Supermembranes with Winding

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ABSTRACT

We study supermembranes in the light-cone gauge in eleven flat dimensions with compact directions. The membrane states are subject to only the central charges associated with closed two-branes, which, in this case, are generated by the winding itself. We present arguments why this winding leaves the mass spectrum continuous. The lower bound on the mass spectrum is set by the winding number and corresponds to a BPS state.

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Supersymmetric matrix models that correspond to the reduction of supersymmetric gauge theories to one (time) dimension [1], are relevant for a variety of problems. First of all, with an infinite-dimensional gauge group consisting of the area-preserving diffeomorphisms of a two-dimensional surface of certain topology, the model describes the quantum mechanics of a supermembrane [2] in the light-cone gauge [3, 4]. The group of area-preserving diffeomorphisms (or, at least, a simple subgroup) can be approximated by SU(N) in the large-N limit [5, 4]. More recently, it was discovered that the collective dynamics of D-branes [6] is also described by various dimensional reductions of supersymmetric U(N) gauge theories, where N now corresponds to the number of branes [7]. Supersymmetric U(N) quantum mechanics thus describes the dynamics of N D-particles [8].

These two phenomena are related in the context of M-theory [9], the conjectured eleven-dimensional theory, which, upon compactification of the eleventh dimension to a circle, yields type-IIA string theory. From a ten-dimensional viewpoint the Kaluza-Klein states emerging from the compactification of eleven-dimensional supergravity carry a Ramond-Ramond charge, just as generic D-branes [10]. The Kaluza-Klein states with lowest nonzero charge can thus be identified with the D-particles, which, from the perspective of IIA supergravity, can be identified as extremal black holes [11, 12]. The supermembrane can presumably be viewed as the result of the collective dynamics of arbitrarily large numbers of D-particles. This would then naturally explain the continuum of the supermembrane mass spectrum [13].

These and other considerations led to the proposal that the degrees of freedom of M-theory are in fact captured in the large-N limit of supersymmetric matrix models [14]. Viewed as a theory of D-particles it has been shown that the long-distance interactions between two particles is consistent with eleven-dimensional supergravity [8, 14]. By T-duality arguments one can define compactified versions of the supersymmetric matrix models [14, 15] and incorporate the effect of virtual winding strings stretched between the D-particles. Recently further light has been shed on the relation between the matrix models and the known string theories [17], which involves second-quantized string states. The appearance of a second-quantized string spectrum seems to fit in with the instability of the supermembrane, which collapses into states of multi-membranes connected by infinitely thin strings of arbitrary length. In the context of the matrix model this phenomenon was stressed already in [14].

One of the crucial aspects of the proposal of [14] is that the supersymmetric matrix models lack Lorentz invariance. It is known that Lorentz invariance is realized in the $N \rightarrow \infty$ limit (at least, classically), although one needs to specify additional data, which can be extracted from the supermembrane [18, 19]. In this paper we therefore study supermembranes in flat, compact target spaces, yielding a different and well-defined viewpoint on the compactified large-N models. Specifically, we construct the supersymmetric gauge theory of area-preserving diffeomorphisms in the presence of winding and show the emergence of central
charges in the supersymmetry algebra. From this, we argue that flat, uncorrected potentials can still be present, leading to a continuous spectrum even for membranes winding around an arbitrary number of compact directions. Unfortunately we were unable to find a generalization of the SU($N$) supersymmetric matrix model regularization of the supermembrane to the winding case.

The actions of fundamental supermembranes are of the Green–Schwarz type [2], exhibiting an additional local fermionic symmetry called $\kappa$–symmetry. We follow the light-cone quantization described in [4]. The supermembrane hamiltonian for eleven spacetime dimensions then takes the form

$$H = \frac{1}{P_0^+} \int d^2\sigma \sqrt{w} \left[ \frac{P^a P_a}{2w} + \frac{1}{4} \{ X^a, X^b \}^2 - P_0^+ \bar{\theta} \gamma^- \gamma_a \{ X^a, \theta \} \right].$$

(1)

Here the integral runs over the spatial components of the worldvolume denoted by $\sigma^1$ and $\sigma^2$. In the above $X^a(\sigma)$ ($a = 1, \ldots, 9$) denote the transverse target–space embedding coordinates lying in $T^d \times \mathbb{R}^{9-d}$ and thus permitting us to have winding on the $d$-dimensional torus $T^d$. Accordingly $P^a(\sigma)$ are their momentum conjugates. In this gauge the light-cone coordinate $X^+$ is linearly related to the world-volume time. The momentum $P^+$ is time independent and proportional to the center-of-mass value $P_0^+$ times some density $\sqrt{w(\sigma)}$ of the spacesheet, whose spacesheet integral is normalized to unity. The center-of-mass momentum $P_0^-$ is equal to minus the hamiltonian (1). Moreover we have the fermionic variables $\theta(\sigma)$, which are 32-component Majorana spinors subject to the gauge condition $\gamma^+ \theta = 0$. And finally we made use of the Poisson bracket $\{A, B\}$ defined by

$$\{A(\sigma), B(\sigma)\} = \frac{1}{\sqrt{w(\sigma)}} e^{rs} \partial_r A(\sigma) \partial_s B(\sigma).$$

(2)

Note that the coordinate $X^-$ itself does not appear in the Hamiltonian (1). It is defined by

$$P_0^+ \partial_r X^- = \frac{P \cdot \partial_r X}{\sqrt{w}} - P_0^+ \bar{\theta} \gamma^- \partial_r \theta,$$

(3)

and implies a number of constraints that will be important in the following.

The light-cone quantization leaves a residual reparametrization invariance under area–preserving diffeomorphisms. They are defined by

$$\sigma^r \to \sigma^r + \xi^r(\sigma) \quad \text{with} \quad \partial_r(\sqrt{w(\sigma)} \xi^r(\sigma)) = 0.$$

(4)

We wish to rewrite this condition in terms of dual spacesheet vectors by

$$\sqrt{w(\sigma)} \xi^r(\sigma) = e^{rs} F_s(\sigma).$$

(5)

In the language of differential forms the condition (4) may then be simply recast as $dF = 0$. The trivial solutions are the exact forms $F = d\xi$, or in components

$$F_s = \partial_s \xi(\sigma),$$

(6)
for any globally defined function $\xi(\sigma)$. The nontrivial solutions are the closed forms which are not exact. On a Riemann surface of genus $g$ there are precisely $2g$ linearly independent non-exact closed forms, whose integrals along the homology cycles are normalized to unity \(^1\). In components we write

$$F_s = \phi(\lambda)_s \quad \lambda = 1, \ldots, 2g.$$  

(7)

The presence of the closed but non-exact forms is crucial for the winding of the embedding coordinates. More precisely, while the momenta $P(\sigma)$ and the fermionic coordinates $\theta(\sigma)$ remain single valued on the spacesheet, the embedding coordinates, written as one-forms with components $\partial_r X(\sigma)$ and $\partial_r X^-(\sigma)$, are decomposed into closed forms. Their non-exact contributions are multiplied by an integer times the length of the compact direction. The constraint alluded to above amounts to the condition that the right-hand side of (3) is closed.

It has been known for quite some time [4] that the light–cone supermembrane can be formulated in terms of a supersymmetric gauge theory of area–preserving diffeomorphisms, emphasizing the membrane’s residual gauge symmetry from the start. Whether this equivalence continues to hold after introducing winding contributions is a priori not obvious. Let us therefore investigate the structure of the gauge theory of the full group of area–preserving diffeomorphisms, consisting of the exact and not-exact transformations in the following.

Under the full group of area–preserving diffeomorphisms the fields $X^a$, $X^-$ and $\theta$ transform according to

$$\delta X^a = \frac{\epsilon^{rs}}{\sqrt{w}} \xi_r \partial_s X^a, \quad \delta X^- = \frac{\epsilon^{rs}}{\sqrt{w}} \xi_r \partial_s X^-, \quad \delta \theta^a = \frac{\epsilon^{rs}}{\sqrt{w}} \xi_r \partial_s \theta,$$

(8)

where the time–dependent reparametrization $\xi_r$ consists of closed exact and non-exact parts. Accordingly there is a gauge field $\omega_r$, which is therefore closed as well, transforming as

$$\delta \omega_r = \partial_0 \xi_r + \partial_r \left( \frac{\epsilon^{st}}{\sqrt{w}} \xi_s \omega_t \right),$$

(9)

and corresponding covariant derivatives

$$D_0 X^a = \partial_0 X^a - \frac{\epsilon^{rs}}{\sqrt{w}} \omega_r \partial_s X^a, \quad D_0 \theta = \partial_0 \theta - \frac{\epsilon^{rs}}{\sqrt{w}} \omega_r \partial_s \theta,$$

(10)

and similarly for $D_0 X^-$. Here we note that the exact vectors generate an invariant subgroup [18]. This follows from the commutator of two infinitesimal transformations corresponding to the vectors $\xi_r^{(1)}$ and $\xi_r^{(2)}$, which yields an infinitesimal transformation defined by

$$\xi_r^{(3)} = \partial_r \left( \frac{\epsilon^{st}}{\sqrt{w}} \xi_s^{(2)} \xi_t^{(1)} \right).$$

(11)

\(^{1}\text{In the mathematical literature the globally defined exact forms are called “hamiltonian vector fields”, whereas the closed but not exact forms which are not globally defined go under the name “locally hamiltonian vector fields”.}\)
The action corresponding to the following lagrangian density is then gauge invariant under the transformations (8) and (9),

\[ L = P^+_0 \sqrt{w} \left[ \frac{1}{2} (D_0 X)^2 + \bar{\theta} \gamma_- D_0 \theta - \frac{1}{4} (P^+_0)^{-2} \{ X^a, X^b \}^2 \right. \]
\[ \left. + (P^+_0)^{-1} \bar{\theta} \gamma_- \gamma_a \{ X^a, \theta \} + D_0 X^- \right], \quad (12) \]

where we draw attention to the last term proportional to \( X^- \), which can be dropped in the absence of winding and did not appear in [4]. The action corresponding to (12) is also invariant under the supersymmetry transformations

\[ \delta X^a = -2 \bar{\epsilon} \gamma^a \theta, \]
\[ \delta \theta = \frac{1}{2} \gamma_+ (D_0 X^a \gamma_a + \gamma_-) \epsilon + \frac{1}{4} (P^+_0)^{-1} \{ X^a, X^b \} \gamma_+ \gamma_{ab} \epsilon, \]
\[ \delta \omega_r = -2 (P^+_0)^{-1} \bar{\epsilon} \partial_r \theta. \quad (13) \]

The supersymmetry variation of \( X^- \) is not relevant and may be set to zero. The full equivalence with the membrane hamiltonian is now established by choosing the \( \omega_r = 0 \) gauge and passing to the hamiltonian formalism. The field equations for \( \omega_r \) then lead to the membrane constraint (3) (up to exact contributions), partially defining \( X^- \). Moreover the hamiltonian corresponding to the gauge theory lagrangian of (12) is nothing but the light–cone supermembrane hamiltonian (1). Observe that in the above gauge theoretical construction the space-sheet metric \( w_{rs} \) enters only through its density \( \sqrt{w} \) and hence vanishing or singular metric components do not pose problems.

We are now in a position to study the full eleven–dimensional supersymmetry algebra of the winding supermembrane. For this we decompose the supersymmetry charge \( Q \) associated to the transformations (13) as follows

\[ Q = Q^+ + Q^-, \quad \text{where} \quad Q^\pm = \frac{1}{2} \gamma^\pm \gamma^\mp Q, \quad (14) \]

to obtain

\[ Q^+ = \int d^2 \sigma \left( 2 P^a \gamma_a + \sqrt{w} \{ X^a, X^b \} \gamma_{ab} \right) \theta, \]
\[ Q^- = 2 P^+_0 \int d^2 \sigma \sqrt{w} \gamma_- \theta. \quad (15) \]

The canonical Dirac brackets are derived by the standard methods and read

\[ (X^a(\sigma), P^b(\sigma'))_{DB} = \delta^{ab} \delta^2(\sigma - \sigma'), \]
\[ (\theta_\alpha(\sigma), \bar{\theta}_\beta(\sigma'))_{DB} = \frac{1}{4} (P^+_0)^{-1} w^{-1/2} (\gamma_+)^{\alpha\beta} \delta^2(\sigma - \sigma'). \quad (16) \]

In the presence of winding the results given in [4] yield the supersymmetry algebra

\[ (Q^+_\alpha, \bar{Q}^+_\beta)_{DB} = 2 (\gamma_+)_{\alpha\beta} \mathcal{H} - 2 (\gamma_a \gamma_+)_{\alpha\beta} \int d^2 \sigma \sqrt{w} \{ X^a, X^- \}, \]
\[ (Q^+_{\alpha}, \bar{Q}^-_{\beta})_{DB} = - (\gamma_a \gamma_+)_{\alpha\beta} P^+_0 - \frac{1}{2} (\gamma_{ab} \gamma_-)_{\alpha\beta} \int d^2 \sigma \sqrt{w} \{ X^a, X^b \}, \]
\[ (Q^-_{\alpha}, \bar{Q}^-_{\beta})_{DB} = -2 (\gamma_-)_{\alpha\beta} P^+_0, \quad (17) \]
where use has been made of the defining equation (3) for \( X^- \). The new feature of this supersymmetry algebra is the emergence of the central charges in the first two anticommutators, which are generated through the winding contributions. They represent topologically conserved quantities obtained by integrating the winding densities \( z^a(\sigma) = \epsilon^{rs} \partial_r X^a \partial_s X^- \) and \( z^{ab}(\sigma) = \epsilon^{rs} \partial_r X^a \partial_s X^b \) over the space-sheet. It is gratifying to observe the manifest Lorentz invariance of (17). Here we should point out that, in adopting the light-cone gauge, we assumed that there was no winding for the coordinate \( X^+ \). In [16] the corresponding algebra for the matrix regularization was studied. The result obtained in [16] coincides with ours in the large-\( N \) limit, in which an additional longitudinal five-brane charge vanishes, provided that one identifies the longitudinal two-brane charge with the central charge in the first line of (17). This requires the definition of \( X^- \) in the matrix regularization, a topic that was dealt with in [18]. We observe that the discrepancy noted in [16] between the matrix calculation and certain surface terms derived in [4], seems to have no consequences for the supersymmetry algebra. A possible reason for this could be that certain Schwinger terms have not been treated correctly in the matrix computation, as was claimed in a recent paper [19].

In order to define a matrix approximation one introduces a complete orthonormal basis of functions \( Y_A(\sigma) \) for the globally defined \( \xi(\sigma) \) of (6). One may then write down the following mode expansions for the phase space variables of the supermembrane,

\[
\partial_r X(\sigma) = X^\lambda \phi(\lambda)_r + \sum_A X^A \partial_r Y_A(\sigma),
\]

\[
P(\sigma) = \sum_A \sqrt{w} P^A Y_a(\sigma),
\]

\[
\theta(\sigma) = \sum_A \theta^A Y_A(\sigma), \tag{18}
\]

introducing winding modes for the transverse \( X^a \). A similar expansion exists for \( X^- \). One then naturally introduces the structure constants of the group of area–preserving diffeomorphism by [18]

\[
f_{ABC} = \int d^2 \sigma \epsilon^{rs} \partial_r Y_A \partial_s Y_B Y_C, \]

\[
f_{\lambda BC} = \int d^2 \sigma \epsilon^{rs} \phi(\lambda)_r \partial_s Y_B Y_C, \]

\[
f_{\lambda \lambda'C} = \int d^2 \sigma \epsilon^{rs} \phi(\lambda)_r \phi(\lambda')_s Y_C. \tag{19}
\]

Note that with \( Y_0 = 1 \), we have \( f_{AB0} = f_{\lambda B0} = 0 \). The raising and lowering of the \( A \) indices is performed with the invariant metric \( \eta_{AB} = \int d^2 \sigma \sqrt{w} Y_A(\sigma) Y_B(\sigma) \) and there is no need to introduce a metric for the \( \lambda \) indices.
By plugging the mode expansions (18) into the hamiltonian (1) one obtains 

\[ \mathcal{H} = \frac{1}{2} \mathbf{P}_0 \cdot \mathbf{P}_0 + \frac{1}{4} f_{\lambda\mu} \mathbf{P}^2 + \frac{1}{4} f^{\nu\lambda\mu
u} \mathbf{P}_0 \cdot \mathbf{P}_0 \mathbf{P}^2 + \frac{1}{4} f^{\nu\lambda\mu
u} \mathbf{P}^2 + \frac{1}{4} f^{\nu\lambda\mu
u} \mathbf{P}^2 \]

the decomposition

+ \frac{1}{2} \mathbf{P}^2 + \frac{1}{4} f^{\nu\lambda\mu
u} \mathbf{P}^2 + \frac{1}{4} f^{\nu\lambda\mu
u} \mathbf{P}^2 + \frac{1}{4} f^{\nu\lambda\mu
u} \mathbf{P}^2

where here and henceforth we spell out the zero-mode dependence explicitly, i.e. the range of values for \( A \) no longer includes \( A = 0 \). Note that for the toroidal supermembrane \( f_{\lambda\mu A} = 0 \) and thus the last three terms in (20) vanish. The second term in the first line represents the winding number squared. In the matrix formulation, the winding number takes the form of a trace over a commutator. We have scaled the hamiltonian by a factor of \( P_0^+ \) and the fermionic variables by a factor \( (P_0^+)^{-1/2} \). Supercharges will be rescaled as well, such as to eliminate explicit factors of \( P_0^+ \).

The constraint equation (3) is translated into mode language by contracting it with \( \epsilon^{rs} \partial (\lambda) \) and \( \epsilon^{rs} \partial \gamma \) respectively and integrating the result over the spacesheet to obtain the two constraints

\[
\varphi_{\lambda} = f_{\lambda\mu
u} (X_\nu \cdot \mathbf{P}_0 + X_\nu \cdot \mathbf{P}_0^+) + f_{\lambda\mu\nu} X_\nu \cdot \mathbf{P}_0^+ = 0,
\]

\[
\varphi_A = f_{\lambda
u} \mathbf{X}_\nu \cdot \mathbf{P}_0 = 0,
\]

\[ \mathcal{M}^2 = 2 \mathcal{H} - \mathbf{P}_0 \cdot \mathbf{P}_0 - \frac{1}{2} (f_{\lambda\mu\nu} X_\nu X_\lambda)^2. \]

\[ (21) \]

taking also possible winding in the \( X^- \) direction into account. Note that even for the non-winding case \( X^a \lambda = 0 \) there remain the extra \( \varphi_{\lambda} \) constraints.

The zero mode contributions completely decouple in the hamiltonian and the supercharges. We thus perform a split in \( Q^+ \) treating zero modes and fluctuations separately to obtain the mode expansions,

\[ Q^- = 2 \gamma_\theta, \quad Q^+ = Q^+_0 + \hat{Q}^+, \]

\[ (22) \]

where

\[ Q^+_0 = \left( 2 P_0^a \gamma_a + f_{\lambda\mu\nu} X^a \lambda X^b \lambda \gamma_{ab} \right) \theta_0, \]

\[ \hat{Q}^+ = \left( 2 P_0^c \gamma_a + f_{\lambda\mu\nu} X^a \lambda X^b \lambda \gamma_{ab} \right) \theta_C + 2 f_{\lambda\mu\nu} X^a \lambda X^b \lambda \gamma_{ab} + f_{\lambda\mu\nu} X^a \lambda X^b \lambda \gamma_{ab} \right) \theta_C. \]

\[ (23) \]

Upon introducing the supermembrane mass operator by

\[ \mathcal{M}^2 = 2 \mathcal{H} - \mathbf{P}_0 \cdot \mathbf{P}_0 - \frac{1}{2} (f_{\lambda\mu\nu} X_\nu X_\lambda)^2, \]

\[ (24) \]
the supersymmetry algebra (17) then takes the form

\[
\{ \hat{Q}^+_{\alpha}, \bar{Q}^+_{\bar{\beta}} \} = \left( \frac{1}{2} \hat{Q}^+_{\alpha}, \bar{Q}^+_{\bar{\beta}} \right) = \left( \gamma_+ \gamma_- \right)_{\alpha\beta} \left( f_{\lambda\lambda'} X^{a\lambda} \left( X^{-\lambda'} P^+_0 + X^{\lambda'} \cdot P_0 \right) \right),
\]

\[
\{ Q^+_{(0)\alpha}, \bar{Q}^+_{(0)\bar{\beta}} \} = \left( \frac{1}{2} Q^+_{(0)\alpha}, \bar{Q}^+_{(0)\bar{\beta}} \right) = \left( \gamma_+ \right)_{\alpha\beta} \left( P_0 \cdot P_0 + \frac{1}{2} \left( f_{\lambda\lambda'} X^{a\lambda} X^{b\lambda'} \right)^2 \right) + 2 \left( \gamma_+ \gamma_- \right)_{\alpha\beta} f_{\lambda\lambda'} X^{a\lambda} X^{b\lambda'},
\]

\[
\{ \bar{Q}^+_{(0)\alpha}, \bar{Q}^+_{(0)\bar{\beta}} \} = \{ Q^+_{(0)\alpha}, \bar{Q}^+_{(0)\bar{\beta}} \} = \{ \hat{Q}^+_{\alpha}, \bar{Q}^+_{\bar{\beta}} \} = 0.
\]

And the mass operator commutes with all the supersymmetry charges,

\[
[ \hat{Q}^+, \mathcal{M}^2 ] = [ Q^+_{(0)}, \mathcal{M}^2 ] = [ Q^-, \mathcal{M}^2 ] = 0,
\]

defining a supersymmetric quantum-mechanical model.

At this stage it would be desirable to present a matrix model regularization of the supermembrane with winding contributions, generalizing the SU(N) approximation to the exact subgroup of area-preserving diffeomorphisms [5, 4], at least for toroidal geometries. However, this program seems to fail due to the fact that the finite-N approximation to the structure constants \( f_{\lambda\beta} \) violates the Jacobi identity, as was already noticed in [18].

Finally we turn to the question of the mass spectrum for membrane states with winding. The mass spectrum of the supermembrane without winding is continuous. This was proven in the SU(N) regularization [13]. Whether or not nontrivial zero-mass states exist, is not known (for some discussion on these questions, we refer the reader to [20]). Those would coincide with the states of eleven-dimensional supergravity. It is often argued that the winding may remove the continuity of the spectrum (see, for instance, [21]). There is no question that winding may increase the energy of the membrane states. A membrane winding around more than one compact dimension gives rise to a nonzero central charge in the supersymmetry algebra. This central charge sets a lower limit on the membrane mass. However, this should not be interpreted as an indication that the spectrum becomes discrete. The possible continuity of the spectrum hinges on two features. First the system should possess continuous valleys of classically degenerate states. Qualitatively one recognizes immediately that this feature is not directly affected by the winding. A classical membrane with winding can still have stringlike configurations of arbitrary length, without increasing its area. Hence the classical instability still persists.

The second feature is supersymmetry. Generically the classical valley structure is lifted by quantum-mechanical corrections, so that the wave function cannot escape to infinity. This phenomenon can be understood on the basis of the uncertainty principle. Because, at large distances, the valleys become increasingly narrow, the wave function will be squeezed more and more which tends to induce an increasing spread in its momentum. This results in an increase of the kinetic
energy. Another way to understand this is by noting that the transverse oscillations perpendicular to the valleys give rise to a zero-point energy, which acts as an effective potential barrier that confines the wave function. When the valley configurations are supersymmetric the contributions from the bosonic and the fermionic transverse oscillations cancel each other, so that the wave function will not be confined and can extend arbitrarily far into the valley. This phenomenon indicates that the energy spectrum must be continuous.

Without winding it is clear that the valley configurations are supersymmetric, so that one concludes that the spectrum is continuous. With winding the latter aspect is somewhat more subtle. However, we note that, when the winding density is concentrated in one part of the spacesheet, then valleys can emerge elsewhere corresponding to stringlike configurations with supersymmetry. Hence, as a space-sheet local field theory, supersymmetry can be broken in one region where the winding is concentrated and unbroken in another. In the latter region stringlike configurations can form, which, at least semiclassically, will not be suppressed by quantum corrections. Obviously, the state of lowest energy for a given winding number is always a BPS state, which is invariant under some residual supersymmetry. Hence in that respect the situation is qualitatively similar to the one without winding.

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