

Lecture I

Master Particle Physics I
Wouter Hulsbergen
September 2, 2013

Lectures PPI

L0 : Introduction

L1 : Particles and Fields (historical overview)

L2 : Wave Equations and Antiparticles

L3 : The Electromagnetic Field

L4 : Perturbation Theory and Fermi's Golden Rule

L5 : Electromagnetic Scattering of Spinless Particles

QED of Spinless Particles:
Scattering Theory and
Cross Sections

L6 : The Dirac Equation

L7 : Solutions of the Dirac Equation

L8 : Spin 1/2 Electrodynamics

QED for
Fundamental Fermions

L9 : The Weak Interaction

(Fermi 4-point scattering: an analogy with QED)

L10: Local Gauge Invariance

(the role of symmetries in interactions)

L11: Electroweak Theory

L12: The Process: $e^+e^- \rightarrow \gamma, Z \rightarrow \mu^+\mu^-$

The Standard Model for
massless particles
 $SU(2)_L \times U(1)_Y$

Material

Lecture notes

- we use lecture notes written by Prof. Dr. Marcel Merk
- will hand out notes at beginning of each lecture
- you can also find those notes on the web:

<http://www.nikhef.nl/~wouterh/Lectures/PPI>

Books (one of these)

- Griffiths, Introduction to Elementary Particles (2008)
- Halzen and Martin, Quarks and Leptons (1984)

(See lecture notes for many other books.)

Exercises, exam

- each lecture is followed by a tutorial session
 - exercises can be found in the lecture notes
 - answers count for final mark!
 - grade (1-10) given by assistents (Panos and Sim)
 - receive 2 bonus points if you hand them in within one week of lecture (at start of lecture, or in my mail box downstairs)
 - average exercise grade counts as $1/3$ of your final mark
- final exam
 - Tuesday October 22th
 - open book exam (but no computers, pets, ...)

Nikhef

OCW

Ministry of Education Culture and Science

Minister: **Jet Bussmaker**

Staatssecretaris: Sander Dekker

NWO

Dutch organisation for Scientific Research

Voorzitter algemeen bestuur: **Jos Engelen**

General director: Hans de Groene

- * Aard- en levenswetenschappen (ALW)
- * Chemische wetenschappen (CW)
- * Exacte wetenschappen (EW)
- * Geesteswetenschappen (GW)
- * Maatschappij- en gedragswetenschappen (MaGW)
- * Medische wetenschappen (ZonMw)
- * **Natuurkunde (N)**
- * Technische wetenschappen (STW)

FOM

Foundation for Fundamental Research of Matter

Director: Wim van Saarloos

Institutes:

- Differ: Plasma physics
director: Richard van de Sanden
- Amolf: Atomic and Molecular Physics
director: Albert Polman
- **Nikhef: Dutch Institute for Particle Physics**
director: **Frank Linde**
- KVI: Nuclear Physics (closed)
director: Klaus Jungmann

Universities

Nijmegen: RU Sijbrand de Jong

Utrecht: UU Thomas Peitzmann

Amsterdam: VU Jo van den Brand

UvA Stan Bentvelsen

+...Groningen, Twente, Leiden, Eindhoven...

Nikhef Collaboration:

Nikhef institute + 4 Universities

Real basis for all particle physics
in the Netherlands

Director: **Frank Linde**

LHC:

Atlas: Stan Bentvelsen

LHCb: Marcel Merk

Alice: Thomas Peitzmann

Astroparticle Physics:

Antares: Maarten de Jong

Auger: Charles Timmermans

Grav Waves: Jo van den Brand

Dark Matter: Patrick Decowski

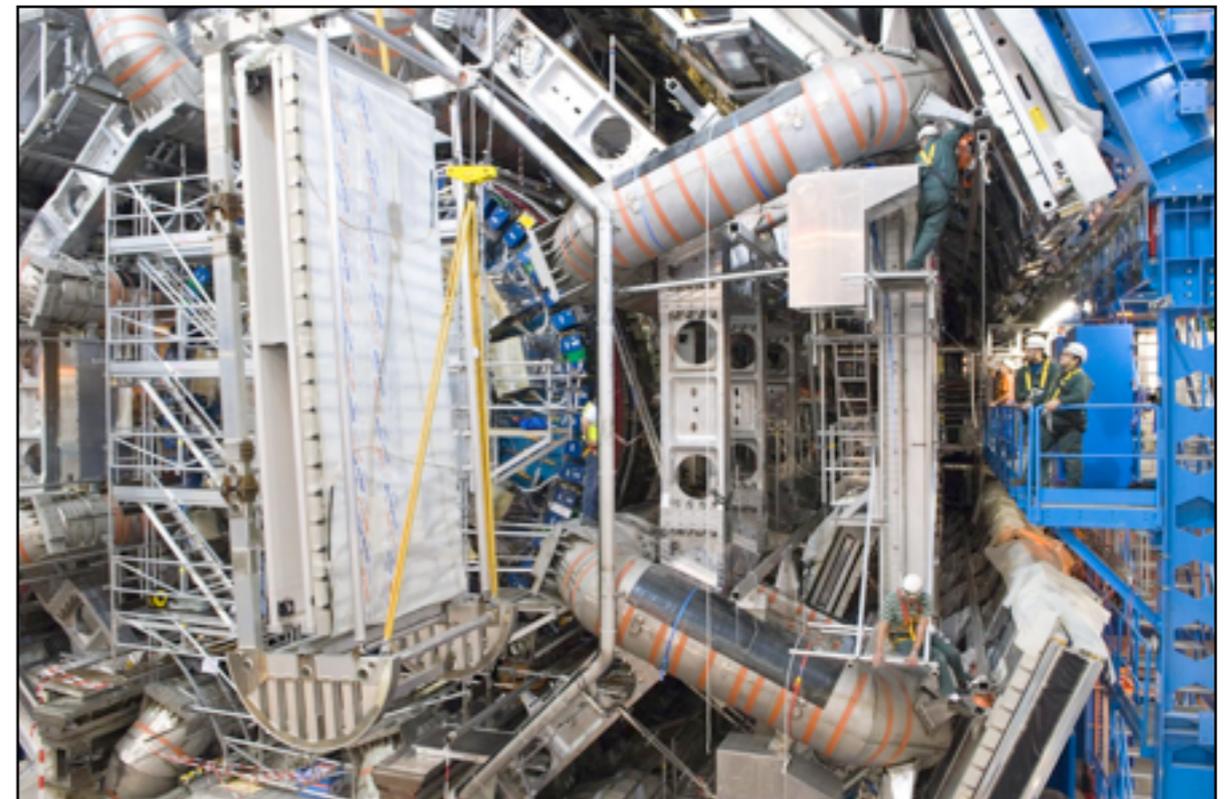
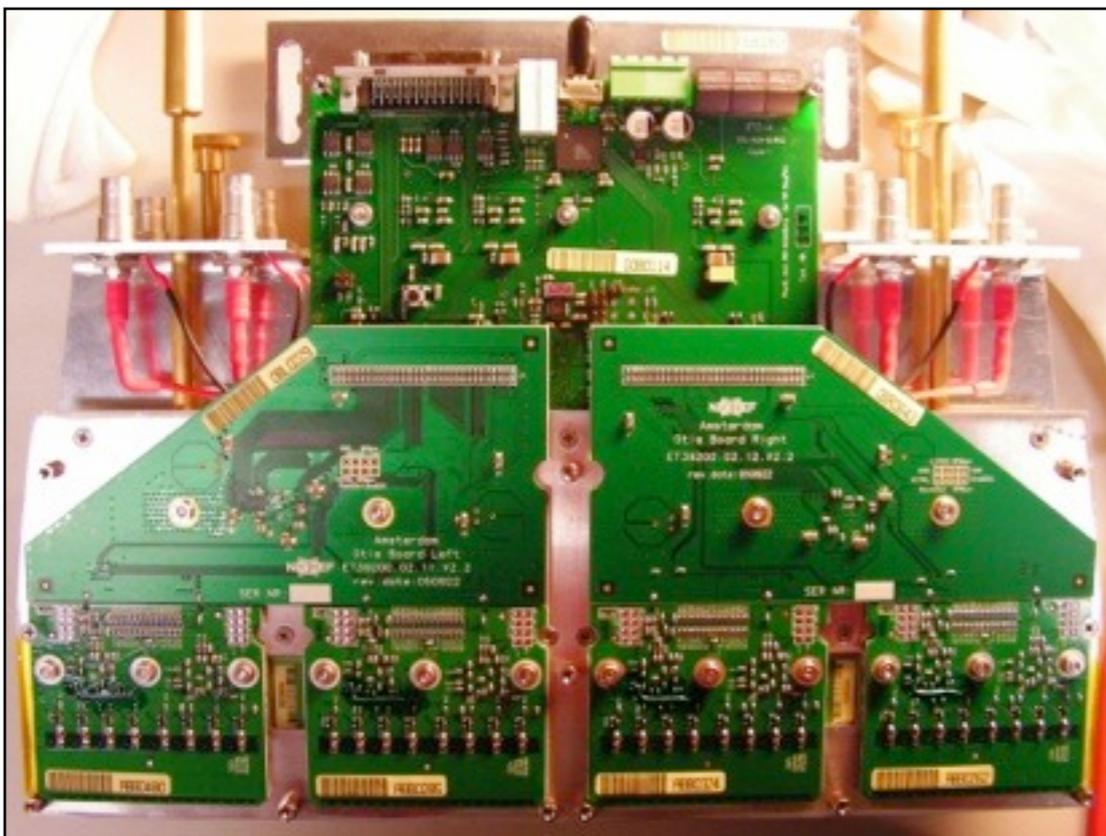
Other:

Theory: Eric Laenen

Grid: Jeff Templon

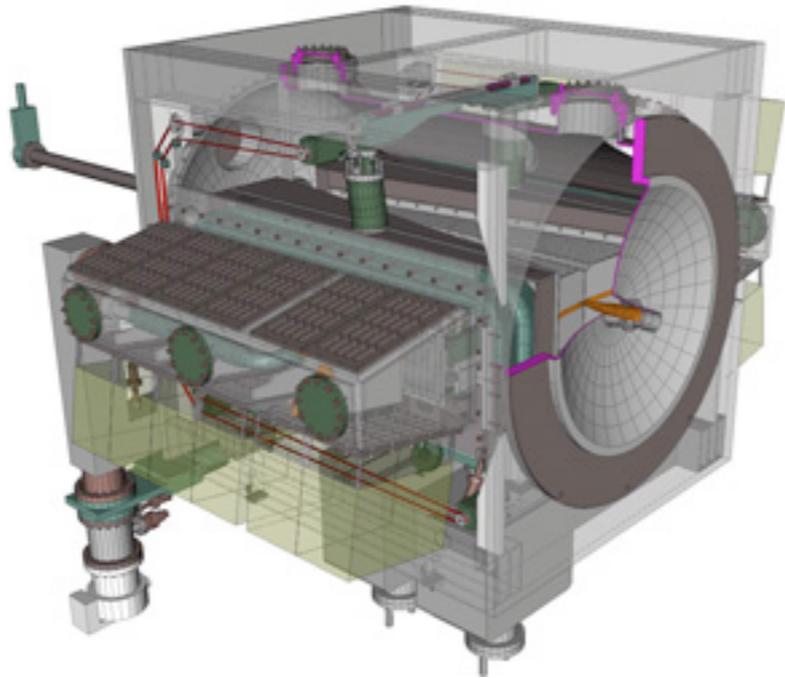
Nikhef

- about 150 physicists: staff, post-docs, PhD students
 - 2/3 on LHC experiments (ATLAS, LHCb, Alice)
 - neutrinos, gravitational waves, dark matter, theory, detector R&D, ...
- engineering
 - electronics
 - mechanics

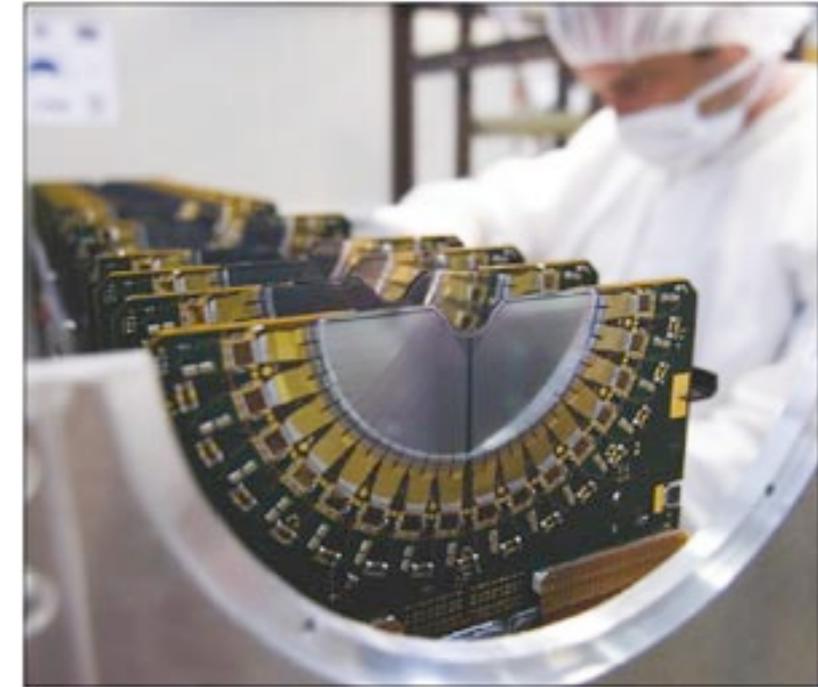
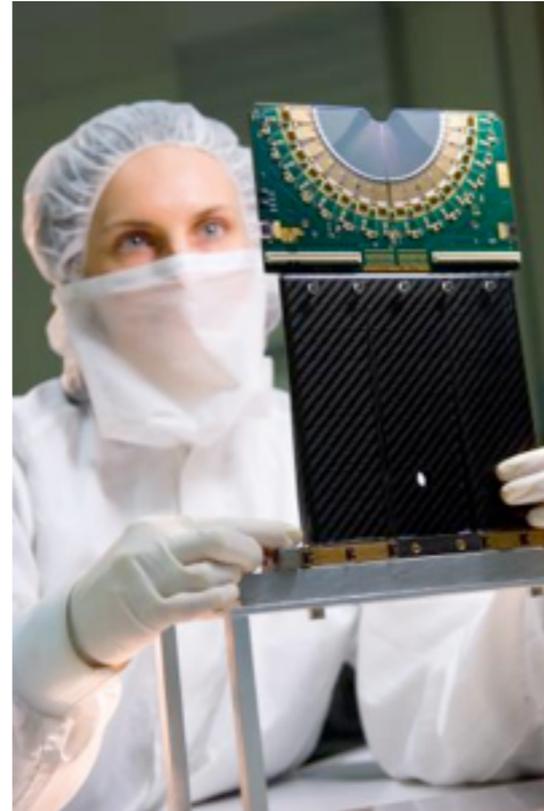


Example: LHCb vertex detector

design



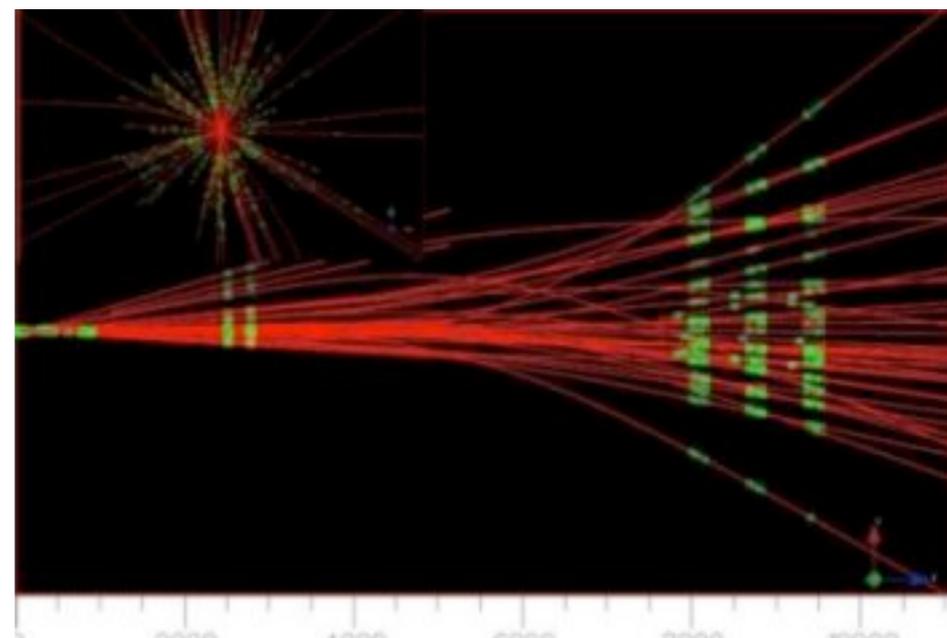
production and assembly



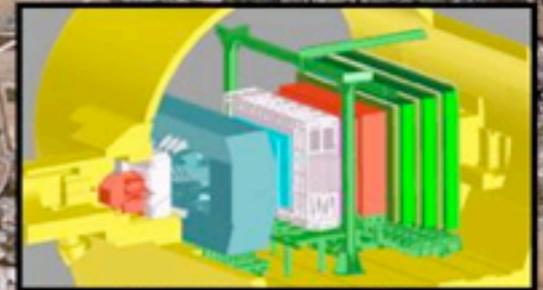
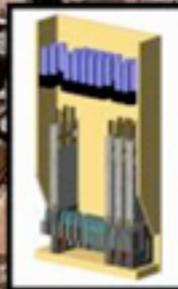
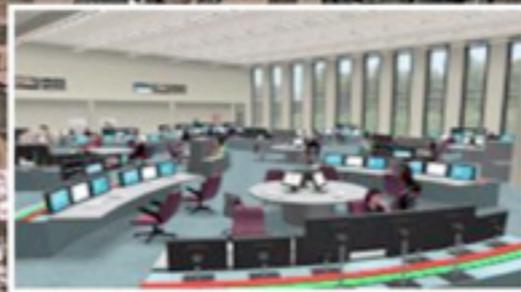
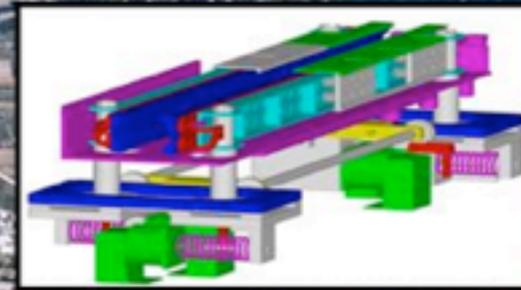
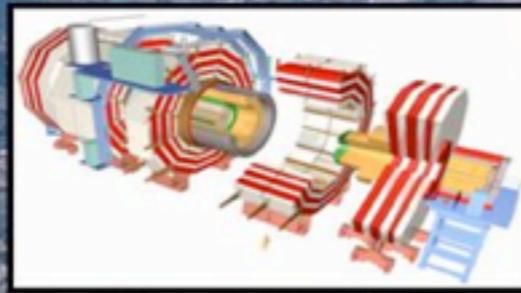
installation in LHCb at cern



physics



LHC



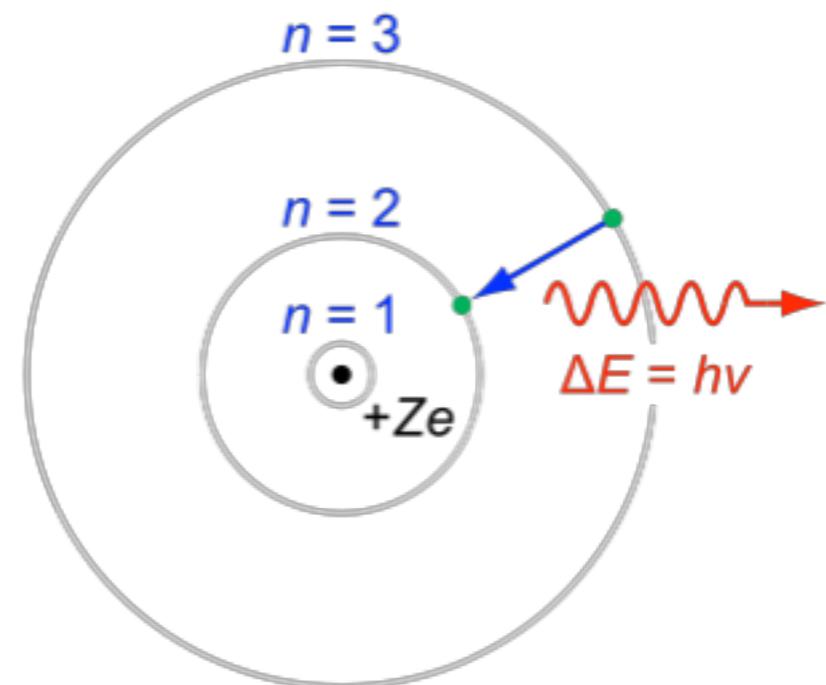
Testing a particle theory

theory (or model)

- particles
- interactions

measurements

- scattering
- decays
- bound states



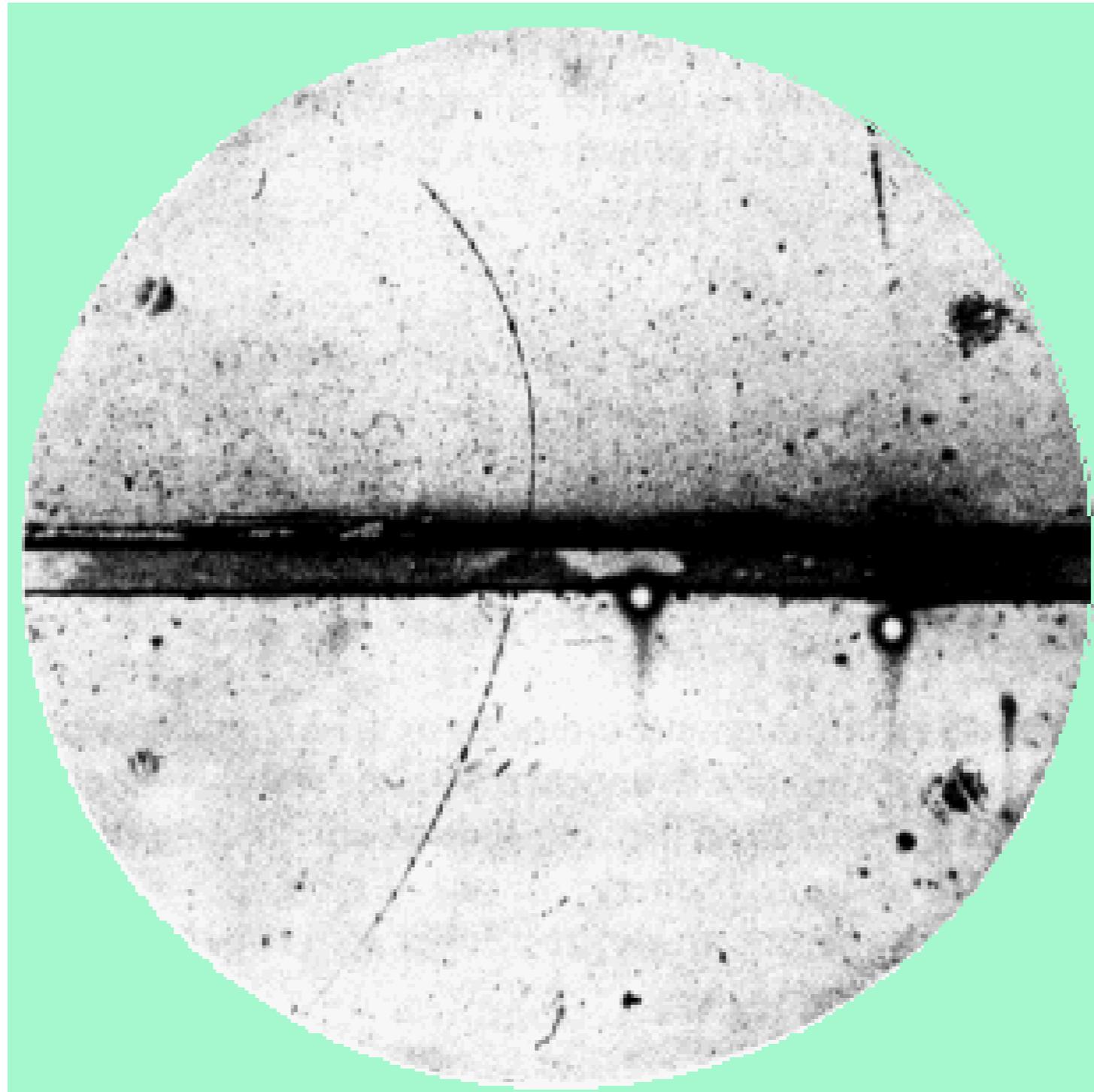


Figure 1.1: The discovery of the positron as reported by Anderson in 1932. Knowing the direction of the B field Anderson deduced that the trace was originating from an anti electron. Question: how?

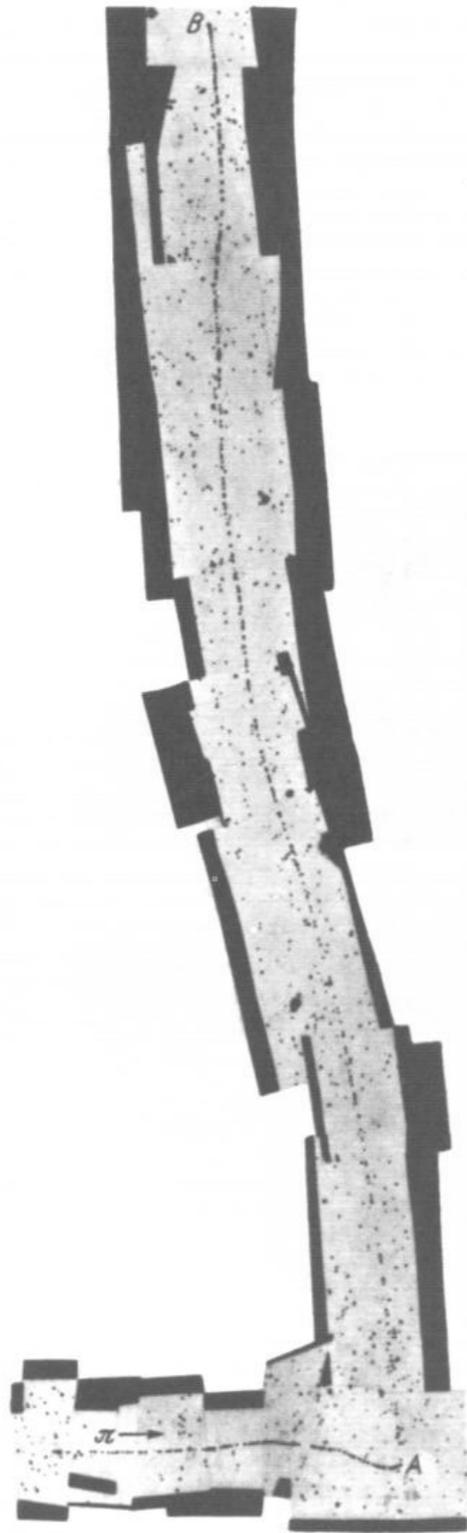
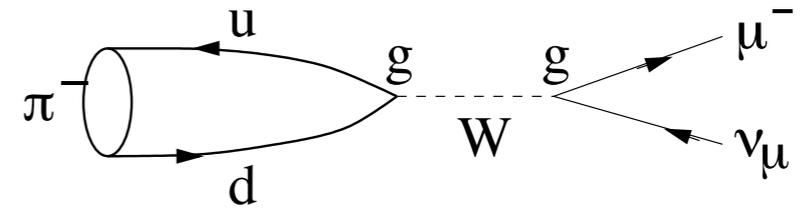
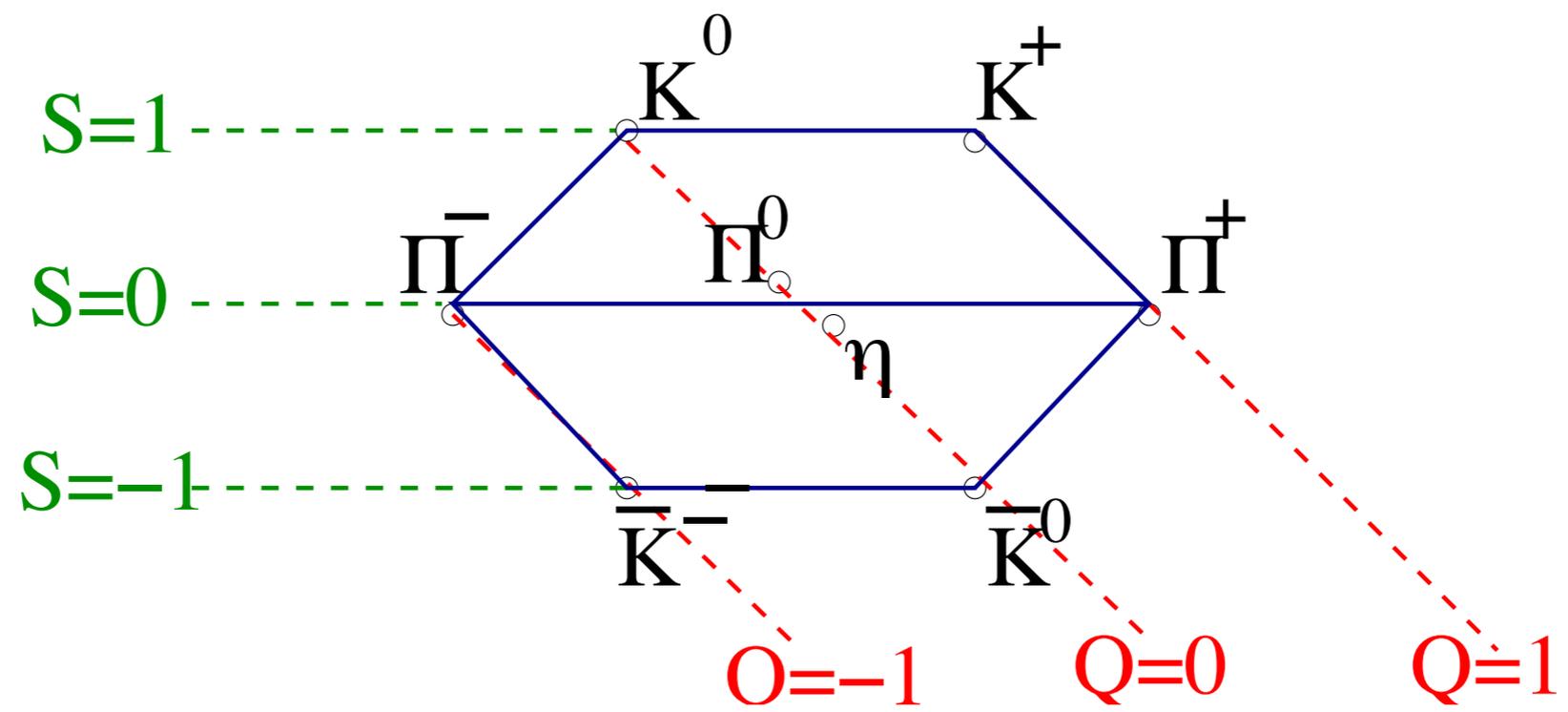
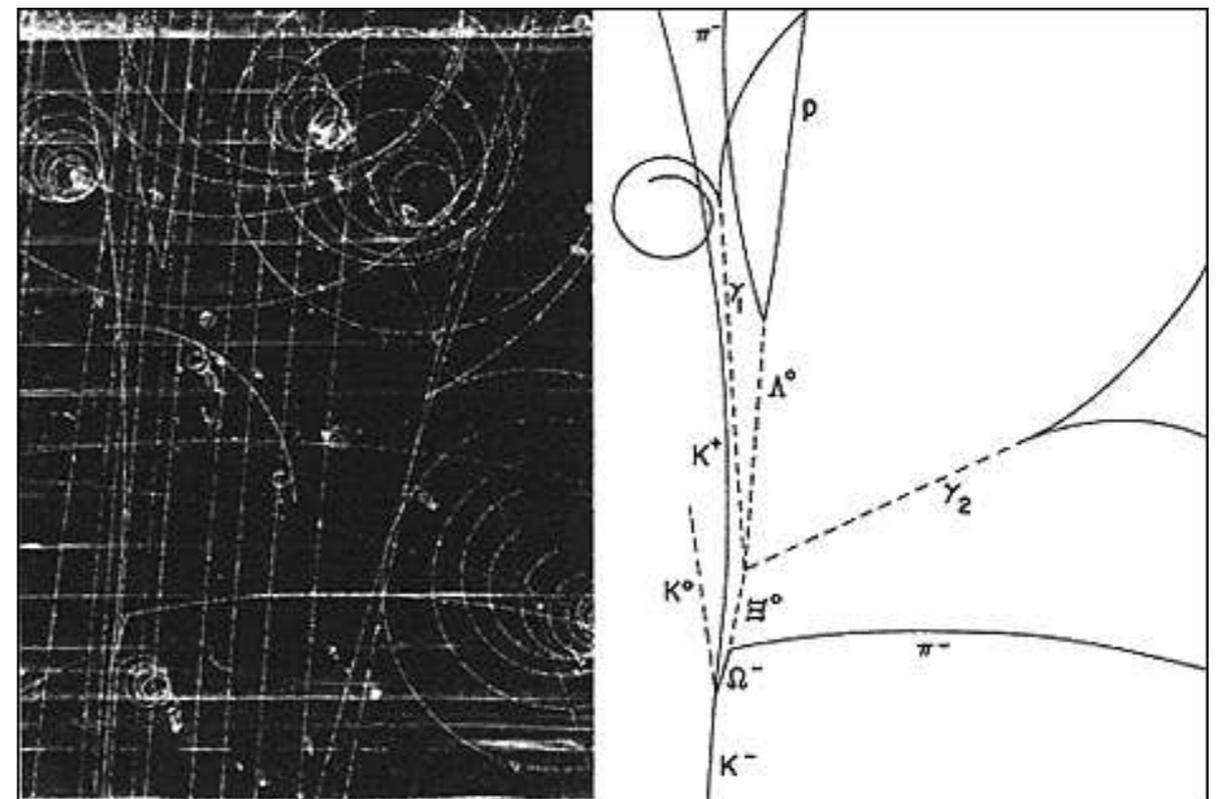
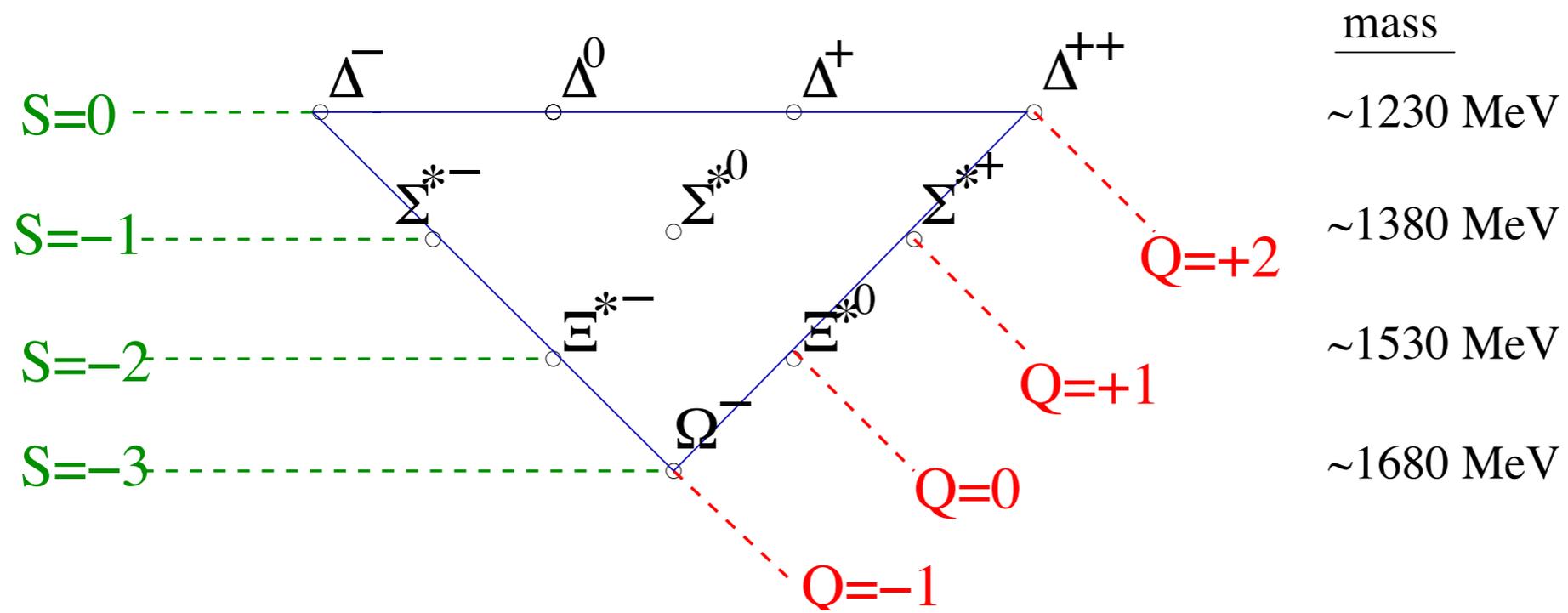


Figure 1.4 One of Powell's earliest pictures showing the track of a pion in a photographic emulsion exposed to cosmic rays at high altitude. The pion (entering from the left) decays into a muon and a neutrino (the latter is electrically neutral, and leaves no track). Reprinted by permission from C. F. Powell, P. H. Fowler, and D. H. Perkins, *The Study of Elementary Particles by the Photographic Method* (New York: Pergamon, 1959). First published in *Nature* **159**, 694 (1947).

electrons we *do* observe are confined to the positive energy states. But if this is true, then what happens when we impart to one of the electrons in the "sea" an energy sufficient to knock it into a positive energy state? The *absence* of the







Lecture 2

Master Particle Physics I

Wouter Hulsbergen

September 5, 2011

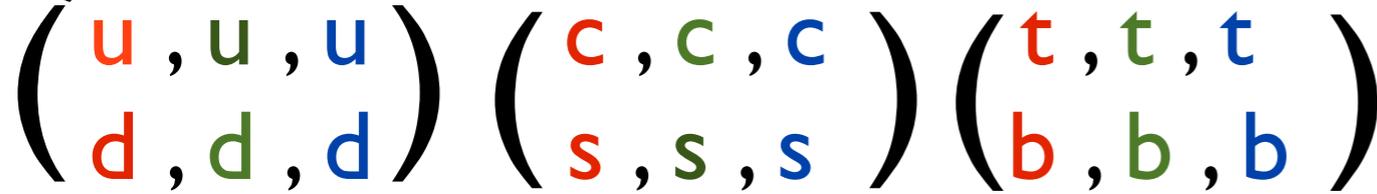
Concepts in lecture 1

- natural units
- observables
- Yukawa potential
- crossing
- strangeness, associated production
- the particle zoo: baryons, mesons, the “eightfold way”
- colour
- the standard model

Standard Model for particles

Fermions (Spin 1/2 particles) :The basic constituents of matter

Quarks:



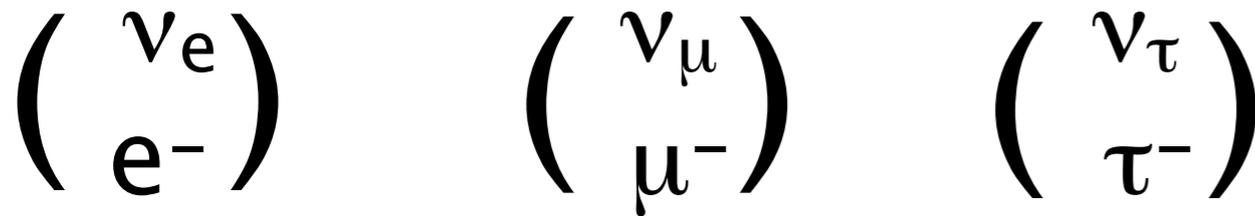
Quarks only occur in colour neutral objects: "Hadrons"

Baryons: qqq

Mesons: $q\bar{q}$

⇒ Hadron particles occur in multiplets

Leptons:

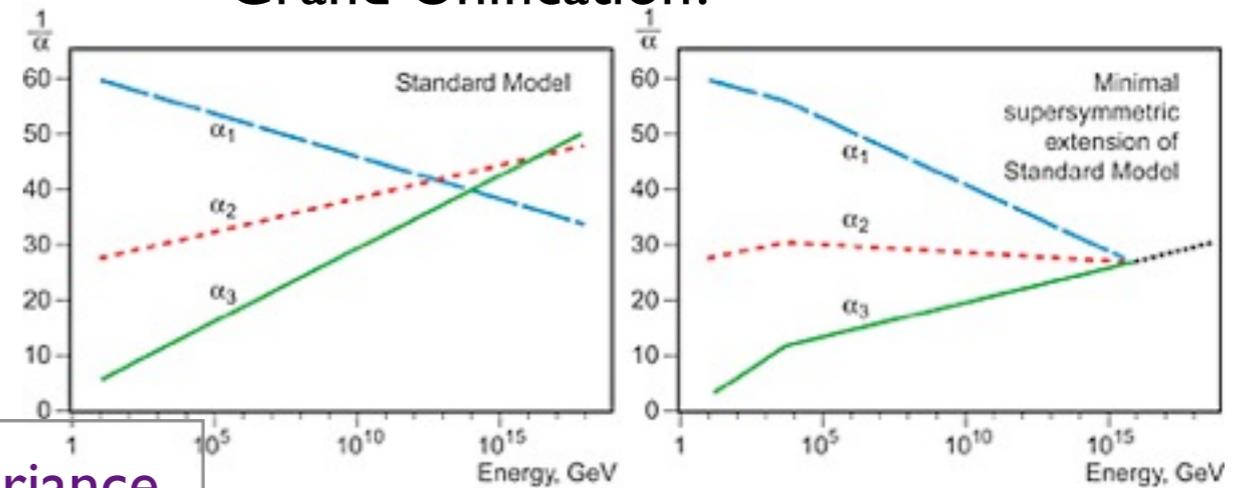


Only 3 generations of fundamental particles are known

Vector Bosons (Spin 1 particles). force carriers

Strong interaction	8 gluons
Weak interaction	W^+ W^- Z^0
Electromagnetic interaction	γ
Gravity	g

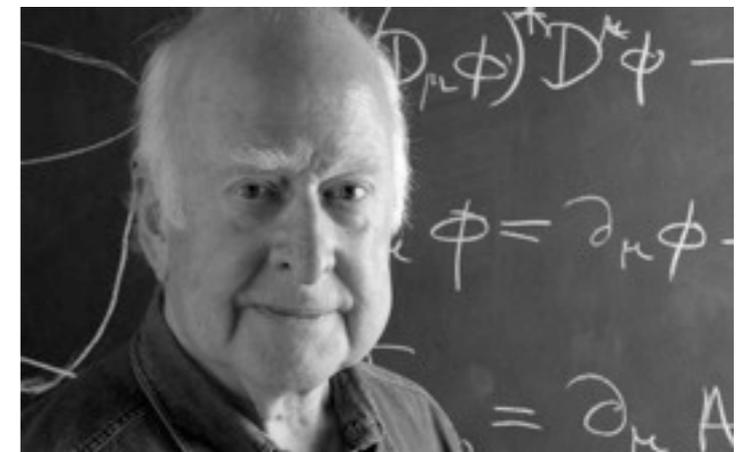
Grand Unification?



⇒ Forces originate from principle of local gauge invariance

Scalar Boson (Spin=0 particle).

Generates fermion masses via the Higgs mechanism



Lecture 3

Master Particle Physics I
Wouter Hulsbergen
September 10, 2011

Matter Waves for Particles without Spin

Non relativistic:

Kinematics:

$$E = \frac{\vec{p}^2}{2m}$$

Quantum Mechanics:

$$E \rightarrow \frac{\partial}{\partial t} \quad \text{and} \quad \vec{p} \rightarrow -i\vec{\nabla}$$

Wave Equation:

$$i\frac{\partial}{\partial t}\psi = \frac{-1}{2m}\nabla^2\psi$$

Relativistic:

$$E^2 = \vec{p}^2 + m^2$$

$$E \rightarrow i\frac{\partial}{\partial t} \quad \text{and} \quad \vec{p} \rightarrow -i\vec{\nabla}$$

$$-\frac{\partial^2}{\partial t^2}\phi = -\nabla^2\phi + m^2\phi$$

Continuity Equation:

$$\vec{\nabla} \cdot \vec{j} = -\frac{\partial \rho}{\partial t} \quad \text{or} \quad \partial_\mu j^\mu = 0$$

Probability density and current:

$$\rho = \psi^*\psi = |N|^2$$

$$\vec{j} = -\frac{i}{2m} \left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^* \right) \frac{|N|^2}{m} \vec{p}$$

$$\rho = i \left(\phi^* \frac{\partial \phi}{\partial t} - \phi \frac{\partial \phi^*}{\partial t} \right) = 2|N|^2 E$$

$$\vec{j} = -i \left(\phi^* \vec{\nabla} \phi - \phi \vec{\nabla} \phi^* \right) = 2|N|^2 \vec{p}$$

Negative Energy solutions: $j^\mu(+e) = 2e|N|^2 (E, \vec{p}) = -2e|N|^2 (-E, -\vec{p})$

The negative energy solution of a particle traveling backwards in time = the positive energy solution of the antiparticle traveling forwards in time

Lecture 4

Master Particle Physics I

Wouter Hulsbergen

September 12, 2011

Matter Waves and EM Field

Matter Waves

Non-relativistic:

$$E = \frac{\vec{p}^2}{2m}$$

$$i \frac{\partial}{\partial t} \psi = \frac{-1}{2m} \nabla^2 \psi$$

$$E \rightarrow i \frac{\partial}{\partial t} \quad ; \quad \vec{p} \rightarrow -i \vec{\nabla}$$

Relativistic:

$$E^2 = \vec{p}^2 + m^2$$

$$-\frac{\partial^2}{\partial t^2} \phi = -\nabla^2 \phi + m^2 \phi$$

Continuity: $\partial_\mu j^\mu = 0$ with: $j^\mu = i (\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*)$

EM Field

Maxwell 1+4
-> continuity

Maxwell Equations:

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= \rho \\ \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} &= 0 \\ \vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} &= \vec{j} \end{aligned}$$

$$\begin{aligned} \vec{B} &= \vec{\nabla} \times \vec{A} \\ \vec{E} &= -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla} V \\ A^\mu &= (V, \vec{A}) \\ j^\nu &= \partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu \end{aligned}$$

Gauge Invariance: $A^\mu \rightarrow A'^\mu = A^\mu + \partial^\mu \lambda$

Lorentz Condition: $\partial_\mu A^\mu = 0$

Coulomb Condition: $A^0 = 0 \quad ; \quad \vec{\nabla} \cdot \vec{A} = 0$

Photon has 2-polarizations!

Bohm - Aharanov Experiment!

Cross Section: $A + B \rightarrow C + D$

$$d\sigma = \frac{(2\pi)^4 \delta^4(p_A + p_B - p_C - p_D) \cdot |\mathcal{M}|^2 \cdot \frac{d^3p_C}{(2\pi)^3 2E_C} \frac{d^3p_D}{(2\pi)^3 2E_D}}{4\sqrt{(p_A \cdot p_B)^2 - m_A^2 m_B^2}}$$

Decay Rate: $A \rightarrow B + C$

$$d\Gamma = \frac{(2\pi)^4 \delta^4(p_A - p_B - p_C) \cdot |\mathcal{M}|^2 \cdot \frac{d^3p_B}{(2\pi)^3 2E_B} \frac{d^3p_C}{(2\pi)^3 2E_C}}{2E_A}$$

Lecture 5

Master Particle Physics I
Wouter Hulsbergen
September 17, 2011

The Story So far...

1) Plane Waves

Kinematics : $E^2 = \vec{p}^2 + m^2$; $E \rightarrow i \frac{\partial}{\partial t}$ $\vec{p} \rightarrow -i \vec{\nabla}$

Klein – Gordon : $(\partial_\mu \partial^\mu + m^2) \phi(x) = 0$; $\phi(x) = N e^{-ip_\mu x^\mu}$

Current : $j^\mu = -ie [\phi^* (\partial^\mu \phi) - (\partial^\mu \phi^*) \phi]$; $\partial_\mu j^\mu = 0$

2) Electromagnetic Field

Maxwell : $\partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = j^\nu$ or $\partial_\mu F^{\mu\nu} = j^\nu$

Gauge Freedom : $A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \lambda$

Lorentz Condition : $\partial_\mu A^\mu = 0 \Rightarrow$ Maxwell : $\partial_\mu \partial^\mu A^\nu = j^\nu$

Plane Waves : $A^\mu = N \mathcal{E}^\mu(\vec{p}) e^{-ip_\mu x^\mu} \Rightarrow 2$ polarizations

3) Scattering: $A_i + B_i \rightarrow C_f + D_f + \dots$

Today: Electromagnetic Scattering

- A particle in a potential
- Spinless π -K scattering

$$d\sigma_{fi} = \frac{W_{fi}}{Flux} d\Phi$$

$$W_{fi} = \lim_{T, V \rightarrow \infty} \frac{|T_{fi}|^2}{TV} ; T_{fi} = -i \int d^4x \psi_f^*(x) V(x) \psi_i(x)$$

$$T_{fi} = -i N_A N_B N_C^* N_D^* (2\pi)^4 \delta^4(p_A + p_B - p_C - p_D) \mathcal{M}$$

$$d\Phi = \sum_{i=1}^N \frac{V}{(2\pi)^3} \frac{d^3 \vec{p}_i}{2E_i} ; Flux = 4 \sqrt{(p_A \cdot p_B)^2 - m_A^2 m_B^2} / V^2$$

Lecture 6

Master Particle Physics I
Wouter Hulsbergen
September 19, 2011

1) Free particle wave equations

K.G.: $(\partial_\mu \partial^\mu + m^2)\phi(x) = 0$

Dirac: $(i\gamma^\mu \partial_\mu - m)\psi(x) = 0$

Plane wave solutions:

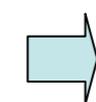
$\phi(x) = Ne^{-ipx}$

$\psi(x) = u(p)e^{-ipx}$

Current: $\partial_\mu j^\mu = 0$

$j^\mu = i[\phi^*(\partial^\mu \phi) - (\partial^\mu \phi^*)\phi]$

$j^\mu = \bar{\psi}\gamma^\mu\psi$



$\rho = 2|N|^2 E$

$\rho = \psi^\dagger\psi \geq 0$

2) Electromagnetic field

Maxwell

$\partial_\mu \partial^\mu A^\nu - \partial^\nu \partial_\mu A^\mu = j^\nu$

or $\partial_\mu F^{\mu\nu} = j^\nu$

Gauge freedom:

$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \lambda$

with $\partial_\nu \partial^\nu \lambda = 0$

Lorentz condition:

$\partial_\mu A^\mu = 0$

$\partial_\mu \partial^\mu A^\nu = j^\nu$

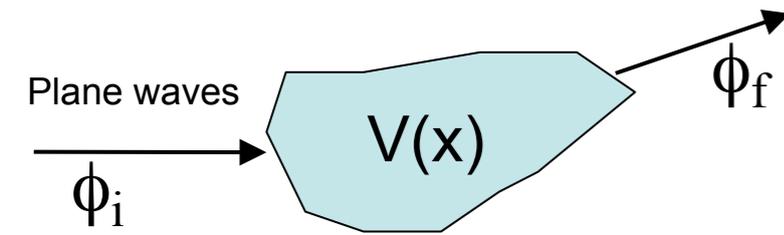
Plane wave solutions:

$A^\mu = N\epsilon^\mu(p)e^{-ipx}$

(2 polarizations since m=0)

3) Scattering Perturbation Theory

A: non-relativistic derivation:



$i\frac{\partial\psi}{\partial t} = (H_0 + V(x,t))\psi$

$\psi = \sum_{n=0}^{\infty} a_n(t)\phi_n(t)e^{-iEt}$

$T_{fi} = a_f(t \rightarrow \infty) = -i\int d^4x \phi_f^*(x)V(x)\phi_i(x)$

1-st order:

$W_{fi} = \lim_{T \rightarrow \infty} \frac{|T_{fi}|^2}{T}$ with

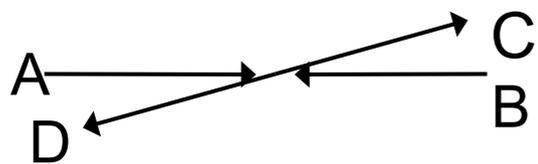
$T_{fi} = -2\pi i V_{fi} \delta(E_f - E_i)$ with

$V_{fi} = \int d^3x \phi_f^*(\vec{x})V(\vec{x})\phi_i(\vec{x})$

B: relativistic extension:

$W_{fi} = \lim_{T,V \rightarrow \infty} \frac{|T_{fi}|^2}{TV}$ and $T_{fi} = -iN_A N_B N_C^* N_D^* (2\pi)^4 \delta^4(p_A + p_B - p_C - p_D) \mathcal{M}$

C: cross section:



$\frac{d\sigma}{d\Omega} = \frac{W_{fi}}{flux} d\Phi$

$flux = 4\sqrt{(p_A p_B)^2 - m_A^2 m_B^2} / V^2$

$d\Phi = \prod_i \frac{V}{(2\pi)^3} \frac{d^3 p_i}{2E_i}$

$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{1}{s} \left| \frac{p_f}{p_i} \right| |\mathcal{M}|^2$

4) Electromagnetic Scattering

$\partial^\mu \rightarrow \partial^\mu - ieA^\mu$

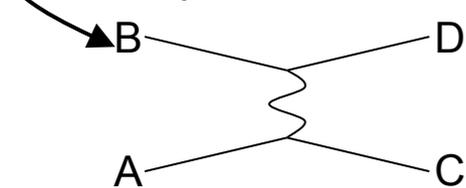
$(\partial_\mu \partial^\mu + m^2)\phi(x) \rightarrow (\partial_\mu \partial^\mu + m^2 + V(x))\phi(x)$; $V(x) = -ie\partial_\mu A^\mu + A_\mu \partial^\mu$

$T_{fi} = -i\int -ie[\phi_f^*(\partial_\mu \phi_i) - (\partial_\mu \phi_f^*)\phi_i] A^\mu d^4x$

$(i\gamma^\mu \partial_\mu - m)\psi(x) \rightarrow (i\gamma^\mu \partial_\mu - m + V(x))\psi(x)$; $V(x) = -e\gamma^0 \gamma_\mu A^\mu$

$T_{fi} = -i\int -e[\bar{\psi}_f \gamma_\mu \psi_i] A^\mu d^4x$

Spin 0 case:



$\phi_i = N_i e^{-ip_i x}$ $\phi_f^* = N_f^* e^{ip_f x}$

$\partial_\nu \partial^\nu A^\mu = -j_{BD}^\mu = -eN_B N_D^* (p_B^\mu + p_D^\mu) e^{i(p_D - p_B)x}$

$A^\mu = -\frac{1}{q^2} j_{BD}^\mu$

$T_{fi} = -i\int j_{AC}^\mu A^\mu d^4x = -i\int j_{AC}^\mu \frac{-g_{\mu\nu}}{q^2} j_{BD}^\nu d^4x = -iN_A N_B N_C^* N_D^* (2\pi)^4 \delta^4(p_A + p_B - p_C - p_D) \mathcal{M}$

$-i\mathcal{M} = ie(p_A + p_C)^\mu \frac{-ig_{\mu\nu}}{q^2} ie(p_B + p_D)^\nu \Rightarrow$ "Feynman rules"

Lecture 7

Master Particle Physics I
Wouter Hulsbergen
September 24, 2011

Particles with Spin = 0

$$E^2 = \vec{p}^2 + m^2$$

Klein Gordon:

$$(\partial_\mu \partial^\mu + m^2) \phi(x) = 0$$

$$\phi(x) = N e^{-ipx}$$

$$(\partial_\mu \partial^\mu + m^2) \phi(x) = 0 \quad \text{:C.C.}$$

$$j^\mu = i [\phi^* (\partial^\mu \phi) - (\partial^\mu \phi^*) \phi] \quad \partial_\mu j^\mu = 0$$

$$\begin{aligned} j_{fi}^\mu &= i [\phi_f^* (\partial^\mu \phi_i^*) - (\partial^\mu \phi_f^*) \phi_i] \\ &= -e N_i N_f^* (p_i^\mu + p_f^\mu) e^{i(p_f - p_i)x} \end{aligned}$$

Transition currents

Particles with Spin = 1/2

$$E^2 = (\vec{\alpha} \cdot \vec{p} + \beta m)^2$$

Dirac:

$$(i\gamma^\mu \partial_\mu - m) \psi(x) = 0$$

with : $\gamma^\mu = (\beta, \beta\vec{\alpha})$; $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$

Q.M.:

$$E \rightarrow i \frac{\partial}{\partial t} \quad ; \quad \vec{p} \rightarrow -i \vec{\nabla}$$

← Solutions →

$$\psi(x) = u(p) e^{-ipx} \quad \bar{\psi} = \psi^\dagger \gamma^0$$

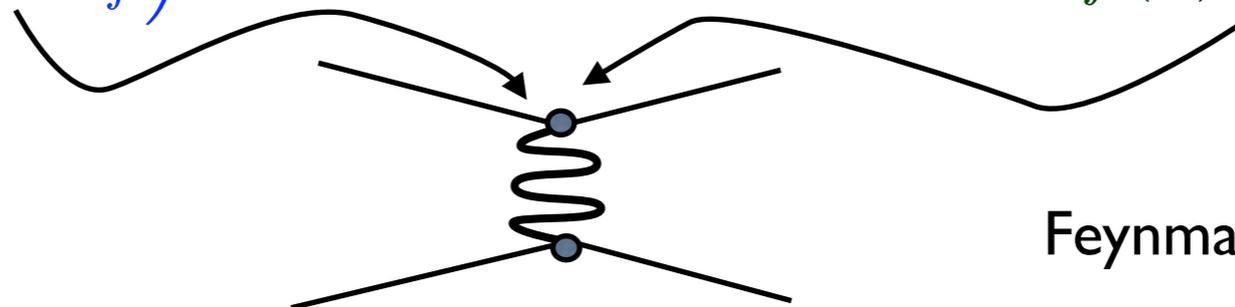
$$\text{Adjoint : } (i\partial_\mu \bar{\psi} \gamma^\mu + m \bar{\psi}) = 0 ;$$

$$j^\mu = \bar{\psi} \gamma^\mu \psi$$

$$j_{fi}^\mu = \bar{\psi}_f \gamma^\mu \psi_i$$

$$= -e \bar{u}_f(p) \gamma^\mu u_i(p) e^{i(p_f - p_i)x}$$

Feynman rules



Lecture 8

Master Particle Physics I

Wouter Hulsbergen

September 26, 2011

Lectures PPI

L0 : Introduction

L1 : Particles and Fields (historical overview)

L2 : Wave Equations and Antiparticles

L3 : The Electromagnetic Field

L4 : Perturbation Theory and Fermi's Golden Rule

L5 : Electromagnetic Scattering of Spinless Particles

QED of Spinless Particles:
Scattering Theory and
Cross Sections

L6 : The Dirac Equation

L7 : Solutions of the Dirac Equation

L8 : Spin 1/2 Electrodynamics

QED for
Fundamental Fermions

L9 : The Weak Interaction

(Fermi 4-point scattering: an analogy with QED)

L10: Local Gauge Invariance

(the role of symmetries in interactions)

L11: Electroweak Theory

L12: The Process: $e^+e^- \rightarrow \gamma, Z \rightarrow \mu^+\mu^-$

The Standard Model for
massless particles
 $SU(2)_L \times U(1)_Y$

Solutions to the Dirac Equation

$$(i\gamma^\mu \partial_\mu - m) \psi(x) = 0 \quad \Rightarrow \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} U_A(p) \\ U_B(p) \end{pmatrix} e^{-ipx}$$

$$\left[\begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} E - \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} p^i - \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} m \right] \begin{pmatrix} u_A \\ u_B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned} (\vec{\sigma} \cdot \vec{p}) U_B &= (E - m) U_A \\ (\vec{\sigma} \cdot \vec{p}) U_A &= (E + m) U_B \end{aligned} \quad U_A = \begin{pmatrix} * \\ * \end{pmatrix} ; \quad U_B = \begin{pmatrix} * \\ * \end{pmatrix} \quad \vec{\sigma} \cdot \vec{p} = \begin{pmatrix} p_z & p_x - ip_y \\ p_x + ip_y & -p_z \end{pmatrix}$$

$\vec{p} = 0$

$$U^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} ; \quad U^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} ; \quad U^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} ; \quad U^{(4)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \cdot e^{-ipx}$$

1) choose :

$$U_A^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} ; \quad U_A^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad U_B^{(1)} = \begin{pmatrix} p_z/(E+m) \\ (p_x + ip_y)/(E+m) \end{pmatrix} ; \quad U_B^{(2)} = \dots \text{ etc}$$

2) choose :

$$U_B^{(3)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} ; \quad U_B^{(4)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad U_A^{(3)} = \dots ; \quad U_A^{(4)} = \dots$$

1) Solutions are orthogonal

2) Normalisation: $N = \sqrt{E + m}$

3) Adjoint:

$$(\not{p} - m) u = 0 \quad \Rightarrow \quad \bar{u} (\not{p} - m) = 0$$

$$(\not{p} + m) v = 0 \quad \Rightarrow \quad \bar{v} (\not{p} + m) = 0$$

4) Completeness: $\sum_{s=1,2} u^{(s)}(p) \bar{u}^{(s)}(p) = \not{p} + m$

5) Helicity: $\lambda = \frac{1}{2} \vec{\Sigma} \cdot \vec{p} ; \quad \vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$

Lecture 9

Master Particle Physics I

Wouter Hulsbergen

October 1, 2011

QED

$$S=0$$

$$S=1/2$$

Wave equation: $(\partial_\mu \partial^\mu + m^2) \phi(x) = 0$

Solution: $\phi(x) = N e^{-ipx}$

Conserved current: $j^\mu = i [\phi^* (\partial^\mu \phi) - (\partial^\mu \phi^*) \phi]$

Perturbation Theory $T_{fi} = -i \int d^4x \phi_f^*(x) V(x) \phi_i(x)$

$(i\gamma^\mu \partial_\mu - m) \psi(x) = 0$

$\psi(x) = u(p) e^{-ipx}$

$j^\mu = \bar{\psi} \gamma^\mu \psi$

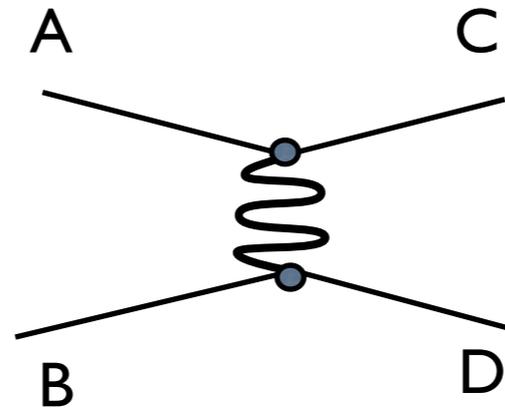
$T_{fi} = -i \int d^4x \psi^\dagger(x) V(x) \psi_i(x)$

Electromagnetic Field: $\partial_\mu F^{\mu\nu} = j^\nu$ with: $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

QED: $\partial^\mu \rightarrow \partial^\mu - ieA^\mu \Rightarrow T_{fi} = -i \int j_\mu^{fi}(x) A^\mu(x) d^4x$

Feynman Rules:

$$-i\mathcal{M} = \begin{matrix} ie(p_A + p_C)^\mu \\ \cdot -ig_{\mu\nu}/q^2 \\ \cdot ie(p_B + p_D)^\nu \end{matrix}$$



$$-i\mathcal{M} = \begin{matrix} ie(\bar{u}_C \gamma^\mu u_A) \\ \cdot -ig_{\mu\nu}/q^2 \\ \cdot ie(\bar{u}_D \gamma^\nu u_B) \end{matrix}$$

Cross Section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \left(\frac{3 + \cos \theta}{1 - \cos \theta} \right)^2 \quad \Leftarrow \quad \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{1}{s} |\mathcal{M}|^2 \quad \Rightarrow \quad \frac{d\sigma}{d\Omega} = \frac{\alpha}{2s} \frac{4 + (1 + \cos \theta)^2}{(1 - \cos \theta)^2}$$

Mandelstam Variables & Crossing:

$$e^+ e^- \rightarrow \mu^+ \mu^- = \frac{\alpha^2}{4s} (1 + \cos^2 \theta)$$

Lecture 10

Master Particle Physics I

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October 3, 2011

Matter Waves: $i(\gamma^\mu \partial_\mu - m)\psi(x) = 0$; $\psi(x) = U(p)e^{-ip_\mu x^\mu}$; $(\not{p} - m)u(p) = 0$

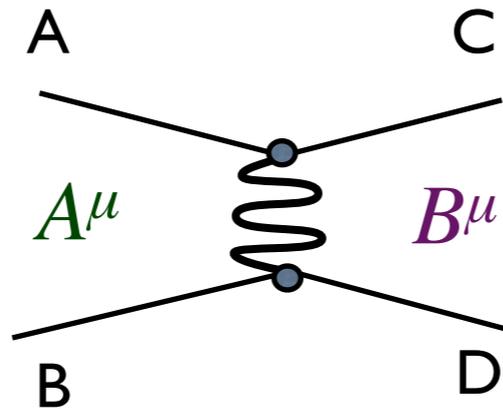
Substitution: $\partial^\mu \rightarrow \partial^\mu - ieA^\mu$; $\partial^\mu \rightarrow \partial^\mu + igB^\mu$

Field Equation: $\partial_\mu \partial^\mu A^\nu - \partial_\mu \partial^\nu A^\mu = j_{EM}^\nu$; $\partial_\mu \partial^\mu B^\nu - \partial_\mu \partial^\nu B^\mu + m^2 B^\nu = j_{Weak}^\nu$

Perturbation Theory $T_{fi} = -i \int j_{EM,\mu}^{fi} A^\mu(x) d^4x$; $T_{fi} = -i \int j_{Weak,\mu}^{fi} B^\mu(x) d^4x$

$j_{EM,\mu}^{fi} = \bar{\psi}_f \gamma^\mu \psi_i$; $j_{Weak,\mu}^{fi} = \bar{\psi}_f \frac{1}{2} \gamma^\mu (1 - \gamma^5) \psi_i$

Matrix Element:
 $-i\mathcal{M} = \begin{matrix} ie(\bar{u}_C \gamma^\mu u_A) \\ \cdot -ig_{\mu\nu}/q^2 \\ \cdot ie(\bar{u}_D \gamma^\nu u_B) \end{matrix}$



$-i\mathcal{M} = \begin{matrix} i\frac{g}{\sqrt{2}}(\bar{u}_C \frac{1}{2} \gamma^\mu (1 - \gamma^5) u_A) \\ \cdot -ig_{\mu\nu}/(M^2 - q^2) \\ \cdot i\frac{g}{\sqrt{2}}(\bar{u}_D \frac{1}{2} \gamma^\nu (1 - \gamma^5) u_B) \end{matrix}$

$e^+ e^- \rightarrow \mu^+ \mu^-$
 $\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} (1 + \cos^2 \theta)$

$\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$
 $\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}$

General Matrix Element: $\mathcal{M} = \sum_{i,j}^{S,V,T,P,A} C_{ij} (\bar{u}_C \mathcal{O}_i u_A) \cdot \text{prop} \cdot (\bar{u}_D \mathcal{O}_j u_B)$
 $S = \bar{\psi}\psi$; $V = \bar{\psi}\gamma^\mu\psi$; $T = \bar{\psi}\sigma^{\mu\nu}\psi$; $A = \bar{\psi}\gamma^5\gamma^\mu\psi$; $P = \bar{\psi}\gamma^5\psi$

Weak Interaction:
 Leptons : $\begin{pmatrix} \nu_e \\ e \end{pmatrix}$; $\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$; $\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \Rightarrow \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V_{PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$
 Quarks : $\begin{pmatrix} u \\ d \end{pmatrix}$; $\begin{pmatrix} c \\ s \end{pmatrix}$; $\begin{pmatrix} t \\ b \end{pmatrix} \Rightarrow \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$

Lecture 11

Master Particle Physics I

Wouter Hulsbergen

October 8, 2011

Symmetries

Lagrangian density is the basic object for physics: $\mathcal{L}(\phi(x), \partial_\mu \phi(x)) = \bar{\psi} (i\gamma^\mu \mathcal{D}_\mu - m) \psi$

Symmetry: Require that the Lagrangian remains invariant under a symmetry operation.

4 Basic symmetry groups:

- Permutation symmetries
- Continuous space-time symmetries (“external” symmetries)
- Discrete symmetries: C, P, T
- Unitary or Gauge symmetries (“internal” symmetries)

Unitary Phase Symmetry U(1)

$$\bar{\psi} (i\gamma^\mu \mathcal{D}_\mu - m) \psi$$

Yang Mills Symmetry SU(2)

$$\psi(x) = \begin{pmatrix} p \\ n \end{pmatrix}$$

Covariant Derivative:

$$\mathcal{D}_\mu = \partial_\mu + iqA_\mu(x)$$

Gauge Transformations:

$$\psi(x) \rightarrow \psi'(x) = e^{iq\alpha(x)} \psi(x)$$

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) - \partial_\mu \alpha(x)$$

(Maxwell gauge invariance)

U(1) Lagrangian:

$$\begin{aligned} \bar{\psi} (i\gamma^\mu \mathcal{D}_\mu - m) \psi &= \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi - q\bar{\psi} \gamma^\mu \psi A_\mu \\ \mathcal{L}_{U(1)} &= \mathcal{L}_{U(1)}^{\text{free}} + j^\mu A_\mu \end{aligned}$$

Covariant Derivative:

$$\mathcal{D}_\mu = \partial_\mu + igB_\mu(x) \quad \text{with} \quad B_\mu(x) = \frac{1}{2} \vec{\tau} \cdot \vec{b}_\mu$$

Gauge Transformations:

$$\psi(x) \rightarrow \psi'(x) = e^{i\frac{1}{2} \vec{\tau} \cdot \vec{\alpha}(x)} \psi(x)$$

$$B_\mu(x) \rightarrow B'_\mu(x) = GB_\mu(x)G^{-1} + \frac{i}{g} (\partial_\mu G) G^{-1}$$

$$\vec{b}_\mu(x) \rightarrow \vec{b}'_\mu(x) = \vec{b}_\mu - \vec{\alpha} \times \vec{b}_\mu - \frac{1}{g} \partial_\mu \alpha(x)$$

“non-Abelian”

SU(2) Lagrangian:

$$\begin{aligned} \bar{\psi} (i\gamma^\mu \mathcal{D}_\mu - m) \psi &= \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi - \frac{g}{2} \bar{\psi} \gamma^\mu \vec{\tau} \psi \vec{b}_\mu \\ \mathcal{L}_{SU(2)} &= \mathcal{L}_{SU(2)}^{\text{free}} + \vec{j}^\mu \vec{b}_\mu \end{aligned}$$

Lecture 12

Master Particle Physics I

Wouter Hulsbergen

October 10, 2011

Standard Model of Electroweak Interactions

Origin of interactions is described via the principle of *local gauge invariance*

Recipe: Take the Lagrangian of *free Dirac particles*:

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}(x) (i\gamma^\mu \partial_\mu - m) \psi(x)$$

and impose that it should *remain invariant* under:

$$U(1)_Y : \psi(x) \rightarrow \psi'(x) = e^{iY\alpha(x)} \psi(x) \quad ; \quad Y = \text{hypercharge} \quad , \quad Q = T_3 + \frac{1}{2}Y$$

$$SU(2) : \psi_L(x) \rightarrow \psi'_L(x) = e^{i\vec{T} \cdot \vec{\alpha}(x)} \psi_L(x) \quad ; \quad \vec{T} = \frac{1}{2}\vec{\tau} = \text{weak isospin} \quad , \quad \psi_L = \left(\frac{1 - \gamma^5}{2} \right) \psi$$

To keep the Lagrangian invariant *compensating gauge fields* must be introduced which transform simultaneously with the Dirac fields:

$$\partial_\mu \rightarrow \mathcal{D}_\mu = \partial_\mu + g' \frac{Y}{2} a_\mu + g \vec{T} \cdot \vec{b}_\mu \quad a_\mu = \text{hypercharge field}$$

$$\mathcal{L} = \mathcal{L}_{\text{free}} - g' \frac{J_Y^\mu}{2} a_\mu - g \vec{J}_L^\mu \cdot \vec{b}_\mu \quad b_\mu^1, b_\mu^2, b_\mu^3 = \text{weak isospin field}$$

$$\text{The physical currents are: C.C.: } W_\mu^+ = \frac{b_\mu^1 - ib_\mu^2}{\sqrt{2}} \quad \text{N.C.: } Z_\mu = -a_\mu \sin \theta_W + b_\mu^3 \cos \theta_W$$

$$W_\mu^- = \frac{b_\mu^1 + ib_\mu^2}{\sqrt{2}} \quad A_\mu = a_\mu \cos \theta_W + b_\mu^3 \sin \theta_W$$

The interaction Lagrangian is:

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{\text{free}} & - \frac{g}{\sqrt{2}} \bar{\psi}_u \gamma^\mu \frac{1}{2} (1 - \gamma^5) \psi_d W_\mu^+ \\ & - \frac{g}{\sqrt{2}} \bar{\psi}_d \gamma^\mu \frac{1}{2} (1 - \gamma^5) \psi_u W_\mu^- \\ & - e Q \bar{\psi} \gamma^\mu \psi A_\mu \\ & - g_z \bar{\psi} \gamma^\mu \frac{1}{2} (C_V^f - C_A^f \gamma^5) \psi Z_\mu \end{aligned}$$

$$e = g \sin \theta_W \quad g' / g = \tan \theta_W$$

$$g_z = g / \cos \theta_W$$

$$C_V^f = T_3^f - 2Q^f \sin^2 \theta_W$$

$$C_A^f = T_3^f$$

$$T_3 = \frac{1}{2} \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} \nu \\ l \end{pmatrix}$$

Feynman Vertices

E.M.:

e^-, μ^-, τ^-

e^+, μ^+, τ^+

f

q

\bar{q}

ν

$\bar{\nu}$

N.C.:

Z^0

f

\bar{f}

$g_z = \frac{g}{\cos \theta_W}$

$C_V^f = T_3^f - 2Q^f \sin^2 \theta_W$

$C_A^f = T_3^f$

C.C.:

W

$g/\sqrt{2}$

ν_{eL}

e_L

u

d'

W

$g/\sqrt{2}$

$\nu_{\mu L}$

μ_L

c

s'

W

$g/\sqrt{2}$

$\nu_{\tau L}$

τ_L

t

b'

$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{CKM} \end{pmatrix} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$

$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{PMNS} \end{pmatrix} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$

PP2

- contents of PP2 course:
 - QCD (Michiel Botje),
 - Higgs (Ivo van Vulpen)
- start: in ~ 3 weeks
- web page QCD part
<http://www.nikhef.nl/~h24/qcdcourse/>
- web page Higgs part
http://www.nikhef.nl/~ivov/lvo_teaching.html