Maximal Entanglement in High Energy Physics

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We analyze how maximal entanglement is generated at the fundamental level in QED by studying correlations between helicity states in tree-level scattering processes at high energy. We demonstrate that two mechanisms for the generation of maximal entanglement are at work: i) s-channel processes where the virtual photon carries equal overlaps of the helicities of the final state particles, and ii) the indistinguishable superposition between t- and u-channels. We then study whether requiring maximal entanglement constrains the coupling structure of QED and the weak interactions. In the case of photon-electron interactions unconstrained by gauge symmetry, we show how this requirement allows reproducing QED. For Z-mediated weak scattering, the maximal entanglement principle leads to non-trivial predictions for the value of the weak mixing angle θW. Our results illustrate the deep connections between maximal entanglement and the fundamental symmetries of high-energy physics.

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Introduction. Entanglement [1] is the key property that pervades many developments in quantum physics. As a paramount example, entanglement is necessary to discriminate classical from quantum physics using Bell inequalities [2]. Entanglement can also be understood as the resource that enables genuine quantum protocols such as cryptography based on Bell inequalities [3] and teleportation [4]. Large entanglement is expected to be present in quantum registers when a quantum algorithm produces a relevant advantage in performance over a classical computer such as Shor’s algorithm [5]. Entanglement also plays a crucial role in condensed matter, where quantum phase transitions in spin chains are characterized by an enhanced logarithmic scaling of entanglement entropy [6], highlighting the relation between entanglement and conformal symmetry.

It is clear that entanglement is at the core of understanding and exploiting quantum physics. It is therefore natural to analyze the generation of entanglement at its most fundamental origin, namely the theories of fundamental interactions in particle physics. If the quantum theory of electromagnetism, QED, would never generate entanglement among electrons, Nature would never display a violation of a Bell inequality. This implies that entanglement must be generated by quantum unitary evolution at the more fundamental level.

A deeper question emerges in the context of high-energy physics. Is maximal entanglement (MaxEnt) possible at all? In other words, are the laws of Nature such that MaxEnt can always be realized? One can imagine a QED-like theory where entanglement could be generated, but in a way which would be insufficient to violate Bell inequalities. Then, it would be formally possible to think of the existence of an underlying theory of hidden variables. On the other hand, if MaxEnt is realized in QED, it is then possible to design experiments that will discard classical physics right at the level of scattering of elementary particles. Taking a step further, one can ask what are the consequences of imposing that the laws of Nature must be able to realize maximally entangled states. Can this requirement be promoted to a principle, and to which extent is it consistent with fundamental symmetries such as gauge invariance?

Here we will show first than in QED only two mechanisms can generate MaxEnt in high-energy scattering of fermions prepared in an initial helicity product state. These are i) s-channel processes where the virtual photon carries equal overlaps of the helicities of the final state particles, and ii) processes which display interference between t and u channels. We will then illustrate the deep connection between maximal entanglement and the structure of the electron-photon interaction vertex in QED. We shall finally analyze the consequences of imposing MaxEnt on the weak interactions.

Some previous works have studied the role of entanglement in particle physics. In Ref. [7] it was shown that orthopositronium can decay into 3-photon states that can be used to perform Bell-like experiments that discard classical physics faster than the standard 2-particle Bell inequality. Bell inequalities have also been discussed in kaon physics [8] and its relation with the characterization of T-symmetry violation [9] as well as in neutrino oscillations [10]. How entanglement varies in an elastic scattering process has been studied using the S-matrix formalism in [11]. Note also recent work on entanglement in Deep Inelastic Scattering [12]. Also, a discussion of quantum correlations in the CMB radiation has been brought to the domain of Bell inequalities [13].
Quantifying entanglement. We shall study here scattering processes involving fermions and photons as incoming and outgoing particles. In the case of fermions (photons) we shall analyze the entanglement of their helicities (polarizations). In both cases, the associated Hilbert space is two-dimensional, and we use $|0\rangle$ and $|1\rangle$ as basis vectors. The quantum state of an incoming or outgoing particle can then be written as

$$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle,$$

where $|\alpha|^2 + |\beta|^2 = 1$ due to normalization. We note that in high-energy scattering a generic outgoing state will involve all possible outcomes of the process being analyzed. The reduction to a two-level system therefore corresponds to a post-selection of results. This is the correct description that delivers the probabilities which we could insert into a Bell inequality, once the final state has been identified.

To quantify entanglement we could use the Von Neumann entropy, but this is not necessary since in our case all possible entanglement measures are related to

$$\Delta = 2|\alpha\delta - \beta\gamma|,$$

known as the concurrence [14]. By construction, $0 \leq \Delta \leq 1$, where the extreme cases $\Delta = 0$ $(1)$ correspond to a product (maximally entangled) state.

Here we shall study scattering processes where the incoming particles are in a product state of their helicities, that is, the incoming particles are not entangled ($\Delta = 0$). We will often work in the high energy limit where helicities and chirality are equivalent, and we will use the basis $|0\rangle_{in} = |R\rangle$ and $|1\rangle_{in} = |L\rangle$, where $R$ and $L$ correspond to right- and left-handed helicities. In general, the outgoing state will be a superposition of all possible helicity combinations, and thus the scattering amplitude of e.g. $RL$ initial helicities, $M_{RL}$, will include each possible combination of outgoing helicities. We will then parametrize scattering amplitudes as

$$M_{RL} = \alpha_{RL}|RR\rangle + \beta_{RL}|RL\rangle + \gamma_{RL}|LR\rangle + \delta_{RL}|LL\rangle,$$

where here $RL$ stand for the incoming helicities, and the kets indicates the values of the outgoing ones.

MaxEnt generation in QED. Let us start our discussion with the analysis of how entanglement is generated in electron-positron annihilation into a muon-antimuon pair, $e^-e^+ \rightarrow \mu^-\mu^+$, described at tree level in QED by a single $s$-channel diagram. In order to analyze entanglement, it is necessary to retain all the helicities in the calculation. As in the rest of this paper, we will work on the center-of-mass frame.

It is convenient to first focus on the current generated at the interaction vertices. If the incoming particles propagate along the $z$-direction, the incoming current associated to two incoming particles in a $RL$ helicity product state will be $\bar{\psi}_l\gamma^\mu u^\perp = 2p_0(0, 1, i, 0)$, where $p_0$ is the electron’s energy. The outgoing particles will be described by a current as a function of $\theta$, the scattering angle. At high energies, we find that the leading contribution only appears for incoming $RL$ (and $LR$) helicities,

$$M_{RL} \sim (1 + \cos \theta)|RL\rangle + (-1 + \cos \theta)|LR\rangle$$

up to a prefactor which is not relevant here. Thus for a scattering angle $\theta = \pi/2$ the final state becomes maximally entangled and proportional to $|RL\rangle - |LR\rangle$, with $\Delta_{RL} = 1$. This result illustrates how MaxEnt can be generated in a high-energy scattering process. While scattering amplitudes in general carry a non-trivial angular dependence, it is always possible to perform the measurement in the specific direction where MaxEnt is obtained, not unlike the way maximally entangled states are obtained in quantum optics by parametric down conversion. Let us also note that the dominant terms in the $e^-e^+ \rightarrow \mu^-\mu^+$ scattering at high energies are easily described by chirality conservation. This is not the case at lower energies, where the emergence of entanglement is more elaborated.

For incoming particles in the $RR$ helicity product state, all terms in the amplitude are suppressed by a power of $p_0$ as compared to the $RL$ case. Nevertheless MaxEnt is found for every angle $\theta$ and incoming momenta $p_0$. An experiment that prepares $RR$ incoming states will therefore always result in MaxEnt.

It is instructive to revisit the computation of the $RL$ case focusing on the currents associated to the virtual photon. The incoming current (in the $z$-direction) corresponds to $j^\mu_{in}(RL) = 2p_0(0, 1, i, 0)$, and at high energies the non-vanishing outgoing currents at $\theta = \pi/2$ read $j^\mu_{out}(RL) = 2p_0(0, 0, -i, -1)$ and $j^\mu_{out}(LR) = 2p_0(0, 0, -i, 1)$. Thus the third component of $j_{in}$ carries equal overlap (with different sign) of the two possible helicity combinations for the outgoing state. In a sense, the photon cannot distinguish between those two options. This is the basic element that leads to MaxEnt generation in $s$-channel processes.

Entanglement can also be generated in QED through a completely different mechanism. Let us consider Møller (electron-electron elastic) scattering, which receives contributions only from $t$- and $u$-channel diagrams. For this process, the computation of the amplitude shows that no entanglement is generated at high energies within each $t$ or $u$ channel separately, and that the only entangled state is produced by their superposition, resulting in $M_{RL} \sim (t/u)|LR\rangle - (u/t)|RL\rangle$, leading to a concurrence

$$\Delta_{RL} \sim \frac{2}{a^2 + t^2} \xrightarrow{t \rightarrow \infty} 1.$$ 

Therefore MaxEnt ($\Delta_{RL} = 1$) is realized when $t = u$, which corresponds again to the scattering angle being $\theta = \pi/2$. The indistinguishability of $u$ and $t$ histories is now at the heart of entanglement. This also implies that
entanglement will not be generated in processes such as \( e^- \mu^- \rightarrow e^- \mu^- \) where the same \( u/t \) interference cannot take place. Including electron mass \( m_e \) effects, the concurrence \( \Delta_{RL} \) reads

\[
\Delta_{RL} = \left| \frac{2tu}{2m_e^2(t-u)^2(2m_e^2-2(t+u)+\frac{tu}{\pi}u^2)+(t^4+u^4)} \right|
\]  

(6)

which shows the more powerful result that, for all energies, the scattering angle \( \theta = \pi/2 \) (when \( t = u \)) leads to MaxEnt, \( \Delta_{RL} |_{\theta=\pi/2} = 1 \) for all \( p_0 \).

In the case of incoming particles in an \( RR \) product state, no entanglement is generated in the high-energy limit, since the amplitude is dominated by the final state which also lives in the \( RR \) sector, as required by helicity conservation. At very low energies on the other hand, the calculation of the concurrence gives

\[
\Delta_{RR} \propto \left| \mu_3 \right| |_{\theta=\pi/2} = 1 + O \left| \frac{p_0^2}{m_e^2} \right|.
\]  

(7)

The combination of Eqs. (5) and (7) illustrates the remarkable fact that two electrons will get always entangled at low energies, irrespective of their initial helicities.

The way in which MaxEnt is generated in QED scattering processes can be studied more thoroughly. It is indeed possible to show that MaxEnt also arises in Bhabha and Møller scattering and in pair annihilation of electron-positron to two photons. It is highly non-trivial that a single coupling, the QED vertex, can take care of generating entanglement in all these processes, and at the same time guarantee that if entanglement is present in the initial state, it will be preserved by the interaction.

MaxEnt as a constraining principle. It is tantalizing to turn the discussion upside down and attempt to promote MaxEnt to a fundamental principle that constraints particle interactions. Such principle would guarantee the intrinsically quantum character of the laws of Nature, allowing Bell-type experiments to be carried out violating the bounds set by classical physics. In this formulation, MaxEnt emerges as a purely information-theoretical principle that can be applied to a variety of problems.

We shall further explore this idea in the context of an unconstrained version of QED (uQED). This is a theory of fermions and photons, obeying the Dirac and Maxwell equations respectively, which can interact via a generic vertex that allows violations of rotation and gauge invariance. For simplicity we shall still impose the C, P and T discrete symmetries. While of course this theory is not realized in Nature, our goals are to determine to which extent imposing MaxEnt constrains this interaction vertex and to verify that QED can be reproduced.

To be specific, we shall replace the QED vertex \( e\gamma^\mu \) with an general object \( eG^\mu \) that can be expanded in a basis of 16 \( 4 \times 4 \) matrices. This unconstrained interaction vertex can be parametrized as \( G^\mu = a_{\mu \nu} \gamma^\nu \), where \( a_{\mu \nu} \) are real numbers and \( a_{ij} = a_{ij} = 0 \) for \( i \neq j \). The QED vertex is recovered for \( a_{00} = a_{11} = a_{22} = a_{33} = 1 \) and \( a_{ij} = 0 \) for \( i \neq j \). The computation of the amplitude \( \mathcal{M}_{RL \rightarrow RL/LR} \) for \( e^- e^+ \rightarrow \mu^- \mu^+ \) scattering in uQED at high energies gives

\[
\mathcal{M}_{RL} \sim (a_{j1} + ia_{j2})(a_{j1} \cos \theta + ia_{j2} - a_{j3} \sin \theta),
\]  

(8)

where the \((-+)\) sign corresponds to the \( RL/\overline{LR} \) final state helicities and the sum over \( j \) is understood. By requiring that MaxEnt is realized in the form \( |RL - \overline{LR} | (\Delta_{RL} = 1) \) at \( \theta = \pi/2 \) we derive the constraint \( (a_{j1} + ia_{j2})a_{j3} = 0 \). Introducing the positive-defined Hermitian matrix \( A_{kl} = a_{kj}a_{lj} \), this condition implies \( A_{13} = A_{23} = 0 \), consistent with QED where all \( a_{ij} = 0 \) for \( i \neq j \). While in general it is not justified to assume that MaxEnt in uQED emerges for the same \( \theta \) as in QED, this example shows the constraints which are obtained from concurrence maximization.

The complete application of the MaxEnt principle to uQED requires the computation of all the scattering amplitudes in the new theory and then the determination of the constraints on the \( a_{\mu \nu} \) coefficients from the maximization of the concurrences. Here we have maximized the sum of the concurrences of four different processes: Bhabha and Møller scattering, \( ee \rightarrow \gamma \gamma \) and \( e^- e^+ \rightarrow \mu^- \mu^+ \), accounting for all initial helicity combinations for product states. The maximization has been performed both over the \( a_{\mu \nu} \) coefficients and over the scattering angle \( \theta \). Full consistency is found between the constraints provided by each of the four processes. The solution to the maximization of the concurrence is found to be

\[
(G^0, G^1, G^2, G^3) = (\pm \gamma^0, \pm \gamma^1, \pm \gamma^2, \pm \gamma^3)
\]  

(9)

which shows that QED is indeed a solution (though not the only one) of requiring MaxEnt for the above subset of scattering processes in uQED. Some of these solutions are equivalent to QED since a global sign can be absorbed in the electric charge.

The solutions Eq. (9) are divided into two groups, those related to QED and those that are inconsistent with QED, for instance because they violate rotation symmetry. The latter solutions cannot be ruled out since the scattering processes considered here cannot determine the overall sign of the \( \gamma^\mu \) matrices, as they always appear in pairs. Including further scattering or decay processes which involve three outgoing particles might remove this ambiguity and eliminate the inconsistent solutions.

MaxEnt in the weak interactions. The mechanism underlying MaxEnt generation in weak interactions is more subtle, due to the interplay between vector and axial currents and between \( Z \) and \( \gamma \) channels. The coupling of the \( Z \) boson to fermions reads

\[
i \frac{g}{\cos \theta_W} \gamma^\mu \left( g_V^f - g_A^f \gamma^5 \right)
\]  

(10)
where the axial and vector couplings are $g_4^j = T_3^j/2$ and $g_V^j = T_3^j/2 - Q_f \sin^2 \theta_W$, and $\theta_W$ is the Weinberg mixing angle. For electrons and muons, $T_3 = -1/2$ and $Q_f = -1$. Beyond tree level, the Weinberg angle runs with the energy and is scheme dependent. The PDG average [13] at $Q = m_Z$ in the on-shell scheme is $\sin^2 \theta_W \approx 0.2234$. In the Standard Model (SM) thus we have that for electrons the vector coupling $|g_V|$, is smaller than the axial one $|g_A|$ by about order of magnitude.

Accounting for the effects of the new axial component in the fermion-boson coupling can be done as follows. We first consider $\epsilon^- \epsilon^+ \rightarrow \mu^- \mu^+$ scattering mediated by a $Z$ boson in the high energy limit, where $m_Z$ is neglected. We define left and right couplings as $g_L = g_V + g_A$ and $g_R = g_V - g_A$, which simplifies the structure of the currents since $j_{in}^{RL} \sim g_R(0,1,i,0)$ and $j_{in}^{LR} \sim g_L(0,1,-i,0)$. The resulting scattering amplitudes are:

$$\mathcal{M}_{LR} \sim (1 + \cos \theta) g_L^2 |LR| + (1 - \cos \theta) g_L g_R |RL|$$

$$\mathcal{M}_{RL} \sim (-1 + \cos \theta) g_R g_L |LR| + (1 + \cos \theta) g_R^2 |RL|$$

and their concurrences for $|\vec{p}| \gg m_Z$ read:

$$\Delta_{LR(RL)} \simeq \frac{\sin^2 \theta |g_L g_R|}{2(c^2 g_L^2 + s^2 g_R^2)} \left( \frac{\sin^2 \theta |g_L g_R|}{2(s^2 g_L^2 + c^2 g_R^2)} \right),$$

where $c = \cos \theta/2$ and $s = \sin \theta/2$ depend on the scattering angle $\theta$. Applying the MaxEnt requirement to the concurrences Eq. [12] we derive a constraint between the couplings $g_R$ and $g_L$ and $\theta$, namely $c^2 g_L \pm s^2 g_R = 0$ ($s^2 g_L \pm c^2 g_R = 0$) for the $LR$ ($RL$) initial states. Note that in general concurrence maximization occurs for different values of $\theta$ for each initial state. This result can be traced back to the $Z \rightarrow f f$ decay, and indeed the decay of any polarization of the $Z$ particle gets maximally entangled under the condition $\sin^2 \theta_W = 1/4$. Thus scattering processes mediated by a $Z$ inherit the entanglement structure from $Z$ decays.

In Fig. 1 we show the maximal concurrence lines ($\Delta = 1$) as a function of the scattering angle $\theta$ and of the coupling ratio $g_R/g_L$ for the two combinations $LR$ and $RL$. We find that both concurrences are simultaneously maximized for $\theta = \pi/2$, where $g_R = \pm g_L$, that is, either $g_A = 0$ or $g_V = 0$. If the axial coupling vanishes $g_A = 0$, we recover the known QED result. The $g_V = 0$ solution, a vanishing vector coupling, corresponds to a Weinberg angle of $\sin^2 \theta_W = 1/4$, in agreement with the experimental value at the $Z$ pole $[15]$ within $\pm 10\%$. Therefore, requesting MaxEnt simultaneously for the two initial state helicities leads either to QED or to a theory which looks surprisingly close to the weak interaction. We have also studied how this result is modified if we include the contribution from $\gamma$ exchange, a non-trivial check since this adds terms to both $RL$ and $LR$ which are independent of $\sin^2 \theta_W$, finding that while the angular dependencies are modified the MaxEnt predictions are preserved.

While the application of MaxEnt to $Z$-boson mediated scattering does not fix completely the coupling structure of the weak interactions, as we mentioned its application to $Z$ decay fixes $g_V = 0$ and thus $\sin^2 \theta_W = 1/4$. The lack of full predictivity of MaxEnt in the full scattering case is due to the freedom to choose different angles for MaxEnt depending on the chirality of the initial particles.

Conclusion. Fundamental interactions generate entangled states using mechanisms based on indistinguishability, consistently with the symmetries of the theory. In this work we have explored the relationship between maximally entangled states and high energy scattering amplitudes in QED and the weak interactions. We found that promoting MaxEnt to a fundamental principle allows one to constrain the coupling structure describing the interactions between fermions and gauge bosons. We also found that MaxEnt in the weak interactions prefers a weak angle $\theta_W = \pi/6$, surprisingly close to the SM value. In the “It from Qubit” viewpoint of J. A. Wheeler, where “all things physical are information-theoretic in origin” [16], the fundamental particle interactions represent a set of operations to implement basic information ideas and protocols. In this framework, MaxEnt arises as a possible powerful information principle that can be applied to different processes, bringing in unexpected important constraints on the structure of high-energy interactions. It is conceivable that extensions of the MaxEnt principle to other processes will lead to new insights in the structure of the SM or on theories of New Physics beyond it.

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FIG. 1: Maximal concurrence line as a function of the angle $\theta$ and the coupling ratio $g_R/g_L$ for $Z$-mediated $\epsilon^- \epsilon^+ \rightarrow \mu^- \mu^+$ scattering for the $LR$ and $RL$ initial state helicities.
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