr>
<tr>
<td>BHLUMI v2.01</td>
<td>0.25</td>
</tr>
<tr>
<td>MC statistics</td>
<td>0.18</td>
</tr>
</tbody>
</table>

**Table A.4:** Summary of the various sources contributing to the systematic uncertainty in the accepted cross section.
Bibliography


Summary

The interactions of elementary particles are successfully described by the Standard Model of Electroweak Interactions. Although the model predicts that elementary particles can be grouped into families of leptons and quarks, it does not predict the number of these families. With the recent discovery of the sixth type of quark at Fermilab, the question arises whether more than the known three families exist. In this thesis an experimental answer to this question is described.

An overview of the Standard Model is presented in Chapter One. The earlier astrophysical and collider measurements which gave upper limits on the number of families are summarized in Chapter Two. At an $e^+e^-$ collider which operates at center-of-mass energies close to the mass of the $Z$ boson, several methods are available to determine the number of families. In particular the production of "Single Photon" events, i.e. the process

$$e^+e^- \rightarrow \gamma + \text{missing energy}$$

is considered in detail: in the Standard Model this reaction is dominated by the radiative production of a $Z$ boson and its subsequent decay into neutrino pairs. The predicted cross section for this process depends almost linearly on the number of different types of neutrino pairs into which the $Z$ can decay. A measurement of this cross section can therefore be used to determine the number of neutrino types. As the Standard Model requires one neutrino species per family of fermions, the number of neutrino species can be translated into a determination of the number of fermion families.

The measurement of the cross section of the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ has been performed at the LEP $e^+e^-$ collider using the data gathered by the $L_3$ detector during the 1992 and 1993 running periods. The experimental apparatus and the collider are described in Chapter Three. Special attention is paid to the electromagnetic calorimeter and the trigger system, two important ingredients of the measurement. The photon, which is the sole detectable particle in this type of events, has typically an energy which is small compared to the LEP center-of-mass energy. The $L_3$ detector is capable of triggering on events containing a single, isolated photon with energies as low as one GeV.

The selection of the candidate events is discussed in Chapter Four. Various sources of background have to be taken into account. The size of the dominant background, the radiative Bhabha process, is set by the the hermeticity of the detector. The hermeticity is estimated from a sample of "Single Electron" events.

In Chapter Five it is shown how the number of neutrino species is obtained from a binned likelihood fit to the energy spectrum of the observed photon. The number of events expected in each bin is calculated as a function of the number of neutrino generations, $N_\nu$. By maximizing the
probability, \( N_\nu \) is determined to be

\[
N_\nu = 3.01 \pm 0.10 \text{ (stat)} \pm 0.07 \text{ (sys)}.
\]

This result excludes a fourth generation of (massless) neutrinos.

The possible contribution of other particles to the production of the “Single Photon” final state has been used to constrain the parameter space available for these new particles. The measurements rule out (at 95% confidence level) a massive fourth neutrino with standard couplings to the Z and a mass of less than 42.8 GeV. In addition, the existence of one generation of the proposed supersymmetric partner of the neutrino, the sneutrino, has been excluded at 95% confidence level if its mass is less than 40.1 GeV. Three generations of mass degenerate sneutrinos are ruled out if their mass is less than 44.1 GeV.
Samenvatting

De interacties van elementaire deeltjes worden met succes beschreven door het Standaard Model van de Electrozwakke Wisselwerkingen. Hoewel het model voorspelt dat elementaire deeltjes in families van leptonen en quarks kunnen worden georganiseerd, voorspelt het model niet het aantal van deze families. De recente ontdekking van het zesde soort quark bij Fermilab leidt tot de vraag of er meer dan de drie bekende families zijn. Een experimenteel antwoord op deze vraag wordt in dit proefschrift beschreven.

Een overzicht van het Standaard Model wordt gepresenteerd in Hoofdstuk 1. De eerdere astrofysische en versneller metingen, die resulteerden in een een bovengrens op het aantal families zijn samengevat in Hoofdstuk 2. Bij een $e^+e^-\to\gamma+\text{ontbrekende energie}$ versneller met een zwaartepuntsenergie die in de buurt ligt van de massa van het $Z$ boson zijn er diverse methoden beschikbaar om het aantal families te bepalen. In dit proefschrift wordt de productie van "Eén Foton" gebeurtenissen, ofwel:

\[
e^+e^- \to \gamma + \text{ontbrekende energie}
\]

is beschreven in detail. In het Standaard Model wordt deze toestand voornamelijk veroorzaakt door de productie van een $Z$ boson tezamen met een foton en het erop volgende verval van het $Z$ boson in een neutrino-antineutrino paar. De voorspelde werkzame doorsnede voor dit process is vrijwel lineair afhankelijk van het aantal soorten neutrinos waarin het $Z$ boson kan vervallen. Een meting van deze werkzame doorsnede kan dus gebruikt worden om het aantal soorten neutrinos te bepalen. Aangezien het Standaard Model precies één soort neutrino per familie vereist, kan dit aantal neutrino soorten worden vertaald in een bepaling van het aantal families van fermionen.

De meting van de werkzame doorsnede voor $e^+e^-\to\nu\bar{\nu}\gamma$ gebeurtenissen is uitgevoerd bij de LEP $e^+e^-\to\nu\bar{\nu}$ versneller met behulp van de gegevens die door het $L_3$ experiment zijn verzameld in 1992 en 1993. De detector en de versneller worden beschreven in Hoofdstuk 3. Bijzondere aandacht wordt besteed aan de electromagnetische calorimeter en de trigger systeem, twee onderdelen die van groot belang zijn bij deze meting. Het foton, het enige waarneembare deeltje in deze gebeurtenissen, heeft een energie die klein is vergeleken met de zwaartepunts energie van de LEP versneller. Een enkel geïsoleerd foton met een energie van 1 GeV is voldoende om de uitlezing van de detector te starten.

De selectie van de gebeurtenissen is beschreven in Hoofdstuk 4. Verschillende achtergrondprocessen moeten in rekening worden gebracht. De grootte van de belangrijkste achtergrond, het Bhabha proces, wordt bepaald door de hermeticiteit van de detector. Deze hermeticiteit is experimenteel bepaald met behulp van "Eén Electron" gebeurtenissen.

In Hoofdstuk 5 wordt aangegeven hoe het aantal neutrinosoorten is bepaald met behulp van een waarschijnlijkheidsoptimalisatie. Het aantal gebeurtenissen dat in elk energie interval wordt verwacht is bepaald als functie van het aantal soorten neutrinos, $N_\nu$. De meest waarschijnlijke
waarde van $N_\nu$ voor het aantal gemeten gebeurtenissen is:

$$N_\nu = 3.01 \pm 0.10 \text{ (stat)} \pm 0.07 \text{ (sys)}.$$ 

Dit resultaat sluit een vierde generatie van (massaloze) neutrinos uit.

De mogelijke bijdragen van andere processen tot de productie van "Één Foton" gebeurtenissen is gebruikt om de parameterruimte die beschikbaar is voor deze processen af te bakenen. De meting sluit een zwaar vierde soort neutrino uit als de massa van dit soort neutrino kleiner is dan 42.8 GeV. Verder is het bestaan van een generatie van de voorgestelde supersymmetriche partner van het neutrino, het sneutrino, uitgesloten als de sneutrino massa kleiner is dan 40.1 GeV. Als er drie types sneutrino zouden zijn met een ontaarde massa, is het uitgesloten dat deze massa kleiner is dan 44.1 GeV.
Acknowledgments

It is my pleasure to thank all members of the $L_3$ collaboration. Without their combined effort the analysis presented in this thesis would not have been possible. I hope that this thesis in turn will contribute to the success of the experiment.

First I would like to thank my advisors Frits Ernê and Frank Linde for their confidence and guidance. Sharing an office at CERN with Frank was quite an experience.

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