Angular orientation reconstruction of a Hall sensor calibration setup

Zdenko van Kesteren
Supervisor: Prof. dr. Frank Linde

14th December 2004
Abstract

In order to achieve precise particle momentum measurements in the ATLAS muon-spectrometer, the magnetic field in the detection volume must be known with high precision. The magnetic field will be measured by Hall probes. These sensors have to be calibrated. This is done by rotating them around two orthogonal axes in an accurately known homogeneous magnetic field. A precise knowledge of the angular orientation of the Hall probe during rotation is required. The main goal of the research described in this thesis is the determination of the angular orientation of the Hall probe and its accuracy.
Acknowledgments

Several people have been helpful and supportive during the time I have spent working on this thesis. Special thanks to my supervisor Frank Linde who guided me in the right direction constructing the analysis and writing the thesis. I would also like to thank Fred Schimmel, Henk Boterenbrood and Jaap Kuijt, for always being ready to help me and to answer my questions. I’m grateful to Martin Woudstra and Niels van Eldik who helped me out with several problems that appeared during the programming. Furthermore, I would like to thank Felix Bergsma whose ideas were the backbone of this research.

Finally, I would like to thank my supportive friends for allowing me to bore them with my enthusiasm for physics. I’m very grateful to my twin sister, who has an endless faith in me. I thank my mother, who managed to keep track of my work during this research even when living in Slovakia, always being ready for me. I’d like to thank my father for his everlasting support and his sincere curiosity in my field of research. And last but certainly not least, I’d like to say thanks to Lisa, who has been around since I’ve started studying physics, for her support.
Contents

1 Introduction ........................................ 5
  1.1 The Standard Model ............................ 5
  1.2 The LHC and its experiments ..................... 6
  1.3 ATLAS ........................................ 9
     1.3.1 The inner detector ....................... 9
     1.3.2 The calorimeters ......................... 10
     1.3.3 The muon spectrometer .................. 12
     1.3.4 Muon momentum measurement ............... 13
  1.4 The calibration of the Hall sensors ............ 13

2 The Calibration Setup ............................... 15
  2.1 Concept .................................... 15
  2.2 Embedded Local Monitor Board .................. 17
  2.3 Temperature management ....................... 17
  2.4 Rotation mechanism ........................... 18
  2.5 The magnetic field ............................ 19

3 Data Acquisition .................................... 21
  3.1 The LabVIEW interface ......................... 21
  3.2 The data files ................................ 22
     3.2.1 Move, Stop and Measure mode ............. 23
     3.2.2 Continuous Move and Measure mode ........ 26

4 Modeling the Calibration Setup .................. 30
  4.1 Theory ..................................... 30
     4.1.1 Hall effect ............................. 30
     4.1.2 Induction ............................... 30
     4.1.3 Integrating Circuit ..................... 31
  4.2 Modeling the calibration setup ................. 32
     4.2.1 Rotation system parameters .............. 33
     4.2.2 Coil probe geometry parameters .......... 34
     4.2.3 Coil electronics parameters ............. 36
  4.3 Model implementation .......................... 37
4.4 Fitting to modeled data ........................................... 39
4.5 Fitting to measured data ......................................... 40
  4.5.1 Normalizing the $\chi^2$ ..................................... 42

5 Position Reconstruction of the Hall Probe ..................... 45
  5.1 The absolute encoders analysis method ....................... 45
  5.2 Coil measurements analysis method ........................... 47

6 Results ........................................................................ 50

7 Conclusion ..................................................................... 53

8 Summary ....................................................................... 54
Chapter 1

Introduction

To explain the scope of the research described in this report, the importance of the calibration of the Hall probes will be discussed in this section. First, some words will be spent on describing the Standard Model of elementary particles. Secondly, the role of particle accelerators such as LHC[1] will be explained, as well as one of its detectors: ATLAS[2]. Finally, the need of precision measurements of the magnetic field in such experiments will be discussed, as well as the means to achieve it.

1.1 The Standard Model

Physicists in general try to describe nature as accurately as possible. Particle physicists try to do this on a fundamental level. In the past few decades a theory known as the Standard Model [3] emerged as a result of the work of many theorists and experimentalists.

The Standard Model describes three families of leptons and quarks, as shown in table 1.1. Ordinary matter is made of particles from the first generation; particles from the other two families were abundant only for a brief moment after the big bang. The quarks and leptons shown in this table are matter particles: fermions with half integer spin. The quarks come in three colour charges: red, green and blue. Each particle in this table has its own antiparticle with opposite electric charge and colour charge.

Interactions between these particles occur via the exchange of force mediator particles, bosons with integer spin as listed in table 1.2. These force mediating particles have

<table>
<thead>
<tr>
<th></th>
<th>1st generation</th>
<th>2nd generation</th>
<th>3rd generation</th>
</tr>
</thead>
<tbody>
<tr>
<td>leptons</td>
<td>q = -1</td>
<td>e^-</td>
<td>μ^-</td>
</tr>
<tr>
<td></td>
<td>q = 0</td>
<td>ν_e</td>
<td>ν_μ</td>
</tr>
<tr>
<td>quarks</td>
<td>q = +2/3</td>
<td>u u u</td>
<td>c c c</td>
</tr>
<tr>
<td></td>
<td>q = -1/3</td>
<td>d d d</td>
<td>s s s</td>
</tr>
</tbody>
</table>

Table 1.1: The Standard Model of elementary particles. q in units of the absolute electron charge.
been observed at collider experiments [4]. The photon couples to electric charge and the gluon to colour charge. To date, the Standard Model has met all experimental tests with great accuracy. Still, an important feature of the theoretical framework remains untested: the mechanism of spontaneous symmetry breaking and the mechanism via which particle masses are incorporated in the Standard Model. This mechanism demands the existence of a scalar field, the Higgs field. As all fields have a mediating particle associated with it, this Higgs field is accompanied by the Higgs particle. Observing this particle and measuring its decay characteristics would give valuable insights in the theoretical framework of particle physics.

The Standard Model requires a total of 25 input parameters: 12 masses of matter particles \( m_e, m_{\mu}, m_{\tau}, m_u, m_d, m_c, m_s, m_t, m_b, m_{\nu_e}, m_{\nu_\mu}, \text{ and } m_{\nu_\tau} \) and 8 parameters to describe the mismatch between mass eigenstates and the eigenstate of the weak interaction of the quarks (4) and leptons (4). Furthermore, one parameters is used to describe the strong coupling \( (\alpha_s) \), three for the electroweak coupling \( (\alpha, M_W \text{ and } M_Z) \) and one for the observable degree of freedom of the Higgs field, the Higgs particle mass \( (M_H) \).

### 1.2 The LHC and its experiments

Measuring the Higgs particle requires a particle accelerator with a centre-of-mass energy higher than that of the Large Electron Positron accelerator (LEP, at CERN, Geneva[4]). The LEP accelerator collided electrons and positrons up to about 105 GeV per beam. This was insufficient to discover the Higgs boson, although it gave constraints on its mass: \( 114 < m_H < 196 \text{ GeV} \) with 95% confidence level [4]. Synchrotron energy losses radiated by the accelerated electrons (and protons) made it impossible to achieve higher centre-of-mass energy.

Higher centre-of-mass energies can be obtained by colliding hadrons. Due to their higher mass the synchrotron radiation losses will not constrain the obtainable energy. The drawback is that hadrons are, in contrast to electrons, composite particles consisting of quarks and gluons (collectively called partons). Unlike the \( e^-e^+ \) collisions the centre-of-mass energy of hadron-hadron collisions is not known because of the underlying parton distributions. In addition, spectator partons will generate jets of many particles due to hadronisation. This makes hadron collisions harder to analyze than \( e^-e^+ \) collisions.

At CERN, the Large Hadron Collider (LHC) is being built in the same 26.7 km circumference tunnel LEP was operating in. The LHC will be operational in 2007. It is
an accelerator which can be operated in different modes: the LHC will accelerate proton beams to energies of 7 TeV per beam. It will also collide beams of heavy ionized nuclei (e.g. lead) with energies up to 1.25 TeV. With such high centre-of-mass energy physics beyond the Standard Model can be revealed, making the LHC a discovery machine.

In contrast to the LEP accelerator, which consisted of one ring accelerating electrons and positrons at the same time, LHC operates with two rings filled with protons. The two beams will meet at intersection points where the particles collide. The first three years the luminosity will be: $10^{33} cm^{-2}s^{-2}$. Later, the interaction luminosity will be $10^{34} cm^{-2}s^{-2}$.

At the points where the beam lines intersect several detectors will measure the collisions of the hadrons:

- **ALICE (A Large Ion Collider Experiment)**
  is a dedicated heavy-ion detector to exploit the physics of nucleus-nucleus interactions. Its aim is to study the physics of strongly interacting matter at extreme energy densities, where the formation of a new phase of matter, the quark-gluon plasma, is expected.

- **LHCb (Large Hadron Collider beauty experiment)**
  is dedicated to study CP-violation which is easiest observed in meson systems containing the b quark. Collisions at LHC will produce these mesons in large numbers.

![Figure 1.1: The LHC and its detectors.](image)
• **CMS (Compact Muon Solenoid)**
  is a general purpose detector. Its aim is to study physics involving the production of the Higgs particle.

• **ATLAS (A Toroidal LHC Apparatus)**
  is a general purpose detector devoted to study physics involving the production of the Higgs particle. In the next section ATLAS will be discussed in more detail.

![ATLAS detector diagram](image)

*Figure 1.2: The ATLAS detector. Yellow: inner detector - Green: electromagnetic calorimeter - Orange: hadronic calorimeter - Blue: muon chambers - Grey: magnet system.*
1.3 ATLAS

Figure 1.2 shows a three dimensional cut-away view of the ATLAS detector, revealing the inner parts. As most of the detectors at colliding beam experiments, ATLAS is built from several cylindrical and disk-like structures. The basic design criteria of the detector include the following:

1. Very good electromagnetic calorimetry for electron and photon identification and measurements, complemented by full-coverage hadronic calorimetry for accurate jet and missing transverse energy measurements;

2. High-precision muon momentum measurements, with the capability to guarantee accurate measurements at the highest luminosity, without using information from the inner tracking detectors;

3. Efficient tracking at high luminosity for high transverse momentum measurements, electron and photon identification, \( \tau \)-lepton and heavy-flavor identification, and full event reconstruction capability at lower luminosity;

4. Large acceptance in pseudo-rapidity \( \eta \) with almost full azimuthal angle \( \phi \) coverage. The azimuthal angle is measured around the beam axis, whereas the pseudo-rapidity relates to the polar angle \( \theta \), the angle with the \( z \) direction. The \( x-y-z \)-frame of the ATLAS detector is defined in the following way: the \( x \)-axis is in the plane of the LHC pointing to the centre of the circle. The \( y \)-axis is in the upward direction perpendicular to the plane of the LHC. The \( z \)-axis is along the beam line, in the direction fixed by the \( x \) and \( y \)-axis using a right-handed coordinate system.

1.3.1 The inner detector

The inner detector (figure 1.3) surrounds the beam line and measures the trajectories of charged particles. The trajectories are bent in a homogeneous 2 T magnetic field generated by a superconducting solenoid. The task of the inner detector is to measure the impact parameter\(^2\), vertex position and provide particle identification. The achieved transverse momentum resolution is in the order of \( \Delta p_T / p_T = 0.04\% \times p_T \oplus 2\% \) (\( p_T \) in GeV/c).

The inner detector consists of three parts: a high resolution silicon pixel detector which is organized in disk and barrel structures. Because this part is mounted nearest to the beam line, the impact parameter resolution is mainly determined by this sub-detector. It contains about \( 1.4 \times 10^8 \) pixels with a size of \( 50 \times 300 \mu m^2 \) and achieves a spatial resolution of \( 15 \mu m \).

The second part of the inner detector is a silicon strip tracker (SCT: Semi Conductor Tracker). These strips with length of 126 mm are stacked in disks in the end-caps and

\(^1\text{pseudo-rapidity } \eta \text{ is a convenient measure of the spatial angle } \theta: \eta \equiv -\ln (\tan \frac{\theta}{2})\).

\(^2\text{The impact parameters is defined as the distance by which the incident protons would have missed each other in the collision, had they continued on their original trajectory.}\)
Figure 1.3: The inner detector layout.

barrels in the central region. The spatial resolution is 23μm of an individual detector element.

The final part of the inner detector is a transition radiation tracker (TRT) surrounding the pixel and strip-detector, containing several layers of gas-filled 4 mm diameter straw drift tubes. The layers of straw tubes are interleaved with radiator material in which transition-radiation photons are produced when very relativistic particles pass through. Together with the SCT the TRT provides the aforementioned high momentum resolution for the tracks.

1.3.2 The calorimeters

Directly outside the solenoid the calorimeters are situated as shown in figure 1.4. The barrel electromagnetic calorimeter (EMCAL) has as absorber accordion shaped lead plates as passive medium with liquid argon (LAr) between the plates as active medium\(^3\). The end-cap electromagnetic calorimeter has copper as absorber in a 'Spanish fan' layout. The whole EMCAL structure covers the pseudo-rapidity regions \(|\eta| < 1.475\) (barrel) and \(1.375 < |\eta| < 3.2\) (end-caps). The energy resolution is \(\Delta E/E = 30%/\sqrt{E} \oplus 0.5\%\) (\(E\) in

---

\(^3\)In the calorimeters, passive media are used as absorbing material, active media are used to detect the traversing particles.
GeV). Presamplers\footnote{The presampler is a detector element between the inner detector and the calorimeter and provides a measurement of the energy of the particle shower. This allows correction for the energy loss inside the inner detector.} are included to improve the energy resolution and to help electron-positron identification.

The hadronic calorimeter (HCAL) consists of a barrel HCAL, hardronic end-cap calorimeters (HEC) and a forward calorimeter (FCAL). The barrel section has iron as absorber with plastic scintillator tiles as active medium. Due to the creation of jets of many particles along the beam line, the detector parts of the end-caps are subject to large amounts of radiation damage. In the end-caps and in the FCAL, more radiation hard technologies are required, as is the case for the EMCAL end-caps. For that reason the radiation-hard LAr is used here as active medium. The end-cap HCALs consist of parallel-plate copper...
absorber. The barrel will cover $|\eta| < 1.7$, the end-caps cover $1.5 < |\eta| < 3.2$ and the FCAL covers $3.1 < |\eta| < 4.9$. The energy resolution of the HCAL is $\Delta E/E = 50%/\sqrt{E} \oplus 3\%$.

1.3.3 The muon spectrometer

The ATLAS muon spectrometer is capable to measure muon momentum accurately completely independent of the inner detector [5]. The toroidal magnetic field is generated by eight large superconducting toroid coils and by the two end-cap toroids. During ATLAS operation, the position of these toroids is measured by a large number of three dimensional Hall probes mounted throughout the muon spectrometer volume. From the position of these magnets the magnetic field is calculated.

Muon momentum is measured with high precision by Monitored Drift Tube chambers (MDT chambers). In the barrel region the chambers are build up like concentric cylinders and in the end-cap regions the drift-tube chambers are placed radially (figure 1.5). In total, about 370,000 MDTs in 1200 chambers are mounted in the muon-spectrometer. A single MDT measures a point on the trajectory of a muon with a precision of about 75 $\mu$m in the bending direction. The barrel and the end-caps consist of three layers of MDT chambers to ensure the muon momentum measurement (see section 1.3.4).

Trigger chambers in the barrel region, the Resistive Plate Chambers (RPCs) and trigger chambers in the end-cap region, Thin Gap Chambers (TGCs), provide the first level trigger and the muon track coordinate in the non-bending direction. Another purpose of these chambers is to identify the bunch crossing by determining the global reference time.

Together these systems allow a muon momentum measurement with a precision ranging from $2 - 3\%$ for $10 - 200$ GeV muons, increasing to about $10\%$ for $1$ TeV muons.

Figure 1.5: Left: the barrel structure of the muon-chambers, right: the end-caps structure.
The muon spectrometer will play a crucial role in the analysis of the $H \rightarrow Z^0 Z^0 \rightarrow \mu^+ \mu^- \mu^+ \mu^-$ decay mode of the Higgs. The spectrometer is also well suited to explore physics beyond the Standard Model whenever energetic muons are involved. It is of great importance to ensure precise muon momentum measurements.

1.3.4 Muon momentum measurement

Muon momentum measurement is based on the measurement of the sagitta of the curved track of the muon. The muon crosses three layers of MDTs giving three points along the track. The sagitta is defined as the distance from the point measured in the middle station to the straight line connecting the points in the inner and outer stations. The precision of the sagitta measurement is a direct measure for the precision of the muon momentum. The actual precision depends not only on the local precision of the points measured in the muon chambers, but also on the accuracy on the relative position of the chambers and the magnetic field in the region traversed by the muon.

Alignment systems monitor the relative chamber positions with high accuracy. In order to deduce the magnetic field with high accuracy, the position of the magnets (i.e. the conductors) are monitored by a large number of three dimensional Hall sensors mounted throughout the spectrometer volume. In order to achieve precision magnetic field measurements, these Hall sensors have to be tested and calibrated. The characteristics of each Hall sensor will be stored in a database.

1.4 The calibration of the Hall sensors

The Hall sensors will be calibrated at CERN with an experimental setup of Felix Bergsma, who has developed a calibration mechanism in which a probe containing a set of Hall sensors is being rotated in a reference homogeneous magnetic field (its magnitude is monitored by a NMR$^5$). The angular position of the Hall sensor with respect to the magnetic field is monitored by measuring the induced voltage of three mutually perpendicular coils mounted on a probe which is being rotated together with the Hall sensors. At NIKHEF, another version of the calibration setup is constructed, adding additional possibilities to determine the angular orientation during rotation. The setup will be discussed in the next section.

$^5$ Nuclear Magnetic Resonance probe.
My research project was the analysis of the data acquired with the NIKHEF calibration setup and includes the determination of the angular position of the rotation. Determining the orientation and its precision requires a detailed understanding of the rotation of the probe in the setup. By describing the rotation by a model and fitting this model to data, the characteristics of the rotation can be deduced. The imperfections of the rotation mechanism are determined by taking them into account in this model. This will be discussed in section 4. The characteristics of the rotation are used to determine the angular position and its precision as will be shown in section 5.
Chapter 2

The Calibration Setup

2.1 Concept

To calibrate the 1200 3D Hall sensors a setup was built at NIKHEF based on the setup developed by Felix Bergsma at CERN[6]. Hall sensor cards are rotated in a 20 cm gap between two poles of a magnet by step motors. This is done at several values of the magnetic field and at different temperatures.

NIKHEF has built a second version of the calibration setup with servo motors instead of a stepping motors [7]. Moreover, this setup offers additional possibilities to determine the angular orientation during the rotation.

1. By means of the induced voltage in three mutually perpendicular coils. The coils are rotated in the homogeneous magnetic field and yield an output signal from which the angular orientation of the probe can be constructed. This method is also used at Felix Bergsma’s calibration setup.

2. By measuring the orientation given by absolute encoders connected to the rotation axes. This method is new compared to Felix Bergsma’s setup.

3. By using a pre-calibrated Hall-sensor card (the so-called motherboard) embedded in the rotating probe. Since the characteristics of this sensor is known, the position of the probe can be extracted from the output signal. This method was proposed by Felix Bergsma but has not been tested yet. In contrast to the other two methods, this method will not be discussed in the scope of this thesis.

The second version of the setup should be at least as accurate as the first design which achieved a resolution in the order of a few $10^{-5}$ rad. However, the new setup was not fully operational yet at the time of writing of this thesis.

A picture of the NIKHEF setup is shown in figure 2.1. In the following sections the different parts of the setup will be discussed. In order to calibrate the Hall sensors with large precision, the position and orientation with respect to the calibration magnetic field must be measured with great accuracy. The calibration includes the deduction of the
Figure 2.1: The 3D Hall sensor calibration setup. The inner picture shows the overall layout, the magnet is shown in red. The upper and lower pictures show close-up detailed parts of the setup.
characteristics of the probes such as the orientation of the Hall plates with respect to each other and temperature dependencies. The position of the probe is given by two values $AX$ and $AY$, the number of revolutions of the $x$-axis and $y$-axis, respectively. These values are read out by absolute encoders attached to the two orthogonal axes of the servomotors of the setup (section 2.4). Since the resolution of these encoders might not be sufficient to calibrate the sensors accurately enough, Felix Bergsma developed another method to measure the position of the Hall sensors. The position of the probes can be deduced from the induce voltage over three mutually perpendicular coils integrated in the rotating probe. From the known magnetic field and the measured induced voltage, the position of the probe can be calculated (see section 4).

2.2 Embedded Local Monitor Board

The B-field sensors and the ADCs of the coils are read out by an ELMB (Embedded Local Monitor Board). This system has an ATmega128L processor with a CANopen interface and was programmed by Henk Boterenbrood. The following components are connected to the ELMB (figure 2.2):

- One calibrated B-field sensor card version 2, calibrated by Felix Bergsma.
- Four B-field sensor cards (version 3). These are the chips to be calibrated (four at a time). Each chip is identified with a DS2401 chip giving an unique 64 bit code.
- Three ADCs to read out a coil. These coils are read out by an ADC type CS5524 which has a maximal sampling frequency of 120 Hz.

The B-field sensor cards each have three Hall sensors $H1$, $H2$, $H3$ and a temperature-sensor (NTC).

2.3 Temperature management

The temperature is controlled and adjusted via a RS232 interface connected to the CAN bus via a CANopen-to-RS232 interface. The temperature controller has an input for a Pt100 and an analogue input connected to the ELMB. The mean value of the NTCs of the B-field sensor cards is taken as input temperature value for the controller.

The controller has two outputs: one for cooling, one for heating. Cooling is achieved via two parallel connected Peltier elements. Heating is achieved with a 150 Watt resistor. A fan in the air-duct is used to distribute the temperature-controlled air through the ELMB. The orientation of the B-field sensor and the coil card influence the airflow throughout the probe.

The coil card is known to dissipate most of the heat due to the presence of the ELMB. In table 2.3 the temperatures measured by the different B-field sensor cards are shown. The cards closest to the coil card (see figure 2.2) measure a slight rise in temperature compared
to the other NTCs. The spread in temperature is due to the rotation-dependent efficiency of the cooling. These effects are reduced by increasing the distance of the \textit{B-field sensor Calibrated} card and the Coil ADC from 7 mm to 10.8 mm and the distance between the \textit{B-field sensor 2} card with respect to the Coil ADC from 5.8 mm to the value of 10.8 mm (see figure 2.2). Table 2.3 shows the effect of the adjustment of the probe structure. The mean values of the NTC measurements are reduced because the air reaches the coil ADC more easily; this has a similar effect on the orientation-dependency of the efficiency of the cooling resulting in a narrower spread in temperature measurements.

### 2.4 Rotation mechanism

The probe is rotated around two orthogonal axes perpendicular to the magnetic field by two DC servo motors. The servo motors are controlled by a motor-controller of DC systems and the controller is connected to the CAN bus. The DC motors have their own relative position encoders with 512 positions per rotation. The motors rotate the two axes via a 1 to 180 delay, i.e. if the motor makes one turn, the axis makes 180 turns. One axis makes

![Image of the probe and its dimensions in the old and new configuration](image)

Figure 2.2: The probe and its dimensions in the old and new configuration; distance between the cards given in mm.
Table 2.1: The temperature read out (in °C) by the different NTCs at a set temperature of 20° C. The left column shows the results of temperature measurements in the old geometry in which the distance from coil card to Hall card is about 7.0 and 5.8 mm. In the new geometry (shown in the right column) these distances have been increased to 10.8 mm.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Old mean</th>
<th>Old σ</th>
<th>New mean</th>
<th>New σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor 0</td>
<td>20.47</td>
<td>0.076</td>
<td>19.38</td>
<td>0.020</td>
</tr>
<tr>
<td>Sensor 1</td>
<td>21.69</td>
<td>0.204</td>
<td>19.79</td>
<td>0.022</td>
</tr>
<tr>
<td>Sensor 2</td>
<td>22.03</td>
<td>0.038</td>
<td>19.86</td>
<td>0.038</td>
</tr>
<tr>
<td>Sensor 3</td>
<td>19.90</td>
<td>0.085</td>
<td>19.12</td>
<td>0.044</td>
</tr>
<tr>
<td>Calibrated sensor</td>
<td>20.79</td>
<td>0.049</td>
<td>19.49</td>
<td>0.025</td>
</tr>
</tbody>
</table>

four revolutions during the calibration, the other five.

The X-axis is connected to the Y-axis. Due to the rotation of the Y-axis an apparent rotation of the X-axis appears which has to be taken into account. In the data files the values of the absolute encoders of the X-axis are stored, these have to be corrected according to formula 2.1:

\[
AX_{actual} = AX_{readout} + \frac{AY}{180}
\]  

(2.1)

where \(AX_{readout}\) is the value registered by the absolute encoder of axis X, \(AY\) the value of the absolute encoder of the Y axis and \(AX_{actual}\) the physical position of the X axis. The X-axis rotates clockwise and the Y-axis counterclockwise, this causes the sign of the correction in formula 2.1 to be positive.

2.5 The magnetic field

At CERN, the magnetic field is generated by a magnet, which can reach values up to 1.2 Tesla. The field is monitored by a NMR of type SENTEC (NMR Tesla meter 1101). The magnet is chosen for the large aperture between the pole shoes and the homogeneity of the field it produces (see figure 2.3). The homogeneity is perfect in the z-direction (perpendicular to the magnet poles) but not entirely homogeneous in the other two directions. The coils were arranged on the card in such a way that the three of them fit within about 20 x 20 mm. The centre of rotation was positioned in such a way that the coils rotated within a sphere of radius \(\approx 12\) mm. Across this radius the magnetic field deviated about \(\Delta B_z \approx 6.0 \times 10^{-6}\) Tesla (along the axis of the magnet) and \(\Delta B_y \approx 3.5 \times 10^{-5}\) Tesla.
Figure 2.3: The strength of the magnetic field as function of the position in the magnet gap.
Chapter 3

Data Acquisition

3.1 The LabVIEW interface

Figure 3.1: The interface of the LabVIEW program used to calibrate the B-sensors.
CHAPTER 3. DATA ACQUISITION

The readout of the ELMB is developed by Henk Boterenbrood. The data-acquisition and interface is written in LabVIEW [8] by Fred Schimmel. The graphical user interface is shown in figure 3.1. The upper left part shows the measurement conditions and allows the user to choose the measuring mode, whether or not to take data during backward rotation and to set to the values of the temperature and magnetic field during data taking. The matrix structure shows the status of the different runs.

The upper right part shows the output of one of the channels in two ways. The left graph shows the output of a channel in a blue-to-red scale as function of the cosine of the absolute encoder readout of both axes. The graph at the right shows the output of the channel as function of the readout of the x-axis. The middle part of the user interface monitors the stability of the magnetic field measured by the NMR and the temperature given by the mean of the five NTCs on the Hall-sensor cards.

The rest of the interface shows the identification of the different Hall-sensor cards, the readout of the absolute encoders and actual information of the run. The name of the file in which the data must be stored and the name of the log file can be given as well.

3.2 The data files

A B-field sensor/Coil-ADC measurement consists of a sequence of conversion commands, to switch between the different channels and channel read-out operations [9]. The calibration setup allows two types of measurements: Move, Stop and Measure mode (MSM) and the Continuous Move and Measure (CMM) mode. The measurement cycles of these data files will be explained in the next two sections.

The two types of data files have identical headers, in which the following is documented:

- The version of the read-out program;
- The time at which the read-out commenced;
- The measuring mode;
- The ID-numbers of the cards to be calibrated.

After the header the data at a user-defined B- and T-value is recorded. During the rotation of the axes, data is taken by a number of measurement cycles. The number depends on the sampling rate and the speed of rotation of the axes. After the run, the axes counter-rotate in order to get to the start position and unwind the cables connected to the ELMB. During this reverse motion, a data run can be taken as well. The data from both these runs account for one element in the “matrix” in the user interface discussed in the previous section. The data of each run is a block with in each column a channel readout and each row a measurement cycle.

Prior to the block of data (in forward mode), the air pressure and the relative humidity of the air at the time that a measurement run commences are recorded.
3.2.1 Move, Stop and Measure mode

The Move, Stop and Measure measurement cycle goes in several steps, yielding the format of the datafile:

1. The servomotors move the probe into position
2. Send a SYNC message triggering absolute encoder read out
   \( \text{AX-}x \ \text{AY-}x \)
3. Read out all Coil channels and send
   \( \text{C1-}x \ \text{C2-}x \ \text{C3-}x \)
4. Read out all B-sensor module H1 Hall-sensor channels and send
   \( \text{H1-}x \ \text{H2-}x \ \text{H3-}x \ \text{H4-}x \ \text{H5-}x \)
5. Send a SYNC message triggering absolute encoder read out
   \( \text{AX-}y \ \text{AY-}y \)
6. Read out all Coil channels and send
   \( \text{C1-}y \ \text{C2-}y \ \text{C3-}y \)
7. Read out all B-sensor module H2 Hall-sensor channels and send
   \( \text{H1-}y \ \text{H2-}y \ \text{H3-}y \ \text{H4-}y \ \text{H5-}y \)
8. Send a SYNC message triggering absolute encoder read out
   \( \text{AX-}z \ \text{AY-}z \)
9. Read out all Coil channels and send
   \( \text{C1-}z \ \text{C2-}z \ \text{C3-}z \)
10. Read out all B-sensor module H3 Hall-sensor channels and send
    \( \text{H1-}z \ \text{H2-}z \ \text{H3-}z \ \text{H4-}z \ \text{H5-}z \)
11. Send a SYNC message triggering absolute encoder read out
    \( \text{AX-T} \ \text{AY-T} \)
12. Read out all Coil channels and send
    \( \text{C1-T} \ \text{C2-T} \ \text{C3-T} \)
13. Read out all B-sensor module T-sensor channels and send
    \( \text{H1-T} \ \text{H2-T} \ \text{H3-T} \ \text{H4-T} \ \text{H5-T} \)
→ repeat cycle
1. The servomotors move the probe into position
2. Send a SYNC message triggering absolute encoder read out
   \( \text{AX-}x \ \text{AY-}x \)

\[ \downarrow \]

An example of a Move Stop and Measure-datafile is shown in figures 3.2 and 3.3. The left part of the plots is the full run while the right part shows a detailed part of the run. The full run lasts about 27 seconds, the zoomed part 5 seconds. In figure 3.2, the upper three plots show the output on the Hall-channels, the number of ADC counts as a function of time. The shape of the plots is characteristic for the projection of the magnetic field in the \( z \)-direction on the Hall-plates in the probe. The superimposed sine-shape is due to the rotation around two axes whose angular frequencies are different. One Hall-plate is more or less aligned with one axes, causing the rotations not to be superimposed. Hall-probe
Figure 3.2: Graphs of the output channels read out from a Move Stop and Measure-datafile. Number of ADC counts versus time.
3.2. THE DATA FILES

Figure 3.3: Graphs of the output channels read out from a Move Stop and Measure datafile. Number of rotations (upper plot), temperature in °C (middle plot) and magnetic field strength in T (lower plot) as function of time.

number 1 inhibits this feature.

The coil ADC-output is not meant to be measured during a MSM-measurement since the motion stops during the measurement. The coil ADC channels are read out anyway. In the lower three plots of figure 3.2, the residual induced signal is shown. The coils are measured four times during one measurement cycle; one can see that the signal is strongest in the first measurement and lowest in the fourth, creating the see-saw shape on a rudimentary double-sine plot.

The absolute encoders are read out four times per measurement cycle, the signal is shown in the first plot of figure 3.3. The second plot of figure 3.3 shows the signal from the NTCs which are read out once every cycle. A slight difference in temperature as function of time can be seen in the signal, which is caused by the orientation-dependent efficiency of cooling of the ELMB due to the air-circulation in the probe. The last plot of figure 3.3 shows the value of the magnetic field which is constant within $10^{-5}$ during the measurement run.
3.2.2 Continuous Move and Measure mode

A Continuous Move and Measure measurement cycle goes as follows:

1. Send a SYNC message triggering absolute encoder read out
   \[ \text{AX-x} \text{ AY-x} \]
2. Read out all Coil channels and send
   \[ \text{C1-x} \text{ C2-x} \text{ C3-x} \]
3. Read out all B-sensor module H1
   Hall-sensor channels and send
   \[ \text{H1-x} \text{ H2-x} \text{ H3-x} \text{ H4-x} \text{ H5-x} \]
4. Send a SYNC message triggering absolute encoder read out
   \[ \text{AX-xb} \text{ AY-xb} \]
5. Read out all Coil channels and send
   \[ \text{C1-xb} \text{ C2-xb} \text{ C3-xb} \]
6. Send a SYNC message triggering absolute encoder read out
   \[ \text{AX-y} \text{ AY-y} \]
7. Read out all Coil channels and send
   \[ \text{C1-y} \text{ C2-y} \text{ C3-y} \]
8. Read out all B-sensor module H2
   Hall-sensor channels and send
   \[ \text{H1-y} \text{ H2-y} \text{ H3-y} \text{ H4-y} \text{ H5-y} \]
9. Send a SYNC message triggering absolute
    \[ \text{AX-yb} \text{ AY-yb} \]
10. Read out all Coil channels and send
    \[ \text{C1-yb} \text{ C2-yb} \text{ C3-yb} \]
11. Send a SYNC message triggering absolute encoder read out
    \[ \text{AX-z} \text{ AY-z} \]
12. Read out all Coil channels and send
    \[ \text{C1-z} \text{ C2-z} \text{ C3-z} \]
13. Read out all B-sensor module H3
    Hall-sensor channels and send
    \[ \text{H1-z} \text{ H2-z} \text{ H3-z} \text{ H4-z} \text{ H5-z} \]
14. Send a SYNC message triggering absolute encoder read out
    \[ \text{AX-zb} \text{ AY-zb} \]
15. Read out all Coil channels and send
    \[ \text{C1-zb} \text{ C2-zb} \text{ C3-zb} \]
16. Send a SYNC message triggering absolute encoder read out
    \[ \text{AX-T} \text{ AY-T} \]
17. Read out all Coil channels and send
    \[ \text{C1-T} \text{ C2-T} \text{ C3-T} \]
18. Read out all B-sensor module T-sensor channels and send
    \[ \text{H1-T} \text{ H2-T} \text{ H3-T} \text{ H4-T} \text{ H5-T} \]
19. Send a SYNC message triggering absolute encoder read out
    \[ \text{AX-Tb} \text{ AY-Tb} \]
20. Read out all Coil channels and send
    \[ \rightarrow \text{repeat cycle} \]

1. Send a SYNC message triggering absolute encoder read out
   \[ \downarrow \]
   \[ \text{AX-x} \text{ AY-x} \]

There is always an extra set of Coil-ADC data between subsequent B-field sensor ADC readout; this extra period is used to initiate the next B-field sensor ADC conversions,
which are then read out after the next conversion of the Coil-ADC completes.

Figures 3.4 and 3.5 show the plots of the several channels in the Continuous Move Measure-datafile. In figure 3.4, the right column shows a zoomed in part of the entire range of one run (on the left). The detailed part covers 5 seconds, the full run takes about 180 seconds. The upper three plots show the three Hall channel signals.

The lower three plots of figure 3.4 show the signal from the three coil ADCs, ADC counts as function of time. Again, the superimposed sine characteristic can be seen, as well as the alignment of one of the coils with one axis. The coil ADCs are read out eight times more often than the Hall probes since one Hall signal readout is accompanied with $4 \times 2$ coil measurements.

The absolute encoders are read out at the same rate as the coil ADCs, as shown in the upper plot of figure 3.5. The NTCs are read out at the same rate as the Hall sensors and shown in the middle plot of figure 3.5. The lower plot of figure 3.5 shows the value of the magnetic field which is held constant with a precision of $10^{-5}$ during the measurement run.

Most of the data taken in the period of December 2003 until September 2004 appeared not to be usable due to jitters in the data. The measurement cycles inhibit an eight-point periodic hitch which makes fitting a model to the data difficult. One datafile, SensCal_20040329-1530.txt, is not jittered and consists of 24 measurement runs. Data from these runs have been used in the analysis described in section 4. However, if a NaN-value is encountered in a data run (Not a Number) the run is considered not valid, since the absence of measured values indicate an error in the measurement cycle. An error in the cycle can propagate throughout the data run, messing up the order shown in the former two sections. Non-valid data runs are not used in the following analysis.
Figure 3.4: Graphs of the output channels read out from a Continuous Move Measure-datafile. Number of ADC counts versus time.
Figure 3.5: Graphs of the output channels read out from a Continuous Move Measure-datafile. Number of rotations (upper plot), temperature in °C (middle plot) and magnetic field strength in T (lower plot) as function of time.
Chapter 4

Modeling the Calibration Setup

To determine the angular orientation of the Hall probe, a model has been developed to
describe the rotation in a homogeneous magnetic field of the coils in the calibration setup
described in section 2. The model describes the behaviour of the coils rotating in a homo-
genous magnetic field and is fitted to the data from three coils in the Hall probe. In this
section the details of the model and the fit procedure are described. The quality of the fit
procedure is tested on simulated data.

4.1 Theory

First, in this section the principles of the Hall probes in the calibration setup will be
explained. Secondly, the theory describing coils rotating in a homogeneous magnetic field
will be discussed in the following sections.

4.1.1 Hall effect

When an electric current $I$ is passing through a metal plate which is placed in a magnetic
field $B_\perp$ perpendicular to the plate, a potential difference appears between opposite points
on the edges of the plate. This Hall voltage $V_H$ can be written in terms of the current:

$$ V_H = \frac{I B_\perp}{q n d} $$  (4.1)

where $q$ is the charge of the carrier, $n$ the density of charge carriers in the metal plate and $d$
the thickness of the plate. The Hall voltage is proportional to the magnetic field component
perpendicular to the plane of the current. All three components of the magnetic field can
be measured by measuring the Hall voltages on three perpendicular plates.

4.1.2 Induction

Electromagnetic induction is a phenomenon which was discovered around 1830 by Micheal
Faraday and Joseph Henry. The Faraday-Henry law of induction states that in any closed
circuit placed in a time-dependent magnetic field there is an induced electromotive force whose value is equal to the negative of the time rate of change of the magnetic flux through the circuit [10].

The magnetic flux through an area is given by:

\[ \Phi = \int_A \vec{B} \cdot d\vec{A} \quad (4.2) \]

Where \( \vec{B} \) is the magnetic field and \( \vec{A} \) the vector describing the area enclosing the magnetic field (\( \vec{A} \equiv A\vec{n} \), where \( \vec{n} \) is the normal vector of the surface of the area enclosed by the windings of the coils). When a coil with \( N \) windings with each winding having an area \( A \) is being rotated in a constant magnetic field \( B \), a voltage will be induced over the coil:

\[ V_{\text{ind}} = -\frac{d\Phi}{dt} \quad (4.3) \]
\[ = -N B A \frac{d}{dt} \cos(\theta(t)) \quad (4.4) \]

Where \( V_{\text{ind}} \) is the induced voltage over the coil and \( d\Phi/dt \) the change in the magnetic flux. The angle between the magnetic field lines and the normal of the coil is given by \( \theta(t) \), as is shown in figure 4.1.

![Figure 4.1: A coil in a homogeneous magnetic field.](image)

### 4.1.3 Integrating Circuit

The readout of the induced voltage over the coils will be measured over an integrating circuit to reduce high-frequency tremors and jitters. In this section the mathematical description of the RC circuit, as shown in figure 4.2, will be discussed [11].

Kirchhoff law states that the voltages over the components of the circuit adds up to zero: \( V_R(t) + V_C(t) + V_{\text{ind}}(t) = 0 \), where \( V_R(t) \) is the voltage over the resistor, \( V_C(t) \) the voltage over the capacitor and \( V_{\text{ind}}(t) \) the (induced) voltage over the coils. \( V_R(t) = R \cdot I(t) \) and \( I(t) \) can be expressed in terms of the voltage over the capacitor: \( I(t) = dQ(t)/dt = C dV_C(t)/dt \).

To calculate the voltage over the capacitor the following differential equation has to be solved:

\[ RC \frac{dV_C(t)}{dt} + V_C(t) = -V_{\text{ind}}(t) \quad (4.5) \]

Solving the homogeneous differential equation \( (V_{\text{ind}}(t) \equiv 0) \) gives:

\[ \frac{dV_C(t)}{dt} = -\frac{V_C(t)}{RC} \Rightarrow V_C(t) = Ae^{-t/RC} \quad (4.6) \]
To solve the inhomogeneous differential equation the result of 4.6 is used, assuming that \( \alpha \) is time-dependent (\( \alpha(t) \)). The time-derivative of \( V_C(t) \) will be:

\[
\frac{dV_C(t)}{dt} = e^{-\frac{t}{RC}} \frac{d\alpha(t)}{dt} - \frac{\alpha(t)}{RC} e^{-\frac{t}{RC}} \tag{4.7}
\]

Substituting 4.7 in the inhomogeneous differential equation 4.5 gives rise to an expression for \( \alpha(t) \):

\[
RC \frac{d\alpha(t)}{dt} e^{-\frac{t}{RC}} = -V_{\text{ind}}(t) \rightarrow \tag{4.8}
\]

\[
\frac{d\alpha(t)}{dt} = -\frac{e^{-\frac{t}{RC}}}{RC} V_{\text{ind}}(t) \rightarrow \tag{4.9}
\]

\[
\alpha(t) = \alpha_0 - \int_0^t \frac{e^{-\frac{\tau}{RC}}}{RC} V_{\text{ind}}(\tau) \, d\tau \nonumber
\]

With \( V_C(t) = \alpha(t) e^{-\frac{t}{RC}} \) and choosing an appropriate integration constant an expression for the voltage over the capacitor is found (with \( V_C(t = 0) \equiv V_0 \)):

\[
V_C(t) = V_0 e^{-\frac{t}{RC}} - \frac{e^{-\frac{t}{RC}}}{RC} \int_0^t V_{\text{ind}}(\tau) e^{-\frac{\tau}{RC}} \, d\tau \tag{4.11}
\]

The first term of this equations will disappear when measurements are taken at times \( t \gg RC \).

### 4.2 Modeling the calibration setup

In the model, each coil is represented by a unit vector normal to the area enclosed by the windings of the coil. Note that the normal vectors of the three coils in the probe are independent of eachother and almost, but not exactly, orthogonal. To the Hall probe, two rotation axes are assigned, representing the rotation driven by the servo motors in the calibration setup. The rotation of a coil in the homogeneous magnetic field is modeled as the rotation of the coil normal vector around each of the rotation axes.

The projection of the magnetic field on the unit-vectors gives a measure for the magnetic flux through the coils. The time-derivative gives a measure of the voltage measured by the coils. The coils are integrated in a RC-circuit from which the voltage over the capacitor will be shown in the data files (see formula 4.11).

The following nomenclature is used (see figure 4.3):

- The \( z \)-axis is defined as the direction of the magnetic field:

\[
\vec{B} \equiv B \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{4.12}
\]
the $y$-axis is perpendicular on the $z$-axis in the plane of $\vec{B}$ and the first rotation axis ($\vec{\Omega}_1$). The $x$-axis is defined by the $y$ and $z$-axis in a right-handed coordinate system.

- The rotational axes are denoted by $\vec{\Omega}_1$ and $\vec{\Omega}_2$, with corresponding angular velocities $\Omega_1$ and $\Omega_2$.

- The normal vectors of the coils are denoted as $\hat{n}_1$, $\hat{n}_2$ and $\hat{n}_3$.

The parameters enter the model in three different ways: the description of the rotation system, the geometry of the coil probe and the electronics of the integrating circuit of the individual coils.

The flux through the coils depends solely on the orientation of the coils since the magnetic field is homogeneous and constant in time and position. The orientations of the coils are described by the normals of the coils. The area of the coils and the number of windings will determine the signal amplitude, together with the magnitude of the magnetic field and the electronic gain.

### 4.2.1 Rotation system parameters

The measured signal is proportional to the rate of change of the magnetic flux in a coil (formula 4.4). The time-dependence of the orientation of the coils is given by formula 4.13.
describing the rotation of \( \hat{n}_i \) around \( \Omega_1 \) and \( \Omega_2 \):

\[
\hat{n}_i(t) = R_{\Omega_1}(\Omega_1 t) R_{\Omega_2}(\Omega_2 t) \hat{n}_i(t = 0)
\]  

(4.13)

where \( R_{\Omega_1} \) denotes the rotation around the \( \Omega_1 \)-axis and \( R_{\Omega_2} \) the rotation around the \( \Omega_2 \)-axis. The rotation of a coil around \( \Omega_1 \) is constructed by three rotations: first, a rotation of the \( \Omega_1 \)-axis around the \( x \)-axis \( (R_x(-\Theta_1)) \) followed by a rotation around the \( y \)-axis \( (R_y(\Omega_1 t)) \). The final rotation is around the \( x \)-axis in the opposite direction \( (R_x(\Theta_1)) \):

\[
R_{\Omega_1}(\Omega_1 t) = R_x(\Theta_1) R_y(\Omega_1 t) R_x(-\Theta_1)
\]  

(4.14)

A similar operation is done to construct the rotation of a coil around the \( \Omega_2 \)-axis, although the transformation to the \( x \)-axis invokes two angles \( (R_y(\Theta_2) \) and \( R_x(\Phi_2) \)), as shown in figure 4.3 and formula 4.15, since the \( x \)-axis is not in the plane of \( \vec{B} \) and \( \vec{\Omega}_2 \):

\[
R_{\Omega_2}(\Omega_2 t) = R_y(\Theta_2) R_x(\Phi_2) R_x(\Omega_2 t) R_x(-\Phi_2) R_y(-\Theta_2)
\]  

(4.15)

The rotation matrices \( R \) are:

\[
R_x(\alpha) = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos(\alpha) & \sin(\alpha) \\
0 & -\sin(\alpha) & \cos(\alpha)
\end{pmatrix}
\]  

(4.16)

\[
R_y(\alpha) = \begin{pmatrix}
\cos(\alpha) & 0 & -\sin(\alpha) \\
0 & 1 & 0 \\
\sin(\alpha) & 0 & \cos(\alpha)
\end{pmatrix}
\]  

(4.17)

\[
R_z(\alpha) = \begin{pmatrix}
\cos(\alpha) & \sin(\alpha) & 0 \\
-\sin(\alpha) & \cos(\alpha) & 0 \\
0 & 0 & 1
\end{pmatrix}
\]  

(4.18)

4.2.2 Coil probe geometry parameters

The coils are mounted on the probe with their normal vectors \( \hat{n}_i \) almost mutually perpendicular. The start position of the normal vectors is described by two sets of angles. One set, the orthogonality-angles, are used to define the orientation of the vectors \( \hat{n}_i \) with respect to each other. The other set of three angles describes the rotation of the ensemble of normal vectors \( \hat{n}_i \) in the \( x-y-z \)-frame. The orthogonality-angles are given by:

\[
cos(\theta_{12}) = \hat{n}_1 \cdot \hat{n}_2
\]  

(4.19)

\[
cos(\theta_{13}) = \hat{n}_1 \cdot \hat{n}_3
\]  

(4.20)

\[
cos(\theta_{23}) = \hat{n}_2 \cdot \hat{n}_3
\]  

(4.21)

The \( \hat{n}_1 \)-vector is aligned with the \( x \)-axis as starting point for one coil-normal. The other two vectors (the normals of the other two coils) are defined by use of the orthogonality-angles
(see figure 4.4):

\[
\hat{n}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} 
\]

\[
\hat{n}_2 = \begin{pmatrix} \cos(\theta_{12}) \\ \sin(\theta_{12}) \\ 0 \end{pmatrix} 
\]

\[
\hat{n}_3 = \begin{pmatrix} \cos(\theta_{13}) \\ \frac{\cos(\theta_{23}) - \cos(\theta_{12})\cos(\theta_{13})}{\sin(\theta_{13})} \\ \sqrt{1 - \cos^2(\theta_{13}) - \left[ \frac{\cos(\theta_{23}) - \cos(\theta_{12})\cos(\theta_{13})}{\sin(\theta_{13})} \right]^2} \end{pmatrix} 
\]

The set of unit-vectors \((\hat{n}_1, \hat{n}_2, \hat{n}_3)\) are given a starting position in space by a rotation around the \(\hat{n}_1\) unit vector (\(\equiv\) a rotation around the \(x\)-axis), followed by a transformation assigning \(\hat{n}_1\) a position to in space (see figure 4.5). The whole transformation which is used to give the set of coil-normals at start position is:

Figure 4.4: The definition of the unit-vectors and their orthogonality used in the model.
Figure 4.5: A start position is given to the coil probe by first rotating around \( n_1 (\equiv x\text{-axis}) \), then by giving \( n_1 \) a position in space.

\[
\begin{align*}
n_i(t = 0) &= R_{\text{start position}} n_i \\
R_{\text{start position}} &= R_y(\theta_1) R_z(\phi_1) R_x(\theta_2)
\end{align*}
\]

### 4.2.3 Coil electronics parameters

By differentiating expression 4.13 with respect to time, the change of flux in the coils is calculated. The induced voltage is given by the contraction of this quantity with the magnetic field:

\[
V_{\text{ind}}(t) = |\vec{A}| \cdot \frac{d\vec{n}(t)}{dt}
\]

where \(|\vec{A}|\) denotes the area of the coil. The induced voltage is proportional to the area of the coil. An integrating RC circuit is used to smooth the signal. The RC time of the circuit is approximately 1 second (the precise value is a free parameter in the model). The voltage over the capacitor in the circuit is measured (see 4.11) and the value is stored in the data file. The voltage is given by:

\[
V_C(t) = \mathcal{P} - \mathcal{G} \times e^{\frac{t}{\tau}} \int_{t_{\text{window}}}^{t} V_{\text{ind}}(t') e^{\frac{t'}{\tau}} dt'
\]

where \( \tau_i \) is the RC-time-constant of the integrating circuit of coil \( i \). Parameter \textit{mathcal P}_i is the electronic pedestal value and \( \mathcal{G}_i \) is the electronical gain factor of coil \( i \). The proportionality of the area of coil \( A_i \) with the voltage is included in the gain factor. Formula 4.28 is only valid when measuring at times \( t > > \tau_i \) s to get rid of 'memory charges' on the capacitor (the first term in formula 4.11 will then disappear). In the same fashion, the value of \textit{window} will be taken larger than \( \tau_i \).

Finally, in the model, the measured voltage for coil \( i \) depends on many parameters:

\[
V_i(t) = V_i(t; |\vec{B}|, \mathcal{G}_i, \mathcal{P}_i, \tau_i, \Theta_1, \Theta_2, \Phi_2, \Omega_2, \Omega_1, \theta_{12}, \theta_{13}, \theta_{23}, \phi_1, \phi_2)
\]
i.e. the output of the calculated ADC voltage of a certain coil depends on coil-specific parameters as well as on general parameters. A summary of all the parameters in the model is given by Table 4.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>quantity</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotation system</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Theta_1$, $\Omega_1$</td>
<td>2</td>
<td>parameters to describe one rotation axis: $\Theta_1$ the angle between $\Omega_1$ and the y-axis, $\Omega_1$ the angular velocity of the rotation.</td>
</tr>
<tr>
<td>$\Theta_2$, $\Phi_2$, $\Omega_2$</td>
<td>3</td>
<td>parameters to describe the other rotation axis: $\Theta_2$ and $\Phi_2$ the angles between $\Omega_2$ and the x-axis, $\Omega_2$ the angular velocity of the rotation.</td>
</tr>
<tr>
<td>Coils geometry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_{12}$</td>
<td>1</td>
<td>the angle between $\tilde{n}_1$ and $\tilde{n}_2$</td>
</tr>
<tr>
<td>$\theta_{13}$</td>
<td>1</td>
<td>the angle between $\tilde{n}_1$ and $\tilde{n}_3$</td>
</tr>
<tr>
<td>$\theta_{23}$</td>
<td>1</td>
<td>the angle between $\tilde{n}_2$ and $\tilde{n}_3$</td>
</tr>
<tr>
<td>$\theta_1$, $\phi_1$, $\theta_2$</td>
<td>3</td>
<td>three angles describing the start-orientation of the coil ensemble in space.</td>
</tr>
<tr>
<td>Coil electronics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_i$</td>
<td>3</td>
<td>the gain factor between the flux change and the induced voltage of coil $i$.</td>
</tr>
<tr>
<td>$P_i$</td>
<td>3</td>
<td>the pedestal voltage on the ADC of coil $i$.</td>
</tr>
<tr>
<td>$\tau_i$</td>
<td>3</td>
<td>the RC-times of the circuits in the readout of the coil $i$.</td>
</tr>
<tr>
<td>total</td>
<td>20</td>
<td>parameters to describe the coils and rotation system</td>
</tr>
</tbody>
</table>

Table 4.1: The twenty parameters used to describe the coils and rotation system.

### 4.3 Model implementation

The model gives rise to three formulas describing the measured voltages $V_1(t)$, $V_2(t)$, $V_3(t)$ which have to be compared with three corresponding datasets $C_1(t)$, $C_2(t)$, $C_3(t)$. The model is implemented in a C++ class Model. This class uses the HepMatrix class from the CLHEP-library [12] to describe the rotation-matrices that rotate the three unit vectors $\tilde{n}_i$, for which the HepVector class from the same library is used. The parameters are given as input stored in a HepVector class. The Model class uses several member functions in order to yield a calculated voltage, given a set of parameters:

- **Model::Evaluate(t)** returns the orientation of the normal vectors of the coils at time $t$ contracted with the magnetic field, giving the flux.
• **Model::Differentiate** \((t, \delta t)\) returns the change in flux at time \(t\), the numerical differentiation performed according to ([13], 5.7.5):

\[
\frac{f(t + \delta t) - f(t - \delta t)}{2\delta t} \approx \frac{df}{dt} \tag{4.30}
\]

• **Model::Integrate** \((t_{\text{min}}, t_{\text{max}})\) returns the value of the voltage over the capacitor in the RC-circuit (as given in formula 4.28) at time \(t_{\text{max}}\), with \(t_{\text{max}} - t_{\text{min}} \gg \tau_{\text{RC}}, t_{\text{max}} - t_{\text{min}} = 15\) s. To calculate the integral numerically, the following algorithm ([13], 4.1.11) is used:

\[
h\left[\frac{1}{2}f(t_{\text{min}}) + f(t_{\text{min}} + h) + f(t_{\text{min}} + 2h) + \cdots\right] + f(t_{\text{max}} - 2h) + f(t_{\text{max}} - h) + \frac{1}{2}f(t_{\text{max}}) \approx \int_{t_{\text{min}}}^{t_{\text{max}}} f(t)\,dt \tag{4.31}
\]

where \(h = 1/40\) s is the spacing between the abscissas. The value of \(h\) ensures that the sampling rate of the evaluation of the integral is sufficiently smooth.

• **Model::Calculate** \((t)\) returns the result of the integrated value of the voltage over the capacitor in the RC-circuit at time \(t\), with pedestal values and gain factors included. The method returns the values for each coil in the form of a HepVector \(V_1(t), V_2(t), V_3(t)\).

The model is fitted to data using the C++ TMinuit minimization package of ROOT[14]. This package minimizes a user-defined function, in this case a \(\chi^2\) function, given by:

\[
\chi^2 = \chi_1^2 + \chi_2^2 + \chi_3^2 \tag{4.33}
\]

with \(\chi_i^2\) defined by:

\[
\chi_i^2 = \sum_{n=0}^{\text{points}} \left( \frac{V_i\text{ measured}(t_n) - V_i(t_n; \vec{p})}{\sigma_n} \right)^2 \tag{4.34}
\]

where \(\vec{p}\) is the array of parameters used to describe the model \(V_i(t)\) (see equation 4.29).
4.4 Fitting to modeled data

The fit procedure is tested with simulated data. The data is generated using the model described in section 4.2. The geometry of the setup is known to a certain level of precision, which allows the parameters of the model to be estimated so that a realistic dataset is generated.

In order to make the fit procedure faster, five parameters are fixed. The angular velocities around the $\Omega_1$ and $\Omega_2$ axes are known with sufficient accuracy from a linear fit to the absolute encoder data. The values of parameters $\Omega_1$ and $\Omega_2$ are set to $-0.164189$ and $0.206137$ rad/s. The assumption is made that the model is not sensitive to the RC-*times* parameters $\tau_i$ and that these parameters could be fixed during the fit, without losing much precision. Later, this assumption is tested and proved to be not fully correct. Nevertheless, the analysis described in this section and the following sections (4.4, 4.5, 5) is based on a fit with 15 parameters, with fixed RC times and angular velocities. Since the model with fixed RC times appeared not to describe the measurements, the results of the fit are less reliable than was hoped for. The techniques of determining the instantaneous angular orientation of the Hall probe described in chapter 5 do not suffer from this assumption, although the values of the reconstructed angles will not be valid.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>in generated data</th>
<th>Clean Fit value</th>
<th>Noise Fit value</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta_1$</td>
<td>$1.00 \times 10^{-3}$</td>
<td>$1.00 \times 10^{-3}$</td>
<td>$1.00 \times 10^{-3}$</td>
<td>$8.54 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\Theta_2$</td>
<td>$2.00 \times 10^{-3}$</td>
<td>$2.00 \times 10^{-3}$</td>
<td>$2.00 \times 10^{-3}$</td>
<td>$1.24 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\Phi_2$</td>
<td>$3.00 \times 10^{-3}$</td>
<td>$3.00 \times 10^{-3}$</td>
<td>$3.00 \times 10^{-3}$</td>
<td>$4.21 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>$1.00 \times 10^{6}$</td>
<td>$1.00 \times 10^{6}$</td>
<td>$1.00 \times 10^{6}$</td>
<td>$1.28 \times 10^{1}$</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>$2.00 \times 10^{6}$</td>
<td>$2.00 \times 10^{6}$</td>
<td>$2.00 \times 10^{6}$</td>
<td>$9.75 \times 10^{2}$</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>$-3.00 \times 10^{6}$</td>
<td>$-3.00 \times 10^{6}$</td>
<td>$-3.00 \times 10^{6}$</td>
<td>$1.27 \times 10^{2}$</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>$1.00 \times 10^{3}$</td>
<td>$1.00 \times 10^{3}$</td>
<td>$1.00 \times 10^{3}$</td>
<td>$2.25 \times 10^{1}$</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>$2.00 \times 10^{3}$</td>
<td>$2.00 \times 10^{3}$</td>
<td>$2.00 \times 10^{3}$</td>
<td>$1.85 \times 10^{3}$</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>$3.00 \times 10^{3}$</td>
<td>$3.00 \times 10^{3}$</td>
<td>$3.00 \times 10^{3}$</td>
<td>$5.53 \times 10^{1}$</td>
</tr>
<tr>
<td>$\theta_{12}$</td>
<td>$1.581$</td>
<td>$1.581$</td>
<td>$1.581$</td>
<td>$5.19 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\theta_{13}$</td>
<td>$1.578$</td>
<td>$1.578$</td>
<td>$1.578$</td>
<td>$1.62 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\theta_{23}$</td>
<td>$1.590$</td>
<td>$1.590$</td>
<td>$1.590$</td>
<td>$4.97 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>$5.00 \times 10^{-3}$</td>
<td>$5.00 \times 10^{-3}$</td>
<td>$5.00 \times 10^{-3}$</td>
<td>$1.75 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>$-5.00 \times 10^{-3}$</td>
<td>$-5.00 \times 10^{-3}$</td>
<td>$-5.00 \times 10^{-3}$</td>
<td>$4.34 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>$1.00 \times 10^{-2}$</td>
<td>$1.00 \times 10^{-2}$</td>
<td>$1.00 \times 10^{-2}$</td>
<td>$4.97 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 4.2: The parameters used for testing the model and the fit procedure. The first column shows the name of the parameters, the second column the values of the parameters used to generate the coil output. The third column shows the results from a fit to the generated data in the case that no noise is added to the modeled data. The fourth column shows the fit results in the case that noise with a Gaussian distribution was added to the voltages of the generated data.
CHAPTER 4. MODELING THE CALIBRATION SETUP

The modeled data of the three coils is shown in figure 4.6. In the following sections, the parameters $\Omega_1$ and $\Omega_2$ are fixed since their value is known with a large accuracy, obtained from a linear fit through the absolute encoder data. The input parameters are shown in the left column of table 4.2.

Figure 4.6: The generated data. The x-axis shows time, the y-axis the coil output in ADC-counts.

Several fit-runs are made to test the fit procedure. In the first run the start values of the parameters are set to the values used to generate the data. A $\chi^2/DOF = 0$ is returned, as expected, showing that the fit procedure works correctly.

Figure 4.7: The generated data without noise (black) shown together with the fit results (red).

In the second fit-run start values differed slightly from the values used to generate the data. The fitted values of the parameters are found to match the input parameters well and a $\chi^2/DOF = 2.24 \times 10^{-5}$ is returned. The fitted values of the parameters are shown in the column Clean Fit of table 4.2. Figure 4.7 shows the fitted data (red) superimposed on the generated data (black). Since the values of the fitted parameters are very close to those used to generate the data the results match the data perfectly.

The third and last fit-run is performed by assigning start values slightly different from the values used to generate the data, as in the second fit-run. Then, the generated data is smeared with a Gaussian distribution to simulate noise on the coil ADC voltages. The RMS of the distribution of noise on the measured data is taken as the width of the Gaussian distribution used to generate the noise.

The width of the distribution is used to normalize the $\chi^2$, yielding a value of $\chi^2/DOF = 0.957$. The RMS of the distribution of the residuals match the width of the distribution of the Gaussian smear of the generated data (see table 4.3 and figure 4.8). The fitted values of the parameters of the Noised Fit are shown in the right column of table 4.2.

4.5 Fitting to measured data

In this section the results of a fit of the model to measured data is described. As a dataset, the file
4.5. FITTING TO MEASURED DATA

Figure 4.8: The generated data (black) shown together with the fit results (red). The shape of the residuals match the distribution of noise added to the generated data.

<table>
<thead>
<tr>
<th>Noise modeled</th>
<th>RMS residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>coil 1</td>
<td>13.42</td>
</tr>
<tr>
<td>coil 2</td>
<td>1075</td>
</tr>
<tr>
<td>coil 3</td>
<td>32.86</td>
</tr>
</tbody>
</table>

Table 4.3: The RMS of the generated noise compared with the RMS of the residuals.

SensCal_CMM20040323-1530.txt is used. The file consists of 24 runs, corresponding to 12 'matrix'-elements of the table of the LabVIEW interface shown in figure 3.1. The values of the magnetic field are 1.45, 1.2, 0.9 and 0.6 Tesla. Data is taken at temperatures of 17, 20 and 23°C. The 24 runs consist of the 12 combinations of the values of the magnetic field and temperature times two, since data are taken both in forward and reverse mode. Other data files were not suitable for a fit because of incomplete runs and jittered runs (see section 3.2.2).

The 15 parameter model is fit to data from eight runs. Only forward data is used, not all of them were suitable to be used in the fit, since these runs were not valid (see section 3.2.2). The results of the fit are shown in figure 4.10 and table 4.4.

Table 4.4 shows the mean value as result of fits over 8 runs of data. The column spread shows the RMS of the distribution of fitted values. The column Mean accuracy shows the mean of the fit precision per parameter. The spread in the values is larger than the mean fit precision. Moreover, the mean error on the parameter values is smaller (about one order of magnitude) than the errors on the parameter values obtained by fitting generated data. Generated data is expected to match the model better than measured data, allowing the parameters to be obtained with higher accuracy. This is another indication that fitting with 15 parameters is not reliable enough.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fit results</th>
<th>Mean accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>spread</td>
</tr>
<tr>
<td>$\Theta_1$</td>
<td>$9.46 \times 10^{-4}$</td>
<td>$4.81 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\Theta_2$</td>
<td>$-2.44 \times 10^{-1}$</td>
<td>$6.45 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\Phi_2$</td>
<td>$1.63 \times 10^{-2}$</td>
<td>$1.15 \times 10^{-4}$</td>
</tr>
<tr>
<td>$G_1$</td>
<td>$7.40 \times 10^{5}$</td>
<td>$2.35 \times 10^{3}$</td>
</tr>
<tr>
<td>$G_2$</td>
<td>$5.65 \times 10^{5}$</td>
<td>$1.48 \times 10^{3}$</td>
</tr>
<tr>
<td>$G_3$</td>
<td>$-9.22 \times 10^{5}$</td>
<td>$2.55 \times 10^{3}$</td>
</tr>
<tr>
<td>$P_1$</td>
<td>$-1.23 \times 10^{5}$</td>
<td>$1.30 \times 10^{4}$</td>
</tr>
<tr>
<td>$P_2$</td>
<td>$-6.03 \times 10^{4}$</td>
<td>$6.69 \times 10^{2}$</td>
</tr>
<tr>
<td>$P_3$</td>
<td>$-2.35 \times 10^{4}$</td>
<td>$1.52 \times 10^{4}$</td>
</tr>
<tr>
<td>$\theta_{12}$</td>
<td>$1.59$</td>
<td>$3.21 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\theta_{13}$</td>
<td>$1.54$</td>
<td>$9.10 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\theta_{23}$</td>
<td>$1.54$</td>
<td>$1.54 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\theta_{1}$</td>
<td>$-2.74 \times 10^{-1}$</td>
<td>$6.25 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>$4.53 \times 10^{-2}$</td>
<td>$3.28 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>$1.14$</td>
<td>$7.62 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 4.4: Mean fit results from a fit to data from eight data runs. The spread in the mean values is larger than the mean fit accuracy.

### 4.5.1 Normalizing the $\chi^2$

The uncertainties on the measurements ($\sigma_n$ in formula 4.34) are introduced by looking at the noise on the coil ACDs. The noise is measured by measuring whilst having the ADCs in a short circuit. The noise on the three ADCs is shown in figure 4.9. The plots show several features which make them difficult to use to normalize the $\chi^2$ with.

First, the noise is not entirely random, some periodic structure is present in the data. An explanation for this feature is that the ADC might measure a signal due to induction in loops of the copper prints of the electronics of the coil card which are rotated in the magnetic field.

Secondly, the signal on the ADC of coil 2 behaves different than the signals on the other two ADCs. The noise level has an offset which is about 10 times larger than the offset of the other ADCs. The RMS of the signal on the ADC of coil 2 differs three orders of magnitude with respect to the signal of the ADCs of coil 1 and 2.

Finally, both the signal of ADC coil 2 and ADC coil 3 show the eight-point periodic hitch discussed in section 3.2.2.
4.5. FITTING TO MEASURED DATA

Figure 4.9: The noise levels measured at the condition when the coil ADCs are in a short circuit. It is visible that the ADC measures things other than random noise. The ADC of coil 2 has an RMS in the order of $10^3$ larger than the other two ADCs.
Figure 4.10: The results from a 15-parameter fit to data from eight runs.
Chapter 5

Position Reconstruction of the Hall Probe

The main goal of the research described in this thesis is to deduce the angular orientation of the Hall probe at any time during data acquisition. The orientation is described with polar angles $\theta$ and $\phi$. These angles point out the position of the rotating probe, with reference vector $\vec{x}$ defined to be in the direction of the magnetic field when $\theta = 0$ and $\phi = 0$. With the use of the fitted values of the model parameters, the angles can be obtained from the data files by two means:

- With use of the absolute encoder readout as a direct measure of the position of the probe.
- By calculating the position of the probe from the measured values of the three coils.

For the analysis described in the next sections the following data set was used: SensCal_CMM20040323-1530.txt at $T = 17^\circ C$ and $B = 1.0$ Tesla.

5.1 The absolute encoders analysis method

In the data files the encoder readout is given as $AX$ and $AY$, the number of revolutions of the axes given by the absolute encoders. The $AX$ and $AY$-axes correspond to the $\Omega_2$ and $\Omega_3$ axes in the model respectively (see figure 4.3). A correction to the position of the $AX$-axis has to be applied because of the fact that the $AX$-axis is connected to the $AY$-axis (see section 2.4), which gives rise to an apparent rotation in the opposite direction of the $AX$-axis rotation proportional to the rotation of the $AY$-axis. Since the two axes rotate in opposite directions this correction has to be added to the readout value of $AX$. Then, the absolute encoder readout values are converted to input angles rather than number of revolutions:

\[ \psi_x = \left( AX + \frac{AY}{180} \right) 2\pi \text{ rad} \quad (5.1) \]
\[ \psi_y = AY \ 2\pi \text{ rad} \quad (5.2) \]
CHAPTER 5. POSITION RECONSTRUCTION OF THE HALL PROBE

In order to obtain the instantaneous angles $\theta$ and $\phi$ from the input angles $\psi_x$ and $\psi_y$ the values of the parameters describing the non-orthogonality of the two axes and the value of the magnetic field must be known. The parameters corresponding to this non-orthogonality are $\Theta_1$, $\Theta_2$ and $\Phi_2$ from table 4.1. The parameter values taken are those derived from the fit of the model to the same data set, SensCal.CMM20040323-1530.txt at $T = 17$ °C and $B = 1.0$ Tesla.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta_1$</td>
<td>$9.55 \times 10^{-4}$</td>
<td>$1.26 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\Theta_2$</td>
<td>$-2.55$</td>
<td>$6.19 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\Phi_2$</td>
<td>$4.58 \times 10^{-3}$</td>
<td>$5.84 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

The calculation of the instantaneous position of the probe is purely geometrical. Given the input angles derived from the absolute encoder readout, the position of the probe $\vec{x}$ is given by the equation:

$$\vec{x} = R_{\psi_y} \cdot R_{\psi_x} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$  \hspace{1cm} (5.3)

With $R_{\psi_y}$ and $R_{\psi_x}$ given by equations 4.15. The instantaneous angles $\theta$ and $\phi$ are the polar coordinates of the position vector of the probe $\vec{x}$:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sin(\theta)\cos(\phi) \\ \sin(\theta)\sin(\phi) \\ \cos(\theta) \end{pmatrix} \rightarrow \begin{array}{l} \theta = \arccos(z) \\ \phi = \arctan(y/x) \end{array} \hspace{1cm} (5.4)$$

The transformation from the absolute encoder coordinates $\psi_x$ and $\psi_y$ in data point $n$ to spherical coordinates can be written in vector notation[15]:

$$\begin{pmatrix} \theta \\ \phi \\ \Theta_1 \\ \Theta_2 \end{pmatrix} = A_n \begin{pmatrix} \psi_x \\ \psi_y \\ \Theta_1 \\ \Theta_2 \end{pmatrix}_{\psi_x,\psi_y,n} \hspace{1cm} (5.5)$$

with $A_n$ the transformation matrix in data point $n$:

$$A_n = \begin{pmatrix} \frac{\partial \theta}{\partial \psi_x} & \frac{\partial \theta}{\partial \psi_y} & \frac{\partial \theta}{\partial \Theta_1} & \frac{\partial \theta}{\partial \Theta_2} \\ \frac{\partial \phi}{\partial \psi_x} & \frac{\partial \phi}{\partial \psi_y} & \frac{\partial \phi}{\partial \Theta_1} & \frac{\partial \phi}{\partial \Theta_2} \end{pmatrix}_{\psi_x,\psi_y,n} \hspace{1cm} (5.6)$$

A C++ class is constructed to calculate the spherical coordinates from absolute encoder data. This class, AbsEnc, also calculates the covariance matrix of the spherical coordinates making use of the transformation matrix.

$$C_{\theta,\phi,n} = A_n^T C_{\psi_x,\psi_y,n} A_n^T \hspace{1cm} (5.7)$$
5.2. COIL MEASUREMENTS ANALYSIS METHOD

Where \( \mathbf{C}_{\psi_x,\psi_y} \) is the covariance matrix of the input angles and the parameters:

\[
\mathbf{C}_{\psi_x,\psi_y} = \begin{pmatrix}
\text{cov}(\psi_x, \psi_x) & 0 & 0 & 0 & 0 \\
0 & \text{cov}(\psi_y, \psi_y) & 0 & 0 & 0 \\
0 & 0 & \text{cov}(\Theta_1, \Theta_1) & \text{cov}(\Theta_2, \Theta_1) & \text{cov}(\Phi_2, \Theta_1) \\
0 & 0 & \text{cov}(\Theta_1, \Theta_2) & \text{cov}(\Theta_2, \Theta_2) & \text{cov}(\Phi_2, \Theta_2) \\
0 & 0 & \text{cov}(\Theta_1, \Phi_2) & \text{cov}(\Theta_2, \Phi_2) & \text{cov}(\Phi_2, \Phi_2)
\end{pmatrix}
\]

The input angles and the parameters are expected to be independent. The covariances of the input parameters are obtained from the fit procedure. The variances of the input angles are calculated from the residuals of a linear fit through the absolute encoder data set. The errors on the calculated angles \( \theta \) and \( \phi \) are calculated from the variances in the covariance matrix \( \mathbf{C}_{\theta, \phi} \).

5.2 Coil measurements analysis method

The second way to determine the instantaneous angles of the position of the probe is by calculating the angles by means of the values measured with the three coils. This method relies on the knowledge of the values of all the parameters.

For each given set of measurements \([C1, C2, C3]\) a position \((\theta, \phi)\) can be found on the unit sphere on which the trajectory of the probe reference vector \(\vec{x}\) lies. This boils down to solving three equations with two variables:

\[
\begin{align*}
C_{1\text{model}}(\theta, \phi) - C1 & = 0 \\
C_{2\text{model}}(\theta, \phi) - C2 & = 0 \\
C_{3\text{model}}(\theta, \phi) - C3 & = 0
\end{align*}
\]

It is more practical to solve these three equations with only one variable, the reconstructed time value \(t_{rec}\):

\[
\begin{align*}
C_{1\text{model}}(t_{rec}) - C1 & = 0 \\
C_{2\text{model}}(t_{rec}) - C2 & = 0 \\
C_{3\text{model}}(t_{rec}) - C3 & = 0
\end{align*}
\]

The values \( \theta \) and \( \phi \) can be calculated using the \textit{AbsEnc} class with as input angles

\[
\begin{align*}
\dot{\psi}_x & = \Omega_2 \cdot t_{rec} \\
\dot{\psi}_y & = \Omega_1 \cdot t_{rec}
\end{align*}
\]

This can be done since the angular velocities \( \Omega_1 \) and \( \Omega_2 \) are well known from the fit (and more importantly, their relative magnitude), limiting the values of \( \theta \) and \( \phi \) to a definite trajectory rather than the whole surface of the unit sphere. Solving equations 5.12, 5.13 and 5.14 with one parameter \( t_{rec} \) is less cumbersome than solving equations 5.9, 5.10 and 5.11.
with two parameters $\theta$ and $\phi$. Another advantage is that the class *Model* (see section 4.3) can be used to calculate $t_{rec}$. This is the same class used to model the coils, assuring that the same configuration of the model is applied in the analysis as well as in the fit of the parameters.

The three equations are solved by fitting $t_{rec}$ which minimizes the following expression of $\chi^2$:

\[
\chi^2 = \chi_1^2 + \chi_2^2 + \chi_3^2, \quad \text{where} \quad \begin{cases} 
\chi_1^2 = (C_{1,\text{model}}(t_{rec}) - C1)^2 \\
\chi_2^2 = (C_{2,\text{model}}(t_{rec}) - C2)^2 \\
\chi_3^2 = (C_{3,\text{model}}(t_{rec}) - C3)^2
\end{cases} \tag{5.17}
\]

assuring that the minimum will be such that the value of $t_{rec}$ reaches the roots of the three equations. Due to the sinusoidal shape of the equations, the $\chi^2$ profile is multi-peaked making it difficult to find the correct value of the reconstructed time by fitting with *Minuit* (see figure 5.1). In order to force the values of the reconstructed times to be continuous over the whole data set, the peak closest to the value of $t_{rec}$ in the preceding point is favored in the fit by adding a term to the $\chi^2$:

\[
\chi_i^2 = \chi_1^2 + \chi_2^2 + \chi_3^2 + \text{weight} \cdot (t_{rec} - t_{rec,\text{previous}})^2 \tag{5.18}
\]

![ChI2 profile](image)

Figure 5.1: The profile of the $\chi^2$ for a set measured coil values as function of the possible values of $t_{rec}$. The shape of this distribution has many local minima which makes finding the correct minimum complicated.
5.2. COIL MEASUREMENTS ANALYSIS METHOD

After acquiring the set of reconstructed times, the same method described in the previous section is applied to calculate the instantaneous angles of the position of the probe:

\[
\bar{x} \equiv \mathbf{R}_{f_1}(\Omega_1 \cdot t_{\text{rec}}) \cdot \mathbf{R}_{f_2}(\Omega_2 \cdot t_{\text{rec}}) \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{5.19}
\]

A class \textit{CoilRoot} is built, similar to \textit{AbsEnc}, to calculate the angles and their uncertainty using the following transformation and covariance matrix:

\[
\begin{pmatrix} \theta \\ \phi \end{pmatrix}_{\theta_n,\phi_n} = A_n \begin{pmatrix} t_{\text{rec}} \\ \Theta_1 \\ \Theta_2 \\ \Phi_2 \end{pmatrix}_{\psi_{x,n},\psi_{y,n}} \tag{5.20}
\]

\[
A_n = \begin{pmatrix}
\frac{\partial \theta}{\partial t_{\text{rec}}} & \frac{\partial \theta}{\partial \Theta_1} & \frac{\partial \theta}{\partial \Theta_2} & \frac{\partial \theta}{\partial \Phi_2} \\
\frac{\partial \phi}{\partial t_{\text{rec}}} & \frac{\partial \phi}{\partial \Theta_1} & \frac{\partial \phi}{\partial \Theta_2} & \frac{\partial \phi}{\partial \Phi_2}
\end{pmatrix}_{t_{\text{rec}},n} \tag{5.21}
\]

\[
C_{t_{\text{rec}},n} = \begin{pmatrix}
cov(t_{\text{rec}},t_{\text{rec}}) & 0 & 0 & 0 \\
0 & cov(\Theta_1,\Theta_1) & cov(\Theta_2,\Theta_1) & cov(\Phi_2,\Theta_1) \\
0 & cov(\Theta_1,\Theta_2) & cov(\Theta_2,\Theta_2) & cov(\Phi_2,\Theta_2) \\
0 & cov(\Theta_1,\Phi_2) & cov(\Theta_2,\Phi_2) & cov(\Phi_2,\Phi_2)
\end{pmatrix}_n \tag{5.22}
\]

The uncertainty of the reconstructed time is obtained from the time-reconstruction fit. The covariances of the parameters are calculated from the covariances yielded by the fit.
Chapter 6

Results

The instantaneous angles $\theta$ and $\phi$ of one measurement run are shown in figure 6.1. In the upper two plots the values of the angles are shown versus time, monitoring the position of the probe over the whole data run. The two reconstruction methods gave identical results.

In the lower two plots the corresponding uncertainties in the values of the angles at each position are shown. It is shown that the coil measurement method yields smaller errors than the absolute encoder method. The distribution of the uncertainty in $\theta$, sharp peaks occur, coinciding with times at which the value of $\phi$ reach zero.

Table 6 shows the mean values of errors in the angles from the full data set. The method of analysis which makes use of the coil measured data has a larger precision than the method which makes use of the absolute encoder readout. The goal was to achieve a precision in the order of $10^{-5}$ which is two orders of magnitude higher than achieved by means of these methods.

<table>
<thead>
<tr>
<th>angle</th>
<th>mean error (rad)</th>
<th>\hspace{1cm}</th>
<th>mean error (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>absolute encoder method</td>
<td>coil measurements method</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>$3.77 \times 10^{-3}$</td>
<td>$9.79 \times 10^{-4}$</td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>$2.95 \times 10^{-3}$</td>
<td>$6.24 \times 10^{-4}$</td>
<td></td>
</tr>
</tbody>
</table>

In figure 6.2 the path of the position vector of the probe is shown for both series of instantaneous angles. This figure shows the movement of the probe through space. It is shown that both methods of reconstructing the angles yield the same path of the probe.
Figure 6.1: The angles reconstructed by the two methods together with their uncertainty. In black the absolute encoder method, in red the coil measurements method.
Figure 6.2: The trajectory of the reference vector of the probe.
Chapter 7

Conclusion

The precision of the reconstructed angular orientation of the probe during the measurement cycle is in the order of mrad, which is two orders of magnitude less precise than the desired value. The reasons for this larger error on the analysis can be the following:

- The errors of the fitted values of the parameters obtained by the fit of the model to the coil data are too large. The data from the coils is unreliable due to a eight-point periodic hitch (see section 3.2.2). Although this jitter is not present in the analyzed data set, there is serious doubts whether the data is handled in the correct way in any of the data files.

- The unexpected behavior of the second ADC of the coil card, which is not understood, causes an ill-estimated error on the coil readout data which makes it difficult to interpret the fit results and the errors on the parameters difficult. The values of the parameters are used in two methods of angle reconstruction and influence the precision obtained significantly.

- The rotation mechanism has been recently adjusted. The motors driving the rotation of the axes now run faster and a larger delay ensures that the axes have the same angular velocity as before. This adjustment yields a better resolution on the absolute encoder data and might improve the resolution of the reconstructed angles.

- The model may contain incorrect concepts or miss features which are overlooked, causing the model to not really describe the data correctly.

- The fit procedure with five parameters of the model fixed (the angular velocities and the RC-times of the coil circuits) does not yield reliable values of the parameters. For better results, the RC-times parameters should be released.
Chapter 8

Summary

In order to achieve precise particle momentum measurements in the ATLAS muon-spectrometer, the magnetic field in the detector volume must be known with high precision. The magnetic field will be measured by Hall probes, which have to be calibrated. This is done by rotating them around two orthogonal axes in an accurately known homogeneous magnetic field at several temperatures and several magnetic field strengths. In order to calibrate the Hall probes properly, the angular orientation of the sensors must be known throughout the measurement with high precision.

The calibration setup offers several possibilities to determine the angular orientation of the probe. Among the Hall probes, three mutually perpendicular coils are rotated in the magnetic field. The induced voltage over the coils is a measure for the position of the coils and hence of the Hall sensors. An other way to determine the angular orientation of the probe is by the read out of the absolute encoders mounted on the two axes over which the probe is rotated. From either the absolute encoder data or the coil measurements, the angular orientation of the Hall probe can be reconstructed at any time during calibration.

Both methods rely on the precise knowledge of the geometry of the calibration setup. To determine this, in particular to determine parameters such as the orthogonality of the axes and the magnetic field with respect to each other, a detailed model of the calibration setup is developed and the model is fit to the coil measurements to obtain the values of the model parameters.

With the values of the parameters, the angular orientation of the probe is achieved from the measurements by the two methods. The angular orientation reconstruction based on the absolute encoder read out and the analysis based on the coil measurements yield results which are comparable. The analysis based on the coil measurements has a better resolution compared to the absolute encoder method: a mean error on $\theta$ of $2.95 \times 10^{-3}$ rad and on $\phi$ of $6.24 \times 10^{-4}$ rad. The goal was to obtain a precision in the order of $10^{-5}$, which is not reached.
Bibliography


