On Branching Ratio Measurements of $B_s$ Decays

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Abstract

We have just entered an era of precision measurements for $B_s$-decay observables. A characteristic feature of the $B_s$-meson system is $B^0_s\rightarrow\bar{B}^0_s$ mixing, which exhibits a sizable decay width difference. The latter feature leads to a subtle complication for the extraction of branching ratios of $B_s$ decays from untagged data samples, leading to systematic biases as large as $O(10\%)$ that depend on the dynamics of the considered decay. We point out that these effects can be included through the effective $B_s$ decay lifetime, which requires time information of the untagged data sample. We also address experimental issues in these measurements, and advocate the use of the $B_s$ branching ratios, as presented in this note, in particle listings.
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The difference between these two branching ratio concepts involves $y_s$, and is specific for the considered $B_s$ decay, thereby involving non-perturbative “hadronic” parameters (which suffer from large theoretical uncertainties), CP-violating weak decay phases, and the $B^0_s - B^0_s$ mixing phase $\phi_s$. However, measuring the effective lifetime of the considered $B_s$ decay, the effect can be included in a clean way, i.e. without the need to refer to methods for dealing with non-perturbative physics.

In experimental analyses, this subtle effect has so far been neglected or only been partially addressed; examples are the branching ratio measurements of the $B_s \rightarrow K^+ K^-$, $B_s \rightarrow J/\psi f_0(980)$, $B_s \rightarrow J/\psi K_S$, $B_s \rightarrow D^+ \pi^-$ and $B^0_s \rightarrow D^- \pi^+$ decays by the LHCb, CDF, DØ and Belle collaborations.

I. INTRODUCTION

Weak decays of $B_s$ mesons encode valuable information for the exploration of the Standard Model (SM). The simplest observables are branching ratios, which give the probability of the considered decay to occur. Measurements of $B_s$ branching ratios at hadron colliders, such as Fermilab’s Tevatron and CERN’s Large Hadron Collider (LHC), would require knowledge of the $B_s$ production cross-section, which presently makes absolute branching-ratio measurements impossible. Hence experimental control channels and the ratio of the $f_s/f_u,d$ fragmentation functions, describing the probability that a $b$ quark hadronizes as a $B_s$ meson, are required for the conversion of the observed number of decays into the branching ratio.

At $e^+e^-$ $B$ factories operated at the $\Upsilon(5S)$ resonance, the total number of produced $B_s$ mesons is measured separately and subsequently also allows for the extraction of the $B_s$ branching ratio from the observed decays.

A key feature of the $B_s$-meson system is $B^0_s - B^0_s$ mixing, which leads to quantum-mechanical, time-dependent oscillations between the $B^0_s$ and $\bar{B}^0_s$ states. In contrast to the $B_d$ system, the $B_s$ mesons exhibit a sizable difference between the decay widths of the light and heavy mass eigenstates, $\Gamma^{(s)}_L$ and $\Gamma^{(s)}_H$, respectively. Currently the most precise measurement is extracted from a time-dependent analysis of the $B^0_s \rightarrow J/\psi \phi$ channel by the LHCb collaboration:

$$y_s \equiv \frac{\Delta \Gamma}{2 \Gamma_s} \equiv \frac{\Gamma^{(s)}_L - \Gamma^{(s)}_H}{2 \Gamma_s} = 0.088 \pm 0.014,$$

where

$$\tau_{B_s}^{-1} \equiv \frac{\Gamma^{(s)}_L + \Gamma^{(s)}_H}{2} = (0.6580 \pm 0.0085) \text{ps}^{-1} \tag{2}$$

is the inverse of the $B_s$ mean lifetime $\tau_{B_s}$.

In view of the sizable decay width difference, Eq. (1), special care has to be taken when dealing with the concept of a branching ratio. We shall clarify this issue and give an expression, allowing us to convert the experimentally measured $B_s$ branching ratio into the corresponding “theoretical” branching ratio. The latter is not affected by $B^0_s - B^0_s$ mixing and encodes the information for the comparison with branching ratios of $B^0_s$ decays, where the relative decay width difference at the $10^{-3}$ level can be neglected, or branching ratios of $B^+_{s0}$ modes.

II. EXPERIMENT VERSUS THEORY

What complicates the concept of a $B_s$ branching ratio is the fact that the untagged decay rate is the sum of two exponentials:

$$\langle \Gamma(B_s(t) \rightarrow f) \rangle = \Gamma(B^0_s(t) \rightarrow f) + \Gamma(\bar{B}^0_s(t) \rightarrow f) = R^f \Gamma e^{-\Gamma^{(s)} f,t} + R^f e^{-\Gamma^{(s)} f,t}, \tag{3}$$

corresponding to two mass eigenstates with different lifetimes. Using Eq. (1), the untagged rate can also be written as

$$\langle \Gamma(B_s(t) \rightarrow f) \rangle = \left( R^f + R^f \right) e^{-\Gamma_{f,t}} \times \left[ \cosh \left( \frac{y_s t}{\tau_{B_s}} \right) + A^{f}_{\Delta \Gamma} \sinh \left( \frac{y_s t}{\tau_{B_s}} \right) \right], \tag{4}$$

where the theoretical branching ratio $R^f$ can be extracted from untagged data samples, leading to systematic biases as large as $\mathcal{O}(10\%)$ that depend on the dynamics of the considered decay. We point out that these effects can be included through the effective branching ratio. The latter is not affected by $B^0_s - B^0_s$ mixing and encodes the information for the comparison with branching ratios of $B^0_s$ decays, where the relative decay width difference at the $10^{-3}$ level can be neglected, or branching ratios of $B^+_{s0}$ modes.

The difference between these two branching ratio concepts involves $y_s$ and is specific for the considered $B_s$ decay, thereby involving non-perturbative “hadronic” parameters (which suffer from large theoretical uncertainties), CP-violating weak decay phases, and the $B^0_s - B^0_s$ mixing phase $\phi_s$. However, measuring the effective lifetime of the considered $B_s$ decay, the effect can be included in a clean way, i.e. without the need to refer to methods for dealing with non-perturbative physics.

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where

$$A_{\Delta \Gamma}^f \equiv \frac{R_H^f - R_L^f}{R_H^f + R_L^f} \quad (5)$$

is a final-state dependent observable.

In experiment it is common practice to extract a branching ratio by determining the total event yield, ignoring information on the particles’ lifetime. The experimental branching ratio can thus be defined as follows [11]:

$$\text{BR}(B_s \rightarrow f)_{\exp} \equiv \frac{1}{2} \int_0^\infty \langle \Gamma(B_s(t) \rightarrow f) \rangle \, dt \quad (6)$$

$$= \frac{1}{2} \left[ \frac{R_H^f}{\Gamma(H^s)} + \frac{R_L^f}{\Gamma(L^s)} \right] = \frac{\tau_B}{2} \left( R_H^f + R_L^f \right) \left[ 1 + A_{\Delta \Gamma}^f y_s \right].$$

Note that this quantity is the average of the branching ratios for the heavy and light mass eigenstates.

On the other hand, what is generally calculated theoretically are CP-averaged decay rates in the flavor-eigenstate basis, i.e.

$$\langle \Gamma(B_s(t) \rightarrow f) \rangle|_{t=0} = \Gamma(B_s^0 \rightarrow f) + \Gamma(\bar{B}_s^0 \rightarrow f). \quad (7)$$

This leads to the following theoretical definition of a branching ratio:

$$\text{BR}(B_s \rightarrow f)_{\text{theo}} \equiv \frac{\tau_B}{2} \langle \Gamma(B_s^0(t) \rightarrow f) \rangle|_{t=0}$$

$$= \frac{\tau_B}{2} \left( R_H^f + R_L^f \right). \quad (8)$$

By considering $t = 0$, the effect of $B_s^0 - \bar{B}_s^0$ mixing is “switched off”. The advantage of this $B_s$ branching ratio definition, which has been used, for instance in Refs. [11] [12], is that it allows a straightforward comparison with branching ratios of $B_d^0$ or $B_s^0$ mesons by means of the $SU(3)$ flavor symmetry of strong interactions.

The experimentally measurable branching ratio, Eq. (6), can be converted into the “theoretical” branching ratio defined by Eq. (8) through

$$\text{BR}(B_s \rightarrow f)_{\text{theo}} = \left[ \frac{1 - y_s^2}{1 + A_{\Delta \Gamma}^f y_s} \right] \text{BR}(B_s \rightarrow f)_{\exp}. \quad (9)$$

In the case of a vanishing decay width difference, the theoretical and experimental branching ratio definitions are equal.

Inspection of Eq. (9) reveals that $y_s$ and $A_{\Delta \Gamma}^f$ are required for the translation of the experimental branching ratios into their theoretical counterparts. Ideally, the latter quantities should eventually be used in particle compilations, in our opinion.

The decay width parameter $y_s$ is universal and has already been measured, as summarized in Eq. (1). In Fig. 1 we illustrate Eq. (9) for a variety of values of $A_{\Delta \Gamma}^f$ and observe that differences between $\text{BR}(B_s \rightarrow f)_{\text{theo}}$ and $\text{BR}(B_s \rightarrow f)_{\exp}$, as large as $O(10\%)$ may arise for the current value of $y_s$.

The simplest situation corresponds to flavor-specific (FS) decays such as $B_s^0 \rightarrow D_s^- \pi^+$, where $A_{\Delta \Gamma}^{FS} = 0$ and the correction factor is simply given by $1 - y_s^2$.

However, if both the $B_s^0$ and the $\bar{B}_s^0$ mesons can decay into the final state $f$, the observable $A_{\Delta \Gamma}^f$ is more involved and depends, in general, on non-perturbative hadronic parameters, CP-violating weak decay phases, and the $B_s^0 - \bar{B}_s^0$ mixing phase $\phi_s$. Assuming the SM structure for the decay amplitudes and using the $SU(3)$ flavor symmetry to determine the hadronic parameters from relations to $B_d$ decays, theoretical analyses of $A_{\Delta \Gamma}^f$ were performed for the final states $J/\psi \phi$ [12], $K^+ K^-$ [13], $J/\psi f_0(980)$ [14], $J/\psi K_S$ [15] and $D_s^+ D_s^-$ [16].

### III. USING LIFETIME INFORMATION

The simplest possibility for implementing Eq. (9) is to use theoretical information about the $A_{\Delta \Gamma}^f$ observables. However, this input can be avoided once time information of the untagged $B_s$ decay data sample becomes available. Then the effective lifetime of the $B_s \rightarrow f$ decay can be determined, which is theoretically defined as the time expectation value of the untagged rate [17]:

$$\tau_f \equiv \frac{\int_0^\infty t \, \langle \Gamma(B_s(t) \rightarrow f) \rangle \, dt}{\int_0^\infty \langle \Gamma(B_s(t) \rightarrow f) \rangle \, dt}$$

$$= \frac{\tau_B}{1 - y_s^2} \left[ 1 + 2 A_{\Delta \Gamma}^f y_s + y_s^2 \right] \quad (10)$$

The advantage of $\tau_f$ is that it allows an efficient extraction of the product of $A_{\Delta \Gamma}^f$ and $y_s$. Using the effective
relative changes as large as 10%.

The prominent decay $B_s^0 \rightarrow \mu^+\mu^-$ is very sensitive to New Physics [20]. A similar analysis can also be performed for this channel, where a measurement of the effective $B_s^0 \rightarrow \mu^+\mu^-$ lifetime may actually open a new window to the physics lying beyond the SM [21].

### IV. $B_s \rightarrow VV$ DECAYS

Another application is given by $B_s$ transitions into two vector mesons, such as $B_s \rightarrow J/\psi\phi$ [22], $B_s \rightarrow K^{*0}\bar{K}^{*0}$ [23] and $B_s \rightarrow D_s^+D_s^-$ [8]. Here an angular analysis of the decay products of the vector mesons has to be performed to disentangle the CP-even and CP-odd final states, which affects the branching fraction determination in a subtle way, as recognized in Refs. [23, 24]. Using linear polarization states $\parallel$ with CP eigenvalue $\eta_k = +1$ and $\perp$ with CP eigenvalue $\eta_k = -1$ [25], the generalization of Eq. (9) is given by

$$BR_{\text{VV}}(B_s \rightarrow VV) = (1 - y_s^2) \left[ \sum_{k=0,\parallel,\perp} \frac{f_{VV,k}^{\exp}}{1 + y_s A_{\Delta^f,k}^{\text{SM}}} \right] BR_{\exp}^{VV},$$

where

$$f_{VV,k}^{\exp} = \frac{BR_{\exp}^{V,V,k}}{BR_{\exp}^{VV}} = \frac{\sum_{k=0,\parallel,\perp} f_{VV,k}^{\exp}}{BR_{\exp}^{VV}},$$

and $BR_{\exp}^{VV} = \sum_{k=0,\parallel,\perp} BR_{V,V,k}^{\exp}$ so that $\sum_{k=0,\parallel,\perp} f_{VV,k}^{\exp} = 1$. As discussed in Ref. [17], assuming the SM structure at the decay amplitude level, we can write

$$A_{\Delta^f,k}^{\text{SM}} = -\gamma_k \sqrt{1 - C_{V,V,k}^2} \cos(\phi_s + \Delta \phi_{V,V,k}),$$

where $C_{V,V,k}$ describes direct CP violation, $\phi_s$ is the $B_s^0 - \bar{B}_s^0$ mixing phase, and $\Delta \phi_{V,V,k}$ is a non-perturbative hadronic phase shift. The expressions given in Ref. [23] for the $B_s \rightarrow K^{*0}\bar{K}^{*0}$ decay take the leading order effect of $y_s$ into account, and assume $\phi_s = 0$ and negligible hadronic corrections.

The generalization of Eq. (11) is given by

$$BR_{\text{theo}}^{VV} = \sum_{k=0,\parallel,\perp} \left[ 2 - (1 - y_s^2) \frac{f_{VV,k}^{\exp}}{BR_{\exp}^{V,V,k}} \right] BR_{\exp}^{V,V,k},$$

and does not require knowledge of the $A_{\Delta^f,k}^{\text{SM}}$ observables.

### V. EXPERIMENTAL ASPECTS

Additional subtleties arise in the experimental determination of effective lifetimes and $B_s$ branching ratios. It is experimentally impractical to measure the time expectation value $\tau_f$ of the untagged rate as given by Eq. (10). Instead, the effective lifetime is commonly extracted by fitting a single exponential to the untagged rate $\tau_{\exp}^{B_s}$ [19, 26], which in general is described by two exponentials (see Eq. (4)). Due to detector effects on the one hand and the chosen fit criterium on the other, this fitted lifetime will differ from the analytic expression given in Eq. (10) (and Ref. [26]), depending on the values of $A_{\Delta^f,k}$ and $y_s$. However, for the measured value of $y_s$ in Eq. (11), the difference is always found to be less than 0.5%.

Another subtlety concerns the loss of lifetime information at hadron collider experiments. Specifically, an analysis of $B_s$ decays typically involves selection criteria that use the flight distance of the $B_s$ meson, or the
impact parameter of its decay products, in order to suppress a large number of background events. By rejecting short-living \(B_s\) meson candidates, the relative amounts of \(B_{sL}\) and \(B_{sH}\) mesons in the remaining data sample is altered, resulting in a biased result of the branching fraction determination. A correction can be obtained from simulation, but this requires a priori assumptions of the values for \(y_s\) and \(A_{A\Gamma}\).

The dependence of this correction on the particular value of \(y_s\) and \(A_{A\Gamma}\) is reduced when the effective lifetime is used. For example, the dependence of the branching fraction correction on the value \(A_{A\Gamma}\) is at the order of 1% if only events with \(t_{BF} > 0.5\) ps are considered, using the time-dependence of the \(B_s\) candidates by fitting for the effective lifetime with a single exponential.

With respect to current statistical uncertainties, both experimental systematic effects are still negligible.

VI. CONCLUSIONS

The established width difference of the \(B_s\) mesons complicates the extraction of branching ratio information from the experimental data, leading to biases at the 10% level that depend on the specific final state. On the one hand, these effects can be included through theoretical considerations and phenomenological analyses. On the other hand, it is also possible to take them into account through the measurement of the effective \(B_s \to f\) decay lifetimes, which is the preferred avenue. So far, these effects have not, or only partially, been included and we advocate to use the converted branching ratios for the compilation in particle listings.

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