Proper Time Resolution Model for $B_s^0 \rightarrow J/\psi \phi$ in the LHCb Experiment

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Abstract:

A study on the proper time resolution model for $B^0_s \rightarrow J/\psi \phi$ in the LHCb experiment is performed on Monte Carlo data. A re-definition of the primary vertex is needed, for wrong inclusion of daughter tracks of the signal B meson under study in the determination of the primary vertex leads to a bias as function of the proper time. After removal of these tracks, followed by a complete reconstruction and deterministic annealing refit of the primary vertex, the bias is no longer present and the resolution is constant as function of the proper time. The resolution is described by a sum of three Gaussian distributions. It was found that reconstructed data from the decay channel $B^0 \rightarrow J/\psi K^*$ can be used to obtain the resolution model parameters for $B^0_s \rightarrow J/\psi \phi$. 
1 \textbf{CP Violation in } B_s^0 \rightarrow J/\psi \phi

Covered in this chapter is a short overview of the Standard Model fundamental particles and interactions, followed by a description of CP violation within the CKM picture. The technicalities of $B_s^0 - \bar{B}_s^0$ mixing and the decay $B_s^0 \rightarrow J/\psi \phi$ are presented as well as CP violation in $B_s^0 \rightarrow J/\psi \phi$ due to interference. Finally, an angular analysis in the transversity basis is touched upon for disentangling the CP-even and CP-odd final state contributions.

1.1 Standard Model

Particle physics studies the elementary constituents of matter and radiation, and their interactions. Research in this field has led to the construction of the Standard Model, a quantum field theory describing three of the four known fundamental interactions between the elementary particles, see Figure 1.1. The depicted force carriers are each affiliated with one of the interactions. Photons carry electromagnetism, strong interaction is conveyed by gluons and W and Z bosons are responsible for the weak interaction. The Standard Model (SM) predicts the existence of a particle known as the Higgs boson, which has yet to be discovered. For each fundamental particle (grouped together as fermions) there exists an anti-particle.

In this view the fundamental particles, quarks and leptons, are divided into three generations. Ordinary matter around us is made up solely of particles from the first generation. Interacting with the four forces, they account for the broad diversity observed in our everyday world.

The Standard Model contains 18 numerical values that have had to be determined experimentally. This and the large scale differences between the fundamental interactions are reasons to look for theories beyond the Standard Model in which these parameters can be derived from a more basic principle and the different interactions are described as different aspects of the same unifying force.

Figure 1.1: Standard Model. All, except for the Higgs boson, have been discovered experimentally. The electron, muon and tau carry a negative electric unit charge $e$. Up-type quarks $(u,c,t)$ have $+\frac{2}{3}e$ charge and the down-type $-\frac{1}{3}e$. Neutrinos have zero electric charge, thus they don’t interact electromagnetically. All particles interact through the weak interaction. The strong interaction is only effectuated between quarks.
1.2 Baryogenesis

Our present universe consists of matter created around 13.7 billion years ago [1] during the big bang. Shortly after being created, nearly each particle recombined with its anti-particle, causing them to annihilate into quanta of energy, photons. This explains the extremely small present ratio of baryons\(^1\) to photons: \((6.5^{+0.4}_{-0.3}) \times 10^{-10}\), as observed by the WMAP experiment in the cosmic microwave radiation spectrum [2].

Strangely enough, no antimatter is observed [3], indicating that somewhere during the process of creation, interaction and cancellation a microscopic asymmetry is present which favors matter over antimatter by a ratio of \(10^9\) to \(10^9 + 1\). This asymmetry between baryons and anti-baryons is termed \textit{Baryogenesis}. In 1967, Andrei Sakharov issued a set of three necessary conditions that a baryon-generating interaction must satisfy to produce matter and antimatter at different rates [4]. These “Sakharov conditions” are:

1. Baryon number violation.
2. Violation of C- and CP-symmetry.
3. Interactions out of thermal equilibrium.

The violation of baryon number seems obvious for one wants to provide an asymmetry between baryons and anti-baryons. Similarly, for a surplus of matter over antimatter, the C-operator, which transforms matter into antimatter and vice versa (flipping the sign of all internal quantum numbers) has to be asymmetric.

The need for an invariance under a CP transformation, a charge together with a parity (P) transformation (change in sign of all spatial coordinates), directly follows from quantum field theory if one imposes a baryon number violation [5]. Or more intuitively; if CP is not violated, the creation of an excess of either matter or anti-matter for left-handed particles is cancelled by the opposite effect for right-handed particles.

Thirdly, the need for interactions out of thermal equilibrium, a difference between reactions and their time-reversed back reactions, is required for any asymmetry, reference [5] offers a pedagogical treatment of this interesting subject.

1.3 Weak interaction and the CKM matrix

An asymmetry under CP transformations is incorporated in the Standard Model through the charged current weak interaction. This interaction changes a quark into another type of quark and does the same for leptons.

In 1957 the weak interaction was confirmed by experiments to maximally violate parity, [6], [7]; it only couples to left-handed quarks and leptons. In the Lagrangian the \(\gamma^\mu(1 - \gamma^5)\) term (“vector minus axial vector” or “left-handed”) projects out the left-handed particles and right-handed anti-particles:

\[
\mathcal{L}^{CC} = -\frac{g}{\sqrt{2}}(\bar{u}, \bar{e}, \bar{d})\gamma^\mu(1 - \gamma^5)\frac{1}{2}V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} W^+_{\mu} + h.c. \tag{1.1}
\]

\(^1\text{Baryons:\: Particles made up of three quarks (e.g. protons and neutrons)}\)
The quarks shown in Figure 1.1 are mass eigenstates of the total Hamiltonian. This "quark-space" is rotated to obtain the eigenstates of the weak Hamiltonian, the weak eigenstates. This rotation that appears in the Lagrangian (1.1) is written as a 3 \times 3 complex matrix, known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

\[
\begin{pmatrix}
V_{td} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
= \begin{pmatrix}
|d\rangle & |s\rangle & |b\rangle
\end{pmatrix}

\begin{pmatrix}
|d'\rangle \\
|s'\rangle \\
|b'\rangle
\end{pmatrix}

\]

Figure 1.2: Left: The CKM matrix connects the mass eigenstate of the down-type quarks (\|d\rangle, |s\rangle, |b\rangle) with their electroweak eigenstates (\|d'\rangle, |s'\rangle, |b'\rangle), each a superposition of the down-type quarks.

Right: Charged-current quark interactions through W exchange for \(b \to t\) and the CP conjugate process. The CKM element \(V_{tb}\)

Making use of unitarity constraints \(V_{CKM}^\dagger \cdot V_{CKM} = 1 = V_{CKM} \cdot V_{CKM}^\dagger\), the "standard parametrization" advocated by the Particle Data Group (PDG) [8] is used to write the CKM matrix in terms of real angles \(\theta_{ij} (i, j = 1, 2, 3)\) and one complex angle \(\delta_{13}\):

\[
V_{CKM} = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\
-s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13}
\end{pmatrix},
\]

where \(s_{ij}\) and \(c_{ij}\) mean \(\cos(\theta_{ij})\) and \(\sin(\theta_{ij})\) respectively.

The CKM elements have an absolute value between 0 and 1, subject to a hierarchy; on-diagonal elements are close to 1 and the further away from the diagonal, the closer to 0. The so-called Wolfenstein parametrization [9] describes this hierarchy by setting \(V_{us}\) equal to \(\frac{1}{2}\) and expanding the CKM matrix in powers of \(\lambda\):\nsuch that \(1 - \frac{1}{2} \lambda^2 - \frac{1}{6} \lambda^4 \approx 1\). This parametrization serves as an important tool for phenomenological considerations and is truncated at order \(\lambda^4\), but can be expanded to any desired higher order [10].

Unitarity of the matrix leads to 9 equations, 6 of these are orthogonality equations (1.4 - 1.9) which can be represented by triangles in the complex plane (Fig. 1.3).

Physics in the SM is invariant under rephasing of the quark fields:

\[
|\phi\rangle \to e^{i\alpha}|\phi\rangle,
\]

but the CKM elements are not. The surface size of these triangles and their angles are invariant under such a rephasing, which makes them subject of most CKM studies. Each triangle surface size is equal to \(\frac{1}{2} J\), with \(J\) the so-called Jarlskog parameter, a measure of the amount of CP violation in the Standard Model, [11]. Only two triangles (1.6 and 1.9, Fig. 1.3) have sides of similar size of order \(\lambda^3\), all other triangles are squashed, making it experimentally more difficult to measure such a small angle.
involved (squashed triangles also have two large angles of \( \sim 90 \) deg., but a side length of nearly 0). The triangle belonging to equation 1.6 plays a central role in our understanding of the CKM picture and previous measurements of its angles plus the unitarity constraints provides us values of all other angles, as predicted by the Standard Model. However, it is important to overcontrain the CKM picture. New Physics, with its addition of new particles and/or interactions, will most likely add extra terms to the charged-quark interaction amplitudes and thus affects the CKM elements. The decay \( B^0_\text{s} \to J/\psi \phi \) is a measure for the angle \( \beta_s = 2\delta \gamma \), drawn in Figure 1.3, it is of order \( \lambda^2 \approx -0.04 \) in the Standard Model. Several proposed models that go beyond the Standard Model predict a non-SM value for \( \beta_s \) due to New Physics [12], [13]. A recent study which combined results from the DØ and CDF detectors at Fermilab has found a 3\( \sigma \) deviation of \( \beta_s \) from the SM value [14].

### 1.4 Mixing of neutral mesons

Particles composite of two quarks with opposite color charge are known as mesons. There is no conservation law preventing mesons with zero charge to transform into their own antiparticle. Strong and electromagnetic interactions cannot change the flavor (\( F \)) of quarks, only the weak interaction can. Therefore, meson oscillations (\( \Delta F = 2 \)) and decays (\( \Delta F = 1 \)) are driven by this force. The following describes how \( B^0_\text{s} \) mesons (which contain a \( b \) and \( s \) quark) oscillate into their own anti-particles by means of the weak interaction.

\[
\begin{align*}
V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* &= 0 \\
O(\lambda^2) & O(\lambda^2) & O(\lambda^2) \\

V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ls}V_{lb}^* &= 0 \quad (1.5) \\
O(\lambda^2) & O(\lambda^2) & O(\lambda^2) \\

V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* &= 0 \quad (1.6) \\
(\rho + \eta)A\lambda^3 & -A\lambda^3 & (1-\rho-\eta)A\lambda^3 \\

V_{us}V_{cd} + V_{cs}V_{eb} + V_{ls}V_{tb} &= 0 \quad (1.7) \\
O(\lambda^2) & O(\lambda^2) & O(\lambda^2) \\

V_{cd}V_{td} + V_{cs}V_{ts} + V_{tb}V_{tb} &= 0 \quad (1.8) \\
O(\lambda^2) & O(\lambda^2) & O(\lambda^2) \\

V_{ub}V_{td} + V_{cs}V_{ts} + V_{ub}V_{tb} &= 0 \quad (1.9) \\
(1-\rho-\eta)A\lambda^3 & -A\lambda^3 & (\rho + \eta)A\lambda^3 \\
\end{align*}
\]
and can do so into intermediate virtual states or into real final states. Because $B^0_s$ and $B^0_s$ share common final states, a non-zero amplitude arises for $B^0_s \leftrightarrow B^0_s$ transitions. The time evolution of an initial $|B^0_s\rangle$ or $|B^0_s\rangle$ state can be written as:

$$|\psi(t)\rangle = a(t)|B^0_s\rangle + b(t)|B^0_s\rangle + c_1(t)|n_1\rangle + c_2(t)|n_2\rangle + \ldots$$

(1.11)

where $n_i$ are final states to which $|B^0_s\rangle$ and/or $|B^0_s\rangle$ may decay. The wave function satisfies the Schrodinger equation:

$$i\frac{d}{dt}|\psi(t)\rangle = H_{tot}; \quad H_{tot} = H_S + H_{EM} + H_W$$

(1.12)

Starting with a $B^0_s$ or $B^0_s$ meson, the following initial conditions ($t = 0$) apply:

$$|a(t)|^2 + |b(t)|^2 = 1; \quad \sum_i |c_i(t)|^2 = 0$$

(1.13)

To solve for $|B^0_s\rangle$, use is made of the Wigner-Weiskopff approximations [15]; final state interactions are dominated by the strong interaction, on a time scale much shorter than what we are interested in. This has the fortunate effect that $c_i(t)$ can be written in terms of $a(t)$ and $b(t)$, see for example [16].

After these assumptions and approximations, second-order perturbation theory in the weak interaction leads to the following effective Hamiltonian time evolution:

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = H_{eff}\begin{pmatrix} a \\ b \end{pmatrix} = (M - i\frac{\Gamma}{2})\begin{pmatrix} a \\ b \end{pmatrix}$$

(1.14)

where $H_{eff}$, $M$ and $\Gamma$ are complex hermitian $2 \times 2$ matrices.

$\Gamma$ and $M$ are given by sums over intermediate states, which are real (physical) for $\Gamma$ and virtual (off-shell) for $M$.

Diagonalizing $H$, using $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$ from CPT invariance [16], gives the mass eigenstates $B_{s,H}$ and $B_{s,L}$ ($H$ for ‘Heavy’ and $L$ for ‘Light’), which are admixtures of the flavor eigenstates $B^0_s$ and $B^0_s$:

$$(M_{11} - i\frac{\Gamma}{2} - \lambda)^2 - H_{12}H_{21} = 0$$

(1.15)

Leading to:

$$\lambda_{\pm} = (M_{11} - i\frac{\Gamma}{2}) \pm \sqrt{H_{12}H_{21}}$$

(1.16)

Solving the eigenvalue (ev) problem $H_{eff}ev_{\pm} = \lambda_{\pm}ev_{\pm}$ gives:

$$ev_{+} = \begin{pmatrix} 1 \\ \sqrt{H_{21}H_{12}} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{\sqrt{M_{12}^2 - \frac{1}{4}\Gamma_{12}^2}} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{\sqrt{1 - \frac{1}{4}\frac{\Gamma_{12}}{\Gamma_1}}} \end{pmatrix}$$

(1.17)

and:

$$ev_{-} = \begin{pmatrix} 1 \\ -\sqrt{H_{21}H_{12}} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{\sqrt{M_{12}^2 - \frac{1}{4}\Gamma_{12}^2}} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{\sqrt{1 - \frac{1}{4}\frac{\Gamma_{12}}{\Gamma_1}}} \end{pmatrix}$$

(1.18)
The normalized eigenstates of the weak interaction are:

\[ \frac{q}{p} = \sqrt{\frac{M'_{12} - \frac{i}{2} \Gamma'_{12}}{M'_{12} - \frac{i}{2} \Gamma'_{12}}} \]  

(1.19)

The normalized eigenstates of the weak interaction are:

\[ |B_{\pm}\rangle = \frac{1}{\sqrt{1 + |\frac{q}{p}|^2}} (|B_s\rangle \pm \frac{q}{p} |\bar{B}_s\rangle) \]  

(1.20)

Here a convention is introduced. The weak eigenstates can be labeled by their masses (heavy and light), or their lifetime (long and short). It is custom to label the \( B_s \) mass eigenstates by their mass: \( B_- \equiv B_{s,H} \) and \( B_+ \equiv B_{s,L} \), and the mass difference is chosen to be positive by definition \( \Delta m = m_H - m_L = (m_- - m_+) \). In this case the sign of \( \Delta \Gamma \), defined as \( \Gamma_H - \Gamma_L = (\Gamma_- - \Gamma_+) \), carries physical significance.

Now we can go back to the \( |B_s\rangle - |\bar{B}_s\rangle \) description and determine the time evolution of an initial \( |B_s\rangle \) or \( |\bar{B}_s\rangle \) meson. Subtracting and adding:

\[ |B_{s,L}(t)\rangle = \frac{1}{\sqrt{1 + |\frac{q}{p}|^2}} [|B_s(t)\rangle + \frac{q}{p} |\bar{B}_s(t)\rangle] = e^{-i(m_L - \frac{1}{2} \Gamma_L)t} |B_{s,L}(0)\rangle = e^{-i\mu_L t} |B_{s,L}(0)\rangle \]  

(1.21)

and

\[ |B_{s,H}(t)\rangle = \frac{1}{\sqrt{1 + |\frac{q}{p}|^2}} [|B_s(t)\rangle - \frac{q}{p} |\bar{B}_s(t)\rangle] = e^{-i(m_H - \frac{1}{2} \Gamma_H)t} |B_{s,H}(t)\rangle = e^{-i\mu_H t} |B_{s,H}(0)\rangle \]  

(1.22)

gives:

\[ |\tilde{B}_s(t)\rangle = \frac{\sqrt{1 + |\frac{q}{p}|^2}}{2q/p} (|B_{s,L}(t)\rangle - |B_{s,H}(t)\rangle) \]

\[ = \frac{\sqrt{1 + |\frac{q}{p}|^2}}{2q/p} (e^{-i\mu_L t} |B_{s,L}(0)\rangle - e^{-i\mu_H t} |B_{s,H}(0)\rangle) \]

\[ = \frac{e^{-i\mu_L t} - e^{-i\mu_H t}}{2q/p} |B_s(0)\rangle + \frac{e^{-i\mu_L t} + e^{-i\mu_H t}}{2} |\bar{B}_s(0)\rangle \]

\[ = g_-(t) \frac{D}{q} |B_s(0)\rangle + g_+(t) |\bar{B}_s(0)\rangle \]

where \( g_\pm(t) \) is defined as \( \frac{1}{2}(e^{-i\mu_L t} \pm e^{-i\mu_H t}) \).

Similarly:

\[ |B_s(t)\rangle = g_+(t) |B_s(0)\rangle + g_-(t) \frac{q}{p} |B_s(0)\rangle \]  

(1.23)

For an initial produced B meson, the probability to decay as a B or \( \bar{B} \) meson at time \( t \) are then given by:

\[ |\langle \tilde{B}_s | \tilde{B}_s(t) \rangle |^2 = |g_+(t)|^2 \]

\[ |\langle B_s | B_s(t) \rangle |^2 = \frac{|q/p|^2}{2} |g_-(t)|^2 \]

\[ |\langle \tilde{B}_s | \tilde{B}_s(t) \rangle |^2 = \frac{|p/q|^2}{2} |g_-(t)|^2 \]

\[ |\langle \tilde{B}_s | \tilde{B}_s(t) \rangle |^2 = |g_+(t)|^2 \]  

(1.24)
\[ |g_\pm(t)|^2 = \frac{1}{4}(e^{-i\nu_L t} \pm e^{-i\nu_H t})(e^{i\nu_L t} \pm e^{i\nu_H t}) = \frac{1}{2} e^{it}\left[\cosh\left(\frac{\Delta\Gamma}{2} t\right) \pm \cos(\Delta m_s t)\right] \]

(1.25)

Where

\[ \Gamma = \frac{\Gamma_L + \Gamma_H}{2} \]

(1.26)

Figure 1.4: Probability for a pure \( B_s \) at production, to decay either as a \( B_s \) or as a \( \bar{B}_s \) as given by equations 1.24. The values for \( \Delta m, m, \Delta\Gamma \) and \( \Gamma \) are taken from [8].

The two dominant Feynman diagrams contributing to mixing are shown in Figure 1.5. From (1.24) it follows that the probability for a \( B_s \) to oscillate into a \( B_s^0 \) differs from the CP conjugated process in case \( |\frac{q}{p}| \neq 1 \), called \( CP \) violation in mixing. Taking into account only the dominant mixing contribution depicted in Figure 1.5:

\[ \frac{q}{p} = \frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}} = |\frac{q}{p}| e^{i\phi_s} \]

(1.27)

\( \phi_s \) is the \( B_s^0 \) mixing phase and the angle from the unitarity triangle 1.3. This phase is of no influence on the absolute value of \( \frac{q}{p} \) and can only be measured through interference with another phase from a different process. For the \( B_s - \bar{B}_s \) system \( |\frac{q}{p}| \) is assumed to be 1, as predicted by the SM.

New Physics contributions to \( B_s \) might alter \( \frac{q}{p} \) such that its absolute value differs from 1. One way to measure this is by studying the different decay rates for \( B_s^0 \rightarrow D_s^- + \pi^+ \) and \( \bar{B}_s^0 \rightarrow D_s^+ + \pi^- \).

Figure 1.5: The Standard Model allows \( B_s \) mesons (matter particles) to change into \( \bar{B}_s \) mesons (antimatter particles) and vice versa. These transitions lead to particle mixing. Due to the complex nature of the CKM entries, the amplitude for \( B_s^0 \rightarrow \bar{B}_s^0 \) has a different phase than the amplitude for \( \bar{B}_s^0 \rightarrow B_s^0 \).
1.5 The decay $B^0_s \rightarrow J/\psi \phi$

Both $B^0_s$ and $\bar{B}^0_s$ can decay to $J/\psi \phi$ with amplitudes: $A_f = \langle f | T | B_s \rangle$ and $\bar{A}_f = \langle f | T | \bar{B}_s \rangle$ respectively. The leading Feynman diagrams are drawn in Figure 1.6.

Next to leading order diagrams involve gluonic interactions (also known as penguin diagrams), which have the same complex phase from the CKM elements and can therefore be neglected in the rest of this thesis.

The time-dependent decay amplitudes are:

$$A_f(B_s(t) \rightarrow f) = \langle f | H_{eff} | B_s \rangle = g_+(t)A_f + \frac{q}{p}g_-(t)\bar{A}_f$$

$$A_f(\bar{B}_s(t) \rightarrow f) = \langle f | H_{eff} | \bar{B}_s \rangle = g_+(t)\bar{A}_f + \frac{p}{q}g_-(t)A_f$$

Squaring these, we find the corresponding decay rates:

$$\Gamma(B_s(t) \rightarrow f) = |A_f|^2 (|g_+(t)|^2 + |\lambda_f|^2|g_-(t)|^2 + 2R[\lambda_f g_+(t)g_-(t)])$$

$$\Gamma(\bar{B}_s(t) \rightarrow f) = |A_f|^2 \frac{p}{q}(|g_-(t)|^2 + |\lambda_f|^2|g_+(t)|^2 + 2R[\lambda_f g_+(t)g_-(t)])$$

Where

$$\lambda_f \equiv \frac{q\bar{A}_f}{pA_f}$$

Using equation 1.25 and:

$$g_+(t)g_-(t) = \frac{1}{2}e^{-\Gamma t}[\sinh(\frac{\Delta \Gamma}{2}t) + i \sin(\Delta m_s t)]$$

The decay rates can be expressed as:

$$\Gamma(B_s(t) \rightarrow f) = |A_f|^2 \frac{1}{2}e^{-\Gamma t}[(1 + \lambda_f^2)\cosh(\frac{\Delta \Gamma}{2}t) + (1 - \lambda_f^2)\cos(\Delta m_s t)$$

$$-2R\lambda_f \sinh(\frac{\Delta \Gamma}{2}t) + 2i\lambda_f \sin(\Delta m t)]$$

$$\Gamma(\bar{B}_s(t) \rightarrow f) = |A_f|^2 \frac{p}{q}e^{-\Gamma t}[(1 + \lambda_f^2)\cosh(\frac{\Delta \Gamma}{2}t) - (1 - \lambda_f^2)\cos(\Delta m_s t)$$

$$-2R\lambda_f \sinh(\frac{\Delta \Gamma}{2}t) - 2i\lambda_f \sin(\Delta m t)]$$

Given $B \rightarrow f$ and $\bar{B} \rightarrow f$, with decay amplitudes $A_f$ and $\bar{A}_f$, direct CP violation, or CP violation in decay, occurs when

$$|\frac{A_f}{\bar{A}_f}| \neq 1$$
a phenomenon also possible for charged mesons. For $B_s^0 \rightarrow J/\psi \phi$, $|\frac{A_f}{A_f}|$ is assumed to be 1, as predicted by the SM.

### 1.6 Interference in $B_s^0 \rightarrow J/\psi \phi$

In this thesis both $|\frac{q}{p}|$ and $|\frac{A_f}{A_f}|$ are 1, thus $|\lambda_f| = 1$. However, CP violation can still occur, through the interplay between mixing and the ensuing decay of the $B_s^0$ meson to the final state $f$, which causes an interference between the complex phases.

When a $B_s^0$ meson (or its anti-particle) decays into $J/\psi \phi$, the decay amplitude picks up a phase, $\phi_s$, due to possible mixing and an additional phase, $\phi_D$, through the decay. SM predictions for these phases are $O(-0.04)$ rad. and $O(\lambda^6) \sim 0$ for $\phi_s$ and $\phi_D$ respectively. The final state $J/\psi \phi$ is a CP eigenstate with eigenvalues $f = \pm 1$ defined by:

$$|\bar{f}\rangle = (CP)|f\rangle = \eta_f|f\rangle \quad (1.37)$$

The CP-odd ($\eta_f = -1$) and even ($\eta_f = +1$) eigenstates are identical, but for their angular momentum. Experimentally, one measures a mixture of both CP eigenstates, which are disentangled through an angular analysis at the cost of statistics as discussed in section 1.7.

Using Equations 1.34 and 1.35 and $|\frac{q}{p}| = 1$:

$$\Gamma(B_s \rightarrow f)(t) - \Gamma(\bar{B}_s \rightarrow f)(t) = |A_f|^2 e^{-\Gamma t} \{(1 - |\lambda_f|^2) \cos \Delta mt + 2i\lambda_f \sin \Delta mt\} \quad (1.38)$$

$$\Gamma(B_s \rightarrow f)(t) + \Gamma(\bar{B}_s \rightarrow f)(t) = |A_f|^2 e^{-\Gamma t} \{(1 + |\lambda_f|^2) \cosh \frac{\Delta \Gamma t}{2} - 2\Re \lambda_f \sinh \frac{\Delta \Gamma t}{2}\} \quad (1.39)$$

The decay amplitudes ratio and mixing ratio $\frac{q}{p}$ with their phases explicitly written [16]:

$$\frac{\bar{A}_f}{A_f} = \eta_f e^{-i(\phi_D + \phi_{CP})} \quad (1.40)$$

$$\frac{q}{p} = e^{i(2\phi_s + \phi_{CP})}, \quad (1.41)$$

where $\phi_{CP}$ is an arbitrary phase resulting from rephasing invariance of the quark fields under a CP transformation ((CP)$^2 = 1$):

$$CP|B\rangle = e^{i\varphi}|B\rangle; \quad CP|\bar{B}\rangle = e^{-i\varphi}|\bar{B}\rangle \quad (1.42)$$

$\lambda_f$ (Eq. 1.32) is given by the product of the above:

$$\lambda_f = e^{i(2\phi_s + \phi_{CP})}\eta_f e^{-i(2\phi_D + \phi_{CP})} = \eta_f e^{2i(\phi_s - \phi_D)} \quad (1.43)$$

$$\Gamma \lambda_f = \eta_f \sin(2\beta_s); \quad R \lambda_f = \eta_f \cos(2\beta_s) \quad (1.44)$$

$2\beta_s = \phi_s - \phi_D \approx \phi_s$. 

9
The time-dependent CP asymmetry is defined as the ratio of Equations 1.39 and 1.39:

\[
A_{CP}(t) = \frac{\Gamma(B_s \to J/\psi \phi)(t) - \Gamma(\bar{B}_s \to J/\psi \phi)(t)}{\Gamma(B_s \to J/\psi \phi)(t) + \Gamma(\bar{B}_s \to J/\psi \phi)(t)} = \frac{I_\lambda_f \sin \Delta mt}{\cosh \frac{\Delta t}{2} - R_\lambda_f \sinh \frac{\Delta t}{2}}
\]

(1.45)

after substituting Equations 1.44:

\[
A_{CP}(t) = \frac{\eta_f \sin 2\beta_s \sin \Delta mt}{\cosh \frac{\Delta t}{2} - \eta_f \cos 2\beta_s \sinh \frac{\Delta t}{2}}
\]

(1.46)

This is a measure for the difference between process and its CP transformed equivalent as well as for the angle \( \beta_s \). A value different from zero implies CP violation.

1.7 Angular analysis

The \( B^0_S \) meson is a scalar meson (\( S = 0 \)), decaying into two vector mesons (\( S = 1 \)). Therefore, three possible angular momentum states are possible: \( L = 0 \), \( L = 1 \) and \( L = 2 \). For \( L = 0 \) or \( L = 2 \), \( J/\psi \phi \) is CP-even and CP-odd for \( L = 1 \). The angular momentum for each final state is impossible to measure in the LHCb detector, but fortunately one can use an angular analysis to statistically disentangle the CP states contributions [17], [18]. In the transversity basis, Fig. 1.7, the decay amplitudes can be decomposed in linear polarisations of \( j^\perp \) : \( A_0 \), \( A_\perp \) and \( A_\parallel \). Amplitude \( A_0 \) is the longitudinal component with respect to the flight direction of \( j^\perp \). The two transverse components, \( A_\parallel \) and \( A_\perp \) are respectively parallel and perpendicular to each other. \( A_0 \) and \( A_\parallel \) are associated with the CP-even \( L = 0 \) and \( L = 2 \) states, \( A_\perp \) with the CP-odd \( L = 1 \) contribution.

For each measured decay the angle \( \theta_{tr} \) as illustrated in Figure 1.7 is reconstructed. The time dependent differential decay rate is expressed as:

\[
\frac{d\Gamma(t)}{d\cos \theta_{tr}} \propto (|A_\parallel(t)|^2 + |A_0(t)|^2)(1 + \cos^2 \theta_{tr}) + (|A_\perp(t)|^2) \sin^2 \theta_{tr}.
\]

(1.48)

Or, by expanding the time dependent expressions:

\[
\frac{d\Gamma(t)}{d\cos \theta_{tr}} \propto (1 - R_\perp)[(1 + \cos \beta_s)e^{-\Gamma_{L,t}} + (1 - \cos \beta_s)e^{-\Gamma_{H,t}}
\]

\[
+ 2e^{-\Gamma_{L,t}} \sin(\Delta m_s t) \sin 2\beta_s \frac{1}{2}(1 + \cos^2 \theta_{tr})
\]

\[
+ R_\perp[(1 - \cos \beta_s)e^{-\Gamma_{L,t}} + (1 + \cos \beta_s)e^{-\Gamma_{H,t}}
\]

\[
- 2e^{-\Gamma_{L,t}} \sin(\Delta m_s t) \sin 2\beta_s \sin^2 \theta_{tr})
\]

(1.49)

and for the \( B^0_S \) decay rate:

\[
\frac{d\Gamma(t)}{d\cos \theta_{tr}} \propto (1 - R_\perp)[(1 + \cos \beta_s)e^{-\Gamma_{L,t}} + (1 - \cos \beta_s)e^{-\Gamma_{H,t}}
\]

\[
- 2e^{-\Gamma_{L,t}} \sin(\Delta m_s t) \sin 2\beta_s \frac{1}{2}(1 + \cos^2 \theta_{tr})
\]

\[
+ R_\perp[(1 - \cos \beta_s)e^{-\Gamma_{L,t}} + (1 + \cos \beta_s)e^{-\Gamma_{H,t}}
\]

\[
+ 2e^{-\Gamma_{L,t}} \sin(\Delta m_s t) \sin 2\beta_s \sin^2 \theta_{tr})
\]

(1.50)
Figure 1.7: The transversity basis is constructed in the rest frame of the $J/\psi \phi$ meson. The xy plane is spanned by the kaon pair and the z axis is chosen such to make it a right handed coordinate system. The angle between the z axis and the projection of the momentum direction of positive muon onto the xy plane is known as the transversity angle, $\theta_{tr}$.

$R_\perp$ is defined as the the fraction of CP-odd component at time $t = 0$, defined as:

$$R_\perp = \frac{|A_{\perp}(0)|^2}{|A_{\perp}(0)|^2 + |A_{\parallel}(0)|^2 + |A_0(0)|^2}$$  \hspace{1cm} (1.51)

which is to be measured in an untagged analysis (no separation between $B_s^0$ and $\bar{B}_s^0$):

$$\frac{d\Gamma(t)}{d\cos \theta_{tr}} + \frac{d\Gamma(t)}{d\cos \theta_{tr}} \propto (1 - R_\perp)[(1 + \cos \beta_s)e^{-\Gamma_{\perp}t} + (1 - \cos \beta_s)e^{-\Gamma_{\parallel}t}](1 + \cos^2 \theta_{tr})$$

$$+ R_\perp[(1 - \cos \beta_s)e^{-\Gamma_{\perp}t} + (1 + \cos \beta_s)e^{-\Gamma_{\parallel}t}]\sin^2 \theta_{tr}.$$  \hspace{1cm} (1.52)

Since $\beta_s$ is expected to be small in the SM, the $(1 - \cos \beta_s)$ terms are usually neglected. Thus the $B_H$ meson is associated with the $(1 - R_\perp)$ (or CP-even) term and $B_L$ with the $R_\perp$ (CP-odd) term.


2 The LHCb Experiment

2.1 B-Physics at the LHC

The Large Hadron Collider (LHC) is a particle accelerator located at CERN\textsuperscript{1}, currently in its final stage of construction. Once operational it will accelerate and collide protons at a center-of-mass energy of 14 TeV, making it the world’s largest and highest-energy particle-accelerator.

Along the particle beam, four dedicated experiments are under construction, one of them the Large Hadron Collider beauty (LHCb) experiment, a detector designed for high-precision B-physics.

After full injection, the particle beam in the 27 km long accelerator is filled with 2808 bunches of $10^{11}$ protons each, separated at 25 ns. When the beams are focused in an interaction point to collide head-on, on average 27 proton-proton collisions are produced per bunch crossing. By defocusing the beams at the LHCb interaction point this number is reduced to 0.53 inelastic collisions on average \cite{19}. The reason for doing so, is that the decay vertex of a B meson must be matched with its point of production in order to reconstruct the decay path. Multiple \textit{pp} collisions within the same bunch crossing, referred to as pile-up, increase the identification uncertainty.

LHCb studies B mesons which originate from produced $b\bar{b}$ pairs from the smashed protons. Due to confinement, both the $b$ and $\bar{b}$ quark will combine with other quarks to form hadrons. Due to the high center-of-mass energy of the LHC, all known $b$-hadrons are produced.

As is seen in Figure 2.1, two produced B hadrons from the $b\bar{b}$ pair predominantly fly in the same forward or backward direction, under a relatively small polar angle. For this reason the LHCb detector, discussed in the next session, covers a limited radial space close to the beam pipe.

2.2 Layout of the LHCb detector

The LHCb \textsuperscript{20} is a 20m long single-arm forward spectrometer with a radial acceptance in the horizontal plane between 10-300 mrad and 10-250 mrad in the vertical plane. The setup is shown in Figure 2.2. Centered in the picture is the di-pole magnet, with its field oriented such that charged particles from the interaction point bend in the horizontal plane. The magnet has an integrated bending power of 4 Tm. In the following sections a short description of the different subdetectors is given, categorized into tracking and particle identification (PID) detectors.

\textsuperscript{1}The European Organization for Nuclear Research, http://www.cern.ch
Figure 2.1: Production angles of $b\bar{b}$ pairs at LHCb. The $b\bar{b}$ pairs are predominately produced in the same direction, at small angles with respect to the beam pipe.

Figure 2.2: The LHCb with the different subdetectors in the horizontal plane, or bending plane of the magnet.
2.2.1 Tracking detectors

In the realm of rapid meson oscillations and time dependent CP violation, high precision B vertex and momentum reconstruction is crucial. To accomplish this, LHCb makes use of the following state of the art tracking detectors:

**Vertex Locator** Starting at a 8 mm distance to the interaction point, the Vertex Locator (VELO) is equipped with 21 stations of silicon strip detectors along and perpendicular to the beam axis. In each station two detection planes are mounted back-to-back, measuring the radial and angular position of traversing charged tracks. The VELO serves to measure decay lengths and impact parameters of charged tracks. Because of the high resolution of 6 to 18 μm (strip pitch increases as the radius gets larger) and being placed close to the proton beam, primary and secondary vertices (origin and decay respectively) are obtained with the required high precision.

**Trigger Tracker** The Trigger Tracker (TT) is placed between the RICH1 detector and the beginning of the magnet. It consists of two stations separated by a 27 cm distance. Two layers of silicon strips are placed under a relative stereo angle at each station. The presence of the small magnetic field is exploited to gather rough estimates of the charged particles momenta. This information is used in an early stage of the trigger algorithm. Events that passed the trigger criteria are stored for offline analysis. Here, TT measurements are used for reconstructing long-lived neutral particles that decayed outside the VELO.

**Inner and Outer Tracker** On the downstream side of the magnet three tracking stations are placed, each containing an Inner Tracker (IT) and Outer Tracker (OT) module. Like the TT, the IT is a silicon strip detector, placed around the beam pipe with the same relative stereo angle setup as the TT. OT modules are straw tube detectors placed around the IT units. Track segments from the OT and IT are matched with segments before the magnet to obtain the curvature in the magnetic field for identifying the momentum of a charged particle.

2.2.2 PID detectors

The particle identification system contains detectors to perform π/K separation e, γ and hadron identification and finally μ identification.

**Ring Imaging Cerenkov Detectors** The main purpose of the Ring Imaging Cerenkov (RICH) detectors is to separate charged pions from charged kaons. The RICH system is divided into two detectors. RICH1 is placed in between the VELO and the TT, RICH2 is situated between the IT and OT modules and the calorimeter. Charged particles traversing the RICH detectors emit Cerenkov radiation when their velocity is greater than that of light in the same medium: \( v > c/n \), where \( n \) is the refraction index of the medium traversed. A cone of light forms under an angle \( \theta_C \) with respect to the trajectory of the particle. This angle is a direct measure for the particle’s velocity, \( \beta = \frac{v}{c} \):

\[
\cos(\theta_C) = \frac{1}{n\beta} \tag{2.1}
\]
Together with the momentum information obtained by track reconstruction, the mass of the particle and thus the particle itself is identified. Via spherical and flat mirrors, Cerenkov light is projected as a circle onto a plane of photon detectors. The radius of the circle is proportional to the angle $\theta_C$.

The refractive index $n$ is used to set a threshold for velocities, optimized in RICH1 to identify low momentum particles and in RICH2 for high momentum particles. The momentum range covered by RICH1 and RICH2 together ranges from 2 to 100 GeV.

**Calorimeters** The calorimetry system is made up out of an electromagnetic calorimeter (ECAL) and a hadron calorimeter (HCAL). The ECAL is placed closest to the interaction point and HCAL further downstream, adjacent to the ECAL. All charged particles apart from muons are fully absorbed in the calorimeter system. This dense apparatus identifies particles and measures their energy as they interact with the detector material, setting off a cascade of secondary particles called a “particle shower”. Scintillation detectors measure the amount of light produced in such a shower, proportional to the incident particle’s energy. ECAL is important for identification of electrons as well as neutral pions and prompt photons.

A scintillator pad detector (SPD) is placed before the ECAL to detect charged particles before entering the calorimeter. Directly after traversing the SPD, particles encounter the pre-shower (PS) system: a thin lead plate followed by a scintillator pad layer. The lead plate causes an early shower, detected by the following scintillator. Early shower development is a clear distinction between electron and charged pion showers.

Most hadrons are not completely absorbed in the ECAL and continue their path into the HCAL, a sampling device made out of iron and scintillating tiles. Again, a particle’s energy is measured by the detected light in the scintillators, caused by a cascade of secondary particles.

**Muon System** Muons are the only charged particles able to traverse the calorimeter system. They are detected in the muon system, consisting of five stations built from Multi Wire Proportional Chambers (MWPC). Four stations are placed after the HCAL, separated by 80 cm thick steel plates. One station is placed in front of the calorimeter, as multiple scattering in the calorimeter decreases the resolution on the muon tracks.

Muon information is used by the trigger to identify high momentum muons. In the offline reconstruction the muon system is used to identify muons in the tracks found by the tracking system.

**2.2.3 Trigger**

At a LHC collision rate of 40 MHz, only a small percentage of all events can be written to tape. On average, about one in every 160 events contains a B-meson. To store only those events which contain interesting physics signatures for offline analysis, efficient cuts must be applied. Collisions follow each other every 25 ns, providing a very short time span in which to decide whether or not an event is potentially interesting.

The LHCb trigger can be divided into the Level-0 Trigger and the High Level Trigger:

**Level-0 Trigger** The Level-0 (L0) Trigger aims to reduce the event rate from 40 MHz to
1 MHz. It cuts away events with more than one pp interaction, uses calorimeter information to select particles with large transverse energy and looks for high momentum muons by consulting the muon systems.

The L0 Trigger is implemented in hardware and decides upon keeping or getting rid of an event in 4 $\mu$s after the bunch crossing. The trigger efficiencies for selected B-decays range from 40% to 90%, depending on the type of $B$ decay.

After a positive decision, the event is passed on to the High Level Trigger together with L0 information.

**High Level Trigger** The High Level Trigger (HLT) is a software trigger executed on a CPU farm. Depending on which trigger in L0 accepted the event, a specific HLT trigger strategy is followed. The HLT uses the information from all subdetectors and can run reconstruction algorithms. The HLT lowers the event rate to 2 kHz which is stored on tape. $B$ decay channel efficiencies for the HLT are between 40% and 80%.
3 Proper Time Resolution Model

The purpose of this study is to construct a resolution model which accurately describes the detector’s uncertainty for the proper time \( t \) of the \( B^0_s \) meson in the decay \( B^0_s \rightarrow J/\psi \phi \). Colliding proton bunches, events, are generated by using Monte-Carlo (MC) techniques which are simulated to occur in the LHCb detector. The produced data files are ready for reconstruction and analysis in the same manner as acquired data once the detector is operational.

The generated ("true" or "MC") parameters are compared with the reconstructed ("rec") ones, their difference (residual) is a measure for the detector’s resolution. The residual distribution of the proper time \( t_{\text{rec}} - t_{\text{true}} \), for each reconstructed event \( i \), is analyzed taking into account the per-event-error \( \sigma_{t_{\text{rec}}} \). \( \sigma_{t_{\text{rec}}} \) is the calculated uncertainty from the reconstruction, based on the track covariance matrices and characteristics of the decay such as its topology and decay region in the detector.

3.1 Motivation

The need for a faithful description of the detector’s uncertainty of the \( B^0_s \) meson’s proper time is described in the following. The decay \( B^0_s \rightarrow J/\psi \phi \) provides a measure for the angle \( \beta_s \) as explained in Chapter 1 and in particular Equation 1.47. Figure 3.1 shows the proper time probability distribution functions (pdfs) of the \( B^0_s \) and \( \bar{B}^0_s \) mesons, decaying into the CP even final state \( J/\psi \phi \). Below this, the asymmetry given by Equation 1.46 is depicted. The right side shows the same proper time pdfs and asymmetry, when measured by a detector that has a non-biased Gaussian proper time resolution, with \( \sigma = 60 \text{ fs} \). A non-zero resolution decreases the amplitude of the asymmetry \( \propto \sin^2(2\beta_s) \). Knowledge of the true resolution is therefore crucial to correct for this effect.

The effect of a non-zero resolution on a parameter’s distribution can be simulated by convoluting the true value pdf with the appropriate resolution model. In case of an exponential decay convoluted by a single Gaussian resolution function:

\[
\frac{1}{\tau} e^{-\frac{t}{\tau}} \otimes \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(t-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}\sigma\tau} \int_{-\infty}^{\infty} e^{-\frac{t'}{\tau}} e^{-\frac{(t'+\mu-\tau)^2}{2\sigma^2}} \, dt'
\]

(3.1)

Once LHCb is operational, the \( B^0_s \) proper time distribution convoluted with a resolution model will be fitted to the measured data to obtain the resolution model. However, as illustrated in Figure 3.2, the distortion of a decay distribution without oscillations by resolution effects is only manifest for proper time values near \( t = 0 \). As the CP asymmetry shows a sinusoidal behavior as function of the proper time, information on the resolution for proper time values significantly larger than zero is equally essential. This information is acquired by a MC study and a constant resolution model is preferable for no modeling of parameters is needed.
Figure 3.1: *Left:* Pdfs for $B^0_s \to J/\psi \phi$ (CP even) and $\bar{B}^0_s \to J/\psi \phi$(CP even) and the corresponding asymmetry. *Right:* Same pdfs and asymmetry, convoluted with a $\sigma = 60$ fs Gaussian to simulate resolution effects.

Figure 3.2: Proper time pdfs for $\tau = 3$ ps, with (dashed line) and without (solid line) resolution effects (Gaussian with $\sigma = 40$ fs).

### 3.2 Software

MC data generation is a CPU intensive and time consuming process. Once in several years a so-called Data Challenge (DC) dataset is produced by the LHCb collaboration for miscellaneous event types, all under the same conditions such as detector geometry and response, digitization procedure and reconstruction algorithms. For this study use is made of the DC’06 dataset.

The software framework used is GAUDI, an object oriented C++ environment which encompasses all data production and analysis applications. For the production of data GAUSS [21] is responsible for event generation and detector simulation, BOOLE [22] takes care of the detector digitization and BRUNEL [23] reconstructs tracks and identifies particles.
The produced events are stored in the form of Data Summary Tape (DST) files, on which DAVINCI \cite{24}) runs the selection and offline analysis. DST files used for this study contain per event at least one $B^0_s \rightarrow J/\psi (\mu^- \mu^+) \phi (K^+ K^-)$ (or charge-conjugated) decay that is completely in the detector’s acceptance. The influence of background events is not included. Approximately $1.6 \cdot 10^6$ signal events are analyzed, which roughly corresponds to 2.5 years of data taking.

### 3.3 Event selection

The reconstruction of $B^0_s \rightarrow J/\psi \phi$ starts with the measured charged tracks of the two oppositely charged muons and two oppositely charged kaons. Muons are recombined to a $J/\psi$ meson, kaons to a $\phi$ meson. Together they reconstruct the $B^0_s$ meson.

Cuts on the reconstructed parameters are applied, optimized for a favorable signal over background ratio. Figure 3.3 shows an average event in the tracking system; out of this proverbial haystack the interesting tracks, if present, need to be filtered out.

B mesons are relatively heavy and therefore its daughter particles will typically have a larger transverse momentum ($p_T$) than prompt particles or daughter particles of non-B-particles. A secondary vertex displaced from the interaction point (primary vertex) is an indication for a long-lived particle such as a B-meson. But such a requirement will influence the proper time distribution and cut away the region for short lifetimes, which is needed for obtaining the resolution as discussed in section 3.1.

Charged particle tracks are predominately pions, which account for 73% of charged tracks. Kaons and muons make up respectively about 16% and 0.5% of all charged tracks in a $b$-inclusive event. To separate them from pions, information from the PID detectors is used to calculate a likelihood value for a given particle hypothesis $x$. The $PID(x)$ value is defined relatively to a pion hypothesis:

$$PID(x) = \Delta \ln L(x - \pi) = \ln L(x) - \ln L(\pi) = \ln \left( \frac{L(x)}{L(\pi)} \right)$$ (3.2)

Where $L(x)$ is the combined likelihood information from the PID detectors.

Details on the cuts applied are given below:

$J/\psi$

- A transverse momentum cut on both $\mu^+$ and $\mu^-$: $p_{T, \mu^\pm} > 1500$ MeV
- A $\chi^2$ cut on the $J/\psi$ vertex reconstruction: $\chi^2_{J/\psi} < 6$
- A PID cut on both muons: $PID(\mu^\pm) > -1$
- A mass cut on $J/\psi$: $|m_{J/\psi, \text{rec}} - m_{J/\psi, \text{PDG}}| < 85$ MeV

$\phi(1020)$

- A transverse momentum cut on both $K^+$ and $K^-$: $p_{T, K^\pm} > 750$ MeV
- A PID cut on both kaons: $PID(K^\pm) > -1$
- A $\chi^2$ cut on the $\phi$ vertex reconstruction: $\chi^2_{\phi/\psi} < 40$
- A mass cut on $\phi$: $|m_{\phi, \text{rec}} - m_{\phi, \text{PDG}}| < 28$ MeV
Figure 3.3: A typical event in the bending plane of the LHCb detector, together with a zoom-in view of the VELO-TT region.

For the $B_s^0$ meson a mass and $\chi^2$ vertex cut are applied:

$$|m_{B_s^0,\text{rec}} - m_{B_s^0,\text{PDG}}| < 100$$
$$\chi^2_{B_s^0} < 22.5$$

After these selections, $2.4 \times 10^5$ events are reconstructed and matched with their MC partners which effectively excludes any mis-reconstructed $B_s^0 \rightarrow J/\psi\phi$ decays. All reconstructed daughter particles of the $B_s^0$ meson must match with their MC counterpart for the decay to be truth-matched. Figure 3.4 shows the reconstructed masses for the composite particles after the applied cuts.

Figure 3.4: Reconstructed invariant masses. From left to right: $B_s^0$, $J/\psi$ and $\phi$

A fit of Equation 1.52 on the untagged MC data for $t_{\text{true}}$ is applied to inspect whether the selections introduce a bias on the proper time. Figure 3.5 shows the projections of the fit on the $t_{\text{rec}}$ and $\cos \theta_{tr}$ data distributions together with the Pull values between the fit
and data points for the proper time distribution, defined as:

$$\text{Pull} = \frac{\text{data}(t_{\text{rec}}) - \text{fit}(t_{\text{rec}})}{\text{data}(t_{\text{rec}})\text{stat. error}}.$$  \hspace{1cm} (3.3)

Data was generated with values $(1 - R_\perp) = 0.84$, $\Delta \Gamma = 0.0685$ and $\tau_{B^0} = 1.461$ ps. All three fit values show a significant deviation from their generated values. This is either introduced by the selections or by a possible correlation between reconstruction efficiencies and the proper time.

![Figure 3.5: The true proper times and $\cos \theta_{t_r}$ values with their fit values, shown exponentially together with the Pulls and Residuals.](image)

### 3.4 Control channel: $B^0 \rightarrow J/\psi K^*$

Besides the channel $B^0_s \rightarrow J/\psi$, also the decay channel $B^0 \rightarrow J/\psi(\mu^+\mu^-)K^*(K^+\pi^-)$ is studied. It is considered a control channel for $B^0_s \rightarrow J/\psi$, since it has the same scalar to two vector mesons topology and the $B^0$ meson has a comparable lifetime. $B_d \rightarrow J/\psi K^*$ has a larger branching ratio and $\Delta \Gamma = 0$, making the fit easier. Therefore this channel is favored for obtaining the resolution model on real data from LHCb, assuming both models are equal.

$2.16 \times 10^6$ generated events were analyzed, out of which $3.99 \times 10^5$ full $B^0 \rightarrow J/\psi K^*$ decays passed the applied cuts (shown in Table 3.1) and were truth-matched.

An exponential decay is fitted to the $t_{\text{true}}$ distribution, Fig. 3.6, which shows a small deviation from the lifetime value used for MC data generation ($\tau_{B^0,M\text{C}} = 1.536$ ps). This indicates the presence of a bias.
### Table 3.1: Selection cuts for $[B^0 \rightarrow J/\psi K^+]cc$

|                | max $|M_{rec} - M_{PDG}|$ | max $p_T$ | max vertex $\chi^2$ |
|----------------|---------------------------|-----------|---------------------|
| $J/\psi(\mu^+\mu^-)$ | 50                        | 1000 MeV  | 9                   |
| $K^+(K^+\pi^-)$    | 150                       | 1000 MeV  | 9                   |
| $B^0_d$             | 50                        | -         | 27                  |

3.5 Proper time

The proper time of a $B^0_s$ is the time between its creation and decay measured in its own reference frame. A constrained $\chi^2$ fit, described in [25], determines the $B^0_s$’s reconstructed proper time together with an estimated error. The measured 9 observables $O$ are the primary vertex position $\vec{x}$, the secondary vertex position $\vec{v}$ (point of decay) and the measured momentum of the $B^0_s$ meson at the decay point:

$$O = (\vec{v}, \vec{x}, \vec{p})$$  

(3.4)

The fit also uses the nominal (not the reconstructed) mass $m$ of the $B^0_s$ meson. The relation used to acquire the proper-time is:

$$\vec{x} = \vec{v} - t_{rec} \cdot \frac{\vec{p}}{m}.$$  

(3.5)

It is assumed that $\vec{v}$ and $\vec{x}$ are independent, i.e. no track was used in the determination of both the primary and the secondary vertex. Then the particle’s trajectory can be
described by a set of 7 parameters:

$$\mathcal{P} = (\tilde{v}, \tilde{p}, \tilde{t}_{rec})$$

(3.6)

which are to be obtained by minimizing the following $\chi^2$:

$$\chi^2(\mathcal{P}) = \mathcal{R}^T W_O \mathcal{R}.$$  

(3.7)

$\mathcal{R}$ describes the difference between the observables (measurements) and their predictions from the parameters:

$$\mathcal{R} = \begin{pmatrix} \tilde{v} - \bar{v} \\ \tilde{p} - \bar{p} \\ \tilde{x} - (\bar{v} - \bar{t}_{rec} \cdot \bar{p} / \bar{m}) \end{pmatrix}$$

(3.8)

and $W_O$ serves as the weight matrix that also takes into account the correlation between $\tilde{v}$ and $\tilde{p}$:

$$W_O = \begin{pmatrix} W_{\tilde{v}} & W_{\tilde{v},\tilde{p}} & 0 \\ W_{\tilde{v},\tilde{p}} & W_{\tilde{p}} & 0 \\ 0 & 0 & W_{\tilde{x}} \end{pmatrix}.$$ 

(3.9)

The elements of $W_O$ are provided by the reconstruction of the particle and the primary vertex.

After minimization, the second-order derivatives of the $\chi^2$ with respect to a parameter equals the error on that parameter. Figure 3.7 shows the reconstructed decay time $t_{rec}$ and estimated error $\sigma_{t_{rec}}$ distributions.

![Figure 3.7: Reconstructed proper-time (left) and estimate proper-time error (right) distributions.](image)

### 3.6 Resolution models

The resolution model $R(x)$ describes the residual distribution for the proper-time: $x^i = t_{rec}^i - t_{true}^i$, conditional with respect to the per-event-errors. In the absence of a bias in
the detector and reconstruction procedure, the central limit theorem states that $R(x)$ is approximated by a single Gaussian:

$$R(t_{\text{rec}} - t_{\text{true}} = x, \sigma_{t_{\text{rec}}}) = e^{-\frac{1}{2} \left( \frac{x - t_{\text{rec}}}{{\sigma_{t_{\text{rec}}}}} \right)^2} \sqrt{2\pi} \sigma_{t_{\text{rec}}}, \quad (3.10)$$

in case the many different noise factors that contribute to a non-zero residual are independent. For the LHCb experiment this is a fair approximation, but since the observation of a time-dependent CP-asymmetry necessitates precise knowledge of $R(x)$, more variables are introduced to describe the residual distribution more precisely.

An extra parameter $\mu$ to account for a bias is introduced and a scale factor $SF$ as an overall correction for the per-event-errors. Furthermore $R(x)$ is given more degrees of freedom as a sum of two Gaussians:

$$R(t_{\text{rec}} - t_{\text{true}} = x, \sigma_{t_{\text{rec}}}) = f_1 e^{-\frac{1}{2} \left( \frac{x - \mu_1}{\sigma_{t_{\text{rec}}}} \right)^2} + \frac{1}{\sqrt{2\pi}SF_1 \sigma_{t_{\text{rec}}}} + (1 - f_1) e^{-\frac{1}{2} \left( \frac{x - \mu_2}{\sigma_{t_{\text{rec}}}} \right)^2} \sqrt{2\pi}SF_1 \sigma_{t_{\text{rec}}}, \quad (3.11)$$

or three Gaussians:

$$R(x, \sigma_{t_{\text{rec}}}) = f_1 e^{-\frac{1}{2} \left( \frac{x - \mu_1}{\sigma_{t_{\text{rec}}}} \right)^2} \sqrt{2\pi}SF_1 \sigma_{t_{\text{rec}}} + f_2 e^{-\frac{1}{2} \left( \frac{x - \mu_2}{\sigma_{t_{\text{rec}}}} \right)^2} \sqrt{2\pi}SF_2 \sigma_{t_{\text{rec}}} + (1 - f_1 - f_2) e^{-\frac{1}{2} \left( \frac{x - \mu_3}{\sigma_{t_{\text{rec}}}} \right)^2} \sqrt{2\pi}SF_3 \sigma_{t_{\text{rec}}}, \quad (3.12)$$

Inspired by the data and a previous resolution study for the decays $B^+ \rightarrow J/\psi K^+$ and $B^0 \rightarrow J/\psi K^*$ [26], the following model was used as well:

$$R(x, \sigma_{t_{\text{rec}}}) = f_1 e^{-\frac{1}{2} \left( \frac{x - \mu_1}{\sigma_{t_{\text{rec}}}} \right)^2} \sqrt{2\pi}SF_1 \sigma_{t_{\text{rec}}} + (1 - f_1) \frac{1}{\sqrt{2\pi}SF_2 \sigma_{t_{\text{rec}}}} \{ e^{-\frac{1}{2} \left( \frac{x - \mu_2}{\sigma_{t_{\text{rec}}}} \right)^2} \otimes e^{\frac{x - \mu_2}{\sigma_{t_{\text{rec}}}}} \}. \quad (3.13)$$

The second term is a convolution of a Gaussian with an exponent for negative residual values. This is added to describe the non-negligible down-stream bias in the primary vertex reconstruction due to the inclusion of $B^0_s$ meson daughter particles.

The three models have 5, 8 and 6 free parameters respectively.

### 3.7 Per-event-errors

The per-event-error is an extra observable, which has to be taken into account, as well as its probability distribution [27]. In $R(x)$ the $\sigma$ parameter is substituted by the estimated proper time error for each event. $R(t_{\text{rec}} - t_{\text{true}})$ now becomes $R(t_{\text{rec}} - t_{\text{true}}, \sigma_{t_{\text{rec}}})$ which is conditional on the probability distribution of $\sigma_{t_{\text{rec}}}$, $P(\sigma_{t_{\text{rec}}})$:

$$R(t_{\text{rec}} - t_{\text{true}}, \sigma_{t_{\text{rec}}}) = R(t_{\text{rec}} - t_{\text{true}}|\sigma_{t_{\text{rec}}}) \times P(\sigma_{t_{\text{rec}}}), \quad (3.17)$$

$P(\sigma_{t_{\text{rec}}})$ is a pdf constructed from the measured $\sigma_{t_{\text{rec}}}$ values. A technique called Kernel Estimation, “KEYS”, provides an un-binned and non-parametric estimate of the pdf from
a set of data. A Gaussian is generated for each measured value, centered at this value. The sum of all Gaussians is taken as $P(\sigma_{\text{true}})$ [28]. Figure 3.8 shows the $\sigma_{\text{rec}}$ distribution together with the $P(\sigma_{\text{true}})$ pdf.

A positive correlation between $\sigma_{\text{rec}}$ and $t_{\text{rec}}$ was found [39]. The underlying process of this effect is as of yet unknown and the correlation has not been taking into account in this study.

### 3.8 Determination of parameters on residuals

After the selection of events, the true proper time distribution is sliced into nine bins chosen such that all bins have an equal amount of entries. The $t_{\text{rec}} - t_{\text{true}}$ distribution for each bin $k$ is plotted and fitted with the residual function $R(t_{\text{rec}} - t_{\text{true}}, \sigma_{\text{rec}}) \times P(\sigma_{\text{rec}})_k$ (Eq. 3.17). For each bin the parameters of $R(t_{\text{rec}} - t_{\text{true}}, \sigma_{\text{rec}})$ are obtained by a two-dimensional unbinned maximum likelihood fit to $\{(t_{\text{rec}} - t_{\text{true}})^i, \sigma_{\text{rec}}^i\}$, by maximizing the logarithmic likelihood:

$$
\log(L) = \sum_{i=1}^{N} \log R((t_{\text{rec}} - t_{\text{true}})^i, \sigma_{\text{rec}}^i) \mid \\
\mu_1, SF_1, f_1, \mu_2, SF_2, f_2, \ldots).
$$ (3.18)

Parameters for different bins are compared for constantness. As an indication for the quality of the fit, the Residuals (absolute difference between fit and data points) and Pulls (Eq. 3.3) between data points and the fit curve are plotted (Fig. 3.10) as a function of $x = t_{\text{rec}} - t_{\text{true}}$. A positive correlation is present.
Figure 3.11 shows the parameters of $R(x, \sigma)$ after the fit for the different bins in proper time. These results show no constant behavior for the resolution model and for the third, fourth and fifth proper time bins, the residual data shows an exponential tail for negative values. After separately investigating the ingredients of the proper time as in Equation 3.5, this exponential behavior showed up in the projection of the primary vertex (PV) on the momentum direction. A redefinition of the PV seems necessary and is discussed in the following section.
Figure 3.10: Residual distributions. The projections of $R(x = t_{\text{rec}} - t_{\text{true}}, \sigma)$ on $x$, are given by the curves.
Figure 3.11: $\mathcal{R}(x, \sigma)$ parameters and their errors for each proper time bin.
3.9 Redefining the primary vertex

The upstream bias of the PV as a function of proper time is caused by wrongly including the tracks of daughter particles of the $B_0^0$ meson in the construction of the PV. By including tracks that originate from the secondary vertex (SV) the PV will be pulled towards the SV, leading to shorter reconstructed proper times, as illustrated in Figure 3.12.

This phenomenon also explains the effect to occur as a function of proper time. For $t_{true} \approx 0$, the inclusion of final state particles does no harm, since the PV position roughly equals the position of the SV. A larger proper time corresponds to a longer average decay distance, decreasing the probability of the inclusion of the daughter particles. Therefore a bias is expected somewhere inbetween, as is indeed observed in Figure 3.10.

Comparison of the tracks used in the PV construction and the daughter tracks of the signal $B_0^0$ meson, indeed shows that final state tracks are included. The distribution of the number of final state tracks of the $B_0^0$ meson under study that were used in the PV construction per event is seen in Figure 3.13, as well as the number of these tracks as a function of proper time. As expected, the number of wrongly included tracks decreases for larger proper time values.

All daughter tracks of the signal $B_0^0$ meson that are used in the construction of the PV are removed from the track collection that belongs to the PV and the updated collection is used to refit the PV. After this, the residuals are plotted for the time bins and fitted with $\mathcal{R}(x, \sigma)$, see Figures 3.14 and 3.15.
Figure 3.14: Residual distributions after removal of wrongly included tracks. The projections of $\mathcal{R}(x = t_{\text{rec}} - t_{\text{true}}, \sigma)$ on $x$, are given by the curves.
Figure 3.15: $\mathcal{R}(x, \sigma)$ parameters after removal of wrongly included tracks.
3.10 Comparison of PV definitions and resolution models

The obvious bias has vanished and the parameters show a more constant behavior. Because of the correlation between the parameters this is not easy to interpret for constantness. As an easier approach to interpret, all data is used for one residual fit, conditional on the entire $\sigma_{t_{\text{rec}}}$ distribution, to provide the resolution model parameters. The obtained resolution model is made conditional on the separate $\sigma_{t_{\text{rec}}}$ distributions for each time bin. These new conditional models are superimposed on the data of the corresponding time bins. A $\chi^2$ value is calculated for each bin between the superimposed curve and the data points, divided by the number of points $N$, as a measure of how well the data is described:

$$\chi^2 = \frac{1}{N} \sum_i (\text{data}(t_{\text{true}}, i) - \text{curve}(t_{\text{rec}}))^2$$  (3.19)

The DaVinci framework provides several algorithms to redefine the PV after removal of the wrong tracks:

- A total reconstruction of the PV.
- A least square refit of the PV, minimizing the total $\chi^2$ between the PV position and the contributing tracks. In case the uncertainties on the tracks are non-biased and Gaussian distributed, this method is most accurate.
- A deterministic annealing refit of the PV. A Kalman filter weighing each track by its $\chi^2$ contribution to the PV.

An overall low $\chi^2$ value is preferred, indicating that a removal of the daughter tracks followed by a complete reconstruction and deterministic annealing refit of the PV is the favorable PV definition for all three resolution models (Fig. 3.16 and 3.17).

![Figure 3.16: $\chi^2$ values for all four PV definitions (Left) and for removed tracks only (Right). The resolution model is a sum of three Gaussians](image)

The same procedure is applied to determine which resolution model shows best results. From Figure 3.18 it is concluded that a three Gaussian resolution model is preferable. The two Gaussian models with and without an exponential component produce the same $\chi^2$
Figure 3.17: $\chi^2$ values for a resolution model consisting of two Gaussians (Left) and two Gaussians plus Exponential (Right).

values, implying both models are equal. Thus the exponential component is zero after track removal.

Figure 3.18: $\chi^2$ values for the three different models.
Figure 3.19: $\chi^2$ values for all four PV definitions (Left) and for removed tracks only (Right) for $B^0 \rightarrow J/\psi K^*$. The resolution model is a sum of three Gaussians.

For $B^0 \rightarrow J/\psi K^*$ the PV definition after track removal followed by a PV reconstruction and deterministic annealing refit (Figures 3.19).

To test whether the resolution model parameters obtained from $B^0 \rightarrow J/\psi K^*$ describe the residual distributions for $B^\ast \rightarrow J/\psi \phi$, the model is again fitted to all data for $B^0 \rightarrow J/\psi K^*$. Then a new model is created for $B^\ast \rightarrow J/\psi \phi$ with the parameters obtained from the the performed fit on $B^0 \rightarrow J/\psi K^*$ data. This new model is made conditional on the separate $\sigma_{\text{true.J/\psi \phi}}$ distributions and superimposed (not fitted) on the $B^\ast \rightarrow J/\psi \phi$ data, see Figures 3.20 and 3.21. Pulls are roughly within two standard deviations, indicating that the data is well described.
Figure 3.20: $B^0 \rightarrow J/\psi \phi$ residual data points and the resolution model from the $B^0 \rightarrow J/\psi K^*$ residual fit. Residuals and pulls are shown.
Figure 3.21: $\chi^2$ values between the $B^0 \to J/\psi\phi$ residual data points and the resolution model from the $B^0 \to J/\psi K^*$ residual fit.
3.11 Determination on reconstructed data only

The actual resolution model will be obtained from the reconstructed proper time values and their per-event-errors only. A decay pdf as function of the proper time and only $\tau_{B^0}$ as a free parameter is convoluted with the three Gaussian resolution model as function of $t_{\text{rec}}$ and $\sigma_{t_{\text{rec}}}$ only. The obtained pdf is then made conditional on the $\sigma_{t_{\text{rec}}, J/\psi K^*}$ distribution:

$$\frac{1}{\tau_{B^0}} e^{-\frac{t_{\text{rec}}}{\tau_{B^0}}} \otimes R(t_{\text{rec}}|\sigma_{t_{\text{rec}}}) \times P(\sigma_{t_{\text{rec}}})$$

Equation 3.20 is fitted to the reconstructed time to obtain the parameters (Table 3.2 and Fig. 3.22). From these parameters the new resolution models, conditional on the $\sigma_{t_{\text{rec}}}$ distributions for the appropriate time bins are constructed and compared with the $B_{s}^0 \rightarrow J/\psi \phi$ residuals, as seen in Figure 3.23.

<table>
<thead>
<tr>
<th></th>
<th>SF_1</th>
<th>SF_2</th>
<th>SF_3</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>$\mu_3$</th>
<th>$f_1$</th>
<th>$f_2$</th>
</tr>
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<td>1.902</td>
<td>25</td>
<td>$1.659 \times 10^{-3}$</td>
<td>$2.324 \times 10^{-3}$</td>
<td>0.361</td>
<td>0.900</td>
<td>0.0844</td>
</tr>
<tr>
<td>Error</td>
<td>0.022</td>
<td>0.245</td>
<td>0.068</td>
<td>$0.844 \times 10^{-3}$</td>
<td>0.011</td>
<td>0.0046</td>
<td>0.013576</td>
<td>0.0135</td>
</tr>
</tbody>
</table>

Table 3.2: fit results

The $\chi^2$ values are around 1 and the pulls are within 2 standard deviations for most bins. Therefore, it can be concluded that the resolution model can be obtained from $B_{s}^0 \rightarrow J/\psi K^*$ data to accurately describe uncertainties for $B_{s}^0 \rightarrow J/\psi \phi$.

![Figure 3.22: Fit on the reconstructed proper time distribution of the $B_{s}^0$ meson.](image)
Figure 3.23: $B^0 \rightarrow J/\psi \phi$ residual data points and the resolution model from the $B^0 \rightarrow J/\psi K^*$ fit for reconstructed proper times only, projected on the residuals. Residuals and pulls are shown.
Figure 3.24: $\chi^2$ values between the $B_0^0 \to J/\psi \phi$ residual data points and the resolution model from the $B^0 \to J/\psi K^*$ fit for reconstructed proper times only, projected on the residuals.
4 Summary and Conclusion

To correct for resolution effects on the sought-after value of $\sin 2\beta_s$, a resolution model is to be acquired from the reconstructed data of $B^0_s \to J/\psi\phi$.

Proper time values around $t_{rec} \approx 0$ provide this information, which is needed to correct for larger values as well. Therefore the resolution model at small proper time values has to provide a resolution model, representative for larger values as well. This has been studied with MC data where a bias was spotted in the residuals, varying as a function of proper time. This is caused by daughter tracks of the signal B under study, that were wrongfully taken into account in the determination of the PV.

Several re-definitions of the PV where investigated, where a total reconstruction of the PV after removal of the daughter tracks followed by an deterministic annealing refit provided the most constant resolution model and described data best.

The study was performed for three different resolution models. A sum of three Gaussians had shown to best describe the residuals.

The decay channel $B^0 \to J/\psi K^*$ has similar decay features as $B^0_s \to J/\psi\phi$ and a significant larger branching ratio. For this reason a resolution model obtained from $B^0 \to J/\psi K^*$ has been compared with $B^0_s \to J/\psi\phi$ data. The model described the $B^0_s \to J/\psi\phi$ proper time values well, such that it can be concluded that once LHCb becomes operational, this channel should be investigated to acquire the resolution parameters.
Bibliography


[22] The BOOLE Project. 

[23] The BRUNEL Project. 


