We show that a resummation of infrared logarithms is needed to obtain a sensible theoretical description of dijet rates when symmetric cuts are applied to the transverse energies of both jets. We also present the next-to-leading logarithmic (NLL) resummation we carried out for DIS production of two jets selected with the cone algorithm.

1 Introduction

It was observed some time ago [1,2] that next-to-leading order (NLO) QCD calculations are not able to describe dijet rates measured experimentally in the whole of the dijet phase space [3,4]. In particular in [2] a study was performed of the cross section \( \sigma(\Delta) \) for producing two jets with transverse energies \( E_{t1} > E_{t2} \), \( E_{t2} > E_{\text{min}} \) and \( E_{t1} > E_{\text{min}} + \Delta \). There it was observed that \( \sigma(\Delta) \) at NLO was finite for \( \Delta = 0 \) (symmetric cuts), but in the vicinity of that point the slope \( \sigma'(\Delta) \) became unphysically positive and infinite. This pathological behaviour can be attributed to the fact that incomplete real-virtual cancellations give rise to large logarithms \( \ln Q/\Delta \) (with \( Q \) the hard scale of the process) at all orders in the perturbative expansion for \( \sigma'(\Delta) \). However, an all-order resummation of such logarithms restores the correct physical behaviour for the slope, as will be discussed in the following.

2 Resummation of the jet transverse energy difference

To understand the physical origin of the divergent behaviour of \( \sigma'(\Delta) \) we first observe that this quantity can be related to the differential cross section in the transverse energy of the highest \( E_t \) jet:

\[
\sigma'(\Delta) = -\frac{d\sigma}{dE_{t1}}(E_{t1} = E_{\text{min}} + \Delta).
\]

Being the opposite of a physical cross section, the slope has to be always negative, as is observed experimentally [3,4].

However, if we emit an arbitrary number of soft gluons \( k_i \), and define \( E_{t,\text{jet}} = \left| \sum_{i\in\text{jet}} \vec{k}_{t_i} \right| \), in a recombination scheme that adds transverse momenta vectorially, we have, from momentum conservation,

\[
E_{t1} - E_{t2} = \left| \sum_{i\notin\text{jets}} k_{xi} \right|,
\]

where \( k_x \) is the component of \( \vec{k}_t \) parallel to the jet axis (which can be taken as the
Jets are selected with the variant of the cone algorithm proposed in \cite{11} with an where $\text{annihilation},$ both in the cone algorithm \cite{9} and in the inclusive $Q$ production in DIS. Here we present numerical results corresponding to $R$ production in DIS. The explicit expression for $R$ like $E$ softer gluon outside. Such contributions are typical of non-global observables \cite{8} logarithms arise also when soft gluons inside the jets emit coherently a relatively (including its multiple SC splittings) and exponentiating the result. However, NL only in the large-$R$ factorise from the leading order value for the slope $\sigma'_q(\Delta)$ as follows:

$$\sigma'(\Delta) \simeq \sigma'_q(\Delta) \cdot \Sigma(\Delta), \quad \Sigma(\Delta) \equiv \int dq_x S(q_x) \Theta(\Delta - |q_x|).$$ \hspace{1cm} (3)

Assuming that multiple SC emissions are distributed according to a ‘random walk’ \cite{5} around $q_x = 0$ leads to $\Sigma(\Delta) \sim \Delta/Q$ for small $\Delta$. The resulting slope is finite and negative defined as expected. QCD radiation is a particular form of random walk, giving rise to the following expression for $\Sigma(\Delta)$ (see \cite{6}):

$$\Sigma(\Delta) = \frac{2}{\pi} \int_0^\infty \frac{db}{b} \sin(b\Delta) e^{-R(b)},$$ \hspace{1cm} (4)

where the positive defined ‘radiator’ $R(b)$ acts as an effective cutoff on the $b$ integral. Its perturbative expansion can be reorganised as follows:

$$-R(b) = Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots, \quad L = \ln b,$$ \hspace{1cm} (5)

where $Lg_1(\alpha_s L)$ resums leading logarithms ($\alpha_s^n L^{n+1},$ LL), $g_2(\alpha_s L)$ resums next-to-leading logarithms ($\alpha_s^n L^n,$ NLL) and so on. The NLL expression for $R(b)$ reads

$$R(b) = R_{\text{inc}}(b) + R_{\text{soft}}(b) + R_{\text{jet}}(b) + R_{\text{ng}}(b).$$ \hspace{1cm} (6)

$R_{\text{inc}}(b)$ collects contributions from radiation that is soft and collinear to the incoming parton(s). Its expression is analogous to the well-known Sudakov exponent in Drell-Yan transverse momentum distribution \cite{7}. The term $R_{\text{soft}}(b)$ accounts for coherence of QCD radiation arising from interference among soft gluons at large angles. Both $R_{\text{inc}}(b)$ and $R_{\text{soft}}(b)$ are universal, in the sense that they do not depend on the details of the jet algorithm. However, $R_{\text{soft}}(b)$ should be corrected by taking into account the fact that not all soft gluons contribute to $E_{t1} - E_{t2},$ but only those outside the jets. The correction is provided by the additional terms $R_{\text{jet}}(b)$ and $R_{\text{ng}}(b),$ both of which depend on the jet algorithm. $R_{\text{jet}}(b),$ as well as $R_{\text{inc}}(b)$ and $R_{\text{soft}}(b),$ can be computed by simply considering the emission of a single SC gluon (including its multiple SC splittings) and exponentiating the result. However, NL logarithms arise also when soft gluons inside the jets emit coherently a relatively softer gluon outside. Such contributions are typical of non-global observables \cite{8} like $E_{t1} - E_{t2},$ and are embodied in $R_{\text{ng}}(b).$ Their resummed expression, known only in the large-$N_c$ limit, has been already computed for two outgoing jets in $e^+e^-$ annihilation, both in the cone algorithm \cite{9} and in the inclusive $k_t$ algorithm \cite{10}.

We now discuss the phenomenological impact of such a resummation for dijet production in DIS. The explicit expression for $R(b)$ in this specific case can be found in \cite{11}. Here we present numerical results corresponding to $Q = 20\text{GeV},$ $x_B = 0.01,$ where $-Q^2$ is the virtuality of the photon and $x_B$ the usual Bjorken variable. Jets are selected with the variant of the cone algorithm proposed in \cite{11} with an
opening angle $\delta = 0.3$, minimum transverse energy $E_{\text{min}} = 10\text{GeV}$ and rapidity $|\eta| < 1$ in the Breit frame. In fig. 1 we plot the ratio $D(\Delta) \equiv \sigma'(\Delta)/\sigma'_0(\Delta)$, with the slope $\sigma'(\Delta)$ computed at NLO in the SC approximation (lower curve) and NLL resummed (upper curve). As can be seen, while the NLO curve diverges for $\Delta \ll Q$, the resummed curve stays finite, vanishing as $\Delta/Q$. The numerical analysis of [6]

![Figure 1](image.png)

Figure 1. The ratio $D(\Delta) = \sigma'(\Delta)/\sigma'_0(\Delta)$. See [6] for details.

shows also that the impact of the non-global piece $R_{\text{ng}}(b)$ turns out to be negligible in the whole range of $\Delta$ values considered. This is consoling in view of the fact that its expression is known only in the large-$N_c$ limit.

3 Obtaining the total dijet rate

After a resummation for the slope $\sigma'(\Delta)$ has been computed, and after matching to fixed order calculations, one could obtain the total dijet rate $\sigma(0)$ by simply integrating $\sigma'(\Delta)$ from the maximum kinematically allowed value $\Delta_{\text{max}}$ back to $\Delta = 0$. However, resumming the distribution in the transverse energy difference is not the only way to compute $\sigma(0)$. One can envisage the possibility of studying other observables, which we will discuss shortly.

First of all one can observe that in the massless $E_0$ recombination scheme $E_{11} - E_{12}$ takes contribution also from the invariant masses of the two outgoing jets. This makes the observable global, and therefore within the scope of the automated resummation program CAESAR [12]. However, one should be aware of the fact that, since a mechanism that keeps $E_{11} - E_{12}$ small is transverse momentum cancellation (see eq. (2)), one should expect a divergence in the resummed distribution provided by CAESAR at a critical value $\Delta_c$ (see [12] for a discussion on this point).

Another possibility would be to consider any observable $V$ that vanishes in the limit of two outgoing partons ($E_{11} - E_{12}$ is only an example of such an observable). One could then resum the differential distribution $d\sigma(V)/dV$ and integrate it from $V_{\text{max}}$ to $V = 0$, thus obtaining again the total dijet rate. This has been observed already in [2] for the case of the azimuthal angle $\phi$ between the jets, where $V = \pi - \phi$.
was the quantity to be resummed.

4 Outlook

What we have done so far [6] shows that we have a clear understanding of the physics underlying dijet rates with symmetric $E_t$ cuts. As a mandatory test of our ideas, we plan to extend our calculation in DIS to the $k_t$ algorithm [13], so that we can compare our predictions with existing data [3,4]. Since the data have very small errors, such a comparison would also offer a test of QCD predictions in a three-jet environment, complementary to the event-shape measurements proposed for instance in [14], and, as we believe, less affected by non-perturbative hadronisation corrections. Further developments should include the extension of our results to other hard processes. In particular a study of dijet rates in photoproduction could be exploited to better constrain the gluon distribution $g(x)$ at moderately large $x$, which should be important for the LHC [15].

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References

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