Accessing the Transverse Dynamics and the Polarization of the Gluons inside the Proton at the LHC

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We argue that the study of heavy quarkonia, in particular that of Υ, produced back-to-back with an isolated photon in pp collisions at the LHC is the best—and currently unique—way to access the distribution of both the transverse momentum and the polarization of the gluon in an unpolarized proton. These encode fundamental information on the dynamics of QCD. We have derived analytical expressions for various transverse-momentum distributions which can be measured at the LHC and which allow for a direct extraction of the aforementioned quantities. To assess the feasibility of such measurements, we have evaluated the expected yields and the relevant transverse-momentum distributions for different models of the gluon dynamics inside a proton.

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Introduction.— At LHC energies, the vast majority of hard reactions are initiated by the fusion of two quarks from both colliding protons. A good knowledge of gluon densities is therefore mandatory to perform reliable cross-section predictions, the archetypal example being the $H^0$ boson production. In perturbative QCD (pQCD), the production cross section of a given particle is conventionally obtained from the convolution of a hard parton-scattering amplitude squared and of the collinear parton distribution functions (PDFs) inside the colliding hadrons, $G(x,\mu)$ or $f_1^g(x,\mu)$ for the gluon [1]. The PDF provides the distribution of a given parton in the proton as a function of its collinear momentum fraction $x$, at a certain (factorization) scale $\mu$. Whereas the scale evolution of the PDFs is given by pQCD, experimental data are necessary to determine their magnitude (see e.g. [2]).

This collinear factorization, inspired by the parton model of Feynman and Bjorken, can be extended to take into account the transverse dynamics of the partons inside the hadrons. Different approaches have been proposed (unintegrated PDF, impact factors within $k_T$ factorization, etc.). Out of these, the Transverse-Momentum (TM) dependent factorization is certainly the most rigorous with proofs of factorization for a couple of processes [3–6]. The further advantage of the TM Dependent (TMD) formalism lies in its ability to deal with spin-dependent objects, both for the partons and the hadrons.

Much effort has been made in the last years to extract quark TMD distributions (TMDs in short) inside a proton from low energy data from HERMES, COMPASS or JLab experiments (see e.g. Ref. [7] for recent reviews). On the contrary, nothing is known about the gluon TMDs which rigorously parametrize the transverse motion of gluons inside a proton. For an unpolarized proton, these are the distribution of unpolarized gluons, denoted by $f_1^g$, and the distribution of linearly-polarized gluons, $h_1^g$ [8]. These functions contain fundamental information on the transverse dynamics of the gluon content of the proton [see the interpretation of $h_1^g$ in Fig. 1 (a-b)]. Their study is also motivated by a recently proposed method to determine the spin and parity of the $H^0$ boson [9], which is based on the linear polarization of gluons that appears for non-zero parton TM.

In this Letter, we argue that the LHC experiments are ideally positioned to extract for the first time the gluon TMDs through the study of an isolated photon produced back-to-back with a heavy quarkonium. Furthermore, we show that the yields are large enough to perform such extractions with existing data at $\sqrt{s} = 7$ and 8 TeV.

Reactions sensitive to gluon TMDs.— Several processes have been proposed to measure both $f_1^g$ and $h_1^g$. A potentially very clean probe to extract gluon TMDs is the back-to-back production of a heavy-quark pair in electron-proton collisions, $e p \rightarrow e Q\bar{Q}X$ in which the gluon TMDs appear linearly. Theoretical predictions have been provided at leading order (LO) [10] and next-to-leading order (NLO) [5] in pQCD. However, such a process can only be measured at future facilities (EIC or LHeC), whose realization is at best a decade away.

![FIG. 1. Visualization of the gluon polarization in the TM plane for a positive (a) and negative (b) Gaussian $h_1^{g\perp}$. [The ellipsoid major/minor axis lengths in the plane are proportional to the probability of finding a gluon with a linear polarization in that direction]. (c) Feynman diagram for $p(P_1) + p(P_2) \rightarrow Q(P_\perp) + \gamma(P_\perp) + X$ via gluon fusion at LO in the TMD-factorization formalism.](image-url)
Another process sensitive to gluon TMDs is the back-to-back isolated photon-pair production in proton collisions, \( pp \rightarrow \gamma \gamma X \) [11]. In principle, this process is accessible at RHIC and the LHC but it suffers from a contamination from quark-induced channels, a huge background from \( \pi^0 \)-decays and an inherent difficulty to trigger on such events.

As for the gluon PDF, final states such as a heavy-quark pair or a dijet [10] should also be ideal candidates to probe gluon TMDs. However, once there is a color flow into the detected final state in the partonic-scattering subprocess, one cannot cleanly separate final state interactions of this color flow from the non-perturbative TMD objects due to the non-Abelian characteristics of QCD [12]. This leads to a breakdown of TMD factorization for processes with colored final states. We however note that we do not know of any obvious reason why such complications could be avoided by using another approach, relying for instance on unintegrated PDFs or \( k_T \) factorization with impact factors.

On the other hand, this problem can be avoided in the case of the production of heavy quarkonia, provided that the heavy-quark pair is produced in a colorless state at short distances as in the color-singlet model [13], and that it is not accompanied by other –necessarily colorful– partons. The production of \( C \)-even quarkonia \((\chi_Q, \eta_Q)\) at small TM is one of these cases where the factorization is expected to hold as illustrated by studies both at LO [14] and NLO [15]. At low \( P_{Q_T} \), the production of \( \eta_Q \) and \( \omega_Q \) proceeds without the emission of a final state gluon and the color-octet (CO) contributions [16] are not kinematically enhanced. However, such experimental measurements are particularly difficult since they should be done at low TM, \( P_{Q_T} \ll Q \approx M_Q \), as required by TMD factorization. The hard scale of the process, \( Q \), can only be the mass of the heavy quarkonium, hence \( Q \approx M_Q \). The observation of low \( P_{Q_T} \) \( C \)-even quarkonia is likely impossible with ATLAS and CMS. LHCb may look at these down to \( P_{Q_T} \approx 1 \) GeV, but an unambiguous determination of the gluon TMDs –free of large power corrections in \( P_{Q_T}/Q \)– requires to reach the sub-GeV region. Besides, this would not allow one to look at the scale evolution of the TMDs. Only two ranges can be probed – close to the charmonium and bottomonium masses.

**Back-to-back quarkonium-isolated-photon production.**—

We propose a novel process to overcome these experimental complications and theoretical limitations: the production of a back-to-back pair of a \( ^3S_1 \) quarkonium \( Q (\Upsilon \text{ or } J/\psi) \) and an isolated photon, \( pp \rightarrow Q + \gamma + X \). Compared to the aforementioned processes, it has the advantage of being accessible by the LHC experiments: only the TM imbalance, \( q_t = P_{Q_T} - P_{\gamma} \), has to be small, not the individual TM, for TMD factorization to apply. In addition, the hard scale of the process \( Q \) can be tuned by selecting different invariant masses of the \( Q - \gamma \) pair. This allows one to look at the scale evolution of the TMDs and to greatly increase the \( q_t \)-range where the TMD factorization applies with tolerable power corrections.

Previous studies [17–19] have shown that the CO contributions to inclusive \( Q + \gamma \) production are likely smaller than in the inclusive case \( Q + X \) (see e.g. [20–22]). [The case of \( J/\psi + \gamma \) is however intriguing since a state-of-the-art NLO evaluations using recent NRQCD fits predict negative CO cross-sections [23].] The smallness of CO contributions is crucial since these would violate the TMD factorization.

As studied in [25], the CO contributions are also suppressed w.r.t. the CS ones when one imposes that the \( Q - \gamma \) pair is produced back-to-back, i.e. dominantly from \( 2 \rightarrow 2 \) processes, although the \( gg \) fusion CS contribution (Fig. 1c) scales like \( P_{Q_T}^4 \). Indeed, the \( P_{Q_T}^4 \) (fragmentation) CO contribution only appears for \( q \bar{q} \) annihilation –extremely suppressed at LHC energies– and, incidentally, on the order of the pure QED CSM contribution (as for \( J/\psi + W \) [26]). As regards \( gg \) fusion CO channels, they are subleading in \( P_{Q_T} \), since they come from quark box and \( s \)-channel gluon diagrams, only via \( C = +1 \) CO states, such as \( ^1S_0 \) or \( ^3P_{J} \). [For the \( J/\psi \), these CO states are known to be severely constrained if one wants to comply with \( e^+e^- \) inclusive data [27].] To substantiate this, we have computed the different CS and CO contributions, see Fig. 2. The CS yield is clearly dominant for the \( \Upsilon \) and likely above the CO one for the \( J/\psi \) at the lowest \( Q \) accessible at the LHC (\( P_{Q_T} \gtrsim 10 \) GeV). It is also clear that this process is purely from \( gg \) fusion.

Since QCD corrections to the inclusive production of a quarkonium-photon pair are known to be large for increasing \( P_{Q_T} \) [17, 19], we find it useful to emphasize that the leading-\( P_{Q_T} \) NLO topologies, such as \( t \)-channel gluon exchanges, are in fact absorbed in the evolution of the TMD distribution. Moreover, it is clear that topologies with more than 2 particles in the final state are anyhow suppressed by the back-to-back requirement since they produce \( Q - \gamma \) pair “near” to each other rather than “away”. Finally, the isolation of the photon also favors the LO contribution, where QCD radiations are strictly absent.

![FIG. 2. Direct contributions to the production of an isolated photon back-to-back with a) an \( \Upsilon(1S) \) (resp. b) a \( J/\psi \)) from \( g-\bar{g} \) and \( q-\bar{q} \) fusion from the CS and CO channels as function the invariant mass of the pair, \( Q \). The curves for the \( q-\bar{q} \) fusion are rescaled by a factor 100 (resp. 50). The CO matrix elements we used are very close to those obtained in a recent LO fit of LHC data [24].](image)
A further suppression of CO contributions can be achieved by isolating the quarkonium (see [28]) as done for the photon. The isolation should be efficient at large enough $P_{Q\gamma}$ where the soft partons emitted during the hadronization of the CO heavy-quark pair are boosted and energetic enough to be detected. Experimentally, this would provide an interesting check of the CS dominance by measuring the (conventional) $q_T$-integrated cross section which should coincide with the parameter-free CSM prediction. This would also confirm that double-parton scattering contributions are suppressed by the isolation criteria. We emphasize that, according to our evaluations, such an isolation is not at all necessary for the $\Upsilon$ case.

Along the same lines, the study of $\Upsilon$ or $J/\psi$ with a Z boson is worth some investigation. According to [29, 30], one expects their back-to-back production at low $P_{Q\gamma}$ to be basically free of CO contributions as well as exclusively from gluon fusion, thus satisfying the requirements for gluon-TMD extraction. The only drawback is that we might have to wait for the LHC luminosity increase to get enough events [30].

**Analytical expression for the $q_T$-dependent cross section.—** We now proceed to the $q_T$ differential-cross-section calculation for the reaction of Fig. 1c. At LO in $\alpha_s$, the reaction proceeds via the gluon-fusion process, $g(k_1) + g(k_2) \rightarrow Q(P_Q) + \gamma(P_\gamma)$, which involves 6 graphs. The corresponding amplitude $M$—obtained as in the collinear case—is squared and convoluted with the TMD correlator as described in Fig. 1c, i.e.,

$$\frac{d\sigma}{dQd\Omega d^2q_T} = \frac{C_0(O^2 - M^2)}{s Q^2 D} \left[ F_1 C[f_1 f_1^*] + F_3 \cos(2\phi_{CS}) C[w_3 h_1^{g\perp g} + x_1 \leftrightarrow x_2] + F_4 \cos(4\phi_{CS}) C[w_4 h_1^{g\perp g}] + \mathcal{O}(Q^4) \right].$$

(3)

where $d\Omega = d\cos\theta_{CS} d\phi_{CS}$ is expressed in terms of Collins-Soper angles [33] and where $Q$, $Y$ and $q_T$ are the invariant mass, the rapidity and the TM of the pair—the latter two to be measured in the hadron c.m.s. frame. The overall normalization is given by $C_0 = 4\alpha_s^2 \alpha_{em}^2 |R_0(0)|^2 / (3M_Q^2)$, where $R_0(0)$ is the quarkonium radial wave function at the origin and $e_Q$ the heavy quark charge. The $F$ factors and the denominator $D$ are given by

$$F_1 = 1 + 2a^2 + 9a^4 + (6a^4 - 2) \cos^2 \theta_{CS} + (a^2 - 1)^2 \cos^4 \theta_{CS},$$

$$F_3 = 4a^2 \sin^2 \theta_{CS},$$

$$F_4 = (a^2 - 1)^2 \sin^4 \theta_{CS},$$

$$D = (a^2 + 1)^2 - (a^2 - 1)^2 \cos^2 \theta_{CS},$$

(4)

where $\alpha = Q/M_Q$. The convolution is defined as

$$C[w Q f g] \equiv \int d^2k_1 \int d^2k_2 \delta^2(k_1 + k_2 - q_T) x(w(k_1, k_2)) f(x_1, k_{1T}^2) g(x_2, k_{2T}^2).$$

(5)

in which the longitudinal momentum fractions are evaluated at $x_{1,2} = \exp(\pm Y) Q/\sqrt{s}$. The weights appearing in the convolutions are given by

$$w_3 \equiv \frac{k_{1T}^2 k_{2T}^2 - 2(q_T \cdot k_{1T})^2}{2M_p q_T^2},$$

$$w_4 \equiv 2 \left( \frac{k_{1T}^2 k_{2T}^2 - (k_{1T} \cdot q_T)(k_{2T} \cdot q_T)}{M_p q_T^2} \right)^2 - \frac{k_{1T}^2 k_{2T}^2}{4M_p^2}.$$  

(6)

We propose the measurement of 3 TM spectra, normalized and weighted by $\cos n\phi$ for $n = 0, 2, 4$:

$$S_{q_T}^{(n)} \equiv \frac{d\phi_{CS} \cos(n \phi_{CS})}{d\phi_{CS}} \frac{d\sigma}{d\phi_{CS} d^2q_T} d\phi_{CS} d\Omega d^2q_T,$$

(7)

where we will take the $q_T^2$ integration in the denominator up to $(Q/2)^2$. These spectra separate out the the 3 terms in Eq. 3:

$$S_{q_T}^{(0)} = \frac{C[f_1 f_1^*]}{2F_1 d\phi_{CS} C[f_1 f_1^*]} ,$$

$$S_{q_T}^{(2)} = \frac{C[f_1 f_1^*]}{2F_1 d\phi_{CS} C[f_1 f_1^*]} ,$$

$$S_{q_T}^{(4)} = \frac{C[w_3 h_1^{g\perp g} + x_1 \leftrightarrow x_2]}{2F_1 d\phi_{CS} C[f_1 f_1^*]} .$$

(8)
It is remarkable to note that the sole measurement of $S_{q_T}^{(0)}$, i.e. of the cross section integrated over $\phi_{CS}$, allows for a clean determination of the unpolarized gluon TMD, $f_{\perp}^{g}$, since $h_{1}^{+g}$ does not enter $S_{q_T}^{(0)}$. If the distributions $S_{q_T}^{(2)}$ or $S_{q_T}^{(4)}$ can also be measured, then the linearly-polarized gluon distribution, $h_{1}^{+g}$, is also accessible.

Numerical results and discussions.— In the literature the term unintegrated gluon distribution (UGD) is widely used, particularly in the field of small-$x$ physics, where it has been studied e.g. in the framework of the Color Glass Condensate (CGC) model [34–37]. UGDs also appear in $k_T$-factorization approaches and as the solution of the CCFM equation [38]. These UGDs are not identical to the gluon TMDs appearing in the TMD factorization formula. For instance, the Weizsäcker-Williams distribution that appears in the CGC model does have the same operator structure as Eq. 2, but with a lightlike gauge link. The regularization of the rapidity divergence is thus different. The CCFM equation does not rely on a gauge-invariant-operator definition; its connection with the TMD formalism is certainly less trivial than sometimes asserted, as in e.g. [39, 40]

We nonetheless think that these UGDs form justifiable Ansätze for gluon TMDs. These are in any case sufficient to evaluate $S_{q_T}^{(0)}$ and address the experimental requirements to learn genuinely new information on the gluon transverse dynamics, i.e. on the underlying physics involved in it.

For $f_{\perp}^{g}$, we have taken the Set B0 solution to the CCFM equation with an initial distribution based on the HERA data from [41, 42], the KMR parametrization from [43] and the CGC model prediction from [34–37]. The first two depend on a factorization scale, taken to be the invariant mass of the pair, $Q$, whereas the last one depends on a saturation scale taken as $Q_s = (x_0/\chi)^{\lambda} Q_0$, with $\lambda = 0.29$, $x_0 = 4 \cdot 10^{-4}$ and $Q_0 = 1$ GeV [44]. We have also used a simple Gaussian parametrization, as done in [45] to describe the intrinsic gluon TM, but with a scaled-up width of $\langle p_{T}^{2} \rangle = (2.5 \text{ GeV})^2$. The resulting TM distribution are shown in Fig. 3a.

For $h_{1}^{+g}$, we use the CGC model prediction of [36, 37] and the maximal value from the positivity constraint $|h_{1}^{+g}| \leq 2 M_{T}^{2}/k_{T}^{2} f_{\perp}^{g}$ [8]. The latter Ansatz is accordance with $k_T$-factorization in which a full gluon polarization is implicit. The resulting $S_{q_T}^{(2,4)}$ are plotted in Fig. 3b and Fig. 3c.

From Fig. 3a, we first conclude that measuring $S_{q_T}^{(0)}$ in bins of 1 GeV should suffice to get a first determination of the shape of the unpolarized gluon distribution. As regards $S_{q_T}^{(2)}$ and $S_{q_T}^{(4)}$, whose magnitude is obviously smaller, one can integrate them over $q_T^2$ (up to $(Q/2)^2$) to get the first experimental verification of a nonzero linearly-polarized gluon distribution. $S_{q_T}^{(2)}$ is here the most promising as we obtain for the integrated distribution $-2.9\%$, $-2.6\%$, $-2.5\%$ and $-2.0\%$ for the Gauss, CGC, SetB and KMR Ansatz respectively, whereas for the $n = 4$ distribution we obtain $1.2\%$, $0.7\%$, $0.6\%$, and $0.3\%$ for the Gauss, SetB, KMR and CGC model respectively. We note that the $q_T$-integrated cross section for $\Upsilon + \gamma$ production in Fig. 2 is about 100 (50) fb/GeV at $Q = 20$ GeV for $\sqrt{s} = 14(7)$ TeV. The 40 fb$^{-1}$ of integrated luminosity already collected at 7 + 8 TeV should be sufficient to measure the $q_T$ shape of $S_{q_T}^{(0)}$, while $S_{q_T}^{(2)}$ could be measured in a single $q_T$-bin.

Conclusion.— The production of an isolated photon back-to-back with a -possibly isolated- quarkonium in pp collisions is the ideal observable to study the transverse dynamics and the polarization of the gluons in the proton along the lines of TMD factorization. The requirement for a heavy quarkonium in the final state suppresses quark-initiated reactions making it a very clean probe of the gluon content of the proton, whereas the large scale set by the invariant mass of the pair allows a TMD-factorized description over the extensive range of $q_T$ and hence an extraction of the gluon TMDs in this range. The expected yields at the LHC experiments are large enough to get the first experimental verification of a nonzero gluon polarization in unpolarized protons and to finally deliver the first extraction of the gluon TMDs in the proton.

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