Rare decays of $b$ hadrons

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Abstract

Rare decays of $b$ hadrons provide a powerful way of identifying contributions from physics beyond the Standard Model, in particular from new hypothetical particles too heavy to be produced at colliders. The most relevant experimental measurements are reviewed and possible interpretations are briefly discussed.

Contribution to Scholarpedia [1]‡ This is the arXiv version with many more references.

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Contents

1 Introduction
  1.1 Key players .................................................. 2

2 Theory .................................................. 3

3 Experiments
  3.1 Short history of $b$-quark physics ............................. 6
  3.2 Present .................................................. 7

4 Main experimental results
  4.1 The decay $b \to s \gamma$ ........................................ 7
  4.2 The decays $B^0 \to \mu^+ \mu^-$ and $B^0 \to \mu^+ \mu^-$ ...................... 9
  4.3 Other leptonic decays ........................................ 10
  4.4 The decays $b \to s \ell^+ \ell^-$ ................................ 11
  4.5 Lepton universality tests ................................... 13

5 Wilson coefficient fits ................................... 14

6 Prospects ........................................ 15

7 Conclusion ........................................ 15

References ........................................ 16
# 1 Introduction

Physics studies fundamental interactions and their effects. At the most basic level, particle physics aims to describe the fundamental blocks of matter and their interactions. A century of research has led to the Standard Model of Particle Physics. It relies on firm theoretical grounds unifying quantum mechanics, special relativity and field theory, and is successful at describing all phenomena measured in particle interactions, whether at low or very high energies, like in collisions of particles produced in large accelerators. Yet, it has an empirical character with many parameters that need to be determined by experiment, and it is incomplete as it does not account for gravity, does not explain the baryon asymmetry in the Universe and does not provide a candidate for dark matter. The Standard Model is therefore believed to be an approximation of a more complete theory that is currently unknown (just like Newton’s laws are an approximation of General Relativity). The primary goal of research in particle physics is to find this more complete theory. In the following “New Physics” is used as a catch-all for any contribution, whether particle or coupling, not included in the Standard Model.

Rare decays of hadrons containing a heavy “beauty” (also called “bottom”) quark, denoted $b$ hadrons, provide a powerful way of exploring yet unknown physics. Small contributions from virtual new particles that are too heavy to be produced at colliders may lead to measurable deviations from the expected properties in the Standard Model. See Inset 1 for an example of a virtual particle.

The study of rare decays is an active field within flavour physics, the field of research studying transitions of quarks or leptons from one species (or “flavour”) to another. This article focuses on rare decays of hadrons containing $b$ quarks. The most prevalently produced $b$ hadrons are the $B^0$ meson composed of a $b$ anti-quark and a $d$ quark, the $B^+$ ($\bar{b}u$) and $B^0_s$ ($\bar{b}s$) mesons, as well as the $Λ^0_b$ ($bud$) baryon. Their masses are in the range $5–6$ GeV/$c^2$, which is about 6 times that of the proton, but well below the mass of the $W$ boson of $80$ GeV/$c^2$. The corresponding antiparticles $\bar{B}^0$, $B^-$, $\bar{B}^0_s$ and $\bar{Λ}^0_b$ are obtained by replacing all quarks by anti-quarks and vice-versa. The study of CP violation involves investigation of differences in the behaviour of particles and antiparticles, and is the subject of a dedicated review in Ref. [2]. The inclusion of charge conjugate processes is implied throughout this document.

Hadrons with $b$ quarks decay most of the time via a $b \to cW^{-*}$ transition, where the asterisk indicates the $W$ boson is virtual. See the article on the Cabibbo-Kobayashi-Maskawa matrix

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**Inset 1: Virtual Particles**

Fundamental particle interactions are mediated by force carriers: The photon for the electromagnetic interaction, the gluon for the strong interaction and the $W^\pm$ and $Z$ bosons for the weak interaction. In the nuclear beta decay a neutron decays to a proton, an electron and a neutrino (Fig. 1). This weak interaction is mediated by a virtual $W^-$ boson, sometimes denoted $W^{-*}$. Here virtual means that the process violates energy and momentum conservation for a very short time, as allowed by Heisenberg’s Uncertainty Principle. The mass of the $W^-$ boson is about 80 GeV/$c^2$, while the neutron-proton mass difference is considerably less: 1.3 MeV/$c^2$. It is said that the $W^-$ boson is “off-shell”.

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**Figure 1:** Feynman diagram of the beta decay of the neutron.
(CKM) \cite{3} for more details. The transitions $b \rightarrow u W^-$ also occur, but are less likely. These two transitions are called “tree decays” as the process involves a single mediator, the $W^-$ boson. An example of a tree $d \rightarrow u W^-$ transition is shown in Fig. [1].

This article describes mostly transitions involving more complicated processes. The quark transitions $b \rightarrow d$ and $b \rightarrow s$ do not happen at tree level in the Standard Model as the $Z$ boson does not couple to quarks of different flavour.

Processes like the rare decay $B_s^0 \rightarrow \mu^+ \mu^-$ proceed via loops as shown in Fig. [2] (sometimes referred to as penguins, a word coined by John Ellis \cite{4,5}). Such processes are rare as the probability of a transition rapidly decreases with the number of electroweak vertices: two in the case of a tree decay, three or four for a loop. Also, the heavier the virtual particles involved, the more suppressed the decay. In the following, decays with probabilities in the range $10^{-4}$ to $10^{-10}$ are discussed.

Some of the most interesting decays are described in Section 1.1. They all have in common the following features:

1. Suppressed decay amplitudes, as predicted by the Standard Model, which may potentially be of the same size as New Physics amplitudes.
2. Sufficiently precise Standard Model predictions for their decay rate, or any other observable of interest.
3. Experimental precision which potentially allows disentangling the Standard Model contribution from other contributions.

A historical example, the decay $K^0_L \rightarrow \mu^+ \mu^-$, is described in Inset 2.

A special category of rare decays is those forbidden in the Standard Model, like lepton- or baryon-number violating decays. In their case the Standard Model prediction is effectively zero, but other models may predict non-zero rates. Any observation would be a sign of New Physics.

Given the impressive success of the Standard Model, New Physics amplitudes are known to be small. Therefore, any search for potentially observable deviations from Standard Model predictions will be facilitated if the Standard Model amplitudes are also suppressed, which is the case in rare decays. Studies of such decay modes require require large data samples to produce enough of the relevant particles. This is referred to as the intensity frontier, as opposed to the energy frontier aiming at producing and studying heavy particles on-shell \cite{10}. The main experiments are briefly described in Section 3.

1.1 Key players

The description of the process of the formation of hadrons out of quarks and gluons, called hadronisation, is difficult and leads to large theoretical uncertainties. Theoretically favoured are thus decays to purely leptonic final states, such as the decay $B_s^0 \rightarrow \mu^+ \mu^-$ (Section 4.2). There is also interest in the charged counterparts of these decays, notably $B^+ \rightarrow \ell^+ \nu$, where $\ell^+$ is any lepton, $e^+, \mu^+, \tau^+$ (Section 4.3). They are generated by a charged $W^+$ current, but have interesting theoretical connections to decays that are induced by loops.
Inset 2: A Historical Example: $K_L^0 \rightarrow \mu^+ \mu^-$

The $K_L^0 \rightarrow \mu^+ \mu^-$ decay is forbidden at tree level and had an important role in opening the field of rare decays studies in the 1960s, as its unexpected non-observation allowed the prediction of the then unknown charm quark by Glashow, Iliopoulos and Maiani ("GIM mechanism") in 1970 [6]. The idea of the GIM mechanism is that this decay only occurs via loops, one involving the $u$ quark and the other the $c$ quark (Fig. 3). The amplitudes of the two loops are of opposite sign, causing complete cancellation in the limit $m_c - m_u = 0$. The non-observation of this decay could be explained by adding a new particle to the theory, the $c$ quark, which was eventually discovered in 1974 [7,8]. This is an example of an observation of New Physics mediated by a new virtual particle. The $K_L^0 \rightarrow \mu^+ \mu^-$ branching fraction is now measured to be $(6.9 \pm 0.1) \times 10^{-9}$ [9]. Nowadays there is a great deal of interest in the $B_s^0$-counterpart of this decay: $B_s^0 \rightarrow \mu^+ \mu^-$, discussed in Section 4.2.

Figure 3: Feynman diagrams of the two loops contributing to the decay $K_L^0 \rightarrow \mu^+ \mu^-$. 

Inclusive decays are also of interest, most notably the radiative decay $b \rightarrow s \gamma$, the electroweak semileptonic decays $b \rightarrow s \ell^+ \ell^-$, and $b \rightarrow s \nu \bar{\nu}$. These are quark-level transitions, which cannot be measured directly as the quarks form immediately hadrons. In experiments exclusive decays are detected, and the inclusive decay is the sum of all contributions. For instance the decay $b \rightarrow s \gamma$ was first observed by its exclusive contribution $B \rightarrow K^* \gamma$ (Section 4.1).

Exclusive decays are experimentally favoured, but come with larger theoretical uncertainties. The decay $B^0 \rightarrow K^{*0} \ell^+ \ell^-$ is a well-known example. While the decay rate is hard to compute precisely, observables describing angular distributions of the decay products can be more precisely predicted (Section 4.4).

Among forbidden decays, lepton flavour violating decays of $b$ and $c$ hadrons, like $B_{(s)}^0 \rightarrow e^+ \mu^+$ or $B^+ \rightarrow K^+ e^+ \mu^+$, or of leptons, like $\mu^+ \rightarrow e^+ \gamma$, $\tau^+ \rightarrow \mu^+ \mu^+ \mu^-$ or $\tau^+ \rightarrow \mu^+ \gamma$ are actively being searched for. Rare charm hadron decays are also being studied, but the experimental sensitivity is presently not sufficient to reach the very low rates predicted in the Standard Model. Finally, research in rare kaon decays is ongoing, though mainly at different experiments than those studying rare charm or beauty hadron decays [9]. These channels are not further discussed in this article.

Recent reviews on rare decays can be found in Refs. [11,12].

2 Theory

This section describes briefly the theoretical framework that is commonly used to study rare decays. It may be skipped by readers mostly interested in experimental results. Its main goal is to define some vocabulary which is commonly used in publications on rare decays.

The common theoretical approach to rare decays is model independent. In flavour physics and in particular in rare decays studies, the underlying physics is parametrised in terms of an
effective Hamiltonian describing the transition amplitude of an initial state $I$ to a final state $F$ following Fermi's Golden Rule \[13,14\]. The partial decay width is written as

$$\Gamma(I \rightarrow F) = \frac{2\pi}{\hbar} |\langle F|\mathcal{H}_{\text{eff}}|I\rangle|^2 \times \text{phase-space}.$$ 

Experimentally, the branching fraction $B$ is measured rather than the decay width. They are related by

$$B(I \rightarrow F) = \frac{\Gamma(I \rightarrow F)}{\Gamma(I, \text{total})} \cdot \frac{1}{\tau_I},$$

where $\tau_I$ is the lifetime of particle $I$ (and natural units with $c = \hbar = 1$ are used).

The Standard Model prediction for any particular transition transition can be inferred from a calculation of the effective Hamiltonian derived from the Standard Model Lagrangian. This Hamiltonian is parametrised in terms of a sum of operators $O_i$ and Wilson coefficients $C_i$

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \sum_i V_{\text{CKM}} C_i O_i,$$

where $V_{\text{CKM}}$ stands for some product of Cabibbo-Kobayashi-Maskawa matrix elements that describe the probability of given transitions between different quark flavours. The operators encompass the information about the Lorentz structure and the Wilson coefficients encode the effects of higher energy scales. In the case of the Standard Model these are the effects of the $W$, $Z$ bosons and top quarks, which are effectively removed from the theory and incorporated in the coefficients.

Any $I \rightarrow F$ decay can be described by this effective Hamiltonian, usually with many terms $< F|O_i|I > = 0$. Thus studying a set of decays will give various constraints on the effective Hamiltonian, permitting global fits to Wilson coefficients. This is briefly discussed in Section 5. This procedure does not simplify the computation of the amplitudes, as the matrix elements $\langle F|O_i|I \rangle$ contain the most difficult parts of the calculation. It provides however a common language that is not dependent on the considered New Physics model.

In particular, calculations of decay rates of exclusive decays with hadrons in the final state ($B^0 \rightarrow K^{*0}\mu^+\mu^-$ for example) are difficult and part of our lack of knowledge needs to be parametrised in heuristic quantities that describe the hadronisation, like form-factors and decay constants. They can be calculated in lattice QCD and, in many cases, can also be determined experimentally. Their discussion is beyond the scope of this document.

The operators $O_{1,2}$ describe the $V-A$ structure of weak decays and first-order corrections. For example, the $W$ boson having been absorbed into the $C_1$ and $C_2$ coefficients, the nuclear beta decay $n \rightarrow p e \bar{\nu}$ is represented by a four-fermion operator as shown in Fig. 4. This is how Enrico Fermi first described the process in 1934 \[15\]. The operators $O_{3-8}$ describe loops involving gluons. They are not of interest for this article.

Of most interest in rare decays are the suppressed operators $O_7$, $O_9$, and $O_{10}$. The operator $O_7$ dominates the radiative decay $b \rightarrow s \gamma$ giving a decay width

$$\Gamma(b \rightarrow s \gamma) = \frac{G_F^2 \alpha_{\text{EM}} m_b^5}{32\pi^4} |V_{ts}^* V_{tb}|^2 |C_7|^2 + \text{corrections},$$
where $\alpha_{\text{EM}}$ is the electromagnetic constant, $m_b$ the $b$ quark mass, and $V_{ij}$ are parameters of the CKM matrix. A measurement of the $b \rightarrow s \gamma$ branching fraction thus provides a direct constraint on $C_7$.

The operators $O_9$ and $O_{10}$ dominate $b \rightarrow q\ell\ell$ transitions, with $O_9$ corresponding to a vector and $O_{10}$ to an axial current. Finally the decays $B \rightarrow \ell^+\ell^-$ are, in the Standard Model, dominated by operator $O_{10}$, with a branching fraction which can be written as

$$
\Gamma(B \rightarrow \ell^+\ell^-) = \frac{G_F^2 M_B^2 m_b^3 f_B^2}{8\pi^5} |V_{tb}^* V_{tq}|^2 \frac{4m_{\ell}^2}{m_B^2} \sqrt{1 - \frac{4m_{\ell}^2}{m_B^2}} |C_{10}|^2 + \text{corrections},
$$

for $B = B^0, B^0_s$ (and $q = d, s$) with $f_B$ the $B$ decay constant and $V_{ij}$ CKM matrix elements. It is to be noted that in the $B^0_s$ case only the heavy mass eigenstate contributes, and hence the $B^H$ decay width must be used to compute the branching fraction $[17,18]$. The $B^0_s \rightarrow \mu^+\mu^-$ branching fraction thus provides a constraint on $C_{10}$. Other operators, labelled $O_P$ and $O_S$, which are negligible in the Standard Model, could also contribute to this decay.

If the $V-A$ structure of weak interactions is not assumed, new primed operators with flipped helicities appear, most notably $O'_7$, and its Wilson coefficient $C'_7$ which generate a right-handed photon in $b \rightarrow s\gamma$ decays.

For a comprehensive review of the effective Hamiltonian used to study rare decays, see Refs. $[19,20]$. A more pedagogical introduction can be found in Chapter 20 of Ref $[21]$. Standard Model expectations of Wilson coefficients and operators have been calculated at next-to-leading order or better $[22-24]$.

There exist many theories beyond the Standard Model providing predictions for Wilson coefficients. Often these values depend on unknown parameters of the theory, as masses of yet unseen new particles. This is particularly the case for supersymmetry, a well-motivated extension of the Standard Model.

3 Experiments

There are essentially two families of experiments studying $b$ hadrons:

$B$ factories are experiments based at $e^+e^-$ colliders operating most of the time at a collision energy near 10.6 GeV, corresponding to the mass of the $\Upsilon(4S)$ resonance, the lightest meson decaying to two $B$ mesons. ARGUS $[25]$ (at DESY, Germany), CLEO $[26]$ (at Cornell, USA), BaBar $[27]$ at SLAC (Stanford, USA), and Belle $[28]$ at KEK (Tsukuba, Japan) are notable examples of such experiments.

Hadron collider experiments operate at a $pp$ or $p\bar{p}$ collider with centre-of-mass energies of several TeV. CDF and D0 were located at Fermilab’s Tevatron (Batavia, USA) $[29]$, ATLAS $[30,31]$, CMS $[32,33]$ and LHCb $[34,35]$ presently operate at CERN’s LHC (Geneva, Switzerland).

Hadron colliders have the advantage of much larger production rates: the production cross-section of $b$ quarks is a factor 500,000 larger $[36]$ at the LHC than at a $B$ factory. The advantage of the $B$ factories is cleanliness. Collision events with a produced $\Upsilon(4S)$ resonance are easy to identify, allowing for high efficiencies and low background levels. In such events only two $B$
Figure 5: History of $B^0 \rightarrow \mu^+ \mu^-$ and $B^0 \rightarrow \mu^+ \mu^-$ limits up to the $B^0_s \rightarrow \mu^+ \mu^-$ observation. Figure from Ref. [41].

Mesons are produced, making the reconstruction of the full collision event possible. In a typical LHC collision only one in hundred collisions produce a $b$ quark pair and the two $b$ hadrons are surrounded by hundreds of other particles. Efficient background fighting techniques are thus essential and have a cost in terms of efficiencies. This is achieved by vertexing which exploits the $b$-hadron flight distance — but this only works for decay modes with at least two charged particles produced at the $b$-hadron decay vertex. The physics programme is also somewhat different: $B$ factories have only access to $B^0$ and $B^+$ mesons (and $B^0_s$ mesons when operating at the $\Upsilon(5S)$ resonance), while hadronic collisions produce all $b$ hadrons, including the $B^0_s$ meson, the $B^{+\mp}$ meson (composed of a $c$ quark and an $b$ anti-quark), and $b$ baryons as the $\Lambda^0_b$ and $\Xi_b$.

3.1 Short history of $b$-quark physics

After the discovery of the $b$ quark at Fermilab through the observation of mesons formed by a $b$ and an $\bar{b}$ anti-quark in 1977 [37] and of the $B$ meson at Cornell [38,39], searches for rare decays of $b$ hadrons rapidly took pace. The first limit on the decay $B^0 \rightarrow \mu^+ \mu^-$ was set by the CLEO collaboration in 1985 [40], the start of a long quest during which the sensitivity was improved by six orders of magnitude, as illustrated in Fig. 5 (See Section 4.2).

The CLEO and ARGUS experiments were located at $e^+e^-$ colliders operating at the $\Upsilon(4S)$ resonance. The same concept was employed and improved by the BaBar experiment and Belle in the first decade of the 21st century. If Cornell was initially able to produce few tens of $BB$ pairs per day, the PEP-II and KEKB accelerators at SLAC and KEK achieved a daily rate of one million $BB$ pairs. In the meantime, experiments at CERN’s LEP $e^+e^-$ collider [42,45] and at Fermilab’s Tevatron proton-anti-proton collider used higher energy collisions to produce and
study all $b$-hadron species \[46,47]. All the above-mentioned experiments have terminated their programme but most still exploit their data set to produce new results. Belle and the associated accelerator complex is presently undergoing a major upgrade and will come back as the Belle II experiment around 2018.

3.2 Present

Nowadays the leadership in $b$ physics is taken by the LHCb experiment. Important contributions also come from the ATLAS and CMS experiments.

ATLAS and CMS are detectors optimised for high-energy processes, such as the discovery of the Higgs boson \[48,49]. They also perform $b$-physics research, most effectively in decays of $b$ hadrons to pairs of muons. This distinct signature allows for efficient selection of these decays during the online filtering phase where a large reduction of the recorded collision rate is required, which is difficult to achieve for decays to electrons or hadrons.

The LHCb experiment on the contrary is optimised for the physics of hadrons containing $b$ and $c$ quarks. It is a single-arm forward detector designed to exploit the relatively large $bb$ production in LHC proton-proton collisions in the forward direction. It includes a tracking system surrounding a dipole magnet whose polarity can be reversed, silicon sensors coming as close as 8 mm to the proton beam and a particle identification system based on Cherenkov radiation. The high-resolution silicon system exploits the typical $b$-hadron flight distances of a few millimetres before their decay to select them. This sets requirements on the number of $pp$ collisions per bunch crossing, defining an upper limit to the total collision rate at which the experiment can operate. Consequently, the luminosity is decreased compared to ATLAS and CMS.

4 Main experimental results

This section presents the main recent experimental results and their interpretation. It starts with a more historical section on the decay $b \to s\gamma$ which had (and still has) an important role in the development of the field.

4.1 The decay $b \to s\gamma$

In the Standard Model the decay $b \to s\gamma$ occurs dominantly via a loop involving the top quark and the $W$ boson (Fig. 6). It has played a very important role in flavour physics from the 1980s \[50]. At the time it was the dependency of the branching fraction on the then unknown top quark mass that was the driving force behind the theoretical calculations and the experimental searches. When $B^0 - \bar{B}^0$ mixing was (at the time surprisingly) observed in 1987, it became clear that the top quark was very heavy. The top quark was eventually discovered at the Tevatron \[51] in 1995 and its mass measured, which determined the Standard Model decay rate of $b \to s\gamma$ to be a few $10^{-4}$.

The first observation of the $b \to s\gamma$ decay actually preceded the top quark observation. In 1993 the CLEO collaboration reported a signal of the exclusive decay $B \to K^*\gamma$ with a branching
fraction of $(4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ \cite{52} (Fig. 7 left), where the first uncertainty is statistical and the second systematic. This opened the quest for the inclusive decay $b \to s\gamma$, i.e. the sum of all exclusive contributions. The branching fraction of this decay is more precisely calculable than its individual exclusive components, like $B^0 \to K^{*0}\gamma$, allowing for more precise comparisons of experimental and theoretical results.

Many experiments located at $e^+e^-$ colliders have performed measurements using different methods. The total rate of any $B$ meson to a photon plus anything (where the photon is not caused by an electromagnetic decay, e.g. $\pi^0 \to \gamma\gamma$ or $\eta \to \gamma\gamma$) can be measured by a sum of exclusive decay modes \cite{53–60}. A fully inclusive approach is also possible but more challenging \cite{61–68}. Only the photon and properties of the rest of the collision event are used to separate signal and backgrounds, mostly originating from non-$B$ decays. At the $B$ factories they are determined from data taken at centre-of-mass energies below the $\Upsilon (4S)$ resonance mass, decays to $\pi^0$ and $\eta$ which are vetoed and modelled from data, and mis-identified photons which are modelled from simulation. This method has the disadvantage of larger backgrounds, but has the advantage not to rely on any modelling of the composition of the hadronic state. It thus comes with smaller theoretical uncertainties. A typical result for the measured photon energy spectrum in inclusive $b \to s\gamma$ gamma decays is shown in Fig. 7 (right). The integral of the spectrum gives the decay rate. The width is related to the momentum of the $b$ quark in the $B$ meson, which can be seen as a two-body system of a $b$ and a light quark, similar to a hydrogen atom.

The world average measured branching fraction is $(3.49 \pm 0.19) \times 10^{-4}$ \cite{9}, which can be compared with the latest theoretical calculation of $(3.36 \pm 0.23) \times 10^{-4}$ \cite{69,70}. The very good agreement of these two values sets very strong constraints on New Physics models, in particular supersymmetry. Although the total branching fraction seems to indicate no discrepancy with the Standard Model prediction, small contributions from New Physics may still occur. The $b \to s\gamma$ decay rate is essentially a measurement of the Wilson coefficient $C_7$ (see Section 2).
In the Standard Model, the left-handed chirality structure of the weak interactions makes the photon emitted in $b \to s \gamma$ decays mainly left-handed. It is interesting to also probe right-handed contributions (sensitive to the Wilson coefficient $C'_9$), which requires determination of the polarisation of the photon. This is challenging as the helicity (or chirality) of the photon cannot be measured directly in the detector. Several methods have been proposed, none of which provides a strong constraint so far. The first and so far only measurement of a non-zero photon polarisation was made only recently by the LHCb experiment in the decay $B \to K^+ \pi^- \pi^+ \gamma$ [71], but the interpretation in terms of the photon chirality is still unclear. The most stringent constraints come from global fits to Wilson coefficients (see Section 5).

4.2 The decays $B^0 \to \mu^+\mu^-$ and $B^0 \to \mu^+\mu^-$

The rare decay $B^0 \to \mu^+\mu^-$ proceeds in the Standard Model by a box-type diagram involving the $W$ and $Z$ bosons and the $t$ quark (Fig. 8). The most recent Standard Model prediction of its branching fraction is $(3.66 \pm 0.23) \times 10^{-9}$ [16], where the uncertainty is dominated about equally by CKM matrix elements and the $B^0_s$ decay constant. In this calculation the branching fraction is evaluated as an average over all decay times [17,18]. In Standard Model extensions, the branching fraction of $B^0 \to \mu^+\mu^-$ could be enhanced, in particular in models containing additional Higgs bosons (Fig. 8, right). The decay $B^0 \to \mu^+\mu^-$ and the even more suppressed decay $B^0 \to \mu^+\mu^-$ have been searched for over three decades, with most recent results from the Tevatron [73,74] and the LHC [75–83] (Fig. 5).

The first observation was reported in 2014 jointly by the CMS and LHCb collaborations [41]. Results from the ATLAS collaboration were published recently [84]. The CMS and LHCb LHC Run 1 data sets were combined in a joint fit to the data of both experiments, published in Nature [41], which is unusual in high energy physics. The fit to the invariant mass distribution of the two-muon system is shown in Fig. 9. The result of the combination is an observation of the $B^0_s \to \mu^+\mu^-$ decay and a small excess over the background for $B^0 \to \mu^+\mu^-$. The measured branching fraction $B(B^0_s \to \mu^+\mu^-) = (2.8 \pm 0.7) \times 10^{-9}$ is consistent with the Standard Model prediction.

For the even more suppressed $B^0$ decay, the result is $B(B^0 \to \mu^+\mu^-) = (3.9 \pm 1.6) \times 10^{-10}$, which is slightly, but not significantly, larger than the Standard Model prediction of $(1.06 \pm 0.09) \times 10^{-10}$ [16].

The ATLAS measurement [84] yields lower but consistent results of $B(B^0_s \to \mu^+\mu^-) = (0.9 \pm 1.3) \times 10^{-9}$ and $B(B^0 \to \mu^+\mu^-) < 4.2 \times 10^{-10}$ at 95% confidence level.

These values set strong constraints on supersymmetry and other New Physics models. More data from the LHC will tell if the excess of $B^0 \to \mu^+\mu^-$ is a statistical fluctuation or an indication of New Physics.

![Feynman diagram](image_url)

Figure 8: Feynman diagram of the (left and middle) dominating Standard Model contributions to $B^0_s \to \mu^+\mu^-$ and (right) a potential contribution in the context of supersymmetry [72].
4.3 Other leptonic decays

Just as \( B_s^0 \rightarrow \mu^+\mu^- \) and \( B^0 \rightarrow \mu^+\mu^- \) are theoretically clean decays, so are their counterparts with neutrinos. The challenge is on the experimental side. The decay \( B^0 \rightarrow \nu\bar{\nu} \) is traditionally labelled as “\( B^0 \rightarrow \text{invisible} \)” as there is no way to experimentally determine the number of neutrinos (or if there were any at all). In the Standard Model the branching fraction is vanishing as it is helicity-suppressed by a factor \((m_\nu/m_{B^0})^3\). Helicity suppression occurs because of the \( B^0 \) meson is spinless, so the two spin-1/2 neutrinos must have opposite spins. For massless neutrinos this would be impossible as neutrinos are always left-handed and antineutrinos right-handed. Only the minute mass of neutrinos (rarely) allows opposite-spin neutrinos to be emitted.

Searches have been performed by the \( B \) factory experiments using the full reconstruction technique (also referred to as “on the recoil”). One \( B \) meson from the \( B\bar{B} \) pair is fully reconstructed and the other is required to leave no trace in the detector. The branching fraction is limited to be less than \( 2.4 \times 10^{-5} \) at 90% confidence level, by the BaBar experiment [85].

Decays to one charged lepton, \( B^+ \rightarrow \ell^+\nu_\ell \), are similarly helicity suppressed, with the strength of this suppression depending on the mass of the charged lepton. These are tree decays where the \( \bar{b} \) and \( u \) quarks in the \( B^+ \) meson annihilate, but rare because of the helicity suppression. Contributions with the \( W^+ \) mediator replaced by a charged Higgs boson could enhance or suppress the branching fraction. These decays have all been searched for by the \( B \) factories using the full reconstruction technique described above.

The \( B^+ \rightarrow \tau^+\nu_\tau \) decay, where the suppression is the weakest, has a predicted branching fraction of \( B^{\text{SM}} = (0.76^{+0.08}_{-0.06}) \times 10^{-4} \) in the Standard Model and a measured rate of \((1.14^{+0.27}_{-0.32}) \times 10^{-4}\) [90], which are in agreement. The more suppressed decays \( B^+ \rightarrow e^+\nu \) and \( B^+ \rightarrow \mu^+\nu \) have not been observed yet, with limits on their branching fraction around \( 10^{-6} \) [91,92].

The decay \( B^0 \rightarrow e^+e^- \) is out of reach in the foreseeable future. Due to the low electron mass, it is even more helicity-suppressed than \( B_s^0 \rightarrow \mu^+\mu^- \). Moreover the study of final states involving electrons at hadron colliders is difficult due to the lower reconstruction efficiency and the poorer
mass resolution (see for instance Fig. 14). When passing through matter, electrons radiate a significant amount of energy by bremsstrahlung. This affects the reconstructed momentum and thus smears all derived quantities, like the invariant mass of the two-electron system.

The expected rate of $B^0_s \rightarrow \tau^+\tau^-$ is considerably larger, but the decay is experimentally challenging due to the difficult $\tau$ lepton reconstruction and associated large backgrounds.

Belle II (and LHCb in the case of $B^0_s \rightarrow \tau^+\tau^-$) are likely to perform improved searches of such decays in the near future.

4.4 The decays $b \rightarrow s\ell^+\ell^-$

The family of decays $b \rightarrow s\ell^+\ell^-$ ($\ell = e, \mu$) is a laboratory of New Physics studies on its own. In the Standard Model these decays are induced by a loop diagram similar to that of $b \rightarrow s\gamma$ (but with a $Z$ component) and a box diagram (Fig. 10). The amplitudes corresponding to these diagrams interfere, which causes complex phenomenology.

The exclusive decay $B^0 \rightarrow K^{*0}\ell^+\ell^-$, with $K^{*0} \rightarrow K^+\pi^-$, provides a rich set of observables with different sensitivities to New Physics, and for which theoretical predictions are available. These observables are affected by varying levels of uncertainties related to the calculation of quantum chromodynamical effects. Yet, selected ratios of observables benefit from cancellations of uncertainties, thus providing a cleaner test of the Standard Model [93–99]. The best known example is the lepton forward-backward asymmetry, explained in more details in Inset 3.

This interesting picture is complicated by a dependence on $q^2$, the dilepton mass squared (Fig. 11). At very low $q^2$, $B^0 \rightarrow K^{*0}\ell^+\ell^-$ behaves like $B^0 \rightarrow K^*\gamma$, with a slightly off-shell
photon decaying to two leptons. The physics is dominated by the $O_7$ operator, as discussed in Section 4.1.

Above, there is an interference of the amplitudes controlled by the $O_9$ and $O_{10}$ operators, related to the $Z$ loop and $W$ box diagrams, respectively. This “low-$q^2$” region between 1 and 6 GeV$^2$/c$^4$ is the most interesting and theoretically cleanest. Beyond this, non-suppressed $c\tau$ contributions (Fig. 10 right) make the picture more complicated and theoretical estimates are less reliable. The observation of high mass resonances above the $\psi(2S)$ meson by the LHCb collaboration [100] is an indication that a lot of care is needed when interpreting the high-$q^2$ region.

The differential decay width with respect to the dilepton mass squared $q^2$, the forward-backward asymmetry $A_{FB}$, and the longitudinal polarisation fraction $F_L$ of the $K^{*0}$ resonance have been measured by many experiments [104–112] with no significant sign of deviations from the Standard Model expectation. The most recent measurement of $A_{FB}$ by the LHCb experiment is shown in Fig. 13 (left). LHCb also studied other angular asymmetries. In particular a local deviation of the $P'_5$ observable (see Inset 3) from the Standard Model expectation is observed around $q^2 \sim 5$ GeV$^2$/c$^4$, see Fig. 13 (right) [103]. Recently, Belle have presented preliminary data that are consistent with the LHCb results [113].

This deviation triggered a lot of interest among theorists, with interpretation articles being quickly [114] submitted to journals. See Refs. [115–120] for a small subset. It is not clear yet if the discrepancy in $P'_5$ is a statistical fluctuation, is due to under-estimated theoretical uncertainties [121–124], or is the manifestation of a new vector current beyond the Standard Model.

Similar measurements have been made in the decays $B \to K\ell^+\ell^-$, $B^0_s \to \phi\mu^+\mu^-$ and $\Lambda^0_0 \to \Lambda\mu^+\mu^-$ [106,112,125–129]. The angular observables are consistent with the Standard

Inset 3: The $A_{FB}$ and $P'_5$ asymmetries

Figure 12: (left) The angles in $B \to K^*\mu^+\mu^-$ and (right) the $P'_5$ asymmetry.

In the decay $B^0 \to K^{*0}\mu^+\mu^-$, followed by $K^{*0} \to K^+\pi^-$, the direction of the four outgoing particles can be described by three angles, shown in Fig. 12 (left). The forward-backward asymmetry $A_{FB}$ is defined as the relative difference between the number of positive and negative leptons going along the direction of the $B^0$ meson in the rest frame of the two-lepton system. This corresponds to an asymmetry in the distribution of the $\theta_\ell$ angle. Similarly, the $K^{*0}$ polarisation fraction $F_L$ depends on the angle $\theta_K$, defined analogously to $\theta_\ell$.

Other asymmetries can be constructed from the other angles or combinations of them. The $P'_5$ asymmetry suggested by Ref. [99] is based on the angles $\theta_K$ and $\phi$. It is defined as the relative difference between the number of decays in the regions in red and blue in Fig. 12 (right), divided by $\sqrt{F_L(1-F_L)}$. Quantities based on several angles are more difficult to measure than single-angle ones as they require a better understanding of the reconstruction efficiencies depending on the kinematics of the outgoing particles.
Model, but there is some tension in the branching fraction measurements, which are on the low side compared to the expectation.

The decay family $b \rightarrow s \nu \nu$ is theoretically cleaner than its charged-lepton counterpart $b \rightarrow s \ell^+ \ell^-$. There are no interferences from $c\bar{c}$ loops as those do not annihilate to neutrino pairs. The main difficulty is on the experimental side and only $B$ factory experiments have attempted looking at such decays using the full reconstruction technique. None have been found and the most stringent limits on the decay rates of $B^0 \rightarrow K^* \nu \nu$ and $B^+ \rightarrow K^+ \nu \nu$ are at the $10^{-5}$ level [130].

4.5 Lepton universality tests

The above-mentioned $b \rightarrow s \ell^+ \ell^-$ measurements have been reported assuming that muons and electrons behave the same way. This assumption, called lepton universality, is built into the Standard Model and has been extensively tested, most notably at LEP experiments. The only Standard Model particle that has different couplings to leptons is the Higgs boson, which couples proportionally to mass.

Yet, surprisingly, the lepton universality ratio $R_K = \frac{B(B^+ \rightarrow K^+ \mu^+ \mu^-)}{B(B^+ \rightarrow K^+ e^+ e^-)}$ has been measured to be $0.745^{+0.090}_{-0.074} \pm 0.036$ by the LHCb experiment [131] in the $1 < q^2 < 6$ GeV/c$^2$ range, which corresponds to a $2.6\sigma$ tension with unity. The Standard Model prediction for this ratio is unity within $10^{-3}$ [132,133] as all hadronic uncertainties cancel in the ratio. Fig. 14 shows the mass peaks of $B^+ \rightarrow K^+ \mu^+ \mu^-$ and $B^+ \rightarrow K^+ e^+ e^-$, highlighting the effect of bremsstrahlung affecting electron reconstruction. A similar deviation is seen by BaBar [134] in the low-$q^2$ region, although with larger uncertainties. Belle do not report the low and high $q^2$ values separately, getting a result consistent with unity [112].

These results can be interpreted as an indication of a new vector current that couples more strongly to muons and interferes destructively with the Standard Model vector current [135-138]. More data are needed to confirm (or not) this result.

In the meantime, similar decays can be used to perform tests of lepton universality in $B^0 \rightarrow K^{*0} \ell^+ \ell^-$, $B_s^0 \rightarrow \phi \ell^+ \ell^-$, $A_b^0 \rightarrow A \ell^+ \ell^-$, $A_b^0 \rightarrow pK^- \ell^+ \ell^-$, which are all accessible by the LHCb experiment, but some will have very limited yields. These measurements are complementary
to that of $B^+ \rightarrow K^+ \ell^+ \ell^-$, as the different spins of the hadronic component probe different New Physics couplings [139].

Given the hints of lepton flavour universality violation between muons and electrons, it seems natural to wonder if such an effect can be seen in processes involving the third generation ($\tau$) lepton. This has been tested in $B \rightarrow D^{(*)} \tau \nu_{\tau}$ decays, comparing to the same decay with muons or electrons instead. Unlike the decays described above, the Standard Model contribution to this decay is not suppressed (and does not match the definition of a rare decay). It proceeds via a tree-level $b \rightarrow cW^-$ transition, with the $W^-$ decaying to a lepton and a neutrino. The Standard Model expectation is that the rates for the decays involving electrons, muons and tau leptons differ only due to phase-space effects (plus small effects due to form factors). The ratios, denoted $R_{D^{(*)}}$, are well predicted theoretically [140–143]. The results for $r_D$ and $r_{D^*}$ from BaBar [144,145], Belle [146–148] and LHCb [149] show an intriguing pattern, with values larger than the Standard Model prediction. This could indicate the presence of new couplings preferring $\tau$ leptons.

## 5 Wilson coefficient fits

This section briefly describes some constraints on Wilson coefficients, as of end 2015. It relies on Section 2.

Several groups have performed model-independent fits of Wilson coefficients, using most of the experimental results presented above. See Refs. [116–118,123,150–157] for a representative subset. The fits differ by the set of experimental results used, the statistical treatment of uncertainties and choices of form factors. Another major difference is the level of trust of computations of quark loops (most notably $c\bar{c}$ loops) incorporated in the fit. Depending on these choices, the determined tension with the Standard Model ranges from one to several standard deviations.
In all cases, the New Physics scenario which is preferred changes the value of the $C_9$ coefficient (adding a non-zero term $C_{9\mu}^{NP}$). This term could then be different depending on the flavour of the involved leptons (introducing $C_{9e}^{NP}$ and $C_{9\mu}^{NP}$), thus breaking lepton universality, see Fig. 15. The data are not conclusive yet, but a tension with the Standard Model point at $(0, 0)$ is visible. The significance of this tension depends on the assumed theory uncertainties.

Another popular model is to assume that the weak interaction $V-A$ structure holds in New Physics and thus to impose $C_{9e}^{NP} = -C_{10e}^{NP}$. The data are consistent with such a hypothesis, but again it is too early to draw conclusions.

Right-handed components are also added in the fits, in particular using asymmetries in $b \to s\ell^+\ell^-$ decays that are sensitive to such effects. Presently there is no evidence for any significant need for right-handed currents.

There is a plethora of model-dependent interpretations of these findings. The deviations can be accommodated by supersymmetry [120], models with new vector bosons [115, 158, 159], two Higgs doublets [160], scalar interactions [119] or leptoquarks [136, 161–163], to name a few.

6 Prospects

At the risk of stating the obvious, rare decays have the advantage of being rare. This ensures that the experimental precision will stay dominated by statistical uncertainties, and thus will not run into a limit imposed by irreducible systematic uncertainties. The theoretically cleanest measurements, like the lepton-universality ratios $R_{Xs}$ and the ratio of $B^0 \to \mu^+\mu^-$ to $B_s^0 \to \mu^+\mu^-$ branching fractions will continue to be of interest as more data are acquired at the LHC and by Belle II. The future will tell us if the deviations from expectations hold and tell us something new about Nature.

Other measurements, like branching fractions (for instance $b \to s\gamma$) have already reached the theoretical precision and more work is needed on this side to allow more precise comparisons of experimental values and Standard Model predictions. Finally, asymmetries in $B \to K^*\ell^+\ell^-$ are in between. If the presently measured central values stay while the uncertainties reduce, we may soon be in the situation of having to understand a very significant deviation with predictions based on the Standard Model. More investigations of theory uncertainties are needed before any conclusion can be reached.

7 Conclusion

Rare decays provide a useful tool to search for physics beyond the Standard Model. Many intriguing results hinting at New Physics come from rare decay measurements at the $B$ factories
or LHCb. These measurements do not tell us straight away which kind of New Physics could cause the seen deviations, but allow for model-independent analyses describing the common features of possible explanations. This is in turn needed for model building. Recent results, especially about $B \rightarrow \mu^+\mu^-$ and $b \rightarrow s\ell^+\ell^-$ decays, have triggered a lot of new models that may be confirmed by the observation of on-shell new particles if those are within reach of present colliders.

The Run 2 of the Large Hadron Collider is now ongoing and the amount of rare $b$-hadron decays collected by the LHC experiments will increase rapidly. Also, the Belle II experiment will start in 2018. Improved measurements of the processes described in this article, and also new complementary measurements, will become available and will lead to improved precision which will be useful in global fits.

Acknowledgements

The authors would like to thank Lydia Roos, Andreas Hoecker and Tim Gershon for useful comments.

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