Resonance Production on Nuclei at High Energies: Nuclear-Medium Effects and Space-Time Picture

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Abstract

The influence of nuclear matter on the properties of coherently produced resonances is discussed. It is shown that, in general, the mass distribution of resonance decay products has a two–component structure corresponding to decay outside and inside the nucleus. The first (narrow) component of the amplitude has a Breit–Wigner form determined by the vacuum values of mass and width of the resonance. The second (broad) component corresponds to interactions of the resonance with the nuclear medium. It can be also described by a Breit–Wigner shape with parameters depending e.g. on the nuclear density and on the cross section of the resonance–nucleon interaction.

The resonance production is examined both at intermediate energies, where interactions with the nucleus can be considered as a series of successive local rescatterings, and at high energies, $E > E_{\text{crit}}$, where a change of interaction picture occurs. This change of mechanisms of the interactions with the nucleus is typical for the description within the Regge theory approach and is connected with the nonlocal nature of the reggeon interaction.

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The study of the propagation of hadrons in nuclear matter is a much studied subject. It is a tool to explore the structure of the nucleus and to probe nuclear matter at large densities. Another perspective, which we adopt here, is to consider the nucleus as a known microscopic laboratory for studying elementary hadronic properties and interactions, which can then be compared to the predictions of models. This is particularly important in the case of unstable hadrons (resonances), where nuclear measurements are the only means to study their interaction. Even when the free elementary process is well known, experiments on a nucleus are necessary to tell us something about the space–time picture of the elementary interaction by providing a measure – typical nuclear distances and time scales. The bulk effect of the target nucleons on the propagation of the hadron is often referred to as 'medium modification' of the particle properties, expressed through an ‘in-medium’ mass and, for a resonance, also width of the hadron.

The creation and propagation of resonances in nuclei has been widely studied over the last decades, both experimentally and theoretically. There are two distinct ways to create a resonance in the collision of a projectile with a nuclear target. In the first, the projectile and a target nucleon form a resonance. This s-channel resonance occurs at a definite projectile energy and shows up as a peak in the energy dependence of the total cross section. An example is the excitation of nucleon resonances, in nuclear photoabsorption, where e.g. for the ∆ resonance the elementary process is $\gamma + N \rightarrow \Delta$. It is common to refer to this type of resonance creation as formation. Recent examples are the nuclear photoabsorption measurements at Frascati [1] and Mainz [3]. They showed that some nucleon resonances that were seen in photoabsorption on hydrogen and deuterium disappear in nuclei, which to a large extent may be due to collision broadening of the resonance. In the second type, called resonance production, other particles besides the resonance emerge after the initiating projectile–nucleon interaction. For example in the nuclear photoproduction of the $\rho$ resonance, the underlying mechanism is $\gamma + N \rightarrow \rho + N$. There is no characteristic energy dependence in the cross section in this case and the resonance has to be identified through the invariant mass of its decay products. The production of a particular resonance in a given process continues to play a role at very high projectile energy, while formation of that resonance is suppressed as soon as one is more than a half width removed from the resonance energy. This is the reason that formation is essentially a phenomenon relevant at low ‘resonant’ energies, while production is important at high energies as well. A recent experiment studying the production of the $\rho$ meson is the measurement of di-lepton spectra at the SPS [4] - [6]. The results were interpreted through a shift of the $\rho$ mass in nuclear matter, as predicted in effective Lagrangian models and in approaches based on QCD sum rules.

Many theoretical treatments of the behavior of resonances in nuclei were done for nuclear matter and then applied to finite nuclei by taking over the ‘in medium’ propagator. A central topic of this paper is the appropriate Green function for finite nuclei. The finiteness of nuclei was also a crucial ingredient in the work of Ericson [7] on the propagation of virtual...
pions in nuclei and our discussion has some common points.

We discuss in this paper the *coherent* diffractive production of hadronic resonances on nuclei such as the photoproduction of vector mesons. In this case a resonance $R$ is produced by the initial particle $a$ on one of target nucleons, $aN \rightarrow RN$, propagates in nuclear matter interacting with other nucleons, $RN \rightarrow RN$, and then decays into a system of particles, $R \rightarrow x_1, x_2, \ldots$. The decay products are registered *outside* the nucleus, but the decay itself can occur both inside or outside the nucleus. For simplicity, we will leave out final state interactions of the decay particles, which would be appropriate for the decay of a $\rho$ resonance into an $e^+e^-$ pair. It will be shown that in the case of *coherent* production, where the nucleus returns to its ground state after the reaction, the mass distribution of decay products contains information on both stages of resonance life, in nuclear matter and in the vacuum. As a result, the mass distribution has to be interpreted not in terms of a single ’in medium’ resonance, but as an amplitude with a two–component structure. These two contributions can separately be parametrized through Breit–Wigner shapes. The first (narrow) peak corresponds to the decay outside the nucleus and is determined by the vacuum values of mass and width of the resonance. The second – typically much broader – peak is due to the interaction of the resonance with the nucleus and can be characterized through changed values of resonance mass and width. Clearly, the relative contribution of the two components depends *e.g.* on the nuclear density, the atomic number $A$ of the target or the life time of the resonance. The interference of these two contributions gives the mass distribution of the decay products a more complicated form and makes it harder to extract the medium modified resonance parameters from the observed cross section.

The most common way to think about the propagation of a hadron in a nucleus originated from the intermediate energy hadron–nucleus scattering. At energies where the projectile wavelength is small compared to the characteristic sizes of the process, the range of the potential and the nuclear dimensions, the process is seen as a succession of individual elementary projectile–nucleon interactions, described by the free amplitude. In particular at higher energies the motion of the nucleons can be neglected, leading to the frozen nucleus picture as embodied in *e.g.* the often used eikonal description.

One might expect that the inherent approximations become better with increasing energy. As was pointed out by Gribov [8],[9] and followed up in detail by Koplik and Mueller [10], this is not the case. For a high energy projectile above a critical energy, $E > E_{\text{crit}}$, the picture changes radically. The fast projectile should be seen as developing into a superposition of multiparticle hadronic states before reaching the target and before the first interaction takes place. That interaction takes place only between the lowest energy component of the projectile and the target. For an elastic scattering process, this multiparticle intermediate state eventually recombines into the original projectile–target state. This corresponds to an interaction of a considerable nonlocality, which makes the traditional approximations leading to the eikonal description incorrect. However, it was shown [8],[9] that, by consistently incorporating all intermediate multiparticle states in the coherent scattering of a stable hadron from a nucleus, one surprisingly enough obtains an expression that has the same structure as the result obtained from a naive application of the eikonal approach. Here we extend these ideas to the production and propagation
of a resonance in a coherent production process on a nucleus, such as the electromagnetic production of vector mesons. It is clearly also relevant to the studies of dense nuclear matter through resonance production in relativistic nuclear reactions.

The paper is organized as follows. In Chapter 2 we discuss the production of resonances at intermediate energies, less than the critical energy, but high enough to use the eikonal approximation. We first give a simple, qualitative explanation why the production amplitude in a finite nucleus must have a two-component structure. We then derive the full expression in the eikonal approximation and show that it displays a two–component structure for the invariant mass distribution of the decay products. In Ch. 3, the situation for energies higher than the critical value, $E > E_{crit}$, is discussed. Using Gribov’s method we show that the final formulas turn out to have the same structure as for the eikonal approach in spite of the completely different space–time picture of the interactions in the two energy regions. In case that only one reaction channel is relevant, the propagating resonance, the formulas are identical. The reader mainly interested in the two-component structure of resonance production in finite nuclei could therefore go from Ch. 2 straight to Ch. 4, where we give some numerical examples for the photoproduction of $\rho$ mesons on nuclei to show the relative importance of the two resonance components. The dependence on the nuclear size is studied and the influence of nuclear correlations on medium modified resonance parameters are considered. A summary and some conclusions are given in Chapter 5. A preliminary version of this work has been presented in Ref. [11].

2. INTERMEDIATE ENERGIES

In this section we consider the coherent production of resonances on a nucleus at energies less than $E_{crit}$, where the interaction of the resonance with the nucleus can be described through the multiple scattering of the resonance from the target nucleons, summarized in an optical potential. The propagation of the resonance in the nuclear medium is then given by the corresponding 'dressed' Green function.

Before discussing this in detail, we first derive the relevant features of the production amplitude in a qualitative fashion for a one-dimensional example, similar to the arguments used in Ref. [7]. We consider a high energy photon incident along the positive $z$ axis with momentum $p$ that strikes an infinite slab of nuclear matter extending from $-z_A$ to $z_A$. Inside the nuclear matter at $z_i$ a $\rho$-resonance is excited. By matching the logarithmic derivatives at $z_A$ and neglecting backward motion of the resonance, a good approximation at high energies, it is simple to see that the wavefunction of the resonance has the $z$-dependence

$$
\Psi_{z_i}(z) = N(z_i)\exp[iP^*(z_A - |z|)]\theta(z - z_A) + \exp[iP(z - z_A)]\theta(z - z_A)
$$

(1)

where $N(z_i)$ is a factor depending on the excitation strength at $z_i$. We assume that the influence of the nucleus on the resonance can be represented through a complex optical potential, $V^*$. Then $P^*$, the momentum of the resonance inside the nuclear medium, is

$$
P^* = \sqrt{p^2 - M_R^2 + iM_R\Gamma + 2M_RV^*} \equiv p - Q^*_R.
$$

(2)
where $M_R$ and $\Gamma_R$ are the free resonance mass and width, respectively. The free resonance momentum outside the nucleus is given by

$$P = \sqrt{\tilde{s} - M_R^2 + iM_R\Gamma} \equiv p - Q_R .$$  \hfill (3)

The resonance decays at point $z_f$ into an electron-positron pair, a plane wave state of momentum $p - q$. The production amplitude is then simply the integral over excitation and decay points,

$$T(p,q) = \int_{z_i}^{z_A} dz_i \int_{z_i}^{\infty} dz_f e^{-i(p-q)z_f} \Psi(z_f)$$

$$\propto \int_{z_i}^{z_A} dz_i N(z_i) \left\{ 1 - \frac{\exp[i2pD^*(z_A-z_i)]}{D^*} + \frac{\exp[i2pD(z_A-z_i)]}{D} \right\} ,$$  \hfill (4)

where

$$2pD^* = q - Q^*_R , \quad 2pD = q - Q_R .$$  \hfill (5)

For large $p$ we have $q \simeq \tilde{s}/2p$, where $\tilde{s}$ is the total energy of the resonance. We can then write the denominators in Eq. (4) as

$$D \simeq \tilde{s} - M_R^2 + iM_R\Gamma_R ,$$

$$D^* \simeq \tilde{s} - M_R^2 + iM_R^*\Gamma_R .$$  \hfill (6,7)

At this stage, we can already read off the relevant features. First, the amplitude is the sum of two Breit-Wigner resonance contributions, consisting of a term with the free and medium modified denominators, $D$ and $D^*$, respectively. There is no true pole in the complex $\tilde{s}$ plane due to the in medium denominator. Furthermore, it can be seen that only in the limit of infinite nuclear matter, $z_A \to \infty$, where the exponential terms vanish, we are left with an amplitude that has a single, medium modified component and a pole in the complex $\tilde{s}$ plane. These are the main results we derive in this Chapter.

We now turn to a more detailed and complete discussion. Rather than working directly within the well eikonal formalism, we briefly rederive it from a diagrammatic approach. This useful in order to better connect to the discussion in Chapter 3. We start with the amplitude for resonance production by a projectile $a$ with momentum $p$ striking a nucleus of mass $A$, neglecting its further interactions with the nucleus. It corresponds to the diagram of Fig.1 and the amplitude has the form

$$T_0(p,q^2,\tilde{s}) = t(R \to X) G_0(\tilde{s}) t_0(s,q^2) F_A(q) ,$$  \hfill (8)

Here $t_0(s,q^2)$ is the amplitude for resonance production on a free nucleon, $G_0(\tilde{s})$ is the propagator of the resonance and $t(R \to X)$ is the amplitude of its decay into the final state $X = \{ x_1, x_2, \cdots \}$ . The momentum transfer to the nucleus equals $q$ and the invariant mass of the resonance is denoted by $\tilde{s} = (p - q)^2$. We assume that the energy is high enough that Fermi motion can be neglected. Therefore, we use in the amplitude $t_0$ the average total energy of the projectile and struck target nucleon, $s = (p + P_A/A)^2$, where $p$ and $P_A$ are four-momenta of the projectile and the nucleus, respectively. In principle $t_0$ also
depends on the invariant mass of the resonance, $\tilde{s}$. The main dependence of $T_0$, Eq. (8), on $\tilde{s}$ comes from the Green function $G_0(\tilde{s})$ and we therefore ignore the weaker dependence in $t_0$; similar considerations are also applied to $t(R \rightarrow X)$. We will only explicitly indicate the $s$ dependence when it is necessary to avoid confusion.

We choose the $z$-axis along the beam direction. For coherent production we obtain the following kinematical conditions, typical for the eikonal approximation at high energies:

$$
\begin{align*}
|q^2| &\approx q^2 \sim R_A^{-2}, \\
q_0 &\sim q^2/2M_A \ll |q|,
q_z &\approx (\tilde{s} - m_a^2)/2|p|,
|q_{\perp}| &\leq R_A^{-1}.
\end{align*}
$$

(9)

The nuclear form factor $F_A(q)$ is the Fourier transform of the nuclear density,

$$
F_A(q) = \int d^3r e^{iq \cdot r} \rho_A(r).
$$

(10)

The propagator of the resonance,

$$
G_0(\tilde{s}) = i \left[ (p - q)^2 - M_R^2 + iM_R \Gamma_R \right]^{-1},
$$

(11)

with the above kinematical approximations and neglecting the $q^2$ term, becomes

$$
G_0(\tilde{s}) = i \left[ 2|p| (q_z - q_R + i\gamma_R/2) \right]^{-1} = \frac{i}{2|p|(q_z - Q_R)}.
$$

(12)

Here

$$
q_R = \frac{M_R^2 - m_a^2}{2|p|}, \quad \gamma_R = \frac{M_R}{|p|} \Gamma_R
$$

(13)

are the minimal momentum transfer to produce the resonance $R$ and the Lorentz-factor reduced width of the resonance, respectively, which we combine into the complex quantity

$$
Q_R = q_R - i\gamma_R/2.
$$

(14)

We will in the following take the point of view of time ordered perturbation theory, where the intermediate propagation of the resonance occurs at fixed energy,

$$
E_0 \approx |p| + m_a^2/2|p|,
$$

(15)

determined by the initial projectile momentum. It will be most convenient to work in the coordinate representation. We therefore Fourier transform with respect to the momentum variables and obtain for propagation by the distance $r = (z, b)$ with $b \cdot p = 0$:

$$
G_0(z, b; E_0) = (2\pi)^{-3} \int dq_z d^2q_{\perp} e^{ipz - iq_z z - iq_{\perp} \cdot b} G_0(p - q; E_0)
$$

$$
= e^{ipz} g_0(z) \theta(z) \delta^2(b),
$$

(16)
where
\[ g_0(z) = \frac{1}{2|p|} e^{-iQ_R z} = (2|p|)^{-1} e^{-iQ_R z}. \]

This Green function has the natural form for a high energy process: forward propagation of the resonance at fixed impact parameter b with an attenuation proportional to \( \gamma_R \). The production amplitude can be written in a way reflecting the space–time sequence of the process (with the initial state on the right):
\[
T_0(p, q^2, \bar{s}) = t(R \rightarrow X) \int d^3r_f \int d^3r_i e^{-i(p-q) \cdot r_f} G_0(r_f - r_i; E_0) U_0(r_i) e^{i p \cdot r_i},
\]
where
\[
U_0(r) = (2\pi)^{-3} \int d^3q e^{i q \cdot r} t_0(q^2, s) F_A(q).
\]

Eq. (18) describes the resonance production on a nucleon at the point \( r_i = (z_i, b) \), its free propagation at constant impact parameter \( b \) to the point \( r_f = (z_f, b) \) and its decay at this point.

The corresponding cross section has the form
\[
\frac{d\sigma}{dq^2 d\bar{s}} = \frac{|t_0(s, q^2)|^2}{(s - M_R^2)^2 + M_R^4 \Gamma_R^2} \frac{dq^2 d\bar{s}}{32\pi^2 p\sqrt{s}} d\tau_X(\bar{s}).
\]

where \( d\tau_X \) is the phase volume for the decay \( R \rightarrow X \). One can see that the distribution \( d\sigma/dq^2 d\bar{s} \) has the usual Breit–Wigner form as a function of \( \bar{s} \).

What happens now when we allow for interactions of the resonance with the nucleus? One might simply expect that one again obtains a Breit–Wigner distribution, but now for an ‘in medium’ resonance with modified parameters \( M_R \) and \( \Gamma_R \). Below, we will show that this is not the case for production of a resonance on a finite nucleus.

To take into account the final state interactions of the resonance \( R \) with the nucleus (see Fig.2), we replace the free Green function, \( G_0 \) in Eq. (18) by the dressed Green function, \( G \), which takes into account the interactions with the residual nucleus. The production amplitude obtained with the full Green function, \( G \), becomes:
\[
T(p, q^2, \bar{s}) = t(R \rightarrow X) \int d^3r_f \int d^3r_i e^{-i(p-q) \cdot r_f} G(r_f, r_i; E_0) U_0(r_i) e^{i p \cdot r_i}.
\]

For the construction of the dressed Green function in the usual multiple scattering picture, we use the elastic resonance–nucleon scattering amplitude, \( t_R(s, q^2) \), normalized according to
\[
2 \text{Im} \ t_R(s, 0) = \sigma_R^{tot}(s).
\]

Taking into account that the range of strong interactions is small compared to the nuclear size one can write approximately for the corresponding ‘optical potential’ that describes the interaction with the nucleons in the nuclear ground state
\[
U_R(r) = (2\pi)^{-3} \int d^3q e^{i q \cdot r} t_R(q^2, s) F_A(q^2) \approx t_R(s, 0) \rho_A(r).
\]
For much of the discussion below the precise form of the interaction does not matter. Other effects that vary approximately as the nuclear density and can be included analogously will lead to similar conclusions.

In coordinate space the dressed Green function has the form

\[
G(r_f, r_i; E_0) = e^{ip(z_f - z_i)} g(z_f, z_i) \theta(z_f - z_i) \delta^2(b_f - b_i) ,
\]

with

\[
g(z_f, z_i) = \sum_{n=0}^{\infty} i^n \int \cdots \int dz_n \cdots dz_1 g_0(z_f - z_n) U_R(z_n, b) \times g_0(z_n - z_{n-1}) \cdots U_R(z_1, b) g_0(z_1 - z_i)
\]

\[
= (2|p|)^{-1} \exp[-iQ_R(z_f - z_i)] \sum_{n=0}^{\infty} \frac{1}{n!} \left[ i \int_{z_i}^{z_f} dz U_R(z, b) \right]^n
\]

and where the integrations over \(z_i\)'s are longitudinally ordered, i.e.

\[
\cdots < z_1 < z_2 < \cdots < z_n < z_f .
\]

It can easily be seen from Eqs.(24) and (25) that for homogeneous infinite nuclear matter with density \(\rho_0\), the full Green function \(G(r_f, r_i; E_0)\) depends only on the difference \(r_f - r_i\). This is due to translational invariance and yields momentum conservation in the multiple scattering process. The full Green function can in this case be obtained from the free one, Eqs.(16) - (17), by the replacement

\[
Q_R \rightarrow Q_R^* = Q_R - t_R(0) \rho_0 ,
\]

or

\[
M_R \rightarrow M_R^* , \quad \Gamma_R \rightarrow \Gamma_R^* ,
\]

where

\[
M_R^2 = M_R^2 - 2|p| \text{Re} t_R(0) \rho_0 ,
\]

\[
M_R^* \Gamma_R^* = M_R \Gamma_R + 2|p| \text{Im} t_R(0) \rho_0 \equiv \gamma_R^* |p| , \quad \gamma_R^* = \gamma_R + \sigma_R \rho_0 .
\]

For infinite nuclear matter, we thus obtain a production amplitude, \(T\), for which the \(\tilde{s}\) dependence is again given by a single Breit-Wigner denominator as in Eq.(20), but now with the medium modified parameters \(M_R^*\) and \(\Gamma_R^*\). From Eq.(30) it is clear that the nuclear matter width of the resonance is larger than the free one, yielding a broader peak of the production cross section. The resonance peak also shifts to a different position, but whether this shift is repulsive or attractive depends on the sign of \(\text{Re} t_R\).

The \(\tilde{s}\) dependence of the cross section for a finite nucleus is already revealed by carrying out the integration over the final coordinate, \(r_f\), in Eq.(21), which amounts to taking the partial Fourier transform of the Green function \(G(r_f, r_i; E_0)\)

\[
G(r_i; p - q; E_0) = \int d^3r_f e^{-i(p - q)r_f} G(r_f, r_i; E_0) .
\]
This function describes the amplitude to find the resonance \( R \), after having been produced at the point \( \mathbf{r}_i \), in a plane wave final state with momentum \( \mathbf{p} - \mathbf{q} \) and with an invariant mass

\[
\tilde{s} = E_0^2 - (\mathbf{p} - \mathbf{q})^2 \approx 2|\mathbf{p}|q_z + m_n^2. \tag{32}
\]

For simplicity, we now assume that we are dealing with a nucleus of radius \( R_A \) and constant nuclear density, \( \rho_0 \)

\[
\rho_A(\mathbf{r}) = \rho_0 \theta(R_A - r). \tag{33}
\]

When Fourier transforming the Green function, Eq.(31), one receives two contributions corresponding to the decay inside and outside the nucleus, \( r_f < R_A \) and \( r_f > R_A \), respectively.

As can be seen from Eq. (25), these contributions can be written in the simple form

\[
G(\mathbf{r}_i; \mathbf{p} - \mathbf{q}; E_0) = C_{in} \cdot G_{in}(\mathbf{r}_i; \mathbf{p} - \mathbf{q}; E_0) + C_{out} \cdot G_{out}(\mathbf{r}_i; \mathbf{p} - \mathbf{q}; E_0), \tag{34}
\]

where

\[
G_{in}(\mathbf{r}_i; \mathbf{p} - \mathbf{q}; E_0) = i \frac{2|\mathbf{p}|(q_z - Q_R^*)}{[q_z - Q_R^*]^{-1}},
\]

\[
G_{out}(\mathbf{r}_i; \mathbf{p} - \mathbf{q}; E_0) = i \frac{2|\mathbf{p}|(q_z - Q_R^*)}{[q_z - Q_R^*]^{-1}}. \tag{35}
\]

The coefficients in Eq.(34) are given by

\[
C_{in}(\mathbf{r}_i; \mathbf{p} - \mathbf{q}, E_0) = \exp(-i(p - q_z)z_i) \{1 - \exp[i(q_z - Q_R^*)(z_A - z_i)]\}
\]

\[
C_{out}(\mathbf{r}_i; \mathbf{p} - \mathbf{q}, E_0) = \exp(-i(p - q_z)z_i) \exp[i(q_z - Q_R^*)(z_A - z_i)], \tag{36}
\]

where \( z_A = \sqrt{R_A^2 - b^2} \) defines the point where the resonance with impact parameter \( b \) leaves the nucleus.

To explicitly display the \( \tilde{s} \) dependence and singularity structure, we re-write this by using the kinematics of the eikonal approximations, Eqs.(12 - 13) and the definitions in Eqs.(27)–(30):

\[
G_{in}(z_i, \tilde{s}, E_0) = \left[ \tilde{s} - M_{R}^2 + iM_R^* \Gamma_R \right]^{-1},
\]

\[
G_{out}(z_i, \tilde{s}, E_0) = \left[ \tilde{s} - M_{R}^2 + iM_R \Gamma_R \right]^{-1}, \tag{37}
\]

and

\[
C_{in}(\mathbf{r}_i; \mathbf{p} - \mathbf{q}, E_0) = \exp(-i(p - q_z)z_i)\left[1 - \exp\left(\frac{i}{2|\mathbf{p}|}(\tilde{s} - M_{R}^2 + iM_R^* \Gamma_R)(z_A - z_i)\right)\right],
\]

\[
C_{out}(\mathbf{r}_i; \mathbf{p} - \mathbf{q}, E_0) = \exp(-i(p - q_z)z_i)\exp\left(\frac{i}{2|\mathbf{p}|}(\tilde{s} - M_{R}^2 + iM_R \Gamma_R)(z_A - z_i)\right). \tag{38}
\]

The above expression for the Green function and its \( \tilde{s} \) dependence contain the central result of this section. As the Green function enters directly into the production amplitude, we see that the production amplitude in finite nuclei is necessarily a superposition of two separate resonance structures: the original, narrower resonance peak due to decay outside
the nucleus, and the broader peak due decay inside the nucleus. Note that the term $G_{in}$ does not have a pole singularity at $\tilde{s} = M^2_R - iM^*_R\Gamma_R$ because the residue of the pole, $C_{in}$, vanishes at this point. The Green function thus has no pole corresponding to a medium modified resonance due the finiteness of the nuclear medium. Nevertheless, for real values of $\tilde{s}$, the amplitude does exhibit a Breit-Wigner structure.

The relative weights of the two components are given by the probability amplitude for the decay inside and outside. If it is known experimentally that one has detected decay products corresponding to decay of the resonance at some distance from the target, only the narrow component will contribute. On the other hand, for an infinitely extended nucleus $z_A \to \infty$ only the broad component is present with a non-vanishing residue. For a finite nucleus, both contribute to the amplitude and interfere with each other in the cross section. As one is usually interested in the medium modified part, both contributions must be carefully separated. This situation is quite different from a coherent nuclear formation reaction $[12]-[15]$, where only the propagation of the broadened resonance (or in medium component) is important.

For a finite nucleus with constant density we can obtain an analytical expression for the forward production amplitude, $T$, by carrying out the integration in Eq. (21) also over the initial coordinate, $r_i$. The result again shows separate contributions from decay inside and outside of the nucleus:

$$T(p, 0, \tilde{s}) = D_{in}(x, y) \cdot \left[ \tilde{s} - M^2_R + iM^*_R\Gamma_R \right]^{-1} + D_{out}(x, y) \cdot \left[ \tilde{s} - M^2_R + iM_R\Gamma_R \right]^{-1},$$

where

$$D_{in, out}(x, y) = t(R \to X)t_0(s, 0)\rho_0 \int d^2b \int_{-z_A(b)}^{z_A(b)} dz_i C_{in, out}(r_i; p - q; E_0),$$

and we have introduced the dimensionless variables

$$x = q z_A = R_A \frac{M^2 - m^2_0}{2|p|}, \quad y = Q^*_R R_A = \frac{M^2_R - iM^*_R\Gamma_R - m^2}{2|p|} R_A.$$

Defining a function

$$K(x) = 3 \int_0^1 \zeta d\zeta e^{-ix\zeta} = \frac{3}{2} \left(1 + ix\right)e^{-ix} - 1 \frac{x^2}{x^2}.$$

we can express the coefficients $D_{in}$ and $D_{out}$ in the following form

$$D_{in}(x, y) = \frac{iAt_0(s, 0)}{2p} t(R \to X) \left[ \frac{K(x) - K(-x)}{x} + \frac{K(-x) - K(-x + 2y)}{y} \right],$$

$$D_{out}(x, y) = -\frac{iAt_0(s, 0)}{2p} t(R \to X) \left[ \frac{K(-x) - K(-x + 2y)}{y} \right].$$

In the discussion above, we did not take into account the contribution of intermediate states with higher mass than the resonance R because of the damping due to the nuclear form factor which takes place at intermediate energies. Before showing examples for resonance
production at intermediate energies, we first discuss the production at high energies. The expressions will turn out to look very similar, even though the space–time picture is entirely different. Assuming that only one intermediate state, the resonance $R$, contributes the expression for the amplitude will be identical to Eq. (43). As we will consider this case in Ch. 4, the reader may proceed directly to these results concerning the two component structure of the amplitude and skip the discussion of the space–time picture at high energies in the next chapter.

3. HIGH ENERGIES

The discussion of resonance propagation in nuclei of the previous chapter cannot be applied at high energies. The nonlocality of the amplitudes $t_R$ and $t_0$ must be taken into account. The interaction at high energies is dominated by intermediate multiple particle production and the amplitudes $t_R$ and $t_0$ are almost purely imaginary due to the dominance of multiparticle intermediate states. Under these circumstances the description of the high energy reaction in terms of reggeon exchanges with the target becomes most natural.

It was already discussed by Gribov [8] that a crucial feature of high energy interactions are the multiparticle fluctuations of the fast hadronic projectile, which are taken into account by reggeon exchange (see Fig.3). Only the low momentum part of the fluctuation will interact with the target. Two body processes, like elastic scattering or diffractive dissociation, only arise as the shadow of these multiparticle processes as required by the unitarity condition. The lifetime of the fluctuations – and thus also the nonlocality of the interaction – is proportional to the hadron energy. For the scattering of hadrons off nuclei it was shown e.g. in Refs.[9], [10] and [16] that there is therefore a critical energy of order $E_{\text{crit}} \sim m_a(\mu R_A)$, above which the length of the fluctuation becomes larger than the nuclear size; here $m_a$ is the projectile mass and $\mu$ a hadronic scale parameter of the order of a few hundred MeV. Two effects become important above this energy. Coherent production of higher mass states, $\sim (m_a + \mu)^2$, is not damped anymore by the nuclear form factor. Secondly, the nonlocality of the interaction of a fast hadron with a target nucleon becomes greater than the nuclear dimension, $R_A$. This latter effect can be understood as follows. In hadron–nucleus scattering at high energies the hadron typically enters into a multiparticle state long before it enters the nucleus and reappears out of such a fluctuation only far outside the nucleus. The space–time picture of resonance production as a localized production and successive rescatterings of the resonance $R$ in nuclear matter, which we used in the previous Chapter, is therefore not applicable anymore. It means that the sequential multiple scattering diagrams of Fig.2 – called planar diagrams because of the topological structure of the upper part – do not dominate anymore. Note that at high energies the wavy lines denoting the interactions with the nucleon stand for the exchange of one or more reggeons (i.e. multiparticle ladders). Intermediate multiparticle states correspond to cuts of these ladders.

While the intermediate coupling to multiparticle states through sequential processes in
finite nuclei is negligibly small at high energies, a more complicated type of coupling to the multiparticle states becomes dominant. This is the simultaneous appearance of fluctuations. An example, a non-planar reggeon diagram, is shown in Fig. 4. The direct calculation of the contributions from such non-planar diagrams would be extremely difficult. Gribov was able to include them in his discussion of high energy scattering and showed that surprisingly the structure of the total scattering amplitude turned out to be analogous to that obtained from the eikonal multiple scattering approach used in the previous chapter. We shall only outline briefly what Gribov’s arguments for hadron–nucleus interactions at high energies imply for resonance production on nuclei. The derivation is based on the analytical properties of the reggeon amplitudes as a function of the complex variables corresponding to the invariant masses of intermediate states, \( \tilde{s}_i = M_i^2 \). This analytic structure reflects the appearance of intermediate states and the nonlocality of the interaction.

For the coherent production of the resonance \( R \) by a high–energy projectile \( a \), we separate in the amplitude the part \( A_{aR} \), which contains all reggeon exchange dynamics, indicated schematically in Fig. 5:

\[
T(p, q^2, \tilde{s}) \sim \sum_{n=1}^{\infty} \int d^2 b \int dz_n \cdots dz_1 \rho_A(z_n, b) \cdots \rho_A(z_0, b) \\
\times \int dq_{n-1,z} \cdots dq_{0z} \exp\left\{ -i \sum_{i=0}^{n-1} q_{iz}(z_{i+1} - z_i) \right\} A_{aR}(p; q_{iz}).
\]

The integrations over the \( z_i \) are longitudinally ordered, and the integrations over the \( q_{iz} \) are carried out from \(-\infty\) to \(+\infty\) as required by the Feynman rules.

The function \( A_{aR}(p; q_{iz}) \), the amplitude for a diffractive \( a \to R \) transition through the interactions with \( n \) target nucleons, can also be considered as a function of the variables \( \tilde{s}_i \). This is possible since the invariant masses \( \tilde{s}_i \) of the \( (n-1) \) intermediate diffractively produced states can in the eikonal limit be expressed in terms of longitudinal momentum transfers, \( q_{iz} \), as was done in Chapter 2:

\[
\tilde{s}_i = (p - q_i)^2 \approx 2 p q_{iz} + m_a^2, \quad i = 0, \ldots, n - 1,
\]

where \( m_a \) is the mass of the incident projectile \( a \).

The important feature of the reggeon-exchange dynamics is a power-like decrease of amplitudes with increasing \( \tilde{s}_i \). It means that integrals over \( \tilde{s}_i \) are convergent, and it is possible to modify integration contours in the complex \( \tilde{s}_i \)-plane by adding to them integrals over large semi-circles which are negligible.

We first discuss the analytical structure of the integrand of the simplest planar diagram, Fig. 6a, and show that at high energies its contribution is negligibly small, as already argued above. On the positive real axis, we see in Fig. 6a a pole singularity, corresponding to a possible stable single-particle state. The branch point corresponds to multiparticle states, contained in the internal structure of the exchanged reggeons (related to the nonlocality of reggeon interactions). The pole due to the resonance \( R \) lies on the unphysical sheet. The presence of only right-hand-side but not left-hand-side singularities is a specific property of
planar diagrams. There are also singularities due to the nuclear form factor in the $s_i$-plane. These singularities occur at $|q_z| \sim R_A^{-1}$ as can be seen by assuming, for example, a form factor of the structure $F_A \sim (1 + q^2 R_A^2)^{-1}$. At high energies, as $|p|$ becomes asymptotically large, they thus correspond to far away singularities, see Eq.(45), and can be neglected. The nuclear form factor singularities are not shown in the figure.

The solid line $C$ in Fig. 5a indicates the desired integral along the real axis, which we now want to rewrite. Combining the contribution from the contour $C$ and the contribution from the upper semi-circle (dashed), we obtain zero, since no singularities are enclosed by the contour. Due to the behavior of the reggeon exchange amplitude for large $s_i$, the contribution from the semi-circle is negligible and thus the desired integral must be zero. It means that at high energies the different singularities of the planar diagram cancel each other when one takes into account all intermediate multiparticle states. At energies lower than the critical one, the nuclear form factor is crucial and prevents this cancellation by damping large-mass contributions. (The form factor induced singularities are situated at small $s_i$ values in this case).

The situation is different for the non-planar graphs shown in Fig. 4. There are now also singularities on the negative real axis, see Fig. 6b. In this case, we can modify the integration contour combining the contributions from $C$ and the closed contour in the lower halfplane. The parts along the negative axis cancel and we obtain a contribution from the contour enclosing the positive $s_i$-axis on the physical sheet. By combining the integrand above and below the cut, only the discontinuity of the amplitude is left. Due to the unitarity relations this corresponds to the contribution of on-shell intermediate states.

In order to deal with physical intermediate states in constructing the total amplitude, the crucial point in the treatment by Gribov consists now in combining the absorptive parts of planar and non-planar contributions, i.e. of diagrams with different topological structures. Thus the integral reduces to the contribution from the absorptive parts in the $s_i$ variables and the final result is thus expressed in terms of a sum or integration over real intermediate states, regardless of the reaction mechanism (planar or non-planar). These states are labeled with momentum or mass values according to Eq.(45). The answer is therefore obtained in the form of multiple scattering between physical states, mediated by reggeon exchange. In general, these amplitudes are nondiagonal, i.e. connect different states. The final formulas thus now include instead of amplitudes $t_0$ and $t_R$ the matrix amplitude $\hat{t}$ which describes all possible diffractive transitions between different states. Similarly, the effective potential becomes a matrix, e.g. $\hat{U}(q) = \hat{t} \cdot F_A(q)$ in its momentum representation. It is convenient also to introduce a diagonal matrix for the intermediate longitudinal momenta for different states $\hat{Q}$ where

$$\hat{Q}_{cd} = \frac{M_e^2 - m_a^2}{2|p|} \delta_{cd}.$$  \hspace{1cm} (46)

It allows one to express the free Green function as a matrix analogue of Eqs. (16) - (17):

$$\hat{G}_0(r) = e^{ipz} \hat{g}_0(z) \theta(z) \delta^2(b),$$  \hspace{1cm} (47)

$$\hat{g}_0(z) = (2|p|)^{-1} \exp(-iQz).$$  \hspace{1cm} (48)
The matrix amplitude describing scattering from the initial projectile to a final hadronic state with nucleus staying in the ground state has the form

\[ \hat{T}^{(n)}(p; b) = i^{n-1} \int \! dz_n \ldots dz_1 dz_0 \hat{U}(z_n, b) \hat{g}_0(z_n - z_{n-1}) \times \]
\[ \times \hat{U}(z_{n-1}, b) \ldots \hat{U}(z_1, b) \hat{g}_0(z_1 - z_0) \hat{U}_0(z_0, b) , \]  

(49)

where \( \hat{U}(z, b) = \hat{t} \cdot \rho(z, b) \). The index '0' on the potential \( U_0 \) denotes initial interaction of the projectile. This incident channel may involve a photon, which will needs not be taken into account as an intermediate state again.

It is convenient not to perform the last \( z \) integration but to define the operator functions \( \hat{F}^{(n)}(z; b) \) and \( \hat{F}(z; b) \),

\[ \hat{T}^{(n)}(b) = \int_{z_A}^{z_A} \! dz \hat{F}^{(n)}(z; b) , \]  

(50)

\[ \hat{F}(z; b) = \sum_n \hat{F}^{(n)}(z; b) . \]  

(51)

The function \( \hat{F}(z; b) \) satisfies the Gribov integral equation [9]

\[ \hat{F}(z; b) = \hat{U}_0(z, b) + i \int_{z_A}^{z_A} \! dz_1 \hat{U}(z, b) \hat{g}_0(z - z_1) \hat{F}(z_1; b) . \]  

(52)

It is surprising that the final formulas obtained by Gribov have the same multiple scattering form as in the simple intermediate energy approach, extended to the whole set of intermediate states which can be produced coherently. However, the simple space–time interpretation of the interaction with the nucleus is lost. Thus, it would be wrong to interpret the final amplitude as the propagation of the resonance or another hadronic state through the nucleus with successive rescatterings off the target nucleons. If the energy is higher than the critical one the correct picture corresponds to simultaneous interactions mediated by reggeon exchange. The range of the interaction taking the initial projectile to its final state is typically larger than the nuclear dimension.

The general solution of Eq. (52) of course involves a complicated matrix problem. However, in the approximation of constant nuclear density \( \rho_0 \) the solution can be found using the method of Laplace transformations. Introducing Laplace transforms for functions \( F(z) \) and \( g_0(z) \),

\[ \hat{F}(\xi) = \int_{z_A}^{\infty} \! e^{-\xi z} \hat{F}(z) , \]  

(53)

\[ \hat{G}_0(\xi) = \int_0^{\infty} \! dz e^{-\xi z} \hat{g}_0(z) = (\xi + iQ)^{-1} , \]  

(54)

we get in the case of semi-finite matter with constant density \( \rho_0 \) a simple algebraic matrix equation instead of integral equation (52) (we suppress for the moment the dependence on \( b \)):

\[ \hat{F}(\xi) = \hat{U}_0/\xi + i\hat{U} \hat{G}_0 \hat{F}(\xi) , \]  

(55)
where $\hat{U} = i\rho_0$.

It has the following solution

$$\hat{F}(\xi) = \frac{\exp \xi z_A}{\xi} \left[ 1 - i\hat{U}(\xi + i\hat{Q})^{-1} \right]^{-1} \hat{U}_0$$

$$= \frac{\exp \xi z_A}{\xi} G(\xi) \hat{U}_0 ,$$

(56)

where

$$\hat{G}(\xi) = \left[ \xi + i\hat{Q}^* \right]^{-1} ,$$

(57)

with

$$\hat{Q}^* = \hat{Q} - \hat{U} .$$

(58)

The function $\hat{F}(z)$ can be expressed through the inverse Laplace transformation

$$\hat{F}(z) = \frac{1}{2\pi i} \int_{C} d\xi e^{\xi z} \hat{F}(\xi) ,$$

(59)

where the integration is performed along a contour which is parallel to imaginary axis at a distance such that all singularities of the integrand are on the left-hand side of it.

The expression for $\hat{F}(\xi)$, Eq. (56), was derived in the approximation of semi-infinite matter, $(-z_A, \infty)$. To get a result valid for nuclear matter of finite size, $(-z_A, z_A)$, Eq. (33),

$$\rho_A(z; b) \propto \theta(z_A - |z|) ,$$

one has to multiply the Laplace inversion of Eq. (56), $F(z)$, by $\theta(z_A - z)$

$$\langle F(z) \rangle = F(z)\theta(z_A - z) ,$$

(60)

where we have denoted the quantities corresponding to the finite-matter case with the brackets $\langle \rangle$:

The general approach of Gribov is easily applied to the special case of resonance production on a nucleus. If one of the intermediate states is a resonance, one simply has to replace $M^2_c$ in the Green function, Eq. (17), by $M^2_R - iM_R\Gamma_R$. To get the amplitude of resonance decay, the matrix element $\{\hat{F}(z, b)\}_{Ra}$ should be convoluted with the free Green function $\{\hat{G}_0(r_f - r)\}_{RR}$ and multiplied with the vertex function $t(R \rightarrow X)$ introduced in the previous chapter that describes the final decay of the resonance:

$$T(p, q, \bar{s}) = \int d^2b e^{-i\mathbf{q}\cdot\mathbf{b}} T(\bar{s}; b) ,$$

$$T(\bar{s}; b) = t_R(R \rightarrow X) \int_{-z_A}^{z_A} dz \int_{z}^{\infty} dz_f \{\hat{g}_0(z_f - z)\}_{RR} \{\hat{F}(z, b)\}_{Ra} e^{i\mathbf{q}\cdot\mathbf{z}_f} ,$$

(61)

(62)

where $q_z = (\bar{s} - m^2_a)/2|\mathbf{p}|$.
Note that the semi-infinite integral over $z_f$ can be seen as a complex Laplace transformation to a variable $\xi$, related to momentum $q_z$, or invariant mass $\tilde{s}$,

$$\xi = (\tilde{s} - m_a^2)/(2i|p|)$$

(compare Eq. (62) and (53)). Therefore the Laplace-transform method is very convenient for the calculation of the resonance mass distribution. The expression (62) has a form of a convolution of two functions, which corresponds to a product of two Laplace transforms in variable $\xi$:

$$T(\tilde{s}; b) = t_R(R \to X)(F(\xi; b))_{aR} \{G_0(\xi)\}_{RR}.$$  

To get the Laplace transform for the truncated function $\langle F(z) \rangle$, one should convolute Eq. (66) with the Laplace transform $\theta_A(\xi)$ of the step function $\theta(z_A - |z|)$:

$$\langle \hat{F}(\xi) \rangle = \int d\xi' \hat{F}(\xi') \theta_A(\xi - \xi'),$$

$$\theta_A(\xi) = \frac{\exp(z_A\xi) - \exp(-z_A\xi)}{\xi}.$$  

For (semi-)infinite nuclear matter the amplitude can be represented as a sum of poles,

$$\hat{F}(\xi) = \sum_i \frac{a_i}{\xi - \xi_i},$$

and the finite-matter amplitude then has the form

$$\langle \hat{F}(\xi) \rangle = \sum_i a_i \theta_A(\xi - \xi_i).$$

The general formalism in the multichannel case can be written in a compact fashion using functions of a matrix argument. An essential point is the ordering of different matrix factors. From the pole structure of the function $\hat{F}(\xi)$ one gets by means of Eqs. (64), (65), and (56)

$$T(\tilde{s}; b) = t_R(R \to X)(\xi + i\hat{Q})^{-1} \left\langle \frac{\exp(z_A\xi)}{\xi}(\xi + i\hat{Q})(\xi + i\hat{Q}^*)^{-1} \right\rangle \hat{U}_0$$

$$= \left[ \hat{Q}(\hat{Q}^*)^{-1} \frac{\exp(z_A\xi)}{\xi} + (\hat{Q}^* - \hat{Q})(\xi + i\hat{Q}^*)^{-1}(\hat{Q}^*)^{-1} \{\exp(z_A\xi) - \exp[-z_A(\xi + 2i\hat{Q}^*)]\} \right].$$

The important feature of hadron–nucleus interactions at high–energies, the possibility for the initial hadron to convert in the course of rescatterings into different hadronic states, is determined by the matrix $\hat{U}$ contained in the propagator $\hat{G}$. Therefore it is necessary to know the structure of the effective potential, $\hat{U} = \hat{t}\rho$, and thus of the elementary diffractive matrix amplitude $\hat{t}$ through diffractive dissociation processes. At present, this information is essentially limited to reactions involving at least one stable hadronic state. It is known that
for stable particles, such as protons or pions, the nondiagonal elements are much smaller than the diagonal ones [18]. There are some arguments for the magnitude of diagonal amplitudes for unstable hadrons: for \( \rho \) mesons they are of the same order as for pions, and for \( \Delta \) isobars comparable to that for nucleons. Resonance production on nuclei can provide further information on the structure of the hadron–nucleon diffraction matrix.

Some conclusions about the structure of the nuclear diffraction amplitude, \( T \), can be obtained at high energies when the free part of the Green function, \( (s - M_R^2 + iM_R\Gamma_R)/2|p| \), may be neglected compared to its density dependent part, \( \hat{U} \):

\[
\hat{G}(\xi) = [\xi + i\hat{Q}^*]^{-1} \approx \hat{U}^{-1}.
\] (70)

As a result, the amplitude is proportional to

\[
\hat{T}(\hat{s},b) \propto (\xi + i\hat{Q})\hat{U}^{-1}\hat{U}_0.
\] (71)

If the projectile particle is a hadron, i.e. \( \hat{U}_0 = \hat{U} \), the resulting \( \hat{T} \) matrix is diagonal. This takes place even if there are non-diagonal hadron–nucleon matrix elements which are not small. Note that while the Green function in nuclear matter is not diagonal (as is \( \hat{U}^{-1} \)), the \( \hat{T} \) matrix becomes diagonal due to a cancellation between the resonance production amplitude and the terms induced by subsequent rescatterings.

To be more precise, not the matrix elements, but the eigenvalues of the matrix \( U \) should be large compared to other terms in the Green function denominator to lead to a diagonal total amplitude. If one of eigenvalues is small for some reason, the situation can be quite different. The interaction with the nucleus looks much simpler in terms of eigenstates of the matrix amplitude of hadronic scattering. The initial hadron can be represented as a linear combination of diagonal states propagating through the nucleus. These diagonal states have, in principle, different probabilities to be absorbed in nuclear matter. As a result, the nucleus works as a filter that only lets hadronic states with the smallest cross section pass.

In the case of photoproduction, the incident photon can be represented as a superposition of different hadronic states. In the vector dominance model the main contribution comes from low-lying vector mesons (\( \rho, \omega, \phi \)). If one of these resonances is detected in the final state, the non-diagonal transitions in intermediate states are not important and the amplitude can be represented in the form of Eq.(39). For photoproduction of higher resonances, such as the \( \rho' \), several mechanisms can contribute: \( <\gamma|\rho><\rho|T|\rho'> \) and \( <\gamma|\rho'><\rho'|T|\rho'> \). They are related to the admixtures of \( \rho \) and \( \rho' \) in the photon wave function, respectively. Although the admixture of \( \rho' \) is small, the second mechanism is diagonal in nuclear transition and therefore increases faster with \( A \) as compared to the first one. Thus, the coherent photoproduction of higher vector mesons on a nuclear target can provide useful information on the hadronic structure of the photon wave function.
Single channel case

We now consider the general amplitude for the case when only a single state, the resonance \( R \), of all intermediate states is essential. In this case Eq. (69) simplifies since it involves only scalar quantities \( Q_R \) and \( Q_R^* \) instead of matrices \( \hat{Q} \) and \( \hat{Q}^* \).

\[
T(\tilde{s}; b) = t_R(R \to X) \frac{t_0(s, 0) \rho_0}{[1 - it_R \rho_0 (\xi + iQ_R)^{-1}]} \frac{\exp(z_A \xi)}{\xi}. \tag{72}
\]

After integration over the impact parameter \( b \) we get for forward resonance production, i.e. when \( q^2 = -q_{z, min}^2 \),

\[
T(p, -q_{z, min}^2, \tilde{s}) = \int d^2 b T(\tilde{s}; b)
\]

\[
= \frac{4}{3} \pi R_A^3 \rho_0 t_0(s, 0) \frac{i}{x} \left[ \frac{K(x) - K(-x + 2y)}{\xi + iQ_R} + \frac{Q_R K(-x) - K(-x + 2y)}{Q_R} \right]. \tag{73}
\]

The function \( K(x) \) and the kinematical variables \( x \) and \( y \) are given in Eqs. (42) and (41), respectively. This expression can be transformed to yield the same amplitude as in Ch. 2, Eq. (39), which was obtained for lower energies. It illustrates that the high energy formulas are a smooth continuation of the intermediate energy expressions, even though the underlying space-time picture is completely different: sequential 'planar' contributions dominate at lower energies and 'non-planar' mechanisms at high energies.

4. EXAMPLES

As was shown in the preceding Chapter, we can simply use the single channel expression with its two component structure over a wider range of energies as long as it is reasonable to neglect the coupling to other channels. We now show some implications of the presence of two resonance components for the invariant mass distribution of the decay products. This is the natural aspect to study if one is interested in the behavior of the resonance in the nuclear medium. We consider a situation with a high momentum \(|p|\) where the production is not damped by the nuclear formfactor, since \( q_{min} \approx (\tilde{s} - m_a^2)/2|p|^{-1} \). This implies that the broadening of the resonance is large compared to its free width. Then in the mass distribution the narrower peak from the decay in the vacuum must compete with the broad in-medium contribution. The interference between these two terms with a Breit–Wigner structure will be important for the mass distribution. In contrast to the usual non-resonant background, the form and relative phase of the in medium 'background' here are determined by the resonance interaction with the medium. (Certainly there will also be a background of non-resonant origin in the actual measurement; we have neglected it here).

We present as an illustration the application of the two–component formula, Eq. (39), to coherent photoproduction of a \( \rho \) meson. In Fig. 7 we show the mass spectra of the \( e^+e^-\).
pair from $\rho$ decay produced at $\theta = 0^0$ in the reaction

$$\gamma A \rightarrow \rho A \rightarrow (e^+ e^-) A$$

(74)

at laboratory momenta $p_{lab}$ of 2 and 5 GeV and for finite nuclei with $A = 50$ and $A = 200$. The cross section $d\sigma/d\tilde{s}$ was calculated using the amplitude from Eq. (39) for a constant nuclear density, Eq. (33), with $\rho_0 = (4/3\pi R_A^3)^{-1}$, $R_A = 1.12 A^{1/3}$ fm. The forward $\rho N$ scattering amplitude was assumed to be purely imaginary and $\sigma_{\rho N}^{tot}$ was taken to be 20 mb. The solid and dotted curves show the contributions of the free and in medium components, respectively. The bold curves represent the total contribution, which includes the interference of the components. The narrow component has a Breit–Wigner structure, which is distorted due to the dependence of the nuclear form factor, $F_A(q_{min})$, on $\tilde{s}$ through $q_{min}$. To partially remove this effect, we have scaled the cross section by a factor $\tilde{s}^2$. The distortion due to the presence of two components in the amplitude is especially visible for the lower energy, $p = 2$ GeV and the heavier nucleus: The in-medium contribution separately can be seen as a broad resonance peak, where the form factor distortion is much more evident. The interference between broad and narrow components is dramatic at 2 GeV and completely changes the form of the mass distribution. It is largely constructive at $\sqrt{\tilde{s}} < M_R$ and destructive at $\sqrt{\tilde{s}} > M_R$. Due to the strong interference the form of the free $\rho$-peak becomes completely distorted and asymmetric at $A = 50$ and even develops two minima near 0.7 and 0.9 GeV for $A = 200$. The more complicated picture for heavy nuclei is again related to the presence of the nuclear form factor, $F_A(q_{min})$, which is more rapidly varying for heavier nuclei. We have tested that the general interference features are not an artifact of the sharp edge of the nuclear density distribution assumed in Eqs. (19) and (23) by repeating the calculations with a smoothly varying density.

The width of the broad component increases linearly with $p$. At 5 GeV it appears only as a small and broad background. The interference of the broad and narrow component changes the form of the $\rho$-peak again, but clearly less than at the lower energy.

We see that at both energies the broad component becomes more important as the nuclear mass number, $A$, increases. The different mass dependence can be made explicit in the following way. We consider a situation when the momentum, $|p|$, is high and the broadening of the resonance is much larger than its free width, $\Gamma^*_R \approx \rho_0 \sigma_R / 2M_R \gg \Gamma_R$. The in-medium contribution then only represents a broad background to the pronounced narrow free peak. In the vicinity of free peak we have

$$x \approx M^2 R_A / |p| \ll 1,$$

$$y \approx -i R_A \rho_0 \sigma_R / 2, \quad |y| \gg 1.$$  

(75)

Expanding $K(x) \approx 3/4 - ix/2$ and $K(x + 2y) \ll K(x)$, one obtains for the two terms in Eq. (73)

$$\langle T_{aR}(\xi) \rangle \approx f_0 \left[ \left( A - \frac{2\pi R_A^2}{\sigma_R} \right) \frac{1}{\xi + i Q^*_R} + \frac{2\pi R_A^2}{\sigma_R} \frac{1}{\xi + i Q_R} \right].$$

(76)

Since $R_A^2 \sim A^{2/3}$, this shows that the medium modified part grows faster with $A$ than the free resonance contribution.
In the above consideration it is easy to estimate the effect of correlations between nucleons [9]. The most important are two–body correlations which can be be incorporated by simply multiplying the Green function \( g_0(z_i - z_{i-1}) \) by a correlation function. This function \( \chi(z_i - z_{i-1}) \) is defined to be unity if correlations are absent and we assume for an estimate a simple behavior corresponding to a hard core with radius \( r_{corr} \),

\[
\chi(z) = \theta(|z| - r_{corr}).
\]  

(77)

Its Laplace transform is

\[
\chi(\xi) = \frac{\exp(-\xi a)}{\xi}.
\]  

(78)

This yields a resonance denominator in (72) of the form

\[
\xi + iQ_R - itR\rho_0 \exp -i(\xi + iQ_R)r_{corr} \approx (1 - \epsilon)(\xi + iQ_R + \frac{1}{2}\rho_0\sigma_R(1 + \epsilon)),
\]  

(79)

where \( \epsilon = \frac{1}{2}\rho_0\sigma_R r_{corr} \). Thus, the inclusion of correlations results in a modification of the resonance width of second-order in the density, \( Q^*_R \rightarrow Q^*_R + \frac{1}{2}\rho_0\sigma_R \epsilon \). For typical values of the parameters entering into \( \epsilon \), the correction to the induced width is not very large: For \( \sigma_R \approx 20 \text{ mb} \) and \( r_{corr} \approx 0.2 \text{ fm} \) one obtains \( \epsilon \approx 0.2 \).

In summary, the amplitude for the production of hadronic resonances on nuclear targets contains two types of components. They correspond to the propagation and decay of the resonance outside and inside nucleus. The narrow component is characterized by the free values of resonance mass and width. The broad 'in-medium' component does not have a true pole in the invariant mass; it has an approximate Breit–Wigner form with parameters depending on the nuclear density and the resonance–nucleon cross sections. Both components have a different \( A \) and energy dependence. In the limit of large \( A \), the ratio \( D_{in}/D_{out} \) of the broad and narrow components is proportional to the nuclear size \( A^{1/3} \). As the width of the broad component increases linearly with energy, the inside/outside ratio at a fixed invariant mass decreases as \( 1/p \). Thus, the most interesting region for the investigation of 'in-medium' effects may be the region of intermediate energies, where there is only moderate damping by the nuclear form factor and the inside to outside ratio is not suppressed. As this ratio is proportional to the fraction of time the resonance spends inside the nucleus, it will be difficult to establish nuclear effects for the narrow, long lived resonances like \( J/\psi \). This makes it hard to extract the 'in-medium' parameters from such experiments in a simple fashion.

5. Summary and Conclusions

Experiments on nuclei are an important way to study the space-time picture of elementary interactions. Through the interaction with the nuclear medium one has a way of measuring the development of the reaction in units of typical nuclear length and time scales. Many recent experiments concern the behavior of unstable elementary particles - resonances - in
nuclei. Ideally one would like to extract as directly as possible the 'in medium' properties of the resonance, e.g. the medium modified resonance parameters of mass and width. They can then be compared to models - be it on the level of quarks or hadrons - for the internal structure of the resonance and its interaction with the target nucleons. In this paper we have looked specifically at the production of resonances. Examples are the production of $\rho$ or $J/\psi$ on nuclei. For simplicity, we left out final state interactions of the decay products in our more qualitative discussion; this is appropriate for the decay of the above resonances into e.g. an $e^+e^-$ pair.

Clearly the relevant kinematical variable for such a study is the invariant mass of the resonance, $\tilde{s}$, which enters into the resonance amplitude and governs the resonant shape of the cross section. However, in general the connection between the incident energy, the variable at our disposal in an experiment, and the invariant mass $\tilde{s}$ of the resonance in the nucleus is not direct in a high energy collision: the nucleus will break up and different fragments will carry of part of the energy, with only a variable fraction left for the resonant state we detect in the end. However, in a coherent process, where the nucleus returns to its ground state, the nuclear formfactor restricts the momentum transfer to the target and energy and invariant mass are closely related. That is why we discussed coherent production processes only.

In a production process, the resonance can decay inside and outside of the nucleus. The relative contribution of these two possibilities depends e.g. on the momentum of the resonance, the size of the nucleus and the resonance lifetime. Both possibilities show up in the total amplitude: the decay outside through a component with the free resonance parameters and an additional medium-modified component for the decay inside the nucleus. The medium modifications can be described - in the simplest picture - as a shift and broadening of the resonance due to interactions with the nuclear medium. As we saw in some illustrative examples, the background contribution drops off fast with energy and we mainly see the free resonance at high energies. This is intuitively clear since the faster the resonance, the smaller its chance for an interaction with the nucleus. A similar qualitative statement also holds for the dependence on nuclear size: the importance of the medium-modified resonance is greater the larger the nucleus. Thus in order to get at medium effects, it is best to stay at intermediate energies and chose a heavy nuclear target.

It is legitimate to talk about the medium-modified component of the resonance in the production amplitude. However, it must be stressed that for a finite nucleus the amplitude doesn’t develop a pole corresponding to an 'in-medium' state. While there is a resonance denominator of the type $\tilde{s} - M_R^2 + iM_R\Gamma_R$, where $M_R$ and $\Gamma_R$ are the medium modified resonance parameters, the amplitude has no pole because, as we showed, the residue of the amplitude at $\tilde{s} = M_R^2 + iM_R\Gamma_R$ is zero. There is only a pole in $\tilde{s}$ due to the free resonance, i.e. a $(\tilde{s} - M_R^2 + iM_R\Gamma_R)^{-1}$ contribution with a non-vanishing residue. The reason behind this is clear: we detect the resonance through its decay products far from the target. Singularities in the $S$-matrix can thus not be due to the interaction in a limited space–time region, as in general the $S$ - matrix has only poles corresponding to asymptotic states. A simple illustration of this statement was shown by considering the production of the resonance in an infinite nuclear medium, $R \to \infty$. In that case the in-medium resonance never gets out
and a true pole does develop. The above general conclusions were first derived in the standard multiple scattering formalism, the eikonal description, and our results can intuitively be understood in this picture. The statements were then shown to apply at intermediate as well as high energies. However, while the final formulas don’t change, we must at high energies radically change our space–time picture of the reaction, analogous to the findings by Gribov for the scattering of a stable projectile from a nucleus. Multi-component intermediate hadronic states, instead of a single hadron, propagate. These components interact simultaneously with the target, instead of sequentially as is the case at lower energies. In the diagrammatic language, the sequential multiple scattering can be represented by planar diagrams. The new element entering into the description at high energies, above the critical energy, are non-planar diagrams with an entirely different singularity structure. Rather than dealing with this new contribution separately, it was discussed how the consideration of planar and non-planar diagrams together, grouped in such a way that the total contributions of real intermediate states with a definite mass enter, leads to a matrix expression with a ‘multiple scattering’ structure that is very similar to the amplitude at \( E < E_{\text{crit}} \). This result is quite surprising given the entirely different underlying space–time picture.

We have seen that in coherent production on a finite nucleus the medium-modified resonance amplitude is necessarily accompanied by a free resonance contribution, which coherently interferes with it in the expression for the cross section. In a production process, we thus cannot directly measure an in-medium resonance shape. The medium-modified component plays a significant role mainly at relatively low energies and for heavy targets, where due to its interference with the free amplitude it can lead to a significant change in the overall \( \tilde{s} \) dependence of the cross section. A qualitative estimate showed that nuclear correlations will only have a moderate influence on the distribution of the resonance decay products. At high energies and on lighter targets, the in-medium contribution quickly becomes a small, broad background that is difficult to identify uniquely. What we see at high energies is mainly the peak due to the free resonance, determined by the vacuum parameters. At these energies, we also must be careful in our interpretation of the process, as a totally different space–time picture applies than at lower energies.

Acknowledgement. The authors are indebted to A.B. Kaidalov for useful discussions and friendly criticism. L.K. thanks T.E.O. Ericson for stimulating discussions with. K.B. and L.K. acknowledge support from grant J74100 of the International Science Foundation, the Russian government and the Russian Fund of Basic Research. M.I.K. was also supported by a grant from the Alexander von Humboldt-Stiftung and by RFBR Grant 94-03068. The work of J.K. is part of the research program of the Foundation for Fundamental Research of Matter (FOM) and the National Organisation for Scientific Research (NWO). The collaboration was made possible by a grant from NWO as well as by INTAS grants 93-0023 and 93-0079.

References


Fig. 1: Lowest order diagram for nuclear resonance production

Fig. 2: 'Planar' diagram for resonance rescattering
Fig. 3: Reggeon exchange diagram as a multiparticle fluctuation: (a) projectile fluctuation and interaction of low momentum component with target; (b) corresponding elastic amplitude represented as reggeon exchange.

Fig. 4: Example of a non-planar diagram.
Fig. 5: (a) General structure of multi-reggeon amplitude; (b) decomposition into planar and non-planar contributions.

Fig. 6. Singularity structure in \( \tilde{s} \)-plane. Formfactor singularities outside of large semi-circles not shown.
Fig. 7: Square of scaled amplitude, $|\tilde{s}T(p, q_{z\text{min}}, \tilde{s})|^2$, as a function of $\sqrt{s}$ (in arbitrary units)