SUDAKOV RESUMMATION FOR ELECTROPRODUCTION
OF HEAVY QUARKS

S. Moch
NIKHEF Theory Group
P. O. Box 41882, 1009 DB Amsterdam, The Netherlands

Abstract
The leading and next-to-leading threshold logarithms of the QCD corrections to electroproduction of heavy quarks in single-particle inclusive kinematics are resummed to all orders in perturbation theory. The resummed cross-section is used to derive the NLO and NNLO results near threshold and their numerical impact on the charm structure function is studied.

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The leading and next-to-leading threshold logarithms of the QCD corrections to electroproduction of heavy quarks in single-particle inclusive kinematics are resummed to all orders in perturbation theory. The resummed cross-section is used to derive the NLO and NNLO results near threshold and their numerical impact on the charm structure function is studied.

The deep-inelastic production of heavy quarks is an important reaction, which is most prominently used for the extraction of the gluon density in the proton. Experimental data on charm production in lepton proton collisions have been published by e.g. EMC, H1 and ZEUS, while calculations up to next-to-leading order (NLO) provide solid foundations for a theoretical description. As a matter of fact, the charm structure function $F_2^c$ is quite sensitive to the effects of partonic processes close to the charm quark pair threshold. This is due to the large gluon density $g(x,\mu)$ at small momentum fractions $x$, which enhances the contribution of soft gluon emission near the elastic limit. In this kinematical region, the QCD corrections are dominated by large Sudakov double logarithms, which have to be resummed to all orders of perturbation theory. This task has been performed up to next-to-leading logarithmic (NLL) accuracy. The technology and some results are briefly presentend here.

We study electron proton scattering with the exchange of a single virtual photon, $Q^2 = -q^2$, and a detected heavy quark in the final state, thus

$$\gamma(q) + P(p) \rightarrow Q(p_1) + X, \quad (1)$$

where $X$ denotes any additional hadrons in the final state and $p_1^2 = m^2$. The Mandelstam invariants, $S' = (p + q)^2 + Q^2$, $T_1 = (p - p_1)^2 - m^2$ and $U_1 = (q - p_1)^2 - m^2$ can be used define $S_4 = S' + T_1 + U_1$, which vanishes near the partonic threshold. The double differential heavy quark structure function $dF_2$ associated to the process eq.(1) may be written as

$$\frac{d^2F_2(x,S_4,T_1,U_1,Q)}{dT_1dU_1} = \sum_{i=q,g} \int \frac{dy}{y} \phi_{i/p}(y,\mu) \omega_{2i}\left(\frac{x}{y}, s_4, t_1, u_1, Q, \mu\right), \quad (2)$$

where $a = 1 + 4m^2/Q^2$. The $\phi_{i/p}$ denote parton distributions in the proton at momentum fraction $y$ and MS-mass factorization scale $\mu$. The functions $\omega_{2i}$...
describe the underlying hard parton scattering processes and depend on associated partonic Mandelstam variables $s', t_1, u_1$ and $s_4$, which are derived from eq. (2) after replacing the proton $P$ by a parton of momentum $k = (x/y)p$. At $n$-th order in perturbation theory, the gluonic hard part $\omega_2g$ in eq. (3) typically depends on singular distributions $\alpha_s^i \ln^{2n-i} (s_4/m^2)/s_4^+$, that have to be resummed. Light initial state quarks will be neglected.

In order to perform this resummation of threshold logarithms, the phase space near the elastic edge is decomposed into various regions, each depending on an infrared safe kinematical weight $w$ space near the elastic edge is decomposed into various regions, each depend-

\[
\int \frac{d^2 F_2(x, S, U_1, Q)}{d\tau_1 d\tau_1} = H_{2g}(S, T_1, U_1) \int dw_1 dw_w \delta \left( \frac{S_4}{m^2} - w_1 - w_w \right)
\]

separating the partonic degrees of freedom for $s_4/m^2 \rightarrow 0$ into different functions of individual weight $w$. The hard function $H_{2g}$ summarizes off-shell short-distances corrections, while $\tilde{\omega}_g/\omega_g$ accounts for effects of collinear gluons and the soft function $S$ for soft, long-wavelength gluons. Replacing the proton in eq.(2) by a gluon and taking Laplace moments, $\hat{f}(N) = \int_0^{\infty} dw \exp[-Nw]f(w)$, gives

\[
\tilde{\omega}_g(N, t_1, u_1) = H_{2g}(S, T_1, U_1) \left[ \frac{\tilde{\psi}_g/\omega_g(N, p \cdot \zeta)}{\phi_g/\omega_g(N, \mu)} \right] S(N, \zeta, \mu) + \mathcal{O}(1/N). \tag{4}
\]

where $\phi_g/\omega_g$ is the usual $\overline{MS}$-distribution from mass factorization. In moment space, the Sudakov logarithms appear as factors $\alpha_s^i \ln^{2n-i} N$, with $i = 0, 1$ for NLL accuracy, and the $N$-dependence in eq. (4) exponentiates for each function individually. All leading logarithms (LL) are exclusively contained in $\psi_g/\omega_g$, which is a gluon distribution at fixed energy defined as an operator matrix element. It depends on a time-like vector $\zeta, \zeta^2 = 1$, that fixes the kinematics by projecting on the proper energy fraction $p \cdot \zeta$ of the proton. At order $\alpha_s$ in $D = 4 - 2\epsilon$ dimensions and axial gauge $n \cdot A = 0$, $n = \zeta$, $\psi_g/\omega_g$ is given by

\[
\psi_g/\omega_g(w, p \cdot \zeta) = \frac{\alpha_s}{\pi} C_A \left\{ \left[ \frac{1}{e} \right]_+ + \left[ \frac{2 \ln w}{w} \right]_+ + \left[ \frac{1}{w} \right]_+ + \left( \frac{4 p \cdot \zeta^2}{\mu^2} - 1 \right) \right\}, \tag{5}
\]

where the remaining collinear poles are cancelled by $\phi_g/\omega_g$. The LL logarithms in $\tilde{\psi}_g/\omega_g$ are resummed in analogy to the Drell-Yan process, while all scale dependence of $\tilde{\psi}_g/\omega_g$ and $\phi_g/\omega_g$ is governed by renormalization group equations (RGE) with anomalous dimensions $\gamma_\psi$ and $\gamma_g/\omega_g$. 

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The soft function $S$ requires renormalization, since it is defined as a composite operator, that connects Wilson lines in the direction of the scattering partons. Its RGE, $\mu(d/d\mu)\ln S(N) = -2 \text{Re} \Gamma_S$, resums all logarithms in $S$ and its gauge dependence cancels precisely the corresponding terms in $\psi_{g/g}$.

The soft anomalous dimension $\Gamma_S$ is a one-dimensional matrix in colour space and to order $\alpha_s$ calculated to be

$$\Gamma_S(\alpha_s) = \frac{\alpha_s}{\pi} \left( \left( \frac{C_A}{2} - C_F \right) (L_\beta + 1) - \frac{C_A}{2} \left( \ln (p \cdot \zeta)^2 + \ln \frac{4 m^2}{t_1 u_1} \right) \right), \quad (6)$$

with $\beta = \sqrt{1 - 4 m^2/s}$ and $L_\beta = (1 - 2 m^2/s)\{\ln(1 - \beta)/(1 + \beta) + i\pi\}/\beta$.

The final result for the hard scattering function $\tilde{\omega}_{2g}$ in moment space resums all large logarithms in single-particle inclusive kinematics up to NLL accuracy. Combining the resummed $\psi_{g/g}$ with the integrated RGE for $S$, we obtain for $\tilde{\omega}_{2g}$

$$\tilde{\omega}_{2g}(N, t_1, u_1) = H_{2g}(S, T_1, U_1)$$

$$\times \exp \left\{ \int_0^\infty \frac{dw}{w} (1 - e^{-Nw}) \left[ \int_{u_2}^1 \frac{d\lambda}{\lambda} A(g) (\alpha_s(\lambda m^2)) + \frac{1}{2} \nu(g) (\alpha_s(w^2 m^2)) \right] \right\}$$

$$\times \exp \left\{ 2 \int \frac{d\lambda}{\lambda} (\gamma(g) (\alpha_s(\lambda^2)) - \gamma_{g/g} (\alpha_s(\lambda^2))) \right\}.$$ 

The first exponent gives the leading $N$-dependence of the ratio $\tilde{\psi}_{g/g}/\phi_{g/g}$ with $\nu(g)(\alpha_s) = 2 C_A \alpha_s/\pi$, $A(g)(\alpha_s) = C_A(\alpha_s/\pi) + (C_A K/2)(\alpha_s/\pi)^2$ and $K = C_A(67/18 - \pi^2/6) - 5/9 n_f$, the latter ones being the well-known.

Using the resummed result for $\tilde{\omega}_{2g}$ in eq. (8) as a generating functional, we reexpand $\tilde{\omega}_{2g}$ to NLO and NNLO. Inverting the Laplace transform and integrating over the phase space in eq. (2) leaves us for $F_2$ with

$$F_2(x, Q) \simeq \frac{\alpha_s(\mu^2) e^{\mu Q}}{4 \pi^2 m^2} \int \frac{1}{ax} dyg(y, \mu) \sum_{k=0}^{\infty} (4 \pi \alpha_s(\mu^2))^k \sum_{l=0}^{k} c_{2g}^{(k,l)}(\eta, \xi) \ln \frac{\mu^2}{m^2}; \quad (8)$$

where $c_{2g}^{(k,l)}$ are the standard gluon coefficient functions, $\eta = (\xi/4)(y/x - 1) - 1$ and $\xi = Q^2/m^2$. In fig. on the left, we plot $c_{2g}^{(k,0)}$ as a function of $\eta$ for $\xi = 4.4$ and compare the LL and NLL approximations up to NNLO. On the right, we plot the charm structure function $F_2^c$ for $x = 0.1(0.01)$, $Q^2 = 10\text{GeV}^2$ as a function of a $y_{max}$-cut on the integral in eq. (3) to estimate threshold sensitivity.
\[ \xi = 0.44 \times 10^{-1} \]

\[ Q^2 = 10 \text{ GeV}^2 \]

Figure 1: Left: The \( \eta \)-dependence of the gluon coefficient functions; exact results for \( c_{2g}^{(0,0)} \) and \( c_{2g}^{(1,0)} \) (solid lines); approximate LL and NLL results for \( c_{2g}^{(1,0)}(c_{2g}^{(2,0)}) \) (dotted and dashed lines). Right: Threshold dependence of the charm structure function \( F_c^2 \) at NLO with the CTEQ4M gluon distribution and \( \mu = m = 1.6 \text{GeV} \); exact results (solid lines); approximate LL and NLL results (dotted and dashed lines).

In general, at order \( \alpha_s \), the NLL logarithms approximate \( F_c^2 \) very well, for moderate values of \( Q^2 \), better than the LL logarithms, while the result for the gluon coefficient function \( c_{2g}^{(2,0)} \) represents the best present estimate of the NNLO corrections. An extensive derivation of the result eq. (8) for \( \tilde{\omega}_{2g} \) and a detailed study of its phenomenological consequences will be given elsewhere.

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References

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