Nucleons and nucleon pairs in nuclei


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Nucleons and nucleon pairs in nuclei


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Abstract. A set of experiments is described that studies the wave functions of neutrons, protons and proton-proton pairs in nuclei. Information on valence neutron wave functions was extracted from the reaction \( ^{16}O(\gamma, p^-) \). Deep-lying proton orbitals in \(^{208}\text{Pb}\) have been studied via a longitudinal/transverse separation of the cross section for the reaction \((e, e'p)\) up to a missing energy of 110 MeV. In the reaction \(^{16}O(e, e'p)^{14}\text{C}\) we have examined the relative contributions of processes driven by one-body and two-body currents by measuring the energy-transfer dependence of the missing momentum distributions. The data obtained for the reaction \(^{3}\text{He}(e, e'pp)\) are compared directly with calculations that treat both the bound-state and the continuum wave functions in one consistent framework.

1 Introduction

In the past decade a large body of data from \((e, e'p)\) experiments performed at NIKHEF and other laboratories has become available. The experiments pertained mostly to valence proton knockout and except for the few-body systems the data have been analyzed in the framework of mean-field theory (MFT). The picture that emerged [1] was that for complex \((A>4)\) nuclei the spectroscopic strength found for knockout of valence protons is about 60-65% of the independent-particle shell-model (IPSM) value (see Fig. 1). A small part (about 10%) of the reduction could be explained by including long-range correlations (LRC), which couple the single-particle degrees of freedom to surface vibrations, in the calculations. However, the largest part of the reduction was ascribed [1, 2] to short-range correlations (SRC), which cause a promotion of particles to energies far above the Fermi edge, with the consequence that the strength corresponding to the knockout of these particles is well outside the domain of the experimental data. For complex nuclei short-range correlations are difficult to include in a MFT framework.

For few-body systems calculations that are based on realistic nucleon-nucleon \((NN)\) potentials, are possible (with Faddeev [3], VMC [4], cluster VMC [5] and Green's function [6] techniques) and the calculated momentum distributions describe the data \((^2\text{H} : [7, 8], ^3\text{He} : [9] and ^4\text{He} : [10, 11])\) well at moderately small missing momenta. At higher momenta the contribution of competing processes like the coupling of the photon to two-body currents...
starts to become important and hence direct indications for the presence of SRC are hard to establish. For complex nuclei attempts to describe the observed reduction of the single-particle strength were mainly based on calculations [12] for infinite nuclear matter (NM). Indeed, the energy dependence of the spectroscopic strength as measured [13] in the reaction 208\textsuperscript{Pb}(e,e'p) could be described reasonably well by adding LRC effects to the strength for infinite nuclear matter (see Fig. 1).

![Fig. 1](image)

**Fig. 1.** Left panel: Spectroscopic strength for proton knockout from valence orbitals in various nuclei as deduced from $(e,e'p)$ data. Right panel: Spectroscopic strength, as measured in the reaction 208\textsuperscript{Pb}(e,e'p) for proton knockout from various shells as a function of the centroid removal energy for each shell. The data are compared to the strength calculated for infinite nuclear matter (NM, solid curve) and to a NM calculation augmented by RPA corrections (NM+RPA, dash-dot curve). All data and curves were normalized to unity for a completely filled shell as given by mean field theory (MFT, dashed curve).

Although the description of the observed reduction in terms of short-range and long-range correlations has been generally accepted, a number of questions remain to be answered.

- Since the strength reduction to about 60-65% was observed for protons it would be important to establish whether a similar reduction would hold for neutrons as well. Such results can be obtained with the quasi-free reaction $(e,e'n)$. However, this type of experiment is rather laborious to perform because of the difficulty to achieve the required energy resolution and to detect neutral particles in a hostile electron beam environment.

Therefore we have attempted [14] to obtain this information from the
reaction $^{16}\text{O}(\gamma, \pi^- p)$ in the quasi-free regime. Assuming that the elementary process is well understood one can extract information on the wave function of the neutron on which the production process takes place.

As discussed above the observed reduction was found for valence protons only. Hardly any accurate information is available on the strength residing in deeper bound orbitals. Therefore, we carried out a $^{208}\text{Pb}(e, e'p)$ experiment in which the kinematic conditions were chosen such that the missing energy regime of 0-110 MeV was covered. In this way we probe not only the energy regime up to about 55 MeV, where according to MFT the single-particle strength is located [15], but also the higher energy region which may contain strength that is shifted there due to long-range and short-range correlations. Since other processes than direct proton knockout may also contribute to the cross section above the two-nucleon knockout threshold, it was also necessary to perform a longitudinal/transverse (L/T) separation. As two-body contributions are dominantly transverse, the longitudinal response, as deducted from the L/T separation, is expected to be mainly sensitive to the direct knockout process.

Because it was realized that the two-nucleon knockout reaction offers a direct probe to study $NN$-correlations, some pioneering experiments on the reaction $(e, e'pp)$ have been carried out at NIKHEF. In this reaction it is assumed that when a virtual photon hits a strongly correlated proton pair this could be signalled by the emission of a high-energy proton in the forward direction and a low-energy proton in the backward direction. First experiments [17, 18] of this kind were carried out with the MEA facility and two home-built scintillator detector systems. Although these detectors already featured appreciably larger solid angles and energy acceptances than conventional magnetic spectrometers the yield of the studied reaction $^{12}\text{C}(e, e'pp)$ was too small to obtain accurate results. Moreover such triple coincidence experiments still suffered from the large contribution of accidental coincidences due to the low duty factor of the MEA beam. With the high duty factor beam from the AmPS ring and two new large-acceptance scintillator detection systems [16] we studied the reactions $^{16}\text{O}(e, e'pp)$ [19, 20] and $^3\text{He}(e, e'pp)$ [21].

We report on the experiments discussed above in sections 2, 3 and 4. They were carried out in the EMIN end station with the beam extracted from the AmPS ring [22] of NIKHEF. A summary is presented in section 5.

2 Valence neutrons in $^{16}\text{O}$

Spectroscopic strengths of neutrons in a complex nucleus can be obtained with the quasi-free reaction $(e, e'n)$. As argued above this type of experiment is hard to perform and therefore we used the reaction $(\gamma, \pi^- p)$, which allows the detection of charged particles in high-resolution magnetic spectrometers. In the present experiment [14] we studied the reaction $^{16}\text{O}(\gamma, \pi^- p)$ where
$^{16}$O was chosen as the target nucleus since its doubly-closed shell structure facilitates the calculations. Furthermore, experimental data on proton spectroscopic factors for the same nucleus are already available [23].

For a description of the cross section the factorized distorted-wave impulse approximation (DWIA) formalism is used, as first developed for the $(\gamma, \pi^- p)$ reaction by Laget [24]. In this formalism the cross section is written as [25]:

\[
\frac{d^3 \sigma}{dT_\pi d\Omega_\pi d\Omega_p} = k \cdot \sigma_{\gamma n \rightarrow \pi p}^{cm} |\phi_p^D|^2,
\]

in which $k$ is a kinematic constant including a recoil term, $\sigma_{\gamma n \rightarrow \pi p}^{cm}$ the elementary cross section for $\Delta$-production on a free neutron in the center-of-mass framework, and $\phi_p^D$ the distorted momentum distribution of the neutron:

\[
\phi_p^D = \int \chi_{n^-}^{(*)} \chi_{p^-}^{(*)} e^{i k \cdot r} \phi_l(r) dr.
\]

In this equation $\chi_{n^-}^{(*)}$ is the distorted wave of the outgoing proton or pion, and $\phi_l$ the wave function of the neutron, with orbital-momentum quantum number $l$. Hence, if one assumes the elementary production amplitude $\sigma_{\gamma n \rightarrow \pi p}^{cm}$ and the treatment of the final-state interaction between the outgoing hadrons and the residual nucleus to be correct, one directly obtains information on the neutron wave function $\phi_l(r)$.

The cross section $\sigma_{\gamma n \rightarrow \pi p}^{cm}$ includes, apart from processes in which a $\Delta$ is excited, also processes where a pion and a proton emerge from the nucleus without an intermediate $\Delta$ being produced, the so called "Born terms". The relative contribution of the $\Delta$-term and the Born terms depends on the chosen kinematics. Especially the angle of the pion with respect to the incoming photon is important. At backward pion angle the Born terms dominate, whereas the $\Delta$ and Born terms are of equal size at forward pion angle. Hence, the proton angular distributions, obtained at backward pion angle, are directly sensitive to the neutron wave function, without much uncertainty about the (possibly medium-modified) $\Delta$-term. The influence of the $\Delta$-term was studied via the pion angular distributions [14], from which a model uncertainty of 20% on the deduced neutron spectroscopic factors was estimated.

The experiment was carried out in the EMIN end station at NIKHEF with an almost continuous beam of 369 MeV electrons from the pulse stretcher ring AmPS. Instead of using a tagger magnet to create a pure beam of real photons, the target was exposed to the electron beam. In this method both real and virtual photons are created in the target itself. Tiator and Wright [26] have described a formalism to extract photoproduction cross sections from such electroproduction data using virtual photon theory. Because the shape of the virtual photon spectrum is well known, the $(\gamma, \pi^- p)$ cross sections can be extracted accurately from end-point fits to the measured yield.

The particles emerging from the target were detected with two high-resolution magnetic spectrometers [27]. Protons were detected in the QDD
spectrometer. Pions were detected in the QDQ spectrometer, which was equipped with an aerogel Čerenkov detector in order to discriminate between pions and electrons [28]. By employing a triple-foil waterfall target, which is described in more detail in Refs. [28] and [29], we achieved an overall missing-energy resolution of about 0.5 MeV. This allowed to extract values for the cross section of individual states in the residual nucleus $^{150}$.

![Graph](image)

**Fig. 2.** Cross sections as a function of proton emission angle measured in the reaction $^{16}$O(γ, p) leading to the ground state (left panel) and the 6.176 MeV excited state (right panel) in $^{150}$O. The curves represent fits to the data, with the rms radius of the neutron wave function and the spectroscopic factor treated as free parameters. The curves were calculated with a non-local version of the pion production operator.

In Fig. 2 the experimental proton angular distributions for the ground state and first excited state are shown together with non-local DWIA calculations [30]. The employed spectroscopic factor $S$ and the radius of the neutron wave function were obtained from a least squares fit to the data. For both the $1p_{1/2}$ shell and the $1p_{3/2}$ shell a good description of the data is obtained. The numerical results for the rms radii of the neutrons and the corresponding spectroscopic factors are shown in table 1, where they are compared to the rms-radii and spectroscopic factors for $1p$ protons in $^{16}$O derived from high-resolution $^{16}$O(e, e'p) data [23].

The rms-radii of the neutron wave functions extracted from the $(\gamma, p)$ data using the non-local DWIA formalism are in good agreement with the values for the proton wave functions. The neutron spectroscopic factors derived from the $(\gamma, p)$ data are consistent with those derived for the proton from the reaction $(e, e'p)$. The value for the $1p_{1/2}$ neutron shell is somewhat low but still consistent with the corresponding result for protons.

The established values demonstrate for the first time that the observed large depletion of proton valence shells in $^{16}$O [23] (which was also observed for a large number of other nuclei [1]), is also present for the neutron va-
lence shells in $^{16}$O. This confirms the explanation for such a depletion that was given [31] in terms of long-range and short-range correlations, which are expected to affect both neutron and proton wave functions similarly.

Table 1. Experimental values of rms-radius and spectroscopic factor $S_o$ for the $1p_{1/2}$ and $1p_{3/2}$ neutron shells in $^{16}$O determined from the present $^{16}$O($\gamma, \pi^-p$) data. The corresponding results for protons, derived from $^{16}$O($e, e'p$) data, are also listed. The errors in the $(e, e'p)$ and $(\gamma, \pi^-p)$ results include experimental statistical and systematic errors and model uncertainties.

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<tr>
<th></th>
<th>$(e, e'p)$</th>
<th>$(\gamma, \pi^-p)$</th>
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<th>$(e, e'p)$</th>
<th>$(\gamma, \pi^-p)$</th>
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<tr>
<td>$1p_{1/2}$</td>
<td>2.90±0.07</td>
<td>2.86±0.17</td>
<td>$1p_{3/2}$</td>
<td>2.74±0.06</td>
<td>2.80±0.13</td>
</tr>
<tr>
<td>$\Sigma 1p/6$</td>
<td></td>
<td></td>
<td>$1p_{1/2}$</td>
<td>1.25±0.06</td>
<td>0.96±0.19</td>
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<tr>
<td></td>
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<td></td>
<td>$1p_{3/2}$</td>
<td>2.65±0.22</td>
<td>2.81±0.56</td>
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<td>$\Sigma 1p/6$</td>
<td>65±4%</td>
<td>63±12%</td>
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</table>

3 Protons in $^{208}$Pb

Previous $^{208}$Pb($e, e'p$) experiments [13, 32, 33] were carried out at missing energies below 30 MeV and without longitudinal/transverse (L/T) separation of the response. Hence, only information on the spectroscopic strength of valence proton orbits was obtained (see Fig. 1:right). In order to obtain information on the deep-lying hole strength in $^{208}$Pb it is necessary to determine the spectral function up to a missing energy of about 55 MeV where in a MFT description of nuclear structure [15] the deepest bound $1s_{1/2}$ state is located. However, for deep lying states configuration mixing appreciably broadens the strength distribution. According to a calculation by Brown and Rho [35] this spreading should amount to about 20 MeV for states around 55 MeV binding energy. Moreover, this distribution has a long tail due to short range correlations which may extend over several hundreds of MeV. Hence, for an accurate study of the strength residing in deep hole states the spectral function should also be measured at energies well above 55 MeV.

Once the missing energy is above the two-nucleon emission threshold (15 MeV for $^{208}$Pb), other processes than direct one-proton knockout may contribute to the experimental cross section. It is expected that the dominant contributions stem from two-nucleon knockout via intermediate $\Delta$ excitation or coupling of the virtual photon to meson-exchange currents. These processes are transverse and hence a longitudinal/transverse separation of the cross section will greatly facilitate the interpretation of the data. The longitudinal response will be virtually free of the two-body contributions, whereas
possible excess strength in the transverse cross section may give new information on the role of the two-body processes. We therefore carried out a new experimental study of the reaction \( ^{208}\text{Pb}(e, e'p) \) measured in an extended \( E_m \) regime.

### 3.1 Experiment

Instrumental limitations and count-rate considerations restricted the measurable \((E_m, p_m)\) region for which an \( L/T \) separation is feasible, to \((0-110 \text{ MeV}, 0-250 \text{ MeV}/c)\). The measurements were carried out with the high-duty factor electron beam extracted from the AmPS ring at NIKHEF. We used two incident energies (462 and 675 MeV) in order to enable the Rosenbluth separation. Electrons and protons were detected with the high-resolution magnetic spectrometers QDD and QDQ in the EMIN end station. A double-foil water-cooled enriched \(^{208}\text{Pb} \) target (total thickness \( 84 \text{ mg/cm}^2 \)) was used that could stand beam currents up to \( 10 \mu\text{A} \). The detected proton energy was kept constant at \( 101 \text{ MeV} \) in order to minimize uncertainties due to final-state interaction. Given the beam energy and the detected proton energy the requirement that the kinematic conditions be parallel completely fixes the kinematics for chosen \((E_m, p_m)\) values. In order to cover, at each energy, the required energy-momentum domain, nine kinematic settings were needed, of which the central \( E_m \) and \( p_m \) values were \((20, 45, 80) \text{ MeV} \) and \((50, 150, 230) \text{ MeV}/c \), respectively. In this way we covered the missing-energy range of \( 0-110 \text{ MeV} \) continuously, and about \( 80\% \) of the missing momentum range \( 0-280 \text{ MeV}/c \). From the measured cross sections we determined via a standard data analysis [36] the experimental spectral function by dividing out \( \sigma_{rp} \), for which we used the current-conserving expression \( \sigma_{rp}^{\text{cl}} \) of de Forest [37]. The data were sorted in \( E_m \) and \( p_m \) bins of \( 1 \text{ MeV} \) and \( 10 \text{ MeV}/c \) wide, respectively.

Preliminary results for the data measured at 462 and 675 MeV incident energy are presented in Fig. 3. Here we show the missing energy distribution averaged over the \( p_m \)-acceptance according to

\[
\bar{S}(E_m) = \int S(E_m, p_m) dp_m / \int dp_m
\]

This representation has the advantage that the experimental data are readily comparable although they do not cover exactly the same \( p_m \) range for both beam energies. The data are compared to a simple model spectral function:

\[
S(E_m, p_m) = T \sum_{\alpha \in F} \left[ \phi_{\alpha}^{\text{HO}} (p_m) \right]^2 \frac{\Gamma (E_m)}{2\pi(\Gamma (E_m) + \Gamma^{\text{HO}}_{\alpha}(E_m))},
\]

where \( n_{\alpha} \) is the occupation for the shell \( \alpha \) extracted from Fig. 1:right. The quantity \( \phi_{\alpha}^{\text{HO}} (p_m) \) represents a Harmonic Oscillator wave function in momentum space and \( T \) is an overall transparency factor. We employed \( T = 0.39 \)
Fig. 3. Preliminary spectral function for the reaction $^{208}$Pb$(e, e'p)$ as determined from the measurements at 675 MeV incident energy and forward angles (left panel) and 462 MeV and backward angles (right panel). The solid curve represents the calculation with the simple model given by Eq. (4). The dashed curve represents a calculation for correlated nuclear matter. The contribution of rescattering, calculated according to Eq. (5), is given by the hatched area. Data and curves were averaged over the $p_m$ range 0-280 MeV/c according to Eq. (3).

as derived from a transparency calculation using a density-dependent pN scattering cross section [34] integrated over the $^{208}$Pb matter density distribution. The energy dependence of the spectral function is modeled as a Lorentzian with an energy-dependent width $\Gamma_0(E_m)$ that was calculated according to the formula given by Brown and Rho [35].

Figure 3 shows that below about 50 MeV the data are described reasonably well by this simple model (solid curves), although the data obtained at the low beam energy, which are more transverse, lie above the curve. Above 50 MeV missing energy we observe up to an order of magnitude more strength than the simple model predicts. The calculated distribution has a tail extending to high missing energies that drops approximately as $1/E_m$ due to the high-energy behavior of the spreading function. Although this spreading already contains part of the effect of correlations in a phenomenological way it seems insufficient to describe the behavior of the spectral function at high energy.

Also shown in Fig. 3 is the spectral function (dashed curves) for correlated nuclear matter as a function of energy and averaged over the same $p_m$ region as the experimental data. Below about 50 MeV, which is the centroid binding energy for states with $k = 0$ in nuclear matter, this infinite nuclear-matter spectral function cannot be compared with the data for the finite nucleus $^{208}$Pb, but above this energy such a comparison is possible. We observe that
3.2 Rescattering

Before drawing any firm conclusions from these data we need to investigate whether the observed strength at high energy is not due to other processes than one-proton knockout. For this purpose we first calculated the rescattering contribution, i.e. the reaction \((e, e'p)\) followed by the reaction \(p + N \rightarrow p + N\), where \(N\) is a neutron or a proton. The rescattering contribution was calculated according to

\[
S^R(E_m, p_m) = \int_{T_d}^{E_d+\omega} dT_i \int_0^{\infty} dr_1 \int_0^{\infty} dr_2 \rho_p(r_1) S(E_1, p_1) \frac{1}{|r_2 - r_1|^2} T(T_i, r_1, r_2) \sum_{N=n, p} \rho_N(r_2) \frac{d^3\sigma_N}{dT_d d\Omega_d} T(T_d, r_2, \infty). \tag{5}
\]

The quantities \(p_\omega\) and \(T_\omega\) denote momenta and kinetic energies respectively, where the subscripts refer to intermediate proton \((i)\), detected proton \((d)\), scattering nucleon \((s)\) and undetected nucleon \((u)\). Production of the intermediate proton by a virtual photon of energy-momentum \((\omega, q)\) takes place at location \(r_1\) with probability \(S(E_1, p_1)\rho_p(r_1)\), where the arguments of the spectral function \(S\) follow from energy-momentum conservation in the first vertex: \((E_1, p_1) = (\omega - T_i, q - p_i)\). A rescattering event occurs at location \(r_2\) with probability \(\rho_N(r_2) d^3\sigma_N/dT_d d\Omega_d\), where the cross section represents scattering of a proton of kinetic energy \(T_i\) off a moving nucleon of kinetic energy \(T_d\), leading to a detected proton of kinetic energy \(T_d\). The latter cross section was deduced from the in-medium cross section for proton-nucleon scattering \([38]\) according to :

\[
\frac{d^3\sigma_N}{dT_d d\Omega_d} = \int_0^{k_F} dP_s \int_0^{k_F} dP_u \sigma_{pN}^{p,N}(E_s, P_s) \delta(E) \delta(P). \tag{6}
\]

It is integrated over scattering nucleons with momenta below the Fermi momentum \(k_F\) and undetected nucleons above \(k_F\) in order to account for Pauli blocking. Furthermore, energy-momentum conservation is required in the second vertex as indicated by the \(\delta\) functions of total energy and momentum, where non-relativistic kinematics were used in combination with a density-dependent dispersion formula \(E = (p^2/2M) + V(p, p)\) for the relation between energy and momentum of the bound nucleons \([34]\). Finally, the transparency factor \(T(T_x, r_1, r_2)\) was calculated as

\[
T(T_x, r_1, r_2) = \exp \left( - \sum_{N=n, p} \int_{0}^{|r_2 - r_1|} d\rho_N(s) g(r_1 - s) \sigma_{pN}^{p,\text{tot}}(T_x) \right). \tag{7}
\]
It indicates the probability for a proton of kinetic energy $T_x$, starting at $r_1$, to arrive at $r_2$. The line integral depends on the density-dependent total proton-nucleon cross section $\sigma_{pN}(T_x)$ in nuclear matter and a two-body correlation function $g$. For the density $\rho(r)$ we used the point nucleon densities derived from the proton charge distribution of $^{208}\text{Pb}$ [39] unfolded for proton size.

The calculated rescattering contributions are shown as the hatched areas in Fig. 3. They are one to two orders of magnitude below the measured data. Hence it can be concluded that rescattering cannot explain the observed enhancement.

![Fig. 4. Same as Fig. 3, but for the longitudinal (left panel) and transverse (right panel) spectral function.](image)

### 3.3 Longitudinal/Transverse separation

In order to perform a Rosenbluth separation of the experimental data we write the $(e,e'p)$ cross section for parallel kinematics as [37]

$$
\frac{d^6\sigma}{dE_e d\Omega_e dE_{e'} d\Omega_{e'}} = \left| P \right| \frac{Q^2}{4\pi^2} \sigma_{\text{Mott}}(E_e, \theta_e) \times \\
(W_L^{\text{FF}}(E_m, P_m, Q^2) + e^{-1}W_T^{\text{FF}}(E_m, P_m, Q^2)),
$$

where the virtual photon polarization is

$$
e^{-1} = 1 + \frac{2q^2}{Q^2} \tan^2\left(\frac{\theta_{e'}}{2}\right).
$$

The longitudinal and transverse spectral functions are defined by
\[ S^L_{exp}(E_m, P_m) = \frac{W^L_{exp}(E_m, P_m; Q^2)}{G^L_E(Q^2)} \] (10)

\[ S^T_{exp}(E_m, P_m) = \frac{W^T_{exp}(E_m, P_m; Q^2)}{(Q/2M)^2 G^T_M(Q^2)} \] (11)

The actual L/T separation was performed for each \((E_m, P_m)\)-bin separately.

![Graph](image)

**Fig. 5.** Energy (left panel) and momentum (right panel) dependence of the difference \(S_T - S_L\) between the separated transverse and longitudinal spectral functions integrated over the indicated regions.

In order to account for Coulomb distortion we used an effective momentum approximation (EMA) [40], i.e., we replace \(P_m\) everywhere by \(P_m^{\text{eff}} = q^{\text{eff}} - P\). The resulting L/T separated responses are shown in Fig. 4. Obviously the error bars on the data are now larger due to error propagation in the separation procedure. It is seen that the simple model is still right for the longitudinal spectral function up to 50 MeV, but the transverse response apparently carries already more strength in this region. Above 50 MeV the longitudinal and transverse responses are equal within error bars as demonstrated in Fig. 5, which shows their difference. Apparently the excess transverse strength manifests itself at low values of the missing energy. Inspection of Fig. 4 reveals that this excess strength is predominantly located at momenta around 200 MeV/c.

### 3.4 Koltun Sum Rule

A further indication that effects beyond the mean-field approach play an important role in the present data is obtained from an evaluation of the Koltun Sum Rule (KSR) [41]. This sum rule, which holds if there are only one-body and two-body forces in the Hamiltonian, is written as
\[ \frac{E_Z}{Z} = \frac{1}{2}((T) - (E)), \]

where the mean kinetic energy \((T)\) and mean removal energy \((E)\) are defined by

\[
\langle T \rangle = \frac{1}{A} \int \int dP_m d(E_m) \frac{P_m^2}{2M} S(E_m, P_m),
\]

\[
\langle E \rangle = \frac{1}{A} \int \int dP_m d(E_m) E_m S(E_m, P_m),
\]

\[
A = \int \int dP_m d(E_m) S(E_m, P_m).
\]

The left hand side of the sum rule equals -3.5 MeV, which is the binding energy per proton obtained from the Weizsäcker mass formula after correction for symmetry and Coulomb terms. The right-hand side was evaluated in the present experimental \((E_m, P_m)\)-domain and amounts to -8 MeV as shown in table 2. We obtain the same value for correlated nuclear matter when integrated over the same domain. Integrating the nuclear-matter spectral function over its full domain we arrive at -13 MeV, which indicates that there are appreciable contributions to the KSR outside our experimental domain. If a similar contribution would appear in the, as yet unmeasured, experimental spectral function, this would make the discrepancy with the left hand side only worse. The preliminary conclusion is that the Koltun Sum Rule does not hold for \(^{208}\text{Pb}\). Such a conclusion was also drawn from earlier experiments on \(^3\text{He} \) \([42]\) and medium-heavy nuclei \([43]\) carried out at Saclay.

**Table 2.** Experimental values for the Koltun Sum Rule (KSR) as determined from the present data (second column), compared to theoretical values evaluated with the simple model of Eq. (4) (column 3 and 4) and correlated nuclear matter (column 5 and 6). The employed integration domains are \((E_m, P_m) = (0-100 \text{ MeV}, 0-260 \text{ MeV/c})\) (limited) and \((0-750 \text{ MeV}, 0-800 \text{ MeV/c})\) (full).

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<th>correlated NM</th>
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<tr>
<td>(&lt; T &gt; ) [MeV]</td>
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<tr>
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<td>26</td>
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<tr>
<td>KSR [MeV]</td>
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<td>-6</td>
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The analysis of this experiment is still in progress. Especially the L/T separation will require further detailed investigation since the treatment of the Coulomb distortion will strongly influence the outcome. Furthermore we
will use in the final analysis a CDWIA calculation [44] which treats the final-state interaction via an optical model potential and uses more realistic MFT wave functions based on a Woods-Saxon potential.

4 Two-proton knockout experiments

Electron-induced two-nucleon knockout reactions at intermediate electron energies are driven both by one-body currents (coupling of the virtual photon to correlated nucleon pairs) and two-body currents (intermediate Δ-excitation and meson exchange currents (MEC)). In addition, final state interactions (FSI) contribute to the cross section. In recent studies [19, 20] of the reaction \(^{16}\text{O}(e,e'pp)^{14}\text{C}\) clear signatures have been observed for direct knockout of strongly correlated proton pairs via one-body currents. We discuss this experiment in section 4.1.

Since continuum Faddeev calculations employing realistic \(NN\)-potentials are currently available for the three-body system [3], an exclusive measurement of the three-body breakup of \(^3\text{He}\) offers the opportunity to compare data to predictions of a microscopic model, in which initial and final states are consistently treated. The relative importance of competing two-proton knockout mechanisms can be investigated by varying the energy and momentum of the virtual photon, as was done in the experiment described in section 4.2.

Both experiments described below were performed in the EMIN end station with the high duty-factor electron beam extracted from the pulse stretcher AmPS. The scattered electrons were detected in the QDQ magnetic spectrometer, and the emitted protons in the large acceptance HADRON3 and HADRON4 plastic scintillator arrays [16], covering solid angles of 240 and 550 msr, respectively. With this system a FWHM resolution in the missing energy could be achieved of about 4-5 MeV. By integration over the relevant excitation energy region we deduce cross sections as a function of the missing momentum \(P_m = q - p_1 - p_2\), in which \(p_1\) and \(p_2\) are the momenta of the outgoing protons and \(q\) is the transferred three-momentum.

4.1 Proton pairs in \(^{16}\text{O}\) observed in the reaction \(^{16}\text{O}(e,e'pp)^{14}\text{C}\)

In a first study [19, 20] the response of \(^{16}\text{O}\) to electron-induced two-proton knockout was measured via the reaction \(^{16}\text{O}(e,e'pp)^{14}\text{C}\). The data were taken at an incident energy of 580 MeV, a momentum transfer \(q\) of 300 MeV/c and a transferred energy \(\omega\) of 210 MeV. We employed a waterfall target [29] and the detectors described above. The achieved resolution allowed to identify for the first time a transition to a discrete state in the residual nucleus, i.e. the ground state of \(^{14}\text{C}\). The first analysis of this spectrum and the associated momentum densities of the center-of-mass motion of the proton pair, indicated that the transitions to the ground state and to the lowest excited \(2^+\) states in \(^{14}\text{C}\)
can be described as a quasi-free two-nucleon knock-out process and that the knocked-out protons are predominantly in a relative $^1S_0$ state.

Fig. 6. Preliminary eight-fold differential cross sections for the reaction $^{16}O(e,e'pp)^{14}C$ as a function of the missing momentum for three values of the energy transfer $\omega$. The upper panel shows the data for the transition to the $0^+$ ground state, the lower panel for the excitation energy region $4 < E_x < 9$ MeV. The solid curves are calculations based on a microscopic model (see text), where the dashed (dotted) curves represent the one- (two-)body contributions.

In order to further investigate this observation we performed new measurements in which the energy transfer was 180 and 240 MeV, respectively. In this way we have studied the reaction in a domain that is farther from and closer to the $\Delta$ resonance, respectively. Consequently we expect, within the model, that the ratio of one- and two-body contributions will change as a function of $\omega$.

In Fig. 6 the data are presented and compared with calculations based on a microscopic model that involves a Dressed RPA two-proton spectral function [6] including short-range correlations calculated with the Bonn-A $NN$-potential. The data measured for the three values of $\omega$ cover approximately the same ranges in $p_m$ and $\gamma_1$, which is the relative angle between the momentum transfer $q$ and the forward emitted proton $p'_1$. This implies that not all data taken for $\omega=210$ MeV were used. Hence, the $p_m$ distribution for this value of the transferred energy differs slightly from the one published in Ref. [20]. For the g.s. transition the data are well described by the model and follow the predicted $\omega$ behavior closely. The strength is apparently dominated by processes driven by one-body currents. In the region $4 < E_x < 9$ MeV,
where the response is dominated by the excitation of the lowest lying $2^+$ states with a centroid energy of 7.7 MeV, the experimental cross section shows a strong dependence on the transferred energy. This trend is not reproduced by the calculations, which predict only a smooth dependence on $\omega$, albeit with different contributions of one- and two-body hadronic currents.

A more detailed analysis is presently being performed, in which the $\gamma_1$-dependence of the cross sections is studied. In the case that the virtual photon momentum is transferred to the proton that is emitted in the forward direction this angle is closely related to the relative momentum between the two protons. Hence, this angular distribution may depend sensitively on the relative angular momentum ($l=0,1,2,\ldots$) between the two protons. Short-range correlations are expected to modify the relative proton-proton wave function strongest when the pair is in an $|l=0$ state. In connection to this analysis the cross section is also being studied as a function of $(p'_1 - q) - p'_2$). Under the assumption that the cross section is dominated by one-body currents and that the virtual photon momentum is transferred to the proton that is emitted in the forward direction this quantity is proportional to the relative momentum of the protons in the initial state.

4.2 Proton pairs in $^3\text{He}$ observed in the reaction $^3\text{He}(e,e'pp)$

The $^3\text{He}(e,e'pp)n$ experiment was performed with an incident electron energy of 540 MeV and a high-pressure (3 MPa) cryogenic (15 K) barrel cell containing the $^3\text{He}$ gas. Data were taken at momentum transfers ($q$) of 305, 375 and 445 MeV/c for an energy transfer ($\omega$) of 220 MeV. At the central $q$ value, data in the continuous $\omega$-range from 170 to 290 MeV were collected. In the reaction $^3\text{He}(e,e'pp)n$ the neutron momentum can be identified with the missing momentum $p_m$. As in the reaction $^{12}\text{O}(e,e'pp)^{14}\text{C}$ the angle $\gamma_1$ between the three-momentum $q$ and the momentum of the forward proton in the final state was used as an independent variable, while the cross section was averaged over the other kinematic quantities.

Here we present the data taken at $\omega = 220$ MeV which were obtained in three kinematic settings, covering largely overlapping regions in $p_m$ and $\gamma_1$. The data shown below are still preliminary, but are not expected to change more than 10% in the final analysis.

The data are compared to continuum Faddeev calculations [3] performed with the Bonn-B $NN$-potential. In these calculations, two-body angular momenta up to $j = 2$ are taken into accounted, and both the $^3\text{He}$ ground state and the $3N$ breakup state are treated in the same framework, while all rescattering effects in the final state are accounted for up to infinite order.

Figure 7.6 shows the differential cross section for the $q=305$ MeV/c and $\omega = 220$ MeV kinematics as a function of the missing momentum. Comparison with the theoretical prediction shows that the data are a factor of about 1.7 larger in the low $p_m$-region, increasing to a factor of about 3 at $p_m \approx 250$ MeV/c.
In Fig. 7: right the cross section is shown as a function of the transferred three-momentum $q$ for a region in the $(\gamma_1, p_m)$ phase space that is common to the detection volumes of the $\omega=220$ MeV kinematic points. The measured cross section decreases by a factor of 5 between $q=305$ and 445 MeV/c. This ratio is similar to the ratio predicted by the electron-proton cross section $K\sigma_{ep}$, which describes the coupling of a photon to an off-shell proton. This supports the picture of the photon coupling to one of the correlated protons, thus causing the second proton to be emitted as well, while the neutron acts as a spectator. At small $p_m$ the results of the full Faddeev calculations show a similar decrease of the cross section as a function of $q$. The underestimation of the data at $q=375$ and 445 MeV/c in the region $150 \leq p_m \leq 250$ MeV/c is similar to the one observed at $q=305$ MeV/c, as shown in Fig. 7. However, in the region $p_m \geq 250$ MeV/c the calculated $q$-dependence is significantly flatter than the experimental one.

Several ingredients of the calculations are currently being investigated as a possible source for the observed discrepancies: the $NN$-potential used, meson-exchange and $\Delta$-contributions and the upper limit of the two-body angular momenta included. Three-body effects might play a role as well, because they likely affect the part of the $^3$He wave function that contains three nucleons with high momentum.
5 Summary

Experiments have been carried out with the AmPS facility at NIKHEF that studied wave functions of neutrons, protons and proton-proton pairs in nuclei.

The spectroscopic strength for \( \frac{1}{2}^{-} \) and \( \frac{3}{2}^{-} \) valence neutron wave functions in \( ^{16}\text{O} \) was determined from proton angular distributions measured in the reaction \( ^{16}\text{O}(\gamma,\pi^{-}p) \). The summed strength for the \( 1p \)-shell amounts to 63\( \pm \)12\% of the IPSM value, comparable to the value 65\( \pm \)4\% obtained for protons in the reaction \( ^{16}\text{O}(e,e'p) \). This result is the first confirmation that the reduction of proton spectroscopic factors, observed for a large range of complex nuclei [1], also applies for neutrons.

Deep-lying proton orbitals in \( ^{208}\text{Pb} \) have been studied by performing a longitudinal/transverse separation of the cross section for the reaction \( ^{208}\text{Pb}(e,e'p) \). The distribution of the strength in the region up to a missing energy of 50 MeV, as deduced from the separated longitudinal response, agrees with a prediction based on a simple model that uses mean-field wave functions. In the model we used occupations for the single-particle orbitals that were taken from a calculation of correlated nuclear matter. Above 50 MeV we observe excess strength that is ascribed to an incomplete description of the spreading due to short-range correlations.

In the reaction \( ^{16}\text{O}(e,e'pp)^{14}\text{C} \) we have examined the relative contributions of processes driven by one-body and two-body currents by measuring the cross section as a function of energy transfer \( \omega \). For the transition to the ground state, which is governed by one-body currents, the \( \omega \)-dependence of the data follows the prediction of a microscopic model. However, the measured \( \omega \)-dependence for excitation of the lowest-lying \( 2^{+} \) states cannot be described by the same model. This may be indicative of an incorrect mixture of one-body and two-body currents in the reaction mechanism, or of an incorrect calculation of the relative proton-proton wave function.

The cross sections for the reaction \( ^{3}\text{He}(e,e'pp) \) have been compared directly with calculations that treat both the bound-state and the continuum wave functions in one consistent framework. The observed momentum distribution of the proton-proton pair is significantly underestimated by this microscopic model, which may be due to the absence of two-body currents in the calculation.

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References