Top-Goldstone coupling spoils renormalization of Higgs inflation

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\textbf{ABSTRACT}

We examine renormalization of Higgs inflation in the context of the full Standard Model. In the fermionic sector of the theory there is a parametrically large top-Goldstone coupling which prevents renormalization of the theory. Using a simplified model with a global U(1) symmetry, a Higgs and a fermion, we show that the one-loop contribution to 4-Goldstone scattering cannot be absorbed in any tree level terms, and hence forbids a consistent renormalization of the theory. Our results apply for large non-minimal Higgs-gravity coupling in the large field regime, and indicate that Higgs inflation is not a predictive theory.

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1 Introduction

In Higgs inflation the Higgs field of the Standard Model (SM) is coupled non-minimally to gravity \[1, 2, 3, 4, 5, 6\]. Apart from this single non-minimal coupling, no new physics is needed to describe inflation and the subsequent period of reheating, and the theory seems to be extremely predictive. But, as we shall discuss in this paper, the interplay of the non-minimal coupling, the Goldstone degrees of freedom and the fermionic sector leads to a one-loop correction that does not cancel and which cannot be absorbed in tree level terms. This renders the theory non-renormalizable and ruins its predictivity in the inflationary regime.

The idea of using the Higgs as the inflaton is an attractive one, not least because it allows one to connect collider observables with measurements of the early universe. Since its inception the model itself has come under a lot of scrutiny and criticism. First, unitarity is lost at high energies and the perturbative theory can only be trusted for energies below the unitarity cutoff \[7\]. Although it is uncertain how to interpret this result as the cutoff is field dependent (according to \[6, 8\], all relevant physical scales are always below the unitarity bound), it is clear that any new physics living at this scale may affect the inflationary predictions. Second, given the currently measured central values for the top and Higgs mass, the Higgs potential becomes unstable around \(10^{11}\) GeV, which would be disastrous for Higgs inflation. However, the verdict is not yet out, as it only takes \(2 - 3\sigma\) deviations to push the instability bound all the way to the Planck scale \[9, 10, 11, 12, 13\]. Finally, there is the recent BICEP2-claim that inflationary gravitational waves have now been seen \[14\], which is however still under debate \[15\]. If BICEP2’s result stands, this will definitively rule out minimal Higgs inflation, which predicts a tensor-to-scalar ratio more than a hundred times smaller than the BICEP2 result. See for example the discussion in \[16\].

Even though all of these claims are still debated, and SM Higgs inflation may still be alive, it is worth noting that constraints may be avoided in modified set-ups with an extended Higgs sector. Moreover, it was recently claimed that for very specific top and Higgs masses even the BICEP2 result can be accommodated \[17, 18, 19\]. In this work we focus on the original Higgs inflation model and study its renormalization properties. Our results apply for large non-minimal coupling, but apart from that they are equally applicable to the various implementations of Higgs inflation.

The renormalization group equations (RGEs) in Higgs inflation have been derived by several groups \[4, 5, 20, 21, 22, 23\], but they differ in details. The main source of disagreement comes from the choice of frame, and the treatment of the Goldstone bosons. In previous work \[21, 25\] we have shown that the Jordan and Einstein frame describe exactly the same physics, and any difference comes from an erroneous comparison of quantities defined in different frames. In this work we will look closer at the one-loop corrections involving the Goldstone bosons, with the aim to clarify the differences in the literature. Our calculations show that some loop corrections are large, and moreover have a background field dependence different from the tree level results. We argue that there is no consistent way to add counterterms that absorb the divergent parts of the loop corrections. Therefore it seems that the effective field theory (EFT) in the inflationary regime is non-renormalizable.\footnote{Higgs inflation is non-renormalizable as the field space metric and potential are non-polynomial. But this does not exclude that the theory is renormalizable in the EFT sense (as is the case in the IR). Our demands are that in the large field regime the theory can be expanded in a small parameter \(\delta\), and that all loop corrections can be absorbed in counterterms order by order. Truncating the theory at some finite order in \(\delta\) gives a renormalizable EFT with a finite number of counterterms.}
The small-field regime of Higgs inflation is where \( \phi_0 \ll m_p/\xi \), with \( \phi_0 \) the value of the background Higgs field, \( \xi \) the non-minimal Higgs-gravity coupling which is of order \( 10^4 \), and \( m_p \) the Planck mass. In this regime the theory is effectively like the SM and therefore renormalizable in the EFT sense. In the mid-field regime \( (m_p/\xi < \phi_0 < m_p/\sqrt{\xi}) \) the Higgs-part of the theory becomes non-renormalizable, dependent on some unknown underlying UV theory. However, since the dominant fermion part of the theory is expected to still be well-behaved, and since this regime covers only a small field range, it is hoped that one can match the running in the small and large field regime with relatively small matching corrections to include the effects of the running in the mid field regime. Naively one could expect that in the large field regime \( (\phi_0 \gg m_p/\sqrt{\xi}, \text{corresponding to inflation}) \) the situation is even worse than in the mid-field regime. Here, however, the potential has an approximate shift symmetry, which can restrict the form of the loop corrections. For the Higgs field in isolation one finds indeed that all one-loop corrections can be absorbed in the parameters of the classical theory, and the EFT is renormalizable.

In [24] we studied the renormalization of the non-minimally coupled Higgs field in isolation, without any gauge or fermion fields, and our findings were in line with the literature. In this work we want to extend this previous analysis to the full SM. At first glance this does not seem to be problematic. Due to the non-minimal coupling to gravity, the coupling of the radial Higgs to both gauge field and fermions is suppressed in the large field regime. One can simply neglect all diagrams with these couplings. For example, loop diagrams with a fermion or gauge boson loop always dominate over the corresponding diagram with a Higgs loop. Effectively the Higgs decouples from the theory. However, the situation for the Goldstone bosons (GBs) is more complex: their coupling to the gauge fields is also suppressed, while the GB-fermion coupling is not. Upon going to unitary gauge, this corresponds to a coupling of the fermion to the longitudinal polarization of the gauge fields, and both the transverse and longitudinal polarizations couple with the usual SM strength to the fermions.

Thus, for the calculation of the dominant quantum corrections in the large field regime, only the GB-fermion needs to be considered as far as Higgs interactions are concerned. To concentrate on this coupling and focus on the essential physics, we decided to calculate the loop corrections in a simplified set-up with a complex, non-minimally coupled Higgs field coupled to a fermion.

All calculations are performed in the Einstein frame. For a discussion of the equivalence of Einstein and Jordan frame, see [24, 25]. One of the main complications in the calculation is that after transforming to the Einstein frame one ends up with non-canonical kinetic terms for the Higgs and Goldstone field. Due to the nonzero curvature of the field space, it is impossible to make a field transformation that brings the kinetic terms to their canonical form. Our approach here is to expand the action around a large classical background value for the inflaton field, and use the formalism of [26, 27] so that this background expansion can be done maintaining covariance in the field space metric. In our calculation we have neglected the time dependence of the background field, as well as FLRW corrections and the back reaction from gravity. These corrections can be taken into account, but we argue that they are subleading in the expansion parameter. Our main result is that in the scattering of an even number of GBs the dominant loop corrections are parametrically larger than the tree level result and cannot be absorbed consistently in counterterms. This renders the EFT non-renormalizable.

The outline of this article is as follows. To set the notation we start in Sec. 2 with a short review of SM Higgs inflation. We estimate the couplings of the Higgs and the GBs to the
other fields in the theory, and show that all are suppressed except for the GB-top coupling. We also review the covariant formalism developed by [26, 27]. In Sec. 3 we study a toy model in which a real Higgs field is coupled to a fermion. In this setup, we can test the covariant formalism, as we can also compute the relevant couplings directly by moving to canonically normalized fields and compare the answers. The main point of the paper comes in Sec. 4, where we study a complex Higgs field coupled to a fermion. After computing the relevant couplings with the covariant formalism, we compute the leading one-loop divergent contributions to the Goldstone boson’s four point function. We find that the one-loop corrections are parametrically larger than the tree-level expressions. We perform some checks to gain confidence in our result. In Sec. 5 we try to absorb the divergent parts of the loop corrections found in the previous section into new counterterms. We find that this procedure spoils the theory of Higgs inflation. We conclude in Sec. 6.

All sign conventions used in this paper follow the QFT textbook by Srednicki [28], except for the sign of the Yukawa interaction terms, which is opposite to Srednicki’s.

2 Higgs inflation

In this section we give a brief overview of Higgs inflation, set our notation and derive expressions for masses and couplings, and review the covariant formalism that will be used in latter sections of the paper.

2.1 Jordan frame Lagrangian

The Jordan frame Lagrangian is (using $-+++$ metric signature)

$$\mathcal{L}_J = \sqrt{-g^J} \left[ \frac{1}{2} m_p^2 \left( 1 + \frac{2 \xi \Phi^\dagger \Phi}{m_p^2} \right) R[g^J] + \mathcal{L}_{J SM}^J \right].$$

The gauge-Higgs part of the SM Lagrangian is

$$\mathcal{L}_{J SM}^J = \mathcal{L}_{J gauge}^J + \mathcal{L}_{J higgs}^J + \mathcal{L}_{J ferm}^J,$$

with

$$\begin{align*}
\mathcal{L}_{J gauge}^J &= -\frac{1}{4} (f_{\mu\nu})^2 - \frac{1}{4} (C_{\mu\nu}^a)^2 - \frac{1}{4} B_{\mu\nu}^2, \\
\mathcal{L}_{J higgs}^J &= -(D_\mu \Phi)\dagger (D^\mu \Phi) - V_J (\Phi^\dagger \Phi), \\
\mathcal{L}_{J ferm}^J &= \bar{Q}_L (i \slashed{D}) Q_L + \bar{u}_R (i \slashed{D}) u_R - y_d \bar{Q}_L \Phi d_R - y_u \bar{u}_R \Phi^\dagger (-i \sigma_2) Q_L + \text{h.c.}.
\end{align*}$$

The first line [2] gives the kinetic terms for the SU(3), SU(2) and U(1) gauge fields respectively.

The second line [3] gives the Higgs $\Phi$ kinetic terms and potential. The Higgs field is a $SU(2)$ complex doublet, which contains the Higgs plus 3 Goldstone bosons (which are eaten by $W^\pm$, $A$ fields) for a total of 4 real degrees of freedom. It is well-known in the literature – see for example [29, 30, 31] – that working in the unitary gauge mixes the orders of a loop expansion. Therefore we explicitly keep the Goldstone bosons. We write

$$\Phi = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \theta_2 + i \theta_1 \\
\phi_0 + \phi + i \phi_3 \end{array} \right)$$

with $\phi_0$ the classical background. The covariant derivative for the Higgs field is

$$D_\mu \Phi = (\partial_\mu + ig C_{\mu}^a \tau^a + i Y g' B_\mu) \Phi$$

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with \( \tau^a = \sigma^a / 2 \) for the doublet representation, and \( Y = 1/2 \) for the Higgs doublet.

Finally, the third line \(^4\) gives the kinetic terms and Yukawa interactions for the SM fermions. To find the dominant renormalization group running of the SM’s couplings we are only interested in the top quark. The covariant derivative is

\[
D_\mu Q_L = (\partial_\mu + ig s \tilde{f}_a \tau^a + ig C_\mu \tau^a + i Y_Q g' B_\mu) Q_L, \\
D_\mu u_R = (\partial_\mu + ig s \tilde{f}_a \tau^a + i Y_u g' B_\mu) u_R.
\] (7)

The hypercharges are \( Y_Q = 1/6 \) and \( Y_u = 2/3 \).

We add no gauge fixing or ghost Lagrangian in the Jordan frame. The standard expression for the gauge fixing term (e.g. \( R_\xi \) gauge) does not work in the Jordan frame, as the Higgs field mixes with gravity. It is simpler to first transform to the Einstein frame and do the gauge fixing there. Since both frames are equivalent \(^{24, 25}\), one could of course deduce the Jordan frame gauge fixing term by an inverse conformal transformation.

### 2.2 Einstein frame Lagrangian

We reach the Einstein frame after a conformal transformation:

\[
\sqrt{-g^E} \left\{ \frac{1}{2} m_p^2 R[g^E] + L_{E_{\text{SM}}}^E \right\}
\] (9)

The Einstein frame Lagrangian becomes

\[
L^E = \sqrt{-g^E} \left[ \frac{1}{2} m_p^2 R[g^E] + L_{E_{\text{SM}}}^E \right]
\]

with \( L_{E_{\text{SM}}}^E = L_{E_{\text{gauge}}}^E + L_{E_{\text{Higgs}}}^E + L_{E_{\text{fermion}}}^E \). We can also add a gauge fixing and ghost Lagrangian, but we do not need its specific form in this paper.

We neglect the expansion of the universe, and take a Minkowski metric. The gauge kinetic terms are conformally invariant and \( L_{E_{\text{gauge}}}^E = L_{E_{\text{gauge}}}^J \) as given in \(^2\). The fermionic kinetic terms also remain of the standard form, although with rescaled masses (via rescaled Yukawa couplings):

\[
L_{E_{\text{fermion}}}^E = \bar{Q} E_L i \bar{\gamma} Q^E_L + \bar{u} E_R i \bar{\gamma} u^E_R - \frac{y_u}{\Omega} u^E_R \Phi \Phi (-i \sigma_2) Q^E_L + h.c + \ldots
\] (10)

where we rescaled the Einstein frame fermion fields \( \psi^E = \psi / \Omega^{3/2} \) to get canonical kinetic terms.

All non-trivial effects of the non-minimal coupling are in the Higgs sector. The Higgs part of the Lagrangian becomes

\[
L_{E_{\text{higgs}}}^E = -\frac{1}{\Omega^2} (D_\mu \Phi \Phi)^\dagger (D^\mu \Phi) - \frac{3\xi^2}{2 m_p^2 \Omega^4} \partial_\mu (\Phi \Phi)^\dagger \partial^\mu (\Phi \Phi) - V_E.
\] (11)

\(^2\)For our purposes it is sufficient to consider the Minkowski theory. Otherwise, for FLRW use \( \gamma^a = \epsilon^a_b \gamma^b \) the Minkowski gamma-matrices, and \( \epsilon^a_b \) the vierbein. To make the derivative covariant with respect to gravity replace \( \partial_\mu \psi \rightarrow (\partial_\mu + W_\mu) \psi \) with \( W_\mu = 1/4 \omega_{ab\mu} \gamma^a \gamma^b \) and \( \omega_{ab\mu} \) the spin connection. Explicitly, in conformal FLRW \( W_0 = 0 \) and \( W_i = (1/2) H \gamma^0 \gamma^i \).
Here we have defined the Einstein frame potential as $V_E = V_J/\Omega^4$. The Higgs kinetic term is non-minimal. Let $\chi_i = \{\varphi_R = \phi_0 + \varphi, \theta_i\}$ run over the Higgs field and Goldstone bosons. Then the metric in field space in component form is

$$L_{\text{higgs}}^E \supset -\frac{1}{2} \gamma_{ij} \partial \chi_i \partial \chi_j = -\frac{1}{2} \left[ \delta_{ij} + \frac{6 \xi^2}{m_p^2 \Omega^4} \chi_i \chi_j \right] \partial \chi_i \partial \chi_j. \quad (12)$$

The curvature on field space $R[\gamma_{ij}] \neq 0$, and the kinetic terms cannot be diagonalized. At most one can diagonalize the quadratic kinetic terms at one specific point in field space (just as in general relativity, where one can go locally to a Lorentz frame, but not in all of space if the curvature is non-zero).

Consider now the electroweak sector. For the gauge bosons the kinetic terms remain canonical in the Einstein frame. As far as the quadratic action is concerned the action for the massive gauge bosons and Goldstone bosons is simply three times the action of the U(1) toy model studied in [31]. In addition we have one massless gauge boson corresponding to unbroken electromagnetism. To see all this explicitly, consider the Higgs kinetic terms

$$-\frac{1}{\Omega^2} |D_{\mu} \Phi|^2 = -\frac{1}{2 \Omega^2} \left[ \frac{1}{2} \partial_{\mu} \phi_R \partial^{\mu} \phi_R + \sum_{a=1}^{3} (\partial_{\mu} \theta_a \partial^{\mu} \theta_a + 2g_a A_a^{\mu}(\phi_R \partial^{\mu} \theta_a - \theta_a \partial^{\mu} \phi_R) + g_a^2 \phi_R^2 A_a^{\mu} A_a^{\mu}) + \ldots \right]. \quad (13)$$

We only wrote the quadratic terms needed to identify the mass eigenstates, denoted by $A^a = \{C_1, C_2, Z, A_\gamma\}$ with

$$Z = \frac{1}{\sqrt{g^2 + g'^2}} (-gC^3 + g'B), \quad A_\gamma = \frac{1}{\sqrt{g^2 + g'^2}} (-g'C^3 - gB) \quad (14)$$

and couplings

$$g_a = \frac{1}{2} \times \left\{ g, g, \sqrt{g^2 + g'^2}, 0 \right\}. \quad (15)$$

(The overall factor arises because Higgs field components have hypercharge 1/2 and SU(2) charges ±1/2.) This corresponds to three massive and one massless field. Note that we wrote the Lagrangian in terms of the mass eigenstates, which correspond to the real and imaginary parts of $W_+$, rather than the complex states $W_{\pm}$.

Inflation takes place for field values $\phi_0 \gg 1/\sqrt{\xi}$. This is the regime that we will study in this paper. In this “large-field” regime we have $\Omega \neq 1$ and $\gamma_{ij} \neq \delta_{ij}$. A suitable expansion parameter that we will use throughout the paper is

$$\delta \equiv \frac{m_p^2}{\xi \phi_0^2} \ll 1. \quad (16)$$

The expansion in $\delta$ is equivalent to an expansion in slow-roll parameters, since $\eta = O(\delta)$ and $\epsilon = O(\delta^2)$.

### 2.3 Masses and interactions

From now on we will work in the Einstein frame, and drop all sub/superscripts $E$. We further set $m_p = 1$, and assume $\xi \gg 1$ (which is the case for standard Higgs inflation).
Figure 1: Particle masses $|m^2(\phi_0)|$ as a function of the Higgs vev $\phi_0$. Shown are the real Higgs mass (red), the Goldstone boson mass (green) and the gauge boson mass (blue). The top mass scales like the gauge boson mass. In addition the Hubble scale $H^2$ is indicated (cyan). The two vertical lines correspond to $\phi_0 = 1/\sqrt{\xi}$ and $\phi_0 = 1/\xi$ respectively. Here we used $\lambda = 0.1, g = 0.5, \xi = 2 \times 10^4$.

### 2.3.1 Masses

To extract the masses we need the quadratic action. We can look at scalars (Higgs and GBs), gauge fields and fermions separately.

For non-canonical kinetic terms the scalar masses can be computed using the covariant generalization of $\partial^2 V(\phi)$:

$$
(m^2)^i_j = \gamma^{ik} D_k D_j V(\phi) = \gamma^{ik} (\partial_k \partial_j V - \Gamma_{lj}^k \partial_l V),
$$

which should be evaluated on the background. For the Higgs and GB masses this gives

$$
m^2_h = 3 \lambda \phi_0^2 (1 + 4 \xi^2 \phi_0^2 - 4 \xi^4 \phi_0^2) = 3 \lambda \phi_0^2, \quad m^2_{\theta} = \lambda \phi_0^2 \frac{1}{1 + 6 \xi^2 \phi_0^2} \approx \lambda \phi_0^2, \quad m^2_{\phi} \approx \lambda \phi_0^2 \frac{1}{6 \xi^2}, \quad m^2_{\phi} \approx \lambda \phi_0^2 \frac{1}{6 \xi^4},
$$

where in the most-right expressions the leading terms in the small-, mid- and large-field regimes are shown. The Higgs mass squared is negative during inflation. The potential is convex, leading to a red tilted spectral index in excellent agreement with the Planck data.

Moving to the gauge bosons, the gluons remain massless. Consider then the electroweak sector. The gauge boson masses can be read of from the $|D_\mu \Phi|^2/\Omega^2$ term in the Lagrangian, which gives

$$
m^2_A = \frac{g_a^2 \phi_0^2}{\Omega^2} = \left\{ g_a^2 \phi_0^2, g_a^2 \phi_0^2, g_a^2 \phi_0^2 \frac{g_a^2}{\xi} \right\}.
$$
Finally we look at the fermionic part of the theory. As we explained before, we are only interested in the top quark, which, thanks to its large Yukawa coupling, gives the dominant fermionic contribution to the running of all masses and couplings. After rescaling the spinor field to make it canonical the net result is that in the Einstein frame the fermion mass is \( m^{E}_\psi = m^J_\psi / \Omega \). For the top quark we get

\[
m^2_t = \frac{y^2_t \phi^2_0}{2 \Omega^2_0} = \frac{1}{2} \left\{ y^2_t \phi^2_0, y^2_t \phi^2_0, \frac{y^2_t}{\xi} \right\},
\]

with \( y_t \) the Yukawa coupling and the factor half a convention. See Fig. 1 for a sketch of how the masses behave as a function of the background \( \phi_0 \).

### 2.3.2 Interactions

Since the fermion and gauge sector are of the standard form, the top-gauge interactions are as in the SM. On the contrary, all Higgs-gauge and Higgs-top couplings are affected by the non-minimal coupling.

As we noted above, it is impossible to diagonalize the kinetic terms and identify the canonically normalized fields. Nevertheless, we can estimate the strength of the various interactions by introducing the “quasi-canonical” fields that diagonalize the quadratic action. Since the field space metric is diagonal when evaluated on the background, the quasi-canonical fields are

\[
\chi^{qc}_i = \sqrt{\gamma^{bg}_{ii}} \chi_i.
\]

This procedure is not exact, but should give us the parametric behavior of the couplings, which is enough to determine which interactions are suppressed. When calculating loop corrections in the next section, we will adopt the formalism developed by [26, 27] that is fully covariant with respect to the field space, and bypasses the need to define canonical fields.

**Gauge-top interaction**  The gauge-top interaction is standard:

\[
\lambda_{ttA} \sim -\partial_t \partial_t A \mathcal{L}|_{0} = g \{1, 1, 1\}
\]

where as before the right three terms on the right correspond to the small-, mid- and large-field regimes.

**Higgs-top interaction**  The Yukawa interaction written out in component fields is\(^3\)

\[
\mathcal{L}_{\text{ferm}} \supset -y_t \bar{t}_R \Phi^I (-i \sigma_2) Q_L + \text{h.c.} = \frac{y_t}{\sqrt{2} \Omega} \left( \bar{t}_R (\theta_2 - i \theta_1) b_L - \bar{t}_R (\phi_R - i \theta_3) t_L \right) + \text{h.c.}.
\]

From this we can read off the Higgs-top coupling

\[
\lambda_{\phi t_L t_R} \sim -\sqrt{\frac{1}{6} \xi \phi_0} \partial_t \partial_t \Phi \partial_{t_i} \mathcal{L}_{\text{ferm}}|_{bg} = \frac{y_t}{\sqrt{2}} \left\{ 1, \frac{1}{\sqrt{6} \xi \phi_0}, \frac{1}{\sqrt{6} \xi^{3/2} \phi_0^2} \right\}
\]

---

\(^3\)The \( tt \) interaction is \( \mathcal{L}_{\text{yuk}} = \frac{y_t}{\sqrt{2} \Omega} \left( \phi_R t + i \theta_3 \bar{t} \gamma^5 t \right) \) with \( t = (t_L, t_R)^T \). The \( bt \) interaction includes left/right handed projectors as well.
where the $\sqrt{\gamma \phi \phi}$ enters as we are interested in the coupling to the quasi-canonical field \[21\]. The coupling to the quasi-canonical Goldstone bosons is of the form
\[
\lambda_{b_L L R b_L} \sim \frac{\gamma_{\theta}}{\sqrt{2}} \{1, 1, 1\}, \quad \lambda_{b_L L R b_R} \sim -i \frac{\gamma_{\theta}}{\sqrt{2}} \{1, 1, 1\}, \quad \lambda_{t_L t_R t_L} \sim i \frac{\gamma_{\theta}}{\sqrt{2}} \{1, 1, 1\}.
\]
(25)

This coupling is unsuppressed in the mid- and large-field regime. Furthermore, we now have fermion couplings to two, three, and four Goldstone bosons as well. These couplings are induced by the non-minimal coupling: it is the presence of the factor of $\Omega$ in the denominator of (23) that generates them.

**Higgs-gauge coupling**  The quasi-canonical Higgs-gauge couplings follows from
\[
\lambda_{\chi_i \chi_j A^2} = -1/4 \sqrt{\gamma \gamma^{ij} D_i D_j \partial_A \partial_A \mathcal{L}} |_{\text{bg}} = \frac{1}{2} \gamma^{ik} (\partial_k \partial_j m^2_A - \Gamma_{kj}^i \partial_l m_A^2) |_{\text{bg}},
\]
(26)

Note the use of covariant derivatives for the Higgs field. This is especially important for the GB-gauge coupling in the large field regime, as the leading term cancels between the derivative and connection term in the expression above — this mirrors what happens in the mass formula for the GB.

Explicitly for the Higgs and GB couplings
\[
\lambda_{\phi^2 A^2} = g^2 \{1, \frac{1 - 12 \phi^4 \xi^3}{36 \phi^4 \xi^4}, -\frac{1}{3 \phi^4 \xi^2}\},
\lambda_{\theta^2 A^2} = g^2 \{1, \frac{1}{6 \phi^2 \xi^2}, -\frac{1}{6 \phi^2 \xi^2}\},
\]
(28)

The cubic derivative coupling is
\[
|\lambda_{\phi \partial_\theta A} | = |\lambda_{(\partial_\phi) \theta A}| = \left| \sqrt{\gamma \phi \phi} \gamma^{\phi \phi} D_\theta \partial_\phi \partial_A \mathcal{L} \right|_0 = g \{1, \frac{1}{\sqrt{6} \phi \xi}, \frac{1}{\sqrt{6} \phi \xi}\},
\]
(29)

which is also suppressed for $\xi \gg 1$.

In addition there are Higgs/GB self-couplings. These are also suppressed, for example $\lambda_{4\phi} \sim -2(2\lambda)/(9\phi^4 \xi^2)$.

In summary, the couplings of the quasi-canonical fields to the gauge and fermion fields are all suppressed, except for the top-Goldstone coupling which remains unsuppressed. This is the most important finding of this section. The contribution of the Higgs sector to the one-loop corrections will be dominated by diagrams involving the top-Goldstone coupling.

To isolate this effect we will study in Sec. 4 the renormalization of a complex Higgs field coupled to a fermion.

### 2.4 Covariant notation

This subsection reviews the covariant formalism introduced in [26] and further worked out in [27]. Given the curvature of field space, it is very convenient to adopt an approach that maintains the covariance of the equations.

\[\text{The GB mass is } m^2_\xi = \left[ \gamma^{\phi \phi} (\partial_\phi V - \Gamma^{\phi \phi}_\phi \partial_\phi V) \right]_0.\]
(27)

where the connection term is now $\Gamma^{\phi \phi}_\phi = (1/2) \gamma^{\phi \phi} (2 \partial_\phi \gamma \phi - \partial_\phi \gamma \phi)$. All $\mathcal{O}(1/(\phi^4 \xi^2), 1/(\phi^4 \xi^3), 1/(\phi^4 \xi^4))$ terms cancel between the two terms, and the first non-zero contribution in the large field expansion is $\mathcal{O}(1/(\phi^4 \xi^4))$. 

9
The Einstein frame action for a complex Higgs field coupled to a fermion is

\[ S = \int d^4x \sqrt{-g} \left[ \frac{m_F^2}{2} R - \frac{1}{2} \gamma_{ab} \partial_{\mu} \phi^a \partial^{\mu} \phi^b - i \bar{\psi} \gamma^5 \psi - V(\phi^a) - \bar{\psi} F(\phi^a) \psi \right]. \]  

(30)

The potential and Yukawa interactions are:

\[ V(\phi^a) = \frac{\lambda}{4} |\phi_0 + \varphi + i \theta|^4, \quad F(\phi^a) = \frac{y}{\sqrt{2}} \frac{\phi_0 + \varphi + i \gamma^5 \theta}{\Omega}, \]

(31)

where we used the notation of footnote 3 for the Yukawa interaction. We expand the Lagrangian around the background \( \phi^a = (\phi_0(t) + \varphi(x,t), \theta(x,t)) \) in the large-field regime (16). The fluctuation fields \( \delta \phi^a = (\varphi, \theta) \) are not in the tangent space at \( \phi^a_0 \), and therefore do not transform as a tensor. We are led to introduce the covariant fluctuation \( Q^a = (\bar{\varphi}, \bar{\theta}) \), which is related to \( \delta \phi^a \) via

\[ \delta \phi^a = Q^a - \frac{1}{2!} \Gamma_{bc}^a Q^b Q^c + \frac{1}{3!} (\Gamma_{be}^c \Gamma_{cd}^e - \Gamma_{bc}^e \Gamma_{de}^c) Q^b Q^c Q^d + ... \]

(32)

Further we define the covariant time derivative

\[ D_t = \frac{d\phi^a}{dt} \nabla_a = \dot{\phi} \nabla_\phi. \]

(33)

Now we can expand the action in covariant fluctuations. We neglect FLRW corrections and the back reaction from gravity, as well as the time-dependence of the background field \( \phi_0 \); we come back to this in Sec. 4.2. After integration by parts and dropping total derivatives, the result is

\[ S = \int d^4x \left[ \frac{1}{2} \bar{D}_t Q^a D_t Q^a - \frac{i}{2} \bar{\psi} i \partial^a \psi + D_t \bar{\psi} \partial^a \psi + (V + V_{\varphi} Q^a + \frac{1}{2!} V_{\varphi \varphi} Q^a Q^b + ...) - \bar{\psi} (F + F_{\varphi} Q^a + \frac{1}{2!} F_{\varphi \varphi} Q^a Q^b + ...) \psi \right]. \]

(34)

All coefficients are evaluated on the background. The subscript with a semi-colon denotes the covariant derivative. Demanding the tadpole to vanish gives the equation of motion of the background field.

Now we can derive the Feynman rules from the above action. First we define the effective couplings

\[ \mathcal{L}_\text{c} = - \lambda_{m\sigma n\theta} \bar{\phi}^m \bar{\theta}^n - y_{m\sigma n\theta} \bar{\phi}^m \bar{\theta}^n \bar{\psi}(i \gamma^5)^\alpha \psi \]  

(35)

with \( \alpha = 1 \) if the number \( n \) = odd, and \( \alpha = 0 \) otherwise (signs are absorbed in the couplings). This means that for a vertex with \( m \) \( \bar{\phi} \)-fields and \( n \) \( \bar{\theta} \)-fields and with or without fermion lines we have, respectively:

\[ V^{(m\sigma n\theta)} = (-i)m!n! \lambda_{m\sigma n\theta}, \quad V^{(m\sigma n\theta2\psi)} = (-i)m!n! y_{m\sigma n\theta} (i \gamma^5)^\alpha. \]  

(36)

For each fermion and scalar propagator we add

\[ -iS = \frac{-i(-k + m_\psi)}{k^2 + m_\psi^2}, \quad -iD_{ab} = \frac{-i}{k^2 + (m^2)^a_b}. \]

(37)

with masses \( m_\psi = F \) and \( (m^2)^a_b = \gamma^a c V_{ab} \). The scalar mass is diagonal, which we used in the scalar propagator.
**U(1) symmetry** The original Jordan frame Lagrangian possesses a U(1) global symmetry which rotates the complex $\Phi$ field (in the SM it would be a gauge symmetry). In the broken phase with $\phi_0 \neq 0$ this symmetry is still non-linearly realized. This non-linearly realized symmetry is still present in the model when expressed in the covariant $Q^a$ variables, but it is represented in an intricate way. The reason is that the $Q^a$-fields and the original fields are related by a non-linear field transformation (32). The U(1) acts on the $Q^a$-fields as

$$\bar{\phi} \to \bar{\phi} - \alpha \frac{\delta}{6\xi} \bar{\theta} + \ldots, \quad \bar{\theta} \to \bar{\theta} + \alpha(1/\sqrt{\delta \xi} + \delta \bar{\phi}) + \ldots, \quad \psi \to \psi - i\frac{\alpha}{2} \gamma^5 \psi$$

where the ellipses denote terms higher order in the fields, and $\alpha$ is the infinitesimal symmetry parameter.

### 3 Real Higgs field plus fermion

Our analysis of Higgs inflation in the full Standard Model has suggested that the danger for renormalizability lies with the top-Goldstone coupling. Therefore we now narrow our analysis down to a theory in which a Higgs field is coupled to a single fermion. In this section, we study the case of a real Higgs field, allowing us to identify the canonically normalized field explicitly. This gives us a handle to check the validity of the covariant approach. In the next section we then move to the case of a complex Higgs field, where we have only the covariant approach at our disposal. Another advantage of this “real Higgs training section” is that we can compare results from three- and four-point scattering with the Coleman-Weinberg approach. That gives a check that approximations and symmetry factors etc. are under control.

#### 3.1 Canonical field

The canonical field $h$ is related to the original field $\phi$ via

$$h = \sqrt{6} \ln \left( \phi \sqrt{\xi} \right) + \sqrt{3/2}, \quad \leftrightarrow \quad \phi = \frac{1}{\sqrt{\xi}} e^{(h-\sqrt{3/2})/\sqrt{6}}. \quad (39)$$

Expanding in the canonical field $h$ the Lagrangian becomes

$$L = -\frac{1}{2} (\partial h)^2 - \frac{\delta \lambda}{\xi^2} \left( -\frac{h^4}{108} + \frac{h^3}{9\sqrt{6}} + \ldots \right) - \frac{y \delta}{\sqrt{\xi}} \left( -\frac{h^4}{108\sqrt{2}} + \frac{h^3}{18\sqrt{3}} - \frac{h^2}{6\sqrt{2}} + \frac{h}{2\sqrt{3}} + \ldots \right) \bar{\psi} \psi$$

$$= -\frac{1}{2} (\partial h)^2 - (\lambda_{4h} h^4 + \lambda_{3h} h^3 \ldots) - (y_{4h} h^4 + y_{3h} y^3) \bar{\psi} \psi + \ldots \quad (40)$$

This results is valid to first order in $\delta = 1/(\xi \phi_0^2)$, as we solved $\phi(h)$ to this order.

Now we can calculate the dominant contributions to the Higgs’ two-, three- and four-point functions and to the Yukawa coupling. Diagrams with a Higgs loop are subdominant compared to diagrams with a fermion loop because $\lambda_{nh} \ll y_{nh}$ and $m_h^2 \ll m^2$.

**Two-point function** There are two diagrams with a fermion loop. The first one has two $y_h$ couplings, and scales as $y_h^2 (k^2 + 6m_\psi^2) = O(\delta^2)$. The dominant loop is with one $y_{2h}$ coupling, which scales as $y_{2h} m_\psi^3 \propto y^4 \delta/\xi^2$. This is $O(\delta)$. However, it is a mass correction, and to do the renormalization of the mass properly, we have to take into account FLRW corrections.
Indeed in a FLRW universe \( m_h^2 = m_h^2 |_{\text{mink}} + \mathcal{O}(H^2) \sim \lambda/\xi^2 \). Hence, the FLRW mass is parametrically bigger than the one loop corrections.

The \( k^2 \)-correction to the kinetic term is \( \mathcal{O}(\delta^2) \), and can be neglected. To leading order
\[
Z_h = 1.
\] (41)

**Three- and four-point function** There are three diagrams, which scale as \( y_3^3 = \mathcal{O}(\delta^3) \), \( y_2h y_h = \mathcal{O}(\delta^2) \) and \( y_3h = \mathcal{O}(\delta) \) respectively. We only have to calculate the latter one. We use dimensional regularization in \( 4 - \epsilon \) dimensions, and find
\[
V_{3h}^{\text{loop}} = (3!) y_{3h} \int \frac{d^4l}{(2\pi)^4} \frac{\text{Tr}(-l + m_\psi)}{l^2 + m_\psi^2} = (3!) y_{3h} 4m_\psi \int \frac{d^4l}{(2\pi)^4} \frac{1}{l^2 + m_\psi^2}
= -i(3!) y_{3h} 4m_\psi^3 \frac{1}{8\pi^2} \frac{1}{\epsilon}.
\] (42)

Here we added an overall minus sign for a fermion loop which is canceled by the minus sign that comes from having an odd number of propagators. We also used \( \text{Tr}(1) = 4 \). The overall factor of \( 3! \) is because of the definitions/normalizations of \( \lambda_3h \), \( y_3h \). Similarly, for the four-point function only the diagram proportional to \( y_{4h} \sim \mathcal{O}(\delta) \) contributes at leading order. It is
\[
V_{4h}^{\text{loop}} = -i(4!) y_{4h} 4m_\psi^3 \frac{1}{8\pi^2} \frac{1}{\epsilon}.
\] (43)

Now we can add the tree level and loop diagrams. For the three- and four-point function this respectively yields
\[
V_{3h} = -i(3!) \left( \lambda_{3h} + y_{3h} 4m_\psi^3 \frac{1}{8\pi^2} \frac{1}{\epsilon} \right),
\]
\[
V_{4h} = -i(4!) \left( \lambda_{4h} + y_{4h} 4m_\psi^3 \frac{1}{8\pi^2} \frac{1}{\epsilon} \right).
\] (44)

Plugging in the expressions for the couplings, and using that \( m_\psi = y/\sqrt{2}\xi \), we find
\[
\frac{y_{4h} m_\psi^3}{\lambda_{4h}} = \frac{y_{3h} m_\psi^3}{\lambda_{3h}} = \frac{y^4}{4\lambda}.
\] (45)

Thus we find that the three- and four-point functions give consistent equations, both yield
\[
V^{nh} \propto \frac{\delta}{\xi^2} \left( \lambda + \frac{1}{8\pi^2} \frac{1}{\epsilon} y^4 \right).
\] (46)

If \( \xi, \phi_0 \) do not run we can absorb it in \( Z_\lambda \):
\[
Z_\lambda = \left( 1 - \frac{1}{8\pi^2} \frac{1}{\epsilon} \frac{y^4}{\delta} \right).
\] (47)

Now let us compare this to the Coleman-Weinberg calculation which gives \footnote{To translate from cutoff regularization to dimensional regularization use \( \ln \Lambda \leftrightarrow \frac{1}{\epsilon} \).}

\[
V = V_{\text{tree}} + V_{\text{CW}} = \frac{\lambda}{4\xi^2} + \frac{1}{32\pi^2} \frac{1}{\epsilon} 4m_\psi^4 = \frac{1}{4\xi^2} \left( \frac{\lambda}{8\pi^2} \frac{1}{\epsilon} y^4 \right),
\] (48)

where the factor 4 comes from the 4 degrees of freedom in a Dirac fermion. Note that as long as \( \delta \) does not run, this is the same result as obtained from the three- and four-point functions (46). Everything is consistent.
3.2 Covariant formulation

We again calculate the three- and four-point interactions. The relevant terms in the Lagrangian are

\[ \mathcal{L} \supset - (V_{\theta(abc)} + \bar{\psi} F_{\theta(abc)} \psi) Q^a Q^b Q^c - (V_{\phi(abcd)} + \bar{\psi} F_{\phi(abcd)} \psi) Q^a Q^b Q^c Q^d \]

\[ = - (V_{\phi\phi\phi\phi} + \bar{\psi} F_{\phi\phi\phi\phi} \psi) \phi^3 - (V_{\phi\phi\phi\phi} + \bar{\psi} F_{\phi\phi\phi\phi} \psi) \phi^3. \]  

The couplings are

\[ \lambda_{3\phi} = \frac{4\delta^{5/2} \lambda}{\sqrt{\xi}}, \quad \lambda_{4\phi} = -8\delta^3 \lambda, \quad y_{3\phi} = 2\sqrt{2} y \delta^{5/2} \xi, \quad y_{4\phi} = -4\sqrt{2} \delta^3 \xi^{3/2}, \]  

up to \( \mathcal{O}(\delta) \). The fermion mass is still \( m_\psi = 1/\sqrt{2\xi} \). Plugging in (45), we get the same result as for the canonical field \( h \).

4 Complex Higgs field plus fermion

After all preliminary exercises we are now ready for the key computation of this paper. We consider a complex Higgs field coupled to a fermion and compute the corrections to \( 4\theta \)-scattering, using the covariant expansion introduced in Sec. 2.4. The tree level interaction is

\[ V^{(4\theta)}_{\text{tree}} = (-i)4! \lambda_{4\theta} = i \frac{2 \lambda \delta^5}{9 \xi^2}, \]  

where in the last step we used \( \lambda_{4\theta} = (1/4!) V_{\theta\theta\theta\theta} = (1/4!)(-2\delta^3 \lambda)/(9\xi^2) \). The 4!-factor is because we defined \( V = \lambda_{4\theta} \bar{\theta}^4 \).

The dominant loop contributions come from the diagrams in Fig. 2 with a fermion loop. We will now calculate each of them in turn.
Diagram 1  A fermion loop with one $y_{4\theta}$ vertex. This gives

\[ V^{(4\theta)} \supset 4! y_{4\theta} \int d^4 l \frac{\text{Tr} \left[ -I + m_\psi \right]}{l^2 + m_\psi^2} = 4! \times 4y_{4\theta}m_\psi \int d^4 l \frac{1}{l^2 + m_\psi^2} \]

\[ = -i4! \times 4y_{4\theta}m_\psi^3 \frac{1}{8\pi^2} \epsilon, \quad (52) \]

with $dl = dl/(2\pi)$. The 4! counts the different ways of assigning the momenta to the external lines. The overall sign is for a fermion loop plus odd Yukawa insertions, and the factor 4 in the second expression comes from the trace.

Diagram 2  A fermion loop with one $y_{3\theta}$ and one $y_\theta$ vertex. This gives

\[ V^{(4\theta)} \supset -3!4y_{3\theta}y_\theta \int d^4 l \frac{\text{Tr} \left[ i\gamma^5 (-I + m_\psi)i\gamma^5 (-I + k) + m_\psi \right]}{(l^2 + m_\psi^2)((l + k)^2 + m_\psi^2)} \]

\[ = 3!4y_{3\theta}y_\theta \int d x \int d^4 l \frac{\text{Tr} \left[ (I + m_\psi)((I + k) + m_\psi) \right]}{(q^2 + D)^2}, \quad (53) \]

with $q = l + xk$ and $D = x(1 - x)k^2 + m_\psi^2$. The overall sign is minus because we have a fermion loop with an even number of Yukawa insertions. There are 4 different permutations (as the single vertex can be one out of 4).

The numerator can be written

\[ \int dx Tr[...] = \int dx Tr \left[ -l^2 - Ik + m^2 \right] \]

\[ = \int dx Tr \left[ -q^2 + k^2(x - x^2) + m_\psi^2 \right] \]

\[ = 4 \int dx \left( q^2 - D + m_\psi^2 \right), \quad (54) \]

where in the first line we dropped all terms with an odd number of gamma matrices, and in the second line dropped terms odd in loop momentum $q$, as these all evaluate to zero.

Plugging back in the diagram evaluates to

\[ V^{(4\theta)} \supset 3! \times 16y_{3\theta}y_\theta \int dx \int d^4 q \frac{q^2 - D + m_\psi^2}{(q^2 + D)^2} \]

\[ = i3! \times 16y_{3\theta}y_\theta \int dx \left( -3D + 2m_\psi^2 \right) \frac{1}{8\pi^2} \frac{1}{\epsilon}. \quad (55) \]

Now use $\int dx D = 1/6k^2 + m_\psi^2$. Then

\[ V^{(4\theta)} \supset -i3! \times 16y_{3\theta}y_\theta \left( \frac{1}{2} k^2 + m_\psi^2 \right) \frac{1}{8\pi^2} \frac{1}{\epsilon}, \]

Diagram 3  A fermion loop with two $y_{2\theta}$ vertices. This gives

\[ V^{(4\theta)} \supset -2!2!3y_{2\theta} \int d^4 l \frac{\text{Tr} \left[ (-I + m_\psi)(-(I + k) + m_\psi) \right]}{(l^2 + m_\psi^2)((l + k)^2 + m_\psi^2)}. \quad (56) \]
The minus sign follows because we again have an even number of fermion insertions. The symmetry factor is 2 for the two vertices, times another factor 3 for the 3 different configurations 12, 34 and 13, 24 and 14, 23. The numerator can be written

\[
\int dx \text{Tr}[\ldots] = \int dx \text{Tr}\left[ I^2 + I\mathbb{k} + m^2 \right]
\]

\[
= -4 \int dx \left[ q^2 + k^2 x(x - 1) - m^2 \psi \right]
\]

\[
= -4 \int dx \left( q^2 - D \right).
\]

Then

\[
V^{(49)} \supset 12 \times 4y_{2\theta}^2 \int dx \int d^4l \frac{q^2 - D}{(q^2 + D)^2}
\]

\[
= 12i \times 4y_{2\theta}^2 \frac{1}{8\pi^2\epsilon} \frac{1}{8\pi^2\epsilon}
\]

\[
= -12i \times 4y_{2\theta}^2 \left( \frac{1}{2} k^2 + 3m^2 \psi \right) \frac{1}{8\pi^2\epsilon}.
\]

**Diagram 4** A fermion loop with one $y_{2\theta}$ and two $y_\theta$ vertices. This gives

\[
V^{(49)} \supset 2!6y_{2\theta}y_{\theta}^2 \int d^4l \frac{1}{(l^2 + m^2)(l + k_1)^2 + m^2((l + k_2)^2 + m^2)}
\]

\[
= -12y_{2\theta}y_{\theta}^2 \int dF_2 \int d^4l \frac{1}{(l^2 + m^2)(l + k_1)^2 + m^2((l + k_2)^2 + m^2)}.
\]

Now the overall sign is plus because of the odd number of Yukawa insertions. The symmetry factor is 2 for the $y_{2\theta}$ vertex, and there are 6 different configurations (the same as for diagram 3, but now with an extra factor 2 as one can distinguish the 2h vertex from the 2 single h vertices). Further $q = l + xk_1 + yk_2$ and $\int dF_2$ is the 2D integration over the Feynman parameters. From the numerator we only have to extract the $q^2$-term; lower powers of $q$ give a finite result and odd powers vanish. We have

\[
\int dF_2 \text{Tr}[\ldots] \supset \int dF_2 \text{Tr}[-m\psi l^2] = \int dF_2 \text{Tr}[-m\psi l^2]
\]

\[
= 4m\psi q^2.
\]

The diagram evaluates to

\[
V^{(49)} \supset -12 \times 4y_{2\theta}y_{\theta}^2 m\psi \int d^4l \frac{q^2}{(q^2 + D)^3}
\]

\[
= -i12 \times 4y_{2\theta}y_{\theta}^2 m\psi \frac{1}{8\pi^2\epsilon}.
\]

**Diagram 5** A fermion loop with four $y_\theta$ vertices. We find

\[
V^{(49)} \supset -3!y_{\theta}^4 \int d^4l \frac{1}{(l^2 + m^2)(l + k_1)^2 + m^2((l + k_2)^2 + m^2))}
\]

\[
= -3!y_{\theta}^4 \int dF_3 \int d^4l \frac{1}{(l + m\psi)((l + k_1)^2 + m\psi)((l + k_2)^2 + m\psi)((l + k_3)^2 + m\psi)}.
\]

\[
= -3!y_{\theta}^4 \int dF_3 \int d^4l \frac{1}{(l + m\psi)((l + k_1)^2 + m\psi)((l + k_2)^2 + m\psi)((l + k_3)^2 + m\psi)}.
\]
The minus sign follows for a fermion loop with an even number of Yukawa insertions. There are 4! permutations of the external legs, divided by a factor 4 for exchange of the vertices, giving 3! different diagrams. In the second line $q = l + xk_1 + yk_2 + zk_3$ and $\int dF_3$ is the 3D integration over the Feynman parameters. From the numerator we only have to extract the $q^4$-term; lower powers of $q$ give a finite result and odd powers vanish.

\[
\int dF_3 \text{Tr}[...] \supset \int dF_2 \text{Tr}[q^4] = \int dF_3 \text{Tr}[q^4] = 4q^4.
\] (63)

The diagram becomes

\[
V^{(4\theta)} \supset -4 \times 3! y_4^4 \int \frac{q^4}{(q^2 + D)^4} = -i4 \times 3! y_4^4 \frac{1}{8\pi^2} \frac{1}{\epsilon}.
\] (64)

**All diagrams combined** Adding all five diagrams, we get for the total one-loop contribution at leading order

\[
V_{\text{loop}}^{(4\theta)} = \frac{1}{8\pi^2} \frac{1}{\epsilon} 16 \left[ -6y_{4\theta}m_\psi^2 - 6y_{3\theta}y_\theta m_\psi^2 - 9y_{2\theta}y_\theta^2 m_\psi^2 - 3y_{2\theta}^2 y_\theta^2 m_\psi - \frac{3}{2} y_\theta^3 \right] + \frac{1}{8\pi^2} \frac{1}{\epsilon} 16k^2 \left[ 0 - 3y_{3\theta}y_\theta m_\psi^2 - \frac{3}{2} y_{2\theta}^2 m_\psi^2 + 0 + 0 \right]
\]

\[
= -6\delta^2 \frac{y^4}{8\pi^2} \frac{1}{\epsilon} \frac{1}{\epsilon} + \frac{\delta}{2} \frac{k^2 y^4}{8\pi^2} \frac{1}{\epsilon}.
\] (65)

where in the last step we used

\[
m_\psi = \frac{y}{\sqrt{2} \xi}, \quad y_\theta = \frac{y\sqrt{\delta}}{\sqrt{2}}, \quad y_{2\theta} = \frac{y\delta\sqrt{\xi}}{2!\sqrt{2}}, \quad y_{3\theta} = -\frac{y\delta^{3/2}\xi}{3!\sqrt{2}}, \quad y_{4\theta} = \frac{y\delta^2\xi^{3/2}}{4!\sqrt{2}}.
\] (66)

To compute $\lambda$’s beta-function $\beta_\lambda$ we also need to calculate $Z_\theta$. The two-point diagrams with fermion loops dominate; there are two of them. The first one is with a $y_{2\theta}$-vertex. Here the calculation is analogous to diagram 1 above, so there is no $k$-dependence in the result. Hence this gives a mass correction but no wave function correction. The other diagram is the usual one with two $y_\theta$-vertices, which gives

\[
\Pi_\theta = -\frac{y_\theta^2}{8\pi^2} \frac{1}{\epsilon} (k^2 + 2m_\psi^2),
\] (67)

from which $Z_\theta$ can be extracted.

We conclude that the dominant term in one-loop $4\theta$ scattering does not cancel, as it is larger than the tree level result. The $k$-dependent contribution does not cancel either, but correspond to higher order kinetic terms of the form $\sim \bar{\theta}^2(\partial \bar{\theta})^2$. The result seems to challenge the renormalizability of this toy model, and of Higgs inflation in the full Standard Model as well. Before discussing the implications of our result, however, we will first have a look at its generality, and check the approximations we have made.
4.1 Goldstone n-point functions

In the previous section we calculated the four-point scattering amplitude for the Goldstone bosons, and showed that the one-loop result is parametrically larger than the tree level result, and moreover has a different background field dependence. This is not specific to the four-point function but is the case for all Goldstone $n$-point functions. The tree-level $2n$-self interactions scale as

$$V_{\text{tree}}^{(2n\theta)} \propto \lambda^{2n\theta} \sim \frac{\lambda}{\xi^2} \delta^{1+2n},$$

whereas the one-loop diagram scales as

$$V_{\text{loop}}^{(2n\theta)} \sim y_{2n\theta}m_\psi^3 \frac{1}{\epsilon} \sim y^4 \delta^n \xi^{n-2} \frac{1}{\epsilon}.$$  \hspace{1cm} (69)

Note that for odd $n$ both the tree-level and one-loop corrections vanishes.

In particular, at lowest order in $\delta$ the problem arises for the two-point function. Focusing on the effective mass (and neglecting the $k$-dependent terms) the tree-level two-point interactions gives

$$V_{\text{tree}}^{(2\theta)} = (-i)2! \left( m_\sigma^2 + O(H^2) \right) = (-i)2! \left( \frac{\lambda}{6\xi^2} \delta^3 + O\left( \frac{\lambda}{4\xi^2} \right) \right).$$  \hspace{1cm} (70)

Here the $m_\sigma^2 = 2\lambda_2$-term is the Minkowski result; in an FLRW universe the corrections to the mass are of order $H^2$. Two diagrams contribute to the one loop result, namely the analogue of diagram (1) and (5) in Fig. 2 for two external Goldstone boson lines. The result is

$$V_{\text{loop}}^{(2\theta)} = (-i)2! \left( y_\theta m_\psi^2 + 4y_{2\theta}m_\psi^3 \right) \frac{1}{8\pi^2\epsilon} = (-i)2! \left( \frac{\delta}{4\xi} - \frac{\delta}{2\xi} \right) \frac{1}{8\pi^2\epsilon}. $$  \hspace{1cm} (71)

We thus see that also for the two-point function, the one-loop divergencies are at a different order in $\delta$ and cannot be absorbed in the existing tree level results (also when FLRW corrections are taken into account).

4.2 Checks

In this subsection we list a number of checks that we did on the computation that we performed in the previous section.

Our result is independent of the field definitions used. This was shown explicitly in Sec. 3 where we computed the four-point scattering amplitude for the Higgs field both using the canonically normalized Higgs fields, as well as the covariant Higgs fluctuation $Q^1 = \varphi$. The ratio of tree-level and one-loop result is field independent. The only difference is an overall factor for the scattering amplitude, but this is expected as different fields are on the external lines.

We only computed the diagrams with a fermion loop, as these give the dominant contribution to $4\theta$-scattering. This is because the Goldstone-fermion couplings are unsuppressed (when written in canonical fields) as opposed to Goldstone-Higgs (self) interactions, as shown in Sec. (2.3). Indeed, compare for example the four-point Goldstone-fermion coupling with the Goldstone-Higgs couplings:

$$\lambda_{\partial\theta\bar{\psi}\psi} \sim y\delta\sqrt{\xi}, \quad \lambda_{2\partial\psi} \sim \frac{\lambda}{4\xi}. $$  \hspace{1cm} (72)
We have neglected the time-dependence of the background, FLRW corrections as well as the back reaction from gravity. Not only are these corrections subleading in δ, as we show below, but they would not affect the calculation anyway (unless parametrically larger than the static Minkowski result). The reason is that all of the effects mentioned above only affect the interactions and the propagator of the Goldstone and Higgs fields — which do not enter the calculation of the dominant fermion loop diagrams — but not the fermion interactions. For example, there are higher order kinetic terms for the Higgs and Goldstone but not for the fermion. The Higgs propagator is corrected because the field space metric is field, and thus time, dependent, but this is not the case for the fermion propagator. The scalar mass is 

\[ m_\phi^2 \ll H^2, \]

and thus gets large corrections in FLRW; the fermion mass is not corrected and moreover \( m_\psi \gg H \).

The reason that the corrections from gravity and time-dependence are small (except for the FLRW correction to the GB/Higgs mass) is that during inflation they are suppressed by slow roll parameters \( \eta \sim \delta \) and \( \epsilon \sim \delta^2 \). We can estimate \( \dot{\phi}_0^2 \)-corrections using the slow roll approximation. Consider the canonical classical Higgs field. Its equation of motion is

\[
\dddot{h}^0 + 3H \dot{h}^0 + V_{h} = 0.
\]

In the slow-roll approximation the \( \dddot{h}^0 \)-term is a factor \( \delta \) smaller than the last two terms. Rewriting in terms of the Jordan frame field \( \phi^0 \) gives

\[
\dot{\phi}^0 + \sqrt{\frac{\delta}{\gamma}} + \frac{3H}{\sqrt{\delta}} \dot{\phi}^0 + \sqrt{\frac{\gamma}{\delta}} V_{\phi} = 0.
\]

Explicitly we find

\[
\dot{\phi}^2_0 + \left( \frac{V_{\phi}}{3H\gamma_{\phi\phi}} \right)^2 = \frac{\delta \lambda}{27 \xi^2},
\]

up to \( O(\delta) \) corrections.

Now consider first the quadratic action with the mass terms

\[ S \supset \frac{1}{2} \int d^4 x a^3 \left[ D_t Q_a D_t Q^a - \partial^i Q_a \partial_i Q^a - M_{ab} Q^a Q^b + \bar{\psi} (i \partial - F) \psi \right]. \]

The fermion mass is unaffected by time dependence and going to FLRW. The scalar mass matrix is

\[ M_{ab} = V_{ab} - R_{cab} \dot{\phi}_0 \dot{\phi}_b - 1 \frac{a^3}{a^3} D_t \left( a^3 \frac{\partial^i}{H} \phi_a \phi_b \right) \]

evaluated on the background. The second term is from time dependence, and the last term is the FLRW correction including the back reaction from gravity. The mass term is diagonal, and we find:

\[
M_{\phi\phi} = V_{\phi\phi} - R_{\phi\phi\phi\phi} \dot{\phi}_0^2 - \frac{\gamma_{\phi\phi}}{\gamma_{\phi\phi}} \left( \dot{\phi}_0^2 (1 - H/H^2) + 2D_t \phi_0 \phi_0 \right) = V_{\phi\phi} (1 + O(\delta)),
\]

\[
M_{\theta\theta} = V_{\theta\theta} - R_{\theta\phi\theta\phi} \dot{\phi}_0^2 = V_{\theta\theta} (1 + O(\delta)).
\]

Note that the usual Hubble squared corrections to the scalar mass are hidden in the kinetic term for the scalars, and are not included in \( M \). Here we used the slow roll approximation \[\text{[74]}\] to estimate the size of the corrections.

We can also consider the cubic interaction

\[ L \supset -\frac{1}{3!} \left( V_{abc} - R_{(a|de|b|c)} \dot{\phi}_0^2 \dot{\phi}_d \dot{\phi}_e \right) Q^a Q^b Q^c + 4R_{(a(bc)} \dot{\phi}_0^2 D_t Q^a Q^b Q^c + \frac{\dot{\phi}_0}{H} D_t Q_a D_t Q^a + ... \]

(79)
The terms involving the Ricci tensor stem from time dependence. There are several FLRW corrections. Following reference [27], the dominant term seems to be the correction to the kinetic terms. This is suppressed as \( \dot{\phi}_0 / H = \mathcal{O}(\sqrt{\delta/\xi}) \). The first term involving the Ricci tensor is also \( \delta \)-suppressed. To show the same for the second Ricci tensor term, use integration by parts to write

\[
4R_a^{(bc)} \dot{\phi}_0^c D_t Q^a Q^b Q^c = 2R \dot{\phi}_0(D_t Q^1 Q^2 Q^2 + 2Q^1 Q^2 D_t Q^2) = -2(R \ddot{\phi}_0 + dR \dot{\phi}_0^2)Q^1 Q^2 Q^3 \quad (80)
\]
as a cubic \( Q^a Q^b Q^c \) interaction. This term is \( \delta \) suppressed as well.

In the appendix of reference [27] we see that the scalar propagator is corrected if \( \dot{\phi}_0 \neq 0 \) is time dependent; we also expect \( \mathcal{O}(H^2) \) corrections to the FLRW mass. This does not affect our calculation.

5 Absorbing the loop corrections

We have found that the one-loop \( \bar{\theta}^4 \)-scattering is parametrically larger than the tree-level result, and moreover cannot be absorbed in the counterterms of the tree-level Higgs inflation potential. From the first statement, one may be inclined to conclude that perturbativity is lost, as one-loop corrections overwhelm the tree-level result. It should be noted though that the two- and higher loop diagrams are all higher order in \( \delta \) than the one-loop correction. Let us then focus on the second part of the statement, that all divergencies cannot be absorbed.

So far we have restricted the discussion to only quadratic and quartic terms in the Jordan frame Higgs potential, but in principle all operators that are higher order in the fields can (and should) be added. These higher order terms are irrelevant in the infrared, and the usual Standard Model Higgs potential is retrieved. However, in the large field regime during inflation, Higgs inflation suffers from the same UV problems as all other chaotic models with \( \phi_0 > 1 \) (or equivalently \( \xi \delta < 1 \)), namely that all higher order terms are large unless their coefficients are tuned to zero.

The loop-induced term \( V_{\text{loop}}^{(4\theta)} \) has parametric order \( \delta^2 \) and cannot be absorbed by any existing tree-level terms, but may be absorbed in higher order terms. To that end, consider modifying the Jordan frame potential:

\[
V_J = \lambda (\Phi^4 - v^2 / 2)^2 + \lambda_8 |\Phi|^8. \quad (81)
\]
The new eighth-power term induces the following operators for the covariant variables:

\[
V \supset \frac{\lambda_8}{\xi^3 \delta} \ddot{\theta}^2 + \frac{16 \lambda_8}{\xi^2} \dot{\theta}^4 + \frac{\lambda_8}{24 \xi^4} \theta^2 + \frac{5 \lambda_8 \delta^2}{72 \xi^4} \bar{\theta}^4. \quad (82)
\]

We see the appearance of a \( \bar{\theta}^4 \) term with a \( \delta^2 \) coefficient, which is parametrically the correct form to cancel the \( 4 \theta \) loop correction. The value of \( \lambda_8 \) will be chosen to cancel \( V_{\text{loop}}^{(4\theta)} \). The addition of this term to the Jordan frame action retains all symmetries of the original action.

However, using this eighth-power term to cancel the loop correction introduces further problems: the term also introduces new contributions to \( \ddot{\theta}^2 \) and \( \dot{\theta}^4 \) (and also other powers, but they are not important for the discussion), but no corresponding one-loop divergencies. One cannot at the same time absorb the divergencies in the \( \bar{\theta}^4 \)-channel, while keeping \( \bar{\theta}^4 \)-scattering finite. There is no consistent way to absorb all infinities, and the theory is non-renormalizable.
One may try to find other terms to add to the Jordan frame which give a tree-level term proportional to $\delta^2 \theta^4$, without introducing the $\dot{\phi}$-terms in [82] as well. But it is difficult to see how this would be possible without breaking the global U(1) symmetry (in the SM one cannot explicitly break the electroweak symmetry). Indeed, any Jordan-frame term that is U(1) invariant and — when transformed to the Einstein frame and written in terms of the covariant fields — contains a $\bar{\theta}^4$ should also, due to [32], contain low powers of $\bar{\phi}$.

Apart from the leading terms in $\delta$ there are also subdominant $\delta$ contributions to the one-loop process that need to be absorbed in the original counterterms. It is doubtful whether or not this can be done, as the $\lambda$ counterterm is fixed by the $4\bar{\phi}$-scattering and there is no additional freedom. It is hard to check this explicitly, as at subleading order one needs to take time-dependence of the field and the backreaction from gravity into account.

Our conclusion is that the loop corrections to $4\bar{\theta}$-scattering cannot be absorbed into a counterterm without breaking the symmetry, and hence, by definition, the theory is not renormalizable, not even in the EFT sense. Adding a $|\Phi|^8$ term respects the symmetries and contains the correct tree level term to absorb the $4\bar{\theta}$-divergence, but this cannot be done while keeping all other terms in [82] finite.

6 Conclusion

In this paper, we have looked at the renormalization of Higgs inflation. We have worked in the large field regime, in the large-$\xi$ limit, in which the theory is widely believed to be renormalizable. For simplicity, and to isolate the important effects, we have analyzed a theory of a non-minimally coupled Higgs field and a fermion. We have used the covariant approach introduced in [26, 27] to take into account the non-minimal kinetic terms for the complex Higgs field.

The main result of our work is that the dominant one-loop corrections to the Goldstone’s n-point functions, which are the diagrams that involve fermion loops, cannot be absorbed in the parameters of the classical theory because of a different dependence on the background field. Moreover, the one-loop corrections are parametrically larger (in the expansion in slow roll parameters) than the tree-level result. This is caused by the large (unsuppressed) Yukawa couplings of the top quark to the Goldstone bosons.

We have argued that this result spoils the renormalization of our toy model, and therefore of Higgs inflation in the Standard Model. The divergencies found here cannot be removed by (gauge symmetry respecting) counterterms. Therefore it seems that even if the theory of an isolated real Higgs inflaton is perfectly renormalizable (in the EFT sense), there exists no consistent way of generalizing to a complex Higgs field coupled to a fermion.

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