Quark mixing in the discrete dark matter model

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We consider a model in which dark matter is stable as it is charged under a $Z_2$ symmetry that is residual after an $A_4$ flavour symmetry is broken. We consider the possibility to generate the quark masses by charging the quarks appropriately under $A_4$. We find that it is possible to generate the CKM mixing matrix by an interplay of renormalisable and dimension-six operators. In this set-up, we predict the third neutrino mixing angle to be large and the dark matter relic density to be in the correct range. However, low energy observables – in particular meson-antimeson oscillations – strongly limit the available parameter space.

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I. INTRODUCTION

Nowadays, there is strong observational evidence of the existence of dark matter (DM) \cite{1, 2}. Many experiments are currently looking for direct or indirect observation of a dark matter candidate \cite{3–5}. Among all the possible DM candidates stable cold dark matter (CDM) ones have been discussed in many Standard Model (SM) extensions. Typically its stability can be secured by introducing a parity, under which the CDM candidate is odd. This parity is often introduced \textit{ad hoc} with the sole purpose of making the DM candidate stable or it is related to a parity added to the theory by hand.

Recently, a model \cite{6–8} (to be referred to as DDM) was proposed that relates this DM parity to the residual symmetry of a spontaneously broken flavour symmetry. Flavour symmetries became popular after the discovery of particular patterns in neutrino mixing, since they can reproduce the observed structures from symmetry principles.

In DDM, only the lepton sector was studied. In this work, we will consider a simple way to add quarks to the model. We will show that this results in a diagonal CKM quark mixing matrix at the renormalisable level, but that non-renormalisable operators can generate correction to this in order to reproduce the observed mixing patterns. It is not the first time that a discrete lepton non-Abelian flavour symmetry is extended to the quark sector \cite{9–27}. However, contrary to the majority of the cases in our scenario we do not have \textit{flavons}, that is heavy flavour scalar SM singlets; instead SM scalar doublets transform non trivially under the flavour symmetry. Recently a similar set-up has been proposed in \cite{28}.

We will see that in our scenario we can predict the third \textit{neutrino} mixing angle to be in the near-future experimental sensitivity and that we have a strong enhancement of meson-antimeson oscillations. Nevertheless we have only a limited effect on the calculation of the relic density of the DM candidate.

II. THE MODEL

As in DDM, we assign matter fields to irreducible representations of $A_4$, the group of even permutations of four objects, isomorphic to the symmetry group of the tetrahedron. The group $A_4$ is often used in flavour model building as it naturally allows the neutrino mixing to be of the tribimaximal type \cite{29}, that fits the observed data very well. The properties of the group $A_4$ are summarized in the appendix.

The representations of the leptons and the Higgs fields are as in DDM. In particular, there is an $SU(2)$-doublet Higgs $\hat{H}$ in trivial representation of $A_4$ and we assume three extra copies of the Higgs $\eta = (\eta_1, \eta_2, \eta_3)$ transforming as a triplet. There are also four neutrinos: three of them transform as an $A_4$-triplet $N_T = (N_1, N_2, N_3)$ and a fourth transforms as a singlet $N_4$. We assume that all quarks transform in the same way as the charged leptons. Both left-handed and righthanded fields transform as one dimensional representations of $A_4$, with different representations.
over the generations: the first generation is taken to transform as 1, the second generation as $1'$ and the third generation as $1''$. They thus transform non-trivial under the $Z_3$ subgroup of $A_4$, but are uncharged under the $Z_2$ subgroup. All matter and Higgs assignments of our model are summarized in Table I.

<table>
<thead>
<tr>
<th>$SU(2)$</th>
<th>$Q_1$</th>
<th>$Q_2$</th>
<th>$Q_3$</th>
<th>$q_{R_1}$</th>
<th>$q_{R_2}$</th>
<th>$l_e$</th>
<th>$L_\mu$</th>
<th>$L_\tau$</th>
<th>$l_{Re}$</th>
<th>$l_{R_\mu}$</th>
<th>$l_{R_\tau}$</th>
<th>$N_T$</th>
<th>$N_4$</th>
<th>$H$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_4$</td>
<td>1</td>
<td>1'</td>
<td>1''</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

**TABLE I: Summary of relevant model quantum numbers.** $q = u, d$.

The resulting Yukawa Lagrangian for the leptons is unchanged

$$
\mathcal{L}_l = y_e \overline{\nu}_e l e_R \tilde{H} + y_{\mu} \overline{\nu}_\mu l \mu_R \tilde{H} + y_{\tau} \overline{\nu}_\tau l \tau_R \tilde{H} + \\
y_e^* \overline{\nu}_e l e_R (N_T \tilde{\eta})_1 + y_{\mu}^* \overline{\nu}_\mu l \mu_R (N_T \tilde{\eta})_1' + y_{\tau}^* \overline{\nu}_\tau l \tau_R (N_T \tilde{\eta})_1'' + \\
y_4^* \overline{\nu}_e N_4 \tilde{H} + M_1 N_T N_T + M_2 N_3 N_4 + h.c.,
$$

with $\tilde{\eta} = i \sigma_{2y}^\dagger$.

For the quarks, the lagrangian reads

$$
\mathcal{L}_q = y_u \overline{Q}_1 \tilde{H} u_{1R} + y_d \overline{Q}_2 \tilde{H} d_{2R} + y_t \overline{Q}_3 \tilde{H} t_{3R} + \\
y_u^* \overline{Q}_1 \tilde{H} d_{1R} + y_d^* \overline{Q}_2 \tilde{H} u_{2R} + y_t^* \overline{Q}_3 \tilde{H} t_{3R} + h.c.,
$$

where $\tilde{H}$ stands for $\tilde{\tilde{H}}$, defined analogously to $\tilde{\eta}$.

At the renormalisable level, both up- and downquark matrices are diagonal with all masses given by $m_i = y_i v_H / \sqrt{2}$, with $v_H / \sqrt{2}$ the vacuum expectation value of the $A_4$-singlet Higgs field. As we don’t aim to explain the hierarchy of the quark masses, with $m_u \approx 10^{-5} m_t$, we assume an hierarchy in the Yukawa couplings, with most Yukawa coupling being small to very small. For instance a Froggatt-Nielsen symmetry [30] could make this more natural.

As discussed in more detail in [6], electroweak symmetry is broken by the vacuum configuration

$$
\langle \tilde{H}^0 \rangle = v_H / \sqrt{2}, \quad \langle \eta_1^0 \rangle = (v_\eta / \sqrt{2}, 0, 0).
$$

We write the ratio between the vev of $H$ and $\eta$ as $\tan \tilde{\phi}$ and obviously, their squares sum to $(246 \text{ GeV})^2$. The vev of $\eta$ breaks the $A_4$ group into its subgroup $Z_2$, generated by $S$, that is diagonal in the three dimensional representation: $S = \text{Diag}(1, -1, -1)$. The $Z_2$ symmetry thus acts on the $A_4$ triples fields in the following way:

$$
Z_2 : \begin{align*}
N_2 & \rightarrow -N_2, & h_2 & \rightarrow -h_2, & A_2 & \rightarrow -A_2, \\
N_3 & \rightarrow -N_3, & h_3 & \rightarrow -h_3, & A_3 & \rightarrow -A_3,
\end{align*}
$$

where $N_{2,3}$ are the component of the triplet $N_T$ and $h_{2,3}$ and $A_{2,3}$ are respectively the CP-odd and CP-even components of the Higgs doublet $\eta_{2,3}$.

The residual $Z_2$ symmetry is responsible for the stability of the lightest combination of $h_2$, $h_3$, $A_2$ and $A_3$ which is the dark matter candidate. Indeed the $Z_2$-odd candidate may couple only to heavy right-handed neutrinos and not to the SM charged fermions, that are $Z_2$-even. Such a scalar dark matter candidate is potentially detectable in nuclear recoil experiments [4] [5].

The four $Z_2$-even components of the $Z_2$-even Higgs fields $\tilde{H}$ and $\eta_1$ we will call $H_0$, $H_1$, $A'_0$ and $A'_1$ in accordance with [8]. They give rise to two scalars $H$ and $H_0$, one pseudoscalar $A_0$ and the neutral Goldstone boson of electroweak symmetry breaking.

As mentioned above, the charged leptons and quarks transform non-trivially under the $Z_2$ subgroup of $A_4$ generated by $T$. The vev configuration (5) clearly breaks this $Z_3$. Still, at the tree level, quark and charged lepton masses preserve the $Z_3$ invariance thanks to the scalar charge assignments. As we will see in the next section $Z_3$ breaking effects appear at next to leading order (NLO) level giving rise to the quark mixing matrix.
III. QUARK MIXING

Quark masses at the tree level are given by equation (2). This gives rise to diagonal quark mass matrices and the CKM matrix \( V_{\text{CKM}} = (V_{\text{CKM}}^c)^T V_{\text{L}}^d \) is simply the identity matrix.

The gauge and flavour charge assignments in the Higgs sector allow the construction dimension six operators for the down-type quark masses that contain \( H, \eta \) and their conjugates. There are three ways to contract the \( SU(2) \) indices, represented by brackets in the equation below

\[
\sum \frac{f_{ij}}{\Lambda^2} (Q_i \eta) d_j (\eta^\dagger) + \frac{f'_{ij}}{\Lambda^2} (Q_i \eta) d_j (\eta^\dagger) H + \frac{f''_{ij}}{\Lambda^2} (Q_i \eta) d_j (\eta^\dagger) \eta',
\]

The contraction of \( A_i \) indices between the two \( \eta^{(i)} \) triplets (see equation (A3)) is such that it generates the right type of singlet (1, 1’ or 1’’) to match the charges for \( Q_i \) and \( d_j \). It is important to note that this is possible for any combination of \( i \) and \( j \) due to the product rules of \( A_i \). \( \Lambda \) is the cut-off scale, up to which we accept the theory to be valid and the \( f \) couplings are dimensionless. Analogous dimension-6 operators can obviously be constructed for up-type quarks and charged leptons.

The mass term Lagrangian \( \mathcal{L} \) and the effective couplings \( \mathcal{O} \) generate the effective mass matrix for down-type quarks

\[
M_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} + \frac{v_H v_\eta^2}{\Lambda^2} \begin{pmatrix} h_{dd} & h_{ds} & h_{db} \\ h_{sd} & h_{ss} & h_{sb} \\ h_{bd} & h_{bs} & h_{bb} \end{pmatrix} + \mathcal{O}(1/\Lambda^2),
\]

where \( h_{ij} = (f_{ij} + f'_{ij} + f''_{ij})/2\sqrt{2} \). Analogous expressions again hold for up-type quarks and charged leptons.

Now the crucial question is how large the cut-off scale \( \Lambda \) is. In principle, this is a scale we are free to set. Only using 'naturalness' and 'finetuning' arguments, we can find a range for it. We will give two arguments, both pointing to a scale of 1 to 10 TeV.

In the first argument, we demand that there should not be more than 10 to 100% corrections to the Higgs from one-loop corrections to the Higgs propagator with the fermions and the (new) scalars of the theory. These corrections are typically of the order \( \Lambda^2/(4\pi)^2 \) and requiring them to be not too large with respect to \( v^2_{\text{ew}} = (246\text{GeV})^2 \) indeed gives \( \Lambda \lesssim (1 \text{ to } 10) \text{ TeV} \).

Interestingly, we find the same scale from an argument where we require the dimensionless parameters \( h \), in particular \( h_{ds} \), to be of order 1. The off-diagonal terms in equation (6) are responsible for generating the quark mixing as parameterized by the CKM matrix. As we don’t have information about the size of the dimensionless parameters \( h \), we assume them to be of order 1, which can be seen as the most natural assumption for dimensionless parameters.

Under this assumption, the absolute values of the corrections to the leading order elements of the mass matrix are of the same order for the up-type quark matrix and the down-type quark matrix. However, due to the much larger elements of the leading order up-type quark mass matrix, the effects on quark mixing are dominated by the down-type quark contributions. This allows us to estimate the order of magnitude of the cut-off scale.

Now the (1 2) element of equation (6) should be of order \( \lambda_C m_s \) in order to reproduce the Cabibbo angle.

\[
h_{ds} \frac{v_H v_\eta^2}{\Lambda^2} = \lambda_C m_s,
\]

This gives

\[
\Lambda^2 = h_{ds} \frac{v_H v_\eta^2}{\lambda_C m_s} = h_{bd} \frac{v_\beta^3}{(\tan^2 \beta)(1 + \frac{1}{\tan^2 \beta})^{3/2}} \lambda_C m_s = [(1 \text{ to } 10) \text{ TeV}]^2,
\]

depending on the exact values of \( h_{ds} \) and \( \tan \beta \), which we have taken between 0.1 and 1 and between 0.1 and 10 respectively. Due to the large bottom mass, the effect of \( h_{db} \) and \( h_{sb} \) of the same size as \( h_{ds} \) on \( \theta_{13} \) and \( \theta_{23} \) is relatively minor and these angles are thus naturally smaller than the Cabibbo angle and indeed in a large part of parameter space, we can fit them to their measured values.

The analogue of the dimension 6 operator affects the lepton mixing. In \( \mathcal{O} \), it was shown that at leading order, the lepton mixing matrix has zero \( \theta_{13} \)-angle and large \( \theta_{12} \) and \( \theta_{23} \) mixing angles, although these do not necessarily
fit in a mixing pattern such as tribimaximal or bimaximal mixing. The fact that the down-type quark and charged lepton mass matrices are alike (at least at leading order) suggests that the matrices that diagonalize them, $V_L^d$ and $V_L^e$, are also similar. We thus expect a large angle (of the order of the Cabibbo angle) in the (1 2) sector of $V_L^e$, this affects all three angles. In particular, we expect a Cabibbo-sized correction to the $\theta_{13}$-angle. Although $\theta_{13}$ has not been measured yet, the current upper bound still allows an angle of this size, and some fits actually favour this. In any case, we predict that non-zero $\theta_{13}$ should be measured by next generation experiments, such as Daya Bay and Double Chooz.

We end this section with a comment on the scale of the neutrino seesaw. In the DDM model, neutrino masses are assumed to originate from the type-I seesaw

$$m_\nu = -m_{D_{3\times 4}} M_{R_{4\times 4}}^{-1} m_{D_{3\times 4}}^T = -\frac{v_\eta^2}{2 M_1} \begin{pmatrix} (y_1^e)^2 + \frac{(y_2^e)^2 M_1}{M_2} y_1^e y_2^e y_1^\nu y_3^\nu & (y_2^e)^2 y_2^e y_3^\nu \\ (y_1^e)^2 y_1^e y_2^e & (y_2^e)^2 y_2^e \end{pmatrix}.$$  \hspace{1cm} \text{(9)}$$

This has two non-zero eigenvalues, that are of the order $y_1^e y_2^e v_\eta^2/M_\nu$. There can be a low energy seesaw, where we identify the mass scale of the right-handed neutrinos to $\Lambda$ if the neutrino Yukawa couplings are not too small $y_\nu = 10^{-(4\div 5)}$. Lastly, next-to-leading order effects lift the mass of the lightest neutrino away from zero. Still, it is suppressed with respect to the other neutrino masses by a factor $v_\eta/\Lambda$.

\section{FCNC}

The inclusion of the operators eq. \text{[5]} gives rise to tree level FCNC processes mediated by the $Z_2$-even scalar ($H_0$ and $H$) and pseudoscalar ($A_0$). In the mass eigenstate basis, the trilinear couplings read

$$1/(2\sqrt{2}) \bar{F}_{Li} f_{Rj} H \nu_0^2 v_h ([U_{HH_0}]_{11}/v_h + 2[U_{HH_0}]_{21}/v_\eta)(f_{ij} + f_{ij}^*/f_{ij}^{'}) ,$$

$$1/(2\sqrt{2}) \bar{F}_{Li} f_{Rj} H_0 \nu_0^2 v_h ([U_{HH_0}]_{12}/v_h + 2[U_{HH_0}]_{22}/v_\eta)(f_{ij} + f_{ij}^*/f_{ij}^{'}) ,$$

$$\pm 1/(2\sqrt{2}) \bar{F}_{Li} f_{Rj} A_0 [U_{GA_0}]_{12}(f_{ij} + f_{ij}^*/f_{ij}^{'})/v_h + 2[U_{GA_0}]_{22} f_{ij}^*/v_\eta,$$ \hspace{1cm} \text{(10)}$$

where in the last equation $+$ is for up-quarks and $-$ for down-quarks and charged leptons as well. $U_{HH_0}$ and $U_{GA_0}$ are the matrices that relate the scalar and pseudoscalar mass and interaction eigenstates; in particular $U_{GA_0}$ is a rotation matrix over the angle $\beta$ \text{[8]}.

In this section, we will focus on meson-antimeson oscillations \text{[34]}, as these are among the most constraining tests for new physics. In particular, we find that they are more constraining than the often-discussed K-meson decays. In our model, the Standard Model box diagrams are accompanied by new tree diagrams; see figure \text{[1]}. Indeed we

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Feynman diagrams for $B$ mesons oscillations in the SM (left) and in our model (right); similar diagrams can be drawn for $B_s$ and $K$ mesons.}
\end{figure}

find that the meson-antimeson oscillations severely reduce the parameter space of our model. As mentioned in the previous section, the CKM matrix can dominantly originate from corrections to the up-type quark mass matrices or
the down-type quark mass matrices and we mentioned that dominance of the latter is more natural. Indeed we find that if corrections of the former type dominate or even if there is no dominance of one of the two, $\Delta M_D$ of D meson oscillations is much larger than the experimental bound and this scenario should be excluded as shown in figure 2.

FIG. 2: $D$ meson oscillations in a scenario where the CKM matrix is generated dominantly by corrections to the up-type quark mass matrix.

In case the CKM matrix mostly originates from corrections to the down-type quark mass matrix, we find that in particular the bounds from $\Delta M_K$ and $\Delta M_{B_s}$ are rather strong. In figure 3 we show the contribution of the new diagrams to these as well as to $\Delta M_{B_d}$ as function of the lightest Higgs mass. We see that in almost all of parameter space the points are above the lower dashed line, which indicates the current experimental value [35] that is well described by the Standard Model box diagrams [36, 37]. Now to compare with the experimental data we should consider the amplitude for the process $q_1 \bar{q}_2 \rightarrow q_2 \bar{q}_1$, being $q_1, q_2$ the quarks that constitute the meson, taking into account both the SM contribution, namely $A_{SM}$, and the new contribution present in our DDM model, that we indicate as $A_{DDM}$. Thus the total $\Delta M_X$, with $X = K, B_s, B_d$ will be

$$\Delta M_X \propto |A_{SM} + A_{DDM}|^2.$$  \hfill (11)

Since $A_{SM}$ gives a contribution to $\Delta M_X$ that is close to the experimental bound we have two options: either $A_{DDM} \ll A_{SM}$ or there should be a significant amount of negative interference between the SM and the new diagrams, thanks to the presence of the new phases involved. In the latter case $A_{DDM}$ is roughly allowed to take values up to twice the experimental value. In fig. 3 it corresponds to take all the points below the upper dashed line. We conclude that it is possible to match the meson oscillation constraints, but only if we admit a strong cancelation between the Standard Model diagrams and the new physics diagrams. The need for cancelation diminishes for larger Higgs masses, although there are still no points below the lower line. Indeed, requiring that the new diagrams contribute less than the experimental bound, as is customarily done, the model would be excluded. On the other hand allowing a strong negative interference between the SM and the DDM contributions does not further constrain the scalar spectrum with respect to the analysis done in [8]. Indeed fig. 4 shows that there is no correlation between the bound imposed and the mass of the lightest $Z_2$-even scalar state, even if the number of points allowed significantly reduces with respect to those in [8].

V. DARK MATTER RELIC DENSITY AND DIRECT DETECTION

The operators [5] contribute to the same DM annihilation channel into down-type quarks – we have seen that the up-type quarks are forced to be almost diagonal – obtained by the exchange of the SM Higgs in the s-channel. In [8] it has been shown that the model, without the inclusion of NLO terms, gives rise to an available DM candidate. The relic dark matter density gives constraints on the parameter space of the model. In particular it was found that dark matter mass $M_{DM}$ is in the range 1 - 100 GeV, the region with $M_{DM} < 40$ GeV and mass of the standard
model Higgs $M_H > 400$ GeV is excluded, while for $M_{DM} > 50$ GeV the Higgs mass can go up to about 500 GeV since co-annihilation can be possible. For $M_{DM}$ lighter than about 80 GeV dark matter annihilate (coannihilate) into fermions through exchange of scalar (pseudoscalar and gauge boson) in the s-channel. For masses heavier than about 80 GeV the main channels of annihilation are with $W$ bosons in the final state.

Below we consider the relevance of the inclusion of the dimension-six terms of eq. (5) to the relic density. The operators (5) give an effective quartic coupling of the dark matter with quarks. We therefore study the effects of such a operator only for dark matter mass below 80 GeV since for heavier dark matter mass the main annihilation is into
We recall that the DM candidate is one of the four neutral state (2 scalar and 2 pseudoscalar) arising by the mixing of the neutral components of \(\eta_2\) and \(\eta_3\) that in [8] have been indicated as \(H_m' + i A_m'\) with \(m = 2, 3\). From eq. (5) it is easy to derive the \(H_m', A_m'\) coupling with fermions. For the couple \(H_m'H_m'\) with \(m = 2, 3\) the coupling is proportional to

\[
v_H(f_{ij} + f'_{ij} + f''_{ij})/\Lambda^2 = v_H h_{ij}/\Lambda^2,
\]

while for \(A_m'A_m'\) to

\[
v_H(f_{ij} + f'_{ij} - f''_{ij})/\Lambda^2.
\]

In the case the DM candidate turns out to be the CP-even \(Z_2\)-odd lightest state we may estimate the contribution of the new operator to the total \(\sigma_{\chi\chi \to d\bar{d}}\). Similar conclusions would be obtained considering the case in which the DM candidate is the CP-odd \(Z_2\)-odd state. Moreover we consider only the scattering into down-type quarks since FCNC processes forces the \(h'_{ij}\) to be negligible. In the previous section we have seen that

\[
h'^d_{ij} v_H v_H^2 / \Lambda^2 \sim m_s \lambda_C,
\]

for any couple \((ij) \neq 11\), since for the first family the \(h'^d_{11}\) is required to be smaller to fit down mass and we need

\[
h'^d_{11} v_H v_H^2 / \Lambda^2 \sim m_d.
\]

We may define the \(\lambda^q_{eff}\) of the four points interactions \(\chi\chi d_i\bar{d}_j\) as

\[
\lambda^q_{eff} \sim h'^d_{ij} v_H v_H^2 / \Lambda^2 \sim m_s \lambda_C / v_H^2.
\]

For the process \(\chi\chi \to d\bar{d}\) we have to compare the contribution of this operator to the \(\sigma v_{rel}\) with respect to the one arising by exchanging the lightest scalar CP-even, the SM-like Higgs, in the s-channel. Since the SM-like Higgs couples to fermions proportionally to \(y_q \sim m_q/v_H\), it turns out that the new contribution is negligible if

\[
\frac{m_s \lambda_C}{v_H^2} < \frac{m_q A_H}{v_H m_h^2}.
\]

In eq. (17) \(m_h\) is the mass of the lightest CP-even neutral scalar and \(A_H \sim v_W\) the dimensional coupling that controls the interaction of the dark matter with the Higgs doublet \(H\chi\chi\). Since \(v_q \sim v_H \sim m_h \sim A_H \sim v_W\) the new contribution is naturally subleading for the second and third generation. On the contrary for the first generation the new contribution to \(\chi\chi \to d\bar{d}\) is of the same order of the old one and to be conservative we may estimate that this channel will be negligible for values \(M_{DM} \geq 1\) GeV.

NLO terms also give rise to non-diagonal scattering \(\chi\chi \to d\bar{d}\) not included in the analysis done in [8]. In particular the scattering \(\chi\chi \to d\bar{b}\) could be non-negligible for DM masses around a GeV. For this reason we conclude that the previous analysis is not affected in the range \(M_{DM} \geq 5\) GeV. We take this lower bound as a further constrain for the scalar sector parameter space postponing a complete new analysis to the future [8].

However the direct detection is not affected at all by the NLO terms: the quark flavour diagonal scattering contribution are subdominant with respect to the ones mediated by the scalar \(H\) while the off diagonal one could only give rise to processes that are not kinematically allowed such as \(\chi + \mathcal{N} \to \chi + \mathcal{N} + \pi^+ + e^- + \nu\) with \(\mathcal{N}\) a nucleus in the detector bulk.

VI. CONCLUSIONS

Neutrino mixing might be explained by a discrete non-Abelian flavour symmetry. If this symmetry is dynamically broken only in one direction a residual symmetry survives. It is interesting that this residual symmetry may be

responsible for the existence of a stable dark matter candidate. It has already been shown that this set-up can describe the physics in the lepton sector and the dark matter abundance rather well.

It is a natural extension to see if the quark sector can also be described in such scenario’s. In this paper we investigated this possibility in a particular model \cite{6,7}. We found that if we add quarks in the same non-trivial representations of the flavour symmetry as charged leptons, interplay between renormalizable operators and dimension-six operators can generate a realistic CKM matrix in a large portion of parameter space. It is possible to let the CKM matrix dominantly originate from the up-type quark sector as well as the down-type quark sector, although the latter case is more natural.

The new six dimension operators should also be present in the lepton sector. As consequence we have shown that the predictions of the original model, $\theta_{13} = 0$ and $m_3 = 0$, are shifted away from zero, in the former case into the near-future observable region. At the same time for what concerns the quark sector the new operators can lead to new channels for flavour changing neutral currents and we have analysed their effects on meson-antimeson oscillations. We found that D meson oscillations rule out the scenario where the CKM matrix originates mostly from the up sector and $K$ and $B_{s,d}$ meson oscillations are much enhanced as well, with their amplitudes at least as large as corresponding the Standard Model amplitudes. If this is the case the scalar sector behaves exactly as described in \cite{8}.

Although these observations do not rule out the extension of the discrete dark matter model with quarks, honesty forces us to say that this ‘natural extension’ is less natural than we hoped.

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Appendix A: The group $A_4$

All 24 elements of $A_4$ are generated from two elements $S$ and $T$ with $S^2 = T^3 = (ST)^3 = I$. $A_4$ has four irreducible representations, three singlets $1$, $1'$ and $1''$ and one triplet $3$.

We can choose a basis in which $S$ and $T$ can be represented as $(1,1)$, $(1,\omega)$, $(1,\omega^2)$, with $\omega = e^{2\pi i/3}$ for the three one dimensional representations and

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \ ; \quad T = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{pmatrix} ;$$

for the three dimensional representation. $S$ and $T$ on themselves generate the two maximal subgroups of $A_4$, the Abelian $Z_2$ and $Z_3$. The $A_4$ multiplication rules are given by

$$1 \times r = r \text{ for all representations } r,$$

$$1' \times 1' = 1'' \ ; \quad 1'' \times 1'' = 1 \ ; \quad 1' \times 1'' = 1,'$$

$$1' \times 3 = 3 \ ; \quad 1'' \times 3 = 3,$$

$$3 \times 3 = 1 + 1' + 1'' + 3 + 3.$$ (A2)

Representing the two triplets as $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$, the elements of the last product are

$$(ab)_1 = a_1 b_1 + a_2 b_2 + a_3 b_3 ;$$

$$(ab)'_1 = a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3 ;$$

$$(ab)''_1 = a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3 ;$$

$$(ab)_3_1 = (a_2 b_3, a_3 b_1, a_1 b_2) ;$$

$$(ab)_3_2 = (a_3 b_2, a_1 b_3, a_2 b_1),$$

(A3)


