Elliptic Flow Measurement
at ALICE

Meting van elliptische stroming
met ALICE

/met een samenvatting in het nederlands/

Proefschrift

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deep down the rabbit hole
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Introduction

“ [...] Thus grew the tale of Wonderland: Thus slowly, one by one, Its quaint events were hammered out - And now the tale is done, And home we steer, a merry crew, Beneath the setting sun [...]”
Lewis Carrol

The work presented in this thesis is dedicated to the physics of high-energy heavy ion collisions, which offer a very rich playground for studying fundamental properties of strongly interacting matter, such as quarks and gluons, under extreme conditions of energy and density.

From the experimental point of view, quarks are not observed as ‘free’ particles, since the strong force keeps them confined into hadrons. Hadrons are classified as mesons, which are made of quark-antiquark pairs, and baryons, which are made of three quarks. The most common baryons are the proton and the neutrons, which are found in the atomic nuclei of all the stable matter in the universe.

The Quantum Chromo-Dynamics (QCD) successfully accounts for fundamental properties observed in high energy experiments and can correctly describe the spectra and the quark configuration of all known hadrons. However, why ‘confinement’ happens in first place is still an open question of QCD, and the existence of a more crowded configurations of quarks and gluons, who behaves almost as free particles inside a confined volume, is not excluded. In relativistic heavy ion collision there’s a glimpse of the creation of such a state, known as Quark-Gluon Plasma (QGP).

There are experimental evidences of the QGP, mainly based on the collective behavior of the system created in the collision, in particular on its evolution which seems to be well described by relativistic hydrodynamic. A key observable to study the thermodynamic properties of the QGP is the ‘elliptic flow’, i.e. the azimuthal anisotropy in the momenta distribution of the particles produced in the collision, which can be connected to the Equation of State of the system.

ALICE is a dedicated heavy ion detector for the reconstruction of lead-lead collisions at the Large Hadron Collider, being built between the years 2002 and 2008 at CERN. The main purpose of the ALICE experiment is to study the properties of the QGP at collision energies never achieved before.

Unfortunately, the present thesis has been developed when LHC was still under construction, and therefore the entire work presented here is based on simulations.
Efforts have been devoted to both the development of parametrizations of the main observables in Pb-Pb collisions at LHC energy, and the implementation of analysis tools interfaced to the ALICE environment.

The present thesis should be seen as a first example of physics analysis with ALICE, to point out the possible sources of uncertainty in this kind of measurement. More accurate ways to perform the flow analysis should and will be developed in the exciting future of the experiment. Fig.1 shows a full 3D simulation of a heavy ion event, as it will be ‘seen’ by the ALICE detector.

Figure 1. 3D display of a simulated collision in ALICE (picture generated with the Event Display in AliRoot).

The thesis is organized as follows. Chapter 1 gives an overview of the theoretical background of heavy ion collisions, focusing the attention on the concept of ‘anisotropic flow’ and on the extrapolation of $v_2$ to LHC energy. Chapter 2 presents the ALICE detector, and also the software framework used to simulate and analyse the data. In chapter 3 the event plane analysis method is introduced, together with its implementation in the ALICE environment, a brief overview of other analysis methods is also given. Chapter 4 is dedicated to the feasibility of the event plane analysis, considering the presence of non-flow effects as expected at LHC. Chapter 5 shows a complete analysis of simulated data with full detector reconstruction, and studies the possible sources of uncertainty. Finally, chapter 6 draws some conclusions and gives an outlook about how to improve the measurement.
Chapter 1

Heavy Ion Collisions & Anisotropic Flow

Heavy ion collisions are meant to study the physics of nuclear matter under extreme conditions of energy and density, to characterize the fundamental properties of strongly interacting fields.

The major issue of this thesis is the measurement of Elliptic Flow, an observable which provides a test of the initial Equation of State of the produced medium in a domain where perturbative QCD does not apply.

The first section of this chapter will present the general understanding of heavy ion collisions, from the experimental observables to their interpretations and the underlying theory (see sec.1.1). The following section (sec.1.2) will describe the initial condition of the system created in the collision and its description in terms of a Glauber model. Section 1.3 is dedicated to the medium properties, i.e. what has been observed so far by existing experiments and the description of anisotropic flow and in term of a Fourier decomposition. The observed scaling of $v_2$ will be also discussed, and some extrapolations of $v_2$ to LHC energies will be made. The last section (sec.1.4) will briefly introduce the concept of non-flow effects, postponing their detailed study to chapter 4 and 5.

1.1 A hot, dense, nearly perfect liquid

The strong interaction between quarks is described by the Quantum Chromo Dynamics (QCD) in which the color degrees of freedom are introduced.

One of the characteristic features of QCD is that the coupling strength increases with the distance between the interacting quarks. In fact, the interaction becomes so strong that in ordinary matter quarks are permanently confined to colorless hadrons. At large momentum transfer, however, the running coupling constant $\alpha_s(q^2)$ decreases logarithmically, leading to a weak coupling of quarks and gluons called asymptotic freedom. In this regime perturbative QCD (pQCD) can be applied, lead-
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In (approximated) analytical solutions which have been widely tested in high energy physics experiments.

Over the last years, more and more attention has been devoted to the question about how a strongly interacting medium will respond to a dramatic increase of the energy density. Considerable progress has been made by numerically solving the QCD field equation on a space-time lattice, the lattice-gauge calculations. These calculations, which have been refined in recent years [1–4], show a phase transition from ordinary matter to a new state where the color degrees of freedom are released. This new state is called Quark Gluon Plasma (QGP) and is expected to occur at a temperature of about \(175\) MeV and an energy density of \(0.7\) GeV/fm\(^3\) (fig.1.1(a)). Lattice QCD calculations also provide quantitative information about the pressure of the system around its phase transition to this color deconfined state (fig.1.1(b)).

![Figure 1.1](image_url) (a) Lattice QCD results (for 2 and 3 quark flavors) for energy density and pressure as a function of the temperature around the QGP phase transition [2]. The rapid increase of the energy density around \(T_c\) indicates a rapid increase of the degrees of freedom in the system. (b) Pressure vs temperature from lattice calculations, showing that the pressure changes smoothly during the phase transition [4].

In the Big Bang theory of cosmology, the universe undergoes this phase transition at approximately \(10\) \(\mu\)sec after the Big Bang [5]. This phase transition is believed to be now accessible by laboratory experiments. By colliding atomic nuclei at extremely high energy, it is possible to achieve an energy density high enough for the QGP phase transition to take place.

The Relativistic Heavy Ion Collider (RHIC), which has been operational for the last 7 years at the Brookhaven National Laboratory, can collide gold nuclei up to 200 AGeV obtaining an energy density of 10 GeV/fm\(^3\) [6]. The energy density will be about one order of magnitude higher at the upcoming Large Hadron Collider (LHC) at CERN, where lead nuclei will collide at \(\sqrt{s_{NN}} = 5.5\) TeV.

Fig.1.2(a) shows schematically the evolution of the system after the collision. The system is created at \(t = 0\), and after a pre-equilibrium stage (the detailed physics behind this stage is still unclear) the system enters the QGP phase, and keeps on expanding. When the system is cooled down to the chemical freeze-out, the cons-
tituents hadronize into colorless hadrons, but they still elastically interact until the final decoupling and kinetic freeze-out. Then the system is dilute enough to proceed its expansion as a free streaming of particles.

Figure 1.2. (a) Schematic view of the collision in 2 dimensional space-time. (b) Pressure versus energy density for an ideal gas, a hadron resonances gas, and a QGP with a phase transition.

The creation of the hot and dense phase by the RHIC experiments and the discovery that this state seems to behave as a nearly perfect fluid is considered the major physics discovery of 2005 by the American Institute of Physics [5]. This discovery was based on the collective behavior of the produced medium, especially observed in its anisotropic flow. However, it is still heavily disputed [7, 8].

‘Anisotropic Flow’ is a phenomenological term used to describes the collective evolution of the system, observed as an overall pattern which correlates the momenta of the final state particles. This pattern is believed to develop due to the initial asymmetry of the collision and is conserved by the presence of multiple interactions between the system constituents before the kinetic freeze-out, indicating that the system created in a heavy ion collision is definitely different from a superposition of proton-proton collisions.

The underlying physics of anisotropic flow is usually described in terms of a pressure gradient, which is intimately related to the Equation of State of the system (see fig.1.2(b) where this relation is given for the EoS of an ideal gas, of a hadron resonance gas, and for an EoS with phase transition). For this reason the study of flow provides a sensitive tool to characterize the strongly interacting system created in the heavy ion collision.

Condensed matter experiments [10] also show that in a compressed gas of fermions the pressure and energy density reach their maximum in the center of the the

---

1When the collision is not central and the interaction volume is shaped as an almond (see gig.1.3).

2Beside anisotropic flow, there are many other aspects which clearly distinguish AA from pp collisions, from dN/dy to strangeness enhancement to J/Ψ suppression. See the reference [9] for a more comprehensive overview.
system and decrease toward the outside, until they reach a common value close to zero on the system boundary. The different size of the system with respect to the azimuthal coordinate causes the pressure gradient to be larger where the distance between the center and the boundary is shorter, and this azimuthal dependence of the pressure gradient drives the evolution of the system toward an anisotropic expansion.

Fig. 1.3 gives a 3D representation of a non-central collision. The reaction plane is defined by the beam direction and by the direction of the impact parameter $b$ (the $z$ and $x$ axes respectively). The almond in the middle of the figure is the reaction volume where the participating nucleons take part to the interaction, the two half spheres represent the spectator nucleons, flying away from each other more or less along the beam direction.

Figure 1.3. Schematic 3D picture of a non-central collision, showing the reaction plane, the almond shape of the interaction volume (participants) and the spectator nucleons flying away in opposite directions. The coordinate system of the event has the $x$ axes oriented in the direction of the impact parameter, the $z$ axis along the beam, and the $y$ axis completes the cartesian system.

The observed flow mainly consist of a combination of two different patterns: a radial expansion (affecting the thermal spectra of final state particles) and a non-isotropic one (affecting the spatial orientation of particles momenta). The latter arising from the initial spatial asymmetry of the reaction volume.

In non-central collisions the azimuthal distribution of final state particles turns out to be highly anisotropic, therefore it is possible to determine an event plane $\Psi$ with respect to which the angular distribution of particles momenta shows a strong $\cos(n(\phi - \Psi))$ dependence, called anisotropic flow.

The impact parameter is the distance between the centers of the two colliding nuclei, usually called $b$.\footnote{The impact parameter is the distance between the centers of the two colliding nuclei, usually called $b$.}
The standard way to characterize anisotropic flow uses a Fourier expansion of the Lorentz invariant distribution of the outgoing particles [11]:

\[ \frac{E d^3 N}{dp^3} = \frac{1}{2\pi} \frac{d^2 N}{p_T dp_T dy} \left( 1 + \sum_{n=1}^{+\infty} v_n(p_T, y) \cos[n(\phi - \Psi_R)] \right), \tag{1.1} \]

where $\phi$ is the azimuthal angle of each particle and $\Psi_R$ is the reaction plane angle, both measured in the laboratory frame. The first and second coefficient of the expansion, $v_1$ and $v_2$, are called directed and elliptic flow, respectively.

Elliptic flow at mid-rapidity ($\eta \sim 0$) is particularly interesting because it reflects the asymmetry of the region where most of the new particles are produced.

In the current interpretation, flow originates from the rescattering between constituents and the initial spatial eccentricity of the overlap region. The number of interactions (and rescattering) is larger in more central collision while the spatial eccentricity is more pronounced in peripheral collisions. The interplay between these two ingredients dominates the trend of elliptic flow versus centrality.

The huge potential of the measurement of elliptic flow at RHIC (and the fact that this is really ‘first day physics’) leads to the development described in this thesis:

- development of the analysis based on the event plane method, which is a quite straightforward and versatile formalism (see section 3.2),
- implementation of the analysis code within the complex ALICE analysis framework (see section 3.3).

To show the limits of applicability of this approach, study has been done with Monte Carlo simulations for different particle multiplicities and different magnitudes of elliptic flow. To show how different models can be tested by the ALICE experiment, extrapolations of $v_2$ up to LHC energies have been developed.

The experimental effort of many years on the determination of the elliptic flow is summarized in a "universal scaling" of $v_2$ (shown in fig. 1.4), i.e. all existing results can be represented on the same axis [12–14]: $v_2$ is divided by the initial eccentricity of the reaction volume $\epsilon$ (in order to distinguish dynamics from purely geometrical effects) and the ratio $v_2/\epsilon$ is plotted versus the rapidity density of the overlap region, defined as the charged particle multiplicity at mid-rapidity $dN_{ch}/dy$ divided by the transverse area of the overlap $S$ (see section 1.2).

What is surprising is that all experimental data show an almost linear scaling behavior, suggesting a common driving force in the development of elliptic flow. A very recent work [15] suggests that either the QGP fraction of the system or the systems lifetime might drive this scaling. The plot also shows that only at the highest RHIC energy the data are compatible with ideal relativistic hydrodynamic calculations, which are believed to hold also at even higher energies (e.g. LHC).

The systematic uncertainties on fig.1.4 are under intense study nowadays, including the uncertainty on the measured $v_2$ that arise from the presence of non-flow correlations [12] (azimuthal correlations not related to the reaction plane, see sec.1.4) and from the effects of flow fluctuations [16].
Figure 1.4. Elliptic Flow is divided by the eccentricity of the reaction volume to distinguish dynamics from purely geometrical effects, and plotted versus the entropy density of the overlap region [14] (the \( x \) and \( y \) axis have been enlarged to cover the LHC energy range). The hydrodynamic predictions for two different EoS are shown (for an ideal gas EoS I and for a QGP with phase transition EoS Q, see sec.1.3.2), and also a linear fit of the data (see sec.1.3.1).

LHC will provide data points up to a much higher energy, where also an increase in the multiplicity is expected, which will enhance the detectability of the elliptic flow. Moreover, at the higher initial energy density, the system will probably stay longer in the partonic stage, where all the elliptic flow will be generated.

Based on the extrapolation described in sec.1.3, the most central Pb-Pb collision at 5.5 ATeV can be represented on the \( x \) axis of fig.1.4 at \( \frac{1}{S} \frac{dN_{ch}}{dy} \simeq 60 - 80 \), depending on the definition of the transverse area (see sec.1.2).

To extrapolate from the existing data to the magnitude of \( v_2 \) to be expected at LHC energies, two ingredients are leading:

- the geometry of the initial system (eccentricity \( \epsilon \), transverse area \( S \)), calculated with a Glauber model of heavy ion collisions (see section 1.2),

- the EoS of the produced medium, which is needed to transform the initial spatial asymmetry of the system to the momentum anisotropy observed in the final state (see section 1.3).

Two models have been considered to describe the properties of the produced medium and to estimate the final state momentum anisotropy with respect to the initial eccentricity of the reaction volume:

The Microscopic Transport (cascade) Model [17] describes the time evolution of the hadronic/partonic phase by solving a transport equation derived from
1.2 Initial Conditions

In this model, collectivity depends on the interaction cross section between the constituents and the main assumption is that the mean free path is comparable to the system size ($\lambda \gg 0$). Calculations are done in a perturbative way, giving first correction to collisionless limit (free streaming). This approach, also called Low Density Limit approximation, is described in sec.1.3.1.

The Relativistic Hydrodynamic Model [18] describes the evolution of the system (before the kinetic freeze-out) as the expansion of volume elements of a relativistic fluid, the main assumption is that the mean free path is much smaller than the system size ($\lambda \sim 0$). This concept appeared the first time in 1953 in a paper by Landau [19]. The system is described in terms of (classic) macroscopic quantities, such as pressure and energy density, local thermal equilibrium is assumed (thermodynamic) and an Equation of State is required. The $v_2$ coefficient in this approach comes out to be proportional to the speed of sound in the medium times the spatial eccentricity (see sec.1.3.2).

1.2 Initial Conditions

The usual tool to describe the initial state of a heavy ion collision is a Glauber model [20]. For a given pair of colliding nuclei with atomic number $A$ and $B$ (usually called target and projectile), the Glauber model provides a way to calculate the number of nucleon-nucleon interactions and the geometry of the overlap region as a function of the impact parameter $b$ (see fig.1.5).

Glauber calculations can be either optical [20], where nucleon positions are approximated by a smooth distribution (the number of participants is proportional to the geometrical overlap of the two nuclear density functions), or Monte Carlo, where the nucleons are point-like centers randomly distributed inside the nucleus and the probability of each interaction is calculated inside the overlap region proportionally to the nucleon-nucleon cross section [21]. The two approaches lead to similar results over a large range of impact parameters, being different only for the most central and most peripheral collisions [21]. For extremely peripheral collisions ($b \geq 2R_A$) the optical Glauber approach does not provide a good parametrization of the physics of the process, which is then dominated by the random occurrence of single nucleon-nucleon interactions.

However, the study of fluctuations in a Glauber Monte Carlo approach was beyond the purpose of the present thesis (see sec.1.2.1). The extrapolations developed in the following sections are done using the optical Glauber approach.

The Glauber calculation starts with a parametrization of the spatial distribution of the colliding nuclei (defined as the probability to find a nucleon at the radius $r$), which is given by a Wood-Saxon profile:

$$p_A(r) = \frac{\rho_0}{e^{(r-R_A)/\xi} + 1},$$  \hspace{1cm} (1.2)
where \( R_A \) is the radius of the nuclei with atomic mass \( A \) and atomic number \( Z \) (the same radius is taken for protons and neutrons), \( \xi \) is the nuclear surface diffuseness, and \( \rho_0 \) is a normalization factor. The distribution of protons and neutrons are normalized separately in such a way that \( \int \rho_p(r) d\vec{r} = Z \) and \( \int \rho_n(r) d\vec{r} = A - Z \).

In the present calculations the colliding nuclei are \(^{208}\text{Pb}\) and the parameters of the nuclear density distribution (eq.1.2) have been taken from literature (nuclear data [22]): the radius is \( R_A = 6.621 \pm 0.02 \text{ fm} \), and the nuclear surface diffuseness \( \xi = 0.551 \pm 0.01 \text{ fm} \).

The nuclear thickness function is defined as the optical path through the nucleus along the beam direction (\( z \)):

\[
T_A(x, y) = \int_{-\infty}^{\infty} \rho_A(x, y, z) dz .
\] (1.3)

The transverse coordinates for a Glauber calculation are shown in fig.1.5, the \( x \) is oriented in the direction of the impact parameter \( b \) and \( y \) is the direction perpendicular to it.

![Figure 1.5](https://example.com/figure1.5.png)

**Figure 1.5.** Coordinate system of a non-central collision, used for the Glauber calculation. The impact parameter \( b \) is the distance between the centers of the two nuclei.

In non-central collisions, the probability of each binary nucleon-nucleon interaction in the transverse plane is given by the product of the thickness functions of the two nuclei \( A \) (transversally shifted by the impact parameter \( b \)) times the total inelastic nucleon-nucleon cross section \( \sigma_{NN} \):

\[
P_{BC}(x, y; b) = T_A(x + b/2, y)T_B(x - b/2, y)\sigma_{NN} .
\] (1.4)

The energy dependence of Glauber calculations is determined by the nucleon-nucleon inelastic cross section \( \sigma_{NN} (\sqrt{s}) \), which is extrapolated from existing \( pp \)
1.2 Initial Conditions

and $p\bar{p}$ data including the highest energy Tevatron p-p collision (see the current Review of Particle Physics [23] or the PDG website [24]).

According to the value $^4$ used in the ALICE PPR [25] the nucleon-nucleon inelastic cross section, for Pb-Pb at a collision energy $\sqrt{s_{NN}} = 5.5$ TeV, has been set to $\sigma_{NN} = 60$ mb.

However, the main ingredients of the extrapolations given in sec.1.3 are not very sensitive to the chosen value of the cross section (see below).

![Figure 1.6](image)

**Figure 1.6.** Transverse picture of the density distribution of wounded nucleons $N_{WN}$ and binary collision $N_{BC}$ (arbitrary scale) in the optical Glauber calculations, for an impact parameter $b = 7$ fm.

The total number of binary nucleon-nucleon collisions is obtained integrating over the transverse plane (fig.1.6(b)):

$$N_{BC}(b) = \int T_A(x + b/2, y) T_B(x - b/2, y) \sigma_{NN} dxdy, \quad (1.5)$$

where the $x$ axis is oriented in the direction of the impact parameter $b$ and $y$ in the perpendicular one.

The number of ‘wounded nucleons’ is defined as the number of nucleons participating to the production process with at least one collision, and is given by the integral [27] (fig.1.6(a)):

$$N_{WN}(b) = \int T_A(x + b/2, y) \left( 1 - \left( 1 - \frac{\sigma_{NN} T_B(x - b/2, y)}{B} \right)^B \right) dx dy +$$

$$+ T_B(x - b/2, y) \left( 1 - \left( 1 - \frac{\sigma_{NN} T_A(x + b/2, y)}{A} \right)^A \right) dxdy. \quad (1.6)$$

$^4$The value given in the ALICE PPR (pag.1583 of [25]) is $\sigma_{NN} = 57$ mb. Other references quote a higher cross section at the same collision energy [26], however, the actual value of the nucleon-nucleon inelastic cross section at LHC remains an open issue.
For the symmetry of the system (Pb+Pb), in our calculation $T_A = T_B$.

The two panels of fig.1.6 show the density distributions of wounded nucleons and binary collisions. Depending on the choice of one or the other, the impact parameter dependence of the geometrical quantities (such as spatial eccentricity and transverse area) changes significantly (see fig.1.8).

Figure 1.7 shows the impact parameter dependence of the number of binary collision ($N_{BC}$) and the number of participants to the reaction (wounded nucleons $N_{WN}$). As we see, only the number of binary collisions strongly depends on the choice of the nucleon-nucleon cross section, while the number of participant is affected at a level of 1%.

The impact parameter range has been limited to $0 < b < 15$ fm (see fig.1.7), where the upper limit is consistent with almost no interactions $\langle N_{BC} \rangle \simeq 0$.

The Glauber model also provides the geometry of the overlap region, parameterized by the spatial eccentricity and the transverse area of the overlap (both used in fig.1.4). There are different ways to define these geometrical quantities, depending on the chosen distribution (weighting function) used to compute the averages over the $x$ and $y$ coordinates, e.g. geometric overlap, wounded nucleons or binary collisions (the reference [28] gives a few examples of the procedure). Another option attempted in more recent developments makes use of the Color Glass Condensate (CGC) initial conditions, which leads to larger eccentricities and therefore higher flow values [29, 30].

The spatial eccentricity $\epsilon$ is defined in terms of the RMS of the distribution projected on the $x$ and $y$ axes ($\sigma_x = \langle x^2 \rangle - \langle x \rangle^2$, $\sigma_y = \langle y^2 \rangle - \langle y \rangle^2$):

$$
\epsilon \equiv \frac{\sigma_y^2 - \sigma_x^2}{\sigma_x^2 + \sigma_y^2}.
$$ (1.7)
The transverse area of the overlap $S$ (also used in fig.1.4) is defined as:

$$S \equiv \pi \sigma_x \sigma_y,$$

(1.8)

where $\sigma_x$ is the variance along the direction of the impact parameter $b$, and $\sigma_y$ the variance on the perpendicular to it.

Figure 1.8. Impact parameter dependence of eccentricity $\epsilon$ (left) and transverse area $S$ of the overlap region (right), for both the density of wounded nucleons (+) and binary collisions ($\times$). The plots are produced with the same Glauber calculation of fig.1.7 (i.e. 30 steps in impact parameter from 0 to 15 fm).

Fig.1.8 shows the impact parameter dependence of the collision geometry ($\epsilon$ and $S$) obtained from the optical Glauber calculation, of the distribution of wounded nucleons and of binary collisions (the integral along the transverse plane of eq.1.6 and eq.1.5 respectively). Both the distributions have a physical meaning, and the choice of one or the other will be discussed in sec.1.3.

The initial energy density in the transverse plane depends only on the thickness functions $T_A$ and $T_B$ (eq.1.3) and is defined as:

$$E(x, y) = f(T_A(x, y), T_B(x, y)),$$

(1.9)

where $f$ is a function that depends on the initial assumptions (different approaches can be found in literature [31]).

Early thermalization is assumed, giving all the available energy thermalized in a Lorentz-contracted volume [32].

### 1.2.1 Eccentricity in Glauber MC

Recent developments, mainly due to the fluctuations observed in $v_2$ [33], suggest a different definition of eccentricity: the eccentricity of the participants $\epsilon_{\text{part}}$, in which the fluctuations in the position of the participants (wounded nucleons) is explicitly taken into account [34].

---

5The obvious assumption is that elliptic flow follows the initial eccentricity of the system.
The total number of collisions does not just depend on the geometrical overlap of the two nuclei, but has a probability proportional to $\sigma_{NN}$. Due to this, the spatial distribution of the nucleons that are actually participating to the reaction may have a slightly different shape than the geometrical overlap. The effect is much more pronounced in peripheral collisions, where the overlap region (and its thickness) is small and the randomness of binary processes dominates.

Therefore, the ellipse created by participating nucleons may be rotated with respect to the geometrical overlap, so that the minor axis is not oriented along the impact parameter vector $\vec{b}$ (see fig.1.9).

![Figure 1.9](image)

Figure 1.9. Schematic view of a collision of two identical nuclei in the transverse plane. The $x$ and $y$ axes are drawn in the standard way, with $x$ oriented in the direction of the impact parameter $\vec{b}$. The circles indicate the positions of wounded nucleons (participants). Due to fluctuations, the interaction region is shifted and tilted with respect to the standard $(x, y)$ frame, leading to a spatial distribution which is better approximated by an ellipse along the $x'$ and $y'$ axes.

The eccentricity of the participants $\epsilon_{\text{part}}$ can be defined with respect to the standard $x$ and $y$ axes (or any other cartesian system in the transverse plane) as:

$$
\epsilon_{\text{part}} \equiv \sqrt{\frac{(\sigma_y^2 - \sigma_x^2)^2 + 4(\sigma_{xy}^2)^2}{\sigma_x^2 + \sigma_y^2}},
$$

(1.10)

where $\sigma_{xy} = \langle xy \rangle - \langle x \rangle \langle y \rangle$. Note that this expression is reduced to eq.1.7 if the elliptic distribution of the participants has the same direction as the geometrical overlap. Eccentricity fluctuations should also be taken into account, especially in very peripheral events [16, 33, 35]. However, including these effect would have required a Glauber Monte Carlo approach and the development of software tools.
that were not yet available for ALICE. Therefore, in the following, $\epsilon$ always refers to the geometrical eccentricity as defined by eq.1.7.

## 1.3 Medium Properties

In the final state particle spectra, both a thermal and an anisotropic collective component can be observed. The first is due to the thermal motion of the particles in the hot dense system created in the collision, the second is a radial boost due to the asymmetry of the system.

### A thermalized medium

The thermal motion of the system’s constituents is observed in the transverse momentum spectra of the final state particles. The low $p_T$ component of the observed $dN/dp_T$ distribution approximately follows a Boltzmann blackbody spectrum [8]:

$$
\left. \frac{dN}{dp_T} \right|_{y \rightarrow 0} \propto \frac{1}{e^{\frac{p_T - \mu_B}{T_{\text{app}}} \pm 1}},
$$

where $\mu_B$ is the baryon chemical potential, a parameter which accounts for the energy needed to produce the hadrons.

The radial boost, due to the expansion of the system (responsible for the blue shift in the final spectra), can be incorporated into the phenomenological parameter apparent temperature ($T_{\text{app}}$ [36]), which is expressed in terms of the transverse flow velocity $v_T$ [37]:

$$
T_{\text{app}} = T_{\text{f.o.}} + \frac{1}{2} m \langle v_T \rangle^2.
$$

The freeze-out temperature ($T_{\text{f.o.}}$) quantifies the thermal motion of the constituents just before the kinetic freeze-out, when the system decouples and all the particles propagate as free streaming. The transverse flow contributes to the temperature proportionally to the mass of the particle (heavier particles moving at a fixed velocity carry a higher momenta), therefore a multiple fit of identified particle spectra allows to disentangle the two components [38, 39].

However, the distribution of eq.1.11 does not properly reproduce the long tail at high $p_T$ observed in experimental data, which is dominated by non-thermal processes (hard scattering, recombination [40]). To better reproduce the data, the input distribution used in the GeVSim simulations presented in chapter 5 is a phenomenological functional form inspired by the Levy distribution [41] (see sec.5.3 for the details).

### Particle ratios

Assuming that the hadronization process occurs in an equilibrated system composed of non-interacting hadron resonances, hadron yields can be described by a thermal
distribution calculated in a grand canonical ensemble [42]. The relative abundances of hadron species are interpreted in term of statistical hadronization [43, 44]:

\[
n_i = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2}{e^{(E_i(p) - \mu_i)/T_{ch}} \pm 1} dp,
\]

where \(E_i = \sqrt{p_i^2 + m_i^2}\), and \(\mu_i\) is the chemical potential for the creation of a particle \(i = \pi, K, \ldots\). Eq.1.13 gives the yields at the freeze-out time, short-lived particles and resonances need to be taken into account separately to correctly reproduce the particle ratios observed in the final state.

The success of the above distribution in describing RHIC data supports the assumption that the system is in local thermal equilibrium when the hadronization process takes place, at the chemical freeze-out. The chemical freeze-out represents the end of inelastic processes changing the chemical composition of the system and it occurs at an earlier time than the kinetic freeze-out, which is driven by elastic processes (and thus at a hotter temperature \(T_{ch} \simeq 177\) MeV [45]).

The observed thermal \(dN/dp_T\) spectra and the success of statistical hadronization in describing the observed particle yields support the assumption that the system is in (local) thermal equilibrium, and if the system is in local equilibrium then it could show hydrodynamic behavior [8].

In non-central collision the \(dN/d\phi\) distribution is azimuthally anisotropic (see also sec.1.1). This is a phenomenon that has been observed in heavy ion experiments over a wide range of energy, an event plane can be determined on an event basis defining the favorite direction of radiated particles.

The theoretical efforts in interpreting the data collected at RHIC contributed to a robust description of the system in term of relativistic hydrodynamic [18]. However questions are still open, especially with respect to the initial conditions.

Other descriptions are the low density limit approximation (LDL) [17], and numerical implementations of RQMD (Relativistic Quantum Molecular Dynamic) [46–48]. These theoretical models to describe flow also provide the tools to make extrapolations to LHC energies. Extrapolations based on LDL and relativistic hydrodynamic will be described in the following sub-sections (see sec.1.3.1 and 1.3.2). The RQMD model has not been considered in the present thesis because it already produces too little flow with respect to RHIC data.

The charged particle multiplicity is calculated from the number of wounded nucleons using a saturation model for particle production (see sec.1.3.3 for the details).

### 1.3.1 Low Density Limit

Figure 1.4 shows the linear increase of \(v_2/\epsilon\) with the multiplicity (entropy) density \(\frac{1}{S} \frac{dN}{dy}\). A simple extrapolation to LHC can be done by performing a linear fit on the existing data in a range where they appear to be linear (i.e. from \(\frac{1}{S} \frac{dN}{dy} \gtrsim 5\)). The
1.3 Medium Properties

fit is justified by the LDL model (see eq.1.14), but is extended much above the ‘low density’ domain.

The Low Density Limit is a perturbative approximation which describes the first correction to free streaming [36] [49]. It is valid when the particle mean free paths ($\lambda_i \simeq 1/(\sigma \rho)$, where $\sigma_i$ is the cross section of particle $i = \pi, K, ...$ and $\rho$ is the particle density) are larger than the transverse dimensions of the overlap zone.

Under this assumption, particles can escape from the collision zone almost without interacting, and the system behavior is close to free streaming (collisionless limit). The first order correction to free streaming is calculated from particle collisions. Particles initially are produced azimuthally symmetric in momentum space but not in coordinate space, and the interactions with comovers produce an azimuthally asymmetric momentum distribution because of the (azimuthal) spatial asymmetry of the source.

The starting point is the initial condition at formation time. Subsequent scattering between comovers are described by inserting a collision term into to the free-streaming distribution function. The first order correction is calculated as the deviation from cylindrical symmetry, which directly leads to the magnitude of elliptic flow $v_2$ for the particle species $i = \pi, K, p, ...$ (see the reference [36]):

$$v_i^2 = \frac{\epsilon}{16\pi\sigma_x\sigma_y} \sum_j \langle v^{ij} \sigma^{ij}_m \rangle \frac{dN_j}{dy} \frac{v_{i\perp}^2}{v_{i\perp}^2 + \langle v_{j\perp}^2 \rangle},$$  \hspace{1cm} (1.14)

where $v_i$ is the velocity of the particle, $v_j$ of the scatterer (what is used is the transverse velocity $v_{\perp}$ w.r.t. the reaction plane), and $v^{ij}$ is their relative velocity. The averages $\langle .. \rangle$ are taken over the scattered momenta $p_j$. Since it is the momentum transfered in the collisions that deforms the momentum distribution, $\sigma^{ij}_m$ is the *momentum transport cross section* (i.e. the cross section averaged over energy and scattering angle [36]).

From eq.1.14 the elliptic flow is proportional to the eccentricity of the overlap region $\epsilon$, and it vanishes for an azimuthally symmetric source (central collisions).

The integrated value of $v_2$ of this linear extrapolation is calculated as:

$$v_2 \simeq A_{LDL} \frac{\epsilon}{S} \frac{dN}{dy} + B_{LDL}.$$

(1.15)

with $\epsilon$ and $S$ given by the density of nucleons participating to the reaction in a Glauber calculation (eq.1.7 and 1.8 respectively).

The coefficient $A_{LDL} = 0.00614 \pm 0.0001$ and the constant $B_{LDL} = 0.051 \pm 0.002$ are obtained from a linear fit of the highest energy RHIC data 6 (see fig.1.4). The fit has been restricted to only one set of data points because the scaling is not perfect (see the discussion in sec.1.1).

6The fit only includes data points from Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV (i.e. $N_{\text{pts}} = 9$, $\chi^2/\text{DoF} \simeq 9$).
1.3.2 Relativistic Hydrodynamics

Relativistic hydrodynamics is a classical calculation, which describes the system in terms of volume elements of a relativistic fluid [18, 31]. Each ‘fluid cell’ $x$ is characterized by its energy momentum tensor:

$$T^{\mu\nu} = (\mathcal{E}(x) + p(x)) u^\mu(x) u^\nu(x) - p(x) g^{\mu\nu},$$  \hspace{1cm} (1.16)

where $p$ and $\mathcal{E}$ are the pressure and the energy density of the fluid cell, and $u^\mu = \gamma(1,v_x,v_y,v_z)$ is the flow velocity.

The evolution is ruled by conservation laws. Local conservation of energy and momentum are expressed by the equations:

$$\partial_\mu T^{\mu\nu} = 0, \quad (\nu = 0, 1, 2, 3).$$ \hspace{1cm} (1.17)

Since the fluid is made of quanta, it carries few conserved charges $N_i$ (such as electric charge, baryon number, strangeness, etc.), with charge density $n_i(x)$ $(i = 1, ..., M)$ corresponding to charge currents densities $j_i^\mu(x) = n_i(x) u^\mu(x)$. Charge conservation is expressed by the equations:

$$\partial_\mu j_i^\mu = 0, \quad (i = 1, ..., M).$$ \hspace{1cm} (1.18)

Hydrodynamics implies the concept of thermodynamics, in particular an equation of state (EoS) of the system is needed to close the system of differential equations. The above picture provides a set of $4 + M$ differential equations, involving $5 + M$ undetermined fields:

- 3 independent components of the flow velocity $u^\mu(x)$,
- the energy density $\mathcal{E}(x)$,
- the pressure $p(x)$,
- and the $M$ conserved charge densities.

This set of equations is closed by an equation of state which relates the local thermodynamic quantities $p$ and $\mathcal{E}$ (see fig.1.2(b)).

The EoS of strongly interacting particles can, in principle, be calculated by lattice-QCD (see fig.1.1). However those calculations are technically difficult and still lead to large uncertainties [4]. An alternative is to model the system of nuclear matter as a non-interacting gas of hadronic resonances [50].

If the relaxation rate is not fast enough to ensure an almost instantaneous thermalization, the energy momentum tensor and charge current densities must be generalized including dissipative effects (e.g. shear viscosity [51]). The goal of this approach is to provide a more accurate description of heavy-ion collisions by taking into account the deviation from an ideal fluid. First order viscous corrections have been derived [52, 53], however the actual value of the viscosity in a hot QGP is still
1.3 Medium Properties

controversial. A universal lower bound on the viscosity to entropy ratio has been proposed in connection with black-hole physics: \( \eta/s > \hbar/4\pi \) [54], while a recent study of elliptic flow at RHIC suggests that the magnitude of viscous correction is significantly higher than this lower bound \( \eta/s \simeq 0.11 \) to \( 0.19 \hbar \) [55].

The ratio between elliptic flow and the spatial eccentricity of the overlap parameterizes the speed at which a perturbation is propagated through the system. In the hydrodynamic picture, this ratio is proportional to the square of the velocity of sound in the medium: \( v_2/\varepsilon \propto c_s^2 \). The velocity of sound is defined as: \( c_s^2 \equiv \frac{dP}{d\varepsilon} \).

Different equation of states lead to different relations between the pressure and the energy density (see fig.1.1), and therefore to different values of \( c_s \) [56].

The spatial anisotropy appears in the early stage of the collision and it is self-quenching (see fig.1.10), however the elliptic flow \( v_2 \) is conserved during the whole evolution of the system, and therefore carries information on the initial condition [18].

A simple extrapolation can be made by assuming the ratio \( v_2/\varepsilon \) to be constant with respect to the centrality, which is approximately true up to very peripheral collisions [56].

A lower limit on \( v_2 \) is given by the equation of state of a quark gluon plasma which undergoes a soft transition to the hadronic phase (EOS Q). The value of \( c_s^2 \) has been chosen at the limit of non-relativistic regime \( c_s = \sqrt{0.22} \) [57]. According to the initial condition used in [57], the eccentricity is calculated from the entropy density distribution which is proportional to the density of wounded nucleons. Therefore the values of \( v_2 \) versus centrality are obtained scaling \( c_s^2 \) by the eccentricity of the wounded nucleon distribution \( (v_2(b) = 0.22 \times \varepsilon_{WN}(b)) \).

For the upper limit, the equation of state of an ideal gas of massless fermions has been chosen (EOS I), giving \( P = \varepsilon/3 \) (and \( c_s = \sqrt{1/3} \)) [31, 56]. In this case, the eccentricity has been calculated from the density of binary collisions \( \varepsilon_{BC} \) (proportional to the initial energy density [18]), which has on average the same magnitude of \( \varepsilon_{WN}(b) \) but a slightly different centrality dependence (see fig.1.8(a)). Elliptic flow versus centrality is obtained as \( v_2(b) = \frac{1}{3} \times \varepsilon_{BC}(b) \).

\( b = 7 \text{ fm} \)

\( \text{Time} \)

\( 0 \text{ fm/c} \) \hspace{1cm} \( 2 \text{ fm/c} \) \hspace{1cm} \( 4 \text{ fm/c} \) \hspace{1cm} \( 6 \text{ fm/c} \) \hspace{1cm} \( 8 \text{ fm/c} \)

\( x (\text{fm}) \)

\( y (\text{fm}) \)

Figure 1.10. Time evolution of the transverse energy density profile from hydrodynamic calculations [18]. As the system expands anisotropically, the initial eccentricity vanishes.
1.3.3 Charged Multiplicity

In order to estimate the centrality dependence of elliptic flow in Pb-Pb at $\sqrt{s} = 5.5$ ATeV, the charged multiplicity at mid-rapidity ($dN_{ch}/dy|_{y=0}$) must be also extrapolated.

The final particle multiplicity is calculated from the number of participants to the reaction (wounded nucleons) [58], which dominates the ‘soft’ component of the final spectra [59].

The model chosen in the present thesis and is a saturation model for particle production in the soft $p_T$ region, extrapolated from lepton-proton collisions (see the reference [58]).

![Figure 1.11](image)

**Figure 1.11.** (a) Number of produced particle per wounded nucleon with respect to the number of wounded nucleons $N_{WN}$. The LHC prediction (upper band) is calculated from eq.1.19, for comparison, the fit of three different sets of RHIC data is also shown ($\sqrt{s_{NN}} = 19.6, 130$ and $200$ GeV [58]). (b) Charged multiplicity per unit rapidity with respect to the impact parameter at LHC (Pb-Pb at $\sqrt{s_{NN}} = 5.5$ TeV). Values are calculated using the number of wounded nucleons $N_{WN}$, obtained from the Glauber calculations (eq.1.6), into equation 1.19.

The main assumption of this approach is the geometric scaling of hadrons produced at small $x_{Bj}$ observed in lepton-proton data at HERA. Over a wide range of Bjorken $x$ and $Q^2$, the $x$ dependence can be expressed by the saturation momentum $Q_{sat}^2(x)$, so that the data are described in terms of a single variable $Q^2/Q_{sat}^2(x)$. By adding a nuclear dependence in the definition of the saturation momentum, $Q_{sat,A}^2 \propto A^n Q_{sat}^2$, the model perfectly works in fitting RHIC and SPS data at different beam energies, and can be easily extrapolated to LHC [58].

The multiplicity of newly produced (charged) particles per participant, with respect to the collision energy $\sqrt{s_{NN}}$, incorporates the $Q_{sat}^2$ dependence in the Golec-

---

7The term soft refers to the low $p_T$ part of the spectra ($p_T \lesssim 1$ GeV/c), in contrast with the hard component, which refers to hard scattering processes leading to jets and high $p_T$ observables. In heavy ion collisions, the soft component mainly consist of thermalized particles, where the thermalization is a consequence of the multiple scattering in the medium.
1.3 Medium Properties

Biernat and Wusthoff (GBW) parameter $\lambda$ [60]:

$$\frac{1}{N_{\text{part}}} \frac{dN_{\text{ch}}^{AA}}{d\eta} \bigg|_{\eta \sim 0} = N_0 \sqrt{s} \kappa^{\frac{1-\delta}{W_N}},$$

(1.19)

where $\delta = 0.79 \pm 0.02$ is the fit parameter, and $N_0 = 47/2$ is the overall normalization [58]. The GBW parameter (for $R_0^2 = 1/Q_{\text{sat}}^2 = (\bar{x}/x_0)^{\lambda}$ in GeV$^{-2}$ and $x_0 = 3.04 \cdot 10^{-4}$) is $\lambda = 0.288$ [60].

Combining eq.1.19 with the number of participants from the above Glauber calculations, it is possible to estimate the impact parameter dependence of the charged multiplicity at mid-rapidity at LHC (see fig.1.11(b)).

![Figure 1.12. Integrated elliptic flow $\langle v_2 \rangle$ versus charged multiplicity at mid-pseudorapidity ($\sim$ event centrality), extrapolated to LHC with the LDL approximation (the most symmetric band) and with the hydrodynamic parametrization, with $c_s^2 = 0.33$ and 0.22 (the upper and lower band respectively). The uncertainties are calculated by propagating the $3\sigma_R + 3\sigma_w$ uncertainty (on radius and width) from the nuclear data [22] to the calculated eccentricity and $dN_{\text{ch}}/dy$. The uncertainty of the LDL extrapolation also includes the errors on the linear fit (see fig.1.4).](image-url)

Figure 1.12 shows the centrality dependence of the integrated elliptic flow as a function of the charged multiplicity at mid-rapidity for the three extrapolations presented above: the more symmetric curve represent the linear extrapolation of the data in fig.1.4 (see sec.1.3.1), the other two curves are the upper and lower limit in the relativistic hydrodynamic approach, with $c_s^2 = 0.33$ and 0.22 respectively (see sec.1.3.2).

The centrality classes and the exact values which have been used for the simulations are listed in tab.5.1 in the analysis chapter.
More recent developments suggest a slightly different extrapolation of $v_2$ as a function of centrality, which better describes RHIC data [14]. The extrapolation is still based on relativistic hydrodynamic, but it includes viscous deviations [61]. However, for time reasons, the model has not been used in the present thesis.

### 1.3.4 Differential Flow

In the hydrodynamic picture, a detailed comparison between different equations of state is achieved by looking at $v_2$ versus the transverse momentum, for different particle species, in the low-$p_T$ region. In particular, the effect of a phase transition would be less pronounced for lighter particles such as pions compared to protons [62]: at the same collective flow velocity, heavier particles carry a higher momentum and therefore are less affected by the thermal motion (see fig.1.13).

![Figure 1.13. Transverse momentum dependence of $v_2$ for protons and pions [63]. The lines represent hydrodynamic calculations, assuming an EoS with (full) and without (dashed) phase transition.](image)

Elliptic flow studies at RHIC show that by scaling both $v_2$ and $p_T$ by the number of constituent quarks $n_q$ a universal curve is observed [45, 64], suggesting that the partons are the relevant degrees of freedom at least during the earliest stage of the system evolution when most of the elliptic flow is built up. The results for the most common mesons and baryons are shown in fig.1.14.

In the simulations performed for the present thesis, the differential shape of $v_2$ versus $p_T$ has been parametrized as linearly increasing with $p_T$ up to its saturation value at $p_T = 2 \text{ GeV}/c$, after which $v_2(p_T)$ becomes flat [65]. The magnitude of the saturation $v_2$ has been determined for each centrality class in such a way that the integrated $\langle v_2 \rangle$ (over the $dN/dp_T$ spectra of charged hadrons) are the extrapolated values shown in fig.1.12 (see sec.5.3 for the details).

Elliptic flow versus (pseudo)rapidity is assumed to be flat, in agreement with what is normally used in hydrodynamic calculations ($v_2$ shows a plateau at $y \sim \eta \sim$...
1.4 Non-Flow correlations

The elliptic flow observed in the final state arises from the anisotropic expansion of the system, which is due to the initial azimuthal asymmetry of the collision along the direction defined by the reaction plane. Therefore the coefficient $v_2$ quantifies the correlation between the directions of radiated particles and the orientation of the reaction plane.

However the reaction plane is not directly observable in a real experiment (the experimental methods to estimate its direction and the magnitude of elliptic flow will be described in chapter 3), what is measurable experimentally is the ‘event plane’, which is reconstructed from the azimuthally anisotropic particle distribution (see sec.3.2).

The reconstructed event plane approximates the real reaction plane just in case flow is the only source of azimuthal correlation. However, in real experiments, other physics phenomena can affect the spatial distribution of particles trajectories. Due to jet emission, resonance decays and momentum conservation, particles are mutually correlated with no respect to the orientation of the reaction plane. These effects

Figure 1.14. Elliptic flow $v_2$ for identified particles scaled by the number of constituents quarks $n_q$, plotted versus $p_T/n_q$ [45].

0 in the interval $|\eta| \lesssim 1$ [66, 67]) and with existing RHIC data [12, 68]. On the experimental side this means that elliptic flow at $\eta \sim 0$ is estimated by averaging the reconstructed $v_2(\eta)$ over a wide pseudorapidity interval, which in our case could be the entire acceptance of the ALICE central barrel detector ($-0.9 < \eta < 0.9$, see sec.2.1).

Since the aim of the present analysis is the measurement of elliptic flow of unidentified charged particles, no particle type dependence of $v_2$ has been developed in the simulations.

1.4 Non-Flow correlations

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are summarized under the concept of ‘non-flow’, defined as azimuthal correlation between \(k\)-tuples (i.e. pair, triplets, \(\ldots\)) of radiated particles.

Depending on the analysis method, non-flow effects introduce a systematic error in the flow measurement, and non-flow contributions at LHC energies represent a big uncertainty in the flow analysis at ALICE.

In the present thesis, non-flow effects have been simulated using Hijing (see sec.2.2.3), and part of the study has been devoted to characterize their source, and to compare their magnitude to the expected flow signal (defining, in such a way, the applicability limits of the event plane analysis). The details of this study and the analysis results are given in sec.4.1.
Chapter 2

Experimental Setup and Analysis
Framework

ALICE (A Large Ion Collider Experiment [69]) is an experiment dedicated to study heavy ion collisions at the LHC (Large Hadron Collider [70]), located at CERN.

One of the main physics goals of ALICE (and the major issue of this thesis) is the measurement of ‘anisotropic flow’ in Pb-Pb collisions (and in particular elliptic flow, see section 1.1). Flow is a collective phenomenon classified as ‘soft physics’, since its observation requires the ability to reconstruct and identify particles down to very low momentum. Besides soft physics the ALICE program will cover many other physics observables occurring in heavy ion collisions, e.g. jets, heavy quarks, direct photons, HBT interferometry, etc. This lead to the construction of a multi purpose detector combining different detection techniques.

In the first part of this chapter, the ALICE detector will be described, devoting more attention to the sub-detectors directly involved in the flow measurement (see section 2.1). Since the LHC is not yet operational during the development of the present thesis, the analysis presented in the following chapters is entirely based on simulations. Therefore, the second part of this chapter will describe the simulation and analysis framework that has been use (section 2.2). The last section of this chapter describes the procedure for track reconstruction and particle identification implemented in the ALICE software framework, a prototype of the one that will be used during the real experiment (see section 2.3). The final output of the reconstruction algorithm is a data structure (the ALICE Event Summary Data) that constitutes the starting point of the flow analysis, as will be described in chapter 3.

2.1 The ALICE detector at LHC

The heavy ion program at LHC, which is supposed to start after the first p-p run, will collide the largest available nuclei at the highest possible energy (Pb-Pb collision at $\sqrt{s_{NN}} \simeq 5.5$ TeV), and also explore different systems (p-A, A-A) at different beam
energies.

The nominal luminosity of the LHC for Pb-Pb collisions is 1 inverse milli-barn per second, i.e. an event rate of about 8000 minimum bias collisions per second. On average 5% of them will correspond to the most central events with a multiplicity of about 2000 charged particles per unit rapidity (see sec.1.3).

![Figure 2.1. General layout of the ALICE detector [71]. For visibility, the HMPID detector is at 12 o’clock position instead of 2 o’clock position where it will actually be. For the meaning of the abbreviations refer to the text.](image)

This low interaction rate together with the high multiplicity environment lead to the design of slow but highly granular tracking detectors. The soft physics domain requires a wide acceptance tracking device with low material density, immersed in a moderate magnetic field. In addition, particle identification over a wide momentum range is required, which implies the implementation of many different identification techniques (energy loss, time of flight, transition radiation, and Cherenkov light).

ALICE is a general purpose detector to measure and identify hadron, lepton and photons produced in the interaction, from very low to very high transverse momentum ($100\text{MeV}/c < p_t < 100\text{GeV}/c$). It consists of a central detector system, designed to provide full tracking at mid-rapidity ($-0.9 < \eta < 0.9$) over the full azimuth, and several forward detectors.

The experimental setup of ALICE is extensively described in the ALICE Technical Proposal [72] and its addenda [73, 74] and in the ALICE Physics Performance Report [71]. The detector systems are described in the various Technical Design Re-
port (TDR [75–87]). The Trigger System is described in [88], the Data Acquisition System is described in the ALICE-DAQ manual [89].

Tracking and particle identification in the central rapidity region relies on four separate layers of $2\pi$ coverage detectors (ITS, TPC, TRD and TOF) immersed in a uniform magnetic field of parallel to the beam axis. The ALICE experiment is designed to run with three possible configurations of the magnetic field, $B = 0.2, 0.4$ and $0.5$ Tesla (value of $B$ at the center of the ALICE solenoid). The magnitude of the magnetic field affects the transverse momentum acceptance, a stronger magnetic field gives a better resolution at high $p_T$ but worsens the efficiency at low $p_T$. The current default value of the magnetic field is $B = 0.4$ T.

The detector arrangement (and in particular, the Inner Tracking System, see sec.2.1.1) provides high granularity close to the interaction point to reconstruct short-lived resonances, B and D mesons. The magnetic field is generated by the large solenoidal L3 magnet which contains the experiment (see fig.2.1).

The central system is complemented by a high momentum particle identification detector (HMPID [80]) which is a high resolution array of ring-image Cherenkov detectors (located at $|\eta| < 0.6$ with an acceptance of $57.6^\circ$ degrees in azimuth). Photons are reconstructed in a high density crystal photon spectrometer (PHOS [81]) which covers a small $\eta$ slice ($|\eta| < 0.12$) at mid-rapidity and $100^\circ$ in $\phi$. A future upgrade of the experiment foresees an electro-magnetic calorimeter (EMCAL) to be installed over $100^\circ$ azimuthal degrees in the central rapidity region, to help identification of charged leptons and photons.

Muon detection is performed by a forward spectrometer, which covers a high pseudorapidity cone ($-4.0 < \eta < -2.4$) on the negative $z$ side of the central detector (MUON [83]). The muon spectrometer is equipped with an absorber for filtering out hadrons and photons from the interaction, a dipole magnet and two separate arrays of tracking chambers, before and after the dipole magnet, for muon momentum measurement.

To complement the central detection system, other detectors are used to characterize the centrality of the events: a silicon strip forward multiplicity detectors (FMD [87]) for measuring charged particle multiplicity, and a pre-shower photon multiplicity detector (PMD [85]) for measuring the multiplicity and spatial distribution of photons on an event-by-event basis. They are located at the two opposite sides of the interaction point.

The fast trigger signal is provided by an array of scintillators and quartz counters close to the interaction point: the V0 and T0 detectors [88]. The T0 detector, with two arrays of Cherenkov counters placed on both sides of the interaction point, is particularly important because due to its fast response it is used to start the other detectors.

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1This is the default setting of the release v4-04-Rev14 of AliRoot (the one in use for the PDC06 production, including the simulations presented in chapter 5).

2In the laboratory frame, the $z$ axis is defined by the direction of the beam beam pipe (see also sec.2.3).
28 Experimental Setup and Analysis Framework

About 100 meters away from the collision point, a Zero-Degree Calorimeter (ZDC [82]) uses both hadronic and electro-magnetic shower to measure the energy carried away by non interacting nucleons (spectators \(^3\)). The ZDC consists of two distinct quartz fiber calorimeters, one for spectator neutrons, placed at zero degrees relative to the \(z\) axis, and one for spectator protons, placed externally to the beam pipe on the side where positive particles are deflected. In an ideal case, dividing the collected energy by the average energy per nucleon at LHC (i.e. 2.76 TeV/nucleon in a \(^{208}\)Pb beam), it would be possible to immediately estimate the centrality of the collision. In the real experiment not all the spectator nucleons can be detected.

The elliptic flow measurement described in this thesis requires full tracking over \(2\pi\) of azimuthal coverage, which is the domain of the central barrel detector system. Here in the following, the four main components of the central system will be briefly described, their combined track reconstruction will be presented in section 2.3.

2.1.1 ITS

From the interaction point, the first component of the ALICE detector is the Inner Tracking System (ITS [75]), six concentric layers of silicon detectors with a design based on three different Si techniques (fig.2.1.1).

![Figure 2.2](image)

**Figure 2.2.** Layout of the ITS detectors, showing the spatial arrangement of the three layer.

The position and segmentation are optimized for efficient track finding and for a high spatial resolution in a high multiplicity environment.

The high particle density (80 particles/cm\(^2\) at 4 cm from the interaction point) and the requirements on spatial resolution are the main reasons for choosing a Silicon Pixel Detector (SPD) for the innermost two layers. The following two layers are

\[^3\text{From the number of spectators } N_{\text{spec}}, \text{ the number of participating nucleons can be calculated as } N_{\text{part}} = A - N_{\text{spec}}, \text{ where } A \text{ is the atomic number of the colliding nucleus.}\]
2.1 The ALICE detector at LHC

A Silicon Drift Detector (SDD), and where the track densities become lower than one particle per cm$^2$ (> 40 cm from the interaction point) there are two layers of double-sided Silicon Strip Detector (SSD). Both the SDD and the SSD layers have an analog readout for $dE/dx$ measurement, which allows low-$p_T$ particle identification using the Bethe-Bloch model for energy loss.

| Detector | Layer | $r$ (cm) | $\pm z$ (cm) | $|\eta|$ | $\sigma_{r\phi}$ ($\mu$m) | $\sigma_z$ ($\mu$m) | Channels |
|----------|-------|----------|-----------|-------|----------------|----------------|----------|
| SPD      | 1     | 4.0      | 14.1      | 1.98  | 12             | 100            | 3.278.400 |
|          | 2     | 7.2      | 14.1      | 0.9   | 38             | 28             | 6.556.800 |
| SDD      | 3     | 15.0     | 22.2      | 0.9   | 20             | 830            | 43.008   |
|          | 4     | 23.9     | 29.7      |       |                |                | 90.112   |
| SSD      | 5     | 38.5     | 43.2      | 0.9   |                |                | 1.148.928 |
|          | 6     | 43.6     | 48.9      |       |                |                | 1.459.200 |

The ITS has a pseudorapidity acceptance of $|\eta| < 0.9$ for all vertices located within ±5.3 cm from the beam interaction. The first layer of the SPD has a larger pseudorapidity coverage ($|\eta| < 1.98$), so that this part together with the Forward Multiplicity Detectors (FMD) provide a continuous coverage in rapidity for the measurement of charged-particles multiplicity. Information about the ITS is summarized in tab.2.1.

The material budget of the ITS has been kept as low as possible ($X/X_0 \simeq 7\%$ for perpendicular tracks) in order to maximize the efficiency at low momentum, a thick layer at very close distance from the interaction point would act as a shield, preventing tracks from entering the TPC.

2.1.2 TPC

The ITS is surrounded by the large cylindrical volume of the Time-Projection Chamber (TPC [76]), a conventional device in heavy ions experiments, successfully used already by NA49 and STAR.

The field cage has a total volume of about 88 m$^3$ (making it the largest Time-Projection Chamber ever built), and the detector is optimized for an extremely high multiplicity environment, safely overestimated as about 8.000 tracks per unit rapidity ($\sim 20.000$ tracks in the whole TPC coverage [71]). This can be achieved at the rate of 400 minimum bias Pb-Pb collisions per second (400 Hz), and up to 1 kHz for p-p [71].

The TPC is the main tracking device in ALICE, track seeds start in the outer radius of the TPC (see sec.2.3). It has $2\pi$ azimuthal coverage and an acceptance

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The Forward Multiplicity Detectors measures charged-particles multiplicity in the pseudorapidity range $-3.4 < \eta < -1.7$ and $1.7 < \eta < 5.1$. 
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Figure 2.3. Layout of the TPC, showing the orientation of the electric field toward the central membrane ($e^-$ drift to the end-caps).

$|\eta| < \pm 0.9$ for full radial tracking. For partial tracking (tracks not reaching the outer radius of the TPC) an acceptance up to $|\eta| \sim 1.5$ is accessible.

The ALICE TPC is an ideal device for soft physics observables, the momentum resolution is estimated between 1% and 2% for low-momentum tracks ($100 \text{MeV}/c < p_T < 1 \text{GeV}/c$), depending on the magnetic field (see sec.5.1).

The material budget for the TPC is kept low to minimize multiple scattering and secondary particle production. Both field cage and drift gas are made of materials with small radiation length, the material budget of the TPC is $3.5% < X/X_0 < 5%$ for track in the central rapidity acceptance ($|\eta| < 0.9$).

The field cage has a central high-voltage electrode that divides the TPC volume into two parts, and two opposite sets of axial potential dividers (18 field degraders, 1 per sector) to create a uniform electric field in both sides (fig.2.1.2).

The read out chambers are located on the two end-caps of the TPC cylinder, they are standard multi-wire proportional planes with cathode pad readout, segmented in $5\text{.}$

\footnote{The TPC is a cylindrical volume with radius $r = 2.47 \text{ m}$ and elongation in the $z$ direction $l_z = 5 \text{ m}$, centered around the beam-crossing point $(0,0,0)$. Neglecting the displacement of the vertex in the transverse plane (which is of the order of few tens $\mu \text{m}$), the $\eta$ acceptance of the TPC is given by $\eta(z_0) = - \log \left(\tan \left(\frac{\theta(z_0)}{2}\right)\right)$, where $\theta(z_0) = \tan^{-1}\left(\frac{r}{l_z - z_0}\right)$ is the longitudinal angle under which the TPC is seen from the interaction point. For events with main vertex at the center of the cylinder the TPC has a symmetric acceptance $|\eta| \simeq 0.891$.}
2.1 The ALICE detector at LHC

Table 2.2. Synopsis of TPC parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudorapidity coverage</td>
<td>$-0.9 &lt; \eta &lt; 0.9$ for full radial track length</td>
</tr>
<tr>
<td></td>
<td>$-1.5 &lt; \eta &lt; 1.5$ for $1/3$ radial track length</td>
</tr>
<tr>
<td>Azimuthal coverage</td>
<td>$2\pi$</td>
</tr>
<tr>
<td>Radial position (active volume)</td>
<td>$845 &lt; r &lt; 2466$ mm</td>
</tr>
<tr>
<td>Length (active volume)</td>
<td>5000 mm</td>
</tr>
<tr>
<td>Segmentation</td>
<td>$18(\phi), 2(r), 2(z)$</td>
</tr>
<tr>
<td>Pad rows</td>
<td>159 (63 inner pad + 96 outer pad)</td>
</tr>
<tr>
<td>Material budget</td>
<td>$X/X_0 = 3.5$ to $5%$ for $0 &lt;</td>
</tr>
<tr>
<td>Detector gas</td>
<td>$88 \text{ m}^3$ of Ne/CO$_2$ (90%/10%)</td>
</tr>
<tr>
<td>Drift length</td>
<td>$2 \times 2500$ mm</td>
</tr>
<tr>
<td>Drift field</td>
<td>400 V/cm</td>
</tr>
<tr>
<td>Drift velocity, time</td>
<td>$v = 2.84 \text{ cm/\mu s}, t_{max} = 88 \mu s$</td>
</tr>
<tr>
<td>Position resolution ($\sigma$) in $r\phi$</td>
<td>1100 to 800 $\mu$m inner / outer radii</td>
</tr>
<tr>
<td>in $z$</td>
<td>1250 to 1100 $\mu$m</td>
</tr>
<tr>
<td>$dE/dx$ resolution, isolated tracks</td>
<td>5.5%</td>
</tr>
<tr>
<td>$dN/dy = 8000$</td>
<td>6.9%</td>
</tr>
</tbody>
</table>

18 sectors in $\phi$ with 2 readout chambers per sectors (inner chamber $84.1 < r < 132.1$ cm, outer chamber $134.6 < r < 246.6$ cm). In total there are $18 \times 2 \times 2 = 72$ readout chambers, for a total of 159 radial pad rows.

The inactive areas between neighboring inner chambers are aligned with those between neighboring outer chambers to optimize the momentum precision for high-momentum tracks, but has the drawback of creating dead zones in the azimuthal acceptance\(^6\) (the detector is non-sensitive for about $10\%$ in $\phi$).

The analog readout of the TPC allows particle identification by $dE/dx$ measurement, both in the low momentum region, where the expected ionization for particle types is well separated, and at very high $p_T$, due to the relativistic rise in the Bethe-Bloch curve (see sec.2.3). Information about the TPC is summarized in tab.2.2.

\(^6\)Each sector of the TPC covers $\sim 18^\circ$ degrees in $\phi$, with a gap between two neighboring sectors of $\sim 2^\circ$. This results in $\sim 324^\circ$ degrees of azimuthal coverage and $\sim 36^\circ$ of dead area, located at $\phi = n \times 18^\circ \pm 1^\circ$. 
2.1.3 TRD and TOF

The Transition-Radiation Detector (TRD [77]) is located around the TPC (fig.2.1.3 (a)). It provides electron identification in the central barrel for momenta greater than 1 GeV/c by detecting the transition radiation (TR) produced by those particles in the radiator, i.e. the radiation produced by fast particles (with relativistic $\gamma > 1.000$) when crossing a boundary between two materials with different dielectric constants.

In the momentum range from 1 to 10 GeV/c, only electrons (and positrons) are highly relativistic due to their small mass. This process causes a larger release of energy in the detector material (due to TR photons), which allows to separate pions from electrons with a misidentification probability of less than 1%.

The TRD fills the radial space between the TPC and the TOF detector and it also has $2\pi$ azimuthal coverage and a pseudorapidity acceptance $|\eta| < \pm 0.9$. The TRD consists of 6 individual layers, divided into 18 sectors to match the azimuthal segmentation of the TPC. In total there are $18 \times 5 \times 6 = 540$ detector modules, made of a sandwich radiator and a multi-wire proportional readout chamber.

*Figure 2.4.* (a) Cut through the TRD with the TPC inside. (b) TOF sector (super-module), consisting of five modules inside the space frame which surrounds the TRD.

The last layer with $2\pi$ azimuthal coverage in the central barrel is the Time-Of-Flight detector (TOF [78]). Its cylindrical surface covers the central pseudorapidity region ($|\eta| \leq 0.9$) and provides particle identification in the intermediate momentum range (from $0.2 < p_T < 2.5$ GeV/c, see sec.2.3).

The time of flight of detected particles is calculated by the delay between the fast trigger signal given by the T0 detector (minus a fixed $t_{T0} = z_{T0}/c$) and the TOF signal. This allows particle identification by calculating the invariant mass
2.2 The Off-Line Framework

with the relativistic formula:

\[ m = \frac{p_{\text{tot}}}{\beta \gamma}, \]

(2.1)

where the relativistic \( \gamma = \frac{1}{\sqrt{1-\beta^2}} \), and \( \beta \) is calculated from the TOF signal as:

\[ \beta = \frac{l_{\text{trk}}}{c \times t_{\text{TOF}}} \]

(\( l_{\text{trk}} \) is the length of the track, calculated from the track fit and \( c \) is the speed of light). The invariant mass obtained in this way is used to compute the probability of the particle to be of a specific type.

The modular structure of the TOF corresponds to 18 sectors in \( \phi \) (matching TPC and TRD) and to 5 segments in \( z \) (fig.2.1.3(b)). Each TOF module is a Multi-gap Resistive-Plate Chamber (MRPC [90]), which can operate efficiently in extreme multiplicity conditions.

The electric field is high and uniform over the whole gas volume of the detector, any ionization produced by a charged particle passing through will immediately start an avalanche process which will eventually generate the observed signals on the pick-up electrodes. There is no drift time associated with the movement of the electrons to a region of high electric field, therefore the time uncertainty of these devices is only caused by the fluctuations in the growth of the avalanche.

2.2 The Off-Line Framework

Since the analysis presented in this thesis are entirely based on simulated data, the following section describes the simulation and analysis framework in use at ALICE. The ALICE Off-line framework, AliRoot, is a full experimental environment built on top of ROOT.

2.2.1 ROOT

ROOT [91] is a widely accepted software framework for experimental high-energy physics that offers a common set of features and tools for many domains: generation of events, detector simulation, data reconstruction, data storage, analysis and visualization.

It was initially developed in the context of a heavy ion experiment (NA49 at CERN [92]) in year 1995 [93], following the new standards of Object-Oriented programming. The ROOT framework has rapidly taken over most of the old FORTRAN tools still very popular, and has become an essential software of experimental particle physics.

Thanks to the object-oriented approach the system can be easily extended to other domains, e.g. interfaces for remote or distributed analysis (see sec.2.2.4), or the implementation of user defined macros and libraries (the AliFlow package is a good example, see section 3.3).

The built-in C++ interpreter (CINT [94]) provides the possibility to use both
C++ macros and compiled ‘shared object’ libraries. ROOT is in fact a versatile system that can be dynamically extended.

In the ALICE collaboration, ROOT has been adopted as the underlying system for data acquisition, simulation and analysis.

### 2.2.2 AliRoot and the ALICE Off-line Project

Many collaborations have developed their own ROOT based tools to better satisfy specific needs of the experiments. The STAR collaboration is an example of this approach with the implementation of the Star Class Libraries (SCL [96]). A more radical strategy has been adopted by the ALICE collaboration, giving birth to a complete experimental framework named AliRoot.

#### Brief History of AliRoot

A Geant3 based simulation program (gAlice [97]) was originally developed for the Technical Proposal of the ALICE experiment at LHC [72]. It was an object-oriented prototype for data reconstruction, mainly written in FORTRAN and built on top of existing Monte Carlo codes (such as GEANT [98] [99] and FLUKA [100]).

After the publication of the Technical Proposal (TP [72]) in 1995, simulations became an essential tool for the detailed design of the detectors and for the development of the Technical Design Reports (TRD [75–88]) for the various ALICE sub-detectors. It was clear that a substantial upgrade of the gAlice package was necessary. A second version of gAlice was quickly prototyped, still using the ‘Geant3’ simulation program (in FORTRAN) but completely wrapped into a C++ class. This rapid prototyping was possible thanks to the availability of ROOT as framework and to the active support of the ROOT team. The results of this activity was a suitable tool for simulations, which was using at the same time both the advantages of the Object-Oriented programming, and the robustness of the ROOT framework, the output of the simulations were persistent objects that could be stored on disks.

The official adoption of ROOT by the ALICE Off-line Project was in November 1998. As a consequence, new C++ versions of the simulation programs started to be developed, together with the digitization and reconstruction code, that was now based on ROOT as a common framework. Since version 3, the name ‘AliRoot’ was adopted and the simulation and reconstruction code was completely rewritten in C++.

The version of AliRoot that has been used in the present thesis is the release v4-04-Rev14. The entire framework is constantly under development [101].

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7A ‘shared object’ library (with extension ‘.so’) is the standard format of dynamically linked libraries on the Linux platform, usually compiled with ‘gcc’ [95].
2.2 The Off-Line Framework

The AliRoot Framework

AliRoot is a complete experimental framework to simulate, reconstruct and analyze heavy ion data in the ALICE environment.

Heavy ion collisions are simulated using a Monte Carlo event generator (see sec.2.2.3). Using the transport code from Geant [99] they are propagated through the detector response simulation packages, and transformed into digitized signals that match the detector layout of real reconstructed data.

The result of this process is the production of ‘raw data’, i.e. data representing the digitalized output of the ALICE detector, that can be submitted to the event reconstruction chain. Simulated data are then processed in the same way as data from the real experiment, the tracking algorithm fits the reconstructed space points (clusters) in each detector and calculates the particle trajectory. Analog detector signals are also associated to the fitted tracks, and the energy loss and the Time-Of-Flight signals are used to calculate the Bayesian weights for particle identification (see sec.2.3.2).

A good feature of the transport code, as implemented in AliRoot, is that it keeps track of which simulated particles produced a signal in the sensitive volume of the detectors, by associating the particle’s label to every ‘hit’ produced in the detector. At the end of the simulation, the reconstructed tracks can be compared one to one to the original particles that have been simulated, and this is very useful for calculating the reconstruction efficiency and to optimize the analysis cuts.

The simulation process can be summarized in the following steps:

- **Event generation:** The collision is simulated by an event generator, which produces an array of final-state particles with outgoing momenta, propagating from the main vertex of the collision (which can be set at any position along the beam intersection, see sec.5.1.4). This array is called ‘KineTree’, and it is a ROOT TTree structure containing a list of TParticles with their complete kinematic (see sec.2.2.3).

- **Particle transport:** Particles emerging from the interaction are propagated along the direction of their momenta and the transport code (Geant [99]) simulates the interactions with the detector material (particle decays, particle scattering, ionization processes and energy deposition) by calculating the probability of random microscopic processes between the particle and the surrounding. Whenever a secondary particle is produced, it is added to the KineTree and transported as well, the transport code stops when a particle exits the detector volume or when a low energy threshold is reached (the particle stops). During the transport process, the information contained in the TParticle is lost and reduced to that generated by a particles crossing the detector.

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8The ROOT TParticle class is meant to summarize the sensitive information of a physical particle, such as momentum, charge, mass, particle type. For more information see the on-line ROOT documentation [91]
Experimental Setup and Analysis Framework

- **Detector response:** The energy deposited in the detector is then translated into a detector response (a ‘hit’), according to the geometry of the detector and the implemented detection techniques (this is the ideal detector response).

- **Digitization:** The detector response is digitized and formatted according to the output of the front-end electronics and the data acquisition system (DAQ [89]), some smearing of the signal due to electronic noise is applied at this step. The resulting data closely resemble the real output that will be produced by the detector.

- **Event reconstruction:** The reconstruction algorithm fits the reconstructed space points to produce track candidates (AliESDtracks), and retrieves / calculates all the sensitive information available (fit parameters, energy loss, p.I.d. hypothesis), it also extrapolates the interaction vertex and reconstructs neutral decay vertices (see sec.2.3). Each reconstructed event is stored as an ALICE Event Summary Data object (class AliESD).

All the procedure is handled by AliRoot and can be executed at any time using some simple commands and a configuration script (to specify event generator, detector settings and reconstruction parameters). However, a full simulation requires a few hours of computing time, depending on the particle multiplicity and the number of detectors switched on.

### 2.2.3 Event Generators

Since a full and complete description of the processes occurring in heavy ion collisions has not been achieved yet, AliRoot incorporates a few Monte Carlo event generators, specifically implemented to simulate different physics observables.

The analysis described in this thesis made use of two different event generators (both available in the standard release of AliRoot), ‘Hijing’ and ‘GeVSim’. They will be briefly described in the following to sub-sections.

**Hijing**

Hijing (Heavy Ion Jet INteraction Generator [102] [103] [104]) is a multi-purpose heavy ion event generator implemented in FORTRAN and wrapped into a C++ class to be easily incorporated in the AliRoot framework. At the moment, Hijing offers a very good description of jets production in nucleus nucleus collision, incorporating all known physics effects from a superposition of multiple proton-proton collisions, plus some parametrizations of soft physics observables. Its implementation is based on a perturbative QCD inspired model, where multiple mini-jet production is combined together with Lund type model for jet fragmentation [105].

In high-energy nuclear interactions, and especially in relativistic heavy ion collisions, the multitude of hard or semi-hard parton scatterings result in the production
of an enormous amount of jets, and can be described in terms of perturbative QCD (pQCD). Minijets are expected to dominate the transverse energy production in the central rapidity region.

In Hijing, multiple interactions are calculated using Glauber geometry, and a parametrization of the parton distribution function for the nucleus is used to take into account parton shadowing. Jet quenching is modeled using a parametrized energy loss \( \frac{dE}{dz} \) of partons traversing the dense medium. The program uses subroutines of PYTHIA [106] to generate the kinematic variables of each hard scattering process and the associated radiations, and JETSET [107] for string fragmentation. Due to its implementation in terms of pQCD, Hijing is only valid for collisions with center of mass energy \( \sqrt{s_{NN}} \) above 4 GeV per nucleon, which makes it perfectly suitable for LHC collisions.

However, as a superposition of many p-p collisions, Hijing events do not contain any collective effect such as anisotropic flow, while other typical heavy ion observables are added by ad-hoc routines (e.g. the jet-quenching effect [108]). Another disadvantage of Hijing is the particle multiplicity, which is too large with respect to the current predictions for LHC (this problem can be taken care of by rescaling the centrality of the collisions, as it is done in sec.4.3).

In the present thesis, Hijing has been used to simulate the background of flow measurement (i.e. non-flow effects) arising from the presence of jet-like correlations and resonance decays (see sec.4.1). In sec.4.3 and 5.4 collective flow has been added on top of the Hijing simulations by boosting the generated events with the flow After-Burner (see below).

**GeVSim and the flow After-Burner**

GeVSim [109] [110] is a fast and easy to use Monte Carlo event generator, based on the MeVSim [111] event generator developed for the STAR experiment (written in FORTRAN), and re-implemented in C++ for AliRoot.

It does not reproduce the physics of the heavy ion reaction, but simply radiates user defined particle types out of the primary vertex, with a custom momentum spectrum parametrized with respect to \( p_T, \eta \) and \( \phi \). The \( dN/dp_T \) and \( dN/d\eta \) distributions can be expressed analytically or with user defined histograms, while the azimuthal distribution is described by two Fourier coefficients \( v_1 \) and \( v_2 \) (representing directed and elliptic flow, see eq.1.1), which can be expressed as functions of \( p_T \) and \( \eta \).

At the present time, GeVSim offers the simplest way to parametrize anisotropic flow in heavy ion events, just by introducing a modulation in the generated \( dN/d\phi \) distribution with respect to the reaction plane angle. The Fourier expansion of the azimuthal distribution implemented in GeVSim is truncated at the second coeffi-
cient, therefore the azimuthal anisotropy is parametrized as:

$$E \frac{d^3 N}{dp_T^3} = \frac{1}{2\pi} \frac{d^2 N}{p_T dp_T dy} \left[ 1 + V_1(p_T, y) \cos (\phi - \Psi) + V_2(p_T, y) \cos (2[\phi - \Psi]) \right].$$

(2.2)

where $\phi$ is the azimuthal angle of the particles, $\Psi$ is the reaction plane angle, and $V_n(p_T, y)$ ($n = 1, 2$) are the first and second Fourier coefficients.

The Fourier coefficients can be set separately for each particle type, and they can be constants or functions of $p_T$ or $\eta$. In particular, the parametrization used in this thesis is:

$$V_1(p_T, \eta) = 0,$$

$$V_2(p_T, \eta) = \begin{cases} v_{2,1} \cdot \frac{p_T}{p_{T,\text{sat}}} & \text{if } p_T < p_{T,\text{sat}}^\text{sat} \\ v_{2,2} & \text{if } p_T \geq p_{T,\text{sat}}^\text{sat} \end{cases},$$

(2.3)

with $v_2$ (and therefore $v_{2,2}^\text{sat}$) assigned with respect to the centrality of the event (see sec.1.3) and $p_{T,\text{sat}}^\text{sat} = 2$ GeV/$c$. The event plane angle is generated with random orientation (as it will be in real collisions).

Events can be also produced with any other event generator and then boosted with the GeVSim ‘After-Burner’, to add flow on top of an existing array of final state particles. The After-Burner is applied to an existing KineTree, and it distorts the $dN/d\phi$ distribution according to the specified values of $v_1$ and $v_2$, with respect to an event plane angle that must be specified on an event-basis.

In sec.4.3 and 5.4 Hijing simulated events have been boosted with the flow After-Burner, in order to obtain ‘realistic’ heavy ion events with both collective flow and jet-like azimuthal correlations. However, the ‘boost’ is applied on top of the Hijing simulated event, where jet and strong resonance decays already took place (instead, weak and electro-magnetic processes are performed in a later stage by the transport code), therefore in our procedure part of the non-flow effects is probably washed away.

The After-Burner is fed with the same reaction plane generated by Hijing, which is distributed randomly over $2\pi$ in azimuth. The magnitude of $v_2$ is calculated as a function of the impact parameter of the collision (after the proper rescaling) using the hydro parametrization (see sec.1.3).

### 2.2.4 AliEn and LGC

The large amount of data that is going to be produced by ALICE (and more in general by LHC experiments), requires very large storage and computing power. One month of Pb-Pb collision in ALICE will produce roughly 1 Pbyte of data (1 Peta-Byte = 1,000,000 Giga-Bytes). Thus the construction of LHC required the parallel implementation of a computing infrastructure capable of dealing with such a huge amount of data.
The LCG (LHC Computing Grid [112]) is a network based framework for distributing jobs and data over the resources available world-wide (both as CPUs and storage elements).

The ALICE Off-Line collaboration has developed its own way to access this grid, the ALICE Environment (‘AliEn’ [113] [114]). Massive event simulations (e.g. Particle Data Challenges or PDC.xx [101]) are currently produced through this environment, and during the real experiment the grid will provide the computing power for raw-data reconstruction and distributed analysis. AliEn provides a virtual file catalogue (to access distributed data-sets) and different web services such as user authentication, job execution, file transport and performance monitor [115].

During the development of the present thesis, the LCG grid has been used to produce the simulations presented in chapter 5. Some effort has also been devoted to interface the flow analysis package to the AliEn environment, by the implementation of an AliFlowTask for the creation of AliFlowEvents from AliESDs, and their consequent analysis (see sec.3.3). In this way the job can be submitted to the grid through a ROOT task manager (the AliTaskManager) for distributed analysis.

2.3 Track Reconstruction in the Central Barrel Detectors

The central barrel detector system of ALICE mainly consist of tracking devices, charged particles going through leave discrete signal at the space points where they pass (‘clusters’), and a reconstruction algorithm fits these space points into track candidates to reconstruct the particle kinematics. This operation is called track reconstruction or tracking.

The combined track reconstruction in the central barrel system collects information from the different sub-detectors in order to optimize the track reconstruction performance (the details about the tracking procedure are described in chapter 5 of the ALICE Physics Performance Report [25]).

Reconstructed space points are represented in the global coordinate system of ALICE 9, with the $z$ axis along the beam-pipe (oriented in the opposite direction with respect to the muon arm), the $y$ axis pointing upward and the $x$ axis to complete a right-handed cartesian system (it points outward with respect to the LCH circle). The origin is defined by the intersection of the $z$ axis with the central membrane plane of TPC.

The track fitting algorithm uses the Kalman filter [116] [117], a general and powerful method for local track-finding. Tracks are approximated with a ‘helix’ 10

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9Note: this is the global coordinate system of the detector. On an event-by-event basis, the origin of the coordinate system (which tracks and V^0 coordinates refer to) is located at the reconstructed position of the main vertex

10The helix perfectly describes the ideal trajectory of a charged particle moving in a uniform magnetic field, where the Lorentz force acts perpendicularly to the direction of motion.
and parametrized by a set of five parameters, such as the curvature and the angles with respect to the coordinate axes. The Kalman filter performs an iterative fit, by adding the space points found along the trajectory of the helix. The fit parameters are updated at any additional fit point (after some rejection criteria), improving at every step the quality of the fit. The method is suitable for simultaneous track recognition and fitting, and gives the possibility to reject incorrect space points ‘on the fly’. Moreover, the Kalman filter offers a natural way to extrapolate tracks from one detector to another (e.g. from the TPC to the ITS or the TRD). The reconstruction algorithm is fully integrated within the AliRoot framework, and uses the same detector classes involved in the simulation [101].

Track reconstruction is done in three passes:

1\textsuperscript{st}) track finding and fitting inward from the TPC to the ITS: Tracking starts in the outermost pad rows of the TPC, where the space separation between tracks is the largest. Each track seed is calculated from different combinations of pad rows, with and without a primary vertex constraint. Track candidates are then propagated in the TPC using the Kalman filter, the fit continues to the ITS. After all the track candidates from the TPC are assigned to their clusters in the ITS, a special ITS stand-alone tracking procedure is applied to the rest of the ITS clusters to recover the tracks that were not found in the TPC because of the momentum cut-off, dead zones between the TPC sectors, or decays (however, ITS ‘tracklets’ produced in this way have not been considered in the present analysis).

2\textsuperscript{nd}) the track is propagated outward and reconstruction is invoked for all central detectors: At the end of the first pass, an estimate of the track parameters and their covariance matrix \(^{11}\) is obtained in the vicinity of the main vertex. The Kalman filter is then applied in the outward direction starting with the ITS, space points with large \(\chi^2\) contributions are removed from the track fit. Once the outer radius of the TPC is reached, the precision of the track parameters is sufficient to extrapolate the tracks to the TRD, TOF, HMPID and PHOS detectors. Tracking in the TRD is done in a similar way to that in the TPC, tracks are followed till the outer wall of the TRD and the assigned clusters improve the momentum resolution further. Next, the tracks are extrapolated to the TOF, HMPID and PHOS, where they acquire the information for particle identification.

3\textsuperscript{rd}) the track is refitted inward and the ‘best’ track parameters are calculated at the vertex: At last, all the tracks are refitted inward, from their outermost reconstructed space point to the primary vertex (or to the innermost possible radius, e.g. secondary tracks), to each track is associated the analog \(dE/dx\) signal coming from the clusters included in the fit, the TRD signal and the

\(^{11}\)The covariance matrix of the Kalman fit is a \(5 \times 5\) matrix representing the uncertainties of the fit and their correlation.
2.3 Track Reconstruction in the Central Barrel Detectors

Time Of Flight. Tracks that failed the final refit toward the primary vertex are labeled as secondaries and used for the reconstruction of secondary vertices (see section 2.3.3), tracks who succeeded are labeled as constrainable, and both constrained and unconstrained fit parameters are stored.

Reconstructed tracks are stored in an array of combined track objects (class \textit{AliESDtrack}), and saved into the ALICE Event Summary Data file (class \textit{AliESD}). Further information is added to the ESD by the reconstruction algorithm of each detector, e.g. primary vertex position (see sec.2.3.1), particle identification (see sec.2.3.2), reconstructed secondary vertices (see sec.2.3.3).

Within the geometrical acceptance of the central barrel detectors, combined track finding has an efficiency well above 90%. The momentum resolution of the combined tracking (in Pb-Pb collisions) is estimated between 1 and 2.5% for transverse momenta up to 10 GeV/c (see sec.5.1), the angular resolution $\Delta\phi$ is $\sim 0.2$ mrad or even lower at higher momenta (see section 5.1.6 of the ALICE PPR [25]).

The ‘Distance of Closest Approach’ (DCA) to the primary vertex, defined as the extrapolated minimum distance between the fitted helix and the interaction point, has a resolution that depends both on the spatial resolution of the primary vertex (see sec.2.3.1) and on the track precision in the proximity of the interaction point (therefore, the number of reconstructed space points in the ITS). In the case of Pb-Pb collisions, where the main vertex is very well defined, the DCA resolution for tracks having 5-6 clusters in the ITS, is of the order of $\sim 100\mu$m [25] (see also sec.5.2).

2.3.1 Reconstruction of the primary vertex

The primary vertex constraint is used at various steps of the tracking procedure. The reconstruction of the primary vertex position is done using the information provided by the silicon pixel detector (SPD).

Collisions occur in the ‘interaction diamond’, parametrized as a wide Gaussian along the z axis ($\sigma_z = 5.3$ cm), with approximately the width of the beam in the $x$-$y$ plane ($\sigma_{x,y} \simeq 15 \mu$m to 75 $\mu$m, depending on the beam luminosity and lifetime [75]).

The primary vertex algorithm uses the $z$ coordinates distribution of the reconstructed space points in the SPD layers to find the centroid $z_{\text{cen}}$ around which the distribution is symmetric. When the primary vertex is moved away from the center of the detector ($z = 0$) an increasing fraction of hits will be lost and the centroid of the distribution no longer gives the primary vertex position, so the final position is calculated from the the correlation between the two centroids $z_1$ and $z_2$ found in the two layers. This procedure has been developed and validated on AliRoot simulations, and gives a resolution $\sigma_z \simeq 10 \mu$m for Pb-Pb collisions \footnote{Due to the much lower particle multiplicity, in p-p collisions the primary vertex is reconstructed using a different algorithm (which works in 3D). The achieved resolution of both $\sigma_z$ and $\sigma_{x,y}$ varies}. A similar approach

12Due to the much lower particle multiplicity, in p-p collisions the primary vertex is reconstructed using a different algorithm (which works in 3D). The achieved resolution of both $\sigma_z$ and $\sigma_{x,y}$ varies
is applied to the reconstruction of the vertex position in the transverse plane, giving a resolution $\sigma_{x,y} = 25\,\mu m$ [25].

The $x$, $y$ and $z$ coordinates of the primary vertex (in the global ALICE coordinates system) are stored as an AliESDVertex object in the AliESD.

### 2.3.2 Particle identification

Charged particle identification in the central ALICE detector system is done by combining all the information from ITS, TPC, TRD, TOF and HMPID. The particle identification in ALICE follows a ‘Bayesian’ approach [118], the most efficient way to combine information coming from different detecting systems that are efficient in complementary momentum sub-ranges (see figure 2.3.2), and to combine signals of different nature (e.g. $dE/dx$, time-of-flight, transition-radiation).

![Detector efficiency for particle identification at different intervals of momentum, from about 100 MeV/c up to a few GeV/c. The efficiency of the TPC can be extended up to tens of GeV/c, by measuring particle separation in the relativistic rise of $dE/dx$.](image)

A good introduction to bayesian statistic can be found in the references [119] [120] [121]. The ‘Bayesian’ approach differs from the (standard) ‘frequentist’ approach in the definition of probability. In Bayesian statistics the probability is not defined as the frequency of occurrence of an event in a large set of repetitions of identical experiments (as frequentists do), but as the plausibility that a hypothesis is true given the available informations. The ‘probability’ in the Bayesian view is not a property of the random observable, but a quantitative encoding of our state of knowledge about these observables. The main consequence is that, in data analysis, the Bayesian approach can assign probabilities to hypotheses.

Charged particle identification in ALICE implements 5 hypothesis: $e$, $\mu$, $\pi$, $K$ and $p$ (meaning both particles and anti-particles). Each detector class produces the between 50 and 150 $\mu m$, depending on the number of reconstructed tracks [25].
conditional probability density function (or detector response functions) \( r(s|i) \) to observe a signal \( s \) when a particle of type \( i \) \((i = e, \mu, \pi, K, p)\) is detected. It is reasonable to assume that the functions \( r(s|i) \) reflect only properties of the detector and do not depend on other external conditions, like event and track selections.

The probability to be a particle of type \( i \) if the signal \( s \) is observed, \( w(i|s) \), depends not only on the probability density function \( r(s|i) \), but also on the amounts of this type of particles in the considered sample, i.e. the ‘a priori’ probability \( C_i \) to find the particle \( i \) in the detector. The quantities \( C_i \) (the relative concentrations of particles of type \( i \)) do not depend on the detector properties, but reflects the external conditions, like particle ratios and track selections. The underlying assumption of this approach is that \( C_i \) and \( r(s|i) \) are not correlated. The detector response function \( r(s|i) \) can be parametrized using available experimental data, e.g. for each track reconstructed in the TPC, \( r(s|i) \) (where \( s \) is the assigned \( dE/dx \) measurement) is a Gaussian with centroid \( \langle dE/dx \rangle \) given by the Bethe-Bloch formula and width calculated from simulated data.

The probability of each particle hypothesis is given by Bayes formula:

\[
w(i|s) = \frac{r(s|i)C_i}{\sum_{j=e,\mu,\pi,...} r(s|j)C_j}.
\]

(2.4)

This method can be extended to combine P.Id measurements from several detectors, considering the whole system of different contributing detectors as a single block. The combined P.Id weights \( W(i|\bar{s}) \) are calculated in a similar way to eq.2.4:

\[
W(i|\bar{s}) = \frac{R(\bar{s}|i)C_i}{\sum_{k=e,\mu,\pi,...} R(\bar{s}|k)C_k},
\]

(2.5)

where \( \bar{s} = s_{ITS}, s_{TPC}, s_{TRD}, s_{TOF}, ... \) is a vector of the signals registered in the various detectors, \( C_i \) are the ‘a priori’ probabilities to be a particle of the type \( i \) (same as in eq.2.4) and \( R(\bar{s}|i) \) is the ‘combined response function’ of the whole system of detectors.

The ‘a priori’ probabilities \( C_i \) must reflect the relative concentrations of particles of \( i \)-type belonging to the sample of interest. In a simple approach \( C_i \) can be assumed to be equal for all \( i \) (i.e. same amount of \( e^\pm, \mu^\pm, \pi^\pm \), etc.), however in many cases is possible to do better. For instance it is possible to start with equal ‘a priori’ probabilities for all particles, and update those number event by event with the detected particle ratios. This method has been successfully tested on AliRoot simulations and it shows that the ‘a priori’ probabilities quickly converge [122].

### 2.3.3 Secondary vertices

Thanks to the good spatial resolution of the ITS, the ALICE central barrel detector is capable to reconstruct secondary decay vertices \((V^0)\), cascade decays and kink topologies (i.e. a track deviating from its trajectory due to decay into a neutral plus a charged particle).
The $V^0$ finding algorithm is executed after the tracking procedure, and runs over the final $AliESDtrack$ objects stored in the ESD. The algorithm starts with the selection of secondary tracks, e.g. tracks with a too large impact parameter with respect to the primary vertex. Each secondary track is combined with all the other secondary tracks of opposite charge, and different cuts are applied for the positive and the negative track impact parameters. With the helix track parametrization the minimum Distance of Closest Approach (DCA) between the two tracks is calculated, both in 3-dimensions and in the transverse plane, pairs of tracks are rejected if their DCA is larger than a given value.

The reconstructed $V^0$ candidates are then stored in the $AliESD$ as $AliESDV0$ objects. They can be included in the $AliFlowEvent$ and submitted to the correlation analysis (see sec.3.3).
Chapter 3

Flow Analysis in ALICE

This chapter will give an overview of the flow analysis with the Event Plane Method [123], introducing the terminology and describing the strategy from the experimental point of view (sec.3.1 and 3.2). The chapter includes a description of the analysis code as it has been implemented for the ALICE environment (sec.3.3).

Other flow analysis techniques have been also developed (i.e. the Cumulants and the Lee-Yan zeros), and some of them are currently under implementation at ALICE. A brief overview will be given in sec.3.4.1, pointing out the main advantages and disadvantages with respect to the event plane method.

3.1 Aim of the Flow Analysis

As introduced in sec.1.1, in a non central heavy ion collision, the impact parameter \( b \) together with the \( z \) axis (the beam-line) define the Reaction Plane (see fig.1.3). The azimuthal angle between the reaction plane and the plane \( x - z \) (measured in the lab frame \(^1\)) is called \( \Psi^{true} \) or \( \Psi_R \) (see fig.1.5).

Due to the geometry of the collision, the overlap region between the two nuclei has an initial spatial anisotropy. This causes an angular dependence of the pressure gradient (which is larger along the smallest direction of the overlap, i.e. the direction of \( b \)) and therefore the evolution of the system follows an anisotropic expansion: more particles are radiated along the direction of the reaction plane. The asymmetry observed in the final momentum distribution of the radiated particles is called anisotropic flow (see sec.1.1).

A Fourier expansion of the Lorentz invariant distribution of outgoing momenta is the usual way to characterize anisotropic flow [123]:

\[
E \frac{d^3N}{dp^3} = \frac{1}{2\pi} \frac{d^2N}{p_T dp_T dy} \left( 1 + \sum_{n=1}^{+\infty} v_n(p_T, y) \cos [n(\phi - \Psi_R)] \right),
\]

\( ^1\)In the laboratory frame \( z \) is the beam-line direction, \( y \) is the vertical direction, and \( x \) is the third cartesian axis.
where $\phi$ is the azimuthal angle of outgoing particles and $\Psi_R$ is the reaction plane angle, both measured in the laboratory frame (see also eq.1.1).

The Fourier coefficients $v_n$ are then given by:

$$v_n = \langle \cos [n(\phi - \Psi_R)] \rangle .$$

(3.2)

where the average is taken over all particles of all events. For odd harmonics, $v_n$ changes sign between forward and backward rapidity because particle distributions are equal within two hemispheres $\pm y$ (or $\pm \eta$ in symmetric collision) but opposite in sign for global momentum conservation.

![Diagram](image)

**Figure 3.1.** Left: transverse picture of elliptic flow, projected on the transverse plane $(x - y)$ and side picture of directed flow, projected on the beam-vertical plane $(z - y)$. Right: physical meaning of $v_2$ as a modulation of the $dN/d\phi$ distribution with respect to the reaction plane $\Psi$.

We call the first Fourier coefficient $v_1$ directed flow and the second coefficient $v_2$ elliptic flow (see sec.1.1). Figure 3.1(a) gives an intuitive picture of these two observables, showing the effect of $v_2$ on the transverse plane and the effect of $v_1$ on the beam-vertical plane. Fig.3.1(b) shows the physical meaning of $v_2$ as a modulation of the azimuthal distribution $dN/d\phi$ with respect to the reaction plane.

Higher harmonics can also be studied, but their magnitude is much smaller. Recent studies have shown that the ratio $v_4/v_2^2$ is an important observable which provides information about the ideal fluid behavior of the system [124]. However, this thesis is devoted to the study of elliptic flow.

The method applied in the analysis is the Event Plane method, introduced by Danielewicz and Odyniec in 1985 [125] and generalized by Poskanzer and Voloshin [123]. It has been successfully used in many heavy ion experiment, from AGS to SPS and RHIC, and in particular by the STAR collaboration who wrote a specific software package (from which the present analysis code has been developed). The event plane method and its implementation in the ALICE environment are extensively described in the following sections.
The event plane analysis implemented for ALICE can been applied to identified/unidentified charged particle and to neutral strange particles ($K^0$, $\Lambda^0$) reconstructed as neutral secondary vertices from their decay products. However, due to time limits and to continuous changes in the reconstruction framework, the analysis has been limited to unidentified charged particles (see chap.5).

### 3.2 Event Plane Analysis method

The event plane method is a straightforward consequence of eq.3.2, with the only remark that the true (non observable) reaction plane of the collision is replaced by the experimentally reconstructed ‘event plane’.

Therefore, the first step of the analysis is the reconstruction (on an event basis) of the event plane $\Psi$ from the anisotropy of the event itself.

The ‘observed’ event plane $\Psi^{obs}$, also called $\Psi_n$ to emphasize the harmonic used in the calculation, approximates the true reaction plane $\Psi_R$ and can be used as a replacement with the consequences of under-estimating the true particle-plane correlation, but this can be kept under control (see 3.2.1).

The procedure to extract $\Psi^{obs}$ from the emitted particles starts with the reconstruction of the flow vector, also called $\vec{Q}$ vector due to the original notation [123], defined for each event as:

$$
\vec{Q}_n = \left( \frac{\sum_i w_i \cos(n\phi_i)}{\sum_i w_i \sin(n\phi_i)} \right) = Q_n \left( \frac{\cos(n\Psi^{obs}_n)}{\sin(n\Psi^{obs}_n)} \right),
$$

(3.3)

where the sum includes all detected particles.

However, since not all the particles have the same flow $^2$, weight coefficients $w_i$ are there to enhance the contribution of particles with larger flow in order to make the $\vec{Q}$ vector a better defined observable. The choice of optimal weights will be discussed in section 3.2.3, anyway it is always possible to use $w_i = 1$ for all the particles.

For the 1$^{st}$ harmonic event plane (which is used to study odd harmonic coefficients), the weights $w_i$ must have opposite signs in forward and backward rapidity for reflection symmetry $^3$.

The observed event plane angle of the $n^{th}$ harmonic is given by the orientation of $\vec{Q}_n$:

$$
\Psi_n = \frac{1}{n} \arctan \left( \frac{Q^y_n}{Q^x_n} \right),
$$

(3.4)

by construction $\Psi_n \in \left[ -\frac{\pi}{n}, \frac{\pi}{n} \right]$.

The flow coefficients $v_n$ are obtained from the correlation between $\vec{Q}_n$ and the momentum of the emitted particles in the transverse plane. At the $n^{th}$ harmonic,

$^2$E.g. the observed $p_T$ dependence of $v_2$, see sec.1.3.4.

$^3$In symmetric collisions, the particle distribution is equal but opposite in momentum around the center of mass, and the average $\cos(\phi)$ and $\sin(\phi)$ with $\phi \in [0, 2\pi)$ is 0.
this correlation is calculated by averaging the cosine of the difference between the azimuthal angle of the outgoing particle $\psi_i$ and the event plane angle $\Psi_n$:

$$v_{n}^{\text{obs}} = \langle \cos [km(\phi - \Psi_m)] \rangle .$$  \hfill (3.5)

The average is taken over all the selected particles in all events, in the centrality class under study. What is measured in this way is the ‘observed’ flow $v_{n}^{\text{obs}}$, which magnitude is lower than the ‘true’ flow because in general $\Psi_n \neq \Psi_R$.

It is also possible to extract the event plane angle from any harmonic $m$ and use it in the calculation of the flow coefficient $v_n$, with $n \geq m$ and $n = km$ for an integer $k$:

$$v_{n}^{\text{obs}} = \langle \cos [km(\phi - \Psi_m)] \rangle .$$  \hfill (3.6)

In this way the sign of $v_n$ is determined relatively to $\Psi_n$, but the resolution deteriorates as $k$ increases [123]. Due to the low sensitivity to $v_1$ with the ALICE central barrel detector 4, this strategy has not been applied in the present analysis.

The difference between the true $\Psi_R$ and the reconstructed $\Psi_n$ gives the resolution of the event plane, i.e. the accuracy of $\Psi_n$ to reproduce the true orientation of the reaction plane $\Psi_R$.

From the observed $v_2^{\text{obs}}$, the corrected values of the flow coefficients are obtained as:

$$v_n = \frac{v_{n}^{\text{obs}} \langle \cos [km(\Psi_m - \Psi_R)] \rangle}{\langle \cos [km(\Psi_m - \Psi_R)] \rangle} = \frac{\langle \cos [km(\phi - \Psi_m)] \rangle}{\langle \cos [km(\Psi_m - \Psi_R)] \rangle} .$$  \hfill (3.7)

Following the prescription of the event plane method [123], it is possible to experimentally estimate the average $\langle \cos [km(\Psi_m - \Psi_R)] \rangle$ using the sub-events (see also sec.3.2.1).

### 3.2.1 Resolution

The (full-event) resolution of the event plane $\Psi_n$ is defined as the cosine of the difference $\Psi_n - \Psi_R$. For known value of $v_n$, it can be calculated as [123]:

$$\text{res}_{\text{full}} = \langle \cos [km(\Psi_m - \Psi_R)] \rangle = \frac{\sqrt{2}}{2\chi_m e^{-\frac{\chi_m^2}{2}}} \times \left[ I_{k-\frac{1}{2}}(\chi_m^2/4) + I_{k+\frac{1}{2}}(\chi_m^2/4) \right] ,$$  \hfill (3.8)

where $\chi_m = v_m/\sigma$, and $\sigma = \sqrt{\frac{1}{2 M \langle w^2 \rangle}}$ (choosing $w_i = 1$ then $\chi_m = v_m/\sqrt{2M}$, $v_m$ is the true flow). $M$ is the particle multiplicity used in the calculation of $Q$, and $I_x$ are modified Bessel functions of order $x$.

Since the resolution deteriorates as $k$ increases (eq.3.8), elliptic flow is measured better by using the second harmonic event plane $\Psi_2$. Moreover, eq.3.8 is monotonically increasing with $\chi_m \propto v_2\sqrt{M}$. This gives a good resolution for high

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4 The directed flow increase with rapidity [126] [34], therefore $v_1$ is small in the range of acceptance of the present analysis ($|\eta| < 0.9$), giving a poor resolution on $\Psi_1$. 
3.2 Event Plane Analysis method

The sub-event method to calculate the resolution [123] splits the event into two separated equal multiplicity sub-events. They can be randomly chosen or selected by positive/negative pseudorapidity.

For each sub-event the sub-event plane angle $\Psi_n^A$ is calculated in the same way as in 3.3 and 3.4:

$$\Psi_n^A = \frac{1}{n} \arctan \left( \frac{\sum_{i \in A_{sub}} w_i \sin (n \phi_i)}{\sum_{i \in A_{sub}} w_i \cos (n \phi_i)} \right), \quad (3.9)$$

where the sum is restricted to the particles in the sub-event.

The difference $\Delta \Psi_{sub} = \Psi_A - \Psi_B$ already gives the accuracy of the measured sub-event plane (or sub-event resolution, $\text{res}_{sub}$):

$$\text{res}_{sub} = \langle \cos \left[ n (\Psi_n^A - \Psi_R) \right] \rangle = \sqrt{\langle \cos \left[ n (\Psi_n^A - \Psi_n^B) \right] \rangle}. \quad (3.10)$$

At very low resolution ($\langle \cos \left[ n (\Psi_n^A - \Psi_R) \right] \rangle \ll 1$) the equation 3.8 is approximately linear in $\chi_m$, which is proportional to the square root of the multiplicity $M$ used in the calculation. Taking into account that the full-event has twice the multiplicity of the sub-event, a first estimate of the full-event resolution is given by:

$$\langle \cos \left[ n (\Psi_n - \Psi_R) \right] \rangle \approx \sqrt{2 \langle \cos \left[ n (\Psi_n^A - \Psi_n^B) \right] \rangle}. \quad (3.11)$$

For higher values of the resolution ($\langle \cos \left[ n (\Psi_n - \Psi_R) \right] \rangle \approx 1$) this approximation does not hold, and to correctly extrapolate the full-event resolution (with $\chi$ and $\sigma$ of eq.3.8) an iterative process is needed (and it has been implemented in the analysis code): the first estimate of the full-event resolution from eq.3.8 (if $\sqrt{2} \times \text{res}_{sub} < 1$, otherwise the sub-event resolution is used) is applied to $v_{n}^{obs}$ to obtain $v_{n}^\prime$, which is then used to calculate $\chi_n$. From equation 3.8 a new value resolution is calculated and applied again to $v_{n}^{obs}$ to obtain $v_{n}''$, and so on. The iteration goes on until the variation at each step becomes smaller than a lower limit, at that point the procedure stops and the last calculated resolution is taken. It turns out that such a procedure quickly converges, and just a few steps are needed to obtain a stable estimate of the full-event resolution.

3.2.2 Autocorrelation

The flow coefficients $v_n$ are meant to measure the average correlation between each particle and the rest of the event. However, the presence of the particle $i$ in the

---

5 Other ways to split the event into two separate equal multiplicity sub-events can be used, e.g. separating positively and negatively charged particles. Any method could work, as long as no bias is introduced in the azimuthal distribution. In the present analysis only $\eta$ and random sub-events have been used, in the first case particles are simply divided into positive and negative pseudorapidity, in the second case particles are randomly separated into two arrays of equal multiplicity.
calculation of the event plane slightly moves the direction of $\vec{Q}_n$ toward the direction of $\vec{p}_i$, introducing a small but not negligible ‘spurious’ correlation between $\phi_i$ and $\Psi_n$, and therefore, a bias on the flow measurement.

There are two ways to avoid auto-correlation, both implemented in the flow analysis code:

**Sub-event correlation:** the event is split into two sub-events and each particle $i$ is correlated to the event plane angle $\Psi_{n,i}$ calculated from the opposite sub-event. The average $v_n$ is calculated as:

$$v_n = \frac{1}{2} \left( v_n^A + v_n^B \right),$$

where $v_n^A$ and $v_n^B$ are calculated as:

$$v_n^A = \frac{1}{N/2} \sum_{i \in A} \left( \cos \left[ n(\phi_i - \Psi_n^B) \right] \right) \langle \cos \left[ n(\Psi_n^B - \Psi_R) \right] \rangle,$$

where the term at the denominator is the resolution of the sub-event.

**Full-event correlation:** for each particle $i$ the event plane angle $\Psi_{n,i}$ is re-calculated by subtracting the $\vec{p}_i$ from $\vec{Q}_n$. Eq.3.7 is re-written as:

$$v_n = \frac{1}{N} \sum_{i=1}^{N} v_{n,i} = \frac{1}{N} \sum_{i=1}^{N} \left( \cos \left[ n(\phi_i - \Psi_{n,i}) \right] \right) \langle \cos \left[ n(\Psi_n - \Psi_R) \right] \rangle,$$

where $\phi_i$ is the azimuthal angle of the particle $i$, and $\Psi_{n,i}$ is the event plane angle, calculated from a selection of particles that does not contain the particle $i$. The denominator expresses the resolution of the full-event.

As shown, in the first case $v_n^{\text{obs}}$ is corrected by the resolution of the sub-events (equation 3.10), in the second case $v_n^{\text{obs}}$ is corrected by the full-event resolution, calculated from equation 3.8. In other words, for the sub-event correlation $v_2 = v_2^{\text{sub}}/\text{res}_{\text{sub}}$ (eq.3.10), for the full-event correlation $v_2 = v_2^{\text{full}}/\text{res}_{\text{full}}$ (eq.3.8). Since the resolution is proportional to $\sqrt{M}$, $\text{res}_{\text{full}} > \text{res}_{\text{sub}}$ and as well $v_2^{\text{full}} > v_2^{\text{sub}}$. The ratio between $v_n$ and the resolution should compensate for the difference, so that the flow coefficients calculated in both ways are expected to be equal within the statistical error $^6$.

Because the $\vec{Q}_n$ vector is better defined when more particles are used in its calculation (see eq.3.3), the full-event correlation seems to be the best choice, however the sub-event correlation works better in reducing non-flow effects (see section 4.1). Applying the two methods in parallel provides a useful cross-check.

$^6$This may not be true when non-flow effects are present, see section 4.1.
The effect of auto-correlations is larger at lower multiplicity and it becomes smaller when the multiplicity is high (and the bias of a single particle on the direction of $\vec{Q}_n$ becomes less important). However, if also the 'true' flow is small (e.g. central events), the auto-correlations can dominate the measurement.

For simplicity of notation, here and in the following the event plane angle is just written as $\Psi_n$, giving for granted the above discussion.

### 3.2.3 Weights

Weight coefficients $w_i$ are used in the calculation of the $\vec{Q}_n$ vector to make it a better defined observable and increase the resolution of the event plane. Weights should be chosen in such a way to enhance the contribution of particles with higher flow, since they define the direction of $\vec{Q}_n$ better. Ideal weights should be proportional to $v_n$ itself [127].

Experimentally it is observed that the elliptic flow increases with the transverse momentum (high $p_T$ fragments are more likely radiated along the reaction plane) [128], therefore a good choice of the weights for the calculation of $\vec{Q}_2$ can be the transverse momentum itself or some monotonic function $w_i(p_T) \propto p_T$. In the analysis presented in the following chapters, the choice of the weight was determined by the shape of the input $v_2(p_T)$ used in the simulation\(^7\), and therefore:

$$w_i(p_T) = \begin{cases} p_T/p_{T}^{sat} & p_T < p_{T}^{sat} \\ 1 & p_T \geq p_{T}^{sat} \end{cases}$$  \hspace{1cm} (3.15)

with $p_{T}^{sat} = 2 \text{ GeV}/c$. This choice gives a small gain in resolution with respect to unitary weights (see ch.4 and 5).

As already mentioned, for odd harmonics of the event plane the coefficients $w_i$ must change sign for forward/backward rapidity. The weights for the calculation of $\vec{Q}_1$ can be chosen proportional to $y$ or $\eta$ (which change sign in the two opposite hemispheres).

The weight coefficients $w_i$ must also compensate for the azimuthal anisotropy in the detector acceptance, which may add spurious contributions at higher harmonics to the measured flow. However, such kind of correction is very detector specific and can be directly calculated from the observed $dN/d\phi$ distribution of reconstructed data before running the flow analysis (see the following section for the details).

### 3.2.4 Flattening Weights and Reconstruction Efficiency

Due the geometrical arrangement and the segmentation of the detecting volumes (in particular the TPC), the reconstruction efficiency in the ALICE central barrel is $\phi$ dependent.

\(^7\)In the real experiment the choice of weights is done in a later stage: once the shape of $v_2(p_T)$ has been reconstructed by running the analysis with unitary weights, results can be refined by applying weights that are proportional to the observed $v_2(p_T)$. 
Figure 3.2. (a) $dN/d\phi$ distribution of (from top): all generated particles (MC input), reconstructed tracks and reconstructed primary particles in the ESD passing the minimal event plane cuts (see sec.5.3.2), and all reconstructed secondaries in the ESD. The distribution of reconstructed tracks shows the 18 sectors of the TPC. (b) Efficiency correction ($\phi$ weights) calculated with eq.3.16, for all reconstructed tracks passing the minimal cuts, and for reconstructed primaries. Plot generated from the all simulated Hijing + GeVSIm events (see chapter 5 for the simulation details).

The overall $dN/d\phi$ distribution of fig.3.2(a) clearly shows the radial segmentation of the ALICE TPC. The dips in the distribution of reconstructed primaries correspond to the azimuthal coordinate of the cracks between the 18 sensitive pads on the outer walls of the TPC (see sec.2.1.2). The distribution of secondaries shows a double peak in correspondence of each dip, due to the amount of particles produced in the 18 iron bars of field degrader, located between each sensitive pad at the innermost radius of the TPC.

This azimuthal anisotropy in the reconstruction efficiency may introduce a spurious $18^{th}$ harmonic component to the observed particle distribution, biasing the direction of the reconstructed reaction plane. To correct for this effect we assume that the cumulative $\phi$ distribution from a large sample of events is flat in an ideal detector, this is generally true due to the random orientation of the impact parameter of the collision with respect to the laboratory frame.

This $\phi$ dependence of the reconstruction efficiency can be corrected by introducing $\phi$ weights inversely proportional to the azimuthal efficiency of each $\phi$ bin in the reconstructed $dN/d\phi$ distribution. Each track $i$ gets the weight $w_{\phi_i}$ calculated as:

$$w(\phi_i) = \frac{1}{N_{\phi_i}} \times \frac{\sum_{i}^{N_{bins}} N_{\phi_i}}{N_{bins}},$$

(3.16)

where $\phi_i$ is the azimuthal angle at which the track $i$ is emitted, and $N_{\phi_i}$ is the discrete bin in the histogram that contains $\phi_i$. The obtained weights are then used, together with the $p_T$ (or $\eta$) weights (see sec.3.2.3), in the calculation of $\vec{Q}_n$.

The $\phi$ weights must be calculated specifically for the set of cuts in use, to take
3.3 Implementation

into account the reconstruction efficiency of the specific track selection. Moreover, this procedure can be directly applied on real data without any further use of simulations (this is done, for instance, in the event plane analysis at STAR).

However, the azimuthal efficiency is not the same at all transverse momenta, being almost zero for very high \( p_T \) tracks flying along a crack in the TPC. A more precise estimate of the weights should be done in \( p_T \) bins, creating a two dimensional weight array \( w(p_T, \phi) \), but this approach would require a much larger statistic than the one available and therefore the \( \phi \) weights in use for the present analysis have been calculated irrespectively of \( p_T \).

### 3.2.5 Differential & Integrated Flow

In the present analysis, the elliptic flow has been studied as a global property of the whole event (we talk in this case of ‘integrated’ flow), and with respect to the transverse momentum of the particles \( p_T \) (we talk in this case of ‘differential’ flow). The dependence of \( v_2 \) with respect to other kinematic variables, such as \( y \) or \( \eta \), is approximated to be flat (see sec.1.3.4) and it has not been studied further.

Assuming the \( p_T \) bins used in the analysis are small enough (in the order of the detector resolution, see sec.5.1), we can consider the efficiency as approximately constant in each \( p_T \) bin. The differential flow is therefore calculated by restricting the average of equation 3.7 to separate kinematic windows:

\[
v_2(p_T) = \frac{\langle v_2 \rangle_{p_T}}{\text{res}_2} = \frac{\langle \cos[2(\phi - \Psi_2)] \rangle_{p_T}}{\langle \cos[2(\Psi_2 - \Psi_2')] \rangle}.
\] (3.17)

The differential flow coefficients, calculated at each \( p_T \) bin, describe the \( p_T \) dependence of \( v_2 \).

Existing results show that the differential shape of \( v_2(p_T) \) is a monotonically increasing function of \( p_T \) (see sec.1.3.4). In a real experiment the reconstruction efficiency is generally not flat with respect to the transverse momentum, and therefore to correctly calculate the total (integrated) \( \langle v_2 \rangle \) of the event, the particle average must be weighted by the reconstruction efficiency as a function of \( p_T \):

\[
\langle v_2 \rangle = \frac{1}{N_{\text{tot}}} \int_{p_T=0}^{\infty} v_2(p_T) \frac{dN'}{dp_T} dp_T = \frac{1}{\text{eff}N_{\text{obs}}} \sum_{p_T\text{bins}} v_2(p_T) \times \frac{dN_{\text{obs}}}{dp_T} \times \text{eff}(p_T).
\] (3.18)

The contribution to \( \langle v_2 \rangle \) from low \( p_T \) part of the spectra \( (p_T < 100 \text{ MeV}/c) \), where the reconstruction efficiency is \( \sim (0) \) is evaluated by extrapolating both \( v_2(p_T) \) and \( dN/dp_T \) to \( p_T = 0 \). See sec.5.3.4 for a practical example.

### 3.3 Implementation

The event plane analysis has been implemented for ALICE as a collection of ROOT C++ classes, under the name of AliFlow package. Its structure is similar to the
StFlow package [96], widely used for flow measurements by the STAR collaboration, starting from which the AliFlow classes have been developed.

The main object of the analysis is the AliFlowEvent, a high level object built from the ALICE Event Summary Data AliESD and optimized for the flow analysis. The most useful ESD information is extracted and organized into an efficient structure, which then is submitted to the analysis chain.

Unlike the StFlow package, that was built over a more complex framework (such as the Star Class Libraries SCL [96] [129]), the AliFlow package only depends on ROOT, which improves the portability. AliFlowEvents can be created from the KineTrees contained in the kinematic files produced by the event generators, while their creation from the AliESDs only requires some libraries from AliRoot. AliFlowEvents can be stored for later processing, and the analysis can be entirely executed in ROOT.

However, the parallel processing of ESDs and KineTrees and the one to one comparison between reconstructed and simulated particles (from which the efficiency is calculated) need the AliAnalysisTask machinery to be in place [130], and therefore the whole AliRoot framework (see sec.2.2).

![Flow diagram of the production chain. Events are generated, transported and reconstructed within the AliRoot framework (the reconstruction phase uses the same algorithm that will be used on real LHC events). The AliFlowMaker (embedded into an AliAnalysisTaskRL) translates the produced ESDs and KineTrees into AliFlowEvents, on which the flow analysis is later executed (see fig.3.4). Using the same set of cuts applied in the analysis (represented by the small diamond), the efficiency histograms are also filled at this step by a one to one comparison of simulated and reconstructed particles.](image)
3.3 Implementation

Figure 3.4. Flow diagram of the event plane analysis chain. After a first loop, where the $\phi$ weights are calculated from the $dN/d\phi$ distribution of all events, the analysis proceeds event by event and calculates the $Q_2$ vector and the ‘observed’ event plane angle $\Psi_2$ (for full- and sub-events). Selected particles (and $V^0$s) are then correlated to the ‘observed’ event plane to obtain $v_2$ as a function of $p_T$, while the event resolution is calculated from sub-events as described in sec.3.2.1. At the end of the event loop, the observed $v_2(p_T)$ is corrected by the average event plane resolution, and the integrated elliptic flow is calculated taking into account the efficiency corrections, as a function of $p_T$, from the efficiency histogram (see fig. 3.3).

3.3.1 Analysis Strategy

The flow analysis package is organized in two main steps (see fig.3.3 and 3.4):

(i) Flow Maker:

A parser reads the reconstructed Event Summary Data files and the KineTree files produced by AliRoot and creates *AliFlowEvents*. Only the most useful observable for the flow analysis are stored as data members of the *AliFlowEvent* class, i.e. few global event observables (main vertex position, particle multiplicity), the kinematic of the reconstructed tracks ($p_T$, $\eta$ and $\phi$) and the most sensitive variable for selecting good track candidates together with the p.Id. signal from the central barrel detectors. $V^0$ candidates can be also stored in a separate array.
Very loose quality cuts are applied at this step (e.g. only tracks with TPC signal from the ESD, only primary hadrons from the KineTree).

- Data are organized into AliFlowEvent objects, which can be stored on disk for later analysis.
- If the AliESD loop is executed in an AliAnalysisTaskRL or an AliSelectorRL, efficiency corrections are also calculated.

Fig.3.3 gives a schematic view of the creation of AliFlowEvents, starting from an AliRoot simulation.

(ii) Flow Analysis:

The analysis runs over AliFlowEvent objects, it can be executed on the fly during the parsing process, or executed later on stored files. Fig.3.4 gives a schematic view of the event plane analysis starting from the AliFlowEvent.

- A first loop on the event sample produces the flattening $\phi$ weight histogram (see section 3.2.4), this is usually done over the entire sample available.
- A second event loop performs the calculation of the event plane event by event and the correlation analysis, the observed elliptic flow $v_2^{obs}(p_T)$ of selected particles is stored in a profile histogram (ROOT TProfile).
- At the end of the loop, the resolution of the full- and the sub- events is calculated by averaging $\cos(\Delta \Psi_2)$ from all events in the selected centrality class. The observed $v_2(p_T)$ is corrected by the event plane resolution, if efficiency corrections are available, the integrated flow $\langle v_2 \rangle$ is also calculated.

In a single loop, different track selections can be used for the calculation of the event plane, the resulting $v_2$ and event plane resolution are calculated in parallel. The selection of tracks used for the event plane determination must be defined previous to the first loop, to correctly calculate the flattening $\phi$ weights.

A separate selection is applied to the particles entering the correlation analysis (the ones entering the average $\langle \cos 2(\phi - \Psi_2) \rangle$), which can include more strict cuts for the isolation of primaries or for particle identification. The efficiency corrections must be calculated according to the set of cuts used for the correlation analysis.

The complete list of C++ classes that have been implemented for the event plane analysis is given in appendix A, together with a brief description of their purposes.

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8The functionalities of the AliRunLoader (in particular, the access to the AliStack) are needed to make a one to one comparison between reconstructed tracks and simulated particles (see sec.2.3).
3.4 Other Analysis Methods

3.3.2 The AliFlow package

The AliFlow package is included in the standard release of AliRoot\(^9\), and can be compiled together with the AliRoot framework.

The classes related to the analysis (everything except the \textit{AliFlowMakers}) can also be exported and compiled as a standalone ROOT library\(^10\), and loaded into ROOT to execute the analysis over existing \textit{AliFlowEvents}.

A full set of macros, to make the \textit{AliFlowEvents}, to produce $\phi$ weights and to run the analysis, is also included in the package. Every class is provided with an inline documentation that can be compiled with the standard ROOT \textit{THtml} class to produce a HTML layout [131].

However, due to the many recent modifications in AliRoot (e.g. the \textit{AliESD} has been replaced by the \textit{AliAOD}, the \textit{AliAnalysisTaskRL} has been taken out), some of the functionality described above may not be working at the present time. The latest version of AliRoot on which the AliFlow package has been tested to work is v4.04-Rev14.

3.4 Other Analysis Methods

Beside the event plane analysis described in sec.3.2, other methods to extract the flow coefficients $v_n$ from heavy ions data have been developed in the last years. A brief description of these methods will be given in this section, pointing out their main advantages and disadvantages (see sec.3.4.1).

The event plane analysis can be seen as a particular case of two-particle correlation method [132], in this view the analysis can be extended further to $2k$-particle correlations calculated with the ‘cumulant’ method [133], and when $k$ is pushed to infinity we end up with the ‘Lee-Yan Zero’ method [134].

Pair-Correlation method

Since all particles are correlated to the reaction plane, they are also indirectly correlated to each other, the anisotropic flow can therefore be measured by averaging the observed two-particle azimuthal correlations, without previous determination of the event plane angle [132].

In this approach, the integrated flow at the $n^{th}$ harmonic is calculated as:

$$\left\langle v_n \right\rangle^2 = \left\langle \cos \left( n(\phi_i - \phi_j) \right) \right\rangle,$$

where $i$ and $j$ run over all the particles in the event, and the average is taken over the whole centrality class of interest.

---

\(^9\)The package is included into the ‘Physics Working Group 2’ (soft physics) folder, under AliRoot/PWG2/FLOW.

\(^10\)The package is compiled with ‘root-cint’ [94] and ‘gcc’ [95] as a shared object library (named \textit{AliFlow.so}). These kind of libraries can be loaded in ROOT (see sec.2.2.1).
From the integrated flow, the differential flow can be calculated as:

\[ v_n(p_T) = \frac{\langle \cos [n(\phi_i - \phi_j)] \rangle}{\langle v_n \rangle}, \]  

(3.20)

where \( j \) is now limited to a specific \( p_T \) bin, and \( i \) runs over all the particles in the event.

The pair correlation method does not need to include corrections for the detector anisotropy [135]. On the other hand, the method does not reconstruct an event plane.

**Cumulant method**

Eq. 3.19 can be seen as the construction of a two-particle correlator, similar to the 2\textsuperscript{nd} order cumulant. More in general, the cumulant approach considers multi-particle correlations [136].

The cumulant of \( 2k \)-particle azimuthal correlations \( c_n \{2k\} \) (where \( n \) is the harmonic, and \( 2k \) is the order of the cumulant), is a quantity built with all the measured azimuthal correlations up to order \( 2k \):

\[ c_n \{2k\} = \langle e^{in(\phi_1+...+\phi_k'-\phi_{k'+1}-...-\phi_{2k'})} \rangle, \]

(3.21)

with \( k' + k'' \leq 2k \). For \( k = 1 \) the real part of the eq.3.21 reduces to eq.3.19:

\[ \Re (c_n \{2\}) = \Re \left( \langle e^{in(\phi_1-\phi_2)} \rangle \right) = \langle \cos [n(\phi_1 - \phi_2)] \rangle. \]

(3.22)

The advantage of the cumulant of \( 2k^{th} \) order is that it is insensitive to the contribution of lower order correlations, so that only the genuine \( 2k \)-particle correlation remains.

From cumulants is possible to calculate the integrated flow \( V_n \), defined here as the average projection over the reaction plane of the event flow vector \( \vec{Q}_n \) (see also eq.3.3):

\[ V_n = \left\langle \sum_j \cos [n(\phi_j - \Psi_R)] \right\rangle = M v_n. \]

(3.23)

Depending on the order of the cumulant, \( V_n \) is calculated as:

\[ V_n \{2\}^2 = c_n \{2\}, \quad V_n \{4\}^4 = -c_n \{4\}, \quad V_n \{6\}^6 = \frac{c_n \{2\}}{4}, \ldots \]

(3.24)

Differential flow can also be determined with cumulants, for the details refer to [137].

From the practical point of view, the calculation of cumulants starts from the generating function [137]:

\[ G_n(z) = \left\langle \prod_{j=1}^{M} [1 + w_j (z e^{-in\phi_j} + z^* e^{in\phi_j})] \right\rangle, \]

(3.25)
where \( z \) is a complex number and \( z^* \) is its complex conjugate. The product runs over all the particles in each event, and the average is taken over all events. The \( 2k^{th} \) order cumulant is given by the coefficient of \( z^{2k} \) in a series expansion of the logarithm of \( G_n \). The function \( G_n \) is evaluated in a few points in the complex plane around the origin \( z = 0 \), and by taking the logarithm at each of these points it is possible to interpolate the next derivatives of \( \ln(G_n) \) and obtain the cumulants \( c_n \{2k\} \).

The elliptic flow calculated with the \( 2k^{th} \) order cumulant is only affected by non-flow correlations from \( 2k \)-particles, which scales as \( N^{1-2k} \) (\( N \) is the total multiplicity of particles in an event [133]). Therefore, the \( 4^{th} \) order cumulant is already enough to systematically remove all the non-flow effects due to two- and three-particle correlations (e.g. particle decays), but it does not remove any genuine four- or more particles correlations (e.g. jets).

Since flow is a collective effect, higher order cumulants can be preferable to remove non-flow correlations (the \( 2k^{th} \) order cumulant removes non-flow effects due to \( (2k-1) \)-particle correlation), but their calculation become more and more tedious as \( k \) increases. The cumulant method can be used for different orders and compared to each other to cross-check the results \(^{11} \) [141]. Also the cumulant method does not measure an event plane.

Lee-Yan Zero method

Extending the cumulant method to an infinite order cumulant \( c_n \{\infty\} \) leads to the Lee-Yan zero method [134].

Similarly to the cumulant method, a generating function is defined [142]:

\[
G^\theta(r) = \left\langle \prod_{j=1}^{M} [1 + irw_j \cos(n(\phi_j - \theta))] \right\rangle, \tag{3.26}
\]

where \( r \) is a real positive number and \( \theta \) is an angle between 0 and \( \pi/n \). The product involves all the particles in each event, and the average is taken over the events.

The behavior of the zeros in the generating function \( G^\theta \) reflects the presence of collective flow in the system [134]. In particular the first zero of the generating function is directly related to the magnitude of anisotropic flow.

In practice, the Lee-Yan zero method starts with the calculation of \( G^\theta \) for many values of \( \theta \), and for each the first minimum of the absolute value \( |G^\theta(r)| \) is calculated. The value of \( r \) at the minimum \( r^\theta_0 \) is a good approximation of the first zero.

The integrated flow \( V_n \) (as defined in eq.3.23) is then calculated as:

\[
V_n^\theta \{\infty\} = \frac{j_{01}}{r^\theta_0} , \tag{3.27}
\]

\(^{11}\)In the contest of STAR, a comparison between \( \nu_2 \{2\} \) and \( \nu_2 \{4\} \) is used to estimate the magnitude of non-flow effects [138] (see also [139] and [140]).
where \( f_{01} \) is a constant (\( f_{01} = 2.4 \), see [142] and [134]). The integrated flow values calculated in this way are then used to obtain the flow coefficients at different harmonics and the differential flow [142].

The Lee-Yan zero method provides the smallest systematic error of all the methods, practically removing all non-flow effects (from any \( k \)-particle correlation). The main limitation of this method comes from statistical errors, which decrease only logarithmically with the number of events and dramatically depend on the \( \chi_n \) (\( \chi_n \simeq v_n \sqrt{2M} \), see eq.3.8), therefore it is not always applicable.

In some very recent developments, a way to estimate the event plane using the Lee-Yan zero method has been devised, by recasting it in a form similar to the standard event plane analysis [143]. Non-flow correlations are eliminated by using the information from the length of the flow vector, in addition to the event-plane angle.

### 3.4.1 Applicability

A detailed discussion about the sensitivity of the three methods, with an estimate of their systematic and their statistical errors, is given in section V and VII of reference [134]. The main conclusions are summarized in the following.

#### Systematic Error

The main systematic error of flow measurement is due to few-particle correlations (resonance decays, jets, transverse momentum conservation), generally classified as ‘non-flow’ (see sec.1.4). Non-flow effects become more important at low multiplicity and low genuine collective flow (see chap.4). Two-particle correlation methods, as the event plane method itself, cannot disentangle the genuine flow from any other source of azimuthal correlation, therefore the evaluation of the systematic error due to non-flow has been studied in details in sec.4.1 by applying the event plane method to Hijing simulations without flow. On real data (e.g. at STAR) non-flow effects are measured from the difference between \( v_2 \{2\} \) obtained with the event plane method and \( v_2 \{4\} \) obtained with \( 4^{th} \) order cumulant [138].

The \( 2k^{th} \) order cumulant removes non-flow up to \( (2k - 1) \)-particle correlation, therefore the systematic error due to non-flow becomes smaller and smaller when using higher orders cumulants. The minimum systematic error is reached with the Lee-Yan zero method.

Roughly, the relative systematic error of the event plane method is:

\[
\frac{\delta v_n}{v_n} = O \left( \frac{1}{M v_n^2} \right). \tag{3.28}
\]

While for the \( (2k)^{th} \) order cumulant (with \( 2k > 4 \)) the systematic error is:

\[
\frac{\delta v_n}{v_n} = O \left( \frac{1}{M^{2k-1} v_n^{2k}} \right), \tag{3.29}
\]
only if \( v_n < \frac{1}{M^{1 - \frac{1}{2}} - \frac{1}{2}} \), otherwise it becomes of the same magnitude of eq. 3.30. The Lee-Yan zero method has the smallest systematic error, which is approximately:

\[
\frac{\delta v_n}{v_n} = \mathcal{O} \left( \frac{1}{M} \right).
\] (3.30)

Flow is unambiguously identified when it is larger than other spurious correlations, this defines the conditions under which any of the three methods can be applied. Therefore, the condition of applicability of the two-particle correlation method is \( v_n \gg \frac{1}{M^{1/2}} \), it is \( v_n \gg \frac{1}{M^{3/4}} \) for the 4th order cumulant, and \( v_n \gg \frac{1}{M} \) for the Lee-Yan zero method.

**Statistical Error**

The statistical error on flow measured with the three methods depends in general on the charged multiplicity and the magnitude of genuine flow, summarized by the parameter \( \chi_n \simeq v_n \times \sqrt{M} \) (which is related to the resolution of the event plane, see eq.3.8), and on the number of events entering the analysis.

For values of \( \chi_n \gg 1 \) the relative statistical error associated to the three methods is in the order of:

\[
\frac{\delta v_n}{v_n} = \mathcal{O} \left( \frac{1}{\chi_n \sqrt{N}} \right),
\] (3.31)

where \( N \) is the number of events entering the analysis.

The statistical error of the cumulant method becomes much larger for \( \chi < 1 \), and it increases with the cumulant order. The Lee-Yan zero method has the same problem for small values of \( \chi \) and it is not very reliable for \( \chi < 0.5 \). The statistical error of the event plane method always has the order of magnitude of eq.3.31, and a larger sample of events may compensate for low values of \( \chi_n \).

At LHC energy the resolution parameter is expected to be well above 1 on a wide range of centrality (see sec.1.3), so in principle all the methods can be applied.

In the present thesis, only the event plane method has been fully implemented and tested so far for the ALICE environment. The event plane method has been chosen because, due to its easy implementation and low level of abstraction, it is the most appropriate for the first day physics at ALICE:

- the method is quite intuitive and the mathematics behind is simpler than that of any other method, therefore it is easier to check the consistency of the results;

- the method provides a direct estimate of the event plane angle \( \Psi_n \), which is a needed input for other kinds of analysis (e.g. HBT, jets);
• when the analysis is done on simulations, the reconstructed $\Psi_n$ can be easily compared with the input of the simulations to optimize the applied cuts (see sec.5.3.2);

• theoretical predictions show that elliptic flow will be the dominant contribution to the azimuthal anisotropy of the events at LHC energy, minimizing the systematic due to non-flow;

• the resolution parameter $\chi$ is expected to be well above unit at LHC, but even in a worse scenario ($\chi \ll 1$) the event plane method provides the lowest statistical error on the measured $v_n$ (making it the most convenient method to quickly obtain some results);

• since the event plane method cannot disentangle genuine collective flow from non-flow correlations, it can be used on Hijing simulations to experimentally characterize non-flow effects (as it is done in sec.4.1).

However, for a longer term analysis at ALICE, the event plane method will be probably replaced by the more accurate ‘new’ Lee-Yan zero method [143], the implementation of which is currently under development.
Chapter 4

Feasibility of the Event Plane analysis

A large source of uncertainty on the measurement of elliptic flow is due to the unknown magnitude of non-flow effects at LHC energies, i.e. few-particle correlations not related to the reaction plane, such as jets and resonance decays.

In the present approach, non-flow effects have been simulated using Hijing (see sec.2.2.3). A first set of Hijing simulations has been produced with no genuine elliptic flow, in order to study the magnitude of the ‘apparent’ $v_2$ that would be reconstructed with the event plane analysis, and to characterize its multiplicity (centrality) dependence (see sec.4.1).

The magnitude of non-flow reconstructed from the Hijing events has been compared with an extensive set of GeVSim simulations, produced with different combinations of the two most sensitive observables, i.e. the multiplicity and the magnitude of $v_2$, leading to the feasibility ‘grid’ in fig.4.7 (see section 4.2).

Finally, a new set of Hijing simulations has been produced and boosted with the flow After-Burner, to study the interplay between genuine flow and non-flow effects (see sec.4.3).

4.1 Non-Flow estimate with Hijing

Non-flow effects have been studied using Hijing simulations, which by construction have no genuine flow ($v_2 = 0$). This approach offers the possibility to characterize the contribution of non-flow effects alone and to determine their magnitude, by applying the event plane analysis (described in sec.3.2) to the simulated events and comparing the azimuthal correlation between sub-events and between the reconstructed event plane and the simulated reaction plane.

A full detector reconstruction has been excluded because the aim of this preliminary study is to isolate and measure the non-flow correlations generated by Hijing. Therefore, the analysis was executed over the primary particles in the Hijing Kine-
Trees 1.

Hijing, as briefly introduced in sec.2.2.3, includes all known physics effects arising from the superposition of p-p collisions, such as jets, resonances and cascade decays. The implementation of Hijing also includes the possibility to switch on an internal parametrization of jet-quenching effects, which reproduces the energy lost in the medium by the leading parton of the jet.

Figure 4.1. (a) Charged multiplicity in one unit rapidity as a function of the impact parameter of the collision. The arrow on the x axis indicates what we consider ‘most central’ in our rescaled definition of centrality. (b) $dN_{ch}/d\eta$ distribution for 25,000 Hijing events with jet-quenching and resonance decays switched on, showing both the minimum bias distribution (simulated on the full range of impact parameters, $0 < |b| < 16$ fm) and the ‘rescaled’ one (with $7 < |b| < 14.5$ fm). For comparison, the $dN_{ch}/d\eta$ distribution of 25,000 Hijing events, with jet-quenching switched off, is also shown (events simulated on the ‘rescaled’ impact parameter range: $7 < |b| < 14.5$ fm).

Few remarks must be made about the current implementation of Hijing:

- The charged particle multiplicity produced is a factor 2 higher than the current predictions for LHC (see sec.1.3.3). As shown in fig.4.1(a), a way to reduce the produced multiplicity is achieved by rescaling the impact parameter. Therefore, we define ‘most central’ a collision with impact parameter $|b| = 7$ fm (leading to a charged multiplicity $dN_{ch}/d\eta = 2500 \pm 300$), and ‘most peripheral’ a collision with impact parameter $|b| = 14.5$ fm (where the charged multiplicity can be as low as few particles per event: $dN/d\eta \sim 0$). 2.

---

1The KineTree is the list of all simulated particles (see chap.2.2.2). Detector effects are treated in a separate step (see sec.5.1).

2In this set of Hijing simulations, the multiplicity is about 50% higher than the prediction given in sec.1.3.3, leaving room for the large uncertainties of the extrapolations of $dN/d\eta$ in Pb-Pb collisions at LHC. The upper limit on $b$ has the purpose of reducing the number of events with very low multiplicity, since the domain of ultra-peripheral collision is outside the scope of the flow analysis (see also sec.1.3).
4.1 Non-Flow estimate with Hijing

- The rescaling of the impact parameter introduces a clear bias in the event kinematics: a larger impact parameter implies less binary interactions and therefore a lower number of produced jets. Unfortunately the consequences of this approach on non-flow effects are not easy to quantify and the study of particle production processes in Hijing is beyond the purposes of the present thesis. However, a comparison with existing measurements made at STAR (where non-flow is calculated from the difference between $v_2 \{EP\}$ and $v_2 \{4\}$ [139] [140]) shows that the present approach well reproduce both the magnitude and the centrality dependence of non-flow effects.

- The effects of jet-quenching and resonance decays (which can be turned on and off in Hijing) have been studied separately to characterize the contribution of each of them to the observed non-flow. One side effect of jet-quenching is that the multiplicity becomes, on average, $60 - 70\%$ higher, probably due to the fact that the energy lost in the medium by the leading partons is transformed into soft radiation (low $p_T$ particles). For the selected range of impact parameters, fig.4.1(b) shows the multiplicity distribution with and without jet-quenching.

As discussed in sec.3.4 the event plane analysis is equivalent to a two-particle correlation method, where the azimuthal correlation is quantified by:

$$\langle v^2_2 \rangle = \langle \cos [2(\phi_i - \phi_j)] \rangle ,$$

(4.1)

where the average is taken between each pair of particles $i, j$ in the event. By its definition, any kind of few-particles correlation contributes to the reconstructed $v_2$.

According to the definition given in sec.1.4, the correlation between the reconstructed event plane and the true reaction plane (i.e. $\Psi_2^{true} - \Psi_2^{obs}$) due to ‘flow’ can be compared to the observed sub-events correlation $\Delta \Psi_2^{sub} = \Psi_2^A - \Psi_2^B$ to estimate the magnitude of non-flow effects. When the ‘true flow’ is 0, the observed sub-events correlation gives a direct measurement of the ‘non-flow’ contributions.

Fig.4.2 shows the average $\cos [2(\Delta \Psi_2)]$ for four sets of 25000 Hijing events each (with $7 < |b| < 14.5$ fm), testing all possible combinations of jet-quenching and strong resonances decays on/off. The leftmost bin shows the ‘true’ resolution of the 2nd harmonic event plane, defined as:

$$\langle \cos [2(\Psi_2^{true} - \Psi_2^{obs})] \rangle ,$$

(4.2)

while the following 3 bins (bin 2, 3 and 4) show the ‘observed’ resolution, extrapolated from the observed sub-event correlation $\Delta \Psi_2^{sub}$ using an iteration of eq.3.8 (see sec.3.2.1). Approximately:

$$\langle \cos [2(\Psi_2 - \Psi_R)] \rangle \approx \sqrt{2} \langle \cos [2(\Psi_2^A - \Psi_2^B)] \rangle ,$$

(4.3)
Feasibility of the Event Plane analysis

Figure 4.2. True and observed event plane resolution (defined as \( \langle \cos 2 (\Psi_{\text{true}} - \Psi_{\text{obs}}) \rangle \)), see sec.3.2.1), for 4 different sets of 25000 Hijing events (with 7 < |b| < 14.5 fm), in all possible combinations of jet-quenching and strong resonance decays on/off, using different definitions of sub-events (see text for the details). The two plots show the resolution of the event plane calculated without and with \( p_T \) weights respectively.

where \( \Psi^A_2 \) and \( \Psi^B_2 \) are the event plane angles calculated from two equal multiplicity sub-events. This is done for three different choices of sub-events (random sub-events, \( \eta \) sub-events and \( \eta \) sub-events with a gap of 1 unit at mid-pseudorapidity). In fig.4.2(a) the 2\(^{nd}\) harmonic \( \vec{Q} \) vector is calculated with unitary weights, while in fig.4.2(b) \( p_T \) weights are used (see sec.3.2.3).

It can be immediately noticed that the presence of jet-quenching effects introduces a small correlation with the true reaction plane \( \Psi_{\text{true}} \) (the ‘true’ resolution is larger, 1\(^{st}\) bin in fig.4.2(a) and (b)), and as expected, the use of \( p_T \) weights in the calculation of \( \vec{Q}_2 \) enhances the resolution of the event plane for both genuine flow or spurious non-flow effects (see also fig.4.3).

But the most interesting result is the large amount of non-flow correlations that is observed using the event plane method. The figure, however, indicates a way out: due to the fact that jets and decays cause particles to propagate in the same direction in \( \phi-\eta \) \(^3\), by splitting the event into two rapidity or pseudorapidity intervals most of the non-flow correlations are suppressed, and an even better suppression is achieved by choosing the sub-events well separated, e.g. by using a gap at mid-rapidity (but consequently the statistic is reduced).

It is also interesting to note that the presence of a detectable event plane (due, in this case, to the correlation of the products of jet-quenching with the true \( \Psi \)) makes the observed event plane resolution less sensitive to the choice of the sub-events (see second and last bin of fig.4.2(a)). This even better applies when the genuine elliptic flow is larger (see sec.4.3). In other words, for a large genuine elliptic flow signal the non-flow correlations become less important.

\(^3\)See, for instance, chap.6.8 of the reference [25].
Following the prescriptions of the event plane analysis [123], from the observed values of \( \cos \left[ 2 \left( \Delta \Psi_{sub}^2 \right) \right] \) the resolution of the event plane is calculated, which corrects the observed \( v_{2}^{obs} \) to take into account the uncertainty in the determination of the reaction plane (see sec.3.2). In case non-flow is dominating, the ‘apparent’ event plane resolution (which is always small, since the reaction plane is not clearly defined) gives a large boost to the observed \( v_{2}^{obs} \) (which is non-negligible, due to few particle correlation effects) leading to overestimate of the measured elliptic flow.

![Figure 4.3](image-url) Elliptic flow \( v_{2} \) as a function of \( p_{T} \) calculated w.r.t. the true reaction plane (squares) and the observed event plane with and without resolution correction (triangles and circles, respectively), the resolution is calculated from \( \Delta \Psi_{2}^{rand-sub} \). Two sets of 25000 Hijing events are shown (7 < |\( b \) | < 14.5 fm), one with jet-quenching and strong resonance decays switched off (empty markers), the other with everything switched on (full markers).

The transverse momentum dependence of non-flow effects is shown in fig.4.3. The lower sets of points show the shape of \( v_{2} \) as a function of \( p_{T} \) calculated with respect to the true reaction plane. The presence of jet-quenching introduces a small (\( \langle v_{2} \rangle \simeq 0.1\% \)) true flow effect increasing with \( p_{T} \), which is completely absent when medium effects are switched off.

The two sets in the middle represent the \( p_{T} \) dependence of the ‘observed’ non-flow (\( v_{2}^{obs} \) calculated with respect to the observed 2\(^{nd}\) harmonic event plane), which magnitude is roughly the same for both the inputs. Non-flow effects in Hijing are dominated by jet-like correlations, so that the small correlation with the true reaction plane due to jet quenching is washed away almost completely.

The upper sets of points show the reconstructed \( v_{2} \) after applying the resolution correction (calculated from random sub-events). The use of this ‘apparent’ event plane resolution (only due to non-flow) results in very large values of the reconstructed \( v_{2} \). Also the measured \( v_{2} \) slowly increases with \( p_{T} \), showing a saturation value at about 2 GeV/c (where \( v_{2} \sim 10 - 15\% \)). However the increase is not linear, and \( v_{2} \) is very small for \( p_{T} < 1 \) GeV/c, where most of the particles are produced. This leads to an integrated non-flow \( \langle v_{2} \rangle \simeq 1.4\% \), with a difference of \( \pm 0.3\% \) depending on the different settings of hijing \(^4\).

\[^4\) Note that this is a particle-wise average for all the ‘rescaled’ Hijing events, with multiplicities
Feasibility of the Event Plane analysis

Figure 4.4. Non-flow $v_2$ as a function of $p_T$ for 3 centrality classes (for Hijing events with jet-quenching and resonance decays on).

Since non-flow effects in Hijing arise from few-particle correlations such as jets, one might argue that their magnitude is inversely proportional to the total multiplicity. In fact, eq.4.1 states that the presence of many randomly oriented particles washes away the correlation. A clear example of this is found in fig.4.4, where the $p_T$ dependence of $v_2$ is plotted together for 3 well separated centrality classes (most peripheral $0 - 20\%$, mid-central $40 - 60\%$, and most central $80 - 100\%$). The lower multiplicity implies the higher non-flow effect.

The sub-event method provides a way to quantify the amount of non-flow correlations. Fig.4.5(a) shows that the sub-event correlation due to non-flow is approximately independent of the multiplicity of the event, and its magnitude depends on the choice of the sub-events (see caption). Eq.4.3 quantifies the azimuthal correlation within the event and, in case of genuine flow, is used to calculate the resolution of the event plane $^5$ [123].

In the present simulations $v_2 = 0$, therefore the above average is a direct measurement of non-flow effects, which can be expressed by the parameter $\tilde{g}_2$ in eq.4.4 [136]:

$$\langle \cos [2(\Psi_2^A - \Psi_2^B)] \rangle = M_{\text{sub}}(v_2^2 + g_2) = \frac{M}{2} v_2^2 + \frac{1}{2} \tilde{g}_2 . \quad (4.4)$$

The obtained results on $\tilde{g}$ are compatible with the experimental estimate of non-flow made at STAR [139] [140].

Eq.4.4 also makes clear that the non-flow contributions do not simply add to $v_2$, but to $M/2 \times v_2^2$. This is what we observe in fig.4.5(b), i.e. $\langle \tilde{v}_2 \rangle = \sqrt{\tilde{g}_2/M}$, with $\tilde{g}_2$ obtained from $\cos [2(\Delta \Psi_2^{\text{sub}})]$, using random sub-events ($\tilde{g}_2 \simeq 0.07$). The two data sets represent the sub-event correlation method with $\eta$ sub-events and the full-event distributed as in fig.4.1(b). The values of $\langle v_2 \rangle$ are obtained by integrating $v_2(p_T) \times dN/dp_T$ ($v_2^{\text{res}}$ of fig.4.3 convoluted with the generated $dN/dp_T$ spectrum).

$^5$The 2nd harmonic event plane resolution for the sub-events is immediately given by $\text{res}_{2}^{\text{sub}} = \sqrt{\langle \cos [2(\Psi_2^A - \Psi_2^B)] \rangle}$, while the full-event resolution is obtained using eq.3.8 (see sec.3.2.1).
correlation method with random sub-events (see sec.3.2.2). As expected, the sub-event correlation method gives a lower observed $v_2$, which is partially compensated by the lower observed resolution calculated from $\eta$ sub-events.

4.2 Flow simulation with GeVSim

As explained in the previous chapter, the accuracy of the event plane method in reconstructing the elliptic flow coefficient $v_2$ depends on two ingredients, particle multiplicity and magnitude of the elliptic flow. These two quantities are combined together to give the parameter $\chi_2 = v_2 \times \sqrt{2M}$ which is used to calculate the event plane resolution [123]. The event plane method has a relative systematic error on the measured $v_2$ proportional to $1/Mv_2^2 \propto 1/\chi_2^2$ [134]. Unlike the other approaches,

\[ \text{Figure 4.5.} (a) \cos \left[ 2 \left( \Psi_2^A - \Psi_2^B \right) \right] \text{ with respect to the charged particle multiplicity at mid-rapidity, for three different definitions of sub-events (random, } \pm \eta, \text{ and } \pm \eta \text{ with a gap at mid-rapidity). (b) Observed } \tilde{v}_2 \text{ from pure non-flow correlations vs } dN_{ch}/d\eta, \text{ calculated with the sub- and full-event correlation methods (using } \eta \text{ and random sub-events respectively). The dashed line shows the upper limit on non-flow (} \tilde{g}_2 \approx 0.07 \text{ calculated from random sub-events).} \]

Since the magnitude of $\cos \left[ 2 \left( \Delta \Psi_2^{\text{sub}} \right) \right]$ changes significantly for different definitions of sub-events (see fig.4.5(a)), with the proper choice of the analysis settings it is possible to minimize the effects. In particular, splitting the event into positive and negative rapidity suppress most of the non-flow correlation, and even more suppression is achieved by cutting away a slice at mid-rapidity (limiting the analysis to $0.5 < |\eta| < 1$), however this solution reduces the statistic and worsens the resolution of the reconstructed event plane.

In a realistic situation, the best choice is to use the full-event correlation method and to extrapolate the resolution from $\eta$ sub-events (see sec.5.4).

4.2 Flow simulation with GeVSim

As explained in the previous chapter, the accuracy of the event plane method in reconstructing the elliptic flow coefficient $v_2$ depends on two ingredients, particle multiplicity and magnitude of the elliptic flow. These two quantities are combined together to give the parameter $\chi_2 = v_2 \times \sqrt{2M}$ which is used to calculate the event plane resolution [123]. The event plane method has a relative systematic error on the measured $v_2$ proportional to $1/Mv_2^2 \propto 1/\chi_2^2$ [134]. Unlike the other approaches,
the event plane method has (in principle) no lower limit on $\chi^2$. However, when the uncertainty on $v_2$ becomes of the same order of magnitude of the measured value, the method is not reliable any more.

To avoid any bias from theoretical predictions and from the detector acceptance, this feasibility study has been performed on a wide sample of pure Monte Carlo simulations (without full detector reconstruction), produced with different combinations of multiplicity and $v_2$. A total of 49 sets of GeVSim events have been simulated in 7 centrality classes (with $dN/d\eta$ ranging between 100 and 10000 particles per unit rapidity) times 7 different input values of $v_2$ (with $v_2^{\text{sat}} = 1\%$ to $50\%$).

![Image of Figure 4.6](image-url)

Figure 4.6. Observed Elliptic Flow $v_2^{\text{obs}}$ (a) and full-event plane resolution (b), with respect to the charged multiplicity at mid-rapidity. The 7 data sets represent the 7 input values of $(v_2)$ listed on the right side of the plot (see tab.4.1).

The parametrization of $v_2(p_T)$ is described in sec.1.3.4, i.e. $v_2$ increases linearly for $p_T < 2$ GeV/$c$ and becomes constant on its saturation value at $p_T \geq 2$ GeV/$c$. The particle spectrum is an exponential $p_T$ distribution with slope parameter (temperature) $T_0 = 250$ MeV, flat in pseudorapidity (model n.1 in GeVSim, see [109]), generated in the kinematic range $0 < p_T < 10$ GeV/$c$ and $|\eta| < 1$. The charged particle composition is $80\%$ pions, $10\%$ kaons, and $10\%$ protons/anti-protons, with the settings the same for all multiplicities.

In this way, the integrated $v_2$ is linearly proportional to its saturation value:

$$
\langle v_2 \rangle = \frac{1}{N_{\text{tot}}} \sum_{i=1}^{N_{\text{tot}}} v_2^i = \frac{1}{N_{\text{tot}}} \left[ \int_0^{2\text{GeV}/c} v_2^{\text{sat}} \frac{dN}{dp_T} \frac{dN}{dp_T} d\rho_T + \int_2^{10\text{GeV}/c} v_2^{\text{sat}} \frac{dN}{dp_T} d\rho_T \right] = v_2^{\text{sat}} \times k_{\text{sat}} \cdot (4.5)
$$

The above integral gives $k_{\text{sat}} = 1/2.98$. Integrated and saturation values of $v_2$ for the present set of simulations are summarized in table 4.1.

$^7$This is slightly different from the particle composition generated by HiJing (see sec.5.4), however the magnitude of $v_2$ is the same for all particle species, and the detector acceptance is not taken into account.
4.2 Flow simulation with GeVSim

Table 4.1. Saturation $v_2$ and the resulting integrated flow.

<table>
<thead>
<tr>
<th>$v_2^{sat}$ (%)</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle v_2 \rangle$ (%)</td>
<td>0.3</td>
<td>0.7</td>
<td>1.7</td>
<td>3.4</td>
<td>6.7</td>
<td>10.1</td>
<td>16.8</td>
</tr>
</tbody>
</table>

The number of events produced for each sample is inversely proportional to the particle multiplicity (so that the analysis always involves the same number of particles): $N_{evts} \times dN/d\eta = 2 \cdot 10^7$.

The event plane analysis has been applied to charged primary particles ($\pi^\pm$, $K^\pm$, $p$ and $\bar{p}$) with $|\eta| < 1$. Figure 4.6 summarizes the results on the observed $v_2^{obs}$ and the observed event plane resolution. For higher multiplicity and higher $v_2$ the resolution saturates (i.e. $\text{res}_2 \sim 1$) and the measured elliptic flow becomes more accurate (the observed $v_2$ very well approximates the generated one and the resolution correction becomes negligible).

![Figure 4.6](image)

**Figure 4.6.** Saturation of the integrated $v_2$ by the multiplicity $\langle v_2 \rangle$. The dashed line represents the generated $v_2$ for the number of events produced for each sample, inversely proportional to the particle multiplicity (so that the analysis always involves the same number of particles): $N_{evts} \times dN/d\eta = 2 \cdot 10^7$.

The number of events produced for each sample is inversely proportional to the particle multiplicity (so that the analysis always involves the same number of particles): $N_{evts} \times dN/d\eta = 2 \cdot 10^7$.

The event plane analysis has been applied to charged primary particles ($\pi^\pm$, $K^\pm$, $p$ and $\bar{p}$) with $|\eta| < 1$. Figure 4.6 summarizes the results on the observed $v_2^{obs}$ and the observed event plane resolution. For higher multiplicity and higher $v_2$ the resolution saturates (i.e. $\text{res}_2 \sim 1$) and the measured elliptic flow becomes more accurate (the observed $v_2$ very well approximates the generated one and the resolution correction becomes negligible).

![Figure 4.7](image)

**Figure 4.7.** Feasibility of the event plane analysis with respect to the particle multiplicity and the genuine $\langle v_2 \rangle$. The dashed lines represent the observed ‘non-flow’ $\tilde{v}_2 = \sqrt{\tilde{g}_2/M}$, calculated from pure non-flow effects in Hijing using different definitions of sub-events (see sec.4.1), the central value is $\tilde{g}_2 = 0.05$, measured from $\eta$ sub-events. Each marker represents a set of GeVSim simulations, with $M$ and $\langle v_2 \rangle$ given by the $x$ and the $y$ coordinate respectively. The measured $\chi_2 = v_2 \sqrt{2M}$ is compared to $\sqrt{\tilde{g}_2}$ and the shape of the marker is assigned accordingly. The event plane analysis fails when the calculated resolution is imaginary (i.e. $\langle \cos [2 (\Delta \Psi_{2}^{sub})] \rangle < 0$).

Using the reconstructed values of $\chi_2$ from the above simulations (sec.4.2) and the magnitude of non-flow effects estimated with Hijing (sec.4.1) it is possible to draw figure 4.7, which summarizes the feasibility of the event plane analysis for any possible scenarios of multiplicity and $v_2$.

The event plane method only has problems at low values of the integrated $v_2$
Feasibility of the Event Plane analysis

(i.e. $\langle v_2 \rangle \lesssim 2\%$) and at low multiplicities, it works fine elsewhere.

### 4.3 Flow + non-flow

To study the interplay between flow and non-flow effects, a set of 50,000 ‘rescaled’ Hijing events has been produced ($7 < |b| < 14.5$ fm, with jet-quenching and resonance decays on, and no full detector reconstruction) and boosted with the flow After-Burner (see sec.2.2.3) using values of elliptic flow extrapolated from the lowest hydrodynamic estimate (with $c_2^2 = 0.22$, see sec.1.3.2), $v_2$ is assigned to each event with respect to its impact parameter $b$. These KineTrees could represent the input for a realistic scenario, where both flow and non-flow effects are present and $v_2$ has a continuous dependence on the impact parameter.

The centrality class selection is based on the multiplicity of final state particles (as it would be in a real experiment): the $dN/d\eta$ distribution has been divided in ten intervals, each one containing approximately 10% of the total number of events (i.e. 10% of the total inelastic cross section).

Due to the fluctuations involved in the particle production processes (see also sec.5.4), each multiplicity class contains events in a large range of impact parameter, and consequently, with a large spread in $v_2$. This also reproduce a more realistic $v_2^{RMS}$, and therefore a better estimate of the statistical error on the measured $v_2$ (see also sec.5.5).

Fig.4.8(a) shows the integrated values of $v_2^{obs}$ (the observed $v_2$ without resolution correction, see 3.2):

$$\langle v_2^{obs} \rangle = \langle \cos [2(\phi_2 - \Psi_2)] \rangle . \quad (4.6)$$
Table 4.2. Summary table of the 50k Hijing + After-Burner simulations. Per each centrality class, both the input and the reconstructed $v_2$ and resolution are listed.

<table>
<thead>
<tr>
<th>$\sigma_{\text{tot}}$</th>
<th>$v_2^{\text{in}}$</th>
<th>$dN_{ch}/d\eta$</th>
<th>$dN_{ch}/d\eta_{\text{min}}$</th>
<th>$\langle dN_{ch}/d\eta \rangle$</th>
<th>$\langle v_2^{\text{in}} \rangle$</th>
<th>$\langle v_2^{\text{res}^{\text{th}}} \rangle$</th>
<th>$\langle v_2^{\text{res}^{\text{obs}}} \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 - 10$</td>
<td>2.37%</td>
<td>3000</td>
<td>1880</td>
<td>2322</td>
<td>2.37%</td>
<td>0.767</td>
<td>2.60%</td>
</tr>
<tr>
<td>$10 - 20$</td>
<td>5.16%</td>
<td>1880</td>
<td>1350</td>
<td>1603</td>
<td>5.16%</td>
<td>0.934</td>
<td>5.14%</td>
</tr>
<tr>
<td>$20 - 30$</td>
<td>7.29%</td>
<td>1350</td>
<td>960</td>
<td>1147</td>
<td>7.29%</td>
<td>0.956</td>
<td>7.23%</td>
</tr>
<tr>
<td>$30 - 40$</td>
<td>8.86%</td>
<td>960</td>
<td>660</td>
<td>802</td>
<td>8.86%</td>
<td>0.957</td>
<td>8.73%</td>
</tr>
<tr>
<td>$40 - 50$</td>
<td>9.72%</td>
<td>660</td>
<td>440</td>
<td>545</td>
<td>9.72%</td>
<td>0.946</td>
<td>9.65%</td>
</tr>
<tr>
<td>$50 - 60$</td>
<td>9.81%</td>
<td>440</td>
<td>280</td>
<td>355</td>
<td>9.81%</td>
<td>0.914</td>
<td>9.85%</td>
</tr>
<tr>
<td>$60 - 70$</td>
<td>9.02%</td>
<td>280</td>
<td>170</td>
<td>221</td>
<td>9.02%</td>
<td>0.826</td>
<td>9.25%</td>
</tr>
<tr>
<td>$70 - 80$</td>
<td>7.39%</td>
<td>170</td>
<td>100</td>
<td>133</td>
<td>7.39%</td>
<td>0.639</td>
<td>8.18%</td>
</tr>
<tr>
<td>$80 - 90$</td>
<td>5.44%</td>
<td>100</td>
<td>50</td>
<td>74</td>
<td>5.44%</td>
<td>0.393</td>
<td>7.29%</td>
</tr>
<tr>
<td>$90 - 100$</td>
<td>3.56%</td>
<td>50</td>
<td>0</td>
<td>28</td>
<td>3.56%</td>
<td>0.167</td>
<td>8.09%</td>
</tr>
</tbody>
</table>

The figure also shows the ‘expected’ values of the observed $\langle v_2^{\text{obs}} \rangle$, calculated per each centrality class as $\langle v_2^{\text{in}} \rangle$ times the expected resolution $\text{res}^{\text{th}}$, calculated from the input values of $v_2$ and multiplicity using eq.3.8 (see tab.4.2).

Figure 4.9. Measured and simulated $v_2$ vs $dN/d\eta$ for the 50k Hijing + After-Burner events (see tab.4.2), non-flow affects are calculated as the difference between the two values (for a comparison with experimental results at lower energy, see the reference [139]).

Non-flow effects are responsible for the larger magnitude of the reconstructed $v_2^{\text{obs}}$ with respect to the simulated one. The same happens to the resolution of the event plane calculated from sub-events (fig.4.8(b)), where we see the same systematic effect. In middle and most central events ($500 < dN/d\eta < 2000$) the genuine elliptic flow is much higher than the non-flow contribution ($Mv_2^2 \gg \tilde{g}$, see eq.4.4) and therefore the analysis perfectly works. More peripheral events ($dN/d\eta < 500$)
are an example situation where $Mv_2^2 \sim \tilde{g}_2$, and therefore the event plane analysis leads to an incorrect result (lower left corner of fig.4.7).

Finally, fig.4.9 shows the systematic increase in the measured values of $v_2$ due to non-flow effects. The difference between simulated and reconstructed $v_2$ is an estimate of the non-flow contributions to the measurements, assuming the centrality dependence of $v_2$ is described by the hydro extrapolation. For a comparison with experimental results from STAR, see the references [139] and [140].
Chapter 5
Simulations & Results

The event plane analysis has been studied for lead-lead collision at LHC by means of an elaborate set of AliRoot simulations, in order to determine its feasibility with the ALICE detector.

Using the parametrizations described in sec.1.3, particle multiplicity and elliptic flow have been extrapolated to LHC energy under three different assumptions on the impact parameter dependence of $v_2$ (considered as the upper/lower limit of the existing predictions). Using these extrapolations, three sets of fully reconstructed GeVSim events have been produced in a few centrality classes, and the event plane analysis has been optimized including detector effects.

The study of the analysis cuts, their optimization, and the efficiency corrections are discussed in sec.5.1 and 5.2 respectively, while the results of the flow analysis of the GeVSim events are presented in section 5.3 (including an estimate of the systematic error of the measurement).

Using the extrapolation with the lowest $v_2$ (see section 1.3), a set of Hijing events with flow After-Burner has been simulated and fully reconstructed, leading to a complete set of data including both flow and non-flow effects. In this more realistic scenario, the reconstructed values of $v_2$ have been compared to the simulated ones, and the systematic effects due to non-flow have been calculated (see sec.5.4).

5.1 Efficiency study

In a real experiment, the accuracy of the event plane analysis depends on the detector performance. In the present approach, detector effects are quantified by two main 'estimators', which can be both studied using Monte Carlo simulations with full detector reconstruction (as provided by the AliRoot framework, see sec.2.2.2):

- the reconstruction efficiency (i.e. how many primary stable particles are actually reconstructed by the detector),
- and the purity of the sample (i.e. how accurate is the match between the reconstructed tracks and the simulated primary particles).
The aim of the present analysis is to measure both differential and integrated elliptic flow of unidentified charged primary particles produced in the interaction, therefore the track selection is optimized for selecting primary stable hadrons. We define as ‘stable’ a particle that lives long enough to reach the ALICE TPC and can be fully reconstructed [71] (i.e. $\pi^\pm$, $K^\pm$, $p$ and $\bar{p}$).

5.1.1 Efficiency & Purity

For any applied cut, $N_{ESD}$ is the total number of reconstructed $AliESDtracks$ passing the cut, and $N'_{ESD}$ is the number of ‘correctly reconstructed’ primary tracks passing the cut (i.e. tracks which are reconstructed from primary stable hadrons within the same $p_T$ bin of the generated particles, see below). $N'_{MC}$ is the number of primary stable hadrons ($\pi^\pm$, $K^\pm$, $p$ and $\bar{p}$) generated within the acceptance of the detector (i.e. $p_T > 0.1$ GeV/$c$, $|\eta| < 0.9$ and $0 \leq \phi < 2\pi$), see sec.2.1)

![Graphs showing the transverse momentum resolution and the linear approximation of $\Delta p_T$ as a function of $p_T$.]

Figure 5.1. (a) Transverse momentum resolution, defined as $\langle \Delta p_T \rangle / p_T$, where $\langle \Delta p_T \rangle$ is the RMS of the $\Delta p_T$ distribution of charged primary hadrons in the central barrel detector. (b) $\Delta p_T$ as a function of $p_T$ and its linear approximation (at 2.5 RMS, $\sim 99\%$ of the $\Delta p_T$ distribution).

Fig.5.1(a) shows that the relative transverse momentum resolution of the TPC, $\Delta p_T / p_T$, weakly depends on $p_T$ in the momentum range of interest ($p_T = 0.1$ to 10 GeV/$c$), therefore the condition for a track to be reconstructed in the same $p_T$ bin as the simulated primary particle can be approximated as:

$$\Delta p_T < w_0 \times (1 + p_T / \text{GeV}/c),$$  \hfill (5.1)

where $\Delta p_T = |p_T(ESD) - p_T(MC)|$.

The $\Delta p_T$ distribution is roughly Gaussian for each $p_T$, with a longer tail on the right side due to the statistically larger abundance of low momentum tracks.

1Detector cracks are not taken into account in the definition of efficiency and purity.
reconstructed at a higher $p_T$ (however this effect is less then 1% for primary tracks and can be neglected).

The parameter $w_0$ of eq.5.1 is chosen to linearly approximate the observed RMS in $\Delta p_T$ as a function of $p_T$ (fig.5.1(b)) at 2.5$\sigma$ ($\sim$ 2.5RMS in $\Delta p_T$), where 99% of the track candidates are found: $w_0 = 50$ MeV/c (with $p_T$ expressed in GeV/c). With the parameter $w_0$, eq.5.1 defines the minimum bin size in $p_T$, such that the number of particles reconstructed in the wrong bin is negligible ($\lesssim 1\%$) $^2$.

Eq.5.1 approximates the requirement that in the final histograms a reconstructed track enters the same $p_T$ bin as the simulated particle.

The efficiency is defined as the number of primary charged hadrons ($\pi^\pm$, $K^\pm$, $p$ and $\bar{p}$) correctly reconstructed divided by the number of primary charged hadrons generated in the acceptance:

$$\text{eff} = \frac{N'_{ESD}}{N'_{MC}}. \tag{5.2}$$

For a specific particle type, the efficiency is a detector property which depends on the detector configuration, the geometrical acceptance, the reconstruction algorithm and the applied cuts (see sec.5.1.2).

The purity is defined as the number of correctly reconstructed primary tracks divided by the total number of reconstructed tracks within the cut:

$$\text{pur} = \frac{N'_{ESD}}{N_{ESD}}. \tag{5.3}$$

Without considering the experimental determination of the particle identification, the purity quantifies the level of contamination of the reconstructed spectra (e.g. from secondaries and from track reconstructed at the wrong momentum). It depends on the applied cuts and on the simulated input spectra $^3$.

By definition, both efficiency and purity are smaller than 1 (the particles counted by numerator are a sub-set of the ones counted by the denominator). We will consider efficiency and purity differentially (as a function of the transverse momentum $p_T$), and integrated (over the range of interest of the present analysis, i.e. between $p_T = 0.1$ and 10 GeV/c).

The correction to the observed spectra (when the simulated one is given) is expressed by the ratio:

$$\text{corr} = \frac{\text{eff}}{\text{pur}} = \frac{N'_{ESD}}{N'_{MC}} \times \frac{N_{ESD}}{N'_{ESD}} = \frac{N_{ESD}}{N'_{MC}}. \tag{5.4}$$

In a situation of known input spectra (where efficiency and purity could be determined exactly), the original signal is exactly recovered by dividing the reconstructed

$^2$Due to the limited statistic, the present analysis used a $p_T$ bin size at least twice as large as the lower limit given above.

$^3$Both the number of secondaries and the contamination from other $p_T$ bins depend on the number of primary particles generated at each $p_T$. 

spectra by this factor, i.e.:

\[
\frac{d^3 N}{dp_T d\eta d\phi} = \frac{1}{\text{corr}(\eta, p_T, \phi)} \times \frac{dN^{\text{obs}}}{dp_T d\eta d\phi}.
\] (5.5)

In reality such a correction factor can only be determined by simulating a realistic input spectra, which should be modeled with respect to observed experimental data, not available yet in ALICE. Therefore the effect of impurities is absorbed into the systematic error and the only correction applied is the detector efficiency as a function of \(p_T\).

### 5.1.2 Particle Composition

The reconstruction efficiency (and its transverse momentum dependence) is different for different particle species, and the particle composition of the sample is not known and moreover is not constant as a function of \(p_T\).

![Figure 5.2](image_url)

**Figure 5.2.** Top row: Generated and reconstructed \(dN/dp_T\) spectra for pions, kaons and protons in the ALICE central barrel detector \((\eta < 0.9)\). Lower row: Efficiency and purity as a function \(p_T\) for pions, kaons and protons. No particle identification is involved, the cuts applied to the data are discussed in sec.5.2.

Using the Monte Carlo information from the simulations, both efficiency and purity can be studied for different particles separately, showing their different \(p_T\) dependence. Fig.5.2 shows the generated and reconstructed \(p_T\) spectra of pions, kaons and protons produced at \(|\eta| < 0.9\) (top row), and their reconstruction efficiency and purity (lower row).
5.1 Efficiency study

Since the aim of the present analysis is the characterization of elliptic flow of unidentified charged particles, the efficiency corrections are calculated from the overall spectra of reconstructed tracks, without involving the effects of particle identification. Not knowing the particle composition and the shape of the \( \frac{dN}{dp_T} \) distribution for each particle, a way to determine the systematic error of this procedure is by looking at the differences between different predictions for heavy ion events at LHC.

![Graph showing efficiency and purity as a function of \( p_T \) for different simulations.](image)

**Figure 5.3.** Overall efficiency (a) and purity (b) in the ALICE central barrel detector (\( \eta < 0.9 \)) as a function \( p_T \) for two sets of simulations, Hijing and GeVSIm, the absolute value of the difference is shown as well. The same set of cuts is applied to both the samples (see sec.5.2).

The simulations presented in this chapter are produced from two different scenarios, the particle ratio of the GeVSIm events (tab.5.2) are calculated with an implementation of the thermal model for particle production (*Thermus* [144]), while the particle composition of the Hijing events (tab.5.4) is determined by its internal implementation of QCD interactions and hadronization processes [104].

Figure 5.3 shows the overall efficiency and purity as a function \( p_T \) for the two different inputs (Thermus and Hijing). The difference between the two gives an estimate of the systematic error for the a priori unknown particle ratio. However, the difference is very small due to the fact that in both models the majority of particles are pions, and the amount of protons and kaons is only of the order of 10% (this prediction is supported by experimental data from RHIC, see for instance [45]).

The figure also shows that the reconstruction efficiency rapidly saturates to its maximum (\( \text{eff}_{\text{max}} \sim 90\% \)) for \( p_T \gtrsim 1 \text{ GeV}/c \). This is determined by the dominating contribution from pions and a similar behaviour is also observed for protons (see fig.5.2), while the efficiency of kaons reconstruction saturates only at \( p_T \simeq 2 \text{ GeV}/c \) due to decays \(^4\).

\(^4\)Part of the \( K \) mesons with low momentum decays before reaching the TPC (the mean lifetime of the charged kaon is \( \tau_{K^\pm} \simeq 1.238 \times 10^{-8} \text{ sec} \), giving a \( c\tau_{K^\pm} \simeq 3.7 \text{ m} \)). At large momentum (for
The systematic error on the efficiency due to the unknown particle ratios is calculated from the difference between the efficiencies of the two sets of simulations (Hijing and GeVSim). See sec.5.2.2 for the details.

5.1.3 Multiplicity (in)dependence

Figure 5.4 shows that, in agreement with the ALICE PPR [71], the efficiency is almost constant with respect to the particle multiplicity. However, comparing the efficiency of peripheral, mid-central and central events (i.e. from \( \frac{dN}{d\eta} \sim 100 \) to \( \sim 2000 \) tracks per unit rapidity, according to the extrapolation given in sec.1.3) a systematic decrease in the reconstruction efficiency as a function of \( p_T \) can be observed over all the range of interest of the present analysis (0.1 < \( p_T < 10 \) GeV/c).

![Figure 5.4. Track reconstruction efficiency as a function of \( p_T \) at three multiplicities \( \frac{dN}{d\eta} \mid_{\eta<0.5} \sim 100, 500, 2000 \). The absolute value of the difference between the highest and the lowest multiplicity samples is shown as well (data from the GeVSim simulations, see tab.5.1 for the details).](image)

In the present analysis the efficiency is calculated as an average over all produced events. Therefore, the difference in efficiency between the lowest and highest multiplicity events (which is of the order of a few \%) is added to the systematic error on the calculated efficiency. For more details, see sec.5.2.2.

5.1.4 Main Vertex

The nominal \( \eta \) acceptance of the TPC is from −0.9 to 0.9 for collisions at \( z = 0 \) (center of the TPC). However, for the geometrical arrangement of the beam-crossing, the collision can happen anywhere in an ‘interaction diamond’, i.e. in \( p_T > m_{K^\pm}/c \) the kaon lifetime becomes significantly Lorentz dilated.
5.1 Efficiency study

A length of about 30 cm along the $z$ axis (see sec.2.3.1). As the location of the primary vertex changes on an event base, the $\eta$ acceptance of the ALICE TPC is also different for each event. An event at the edge of the interaction region, with main vertex at $z = 15$ cm, will see the TPC with an acceptance $-0.93 \lesssim \eta \lesssim 0.85$ (see sec.2.1.2).

![Figure 5.5. Reconstruction efficiency as a function of the pseudorapidity for a fixed main vertex position (empty markers) and for a vertex position gaussianly distributed in the ‘interaction diamond’ ($-15 \lesssim z \lesssim 15$ cm). The plot is obtained from 2 sets of 1000 fully reconstructed GevSim events, generated with a flat $dN/d\eta$ spectra.](image)

Considering events with a Gaussian distribution of the main vertex position, the overall efficiency rapidly drops above $|\eta| \simeq 0.85$, as we see in fig.5.5. The figure shows the efficiency of primary particles reconstruction, as a function of the pseudorapidity, for two different sets of simulations, produced with fixed and variable primary vertex position. The first set has fixed main vertex position at $z = 0$, the second has the main vertex randomly located along $z$ (in a Gaussian distribution with $\sigma_z = 5.3$ cm).

In a realistic case (the latter), a symmetric $\eta$ cut should be used in a region of flat efficiency (e.g. $|\eta| < 0.85$), to avoid introducing an artificial asymmetry in the event. However, the simulations presented in this chapter have been produced with a fixed primary vertex position at $z = 0$, and moreover the $\eta$ dependence of $v_2$ is parametrized flat on the full $\eta$ range (see chap.1.3).

Therefore, in the present analysis, only a sharp cut at $|\eta| < 0.9$ is applied (to include the widest TPC range with a ‘uniform’ tracking efficiency). The measurements are averaged over all the detectable pseudorapidity interval, and the main vertex position has not been taken into account for the calculation of the systematic error.
5.2 Cut optimization

Cuts are studied with respect to the reconstruction efficiency and the purity of the sample, using as input the full set of GeVSim and Hijing simulations (see sec.5.3 and 5.4).

The aim of the cuts is to isolate primary particles, to allow a clean reconstruction of the differential shape of $v_2(p_T)$ without any further correction and without loosing too much statistic, in order to obtain a good balance between statistical and systematic error.

Detector signal

Our main interest is in measuring elliptic flow of unidentified charged tracks reconstructed in the ALICE central barrel detectors. Therefore, the first cut consist in selecting tracks reconstructed in the TPC, in the pseudorapidity range of full coverage ($|\eta| < 0.9$). In addition we want the track fit to be propagated (at least) to the ITS, so that the extrapolation to the primary vertex becomes more reliable ($\delta_{VTX} \lesssim 100\mu$m, see below). Fortunately the efficiency drops less than 5% when requiring the ITS signal in association to the TPC (see chap.5 of the ALICE PPR [25]).

Since no particle identification is needed for the measurement of unidentified particles flow, the outermost detectors of the central barrel (TRD and TOF) are not part of the present analysis (however, if a particle reaches them, the Kalman filter includes them in the track fit improving the precision, see sec.2.3). Due to the larger distance from the interaction and the smaller $\theta$ coverage, requiring a TRD and TOF signal would introduce a strong cut in $p_T$ and $\eta$ and the overall efficiency would be dramatically reduced from $80 - 90\%$ to less than $60\%$ (see chap.5 on the ALICE PPR [25]).

Constrainability condition, fit $\chi^2$ and number of fit points/max (TPC)

A first selection of primary tracks is realized by the constrainability condition, where a track is defined ‘constrainable’ if the main vertex of the collision can be included as a fit point.

In the reconstruction code, the constrainability of tracks is tested at the third pass of the fit procedure (see 2.3), when the track is refitted from its outermost point inward. The main vertex is fed to the Kalman filter as an additional space point and, if the fit succeeds with an ‘acceptable’ $\chi^2$, the track is labeled as constrainable and the constrained parameters of the track are updated with the last re-fit. The

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5In the present analysis, only full tracks are considered (for which, by definition, the tracking algorithm starts from the TPC, see sec.2.3). Track segments reconstructed by other detector (e.g. ITS ‘tracklets’) are not taken into account.

6From the $\chi^2$ distribution (fig.5.6(a)) we see that is ‘acceptable’ a value of $\chi^2 < 77$ (i.e. $\chi \lesssim 8.8\sigma'$, where $\sigma'$ is the uncertainty on the primary vertex position).
5.2 Cut optimization

Figure 5.6. (a) $dN/d\chi^2$ of all constrainable AliESD tracks (with TPC + ITS signal) and all constrainable primaries. The full histogram represent the differential distribution, while the upper lines represent the integrated number of track for any given cut on the fit $\chi^2$. The number of simulated primaries is shown as well. (b) Integrated efficiency and purity as a function of the cut on the fit $\chi^2$. The total purity is not very sensitive to the applied cut, instead the ratio primaries/all track reconstructed at each $\chi^2$, provides a more sensitive estimator.

The constrainability condition alone is very efficient in removing secondary tracks, while a more strict requirement on the fit $\chi^2$ does not considerably improves the purity of the selection (see fig.5.6(b)).

However, the ratio between constrainable tracks and constrainable primaries as a function of the fit $\chi^2$ (the ratio $dN'_{ESD}/dN_{ESD}$ at each bin of the $dN/d\chi^2$ distribution), becomes 50\% at $\chi^2 = 20$, i.e. less than half of the constrainable tracks reconstructed with $\chi^2 \geq 20$ actually comes from primary particles. Therefore, in the present analysis, the cut $\chi^2 < 20$ has been applied (a wider cut will increase the background more than the signal).

Due the low sensitivity of the efficiency with respect to this cut, the fit $\chi^2$ has not been included in the calculation of the systematic error.

A one-to-one comparison between reconstructed and simulated particles shows a non negligible contribution of ‘double counted’ (or splitted) tracks. The experimental precision of the tracking device and the accuracy of the reconstruction algorithm can cause a single particle going trough the TPC to be reconstructed twice, producing two different track candidates in the AliESD.

This applies both to curved (low momentum) tracks at $\eta \sim 0$, which spiral back toward the primary vertex, and to straight (high momentum) tracks flying across different detector elements which are not perfectly aligned.

A strategy developed at STAR [129] to suppress this effect is to apply a cut over
Figure 5.7. (a) $dN/d(N_{\text{fit}}/N_{\text{max}})$ of all constraining AliESDtracks (with TPC + ITS signal) and all constraining primaries. The full histogram represents the differential distribution, while the upper lines represent the integrated number of track for any given cut on $N_{\text{fit}}/N_{\text{max}}$ in the TPC. The number of simulated primaries is also shown as well. (b) Efficiency and purity as a function of the cut on $N_{\text{fit}}/N_{\text{max}}$ in the TPC.

The number of fit points (from which the track candidate is interpolated) normalized by the number of clusters that the track could produce in the detector. The actual number of space points used for the track fit $N_{\text{fit}}$ is stored in the AliESDtrack object (see sec.2.3) and, in addition, the reconstruction algorithm uses a helix parametrization of the track to estimate the number of clusters $N_{\text{max}}$ that a particle flying along the reconstructed trajectory would give in each detector element. This is particularly important in the TPC, where the number of sensitive elements is large and the fit of each track can include up to 160 space points (sec.2.1.2).

Figure 5.8. Total efficiency and purity (in the range $0.1 < p_T < 10$ GeV/c, $|\eta| < 0.9$) for all the applied cuts, showing the results for both the GeVSim and the Hijing simulations (see sec.5.3 and 5.4 respectively).
5.2 Cut optimization

A cut on the ratio $N_{\text{fit}}/N_{\text{max}} > 0.6$ in the TPC helps in removing the contributions from double counted tracks and slightly improves the purity by $\sim 1\%$ (see fig.5.8). However, both the efficiency and the purity show a very flat dependence with respect to this cut (see fig.5.7); even a $10 - 20\%$ systematic error on the value $N_{\text{fit}}/N_{\text{max}}$ has a negligible effect on the calculated efficiency. Therefore this cut has not been included in the calculation of the systematic error (see sec.5.2.2).

Fig.5.8 summarizes the cuts applied in the present analysis, showing the integrated efficiency and the purity of the selection passing all cuts, separately for the GeVSim and the Hijing simulations. On top of the basic set of cuts (tracks with both TPC and ITS signal, with at least 60\% of the TPC clusters included in the fit and a constrained $\chi^2 < 20$), further cuts can be applied to enhance the purity of the track candidates, e.g. a cut on the distance of closest approach to the main vertex (see below).

**Transverse DCA**

The excellent resolution of the ITS (see sec.2.1.1) allows an extrapolation of the track to the main vertex with a precision of the order of $100\mu$m in the $y - x$ plane, depending on the momentum of the track and on the number of reconstructed clusters, and a bit worse in the $z$ direction $^7$.

The extrapolated distance between the fitted track and the event’s main vertex is called Distance of Closest Approach (DCA). A Gaussian fit of the DCA distribution of primaries in the transverse plane gives $\sigma_{t\text{DCA}} = 160 \mu$m (see fig.5.9), while a fit on the $z$ direction gives $\sigma_{z\text{DCA}} = 430 \mu$m. Due to their intrinsically different precision they are usually considered separately, and the much better resolution of the DCA in the transverse plane suggest the latter as a good parameter for selecting primary particles.

Figure 5.9(a) shows the transverse DCA distribution for all constrainable $^8$ tracks and for constrainable primaries, together with a half Gaussian fit of the latter (with fixed peak position at 0). The integrated efficiency and purity as a function of the $t\text{DCA}$ cut are shown in fig.5.9(b), the purity of the selection is not very sensitive to the applied cut, while the efficiency rapidly drops for a $t\text{DCA}$ cut smaller than few hundreds $\mu$m.

In the present analysis, a cut at $500\mu$m ($\sim 3\sigma_{t\text{DCA}}$) has been applied. Together with the other cuts, this condition results in an integrated purity of primaries higher than 95\%. The detailed $p_T$ dependence of the purity for both the GeVSim and the Hijing sample is shown in fig.5.3(b).

The systematic uncertainty connected to this cut is calculated by assuming an imprecision of $\pm 100 \mu$m on the reconstructed $t\text{DCA}$, as if the $t\text{DCA}$ distribution obtained from the simulation does not correctly reproduce the one measured in the

$^7$See sec.2.3 and the ALICE PPR [25] (at section 5.1.6.3).

$^8$The main vertex is included in the fit.
Figure 5.9. (a) $dN/d^2\text{DCA}$ of all constrainable AliESDtracks (with TPC + ITS signal, $\chi^2 > 20$ and $N_{\text{incl}}/N_{\text{max}} > 0.6$) constrainable and primaries. The full histogram represents the differential distribution, while the upper lines represent the integrated number of tracks for any given cut on the transverse DCA (the number of simulated primaries is shown as well). The experimental resolution on the measured $^4\text{DCA}$ is obtained through a Gaussian fit of the transverse DCA distribution of reconstructed primary particles ($\sigma_{\text{DCA}} \approx 160\mu\text{m}$). (b) Efficiency and purity of primaries with respect to the transverse DCA cut.

real experiment. The reconstruction efficiency has been calculated separately for two choices of the $^4\text{DCA}$ cut ($^4\text{DCA} < 500 \pm 100 \ \mu\text{m}$) and the difference is taken as an estimate of the systematic error (see sec.5.2.2).

Low $p_T$ cut and extrapolation

The magnetic field in the ALICE central barrel introduces a low $p_T$ cut in the detector acceptance. Low $p_T$ particles ($p_T \lesssim 100 \ \text{MeV}/c$ for pions) are curved enough to barely reach the TPC, and therefore the track reconstruction efficiency becomes almost zero in the $p_T$ region below 100 MeV/c (see sec.2.1).

The strategy applied in the present analysis is to limit the measurements to the $p_T$ range above 100 MeV/c, and after having measured the particle yield and applied the efficiency corrections, extrapolate the measurements down to $p_T = 0$.

The content of the first bin of the $dN/dp_T$ histogram is estimated as a fraction of the total integral of the reconstructed spectrum: assuming the cumulative $dN/dp_T$ spectrum of charged ‘stable’ hadrons is known, is possible to calculate the ratio between the number of particle produced with $p_T < 100 \ \text{MeV}/c$ ($N_1$) and the number of particle produced between $p_T = 0.1$ and 10 GeV/c ($N_a$):

$$n_{\text{low}} = \frac{N_1}{N_a} = \frac{\int_{0}^{100\text{MeV}/c} dN \frac{dN}{dp_T} dp_T}{\int_{100\text{MeV}/c}^{10\text{GeV}/c} dN \frac{dN}{dp_T} dp_T}. \quad (5.6)$$

In the present analysis, the ratio $n_{\text{low}}$ has been calculated exactly from the (known) input spectra, the values are 0.0406 for GeVSim (with input spectra given by eq.5.10)
and 0.0436 for Hijing. The difference between the two values is used to estimate the systematic error of the method due to the ‘a priori’ unknown shape of the observable $dN/dp_T$ spectrum (see sec.5.2.2).

For a small uncertainty on the ratio $n_{low}$, the statistical error associated to this procedure is comparable to the error of a counting experiment ($\sigma_N = \sqrt{N}$), and therefore, to the statistical error of any other $p_T$ bin:

$$\sigma_{N_1} = n_{low} \times \sigma_{N_a} \simeq \frac{N_1}{N_a} \times \sqrt{N_a} = \frac{N_1}{\sqrt{N_a}} < \frac{N_1}{\sqrt{N_1}} = \sqrt{N_1} \cdot (5.7)$$

In a real experiment, the ratio $n_{low}$ should be obtained from an accurate fit of the reconstructed spectra (the fit can be done just once, assuming the centrality class dependence of the spectra is negligible) and the calculated $n_{low}$ could be used as a reconstruction parameter for recovering the $dN/dp_T$ spectrum from the observed data corrected by the efficiency.

The extrapolation of the $dN/dp_T$ spectrum could be done using a Levy distribution, as suggested by some recent studies at RHIC [41]:

$$\frac{1}{p_T} \frac{dN}{dp_T} = A \cdot \frac{1}{(1 + p_T/(m \cdot T))^\alpha} \cdot (5.8)$$

Or, more precisely, using a weighted sum of three Levy distributions (for $\pi$, $K$ and $p$) with $mT$ in place of $p_T$, since the observed $dN/dp_T$ spectrum is actually the sum of the spectra of all charged stable particles.

A slightly modified version of eq.5.8, incorporating the particle mass dependence, has been used to generate all $p_T$ spectra of the GeVSim events (see sec.5.3).

The extrapolations of the particle spectrum and the elliptic flow to low $p_T$ (see sec.5.3.3) are an essential step to calculate the integrated $v_2$. The result of the extrapolations, for both the GeVSim and the Hijing samples, are shown in sec.5.3.4 and 5.4.3 respectively.

### 5.2.1 Final corrections

One of the goals of the present analysis is to measure the integrated elliptic flow $\langle v_2 \rangle$ at mid-rapidity. This is achieved by taking the average $\cos [2(\phi_i - \Psi)]$ in the kinematic range covered by the detector. The track reconstruction efficiency is not constant as a function of $\eta$, $p_T$ and $\phi$, therefore some corrections are needed to provide a measurement which is not biased by the detector itself.

Efficiency corrections are applied under the assumption that the total momentum spectra of reconstructed tracks $d^3N/d\vec{p}$ can be factorized into the three familiar components $\eta$, $p_T$ and $\phi$. This assumption is not completely true, e.g. straight (high $p_T$) track can easily escape through a crack between two segments in the TPC without being detected at all, while more curved tracks (lower $p_T$) could spiral back into the sensitive volume of the TPC and release enough hits to be reconstructed. However, a full 3D study of the efficiency would require a much higher statistic than the one available.
Simulations & Results

Figure 5.10. (a) Simulated and reconstructed $dN/dp_T$ spectra of the full set of simulations (Hijing + GeVSim). (b) Final efficiency and purity correction factors as a function of $p_T$, calculated over the full set of simulations.

- Since the pseudorapidity dependence of $v_2$ is assumed to be flat, $\eta$ corrections are not taken into account but just an acceptance cut is applied (see the previous sections).

- The geometrical arrangements and the magnetic field in the central barrel introduce a non flat $p_T$ dependence of the efficiency. This is particularly important in the low $p_T$ region ($p_T < 1$ GeV/c) where, due to the exponential shape of the $dN/dp_T$ distribution, most of the particles are produced, and where the differential shape of $v_2(p_T)$ is definitely not flat (see chap.1). Therefore, the $p_T$ dependence of the efficiency needs to be taken into account when calculating the integrated $v_2$ (i.e. the integral of $v_2(p_T)$ convoluted with the corrected $dN/dp_T$ spectra, see sec.3.2.5).

- The azimuthal segmentation of the active elements in the main tracking device (the TPC, see sec.2.1.2) causes a periodic drop in the azimuthal dependence of the efficiency, at $\phi = n \times 2\pi/18$ (see fig.3.2(a)). As described in sec.3.2.4, the implementation of the flow analysis code already incorporates a correction of the $dN/d\phi$ distribution, i.e. $\phi$ weights are used for the determination of the $\vec{Q}$ vector.

The efficiency corrections as a function of $p_T$ are calculated by means of the full set of simulations produced for the present analysis (Hijing and GeVSim with Themis), using the ‘optimal’ set of cuts discussed above. This is done to incorporate some systematic effect due to the ‘a priori’ unknown particle composition as discussed in sec.5.1.2: the difference between the two sets is added to the systematic error (see sec.5.2.2).

The $p_T$ dependence of the reconstruction efficiency (for the two cases, Hijing and GeVSim) are shown in fig.5.3. The combined result (the efficiency correction
5.2 Cut optimization

factor that is used in the analysis) is shown in fig.5.10(b). The integrated efficiency (under the applied set of cuts) for particles between 0.1 and 10 GeV/c is $\langle \text{eff} \rangle = 67\%$. The integrated purity is $\langle \text{pur} \rangle = 95.7\%$.

As described in the previous section, the corrections are applied for $p_T > 100$ MeV/c, while the low $p_T$ part of the spectrum ($p_T < 100$ MeV/c) is extrapolated as a fraction of the observed $dN/dp_T$ distribution corrected by the efficiency.

Considering also the first $p_T$ bin, the total reconstruction efficiency is $\langle \text{eff}_{\text{tot}} \rangle = N'_{MC}/N'_{ESD} = 64.3\%$ (this number is used to scale up the reconstructed multiplicity for the plot of $v_2$ vs $dN/d\eta$, see fig.5.24 and 5.29).

5.2.2 Systematic Error

The systematic uncertainty on the efficiency (as a function of $p_T$) is calculated by varying the most sensitive observables pointed out in section 5.1 (i.e. the particle composition and the multiplicity dependence), and the applied cuts (limiting the discussion to the transverse DCA only, see sec.5.2).

![Figure 5.11. Systematic error on the efficiency, calculated from the uncertainty on the particle composition (sec.5.1.2), the difference in particle multiplicity (sec.5.1.3), and the applied cut on the 'DCA.](image)

Each contribution is obtained from the absolute difference in the calculated efficiency between two (extreme) cases: the systematic uncertainty due to the unknown particle composition is the difference between the two simulated inputs (sec.5.1.2), the uncertainty due to the multiplicity dependence is the difference between the lowest and highest multiplicity events (sec.5.1.3)), the uncertainty due to the applied DCA cut is the difference between a DCA cut at 400 $\mu$m and 600 $\mu$m ($\pm 100$ $\mu$m around the chosen value of 500 $\mu$m). The total systematic uncertainty $\sigma_{\text{eff}}$ is calculated by adding quadratically the three contributions (see fig.5.11):

$$\sigma_{\text{eff}} = \sqrt{\sigma_{\text{mult.}}^2 + \sigma_{\text{p.con.}}^2 + \sigma_{\text{DCA}}^2}.$$ (5.9)
The systematic error on the extrapolation of \( dN/dp_T \) between 0 and 100 MeV/c is calculated as the difference in the extrapolation parameter \( n_{\text{low}} \) between the two sets of simulations (see above): \( \sigma_{n_{\text{low}}}/n_{\text{low}} \simeq 7\% \). It is comparable with the systematic error at low \( p_T \) on the calculated efficiency (for comparison, the value is shown as the first \( p_T \) bin of fig.5.11).

The systematic uncertainty on the reconstruction efficiency \( \sigma_{\text{eff}} \) is used to estimate the systematic error on the measured \( v_2 \) (see sec.5.3.5).

5.3 Genuine flow reconstruction (GeVSim)

This section presents the results of the event plane analysis performed over three different sets of fully reconstructed ALICE events simulated with GeVSim, each set based on a different extrapolation of the centrality dependence of elliptic flow (as presented in sec.1.3).

5.3.1 Simulations details

Events are produced in six centrality classes, with particle multiplicity and width listed in tab.5.1.

The main vertex position has been fixed at \((x, y, z) = (0, 0, 0)\) (see sec.5.1.4). The magnetic field, measured at the center of the ALICE solenoid, is \( \vec{B} = 0.4 \) Tesla.

Table 5.1. Summary table of the 3 sets of GeVSim simulations. Per each centrality class (c.c.), the input values of \( v_2 \), the particle multiplicity (and width), and the number of produced events are listed.

<table>
<thead>
<tr>
<th>c.c.</th>
<th>( dN_{ch}/dp_T ) ± ( \sigma )</th>
<th>( \langle v_2^{\text{LDL}} \rangle )</th>
<th>( N_{\text{evts}} )</th>
<th>( \langle v_2^{\text{hydro}} \rangle )</th>
<th>( N_{\text{evts}} )</th>
<th>( \langle v_2^{\text{hydro2}} \rangle )</th>
<th>( N_{\text{evts}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1922 ± 300</td>
<td>0</td>
<td>1k</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1619 ± 290</td>
<td>5.15</td>
<td>1k</td>
<td>2.35</td>
<td>2.5k</td>
<td>3.5</td>
<td>1.7k</td>
</tr>
<tr>
<td>2</td>
<td>1013 ± 200</td>
<td>11.4</td>
<td>0.4k</td>
<td>5.87</td>
<td>1k</td>
<td>8.81</td>
<td>0.5k</td>
</tr>
<tr>
<td>3</td>
<td>617 ± 160</td>
<td>12.95</td>
<td>0.5k</td>
<td>8.04</td>
<td>1k</td>
<td>12.06</td>
<td>0.5k</td>
</tr>
<tr>
<td>4</td>
<td>213 ± 90</td>
<td>8.8</td>
<td>2k</td>
<td>9.55</td>
<td>2k</td>
<td>14.33</td>
<td>0.8k</td>
</tr>
<tr>
<td>5</td>
<td>42 ± 30</td>
<td>2.75</td>
<td>16k</td>
<td>7.63</td>
<td>7.5k</td>
<td>11.44</td>
<td>6.7k</td>
</tr>
</tbody>
</table>

Particle composition

The particle composition has been calculated using Thermus, a ROOT implementation of the thermal model for particle productions [144]. The chemical freeze-out temperature has been set to \( T_{ch} = 170 \text{ MeV/c} \) and the baryon chemical potential to
Table 5.2. Total and relative relative particle abundances calculated with Thermus (input of the GeVSim simulations).

<table>
<thead>
<tr>
<th>p.type (%/tot)</th>
<th>P.Id.</th>
<th>m GeV/c²</th>
<th>%/tot</th>
<th>%/‘stable’ h⁺</th>
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</thead>
<tbody>
<tr>
<td>pions</td>
<td>π⁺</td>
<td>0.13957</td>
<td>22.5398</td>
<td>39.36</td>
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<tr>
<td>72.2%</td>
<td>π⁻</td>
<td>0.13498</td>
<td>22.5452</td>
<td>39.37</td>
</tr>
<tr>
<td></td>
<td>π⁰</td>
<td>0.13498</td>
<td>27.0965</td>
<td>0</td>
</tr>
<tr>
<td>kaons</td>
<td>K⁺</td>
<td>0.49368</td>
<td>4.05139</td>
<td>7.07</td>
</tr>
<tr>
<td>16%</td>
<td>K⁻</td>
<td>0.49765</td>
<td>4.04341</td>
<td>7.06</td>
</tr>
<tr>
<td></td>
<td>K⁰ₛ</td>
<td>0.49765</td>
<td>3.9437</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>K⁰ₗ</td>
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<td>3.9437</td>
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</tr>
<tr>
<td>nucleons</td>
<td>p</td>
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<td>2.05554</td>
<td>3.59</td>
</tr>
<tr>
<td>8.2%</td>
<td>⚠</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>n</td>
<td>0.939565</td>
<td>2.05242</td>
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<tr>
<td></td>
<td>⚠</td>
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</tr>
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<td>hyperons</td>
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<td>1.937515</td>
<td>0</td>
</tr>
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<td>3.6%</td>
<td>Σ⁺⁺Σ⁻⁻</td>
<td>1.18937</td>
<td>0.528322</td>
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<td>1.21945</td>
<td>0.515834</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Ξ⁻⁺Ξ⁺⁻</td>
<td>1.3217</td>
<td>0.311066</td>
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</tr>
<tr>
<td></td>
<td>Ξ⁰⁺Ξ⁰⁻</td>
<td>1.3148</td>
<td>0.315868</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Ω⁻⁺Ω⁺⁻</td>
<td>1.6724</td>
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<td>0</td>
</tr>
<tr>
<td>heavy mesons</td>
<td>φ⁰</td>
<td>1.101945</td>
<td>0.557406</td>
<td>0</td>
</tr>
</tbody>
</table>

\( \mu_B = 10 \text{ MeV/c} \) (the calculation was done for hadrons only). The resulting particle abundances are listed in tab.5.2.

The relative ratios of the three type of charged primary hadrons considered in the analysis are 78.7% \( \pi^\pm \), 14.1% \( K^\pm \), 7.1% \( p \) and \( \bar{p} \). All events of this set of simulations have been produced with the same particle ratios and input spectra, while the total multiplicity and the magnitude of elliptic flow are assigned with respect to the centrality class.

\( p_T \) and \( \eta \) spectra

In order to reproduce a realistic particle spectrum in the momentum range of interest \( (0 < p_T < 10 \text{ GeV/c}) \), the simulated \( d^3N/dp \) distribution (expressed in the three familiar components \( p_T, \eta \) and \( \phi \)) has been customized with a user defined formula, similar to the Levy distribution in \( m_T \) [41], convoluted with a flat distribution in rapidity \( y \) (which leads to a non flat pseudorapidity distribution) and an azimuthal distribution (with flow) generated by GeVSim.
Figure 5.12. Input spectra, \(dN/dp_T\) (a) and \(dN/d\eta\) (b), of the GeVSim simulations for the three stable charged hadrons (\(\pi^\pm, K^\pm, p\) and \(\bar{p}\)) with the relative ratios given in tab.5.2.

The input \(dN/dp_T\) spectrum is given by the equation:

\[
dN_dp_T = A \cdot \frac{p_T}{(1 + (m_T - m)/n_M \cdot T_0)^n M},
\]

where \(m_T = \sqrt{m^2 + p^2_T}\) is the transverse mass, \(T_0\) is the slope parameter (temperature) and \(n_M = n/m\alpha\) is the modified slope variation parameter. This phenomenological term introduces a weak dependence of the slope variation on the particle mass, so that the the tail of the \(dN_i/dp_T\) distribution becomes particle species dependent (through \(m_i\)) and a single slope variation parameter \(n\) can be used to reproduce the spectra of all particles. The parameters of eq.5.10 are tuned by a fit of the generated spectra of pions, kaons and protons produced by Hijing. The obtained values are \(T_0 = 125\) MeV, \(n = 5\) and \(\alpha = 0.11\).

Fig.5.13 shows a fit of the \(dN/dp_T\) spectra generated by Hijing (the fit is limited to the interval \(0 < p_T < 3\) GeV/c). The fit works quite well at low \(p_T\), but it fails to reproduce the correct slope of the tail of the distribution for \(p_T \gtrsim 4 - 5\) GeV/c. However, fig.5.3 shows that the reconstruction is not very sensitive to the shape of the input spectra, especially at high \(p_T\): the difference in efficiency (purity) between the two inputs (Hijing and GeVSim), due to the effects of bin migration and particle composition, is smaller than a few %.

To save computing time, the range of the simulations has been limited to the central pseudorapidity interval, around the coverage of the ALICE central barrel detectors (\(-1.3 \lesssim \eta \lesssim 1.3\)).

**Elliptic flow \(v_2\)**

Table 5.1 summarizes the simulated values of \(\langle v_2 \rangle\) for the three different parametrizations (named LDL, hydro and hydro2).
5.3 Genuine flow reconstruction (GeVSim)

The differential shape of $v_2(p_T)$ is linearly rising, with saturation value at $p_T = 2$ GeV/$c$. Integrating over the given input spectra of $\pi^\pm$, $K^\pm$, $p$ and $\bar{p}$ (see also sec.4.2) the saturation values of $v_2$ are given by $v_2^{sat} = k_{i\rightarrow s} \langle v_2 \rangle$, with $k_{i\rightarrow s} = 3.85$.

The number of simulated events is chosen so that $v_2^2 \times dN/d\eta \times N_{evts}$ is about constant, this should give roughly the same statistical error on the measured $v_2$ in each class. The ‘constant’ value ($v_2^2 \times dN/d\eta \times N_{evts} \sim 3000$) is determined by the available resources and CPU time. The resulting statistical error is comparable to the systematic uncertainty (see sec.5.3.5).

5.3.2 Event plane determination and resolution study

The width of the reconstructed event plane with respect to the true one is described by the resolution parameter (see sec.3.2.1).

As an example, fig.5.14 shows the $\Delta\Psi$ distribution (modulo $\pi$) for three centrality classes (central, mid-central and peripheral events) of the hydro simulations (see tab.5.1). In the upper part of the figure the difference between the reconstructed event plane and the simulated reaction plane is plotted ($\Delta\Psi^{true} = \Psi^{true} - \Psi^{obs}$), in the lower part the difference between $\eta$ sub-events ($\Delta\Psi^{\eta\text{sub}} = \Psi^{A} - \Psi^{B}$, with $A$ and $B$ equal multiplicity $\eta$ sub-events).

The width of the $\Delta\Psi^{true}$ distribution is not very sensitive to the applied cuts, becoming slightly worse if no cuts are applied. However, in the latter case, the observed $\Delta\Psi^{\eta\text{sub}}$ distributions are narrower due to azimuthal correlations between secondary particles (such as decay products) and this can lead to an overestimate of the event plane resolution (see below).

---

9Due to some failed simulations, this is not always the case (see tab.5.1).

10In absence of non-flow effects the result does not depends on the choice of the sub-events.
Using the iterative procedure implemented in the analysis code (see sec.3.2.1), the event plane resolution is extrapolated from the observed $\Delta \Psi_{n}^{\text{sub}}$. The iteration is based on eq.3.8, here rewritten for $n = 2$:

$$\text{res}_2 = \langle \cos \left[ 2(\Psi_{2}^\text{obs} - \Psi_{2}^\text{true}) \right] \rangle = \frac{\sqrt{\pi}}{2 \sqrt{2}} \chi_2 e^{-\frac{\chi_2^2}{4}} \times \left[ I_0\left(\frac{\chi_2^2}{4}\right) + I_1\left(\frac{\chi_2^2}{4}\right) \right], \quad (5.11)$$

where $I_n$ are modified Bessel functions of order $n$, and $\chi_2 = v_2/\sigma$, $\sigma = \sqrt{\frac{1}{2M} \langle w^2 \rangle}$.

For unitary weights ($w_i = 1$), $\chi_2 = v_2/\sqrt{2M}$.

When the extrapolation is done using only primary particles from the KineTree, the result is in perfect agreement with the ‘true’ resolution $\Psi_{2}^\text{true} - \Psi_{2}^\text{obs}$.

The optimization of the cuts for the reconstruction of the event plane is done by comparing the observed event plane resolution with the ‘ideal’ one, calculated by feeding the input values of $M'$ and $v'_2$ into eq.5.11. Note that $M'$ is the multiplicity used for the calculation of the event plane, i.e. all reconstructible primary particles in the ALICE central barrel ($M' = 1.8 \times dN'/d\eta$), $v'_2$ is the integrated elliptic flow of all primary $\pi^\pm, K^{\pm}, p$ and $\bar{p}$. Figure 5.15 shows the observed event plane resolution (calculated from $\Delta \Psi_{2}^{\eta\text{-sub}}$) with respect to the centrality class for the three sets of GeVSim events, using different track selections.

The observed resolution becomes lower using more strict cuts because of the
5.3 Genuine flow reconstruction (GeVSim)

Figure 5.15. Observed event plane resolution with respect to the centrality class (see tab.5.1 for the simulations details), using different track selections. The ‘ideal’ event plane resolution are shown as well (obtained from the generated distribution of $\cos (2 (\Psi^{true} - \Psi^{obs}))$).

Figure 5.16. Observed event plane resolution, calculated using $p_T$ weights in the definition of $\vec{Q}$, versus centrality class (see tab.5.1 for the simulations details). The plot shows the results using different sets of cuts on the $AliESD$, the ‘ideal’ values of the event plane resolution are shown as well (from $\cos (2 (\Psi^{true} - \Psi^{obs}))$).

reduced statistic (lower $M$), however if no condition is applied to exclude secondary tracks, the observed resolution can be higher than the true one. The effect is more visible in peripheral (low multiplicity) events, where the resolution is far from its saturation (see bin 5 of fig.5.15(a) and (b)). The constrainability condition alone (for TPC + ITS tracks) is enough to obtain a resolution very close to the ‘ideal’ values.

A better event plane resolution is achieved by using $p_T$ weights in the calculation of the $\vec{Q}$ vector (see sec.3.2.3). The use of $p_T$ weights, in fact, reduces the contribution of tracks at low $p_T$, where the purity is lower (see sec.5.1). For the same reason, the resolution becomes less sensitive to the applied cuts (see fig.5.16).

From the above study we can conclude that the best resolution is achieved by selecting constrainable TPC + ITS tracks and using $p_T$ weights.
Figure 5.17. Effects on the full-event resolution \( \Delta \text{Res} = \frac{\text{res}^{\text{obs}}}{\text{res}^{\text{true}}} \) of an uncorrectly reconstructed multiplicity, for five different combinations of \( v_2 \) and \( M \) (from the \textit{hydro} parametrization, see tab.5.1).

From the expression of the \( \vec{Q} \) vector (eq.3.3) we may argue that the presence of randomly distributed secondaries and double counted tracks does not affect the direction of the reconstructed event plane. The two averages:

\[
\langle \cos (n\phi_i) \rangle, \langle \sin (n\phi_i) \rangle
\]

lead to the same central values either by adding randomly distributed \( \phi \) angles \( \langle \cos (n\phi_{\text{rnd}}) \rangle \sim 0 \), or by doubling each term (as it would happen if every track is reconstructed twice).

A possible problem may arise from the full-event plane resolution. The resolution of sub-events, calculated from the difference between \( \Psi_A^2 - \Psi_B^2 \) (see eq.3.10), is safely under control because the average direction of \( \Psi_2 \) does not change in presence of impurities. But the calculation of the full-event resolution involves the observed multiplicity (eq.3.8), and a larger \( M \) would result in an overestimate of the resolution (and therefore, an underestimate of the measured \( v_2 \)).

For a few values of elliptic flow and multiplicity, fig.5.17 shows how the resolution changes with respect to the fraction of impurity in the sample (values \( v_2 \) and \( M \) are taken from the \textit{hydro} parametrization, see tab.5.1).

The integrated purity \(^{11}\) of the basic selection (constraintability condition of TPC + ITS tracks) is 90\% (see fig.5.8). If the purity is weighted with \( p_T \) (using the same weight as in the calculation of \( \vec{Q} \)), the integrated purity becomes \( \sim 93\% \), leading to a systematic error on the observed resolution smaller than 4% in the worst case (peripheral events).

\(^{11}\)Integral of the purity convoluted with the observed \( p_T \) spectra.
5.3 Genuine flow reconstruction (GeVSim)

Figure 5.18. (a) Linear fit of the reconstructed $v_2$ as a function of $p_T$ (eq.5.14), with extrapolation to $p_T = 0$. The input value and the KineTree result are also shown. (b) Evaluation of the first $p_T$ bin ($0 < p_T < 100$ MeV/$c$) and the associated error from the efficiency corrected $dN/dp_T$ spectrum, through the factor $n_{flow}$ (see eq.5.6), a Levy fit of the corrected spectra is also shown (eq.5.8). The full histogram represents the simulated spectrum, the lower set of data is the observed spectrum after the cuts (see sec.5.2) without efficiency correction. Those plots are taken from the centrality class 2 of the hydro simulations (see tab.5.1).

5.3.3 Differential flow of charged particles

The shape of $v_2$ as a function of $p_T$ is an important observable for determining the properties of the Equation of State (see sec.1.3.4). Moreover the study of elliptic flow with respect to the transverse momentum is needed for the evaluation of the integrated $v_2$.

For $p_T$ bins small enough (i.e. in the order of the detector resolution), the reconstruction efficiency can be considered roughly constant within each bin and therefore the differential shape of $v_2$ versus $p_T$ can be measured without taking into account efficiency corrections.

According to the event plane analysis method (see sec.3.2 and [123]), $v_2(p_T)$ is obtained dividing the measured $v_2^{obs}$ by the event plane resolution, calculated as the average $\cos \left[ 2 \left( \Delta \Psi^{sub}_2 \right) \right]$ over the centrality class:

$$ v_2(p_T) = \frac{v_2^{obs}(p_T)}{\langle \text{res}_2 \rangle_{c.c.}} = \frac{\langle \cos \left[ 2(\phi - \Psi_2) \right] \rangle_{p_T \text{bin}}}{\langle \cos \left[ 2(\Psi^{obs}_2 - \Psi^{true}_2) \right] \rangle_{c.c.}}. \quad (5.13) $$

Due to the high purity of the track selection (see fig.5.10(b)), no other systematic corrections are applied to the measured elliptic flow. The (small) effect of impurities is incorporated into the systematic error (see sec.5.3.5).

A linear fit going through the origin (fig.5.18(a)) is used to extrapolate the measurement of $v_2(p_T)$ down to $p_T < 100$ MeV/$c$:

$$ v_2(p_T) = a \times p_T. \quad (5.14) $$
The fit interval is $p_T \in (0.1, 2) \text{GeV}/c$, according to the input of the simulations (see sec.1.3.4).

In a real experiment, where the differential shape of $v_2(p_T)$ is not linear (see for example fig.1.13 [45, 64]), the extrapolation of $v_2(p_T)$ to $p_T = 0$ could be also approximated linearly, due to the very small not-covered $p_T$ range and to the physical constraint $v_2(0) = 0$. The limited number of particle produced at $p_T < 100 \text{MeV}/c$ (3.9% of the total in the present parametrization of GeVSim, and 4.2% in Hijing) ensures that an uncertainty up to 20% on the extrapolated $v_2 |_{p_T<100\text{MeV}/c}$ would give an error on the integrated $v_2$ smaller than 1%.

Figure 5.19. Reconstructed $v_2$ as a function of $p_T$ for the six centrality classes of the LDL sample, including the most central events with $v_2^{12} = 0$. The input and a linear fit of the reconstructed data are also shown.

The three figures (fig.5.19, 5.20 and 5.21) show the reconstructed shape of $v_2$ as a function of $p_T$ in the interval $0 < p_T < 5 \text{GeV}/c$, for the three sets of GeVSim simulations. The input values and a linear fit of the data are plotted as well. Only one set of simulations has been produced for the centrality class 0 (most central events, with $v_2 = 0$), and it is shown in fig.5.19 together with the LDL sample.

As expected, the measured $v_2$ is in perfect agreement with the input values as long as the event plane resolution is close to 1 (which is mostly the case). For the most peripheral events (c.c.5), due to the larger fluctuation in multiplicity ($\sim 80\%$, see tab.5.1), the difference between particlewise and eventwise average is not neg-

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12Charged, ‘stable’ hadrons: $\pi^\pm$, $K^\pm$, $p$ and $\bar{p}$. 

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5.3 Genuine flow reconstruction (GeVSim)

![Graphs showing reconstructed $v_2$ as a function of $p_T$ for the five centrality classes of the hydro sample.](image1)

**Figure 5.20.** Reconstructed $v_2$ as a function of $p_T$ for the five centrality classes of the hydro sample. The centrality class 0 plot has not been repeated (see fig.5.19).

![Graphs showing reconstructed $v_2$ as a function of $p_T$ for the five centrality classes of the hydro2 sample.](image2)

**Figure 5.21.** Reconstructed $v_2$ as a function of $p_T$ for the five centrality classes of the hydro2 sample. Centrality class 0 is omitted (see fig.5.19).
The resolution is calculated from the event averaged $\cos (2\Delta \Psi_2)$, while $v_{2}^{\text{obs}}$ is calculated from the particle averaged $\langle \cos (2 [\Psi_2 - \phi]) \rangle$. Higher multiplicity events add more particle with a larger $v_{2}^{\text{obs}}$, but the event plane resolution (calculated as the average over all the events in the centrality class) gives all the events the same weight causing an overcorrection of the observed $v_2$ and a consequent overestimate of the measured elliptic flow. This effect could be corrected by calculating the resolution as a weighted average over the events, with weights proportional to the selected multiplicity. However the effect is smaller than the statistical error on the measurements, and therefore it has not been taken into account.

### 5.3.4 Integrated $v_2$

The integrated elliptic flow is calculated as the average between reconstructed values of $v_2$ versus $p_T$ (see sec.5.3.3) weighted by the number of particle reconstructed at each $p_T$ of the $dN/dp_T$ distribution:

$$\langle v_2 \rangle = \frac{1}{N_{\text{tot}}} \sum_{p_T \text{bins}} v_2(p_T) \times \frac{dN^{\text{obs}}}{dp_T} \times \text{eff}(p_T).$$

(5.15)

As explained in sec.5.2.1, the measured $p_T$ spectrum must be first corrected by the reconstruction efficiency of the selected sample (see also eq.3.18).

![Figure 5.22. Integrated $v_2$ with respect to the centrality class for the three sets of GeVSim simulations. The plot shows the reconstructed $\langle v_2 \rangle$ from the AliESDs and the results of the event plane analysis on the KineTrees, statistical and systematic errors are also shown (see sec.5.3.5).](image)

The number of particle with $p_T < 100$ MeV/c is extrapolated as fraction of the total integral of $dN/dp_T$ (see sec.5.2):

$$N_{p_T<100\text{MeV}/c} = n_{\text{low}} \times \frac{1}{\text{eff}} N_{\text{obs}},$$

(5.16)

where $N_{\text{obs}}/\text{eff}$ is the integral of the efficiency corrected spectrum observed at $p_T > 100$ MeV/c, and $n_{\text{low}} = 0.0406$ (see eq.5.6). The result is shown in fig.5.18(b), together with the input spectrum from the KineTree for comparison.
5.3 Genuine flow reconstruction (GeVSim)

A linear fit is used to extrapolate the $v_2$ measurement down to $p_T = 0$ (see fig.5.18(a)). The mean value of $v_2$ in the first bin is calculated from the fit function, evaluated at the mean value of the $dN/dp_T$ distribution between 0 and 100 MeV/$c$ (calculated from the fit of the efficiency corrected $p_T$ spectrum, see eq.5.8).

Finally, figure 5.22 shows the integrated $v_2$ with respect to the centrality class for the three sets of GeVSim simulations. As we can see, the simulated values of $\langle v_2 \rangle$ are well reproduced within the statistical (and systematic) error.

5.3.5 Systematic and Statistical Error on the measured $v_2$

The only source of systematic error on the differential shape of $v_2$ as a function of $p_T$ is due to the presence of impurities in the reconstructed spectra (‘impurities’ in each $p_T$ bin includes both secondary particles and primary particles reconstructed at a different $p_T$).

As shown in fig.5.23(a), at low transverse momentum ($p_T \lesssim 1$ GeV/$c$) secondary particles have a larger $v_2$ than primary particles (in a decay, the mother particle produces two daughters with roughly the same flow of the mother but a lower momentum), therefore the presence of contaminations increases the measured value of $v_2$ at lower $p_T$. The opposite effect (contamination with a lower $v_2$) can also happen due to bin-shift, however the effect is completely negligible with respect to the statistical fluctuations (see fig.5.23(a), at $p_T > 2$ GeV/$c$).

The systematic error on $v_2(p_T)$ is calculated from the difference $\Delta v_2$ between the measured $v_2$ of correctly reconstructed primary particles and the measured $v_2$ of the contamination found in the final ESD, weighted by the purity of the selection in each bin. The relative difference $\Delta v_2/v_2$ is large only in the first few bins ($p_T < 500$ MeV/$c$).

The overestimate on the measured $v_2$ at low $p_T$ due to impurity can be expressed as:

$$v_2^{\text{meas}}(p_T) = \text{pur}(p_T) \times v_2'(p_T) + (1 - \text{pur}(p_T)) \times v_2''(p_T).$$

(5.17)

The relative systematic error on the measured $v_2$ is therefore obtained as:

$$\frac{|v_2'(p_T) - v_2^{\text{meas}}(p_T)|}{v_2'(p_T)} = (1 - \text{pur}) \times \frac{\Delta v_2}{v_2'(p_T)}.$$  

(5.18)

Weighting this contribution by the purity of the selected track sample, only the leftmost bin ($100 < p_T < 200$ MeV/$c$), where where the magnitude of $v_2'$ is small and the contamination is large, shows a big systematic error $\sigma_{v_2'/v_2} \simeq 6.5\%$ (see fig.5.23(b)). Otherwise the error is in the order of $1 - 2\%$ on almost all the $p_T$ range of interest, becoming negligible for $p_T > 800$ MeV/$c$.

As a consequence, the integrated $v_2$ is hardly affected by this level of contamination, however it is possible to calculate an upper limit for the systematic error on
Figure 5.23. (a) Reconstructed $v_2/v_2^{sat}$ as a function of $p_T$ for all primary particles and for reconstructed secondaries in the ESD, the difference between the two and the relative contribution to the measured $v_2$ are shown as well. Since the effect is similar for any input value of $v_2$, this plot is produced from the whole set of GeVSim simulations, scaling each centrality class by its saturation $v_2$. (b) Systematic error on the measured $v_2$, calculated as $(1 - pur) \times \Delta v_2/v_2$ (the calculated impurity is also shown).

The result of this procedure is:

$$\frac{\sigma_{v_2}}{\langle v_2 \rangle} = \frac{1}{\langle v_2 \rangle} \left| \langle v_2 \rangle^+ - \langle v_2 \rangle^- \right| \simeq 0.126,$$

which implies a systematic uncertainty on the central $\langle v_2 \rangle$ value of $\pm 6.3\%$. 

\[\langle v_2 \rangle \] due to the presence of impurities:

$$\sigma_{v_2}^{tot} = \sqrt{\sum N_{pT} \sigma_{v_2}^2(p_T)} \lesssim 2.4\% , \quad (5.19)$$

where the sum has been limited to the interval $0.1 < p_T < 1$ GeV/c.

The systematic error on the integrated $v_2$ is dominated by the uncertainty on the efficiency (as a function of $p_T$), calculated in sec.5.2.2. 

The relative systematic error $\sigma_{(v_2)} / \langle v_2 \rangle$ on the integrated flow is calculated as the difference $\sigma_{(v_2)} = |\langle v_2 \rangle^+ - \langle v_2 \rangle^-|$, where:

$$\langle v_2 \rangle^\pm = \frac{1}{N_{tot}} \sum_{p_T\text{bins}} v_2(p_T) \times \frac{dN_{obs}}{dp_T} \times (\text{eff}(p_T) \pm \sigma_{\text{eff}}) , \quad (5.20)$$

divided by the (measured) central value of $\langle v_2 \rangle$.

This also includes the systematic uncertainty on the extrapolation of $dN/dp_T$ between 0 and 100 MeV/c, where the two extremes are given by $N_1 \pm 7\%$ (see sec.5.2.2).

The result of this procedure is:

$$\frac{\sigma_{(v_2)}}{\langle v_2 \rangle} = \frac{1}{\langle v_2 \rangle} \left| \langle v_2 \rangle^+ - \langle v_2 \rangle^- \right| \simeq 0.126 , \quad (5.21)$$

which implies a systematic uncertainty on the central $\langle v_2 \rangle$ value of $\pm 6.3\%$. 

\[\langle v_2 \rangle \] due to the presence of impurities:

$$\sigma_{v_2}^{tot} = \sqrt{\sum N_{pT} \sigma_{v_2}^2(p_T)} \lesssim 2.4\% , \quad (5.19)$$

where the sum has been limited to the interval $0.1 < p_T < 1$ GeV/c.
Figure 5.22 shows that, with the number of events available, the systematic error $\sigma_{\langle v_2 \rangle}$ is large but comparable to the statistical error, calculated as $v_2^{\text{RMS}} / \sqrt{N_{\text{obs}}}$ (where $v_2^{\text{RMS}} = \sqrt{\langle (\langle v_2 \rangle - v_2)^2 \rangle}$).

However the statistical error associated to the present measurements is probably underestimated, due to the fact that the simulations in each centrality class have been produced with a fixed input value of $\langle v_2 \rangle$. Therefore the width of the $v_2$ distribution within each centrality class is smaller than in a real experiment.

An upper limit on the statistical error on the integrated flow, with respect to the number of events available, is given by:

$$
\sigma_{\text{stat}} < \frac{\max(v_2^{\text{RMS}})}{\sqrt{N_{\text{evts}}}}, \quad (5.22)
$$

where $v_2^{\text{RMS}} = \sqrt{\langle (\langle v_2 \rangle - v_2)^2 \rangle}$ is the spread of $v_2$ within a single event.

The maximum spread in $v_2$ is 200% (from particles maximally correlated to the event plane, to particles maximally anti-correlated), and the smallest multiplicity considered in the present analysis is 40 particles per unit rapidity (see tab.5.1 and 5.3), which gives a minimum of about 50 correctly reconstructed primary particles per event in the TPC volume $^{13}$. Therefore, the upper limit $v_2^{\text{RMS}}$ is $\max(v_2^{\text{RMS}}) = 4\%$, giving a $\sigma_{\text{stat}} < 0.04/\sqrt{N_{\text{evts}}}$. The upper limit of the relative statistical error on $v_2$ is given by (for $\langle v_2 \rangle \geq 1\%$):

$$
\max(\sigma_{\text{stat}}/v_2) = \frac{4}{v_2(\%)\sqrt{N_{\text{evts}}}} < \frac{4}{\sqrt{N_{\text{evts}}}}, \quad (5.23)
$$

which becomes less than 4\% as soon as 10,000 events are available, and $\sigma_{\text{stat}} < 0.4\%$ for $N_{\text{evts}} = 1.000.000$ (one day of ALICE run). We can compare eq.5.23 with the values listed in tab.5.5 (see also the discussion in sec.5.5).

5.3.6 Conclusions

Fig.5.24 shows the integrated $v_2$ with respect to the charged multiplicity at mid-pseudorapidity (corrected by the total reconstruction efficiency) for the three sets of GeVSim simulations.

From this plot we can see how well the event plane analysis at ALICE can distinguish between different models describing the underlying physics of elliptic flow, in relation to the error associated to the measurement (both statistical and systematic errors are shown).

However, in these simulations non-flow effects are absent or very low (no jet correlations are there, only decays). In a real experiment they are expected to give a large contribution in the low multiplicity region (see sec.4.1 and 5.4).

$^{13}$This number is approximately given by $N'_{\text{TPC}} \sim 1.8 \times \frac{4N}{d\eta} \times \text{eff}$, with an efficiency about 64\% (see sec.5.2.1).
5.4 Realistic scenario (Hijing + After-Burner)

In section 4.1 Hijing simulated events have been studied to quantify the non-flow correlations originating from jets and particle decays, and in section 4.3 we saw the combined effect of genuine elliptic flow and non-flow correlations.

This section will illustrate an analysis done on a realistic set of data, generated with Hijing plus the flow After-Burner, and fully reconstructed in AliRoot.

5.4.1 Simulations details

Events are produced in twelve centrality classes, each one with a fixed impact parameter. Particle multiplicity, its width, and the magnitudes of the integrated $v_2$ are listed in tab.5.3.

The main vertex position is fixed at $(x, y, z) = (0, 0, 0)$. The magnetic field, measured at the center of the solenoid, is $\vec{B} = 0.4$ T.

Particle composition

The particle composition generated by Hijing is the result of its internal implementation of the hadronization processes [104].

Tab.5.4 shows the relative particle abundances, averaged over all the produced Hijing events. The relative ratios of the three type of charged primary hadrons considered in the analysis are $86.7\% \pi^\pm$, $8.7\% K^\pm$, $4.6\% p$ and $\bar{p}$.

A detailed study of the centrality dependence of the particle ratios has not been carried out, however, due to the implementation of Hijing as a superposition of many $pp$ events, they are approximately constant within the statistical fluctuations of each event.
Table 5.3. Details of the Hijing + After-Burner simulations (generated separately in 12 centrality classes).

<table>
<thead>
<tr>
<th>c.c.</th>
<th>b (fm)</th>
<th>( \frac{dN_{ch}}{d\eta} \pm \text{RMS} )</th>
<th>( \langle v_2^{\text{hydro}} \rangle )</th>
<th>%</th>
<th>( N_{\text{evts}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.0</td>
<td>2528 ± 308</td>
<td>0.0</td>
<td>1k</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7.5</td>
<td>2184 ± 303</td>
<td>1.32</td>
<td>2.2k</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8.0</td>
<td>1860 ± 301</td>
<td>3.26</td>
<td>2.4k</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>8.6</td>
<td>1524 ± 295</td>
<td>4.75</td>
<td>1.4k</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>9.15</td>
<td>1264 ± 249</td>
<td>5.95</td>
<td>1.1k</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>9.7</td>
<td>992 ± 192</td>
<td>7.39</td>
<td>1k</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10.6</td>
<td>652 ± 173</td>
<td>8.72</td>
<td>1k</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>11.5</td>
<td>405 ± 139</td>
<td>9.42</td>
<td>1.3k</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>12.2</td>
<td>253 ± 103</td>
<td>9.44</td>
<td>2.5k</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>13.1</td>
<td>121 ± 62</td>
<td>8.67</td>
<td>5.7k</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>13.6</td>
<td>84 ± 54</td>
<td>6.96</td>
<td>12k</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>14.1</td>
<td>43 ± 33</td>
<td>1.0</td>
<td>8k</td>
<td></td>
</tr>
</tbody>
</table>

\( p_T \) and \( \eta \) spectra

The \( dN/dp_T \) and \( dN/d\eta \) distributions generated by Hijing are shown in fig.5.25 for the three species of charged ‘stable’ hadrons considered in the analysis (the spectra in fig.5.25 are obtained as the sum over the entire sample). The pseudorapidity limits are \(-1.3 \lesssim \eta \lesssim 1.3\).

Figure 5.25. Hijing generated spectra of the three charged ‘stable’ hadrons (\( \pi^\pm, K^\pm, p \) and \( \bar{p} \)): \( dN/p_T \) (a) and \( dN/d\eta \) (b).

As we can see, the \( dN/d\eta \) distribution (fig.5.25(b)) is almost flat. The \( dN/dp_T \) distribution (fig.5.25(a)) has a shape which can be described by eq.5.10 (see fig.5.13).
Table 5.4. Total and relative particle abundances produced by the Hijing simulations (not all the particle species are listed, therefore %/tot does not add up to 100%).

<table>
<thead>
<tr>
<th>p.type (%)/tot</th>
<th>P.Id.</th>
<th>%/tot</th>
<th>%/‘stable’ h±</th>
</tr>
</thead>
<tbody>
<tr>
<td>pions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>38.5%</td>
<td>π⁺</td>
<td>10.91</td>
<td>86.7</td>
</tr>
<tr>
<td></td>
<td>π⁻</td>
<td>10.92</td>
<td></td>
</tr>
<tr>
<td></td>
<td>π⁰</td>
<td>16.6</td>
<td>0</td>
</tr>
<tr>
<td>kaons</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.8%</td>
<td>K⁺</td>
<td>1.01</td>
<td>8.7</td>
</tr>
<tr>
<td></td>
<td>K⁻</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>K⁰ₛ</td>
<td>1.67</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>K⁰ₙₛ</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td>nucleons</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.7%</td>
<td>p</td>
<td>0.69</td>
<td>4.6</td>
</tr>
<tr>
<td></td>
<td>¯p</td>
<td>0.68</td>
<td></td>
</tr>
<tr>
<td></td>
<td>n</td>
<td>0.68</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>¯n</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td>hyperons</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.6%</td>
<td>Λ⁰ , Λ⁰</td>
<td>0.73</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Σ⁺ , Σ</td>
<td>0.7</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Ξ⁻ , Ξ</td>
<td>0.16</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Ω⁺ , Ω</td>
<td>0.001</td>
<td>0</td>
</tr>
<tr>
<td>heavy mesons</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ρ , η , ω , φ⁰</td>
<td>13.2</td>
<td>0</td>
</tr>
<tr>
<td>photons</td>
<td>γ</td>
<td>29</td>
<td>0</td>
</tr>
<tr>
<td>leptons</td>
<td>e⁺ , μ⁺ , τ±</td>
<td>0.35</td>
<td>0</td>
</tr>
</tbody>
</table>

Elliptic flow $v_2$

Unlike the simulations described in sec.4.1 and 4.3, the present set of fully reconstructed events has been produced in 12 separated centrality classes, each one with a fixed value of $v_2$ (determined by the geometry of the collision at a fixed impact parameter, see sec.1.3) but a not constant multiplicity, due to the fluctuations involved in the production processes (implemented in Hijing).

Elliptic flow versus centrality has been parametrized according to the hydrodynamic model with lowest value of $c_s$ (see sec.1.3). Tab.5.3 summarizes the simulated values of $\langle v_2 \rangle$ for the 12 centrality classes of the generated events.

The differential shape of $v_2(p_T)$ is linearly increasing up to its saturation value at $p_T^{sat} = 2$ GeV/c (same as the other simulations). Integrating over the Hijing generated spectra of $\pi^{±}$, $K^{±}$, $p$ and $\bar{p}$, the saturation values of $v_2$ are given by $v_2^{sat} = k_{i→s} \langle v_2 \rangle$, with $k_{i→s} = 4.49$. 

for comparison).
5.4 Realistic scenario (Hijing + After-Burner)

5.4.2 Event plane and resolution

Due to the presence of non-flow effects, the ‘observed’ event plane resolution calculated from $\Delta \Psi_{2}^{sub}$ is higher than the ‘true’ one (i.e. $\langle \cos [2(\Psi_{2}^{obs} - \Psi_{2}^{true})]\rangle$). In fig.5.26 the ‘true’ event plane resolution is compared to the ‘observed’ one(s), calculated using two different definition of sub-events (see also sec.4.3). The present results have been obtained from the KineTree of all simulated primary hadrons ($\pi^{\pm}$, $K^{\pm}$, $p$ and $\bar{p}$), fig.4.8(b) in sec.4.3 shows the same plot versus $dN/d\eta$.

![Figure 5.26](image)

Figure 5.26. The generated distribution of $\cos [2(\Psi_{2}^{true} - \Psi_{2}^{obs})]$ is compared to the result of equation 3.8 (for the simulated values of $M = 1.8 \times dN/d\eta$ and $v_{2}$ respectively) and to the ‘observed’ event plane resolution, calculated from $\eta$ and random sub-events. The histogram shows the KineTree results versus the centrality class (see tab.5.3 for the simulations details).

The observed event plane resolution depends on the choice of the sub-events, being closer to the ‘true’ one for $\eta$ sub-events. Therefore the full-event resolution is extrapolated with the iterative procedure described in sec.3.2.1 using $\eta$ sub-events.

The same set of cuts described in sec.5.3.2 has been applied for the reconstruction of the event plane from the AliESDtracks, the choice of constrainable TPC+ITS tracks with no additional cuts gives the best resolution (i.e. the closest to the ‘optimal’ one, calculated from all primary hadrons in the KineTree).

Fig.5.27 shows the observed resolution calculated from the reconstructed tracks using different sets of cuts, with and without $p_{T}$ weights in the calculation of $Q_{2}$. As expected, the use of $p_{T}$ weights gives a higher resolution (closer to its saturation value), which better reproduces the true one.

The presence of non-flow effects is clearly noticeable when their magnitude is comparable with the magnitude of genuine collective flow, i.e. in most central and most peripheral events (first and last bins respectively).
Figure 5.27. Observed event plane resolution versus centrality class, calculated from $\Delta \Psi_2^{nab}$, using different cuts on the reconstructed AliESDs. The two plots show the results using unitary (a) and $p_T$ weights (b) in the calculation of $\vec{Q}_2$. The ‘optimal’ values (i.e. the observed event plane resolution calculated from primary hadrons in the KineTree) are shown as well (see tab.5.3 for the simulations details).

5.4.3 Differential and integrated flow

The reconstruction of the differential shape of $v_2$ is done in the same way as described in sec.5.3.3. Figure 5.28 show the reconstructed shape of $v_2$ as a function of $p_T$ in the interval $0 < p_T < 5$ GeV/c, for the twelve centrality classes of the Hijing + After-Burner simulations. The input values are also shown.

The measured $v_2$ is in good agreement with the input values in mid-central collisions. The agreement is less accurate in the extreme cases (most central and most peripheral events) due to the fact that the magnitude of non-flow effects becomes comparable to the magnitude of the genuine elliptic flow.

The integrated $v_2$ is calculated as in section 5.3.4. Efficiency corrections are applied to the observed $dN/dp_T$ spectrum (see sec.5.2.1), and the first bin of the $dN/dp_T$ histogram is evaluated as a fraction of the total integral of the corrected spectrum observed at $p_T > 100$ MeV/c (see eq.5.9): $N_1 = n_{\text{low}} \times N_a$, with $n_{\text{low}} = 0.0436$. A linear fit of $v_2(p_T)$ is used to extrapolate the measurement of $v_2$ down to $p_T = 0$.

Fig.5.29 shows the integrated $v_2$ as a function of the charged multiplicity (corrected by the total reconstruction efficiency) for the twelve centrality classes of the Hijing + After-Burner simulations. The statistical error on the measurements is $\sigma_{\langle v_2 \rangle} / \langle v_2 \rangle = 6.3\%$ (see sec.5.3.5).

As we can see, the simulated centrality dependence of elliptic flow is well repro-
Figure 5.28. Reconstructed $v_2$ as a function of $p_T$ for the 12 centrality classes of the Hijing + After-Burner simulations. The input and a linear fit of the reconstructed data are also shown.
Simulations & Results

\[ \eta / d \]

\[ \left. \frac{dN}{d\eta} \right. \]

\[ <v_2> \]

\[ \text{Hijing} \]

\[ \text{hydro} \]

\[ \sigma_{\text{sys}} \]

\[ \Delta v_2 \]

Figure 5.29. Reconstructed \( <v_2> \text{meas} \) versus \( dN/d\eta \) for the Hijing + After-Burner simulations, including statistical and systematic error (see sec.5.3.5). The input values of \( v_2 \) are shown as well, and from the difference between the input and the reconstructed \( <v_2> \) the observed magnitude of non-flow effects is drawn.

duced within the statistical error on a wide range of centrality classes (mid-central events). However, non-flow effects cause the reconstructed \( v_2 \) to be larger than the input one, especially in very peripheral collisions. This is shown by the difference between the input and the reconstructed \( <v_2> \) (see fig.5.29).

5.5 Conclusions

Using the results obtained up to here, it is possible to give an overview of the known sources of experimental uncertainties affecting the elliptic flow measurement at ALICE with the event plane method.

To correctly estimate the statistical uncertainty, it must be taken into account that the simulations presented in this chapter were produced in separate centrality classes, each with a fixed value of \( v_2 \). Therefore the statistical error on \( v_2 \) (calculated from the \( v_2^{\text{RMS}} \), see sec.5.3.5) is under-estimated.

For a more reliable prediction, the statistical errors are extrapolated from a set of simulations produced with a continuum impact parameter distribution \((7 < |b| < 14.5 \text{ fm})\) where \( v_2 \) is assigned to each event with respect to its impact parameter, but the centrality class selection is based on the final particle multiplicity. The Kine-Trees of the Hijing + After-Burner simulations (with no detector reconstruction) presented in sec.4.3 have been used for this purpose, where the centrality dependence of \( <v_2> \) follows the hydro parametrization (see sec.1.3.2).

Events are divided into five centrality classes, defined as 20% of the total inelastic cross section (i.e. 20% of the total integral of the Hijing multiplicity distribution,
with rescaled impact parameter $7 < |b| < 14.5$ fm). The statistical errors on $v_2$ obtained in this way have been scaled to take into account the efficiency of the detector and the applied cuts (only 64.3% of the primary particles are actually reconstructed, see sec.5.2.1).

Table 5.5. Summary table of the errors associated to the elliptic flow measurement (from a sample of 50,000 minimum-bias Hijing + After-Burner events, with elliptic flow from the hydro extrapolation). Centrality classes are defined as 20% of the total inelastic cross section.

<table>
<thead>
<tr>
<th>% C.S.</th>
<th>$\frac{dN_{ch}}{d\eta}$</th>
<th>$\langle v_2^{\text{true}} \rangle$</th>
<th>$\sigma_{\text{stat}}$</th>
<th>$\sigma_{\text{sys}}$</th>
<th>$\sigma_{\text{non-flow}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0−20</td>
<td>$&gt; 1450$</td>
<td>3.67%</td>
<td>0.04</td>
<td>0.20</td>
<td>0.12</td>
</tr>
<tr>
<td>20−40</td>
<td>670−1450</td>
<td>7.87%</td>
<td>0.03</td>
<td>0.43</td>
<td>0.10</td>
</tr>
<tr>
<td>40−60</td>
<td>260−670</td>
<td>9.74%</td>
<td>0.04</td>
<td>0.53</td>
<td>0.01</td>
</tr>
<tr>
<td>60−80</td>
<td>260−100</td>
<td>8.09%</td>
<td>0.10</td>
<td>0.44</td>
<td>0.49</td>
</tr>
<tr>
<td>80−100</td>
<td>$&lt; 100$</td>
<td>4.50%</td>
<td>0.38</td>
<td>0.25</td>
<td>2.88</td>
</tr>
<tr>
<td>0−100</td>
<td>0−2500</td>
<td>6.76%</td>
<td>0.05</td>
<td>0.37</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 5.5 summarizes the three sources of uncertainty that have been considered in the present analysis: statistical error, systematic error, non-flow contributions. The statistical errors ($\sigma_{\text{stat}}$) listed in tab.5.5 are calculated as:

$$\sigma_{\text{stat}} = \frac{\sigma_{\text{RMS}}}{\sqrt{\text{eff}} \times N_{\text{c.c.}}^\text{evts}},$$

(5.24)

where $N_{\text{c.c.}}^\text{evts}$ is the number of events in each centrality class (i.e. $N_{\text{c.c.}}^\text{evts} \approx \frac{1}{5} \times 50,000$). Assuming that 10 minimum bias events per second are reconstructed in the ALICE central barrel detector, this corresponds to less than two hours of heavy ion run at LHC.

We immediately see that, provided few days of heavy ion run, the statistical error becomes negligible with respect to the systematic. Non-flow effects represent a large source of uncertainty only at low multiplicity (most peripheral events), while they could be neglected for mid-central events.
Chapter 6
Conclusions

The last part of the previous chapter gave an overview of the sources of experimental uncertainties on the measurement of elliptic flow, as developed in this thesis. Since $v_2$ is calculated as an averaged quantity, its statistical error scales with the square root of the number of events available ($\sigma_\langle \rangle = \sigma / \sqrt{N}$), therefore in a few days of heavy ion run, the statistical error will become negligible with respect to the systematic uncertainty and to the magnitude of non-flow effects (see sec.5.5).

The systematic error is large mainly due to the way efficiency corrections are calculated, and only a small contribution is due to the presence of impurities (which cause an over-estimate of $v_2$ at low $p_T$).

- A larger sample of simulated events would allow a detailed study of the efficiency with respect to the particle multiplicity, eliminating in this way a large contribution to the systematic error, which is due to the multiplicity dependence of the efficiency (see sec.5.1.3).

- A detailed study of the particle ratios (and their $p_T$ dependence) in Pb-Pb collisions at LHC energy would remove the uncertainty due to the unknown particle admixture, which also contributes to the systematic error the efficiency (see sec.5.1.2).

- However, only a better characterization of the ITS resolution, and the implementation of a fit-points dependent cut, could reduce the systematic error due to the applied cut on the transverse DCA (see sec.5.2).

The error on the measured $v_2$ at low $p_T$ could be reduced by increasing the purity of the selection (but this will also reduce the statistics, especially at low $p_T$, see sec.5.3.5), or by extending the linear fit of $v_2(p_T)$ to extrapolate $v_2$ up to $200 - 300$ MeV/$c$. However, since the actual shape of $v_2(p_T)$ is generally not linear, a better fit function should be modeled on available experimental data.
The contributions due to non-flow correlations can be large (assuming they are well described by HiJING), anyway they mainly affect peripheral events \((dN/d\eta < 200-300)\). At higher multiplicity, and especially in mid-central events, where the genuine elliptic flow is expected to be large, non-flow contributions become less important and they could be neglected for a preliminary flow analysis (see sec.4.3 and 5.4.3).

However, non-flow correlations cannot be completely eliminated by the event plane formalism alone, and therefore other analysis methods should be used. For this reason, both the Cumulants and the Lee-Yan zero methods are currently under implementation in the AliRoot environment.
Appendix A

Class Description

The following is a list of the C++ classes implemented in the AliFlow package, with a brief description of their purpose. The HTML documentation of the AliFlow package can be automatically generated from the source files (with ROOT THtml) or found on the web [131].

**AliFlowEvent**

The *AliFlowEvent* class contains global event variables, such as event and run number, trigger signal, and other event observables such as the signals from the ZDC or the FMD. An object array (ROOT TClonesArray class) stores the reconstructed track candidates (*AliFlowtrack* class, see below), and another array is filled with the reconstructed neutral secondary vertices (*AliFlowV0* class).

The *AliFlowEvent* class inherits from the basic ROOT * TObject*, so that it can be chained into a *TChain* or written to disk in a ROOT file. Due to the reduced amount of informations that are stored, the size of an *AliFlowEvent* object is about 1/10 of the original *AliESD*.

The class implements methods to split the event into random or $\eta$ sub-events and to calculate event-by-event quantities (such as $Q_n$ and $\Psi_n$ of the full- and the sub-event) for a given selection of track candidates, with or without $p_T$ or $\eta$ weights. The class also contain the $\phi$ weight structure as a static pointer, which has to be filled at the beginning of the analysis loop with the calculated $\phi$ weights (see sec.3.2.3). Bayesian ‘a priori’ probabilities for particle identification can be also assigned in this way (see sec.2.3.2).

The *AliFlowEvent* data structure enables the event plane analysis by default and the same data structure can be used to implement the Cumulants and the Lee-Yan zero analysis (see sec.3.4). Some of the methods to calculate the generating function for the cumulants analysis have been ported from the *StFlowEvent* code to the *AliFlowEvent*, however they have not been tested so far.
**AliFlowTrack**

The *AliFlowTrack* class summarizes the information of the *AliESDtracks* stored in the ESD. Data members of this class are the kinematic variables $p_T$, $\eta$, and $\phi$, for both the constrained and the unconstrained fit of the track (see sec.2.3), together with their $\chi^2$ and distance of closest approach to the main vertex.

Track parameters are limited to the four central detectors (ITS, TPC, TRD and TOF, see sec.2.1), for each of them, the number of fit points, number of findable clusters and $dE/dx$ signal (time signature for the TOF) are stored. The Bayesian probability for each particle hypothesis is also stored in an array $4 \times 5$ (detectors $\times$ ALICE p.Id., see sec.2.3.2).

The class also contains a pointer to an array of boolean flags, filled during the loop for the determination of the event plane, that allows to discriminate if a track was included or not in the calculation of $\Psi_n$ for a given selection (its contribution can be then subtracted from $\vec{Q}_n$ to avoid autocorrelation effects, see sec.3.2.2). A similar structure is repeated for the sub-event selection.

**AliFlowV0**

Neutral decay vertices can be stored as *AliFlowV0* objects in a separate *TClones-Array* in the *AliFlowEvent*. The *AliFlowV0* class contains the kinematic variables ($p_T$, $\eta$ and $\phi$), the $V^0$ position with respect to the primary vertex (decay length), the invariant mass, the most probable particle identification hypothesis and some reconstruction parameters, such as the DCA of the 2 tracks at the crossing point and the combined $\chi^2$. The *AliFlowV0* also stores two pointers to the daughter tracks in the *AliFlowTracks* array.

**AliFlowSelection**

The *AliFlowSelection* class is used to select events, tracks for the determination of the event plane, and tracks and $V^0$s for the correlation analysis.

Data members of this class are integer or floating point numbers, defining the interval of acceptance for selecting:

- events (typically to select a particular centrality class, e.g. multiplicity limits at mid-rapidity);
- tracks for the determination of the event plane (e.g. constrainable tracks with TPC + ITS signal), more sets of cuts can be tested in a single run (see sec.3.3.1);
- tracks (and $V^0$s) selection for the correlation analysis (e.g. track candidates with a $^4$DCA < 100 $\mu$m), those are the particles that enter the calculation of $v_2$ (eq.3.6).
An `AliFlowSelection` object must be instantiated at the beginning of the analysis (previous to the flattening $\phi$ weights loop) and filled with the desired set(s) of cuts. Only cuts that are explicitly set in the `AliFlowSelection` object are applied to the analysis.

Once the cuts are defined, the method `AliFlowSelection::Select(* TObject)` returns `true` or `false` whether the event/track/$V^0$ is selected. If more selections are used for the determination of the event plane, the harmonic and selection number must also be specified in the method. The selection of $V^0$ candidates for the correlation analysis also requires an invariant mass cut. The flow coefficients are then calculated within the specified mass range and in two equivalent side-bands, to estimate the flow of the background.

In the present thesis, the event selection is only based on the observed multiplicity (see sec.4.3). The optimal cuts for the determination of the event plane are optimized to achieve the best resolution (see sec.5.3.2), and the cuts applied for the correlation analysis of charged particles are optimized for the selection of primaries (see sec.5.2).

### AliFlowAnalyser

The `AliFlowAnalyser` class performs the event plane analysis over the `AliFlowEvents` (see fig.3.4), and produces a default set of histograms summarizing the results.

The `AliFlowEvent` loop has to be implemented externally, providing more flexibility (such as the possibility to perform on the fly analysis while looping on the `AliESDs`). The class is instantiated at the beginning of the event loop, and an `AliFlowSelection` object must be provided to apply the required cuts (the flattening $\phi$ weight histograms can also be loaded at this step).

The whole execution is driven by three methods:

- **Init** is called just once at the beginning to initialize the histograms and set the analysis parameters (e.g. use of $p_T$ weights, choice of the sub-events);

- **Make** is called per each `AliFlowEvent` in the loop, it performs the event selection, the determination of the event plane(s) of the full- and the sub-events, and fills the profile histograms of $v_2^{obs}$ and $\cos(\Delta\Psi_2^{sub})$;

- **Finish** concludes the analysis by calculating the global resolution with the sub-events method (average is taken over all the selected events), and by correcting the observed flow coefficients. If the efficiency histogram versus $p_T$ is provided, it also calculates the integrated flow.

All the analysis histograms are saved in a ROOT file. Therefore, both the resolution and the efficiency corrections can be also applied in a later stage.
AliFlowConstants

The name-space *AliFlowConstants* has the purpose to store static data members that do not need to be changed during the analysis, e.g. the number of selections in use for the event plane determination, the number of bins of the various histogram, the definitions of centrality classes. Any change on those numbers require the AliFlow package to be recompiled.

AliFlowMakers

The *AliFlowMaker* class is the interface between the ALICE event summary data and the *AliFlowEvent*, i.e. a parser that reads the useful values from *AliESD* objects and organizes them into the *AliFlowEvent* structure.

The *AliFlowKineMaker* class is the interface between the kinematic tree produced by the event generator and the *AliFlowEvent*. The *AliFlowKineMaker* is not a fast event simulator (no smearing is applied on the original particles kinematic, no detector information is produced), it just creates clean *AliFlowEvent* objects that can enter the same analysis chain as the reconstructed events. Most of the data members of the *AliFlowEvent* are left empty or filled with dummy values (100% p.Id. probability, fit $\chi^2 = 1$, ...).

This approach has been very useful to test the functionalities of the event plane analysis on an ideal input, without going through the full reconstruction chain of AliRoot (which can be very time consuming, see sec.2.2.2): an event generator is used to generate events with the chosen flow and particle multiplicity (transport is switched off in AliRoot), an the produced Kinetrees of particles with exact momentum, production vertex and particle Id., are converted into *AliFlowEvents* and submitted to the analysis chain (this is the approach used in chapter 4).

Some very wide quality cuts are applied at this step:

- only *AliESDtrack*s with TPC signal are taken from the *AliESD*;
- only primary particles, or secondaries associated to an *AliESDtrack* (if ‘labels’ are available), are imported from the KineTree.

Both the ‘flow makers’ can be used on the fly, creating the *AliFlowEvents* and directly submitting them to the flow analysis, or the ‘maker’ phase can be splitted from the analysis ‘phase’, by storing the *AliFlowEvents* to disk.

In the latest developments both the *AliFlowMaker* and *AliFlowKineMaker* have been embedded in an *AliAnalysisTask* or an *AliSelector* (see below).

AliFlowTask (ex AliSelectorFlow)

Later developments of AliRoot have added functionalities to run a complete analysis over simulated events (see fig.3.3). The class *AliSelectorRL* (inherited from the ROOT *TSelector*), later replaced by the class *AliAnalysisTaskRL* (inherited from the
ROOT TTTask, which also allows distributed analysis), performs in parallel the loop over AliESDs and KineTrees. For each reconstructed event, the AliStack and the simulated KineTree are also opened to give access to the kinematic information of the generated particles, and by using the 'labels' stored in the AliESDtracks, each track candidate can be compared to the simulated particle that produced the hits in the detector from which the track is fitted (see sec.2.3).

If the AliFlowMakers are executed through an AliAnalysisTaskRL, two AliFlow-Events are created (from both the AliESD and the KineTree), and the connection between particles and tracks is preserved. The reconstruction efficiency and purity can be also studied at this step (see below).

**EffHist, EpHist, CutEff**

Few additional classes have been implemented outside the AliFlow package to study the efficiency of the track reconstruction and the effect of the applied cuts:

- **EffHist** is a class to study the reconstruction efficiency and purity as a function of \( p_T, \eta, \phi \) and particle type for a given set of cuts;

- **EpHist** is a class to study the 'true' and 'observed' event plane resolution as a function of the applied cuts;

- **CutEff** is a class to study the dependence of efficiency and purity with respect to some specific observables (e.g. the 1DCA or the fit \( \chi^2 \)).

Without going into the details of their implementation, the general idea is to provide a structure that allows to easily calculate the amount of primary and secondary particles passing a given set of cuts.

Using the Monte Carlo information from the KineTree, the sensitive distributions (such as \( dN/dp_T \) or \( dN/d\eta \)) of the reconstructed tracks are ordered in a three dimensional array (4 if we also include the particle type), which dimensions are given respectively by the number of applied cuts (\( n \) selections can be used, each one sharpening the cuts), the primary condition (track reconstructed from a primary particle, from a secondary particle, from a double counted primary or from a double counted secondary), and the momentum resolution (track reconstructed inside or outside the \( p_T \) bin of the generated particle, see sec.5.1). The same distributions are generated also from the KineTree of primary particles.

Simple operation between the produced histograms lead to the calculation of the track reconstruction efficiency and purity as function of \( p_T, \eta, \) applied cuts (see sec.5.1 for the definitions). Those classes have been extensively used to produce the results shown in sec.5.1, 5.2 and 5.3.2.

However, due to the recent implementation of a more general 'efficiency framework' in AliRoot, they have not been included in the AliFlow package.
Bibliography


[95] *GCC, the GNU Compiler Collection*, http://gcc.gnu.org/.


[99] *GEANT, Detector Description and Simulation Tool*, http://wwwasdw.web.cern.ch/wwwasdw/geant/.


Summary

This thesis presents a study of elliptic flow in lead-lead collisions, in the context of ALICE (A Large Ion Collider Experiment), a dedicated heavy ion detector installed at the Large Hadron Collider (LHC) at CERN.

In a non-central collision, the term ‘anisotropic flow’ refers to the azimuthal anisotropy in the momenta distribution of the emitted particles, which is usually quantified by a Fourier expansion of the $dN/d\vec{p}$ distribution along the direction of the ‘reaction plane’ (the plane spanned by the impact parameter and the beam-pipe). Elliptic flow, the second coefficient of this expansion, is denoted as $v_2$.

In the current understanding, $v_2$ is a key observable to study the thermodynamic properties and the Equation of State of the system created in the early stage of the collision, where the formation of the Quark Gluon Plasma (QGP) is expected: the final momentum anisotropy can be connected to the spatial eccentricity of the initial state by assuming that the constituents are strongly coupled and the system behaves as a relativistic fluid. The magnitude of $v_2$ with respect to the eccentricity of the collision measures the strength of this coupling.

Unfortunately, this thesis was developed in a period when LHC was not yet operational, and therefore the work was devoted to the implementation of experimentally driven predictions of the main observables in Pb-Pb collisions at LHC energy, and the development of analysis tools to be used in the ALICE environment. The thesis also shows a full example of flow analysis on simulated heavy ion data, and points out the main sources of experimental uncertainties.

The expected values of elliptic flow and charged multiplicity have been extrapolated, for Pb-Pb collision at $\sqrt{s_{NN}} = 5.5$ TeV, in two independent ways (the Low Density Limit approximation and the Relativistic Hydrodynamic model) producing different impact parameter dependences of the elliptic flow. These predictions have been used as an input for simulations in the ALICE off-line framework, to develop and test a flow analysis code. The analysis algorithm is based on the event plane method, already successfully used for flow studies in other heavy ion experiments at lower energy, such as the Relativistic Heavy Ion Collider (RHIC) in Brookhaven, and the NA49 experiment at the Super Proton Synchrotron (SPS) at CERN.

One of the biggest experimental uncertainties in measuring flow at LHC is the magnitude of non-flow effects, i.e. azimuthal correlations between collision products not due to collective flow, and therefore not correlated with the reaction plane. Depending on the analysis method, non-flow effects can introduce a large systema-
tic error in the flow measurement. Non-flow effects have been simulated using Hijing, a heavy-ion event generator which implements all known physics effects from a superposition of proton-proton collisions. Comparison between the expected magnitude of elliptic flow and the estimated magnitude of non-flow contributions defines the applicability of the Event Plane analysis. The study also shows that non-flow effects are less important when the genuine flow or the multiplicity are large, leading to the conclusion that only peripheral reactions are heavily affected by non-flow. The event plane analysis, however, cannot completely disentangle genuine collective flow from non-flow effects, and therefore other methods should be also used (e.g. the Cumulants or the Lee-Yan zero methods).

A large systematic error in the calculation of the integrated $v_2$ is related to the uncertainty on the reconstruction efficiency, due to the accuracy of the input and to the event selection. In particular, a better parametrization of the particle ratios (possibly modeled on experimental data) should be implemented in the simulations, and multiplicity dependent correction factors should be used.

However, the analysis shows that the input values of the simulations can be reconstructed within an accuracy of a few $\%$, leading to the conclusion that the ALICE experiment is an optimal environment to measure elliptic flow, and that the event plane analysis provides an easy and straightforward procedure to perform the measurement on a wide range of centralities and therefore it can be perfectly used to perform ‘first-day’ physics analysis at ALICE.
Samenvatting

In dit proefschrift wordt elliptische stroming van deeltjes in lood-lood botsingen bestudeerd met behulp van ALICE (A Large Ion Collider Experiment), een geavanceerde zware ionen detector die geïnstalleerd is in de Large Hadron Collider (LHC) op het CERN.

In een niet-centrale botsing refereert de term anisotrope deeltjes stroom naar de anisotrope hoekverdeling in de impulsverdeling van de uitgezonden deeltjes. Over het algemeen wordt deze gekwantificeerd door de Fourier-reeks ontwikkeling van de $d^3N/d\vec{p}$ verdeling evenwijdig aan het ‘reactievak’ te nemen (het vlak dat opgespannen wordt door de impactparameter en de bundelrichting). Elliptische stroming, de tweede component van de ontwikkeling, wordt genoteerd als $v_2$. Volgens de huidige opvattingen is $v_2$ een sleutel observabele voor het bestuderen van de thermodynamische eigenschappen en toestandsvergelijking van een systeem dat zich instelt vlak na de botsing, wanneer het ontstaan van een Quark Gluon Plasma verwacht wordt: de uiteindelijke impuls anisotropie kan verbonden worden aan de ruimtelijke excentriciteit van de beginfase, door aan te nemen dat de relevante vrijheidsgraden sterk gekoppeld zijn en dat het systeem zich gedraagt als een relativistische vloeiistof, en dat de grootte van $v_2$ ten opzichte van de excentriciteit van de botsing de sterkte van de koppeling representeren.

Helaas werd dit proefschrift vervaardigd in de periode dat de LHC nog niet in bedrijf was, als gevolg daarvan is het werk toegespitst op het implementeren van experimenteel gedreven voorspellingen van de belangrijkste observabelen in lood-lood botsingen bij LHC energieën en de ontwikkeling van de analyse gereedschappen die gebruikt moeten worden in de ALICE omgeving. Dit proefschrift bevat ook een volledig voorbeeld van de strominganalyse uit gesimuleerde zware ionen data en laat de belangrijkste bronnen van experimentele onzekerheden zien.

De verwachte waarden van de elliptische stroming en multipliciteit van de geladen deeltjes zijn doorgerekend voor lood-lood botsingen met $\sqrt{s_{NN}} = 5.5$ TeV op twee verschillende manieren (de lage dichtheidslimiet benadering en het relativistische hydrodynamische model), dit resulteert in verschillende afhankelijkheden van de impactparameter van de elliptische deeltjes stroom. Deze voorspellingen zijn gebruikt als invoer voor de simulaties in het ALICE offline raamwerk om de analyse code te ontwikkelen en te testen. Het analyse algoritme, gebaseerd op de reactievak-methode, is al succesvol gebruikt voor stromingstudies bij andere zware ionen experimenten met lagere energieën zoals bij de Relativistic Heavy Ion
Collider (RHIC) in Brookhaven en bij het NA49 experiment in de Super Proton Synchrotron (SPS) op het CERN.

Een van de grootste experimentele onzekerheden in het meten van de deeltjes stroom in de LHC is de grootte van de effecten die niet het resultaat zijn van niet-stroming, zoals hoekcorrelaties tussen botsingsproducten door niet collectieve stroming die daardoor niet gecorreleerd zijn met het reactievlek. Afhankelijk van de analyse methode, kunnen de niet-stromings effecten een grote systematische fout in de stromingsmeting veroorzaken. Niet-stromings effecten zijn gesimuleerd met behulp van Hijing, een zware ionen botsingsgenerator waarin alle bekende fysische effecten van proton-proton botsingen geïmplementeerd zijn. De vergelijking tussen de verwachte grootte van de elliptische stroming en de verwachte grootte van niet-stromingseffecten definieert de toepasbaarheid van de reactievlek-methode. Het onderzoek laat ook zien dat niet-stromingseffecten minder belangrijk zijn wanneer werkelijke stroming of de multipliciteit groot zijn, daar uit volgt de dat alleen scherende botsingen zwaar onderhevig zijn aan niet-stromingseffecten. De reactievlek-methode kan echter niet gebruikt worden om de niet-stromingseffecten en de stromingseffecten compleet te isoleren, daarom zullen er ook andere methoden gebruikt moeten worden (zoals de Cumulante of Lee Yang zero methode).

Een grote systematische fout in de berekening van de geïntegreerde $v_2$ is gerelateerd aan de onzekerheid van de reconstructie-efficiëntie, afhankelijk van nauwkeurigheid van de data invoer en de botsingsselectie. In het bijzonder zal een betere parametrisatie van de deeltjes verhoudingen (mogelijk gebaseerd op experimentele data) geïmplementeerd moeten worden in de simulaties en de multipliciteits afhankelijke correctie factoren zullen moeten worden gebruikt.

De analyse laat echter zien dat de invoer waarden van de simulaties gereconstrueerd kunnen worden binnen een marge van een paar procent, dit leidt tot de conclusie dat het ALICE experiment een optimale omgeving is voor de meting van elliptische stroming en dat de reactievlek-methode een makkelijke en inzichtelijke procedure verstrekt om de meting in groot bereik van centraliteiten te bewerkstelling, deze methode kan daarom perfect gebruikt worden in een eerste fysische analyse met behulp van ALICE.
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