Warping and F-term uplifting

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Abstract: We analyse the effective supergravity model of a warped compactification with matter on D3 and D7-branes. We find that the main effect of the warp factor is to modify the $F$-terms while leaving the $D$-terms invariant. Hence warped models with moduli stabilisation and a small positive cosmological constant resulting from a large warping can only be achieved with an almost vanishing $D$-term and a $F$-term uplifting. By studying string-motivated examples with gaugino condensation on magnetised D7-branes, we find that even with a vanishing $D$-term, it is difficult to achieve a Minkowski minimum for reasonable parameter choices. When coupled to an ISS sector the AdS vacua is uplifted, resulting in a small gravitino mass for a warp factor of order $10^{-5}$.

Keywords: dS vacua in string theory, Flux compactifications, Supergravity Models, Supersymmetry Breaking.
1. Introduction

If supersymmetry is realised in nature, it must be broken. Low scale supersymmetry (SUSY) breaking is phenomenologically favoured: it addresses the hierarchy problem, and leads to gauge coupling unification and radiative electroweak symmetry breaking. If indeed the scale of SUSY breaking is low, and the LHC sees the superpartners of the standard model fields and measures their masses, what would this tell us about the underlying microphysics? String theory has tried to answer this question since its early days [1], but only recently has real progress been made thanks to a better understanding of the moduli sector and the associated sources of SUSY breaking.

Cosmological observations indicate that our universe undergoes a phase of accelerated expansion. This motivates the search for de Sitter vacua of string theory. KKLT have found an explicit way of constructing de Sitter solutions with a small cosmological constant in the context of type IIB string theory, using a combination of fluxes, D-branes and non-perturbative effects [2]. In their proposal, background RR and NS fluxes stabilise
the complex structure moduli of a Calabi-Yau compactification at some high scale. The remaining Kähler moduli can then be described by some low energy effective theory. Non-perturbative effects are invoked to stabilise the Kähler moduli. The resulting vacuum is SUSY preserving and anti de Sitter. An anti-D3-brane is added to uplift this solution to a dS vacuum with broken SUSY.

Since the original KKLT paper many variations to their stabilisation mechanism have been put forward. Instead of an D3 brane, a D-term originating from an anomalous U(1) could be used for uplifting [3]. This idea was put in a manifestly gauge invariant form in [4], which circumvents the problems of using D-terms for uplifting [5, 6]. The great advantage of an uplifting D-term over the KKLT proposal is that the whole setup is supersymmetric, with SUSY broken spontaneously in the vacuum; in contrast, the D3 brane employed in KKLT breaks SUSY explicitly. But the price to pay is that the uplifting term is naturally of order one in Planck units, giving rise to a large gravitino mass $m_{3/2} \sim m_{Pl}/(8\pi)$ and thus to high scale SUSY breaking. Uplifting mechanisms using $F$-terms have also been analysed [7, 8, 9], and models exist that are supersymmetric and have a small gravitino mass [10, 11].

In this paper we consider these issues within the context of warped compactifications [12, 13, 14] and study the effects of the warping on the Kähler potential. A central assumption in this work is that the throat dominates the volume of the 6D compactified space. We find that the $F$-terms are warped down while the $D$-terms are warping independent. Hence moduli stabilisation models with consistent $D$-terms and a small positive cosmological constant resulting from a large warping are only obtainable for vanishingly small $D$-terms. The resulting models have an $F$-term uplifting. We apply this setup to the case of string inspired configurations with D7-branes.

The requirement of gauge invariance constrains the possible setup considerably. In a general compactification, the non-perturbative effects needed to stabilise the volume modulus can come from either gaugino condensation on D7-branes or from D3 Euclidean instantons. However, only the former can be made compatible with gauge invariance. The stack of D7s on which gaugino condensation occurs is to be placed in the bulk. It cannot be in the throat, as then the gauge coupling of the Yang-Mills theory living on the D7s is red-shifted, leading to a vacuum with an exponentially small modulus VEV, and the effective field theory breaks down. Gauge invariance of both the superpotential from gaugino condensation and the D-term from fluxes is only possible if there are charged chiral fields living on the branes. This is achieved by considering magnetised D7-branes. In addition to the moduli fields, we should consider the origin of the standard model fields. The standard model quark and lepton (super)fields, which are chiral and in bi-fundamental representations, correspond to strings stretching between stacks of D7-branes [15]. Gauge couplings of order one dictate that the standard model branes are located in the bulk region. As usual, the supersymmetry breaking in the moduli sector is transferred to the superpartners of the standard model fields through gravitational interactions.

We study the string inspired examples where the moduli stabilisation superpotential results from the presence of magnetised D7-branes with a chiral spectrum and gaugino condensation [8]. We find no consistent Minkowski solutions with vanishingly small $D$-
terms, as needed for low scale SUSY breaking. However, starting from a stable AdS vacuum with $D \approx 0$ (for example the SUSY AdS vacuum with $F = D = 0$), we find that an ISS uplifting can be applied leading to small gravitino mass for a reasonable warp factor.

This paper is organised as follows. In the next section we perform the dimensional reduction of a warped space-time from ten to four dimensions, and derive the form of the Kähler potential. In addition, we discuss how the various elements in the effective action — the Kähler potential, the superpotential, the $D$-term, and the gauge couplings — red shift due to the warping. In section 3 we study moduli stabilisation with consistent $D$-term and $F$-term uplifting. In section 4 we apply these results to string inspired $F$-term uplifting models. The case of an ISS uplifting is considered in this section. We end with some concluding remarks.

2. Dimensional reduction

In this section we discuss dimensional reduction in warped compactifications. This allows us to derive the form of the various terms in the low energy effective four dimensional theory, and in particular how these terms are affected by the warping.

2.1 The Kähler potential

The background space-time metric arising in warped IIB theories preserving $N = 1$ SUSY is of the form [12]

$$ds_{10}^2 = \frac{\alpha'}{e^{2A(y)} R^2(x)} g_{\mu\nu} dx^\mu dx^\nu + R^2(x) e^{2A(y)} g_{mn} dy^m dy^n$$

(2.1)

with the length $x^\mu$ parameterising our 4 dimensional world, and the dimensionless $y^m$ the 6 extra dimensions. We scale $y^m$ so that $V_6 \equiv \int d^6 y \sqrt{g_6} = (2\pi)^6$, where $g_6 = \det g_{mn}$. The radius $R(x)$ represents the only Kähler modulus and measures the size of the extra dimensions.

The warp factor $e^{A(y)}$ is present due to the non-zero fluxes on the compactification manifold. The manifold is assumed to possess two different regions. First of all, in the bulk the metric $g_{mn}$ and the warp factor are of order one. In the throat, the metric is shifted in such a way that $e^{A(y)}$ is exponentially large. Throughout this work we assume that the throat dominates the volume of the 6D compactified space. We model the throat as a region akin to the AdS bulk of the Randall-Sundrum model with one preferred direction. Along the throat, the warp factor is essentially independent of the transverse coordinates to the throat direction. In the tip of the throat solution constructed by Giddings, Kachru and Polchinski, $e^{A(y)} \approx e^{2\pi K/3M} g_s$ with M units of R-R flux and K units of NS-NS flux [12]. $g_s$ is the string coupling constant. We also assume that the throat direction realises a foliation of the manifold in such a way that compact cycles in the transverse directions to the throat exist. In particular, we will assume that one can wrap Dp-branes around $(p - 3)$ cycles located at an essentially given value of $y$ along the throat, we will describe these
branes as being localised in the throat. As we will ultimately require the D7-branes to be located in the bulk, this assumption is not essential.

Using the above metric, we obtain the effective four-dimensional theory by integrating over the extra dimensions. The 10-dim Ricci curvature of the full space-time metric contains a term proportional to the curvature $R_4$ of the form $R_{10} \supset (R^2 e^{2A}/\alpha') R_4$. The remaining terms are total derivatives which can be dropped in the effective action. This implies that the dimensionally reduced Einstein-Hilbert action, in the supergravity frame, is

$$\frac{1}{(2\pi)^7 \alpha'^4 g_s^2} \int d^{10} x \sqrt{-g_{10}} R_{10} \supset \frac{1}{2\kappa_0^2} \int d^4 x \sqrt{-g_4} \left( f(T, \bar{T}) R_4 + 6 \partial_T \partial_{\bar{T}} T \partial^\mu \bar{T} \right), \tag{2.2}$$

where $g_4 = \det g_{\mu\nu}$. We have introduced the scale $\kappa_0^2 = \alpha'/g_s$, and the moduli field $T$, which is related to the size of the compactified dimensions

$$\text{Re} \, T = \frac{R_4}{2\pi g_s \alpha'^2}. \tag{2.3}$$

We also introduce $A_0$, defined by $\int d^6 y \sqrt{g_6} e^{4A} = V_6 e^{4A_0}$. Assuming that the warp factor is monotonic along the throat, there will then exist a point $y_0$ such that $A(y_0) = A_0$. Two natural situations can be envisaged. For bulk domination, $A_0 \approx 0$ while for throat domination we have $A_0 \approx A_m [16]$. We will focus on the latter case, which is consistent for $T \ll e^{4A_0} [14]$. We find

$$f(T, \bar{T}) = e^{4A_0} (T + \bar{T}) + \cdots \tag{2.4}$$

to leading order in the gravitational constant.

It is useful to define the Einstein frame, where there are no field dependent couplings in the Einstein-Hilbert action. The Kähler potential there is given by $K = -(3/\kappa_0^2) \ln f$. The 4D Einstein-frame metric is related to the supergravity frame metric by the conformal transformation

$$g_{\mu\nu}^E = f g_{\mu\nu} = \frac{e^{4A_0} R_4}{\pi g_s \alpha'^2} g_{\mu\nu}, \tag{2.5}$$

which brings the gravitational action into the form $(2\kappa_0^2)^{-1} \int d^4 x \sqrt{-g_E} R_4^{(E)}$. Although ultimately we will want to work in the Einstein frame, for the time being, we will remain in the supergravity frame to calculate the corrections to $f$.

### 2.2 Coupling to matter

Let us now introduce matter in this setting. This will induce subleading terms in the Kähler potential. Consider fields living on a Dp-brane wrapped around a $(p - 3)$-cycle of the 6-manifold, with $p = (3, 5, 7, 9)$ in type IIB string theory. Define by $d = (9 - p)/2$ the complex dimension of the complex normal bundle to the $(p - 3)$ cycle. Split the coordinates as $(x^\mu, z^i, z^a)$, $\mu = 0 \ldots 3$, $i = 1 \ldots (p - 3)/2$, $a = 1 \ldots d$. The D-brane action is obtained
from the Dirac-Born-Infeld action at leading order

\[
S_{Dp} \supset -\frac{1}{g_{p+1,YM}^2} \left( \int d^{p+1}x \sqrt{-\check{g}_{p+1}^{\mu\nu} \check{g}^{\rho\sigma}_{p+1} F_{\mu\nu} F_{\rho\sigma} } 
- \int d^{p+1}x \sqrt{-\check{g}_{p+1}^{\rho^{(p+1)}_{ab}} (\Phi^a, \bar{\Phi}^b) \check{g}^{\mu\nu} \partial_\mu \Phi^a \partial_\nu \bar{\Phi}^b } \right) .
\] (2.6)

The gauge coupling constant for the Yang-Mills (YM) field living on the brane is \( g_{p+1,YM}^{-2} = (2\pi \alpha')^2 \tau_p \) with \( \tau_p \) the brane tension; for a BPS brane it is \( \tau_p^{-1} = g_s (2\pi)^p \alpha'(p+1)/2 \). The gauge field is \( A_\mu \) along the non-compact dimensions and the \( \Phi^a \)'s are transverse modes to the brane parameterising the normal bundle, hence the contravariant indices. These fields are promoted to be \( x \)-dependent only. The induced metrics are \( \check{g}_{p+1}^{\rho^{(p+1)}_{ab}} \) and \( \check{g}_{p+1}^{\mu\nu} \) on the normal bundle. Effectively we have \( \check{g}_{\mu\nu,p+1} = \alpha' R^{-2} e^{-2A} g_{\mu\nu} \) and \( \check{g}^{\rho^{(p+1)}_{ab}} = (R^2 e^{2A}/\alpha')^{p+1} g^{\rho^{(p+1)}_{ab}} \). The warp factor is evaluated on the brane and \( g_{ab}^{p+1} \) is the metric induced on the normal bundle, with a Kähler potential \( g_{ab}^{p+1} = \partial_a \partial_b g^{p+1} \).

Dimensional reduction of (2.6) leads to a 4D Yang-Mills action \( g_{YM}^{-2} \int d^4x \sqrt{-g_4 F^2} \) with

\[
\frac{1}{g_{YM}^2} = \frac{R^{(p-3)} \check{V}_{p-3}^{(p-3)}}{2\pi g_s \alpha'(p-3)/2} ,
\] (2.7)

where we have defined \( \check{V}_{p-3} = (2\pi)^{3-p} \int d^{p-3}y \sqrt{\check{g}_{p-3}^{(p-3)A(y)}} \), which is dimensionless in our conventions. We approximate this as \( \check{V}_{p-3} = k_p e^{(p-3)A_b} \), where \( k_p = O(1) \) and \( A_b \) depends on \( p \) on the position of the brane in the compactified dimensions. We will focus on two different situations. First of all, the brane may be in the bulk where the warp factor is close to unity, so \( A_b \approx 0 \). Another situation corresponds to branes in the throat, then \( A_b = A_m \approx A_0 \). Supersymmetry requires the gauge coupling to be the real part of a holomorphic function \( g_{YM}^{-2} = \text{Re } f_G \), implying only a D3/D7 system is supersymmetric. For these two cases

\[
f_G = \begin{cases} 
\frac{k_3}{2\pi g_s} & (p = 3) \\
k_7 T e^{4A_b} & (p = 7)
\end{cases}
\] (2.8)

with \( A_b = 0 \) for bulk branes and \( A_b = A_m \) for branes in the throat. In the D7 case, the warp factor implies that the gauge coupling is very small if the branes live in the throat. As a consequence the branes carrying the standard model fields must live in the bulk.

Notice that, as we assume that both the complex structure moduli and the dilaton have been fixed by the flux induced superpotential (see next subsection), the gauge coupling function is field independent on D3-branes implying that no gaugino masses can appear there. Strictly speaking, the dilaton dependence of the D3 gauge kinetic function \( f_G \propto S = (2\pi g_s)^{-1} \) can lead to gaugino masses when the dilaton has a non-zero \( F^S \) term, i.e., contributes to the supersymmetry breaking. Here we assume that \( F^S = 0 \) and supersymmetry breaking occurs only after dilaton stabilisation leading to gaugino masses on D7-branes. Another advantage of D7s is the possibility of having intersecting branes.
On these intersections, strings stretching between branes are in bi-fundamental representations, which can be matched with the standard model representations in an easier fashion. We will come back to intersecting branes at the end of this section.

Consider matter fields on the branes. Scalar fields on the brane are sections of the normal bundle to the brane belonging to the adjoint $U(N)$ representation for a stacks of $N$ D$p$-branes (multiple gauge groups are present at intersections of D$p$-branes). Their action (2.6) reduces to

$$\frac{\pi g_s}{2} \int d^4x \sqrt{g_4} (T + \bar{T})^{(p-3)/4} \tilde{k}^{(p)}_{ab} \partial \Phi^a \partial \bar{\Phi}^b,$$  

(2.9)

where

$$\tilde{k}^{(p)} = \int \frac{dp_{-3}y}{(2\pi)^{p-3}} \sqrt{g_{p-3}} e^{(p-3)(A-A_0)} g^{p+1}(\Phi^a, \bar{\Phi}^b) \approx e^{(p-3)(A_b-A_0)} k^{(p)}(\Phi^a, \bar{\Phi}^b),$$

(2.10)

with $\tilde{g}^{p+1}_{ab} = (R^2 e^{2A / \alpha'}) \partial_a \partial_b g^{p+1}$ the Kähler metric on the normal bundle, and

$$k^{(p)}(\Phi^a, \bar{\Phi}^b) = \int \frac{dp_{-3}y}{(2\pi)^{p-3}} \sqrt{g_{p-3}} g^{p+1}(\Phi^a, \bar{\Phi}^b).$$

(2.11)

As before $A_b = 0$ for branes located in the bulk, and $A_b = A_m$ for branes in the throat. Including (2.9) in the full supergravity frame action (2.4), we find

$$f = e^{A_A}(T + \bar{T}) - \frac{\kappa_0^2}{6}(T + \bar{T}) e^{A_{A_0}} k^{(7)}(\Phi^a, \bar{\Phi}^b) - \frac{\kappa_0^2}{2\pi g_s} k^{(3)}(\Phi^a, \bar{\Phi}^b).$$

(2.12)

Finally, consider matter fields living at the intersections of D$p$ with D$p'$-branes. These matter fields live on a submanifold of dimension $p + p' - 12$. They are charged under the gauge groups of both stack of branes, and therefore belong to bi-fundamental representations. Their contribution to the Kähler potential can be deduced from the previous result (2.9), with the change $p - 3 \rightarrow p + p' - 12$. On the intersection, the induced metric reads $\tilde{g}^{\rho\rho'}_{ab} = (e^{A} R^2 / \alpha') g^{\rho\rho'}_{ab}$, and $g^{\rho\rho'}_{ab} = \partial_a \partial_b g^{\rho\rho'}$ is the Kähler metric on the normal bundle to the brane intersection. For intersecting D7-branes $p = p' = 7$, located at $y = y_s$, we obtain

$$f = e^{A_A}(T + \bar{T}) - \frac{\kappa_0^2}{6(\pi g_s)^{1/2}} (T + \bar{T})^{1/2} e^{2A_{sm}} k^{(7\cap\bar{7})}(\Phi^a, \bar{\Phi}^b)$$

(2.13)

where $A_{sm} = A(y_{sm})$, and

$$e^{2A_{sm}} k^{(7\cap\bar{7})}(\Phi^a, \bar{\Phi}^b) = \int d^2y \sqrt{g_2} e^{2A} g^{7\cap\bar{7}}(\Phi^a, \bar{\Phi}^b).$$

(2.14)

Having derived the expression for $f$, the Einstein frame Kähler potential is $K = -(3 / \kappa_0^2) \ln(f)$. For a setup with a single D7-brane at $y = y_b$, and two intersecting D7-branes at $y = y_{sm}$, the Kähler potential obtained from (2.12) and (2.13) is

$$K = -\frac{12 A_0}{\kappa_0^2} - 3 \kappa_0^2 \ln \left[ (T + \bar{T}) \left( 1 - \frac{\kappa_0^2}{6} e^{4A_{A_0}} k^{(7)}(\Phi^a, \bar{\Phi}^b) \right. \right.$$

$$\left. - \frac{\kappa_0^2}{6(\pi g_s)^{1/2}} (T + \bar{T})^{-1/2} e^{2A_{sm} - 2A_0} k^{(7\cap\bar{7})}(\Phi^a, \bar{\Phi}^b) \right].$$

(2.15)
This is the main result of this section. In terms of modular weights in the heterotic notations we find
\[ n_{D7} = 0, \quad n_{D3} = -1, \quad n_{D7 \cap D7} = -\frac{1}{2} \tag{2.16} \]
in agreement with the results of [17]. The weights have a geometrical origin.

### 2.3 Fayet-Iliopoulos term

Our model, to be discussed in section 3, possesses an anomalous U(1) symmetry. Stabilising the Kähler moduli is achieved in the presence of a field-dependent Fayet-Iliopoulos. Here we briefly recall the microscopic origin of such a term [18, 19, 20], focusing on how it is affected by warping. The existence of chiral fields leading to non-perturbative superpotentials on the branes is linked to a non-vanishing gauge field along the magnetised brane compactified directions. Hence in this context, potential terms due to the magnetic fields are always present. We will see that these terms arise in the 4d description due an anomalous U(1)\(_X\) symmetry.

It is most convenient to work in the Einstein frame in order to obtain the Einstein frame potential induced by the brane gauge fields. Ignoring all fields except \(T\), the 10D uplift of the Einstein frame metric is
\[ ds^2 = e^{-2A(y)-4A_0} \frac{\pi g_{0 \alpha} / 3}{R^6(x)} g_{\mu \nu} dx^\mu dx^\nu + e^{2A(y)} R^2(x) g_{nm} dy^n dy^m \tag{2.17} \]
which upon dimensional reduction gives the Einstein-Hilbert action \((2\kappa_0^2)^{-1} \int d^4x \sqrt{-g_4} R_4\).

The magnetic flux contribution to the action reads
\[ S_{YM} = -\frac{1}{g_{YM,8}^2} \int d^8x \sqrt{-\tilde{g}_8} \tilde{g}_{mn} \tilde{g}^{mn} F_{nm} F_{lk}. \tag{2.18} \]
Using \(\sqrt{-g_8 \tilde{g}_{mn} \tilde{g}^{mn}} \propto R^{-12} e^{-4A(z)-8A_0}\) gives, for a brane located at \(y = y_b\),
\[ V_{\text{gauge}} = \frac{e^{-4A(y_b)-8A_0}}{2\pi^2 \kappa_0^4 (T + T)^3} \int d^4y \sqrt{g_4(y)} F_{nm} F^{nm}. \tag{2.19} \]
The potential can be identified with a \(D\)-term potential coming from an anomalous U(1) gauge symmetry. Under this U(1) the volume modulus transforms \(T \rightarrow T - i(\delta_{GS}/2)\epsilon\) with \(\epsilon\) the infinitesimal gauge parameter, giving rise to a \(D\)-term \(V_D = (\delta_{GS}^2 K_T)^2/(8\text{Re} f_G)\) with the gauge kinetic function for a D7 \(f_G\) in (2.8) and \(K_T = \partial_T K\) follows from (2.15). It follows that the Green-Schwarz parameter is
\[ \delta_{GS} = \frac{\sqrt{2k_7}}{3\pi \kappa_0^2} e^{-4A_0} \left( \int d^4y \sqrt{g_4(y)} F_{nm} F^{nm} \right)^{1/2}. \tag{2.20} \]
The warping dependence of \(\delta_{GS}\) will also obtained from anomaly mediation conditions (3.8, 3.9). Consistency will then fix the warping dependence the above magnetic field integral.
2.4 Superpotential

The presence of fluxes induces a superpotential for the dilaton and the complex structure moduli of the form [21]

\[ W = \int G_3 \wedge \Omega, \]  
(2.21)

where the 3-form on the compactification manifold can be identified with

\[ \Omega_{nml} = \bar{\eta} \Gamma_{nml} \eta, \]  
(2.22)

with \( \eta \) a Killing spinor corresponding to the remaining supersymmetry. We have introduced \( \{ \gamma_n, \gamma_m \} = 2g_{nm} \) and the warped Dirac matrices \( \{ \Gamma_n, \Gamma_m \} = 2G_{nm} \) with \( G_{nm} = e^{2A}g_{nm} \).

The superpotential can be written as

\[ W = \int d^6y \sqrt{G_6} (\bar{\eta} \Gamma_{nml} \eta) (G_3)^{nml} = e^{3A_0} \tilde{W}_0, \]  
(2.23)

where the constant is

\[ \tilde{W}_0 \approx \int d^6y \sqrt{g_6} (\bar{\eta} \gamma_{nml} \eta) (G_3)^{nml}. \]  
(2.24)

Here and in the following we use the notation that the tilde quantities are independent of the warp factor, whereas the equivalent quantity without a tilde has some warp factor absorbed in it. To get the above equation we used that the \( \gamma \)-matrices are of order one, and in the second line that the integral is dominated by the value of the integrand in the throat.

Let us now include matter on D-branes. The full superpotential is

\[ W = e^{3A_0} \tilde{W}_0 + \tilde{W}_{SM}(\Phi^a) + \tilde{W}_{mod}(T, \Phi^i). \]  
(2.25)

The moduli stabilisation potential \( \tilde{W}_{mod} \), which involves the volume modulus \( T \) and chiral fields \( \Phi^a \) charged under the condensing gauge group, will be discussed in the next section. The standard model lives on intersecting D7s in the bulk, and has a superpotential

\[ \tilde{W}_{SM} = \frac{1}{2} e^{A_{sm}} \bar{\mu}_{ab} \Phi^a \Phi^b + \frac{1}{3} \lambda_{abc} \Phi^a \Phi^b \Phi^c \]  
(2.26)

which contains the Yukawa couplings and a \( \mu \)-term. We perform a Kähler transformation

\[ K \rightarrow K + \frac{12A_0}{\kappa_0^2}, \quad W \rightarrow e^{-6A_0} W \]  
(2.27)

which leaves the \( N = 1 \) supergravity Lagrangian invariant. This removes the constant from the Kähler potential (2.15). Consider further the small field approximation for the standard model fields \( k_{ab}^{(7)(7)} = \delta_{ab} \), and introduce the rescaled fields \( \phi^a = \Phi^a e^{-2A_0} (4\pi g_s)^{1/4} \). This gives a Kähler potential

\[ K = -\frac{3}{\kappa_0^2} \ln \left[ (T + \bar{T}) \left( 1 - \frac{\kappa_0^2}{3} (T + \bar{T})^{-1/2} \sum \ |\phi^a|^2 - \frac{\kappa_0^2}{6} e^{4(A_b - A_0)} k_{(7)} \Phi^i \Phi^j \right) \right], \]  
(2.28)
where we have set $A_{\text{sm}} = 0$ appropriate for SM fields in the bulk. The superpotential is

$$W = e^{-3A_0} \bar{W}_0 + e^{-6A_0} \left( \bar{W}_{\text{SM}}(\phi^a) + \bar{W}_{\text{NP}}(T, \Phi^i) \right)$$

$$\equiv W_0 + \bar{W}_{\text{SM}}(\phi^a) + W_{\text{NP}}(T, \Phi^i). \quad (2.29)$$

As before, we use the notation that the tilde quantities are independent of the warp factor, whereas the equivalent quantity without a tilde has some warp factor absorbed in it. Now $\bar{W}_{\text{SM}} = \frac{1}{2} \mu_{ab} \phi^a \phi^b + \lambda_{abc} \phi^a \phi^b \phi^c$ with

$$\mu_{ab} = e^{-2A_0} (4\pi g_s)^{1/2} \tilde{\mu}_{ab}, \quad \lambda_{abc} = (4\pi g_s)^{3/4} \tilde{\lambda}_{abc}. \quad (2.30)$$

The effective Yukawa couplings $\lambda_{abc}$, and thus the standard model masses, do not depend on the warp factor $A_0$ explicitly. The effective $\mu$ term red-shifts as $e^{-2A_0}$. As this is too large compared to the gravitino mass, we can instead use the Giudice-Masiero mechanism to generate a $\mu$ term [22]. Indeed, the gravitino mass $m_{3/2} = e^{5/2} \kappa_0^3 W \propto e^{-3A_0} W_0$. This opens up the possibility to tune the scale of supersymmetry breaking by “switching on” the warping.

2.5 The supergravity approximation

In this subsection we will discuss mass scales, and the regime where the low energy effective supergravity action is to be trusted.

So far we have expressed all quantities in terms of the fundamental string scale $\kappa_0 = \sqrt{\alpha'} g_s$. We can switch scales and go to Planck units by simply changing by $\kappa_0 \rightarrow \kappa_4$ in all formulae. For an observer located at $y = y_{\text{sm}}$, which we define to be the location of the standard model fields, the 4D Planck mass is

$$m_{\text{Pl}}^2 \equiv \frac{1}{\kappa_4^2} = \frac{f(T_0) e^{2A(y_{\text{sm}})} R(T_0)^2}{\alpha'} = \frac{1}{\alpha'} \left( \frac{8\pi T_0^3}{g_s} \right)^{1/2} e^{4A_0 + 2A(y_{\text{sm}})}, \quad (2.31)$$

where $T_0$ is the vacuum expectation value of $T$. Note that this change of length scale implies a conformal scaling of the spacetime metric [23], $g_{\mu\nu}^E \rightarrow \kappa_0^2 / \kappa_4^2 g_{\mu\nu}^E$, as well as the Kähler metrics [2.11] and [2.14], $k(p) \rightarrow \kappa_0^2 / \kappa_4^2 k_p$. For the remainder of this paper we will make the replacement $\kappa_0 \rightarrow \kappa_4$, and work in units of $\kappa_4$. The Kähler potential becomes

$$K = -\frac{3}{\kappa_4^2} \ln \left( (T + T^*) \left( 1 - \frac{\kappa_0^2}{3} (T + T^*)^{-1/2} \sum_a |\phi^a|^2 - \frac{\kappa_0^2}{6} e^{4(A_0 - A_0)} k^{(7)}(\Phi^i, \Phi^j) \right) \right). \quad (2.32)$$

The supergravity approximation is valid when the Kaluza-Klein masses of the higher dimension model are large and the higher order terms in the supergravity Lagrangian are under control. The lightest KK particles come from the tip of the throat where the length scales of the extra dimensions are maximal. Typically we expect that the KK masses are red shifted

$$m_{KK}^{(n)} \approx n e^{-2A_0} m_{\text{Pl}}$$

for $T = O(1)$ [14]. Thus if $m$ is the typical mass scale in the low energy effective theory, we require $m < e^{-2A_0} m_{\text{Pl}}$. 

Let us now turn to the higher order terms in the Lagrangian. We use $R^n_{10}$ to denote $n$ contractions of the ten-dimensional Riemann tensor symbolically (derivatives of the Riemann tensor can be treated similarly). Typically, upon dimension reduction in the supergravity frame, these terms yield

$$R^n_{10} \approx \sum_{p=0}^{n} c_p e^{(4p-2n)A(y)} R^p_4 R^{n-p}_6$$

(2.34)

for some coefficients $c_p$. $R_6 \approx M_s^2$, where $M_s^2 = 1/\alpha'$ is the string scale. Expanding the ten-dimensional effective action gives

$$S_{\text{eff}} \approx M_s^{10} \int d^{10}x \sqrt{-g_{10}} \sum_{m=0}^{\infty} d_m \frac{R^n_{10}}{M_s^{2m}}$$

$$\approx M_s^4 \int d^4x \sqrt{-g_4} \sum_{p=0}^{\infty} c_p \frac{R^p_4}{M_s^{2p}} \left( \sum_{m=p}^{\infty} d_m \int d^6y \sqrt{g_6} e^{(4p-2m+2)A(y)} \right)$$

(2.35)

with $d_0 = 0$. The sum over $m$ is dominated by the term $m = p$ and scales like $e^{(2p+2)A_0}$. Note that we retrieve the $e^{4A_0}$ behaviour when $p = 1$. Expanding the action, and switching to Planck units gives

$$S_{\text{eff}} \approx m_{\text{Pl}}^4 \int d^4x \sqrt{-g_4} \frac{R_4}{m_{\text{Pl}}^2} \left( 1 + e^{2A_0} R_4 \frac{R_4}{m_{\text{Pl}}^2} + \cdots \right) .$$

(2.36)

Hence the effective action in the supergravity frame after dimensional reduction is a series expansion where higher order terms are suppressed for $m^2 e^{2A_0} \ll 1$, with $m$ the typical low energy scale. This is automatic in the regime where throat KK particles are heavy. Thus, in general, we can neglect the higher order corrections and work in the lowest order of the supergravity approximation. Although $\alpha'$ corrections could help with moduli stabilisation and uplifting, we do not expect them to be useful for large warping. This is the case for the example in Appendix B.

To summarise, we have derived how the various terms in the low energy effective four dimensional theory are affected by the warping. The Kähler potential picks up a constant piece proportional to $A_0$, which originates from the warping dependence of the 6D volume. This translates into a warping down of the $F$-terms. On the other hand as we will find in the next section, the $D$-terms are scaled down by the volume of the 4-cycle, which for bulk branes is warping independent. When trying to find moduli stabilised configurations with a small cosmological constant obtained thanks to a large warp factor, one cannot accommodate large $D$-terms. The $D$-terms must be almost vanishing. The resulting configurations are then $F$-term uplifted vacua. In the next section, we will present string inspired configurations with an $F$-term uplifted minimum; however, we find that lifting the minimum all the way to Minkowski space proves to be difficult.
3. F-term uplifting with consistent D-terms

So far we have considered a stack of branes with gauge group U(N). By separating the branes, i.e., giving VEVs to the scalars in the adjoint representation, one can break the gauge group to a product of U(Ni)’s and U(1)’s. The rank is unaffected by the breaking and the model is non-chiral as the original fields are in the adjoint representation. In more general situations such as orientifold compactifications one can obtain chiral spectra (see [4]). For instance, for a single brane with gauge group U(1), a chiral field arises as the open string joining the brane and its orientifold image. By taking a stack of (N + 1) branes and separating one of the branes from the other ones, one can envisage a gauge group SU(N) × U(1) with a chiral spectrum. Quarks in the fundamental representation of SU(N) arise as open strings joining the stack of N branes to the single U(1) branes, antiquarks appear too as open strings joining the stack of branes to the orientifold image of the U(1) brane. There is also a charged field coming from the open string joining the U(1) brane to its orientifold image [8].

The presence of chiral matter is intimately linked to the existence of an internal flux on the U(1)X brane, i.e., the brane is magnetised. In the following we will consider a supergravity model with such a matter content. In this case, with quarks and antiquarks in the fundamental and anti-fundamental representations, non-perturbative phenomena can occur leading to superpotentials involving composite meson fields charged under the U(1)X [1, 23]. To be concrete, we take N D7-branes, possessing a chiral spectrum of Nf quark pairs {Φi, ˜Φi} with charges qi and ˜qi [4, 13]. In addition there is one SU(N) singlet Ξ whose U(1)X charge we normalise to −1. We take the quark (and antiquark) charges to be equal, so qi = q1 (and ˜qi = ˜q1). The U(1)X is anomalous, the implications of which we will discuss in subsection 3.2.

3.1 Moduli superpotential

The effective, non-perturbative superpotential generated by the gaugino condensation on the stacks of D7-branes is

\[ \tilde{W}_{NP} = \kappa^{-3}(N - N_f) \left( \frac{e^{-8\pi^2 f_G}}{\kappa^2 \det(\Phi_i \bar{\Phi}_j)} \right)^{1/(N - N_f)} \]  

where fG is given by (2.8) for p = 7. The gaugino condensation branes are located in the bulk with A(yb) = Ab. Together with W0, this provides a stabilising potential for the volume modulus T. In addition we introduce a direct coupling between the chiral matter fields \[ \tilde{W}_m = \kappa^{-1+q} \tilde{m} \det(\Phi_i \bar{\Phi}_j)^{1/N_f} \Xi^q \]  

where \( \tilde{m} \) is constant and \( q = q_1 + \tilde{q}_1 \). For simplicity we assume an overall squark condensate \( \chi \), for which \( \lvert \Phi_i \rvert^2 = \lvert \bar{\Phi}_i \rvert^2 = \lvert \chi \rvert^2 e^{4(\Lambda_0 - A_0)}/N_f \). The composite field \( \chi \) then has charge q/2. In the small field approximation \( k^{(7)} = \sum_i (\lvert \Phi_i \rvert^2 + \lvert \bar{\Phi}_i \rvert^2) + \lvert \Xi \rvert^2 \), where we have defined \( \lvert \Xi \rvert^2 = 2\lvert \chi \rvert^2 e^{4(\Lambda_0 - A_0)} \). The Kähler potential then reduces to

\[ K = -\frac{3}{\kappa^2} \ln \left( (T + \bar{T}) \left( 1 - \frac{\kappa^2}{3}(T + \bar{T})^{-1/2} \sum_a \lvert \phi_a \rvert^2 - \frac{\kappa^2}{3} (\lvert \chi \rvert^2 + \lvert \xi \rvert^2) \right) \right) \]  

where
The superpotential relevant for moduli stabilisation is
\[
W = e^{-3A_0}W_0 + e^{-6A_0}\left[\hat{W}_{NP}(T, \chi) + \hat{W}_m(\chi, \zeta)\right]
\equiv W_0 + W_{NP}(T, \chi) + W_m(\chi, \zeta)
\] (3.4)
with
\[
W_{NP} = A \frac{e^{-aT}}{\chi^b} = \frac{e^{-2(3+b)A_0+2bA_b}}{\kappa_4^{b+3}} A \frac{e^{-aT}}{\chi^b},
\]
\[
W_m = m \zeta^q \chi^2 = \frac{e^{-2(1-q)A_0-2(2+q)A_b}}{\kappa_4^{1-q}} 2q/2m \zeta^q \chi^2,
\] (3.5)
and
\[
\tilde{A} = (N - N_f)N_f^{b/2}, \quad a = \frac{8\pi^2k_Ne^{4A_b}}{N - N_f}, \quad b = \frac{2N_f}{N - N_f}.
\] (3.6)
As a result of the warping, the scales \(A\) and \(m\) in the non-perturbative superpotential are no longer at the Planck scale.

The \(F\)-term potential is
\[
V_F = e^{\kappa_4^2 k} \left(D_I W K^{IJ} D_J W - 3\kappa_4^2 |W|^2\right)
\] (3.7)
with \(D_I = \partial_I W + \kappa_4^2 K_I W\).

### 3.2 Anomaly cancellation and \(D\)-term potential

The model possesses an anomalous \(U(1)_X\) symmetry whose anomaly can be cancelled by the Green-Schwarz mechanism if \(T\) transforms non-trivially under \(U(1)_X\). Under a gauge transformation with infinitesimal parameter \(\epsilon\), \(\delta T = -i(\delta_{\text{GS}}/2)\epsilon\), where \(\delta_{\text{GS}}\) is the Green-Schwarz parameter. The corresponding transformations for the quarks and antiquarks living on the magnetised brane, and the \(SU(N)\) singlet, are respectively \(\delta \Phi_i = i\eta_i e^{\Phi_i}\), \(\delta \bar{\Phi}_i = i\bar{\eta}_i e^{\bar{\Phi}_i}\), and \(\delta \Xi = -i\epsilon \Xi\). The required cancellation condition for the \(SU(N)^2 \times U(1)_X\) anomaly is
\[
k_N e^{4A_b} \delta_{\text{GS}} = \frac{N_f}{4\pi^2}(q_1 + \bar{q}_1),
\] (3.8)
and the one for the \(U(1)_X^2\) anomaly is
\[
k_X e^{4A_b} \delta_{\text{GS}} = \frac{1}{6\pi^2}(NN_f(q_1^3 + \bar{q}_1^3) - 1).
\] (3.9)
The \(U(1)_X\) \(D\)-term potential follows from
\[
V_D = (2\text{Re} f_X)^{-1} \left(i\eta^I K_I\right)^2, \quad \text{with } I = \{T, \chi, \zeta\}
\]
and \(\eta = \{-i\delta_{\text{GS}}/2, i\eta/2X, -i\zeta\}\). Here \(\eta^I\) are the infinitesimal gauge transformations of the fields. Hence \(V_D \propto \delta_{\text{GS}}^2/k_X \propto e^{-12A_b}\). For bulk \(D7s\) \(A_b = 0\), and \(V_D\) is independent of the warp factor. This is a crucial result as it implies that, for \(g_{YM} \sim 1\), \(D\)-terms must vanish in order for the gravitino mass to be warped down.

Using (3.3) the \(D\)-term potential is then
\[
V_D = \frac{9\delta_{\text{GS}}^2}{8\kappa_4^2(T + \bar{T})^2 \text{Re} f_X} \left(1 + \frac{(T + \bar{T})}{3\delta_{\text{GS}} Y} \left|q\right|^2 - 2\left|\zeta\right|^2\right)^2
\] (3.10)
with \(q = q_1 + \bar{q}_1\) and
\[
Y = 1 - \frac{\kappa_4^2}{3} \left|X\right|^2 - \frac{\kappa_4^2}{3} \left|\zeta\right|^2.
\] (3.11)
4. Uplifting procedures

As discussed in section 3, we consider an $N = 1$ SUGRA model with the following data:

\[ K = -\frac{3}{\kappa_4^2} \ln \left( (T + \bar{T}) \left( 1 - \frac{\kappa_4^2}{3} \left( |\chi|^2 + |\zeta|^2 \right) \right) \right), \]
\[ W = W_0 + A e^{-aT} \frac{\chi^b}{\chi^b} + m\zeta q \chi^2, \quad (4.1) \]

and the $D$-term potential is given by (3.10), with the gauge kinetic function $f_X = k_X T$. Gauge invariance relates the parameters $bq = a\delta_{GS}$. The warping dependence of the superpotential is given by (3.4,3.5).

In this section we will look for (metastable) vacua of this system, which have zero cosmological constant and low scale SUSY breaking. Notice first that the $D$-term is warping independent whereas the $F$-terms depend on the warping. Guaranteeing a small supersymmetry breaking scale can only be achieved for a small value of the $D$-term contribution. This holds true even in the absence of warping.

The $D$-term can be (partially) cancelled by a non-zero VEV for $\zeta$. This generates a mass term for the $\chi$-fields. If the mass is greater than the other mass scales in the problem the $\chi$-fields can be integrated out. The resulting effective theory involving only $T$ and $\zeta$ has a SUSY preserving $F = D = 0$ AdS minimum. An additional $F$-term sector is needed to break SUSY and lift this minimum to Minkowski. We discuss this in more detail in subsection 4.2 where we add an Intriligator-Seiberg-Shih (ISS) [7] section to the model.

It is clear then that to get a SUSY breaking Minkowski vacuum in the model (4.1), we must be in the regime where the $\chi$-fields are light and cannot be integrated out. This is the approach taken in the next subsection. We will perform a systematic expansion of the potential in a small parameter $\epsilon \propto T\chi^2/\zeta^2$. However, we do not find a consistent solution either analytically or numerically, at least not for credible parameter choices. This strongly suggests that the only way to obtain a Minkowski vacuum with a small gravitino mass in our model with $D$-terms, is by adding an extra $F$-term lifting section.

Readers only interested in a working model can go immediately to subsection 4.2.

4.1 Uplifting and stabilisation with light quarks

As discussed above, we are looking for a $D \approx 0$ minimum allowing for the possibility of low scale SUSY breaking. Furthermore, we assume that the quarks are light and cannot be integrated out. Taking inspiration from [8], we will consider the limit $\chi^2 \ll m_{Pl}^2$ and $T \gg 1$. Then the dominant contributions to the potential are $V_D$ and

\[ V_0 = e^K \left( K^{TT} |D_TW|^2 - 3|W|^2 \right) = \frac{a A e^{-aT}}{2T^2 Y^3} \left( \frac{A T e^{-aT}}{3\chi^2 b} + \frac{W}{\chi^b} \right) \]
\[ V_1 = e^K K^{\chi\bar{x}} |D_{\chi}W|^2 = \frac{3 - \chi^2}{2T^3 Y^2} \left( 2m\zeta q \chi - b \frac{A e^{-aT}}{\chi^b + 1} + \frac{\chi W}{Y} \right)^2. \quad (4.2) \]

The $K^{\zeta\bar{\zeta}} |D_{\zeta}W|^2$ contribution turns out to be subdominant. All warping dependence has been absorbed into the parameters $A, W_0, m$, and we work in units with $\kappa_4^2 = 1$. To
leading order in $\chi$, $V_1$ reduces to the global SUSY potential. Perturbatively, one can find a minimum of the potential around the solutions of $D = 0$ and $\partial_\chi W = 0$ (i.e. $F_\chi = 0$ for a global SUSY theory). Thus the zeroth order approximate solution is

$$\zeta_0^2 = \frac{3\delta_{GS}}{4T_0}, \quad \chi_0^b = \frac{bAe^{-aT_0}}{2m\zeta_0^ q}$$  \hspace{1cm} (4.4)$$

with $T_0$ unspecified at this stage. The above solution (4.4) then implies $W_m^{(0)} = (b/2)W_{NP}^{(0)} = m\epsilon\zeta_0^{2+q}/(qaT_0)$.

Stabilising the volume modulus $T$ is the hard part. Write

$$T = T_0(1 + \epsilon T_1 + \epsilon^2 T_2 + \cdots),$$

and similarly for $\chi$ and $\zeta$, with $\epsilon$ a small expansion parameter. If, following [3], we expand in $\epsilon = \chi^2/\zeta^2$ we find a runaway behaviour for at least one combination of the fields. To avoid this we expand instead in

$$1 \gg \epsilon \equiv \frac{q a T_0 \chi_0^2}{\zeta_0^2} \gg \frac{1}{\sqrt{aT_0}}.$$  \hspace{1cm} (4.5)$$

We expand around the lowest order solution (4.4). The details of this expansion can be found in Appendix A; here we only discuss the main results. The first order corrections to $\chi$, $\zeta$ are fixed by $V_D$ and $V_1$, which then leaves $V_0$ to determine the correction to $T$. To leading order, the remaining terms in the potential (arising from $V_0$) are

$$V_{\text{min}}^{(1)} = \frac{3m^2 \epsilon^3 \zeta_0^{2q}}{8(2+b)a^2 T_0^5} \left\{ (2 + b + 4a + bq) + \frac{aT_0 W_0}{m\epsilon\zeta_0^ q} (8 + 4b + 4a + bq) \right\} T_1 + \cdots$$  \hspace{1cm} (4.6)$$

with

$$V_{\text{min}}^{(1)} = \frac{3m^2 \epsilon^2 \zeta_0^{2q}}{8a^2 T_0^3} \left( 1 + 2\frac{aT_0 W_0}{m\epsilon\zeta_0^ q} \right).$$  \hspace{1cm} (4.7)$$

In general, we see that the above potential has runaway behaviour for $T_1$, indicating that $T_0$ and the solution (4.4) is not close to a minimum. The exception to this is

$$\frac{aT_0 W_0}{m\epsilon\zeta_0^ q} = -\frac{2 + b + bq + 4a}{4(2 + b) + bq + 4a}$$  \hspace{1cm} (4.8)$$

in which case the $T_1$ terms in (4.4) cancel, leaving open the possibility that $T_1$ could be stabilised by higher order terms. The second order results, given explicitly in Appendix A, allow for this possibility. Thus if our solution is to be an extremum the above condition must be satisfied. Note that this implies $W_0 \sim m\epsilon\zeta_0^ q/T_0$, i.e. $W_0 \ll m$; this is natural in a warped background, see (3.4,3.5).

For a special solution satisfying (4.8) the potential at the minimum is $V_{\text{min}}^{(1)}$ (4.7). Picking $T_0$ such that $V_{\text{min}}^{(1)}$ vanishes, and we have a Minkowski vacuum, requires

$$4a \approx 2(2 + b) - bq.$$  \hspace{1cm} (4.9)$$

For the parameters (3.1), with $A_b = 0$, this implies $N \approx 8\pi^2 k_N + N_f q/2$. Satisfying this is only possible for large values of $N$ or small $k_N$. The value of $T_0$ is

$$aT_0 \approx \frac{b + 2}{2} \ln \frac{mq}{(-2W_0)} + \ln \frac{Ab}{2m} + a \ln \frac{4}{3\delta_{GS}},$$  \hspace{1cm} (4.10)$$
Since \((aT_0)\) depends only logarithmically on the parameters it will never be large for moderate values of \(a, b, \) etc., and so it is extremely hard to satisfy the condition (4.5) for which our expansion is valid. Indeed, scanning through parameter space it follows that in the regime of validity of the expansion (4.5), the solution (4.9, 4.10) requires very large gauge groups \(N \sim 10^2 - 10^3\). We reject this possibility on the grounds that the presence of so many D7-branes would back react on the geometry, and invalidate the supergravity results derived in section 3, which motivate this model.

To conclude, we do not find a Minkowski vacuum with \(D \approx 0\) for light quarks, at least not for credible parameter choices. The obstacle to finding a metastable minimum is the stabilisation of the volume modulus \(T\). Since our analytics only cover part of parameter space, we have also searched numerically for \(D \approx 0\) solutions, but with negative results as well. These conclusions are independent of the amount of warping.

4.2 Uplifting with an ISS sector

In this section we consider the case of heavy quarks. \(D \approx 0\) implies \(\zeta \neq 0\), which generates a mass term for the \(\chi\)-fields. If their mass is greater than the other mass scales in the problem the \(\chi\)-fields can be integrated out. The resulting effective potential has a SUSY preserving AdS minimum. We include an ISS sector for the necessary uplifting.

But before we discuss this model in detail let us say a few words about higher order \(\alpha'\) corrections to the Kähler potential. As noted in [20], if these corrections are large the SUSY conditions \(F = D = 0\) cannot be satisfied, and supersymmetry is broken, possibly at a low scale. However, this does not work for our setup for two reasons. First, the resulting minimum in our model is dS. And secondly, the \(\alpha'\) corrections are exponentially small in the presence of warping, and do not play a rôle. More details can be found in Appendix B.

We can integrate out the heavy quark fields by solving the \(F_\chi = 0\) constraint in the global supersymmetry limit, i.e. \(\partial_\chi W = 0\), leading to

\[
\chi^{b+2} = \frac{bA e^{-aT}}{2m \zeta^q} \tag{4.11}
\]

and

\[
W = W_0 + (2 + b) A \left( \frac{2m}{bA} \right)^{\frac{1}{b+2}} \zeta^{\frac{\mu_0}{b+2}} e^{-\frac{2\mu_0}{b+2}T} \tag{4.12}
\]

with \(W_0 \sim e^{-3A_0}\). This system has a SUSY AdS min with \(F_T = F_\zeta = D = 0\): The \(D = 0\) constraint fixes \(\zeta\), whereas \(F_T\) fixes \(T\), and \(F_\zeta = 0\) follows then automatically as a consequence of gauge invariance. The superpotential is dominated by the constant piece \(W_0\). Extra input is needed to break SUSY and lift the solution to Minkowski. For definiteness, we will include an ISS sector [4] for this purpose.

Consider a new contribution to the superpotential

\[
\tilde{W}_{\text{ISS}} = \tilde{h} \text{tr}(\bar{q} M q) - \tilde{h} \mu^2 \text{tr}(M) \tag{4.13}
\]

The warping dependence of the meson source term \(h \mu^2\) is analogous to (2.26); it is left undetermined here and will be discussed later. The quark fields \(q^a_i\) are in the \((N, N_f)\)
representation of $\text{SU}(N) \times \text{SU}(N_f)$, $\bar{q}_i^a$ in the $(\bar{N}, \bar{N}_f)$ and $M_i^j$ in the adjoint representation of $\text{SU}(N_f)$. In a string context, the quark fields come from open strings between $N$ D3 and $N_f$ D7-branes. The meson fields $M$ are naturally interpreted as the position of the D7-branes while the D3-branes are placed at singular points in an orientifold compactification. We consider that these branes are placed in the bulk of the compactification. The Kähler potential follows from the results in section [2.3]. For the D7 meson, the modular weight is 0. For the field joining the D3 and the D7, the brane intersection has codimension 8, 6 directions orthogonal to the D3 and 2 to the D7. Now as the D3 is glued at a fixed point, it is not free to move and therefore there are effectively only 2 degrees of motion as for a D7-brane. Hence the modular weight is 0 too. Expanding in small fields compared to the Planck scale, this gives an additive contribution to the Kähler potential

$$K = |q|^2 + |\bar{q}|^2 + |M|^2.$$  \hspace{1cm} (4.14)

Following the strategy in subsection [2.4], we factor out an overall $e^{4A_0}$ from the Kähler potential, perform a Kähler transformation, and rescale all the brane fields $\phi_i \to e^{2A_0} \phi_i$. This results in a Kähler potential from which all warp factors have been removed; in particular, the ISS fields $q, \bar{q}$ and $M$ have canonical kinetic terms. All warping effects resides in the superpotential, which for the ISS sector becomes

$$W_{\text{ISS}} = h \text{tr}(\bar{q}Mq) - h\mu^2 \text{tr}(M)$$  \hspace{1cm} (4.15)

with $h = \tilde{h}$ is warping independent, and $\mu^2 = e^{-4A_0}\tilde{\mu}^2$. At the global supersymmetry level, the ISS sector has a supersymmetry breaking vacuum

$$M = 0, \quad q_i^a = \bar{q}_i^a = \mu \delta_i^a.$$  \hspace{1cm} (4.16)

The vacuum energy is

$$V_{\text{ISS}} = (N - N_f)\mu^4.$$  \hspace{1cm} (4.17)

Let us now couple the ISS sector to the rest of the model including the consistent $D$-term. As $\mu \ll m_{\text{Pl}}$ and expanding in the small VEVs of the ISS sector, the leading terms in the scalar potential are

$$V = \frac{1}{(T + T)^3} \left[ V(q, \bar{q}, M) + V(T, \zeta) \right]$$  \hspace{1cm} (4.18)

where we have taken into account that $Y \approx 1$ when the VEV of $\zeta$ is small compared to the Planck scale. The potential $V(q, \bar{q}, M)$ is the global supersymmetry potential in the ISS sector. In the large $T$ expansion, the minimum of this potential is obtained by imposing a separate minimum for $V(T, \chi^i)$ and $V_{\text{ISS}}$ (the corrections terms to the $T$ equation coming from the ISS sector are in $1/T^4$). As a result, the cosmological constant at the minimum is given by

$$V_{\text{min}} \approx \frac{(N - N_f)\mu^4 - 3W_0^2}{8T^3}.$$  \hspace{1cm} (4.19)

where we have neglected the $\zeta_0$ contribution. The minimum can be set to zero provided

$$h\mu^2 \approx \sqrt{\frac{3}{N - N_f}}|W_0|.$$  \hspace{1cm} (4.20)
Let us restore the warp factors. The Minkowski minimum is obtained by balancing $h\mu^2$ with $W_0$ in (4.20) above. Since both the left hand side scale $\propto e^{-3A_0}$ this can be achieved only if $h\mu^2$ scales as $e^{A_0}$. If this is not the case, the gravitino mass cannot be warped down. On the contrary, the gravitino mass $m_{3/2} \approx |W_0|/(2T_0)^{3/2}$ can be in the TeV range if $h\mu^2 \sim W_0 \sim 10^{-15}$. Using the warping of $W_0 \propto e^{-3A_0}$, this translates into a constraint on $A_0$:

$$e^{-A_0} \approx 10^{-5}.$$  

We have thus found that a relatively small warping is enough to lead to a small gravitino mass once a consistent $D$-term model is coupled to an ISS sector.

5. Conclusions

In this paper we analysed the low energy supergravity action in warped spacetimes with matter on D3 and (intersecting) D7-branes. We dimensionally reduced the action to find the warping dependence of the various terms in the 4D action. A central assumption in our setup is that the throat dominates the volume of the 6D compactified space. This is valid if the volume modulus $T$, which parameterises the bulk volume $V_{\text{bulk}}^{2/3} \sim T$, is stabilised at moderately large volumes: $T \ll e^{4A_0}$, with $A_0$ the warp factor in the tip of the throat.

The 4D action for matter on bulk D3 and D7-branes (wrapped around 4-cycles which lie entirely in the bulk) is warping independent. The warping only enters via the modulus field $T$, which is a truly 10 dimensional field, and thus feels the total 6D volume of compactified space. This is translated into a scaling down of the $F$-term potential $V_F \propto e^{-6A_0}$ via a Kähler transformation. In contrast, the $D$-term is warping independent.

To study the effect of warping on moduli stabilisation and supersymmetry breaking we applied it to simple string inspired models with or without $\alpha'$ corrections in the Kähler potential [8]. In these models the moduli sector arises from matter on magnetised bulk D7-branes, and has a symmetry group $\text{SU}(N) \times \text{U}(1)$. The chiral matter content is a meson field and a $\text{SU}(N)$ singlet; the volume modulus $T$ is charged under the $\text{U}(1)$. Strong $\text{SU}(N)$ gauge dynamics gives a non-perturbative potential that stabilises the volume modulus at $O(1)$ values. The minimum is uplifted by $F$- and $D$-terms.

The supersymmetric standard model resides on intersecting D7-branes. Gauge couplings of order one dictates that these branes are located in the bulk. Supersymmetry breaking by the moduli sector is transferred to the standard model fields by gravitational interactions. Low scale SUSY breaking with a TeV scale gravitino mass requires both the $F$- and $D$-term to be small compared to the Planck scale. Since the $D$-term is warping independent, this is only possible in models in which the $D$-term (nearly) cancels $D \approx 0$.

In the presented models, it is indeed possible to find $D \approx 0$ solutions. In general the resulting minimum is anti-de Sitter, although for some parameter choices non-zero $F$-terms are enough to raise the minimum to a Minkowski vacuum. Low scale SUSY cannot be achieved by turning on a moderate warping. By coupling to an ISS sector we find that low energy SUSY can be achieved. The AdS vacua is uplifted via the ISS sector for moderate warping. This results in a small gravitino mass for warping of the order $10^{-5}$. 

Acknowledgments

ACD, SCD thank CEA Saclay for their hospitality. ACD and MP are also grateful to the Galileo Galilei Institute and INFN for hospitality and partial support. For financial support, SCD and RJ thank the Netherlands Organisation for Scientific Research (NWO), MP thanks FOM, ACD thanks PPARC for partial support, and PhB acknowledges support from RTN European programme MRN-CT-2004-503369.

A. Uplifting and stabilisation with light quarks

In this Appendix we give the details of the expansion performed in subsection 4.1. Expanding the potential around the zeroth order solution (4.4) to first order in $\epsilon$ (4.5) gives

$$V_D = \frac{9\delta_{GS}^2}{32k_X T_0^3} \left[ \epsilon^2 D_1^2 + O(\epsilon^3) \right], \quad V_1 = \frac{3bm^2\zeta_0^2q}{8a^2T_0^3} \epsilon^2 F_1^2 + \cdots, \quad (A.1)$$

$$V_0 = V_{\min}^{(1)} + \frac{3me^2\zeta_0^2}{8(2+b)aT_0^3} \left\{ m\zeta_0^2 \left[ a\delta_{GS} D_1 - 2bF_1 - (2 + 4a + bq)T_1 \right] 
+ aT_0W_0 \left[ a\delta_{GS} D_1 - 2bF_1 - (8 + 4b + 4a + bq)T_1 \right] \right\} + \cdots \quad (A.2)$$

with

$$D_1 = 2\zeta_1 + T_1, \quad F_1 = (2 + b)\chi_1 + aT_1 + q\zeta_1, \quad (A.3)$$

where we used gauge invariance $bq = a\delta_{GS}$. The ($\cdots$) in the above expressions correspond to $O(m^2\zeta_0^2T_0^{-5}\epsilon^4, m\zeta_0^2T_0^{-4}W_0\epsilon^3, W_0^2T_0^{-3}\epsilon^2)$ terms. As discussed in the main test, the above potential does not stabilise $T_1$. The exception to this if (4.8) is satisfied, leaving open the possibility that $T_1$ could be stabilised by higher order terms the potential. This is only possible if $W_0 \sim m\epsilon\zeta_0^2/T_0$, i.e. if $W_0 \ll m$. From subsection 3.1, we expect $W_0/m \sim e^{3A_0+2(2+q)/(A_0-A_0)}$, which is indeed tiny when $A_0$ is large and $A_b$ is close to zero. Without warping the above conditions will be difficult to fulfill.

The combination $F_1$ is stabilised by the above $F$-terms, and minimising $V_0 + V_1$ with respect to it gives

$$F_1 = \frac{1}{2 + b} \left( 1 + \frac{aT_0W_0}{me\zeta_0^2} \right). \quad (A.4)$$

The combination $D_1$ is stabilised by $V_0 + V_D$ at

$$D_1 = \frac{-2}{3(2+b)} \left( 1 + \frac{aT_0W_0}{me\zeta_0^2} \right) \frac{k_X m^2\zeta_0^2q}{\delta_{GS}aT_0^2} \epsilon \quad (A.5)$$

which, assuming (4.8), is negligible unless $W_0^2 \sim e\delta_{GS}/k_X$. From now on we will assume, for simplicity, that $V_D$ dominates the other parts of the potential, and so $D_1 = 0$.

Setting $V_{\min}^{(1)} = 0$ to obtain a Minkowski vacuum requires (4.9,4.10). The condition (4.3) implies

$$\frac{1}{(aT_0)^{(2+q)/2}} \gg \frac{(-2W_0)}{m} \left( \frac{4}{3bq} \right)^{q/2} \gg \frac{1}{(aT_0)^{(3+q)/2}}. \quad (A.6)$$
From (3.5) we find \( A/m \sim e^{2(b+q)}(A_b-A_0) \). Combining this with \( \delta_{GS} \sim e^{-4A_b} \) we find
\[
aT_0 \sim \left( a - \frac{3bq}{4} \right) A_0 + (4a - bq)(A_b - A_0).
\]

For \( A_b = 0 \), \( W_0/m \sim e^{-(1+2q)A_0} \) and \( aT_0 \sim (bq - 12a)A_0/4 \). To satisfy (A.6) and the other consistency conditions, we need to take \( N \) and \( N_f \) large. For example, consider \( N = 1024 \), \( N_f = 1023 \), \( kN = 1/\pi^2 \). Then determine \( a, b, q, \delta_{GS} \) from (3.6), (3.8) and (4.9); this gives \( a = 8, b = 2046, q = 1.986, \delta_{GS} = 508 \). Taking \( A_0 = 2 \), we then obtain \( aT_0 \approx 1936 \) and \( T_0 \approx 242 \) from (4.10). The expansion parameter in this case \( \epsilon = 0.12 \), which marginally satisfies (4.5). However, taking \( N \sim 10^3 \) D7-branes is not a particularly realistic situation.

For completeness, we also give the higher order expansion of the potential. To next order in \( \epsilon \), we find
\[
V_D = \frac{9\delta_{GS}^2}{32k_X T_0^3} \left[ \epsilon^4 D_2^2 + \mathcal{O}(\epsilon^5) \right] \tag{A.8}
\]
\[
V_F = \text{const.} + \frac{3W_0^2 e^2}{2b(2+b)T_0^2} \left\{ \left[ 4(a-1)^2 + (a-2)(2a-1)b \right] T_1^2 
+ (2+b-2a)b \left[ \frac{aT_1}{4(2+b)} + D_2 \right] + \mathcal{O}(\epsilon) \right\} \tag{A.9}
\]
with \( D_2 = 2\zeta_2 + T_2 - 3T_1^2/4 \). The \( D \)-term part of the potential stabilises \( D_2 \). The above solution will only be a minimum if the coefficient of \( T_1^2 \) in (A.9) is positive, otherwise \( T_1 \) will not be stabilised. We see that for the above parameters, we do indeed have a minimum.

**B. Large volume stabilisation with higher order \( \alpha' \) corrections**

After integrating out the quark fields, the effective superpotential (4.12) has a SUSY AdS min with \( F = D = 0 \). This conclusion can be avoided if higher order \( \alpha' \) corrections to the Kähler potential play a rôle [20]. This allows for the possibility of large volume stabilisations with Re \( T \gg 1 \). In this limit the non-perturbative effects are sub-dominant, and the SUSY preserving condition \( F = 0 \) cannot be satisfied making a Minkowski minimum is possible.

The \( \alpha' \) corrections come from terms of the form
\[
\epsilon^{ABM_1M_2...M_8} \epsilon_{ABN_1N_2...N_8} R_{M_1M_2} N_1N_2 R_{M_3M_4} N_3N_4 R_{M_5M_6} N_5N_6 R_{M_7M_8} N_7N_8 \tag{B.1}
\]
in the 10D action [24]. Upon dimensional reduction it leads to a correction of the Kähler potential, which becomes (after performing the Kähler transformation (2.27)) [20, 24]
\[
K = -\frac{2}{\kappa_4^2} \ln [(T + \bar{T})^{3/2} + \xi] - \frac{3}{\kappa_4^2} \ln \left[ 1 - \frac{k_4^2}{3} |\xi|^2 \right] \tag{B.2}
\]
with
\[
\xi = e^{-4A_0} \tilde{\xi} \tag{B.3}
\]
parameterising the \( \alpha' \) corrections. We see that the higher order curvature term is warped down with respect to the Einstein term, as was shown in subsection 2.5. In the limit
$A_0 \to 0$ our results agree with the literature \cite{13}, and in the limit $\xi \to 0$ it reduces to our previous result (3.3). The size of the corrections diminishes rapidly for large warping. It is thus expected that it cannot play a role in this limit. As we will show now, this is indeed the case.

To analyse the system we make a $1/T_R$ expansion with $T_R = \text{Re} T$. At lowest order the $F$ and $D$-term potential are

$$
V_F = \frac{W_0^2}{(2T_R)^3 Y^3} \left\{ |\zeta|^2 + \frac{3\xi}{2(2T_R)^{3/2}} + \mathcal{O} \left( \frac{\zeta^2, \xi}{T_R^{3/2}} \right)^2 \right\} + V_{\text{non-pert}}
$$

$$
V_D = \frac{9\delta_{\text{GS}}^2}{32 k_X T_R^5} \left( 1 - \frac{4T_R}{3\delta_{\text{GS}} Y |\zeta|^2} \right)^2
$$

with $Y = (1 - |\zeta|^2/3)$ and $V_{\text{non-pert}} = V_F - e^K (K_I K^I - 3) W_0^2$, i.e. $V_{\text{non-pert}}$ includes all terms with non-perturbative $e^{-(2aT)/(b+2)}$ factors. Both terms come in at same order, which gives a $D \neq 0$ minimum, with high scale SUSY breaking. This can be avoided if $W_0^2 \ll 1/T_R$. Then the $D$-term potential dominates, and $\zeta$ will adjust itself to cancel it:

$$
|\zeta_0|^2 = \frac{3\delta_{\text{GS}}}{4T_R}. \tag{B.5}
$$

One linear combination of the phase fields is fixed by the non-perturbative potential, the orthogonal contribution remains massless and gets eaten by the anomalous U(1). $T_R$ is fixed at higher order. Write $\zeta = \zeta_0 [1 + \zeta_1/T_R + \mathcal{O}(1/T_R^2)]$, and expand the potential to lowest order

$$
V_F = \frac{W_0^2}{(2T_R)^3} \left[ \frac{3\delta_{\text{GS}}}{4T_R} \left( 1 + \frac{2}{T_R} \zeta_1 \right) + \frac{3\xi}{2(2T_R)^{3/2}} \right] + V_{\text{non-pert}} + \cdots
$$

$$
V_D = \frac{9\delta_{\text{GS}}^2}{8 k_X T_R^5} \left( \frac{\delta_{\text{GS}}}{8} + \zeta_1 \right)^2. \tag{B.6}
$$

The field $|\zeta|$ is fixed at this and successively higher orders by the condition $D = 0$. Although $|\zeta|$ also appears in the $F$-term potential, it will always be at higher order. The volume modulus is stabilised by the $F$-term potential, which requires $\xi < 0$. The $F$-term potential is minimised at

$$
T_R = \frac{81\xi^2}{128 \delta_{\text{GS}}^2} \tag{B.7}
$$

at a value

$$
V_{\text{min}} = \frac{\delta_{\text{GS}} W_0^2}{96 T_R^4}. \tag{B.8}
$$

The minimum is dS, not a viable cosmological solution. The warp dependence of $\xi$ (B.3) does no help here, as $T_R$ will be small for large warping, and our expansion in $1/T_R$ will break down. The difference between our results and those of \cite{20}, who did find a AdS/Minkowski minimum, is the different modular weight for $\zeta$. 

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References


