A-dependence of hadronization in nuclei

H.P. Blok\textsuperscript{1,2} and L. Lapikás\textsuperscript{2}

\textsuperscript{1}Department of Physics and Astronomy, Vrije Universiteit, de Boelelaan 1081, 1081 HV Amsterdam, The Netherlands
\textsuperscript{2}Nationaal Instituut voor Kernphysica en Hoge-Energie Fysica (NIKHEF), P.O. Box 41882, 1009 DB Amsterdam, The Netherlands

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The $A$-dependence of models for the attenuation of hadron production in semi-inclusive deep-inelastic scattering on a nucleus is investigated for realistic matter distributions. It is shown that the dependence for a pure partonic (absorption) mechanism is more complicated than a simple $A^{2/3}$ ($A^{1/3}$) behavior, commonly found when using rectangular or Gaussian distributions, but that the $A$-dependence may still be indicative for the dominant mechanism of hadronization.

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\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{hadronization.png}
\caption{Illustration of hadronization in a nucleus and used coordinates. The parton is produced at point A, while the hadron is formed at B. The distance $l_f$ has a distribution with an average the formation length $L_f$.}
\end{figure}

The study of hadronization, the process that leads from partons produced in some elementary interaction to the hadrons observed experimentally, is of importance, both in its own right as a study of a non-perturbative QCD process, and in the interpretation of data from experiments that use outgoing hadrons as a tag. The end products of the hadronization process in free space are known from products of the hadronization process in free space are known from $e^+e^-$ annihilation, but very little is known about the space-time development of the process. One way to investigate this is to study the semi-inclusive production of hadrons in deep-inelastic scattering of electrons from a nucleus, where the nucleus is used as a length (time)-scale probe (see Refs. \cite{1,2}).

Even if hadronization is not yet quantitatively understood, it is known that the following processes play a role in lepton production of hadrons in a nucleus (see also Fig. 1). After a quark in a nucleon is hit by the virtual photon, it loses energy by scattering from other quarks and radiating gluons, thus creating quark-antiquark pairs. After some time\textsuperscript{1} and corresponding length $l_f$ colorless (pre)hadrons\textsuperscript{2} are formed. The average value $L_f$ of the formation length has been estimated\textsuperscript{2} based on the Lund model to be typically in the range of 1–10 fm if the virtual-photon energy is in the range 5–30 GeV. Hence, $L_f$ is comparable to the size of a nucleus.

If the hadronization is fast, i.e., the hadrons are produced inside the nucleus, they can be absorbed, which will show up as an ‘attenuation’ of the hadron yield. (In experiments one measures the ratio $R_A$ of the yield on a nucleus with mass number $A$ and the one on a deuteron.) If, on the other hand, the hadronization is stretched out over distances large compared to the size of a nucleus, the relevant interactions will be partonic, involving the emission of gluons and quark-gluon interactions, which also changes the production of hadrons.

Since hadronization, as a non-perturbative QCD process, cannot be calculated from first principles, various models have been developed (see e.g. Refs. \cite{3,4,5,6,7}) to describe hadron production and attenuation in a nucleus. Some models focus on the partonic part, while others only emphasize the hadronic part. In all cases a sizable dependence on the mass number $A$ is predicted. However, often the calculations use simple forms for the matter distribution of the nucleus.

In this Report we investigate the $A$-dependence using realistic matter distributions. We will do this for two schematic models, covering the extremes sketched above. The first one (called model I) assumes a purely partonic mechanism, hadronization occurring outside of the nucleus (point B in Fig. 1 effectively at infinity) and thus absorption of the produced hadrons playing no role. In line with and inspired by the model of Ref. \cite{8} it is assumed that the effect of the partonic mechanism depends, on the square of the density-averaged distance the parton travels within the nucleus from the point where it is created (point A in Fig. 1). On the other hand our second schematic model assumes that a possible atten-

\begin{footnotesize}
\begin{itemize}
\item[$^\dagger$] Electronic address: henkb@nikhef.nl
\item[$^\ddagger$] Electronic address: louk@nikhef.nl
\item[1] For a more detailed discussion of the concept of formation time etc. see Ref. \cite{8}.
\item[2] For the present discussion it is not needed to discriminate between hadrons and prehadrons, so in the remainder we will just talk about hadrons (but see, e.g., Refs. \cite{1,2}).
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\end{footnotesize}
u ration is completely due to absorption of the produced hadron, so nothing happens between the time the parton is produced and the hadron is formed (points A and B in Fig. [1]). With a Glauber approach we assume in this case that the effect depends on the density-averaged distance the hadron travels through the nucleus after it has been formed (see Fig. [1]).

This means we have to calculate the following integrals:

\[
\langle t_f^2 \rangle = \frac{2\pi}{A} \int_0^\infty \int_{-\infty}^\infty b db \int_{-\infty}^\infty dz \rho_A(b, z) \left[ \int_{-\infty}^\infty dz' \rho_A(b, z') \right]^2, \quad \text{(model I) (1)}
\]

\[
\langle t_{II} \rangle = \frac{2\pi}{A} \int_0^\infty \int_{-\infty}^\infty b db \int_{-\infty}^\infty dz \rho_A(b, z) \int_0^\infty dz' L_f^{-1} e^{-(z'-z)/L_f} \int_0^\infty d z'' \rho_A(b, z''), \quad \text{(model II) (2)}
\]

Here the exponential models the distribution of the formation distances \( t_f \), and the matter densities \( \rho_A \) are normalized to \( A \). (By entering these quantities with corresponding dynamical factors into the appropriate formula’s of the models, hadron production cross sections and from these values for the attenuation \( R_A \) can be calculated. However, here we are interested in the \( A \)-dependence (and moreover the used models are extreme and schematic). Under the assumption that the cross section is linear in \( \langle t_f^2 \rangle \) and \( \langle t_{II} \rangle \), which is a good first-order approximation, the \( A \)-dependence of these quantities carries over into the one of the cross sections.)

It can easily be shown that for a nucleus with a mass density distribution described by one scale parameter, as in the case of a rigid sphere or a Gaussian, the value of \( \langle t_f^2 \rangle \) is proportional to the equivalent radius or rms radius squared, which in those cases leads to an \( A^{2/3} \) dependence. In case of model II one finds for \( \langle t_{II} \rangle \) for a rigid sphere an \( A^{1/3} \) dependence when \( L_f = 0 \), and a larger exponent\(^3\) (e.g. about 0.55 for \( L_f = 4 \) fm) at larger \( L_f \), and similar for a Gaussian.

However, neither a rigid sphere, nor a Gaussian is a good representation of the mass distribution of a real nucleus. Therefore we have evaluated \( \langle t_f^2 \rangle \) and \( \langle t_{II} \rangle \) for a realistic distribution, described by a 2-parameter Fermi (Saxon-Woods) form

\[
\rho_A(r) = \rho_0/[1 + e^{-(r-c)/a}]
\]

with parameters \( \rho_0 = 0.170 \) nucleons/fm\(^3\), \( a = 0.5 \) fm, and the value of \( c \) so as to give a nucleus with \( A \) nucleons. (This form gives a reasonably good global description of the mass distribution down to low values of \( A \)). The results are given in Table III and are shown (full curves) in Figs. 2 and 3.

It can be seen that for a realistic matter distribution the \( A \)-dependence of \( \langle t_f^2 \rangle \), if one tries to describe it with the power law \( A^\alpha \), requires a value of \( \alpha \) even larger than \( \alpha = 2/3 \), and that \( \langle t_{II} \rangle \) has values of \( \alpha \) that depend slightly on \( A \) and increase when \( L_f \) increases. Values range from about \( \alpha = 0.40 \) for \( L_f = 0 \) fm to \( \alpha = 0.60 \) for \( L_f = 4 \) fm.

Given these findings, and since it is known that the

3 This effect of taking into account a distribution of formation distances has been noted already in Ref. 8.
parameters for actual nuclei are slightly irregular due to, e.g., shell closures, it is interesting to see the behavior of \( \langle t^2 \rangle \) and \( \langle t_{ii} \rangle \) for real nuclei, since those are used in experiments. For that purpose we have used parameterizations \([9,10]\) of measured charge distributions (since the neutron distribution is very similar the error introduced by using the charge distribution instead of the matter distribution is small, and irrelevant for the conclusions of the present study) for the nuclei \(^2\text{H}, ^4\text{He}, ^{12}\text{C}, ^{16}\text{O}, ^{28}\text{Si}, ^{40}\text{Ca}, ^{48}\text{Ca}, ^{84}\text{Kr}, ^{118}\text{Sn}, ^{132}\text{Xe} \) and \(^{208}\text{Pb}\). The results are also given in Table II and shown as the symbols in Figs. 2 and 3.

A first observation is that while the results for \( A \geq 4 \) scatter around the global curve, the results for \(^2\text{H} \) are way down. This is due to \(^2\text{H} \) being a rather dilute system. Furthermore, it is seen that the results for \(^4\text{He} \) lie slightly above the global curve, which is related to \(^4\text{He} \) being a relatively dense system. Fitting the \( A \)-dependence of the results for the real nuclei including \(^4\text{He} \), one finds an exponent \( \alpha \) of about 0.74 for \( \langle t^2 \rangle \) (see Fig. 2) and values between 0.40 and 0.60 for \( \langle t_{ii} \rangle \) depending on the value of \( L_f \) (see Fig. 3). When \(^4\text{He} \) is not included in the fits, the values of \( \alpha \) become larger by about 0.04. Thus in trying to extract an \( A \)-dependence from experimental data it matters if one uses \(^4\text{He} \) as lowest \( A \) nucleus, or e.g., \(^{12}\text{C} \).

Given these results it would in principle be possible to discriminate on account of the \( A \)-dependence between the two extreme mechanisms that we have used here. However, in practice the process of hadronization in a nucleus most probably will be a combination of these mechanisms, with possibly even different dependences on path lengths in the nucleus than employed here. Nevertheless, finding from experimental data an exponent \( \alpha \) in excess of 0.65 when the estimated \( L_f \) is well below 4 fm, would be an indication that partonic processes play an important role. But whatever the mechanism, the \( A \)-dependence will be an important ingredient, and in comparing experimental results with results from model calculations, the use of realistic density functions for the nuclei actually used in the experiment is essential.
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