Asymmetric Gepner Models

(Revisited)

Bert Schellekens
NIKHEF

String Phenomenology 2009
Warszawa

Landscape archeology:

Recovering the lost results of an abandoned project

CERN–TH.5440/89

New Modular Invariants for N=2 Tensor Products and Four-Dimensional Strings

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and

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ABSTRACT

The construction of modular invariant partition functions of tensor products of $N = 2$ superconformal field theories is clarified and extended by means of a recently proposed method using simple currents, *i.e.* primary fields with simple fusion rules. Apart from providing a conceptually much simpler way of understanding space-time and world-sheet supersymmetry projections in modular invariant string theories, this makes a large class of modular invariant partition functions accessible for investigation. We demonstrate this by constructing thousands of (2,2), (1,2) and (0,2) string theories in four dimensions, including more than 40 new three generation models.

⋆ Work supported in part by the US-Israel Binational Science Foundation, and the Israel Academy of Science.

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Heterotic Strings Considered:

Right	**Left**
NSR	$SO(10) \times E_8$
N=2 minimal k_1	N=2 minimal k_1
N=2 minimal k_2	N=2 minimal k_2
…	…
N=2 minimal k_{n-1}	N=2 minimal k_{n-1}
N=2 minimal k_n	N=2 minimal k_n

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…	…
N=2 minimal k_{n-1}	N=2 minimal k_{n-1}
N=2 minimal k_n	N=2 minimal k_n

$$\sum_i \frac{3k_i}{k_i+2} = 9$$

Heterotic Strings Considered:

Modular invariance: bosonic string map(*)

Right	Left
NSR ⟷	$SO(10) \times E_8$
N=2 minimal k_1	N=2 minimal k_1
N=2 minimal k_2	N=2 minimal k_2
…	…
N=2 minimal k_{n-1}	N=2 minimal k_{n-1}
N=2 minimal k_n	N=2 minimal k_n

(*) Lerche, Lüst, Schellekens, 1986

Heterotic Strings Considered:

World sheet susy: "alignment currents"

Right	Left
NSR	$SO(10) \times E_8$
N=2 minimal k_1	N=2 minimal k_1
N=2 minimal k_2	N=2 minimal k_2
…	…
N=2 minimal k_{n-1}	N=2 minimal k_{n-1}
N=2 minimal k_n	N=2 minimal k_n

Heterotic Strings Considered:

Space-time susy: chiral algebra extension

Right	Left
NSR	$SO(10) \times E_8$
N=2 minimal k_1	N=2 minimal k_1
N=2 minimal k_2	N=2 minimal k_2
…	…
N=2 minimal k_{n-1}	N=2 minimal k_{n-1}
N=2 minimal k_n	N=2 minimal k_n

Start with the diagonal invariant, and modify it with simple currents without insisting on worldsheet or space-time supersymmetry in the left (bosonic) sector.

This gives (2,2), (2,1) and (2,0) heterotic strings with chiral fermions in (16)'s of SO(10) or (27)'s of E_6.

Result

A huge "phone-book" of tables of (2,2) and (2,1) spectra.

For example:

(3,3,3,3,3)									
nr.	N_R	N_L	(S,s)	(S,c)	$(S,0)$	$(V,0)$	(V,v)	(S,v)	order
1	1	1	101	1	330	4	-	-	1
2	1	1	49	5	258	4	-	-	2
3	1	1	21	1	210	4	-	-	4
4	1	1	23	7	222	4	-	-	3
5	1	1	21	17	234	4	-	-	2
6	1	1	13	9	210	4	-	-	3
7	1	0	56	0	252	9	-	48	2
8	1	0	40	0	212	9	-	32	3
9	1	0	32	0	192	9	-	24	4
10	1	0	32	0	178	13	-	20	4
11	1	0	30	2	232	5	-	32	3
12	1	0	26	6	232	5	-	32	2
13	1	0	22	2	212	5	-	24	4
14	1	0	24	8	218	9	-	28	3
15	1	0	24	8	178	13	-	20	3

But: the tables were not published and not properly stored...
(Scanned version and also a new complete set of spectra for the (2,2) case available via my home page, www.nikhef.nl/~t58)

The (2,2) spectra were also computed by Fuchs, Klemm, Scheich and Schmidt, but their results are also lost.

Number of families:

Quantized in certain units Δ for each of the 168 combinations of Gepner models.

The following values occur for the
120, 96, 72, 60, 48, 40, 36, 32, 24, 12, 8, 6, 4 and 0.

There is one known way to get multiples of 3:
Use (1,16,16,16) with exceptional invariants in all three factors with k=16 (Gepner, unpublished).

This allowed us to get 3-family (2,2), (2,1) and (2,0) models with gauge groups SO(10) or E_6 (44 distinct ones)

nr.	N_R	N_L	(S,s)	(S,c)	$(S,0)$	$(V,0)$	(V,v)	(S,v)	currents
1	1	0	16	13	230	4	-	35	[v,(0,0,0),(0,7,1),(16,16,0),(0,0,0)]
2	1	0*	16	13	204	4	-	29	[v,(0,0,0),(0,14,0),(0,10,0),(0,-16,0)]
3	1	1	15	12	201	3	-	-	[v,(0,0,0),(0,0,0),(0,0,0),(0,11,1)]
									[0,(0,0,0),(16,-1,1),(16,-11,1),(0,1,1)]
4	1	1	15	12	185	3	-	-	[s,(1,-2,1),(0,0,0),(0,18,0),(16,-16,0)]
5	1	0	15	12	258	4	-	39	[c,(0,1,1),(0,-2,0),(0,-11,1),(0,1,1)]
6	1	0	15	12	226	4	-	31	[0,(0,2,0),(0,0,0),(0,9,1),(0,4,0)]
7	1	0	15	12	210	4	-	31	[0,(0,0,0),(0,16,10),(0,0,0),(0,1,1)]

....

42	1	0	8	5	208	4	-	21	[v,(1,1,0),(0,-9,1),(0,1,1),(0,-12,0)]
									[0,(0,0,0),(16,10,0),(0,1,1),(0,-6,0)]
43	1	0	8	5	206	4	-	21	[v,(0,0,0),(0,-16,0),(0,1,1),(0,0,0)]
									[v,(1,-1,0),(0,-16,0),(0,1,1),(16,0,0)]
44	1	0	8	5	200	4	-	19	[v,(1,-1,0),(0,9,1),(0,1,1),(0,0,0)]
									[s,(0,1,1),(16,-3,1),(0,0,0),(0,1,1)]

6. Outlook and conclusions

Clearly the method we have advocated in this paper greatly extends the list of four-dimensional string theories accessible to exploration. However, this is by no means all one can do. Up to now we have always kept an unbroken $SO(10) \times E_8$ Kac-Moody algebra on the left. However, just as one can break the left-moving "space-time" and world-sheet supersymmetries, one can break this KM-algebra as well. To do so, one simply starts with characters of some conformal sub-algebra of $SO(10) \times E_8$. Of course one wants to get the full $SO(10) \times E_8$ algebra on the *right*, in order to be able to map this sector to a fermionic one. But this can always be achieved by putting some projection matrices in front of the right-moving characters to add the missing $SO(10) \times E_8$ roots.

This opens the way to constructing string theories whose gauge group is something a bit closer to the standard model than $SO(10)$, perhaps even $SU(3) \times SU(2) \times U(1)^n$ (where n is almost inevitably larger than 1). There is no reason why one could not get 3 generations in such a model, and in fact there could well be many more models than those listed in table III, since the center of the conformal field theory one starts with is even larger. We hope to come back to this in the future.

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The future is now!

(work in progress with Beatriz Gato-Rivera)

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Meanwhile this idea was used by Blumenhagen en Wisskirchen (1996)
See also Kreuzer (2009)

What I (probably) tried in 1989:
Consider $SU(3) \times SU(2) \times U(1)_{30} \times U(1)_{20} \subset SO(10)$

We extend this to SO(10), but only in the fermionic sector, then map it to NSR.

This should give chiral families of $SU(3) \times SU(2) \times U(1)$.

Indeed, it does, but there was a major disappointment:
All these spectra had fractionally charged particles.

This was easily seen to be a very general result.
(Phys. Lett. B237, 363, 1990).

In heterotic strings unification (SO(10) or E_6) seems "natural"
(bosonic string map, spin-connection embedded in E_8)

But one beautiful feature of SU(5) GUTs, an explanation for the observed charge quantization, is lost when one breaks the GUT group in CFT.

This can in principle be avoided:

- Massive or non-chiral fractional charges
- Additional confinement groups
- Higher level affine Lie-algebras
- Non-GUT U(1) normalization
- Other string theories (orientifolds, F-theory ....)

But only in the first case the nice heterotic realization of GUTs would remain more or less intact.

This was too hard to analyse in 1989.

Modular Invariant Partition Function:

$$\sum_{ij} \chi_i(\bar{\tau}) M^{\text{proj}}_{il} \; M_{lk}(J_1, \ldots, J_n) \; \chi_k(\tau)$$

Worldsheet susy
Space-time susy
SO(10) projection

$N \times N$ matrix
for n simple currents

For K minimal models:

$$N = 3 \times 2 \times 60 \times 20 \times \prod_i^K N_i$$

$$n \leq K + 4$$

(3,3,3,3,3)

368.640.000.000

9

Potentially a huge landscape:

For K currents of order p (prime)

(B. Gato-Rivera, A.N. Schellekens, Comm. Math. Phys. 145, 85 (1992))

$$N_{\mathrm{MIPF}} = \prod_{l=0}^{K-1} (1 + p^l)$$

The seven $\mathbf{Z}_5$ factors in $SU(3) \times SU(2) \times U_{30} \times U_{20} \times (k=3)^5$ contribute a factor

1.202.088.011.709.312

This is reduced by at most $5! \times 2^8$ (permutations, outer automorphisms), and enhanced by a factor 8 for $(\mathbf{Z}_3)^2$ and an unknown, huge factor for $(\mathbf{Z}_2)^2 \times (\mathbf{Z}_4)^6$

Some questions that remained unanswered in 1989:

- How is Δ affected by breaking SO(10) and world-sheet supersymmetry?
- Are the fractionally charge particles chiral?
- What do distributions of families look like?
- Can we get three families of SU(3) × SU(2) × U(1)?

Discrete SM features from RCFT

- The Standard Model spectrum can be obtained

```
Gauge group: U(3) x Sp(2) x U(1) x U(1)

Q    7 x (V ,V ,0 ,0 ) chirality 3
U    3 x (V ,0 ,V ,0 ) chirality -3
D    3 x (V ,0 ,V*,0 ) chirality -3
L    9 x (0 ,V ,0 ,V ) chirality 3
E    5 x (0 ,0 ,V ,V ) chirality -3
     3 x (0 ,0 ,V ,V*) chirality 3
     6 x (V ,0 ,0 ,V )
H   10 x (0 ,V ,V ,0 )
     2 x (Ad,0 ,0 ,0 )
     2 x (A ,0 ,0 ,0 )
     6 x (S ,0 ,0 ,0 )
    14 x (0 ,A ,0 ,0 )
    10 x (0 ,S ,0 ,0 )
     9 x (0 ,0 ,Ad,0 )
     6 x (0 ,0 ,A ,0 )
    14 x (0 ,0 ,S ,0 )
     3 x (0 ,0 ,0 ,Ad)
     4 x (0 ,0 ,0 ,A )
     6 x (0 ,0 ,0 ,S )
```

No hidden sector

B-L Massive (axion mixing)

Gauge group:

Exactly SU(3)×SU(2)×U(1)

Supersymmetric standard model spectra from RCFT orientifolds. (Nucl.Phys.B710:3-57,2005)
T.P.T. Dijkstra, L.R. Huiszoon, A.N. Schellekens

Discrete SM features from RCFT

- The Standard Model spectrum can be obtained
- But several unwanted features tend to come out too easily:
  Non-chiral particles
  Number of families
  Fractional charges
  Massless B-L

  Why is this not what we see?
- We have by now quite a bit of “statistical” information about the Standard Model embedded in orientifolds.
- But very little is known about similar questions in heterotic strings (cf. Dienes et. al.)
- This is why it would be nice to have the results of the abandoned 1989 project.

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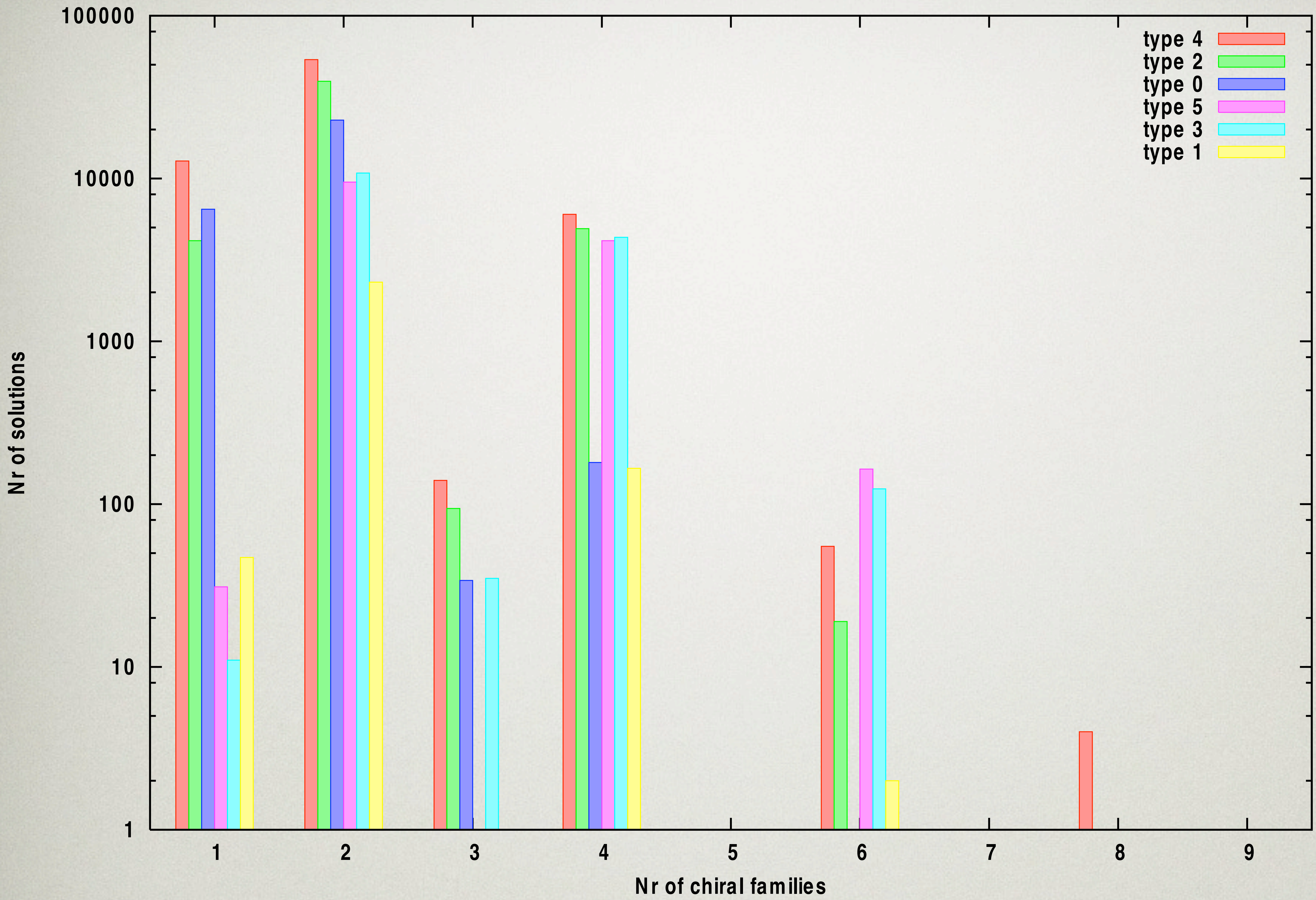
Other sources of inspiration:
- Heterotic mini-landscape
- Free fermionic 3-family models
- Philadelphia sushi bar

The number of families



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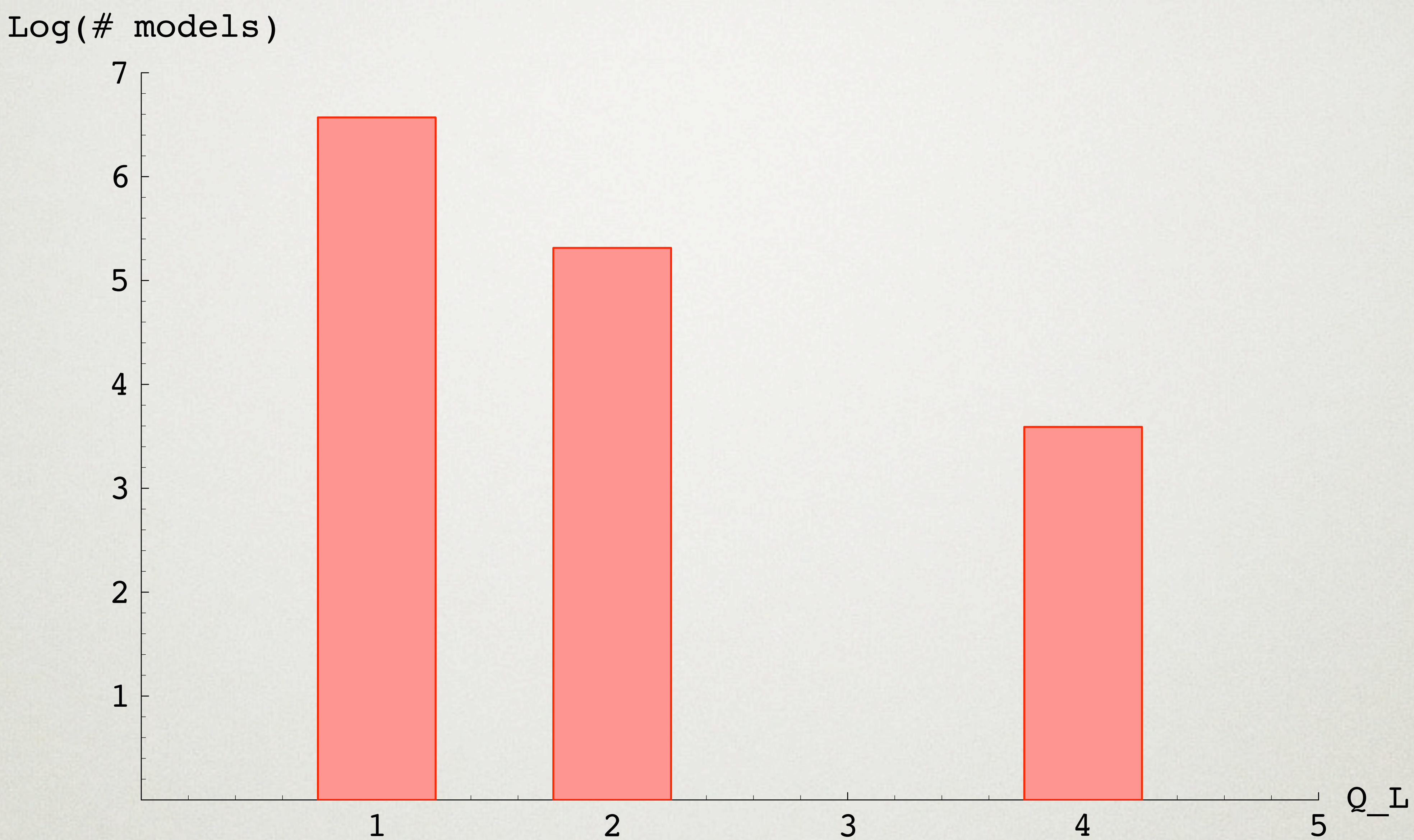
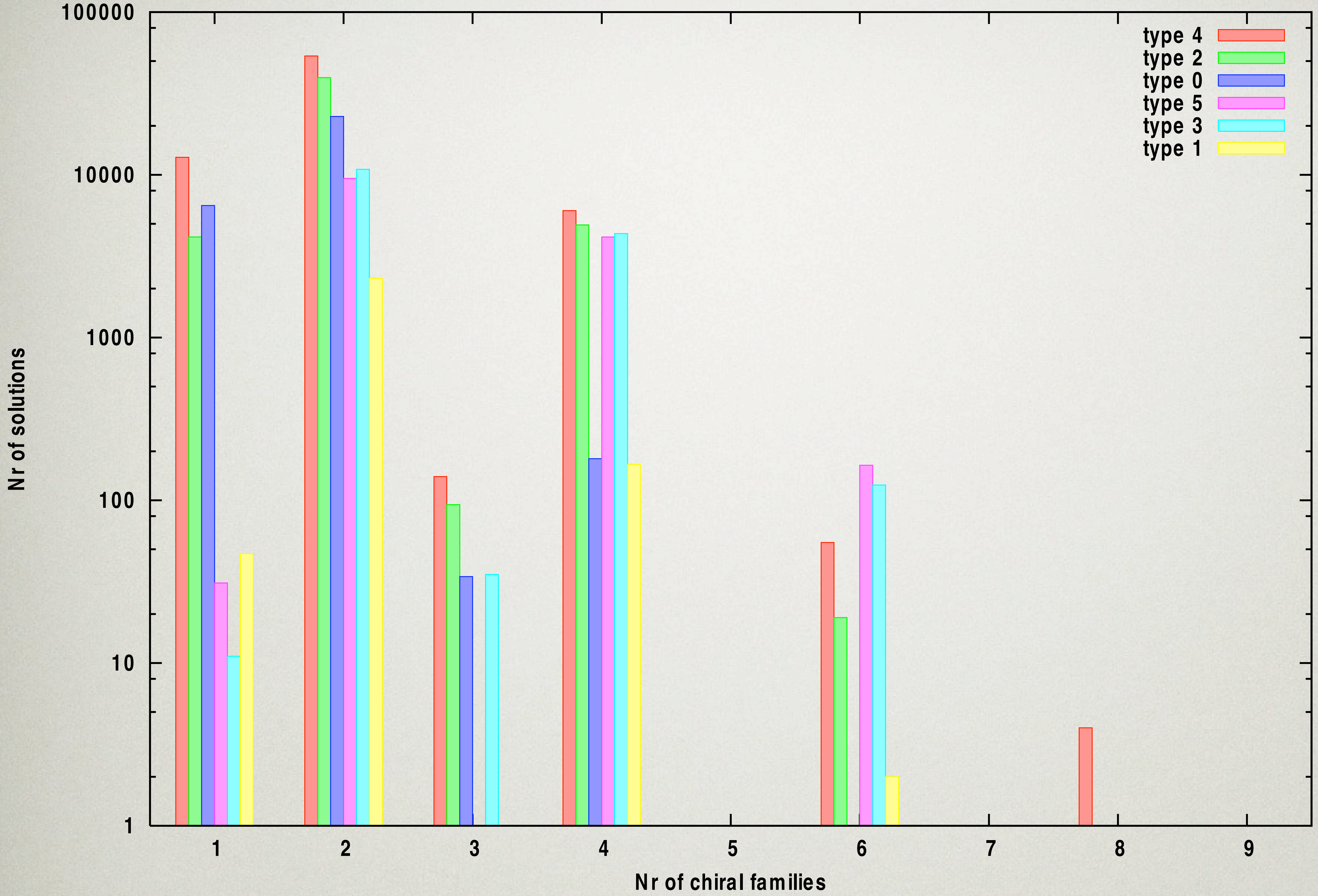
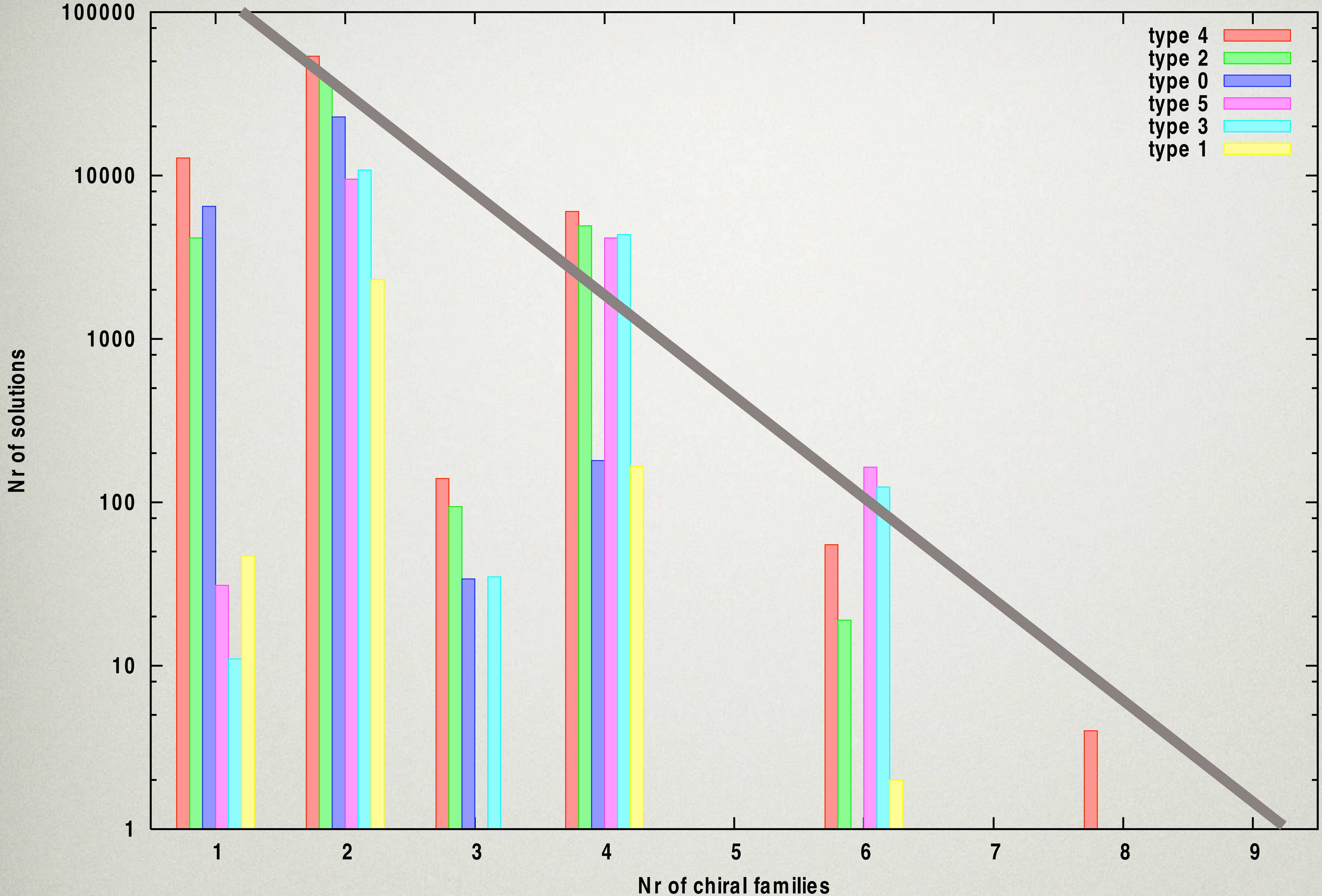
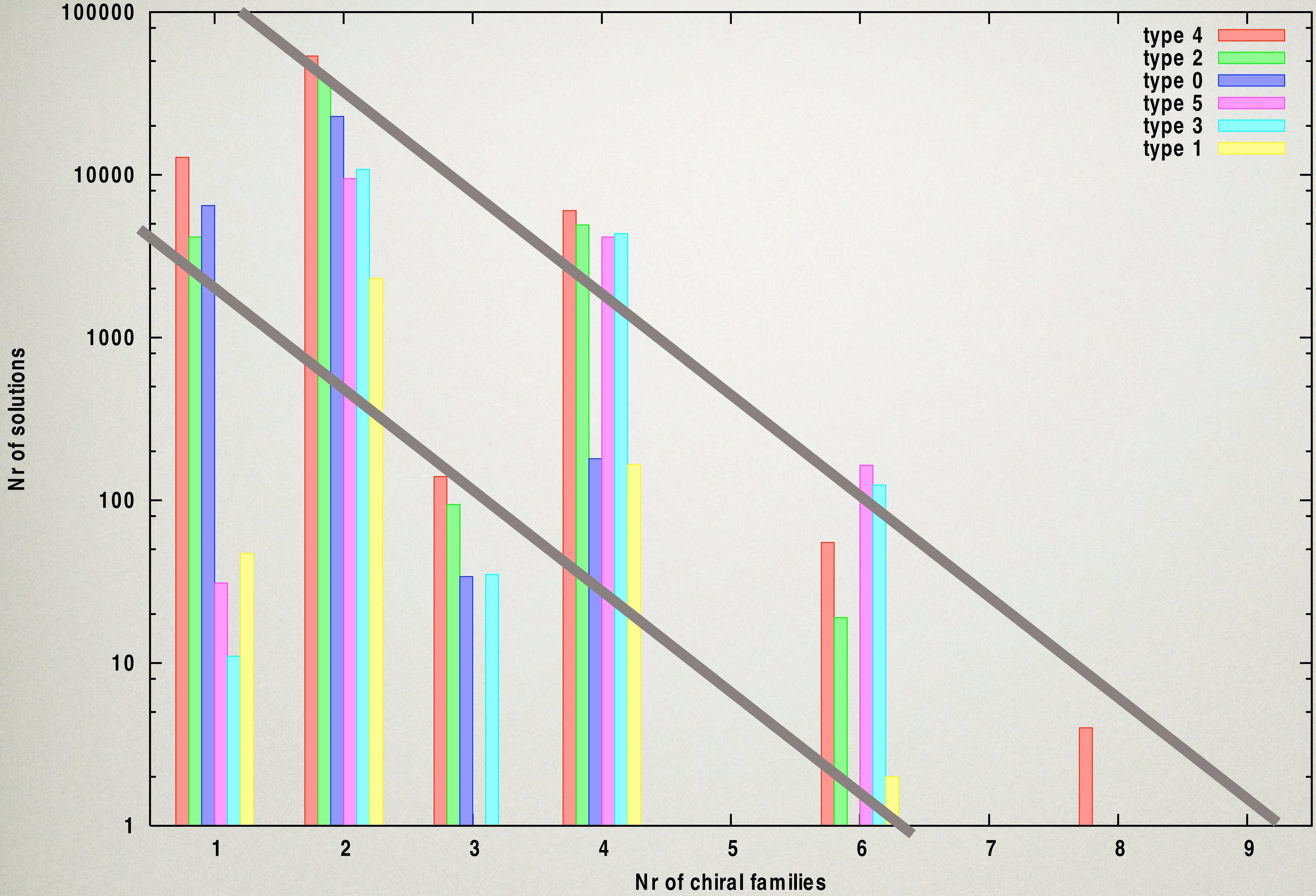


One in a Billion: MSSM-like D-Brane Statistics (JHEP 0601:004,2006)
Florian Gmeiner, Ralph Blumenhagen, Gabriele Honecker, Dieter Lust, Timo Weigand



Sunday, 2 May, 2010


100000
10000
1000
100
10
1
Nr of solutions
1
2
3
4
5
6
7
8
9
Nr of chiral families
type 4
type 2
type 0
type 5
type 3
type 1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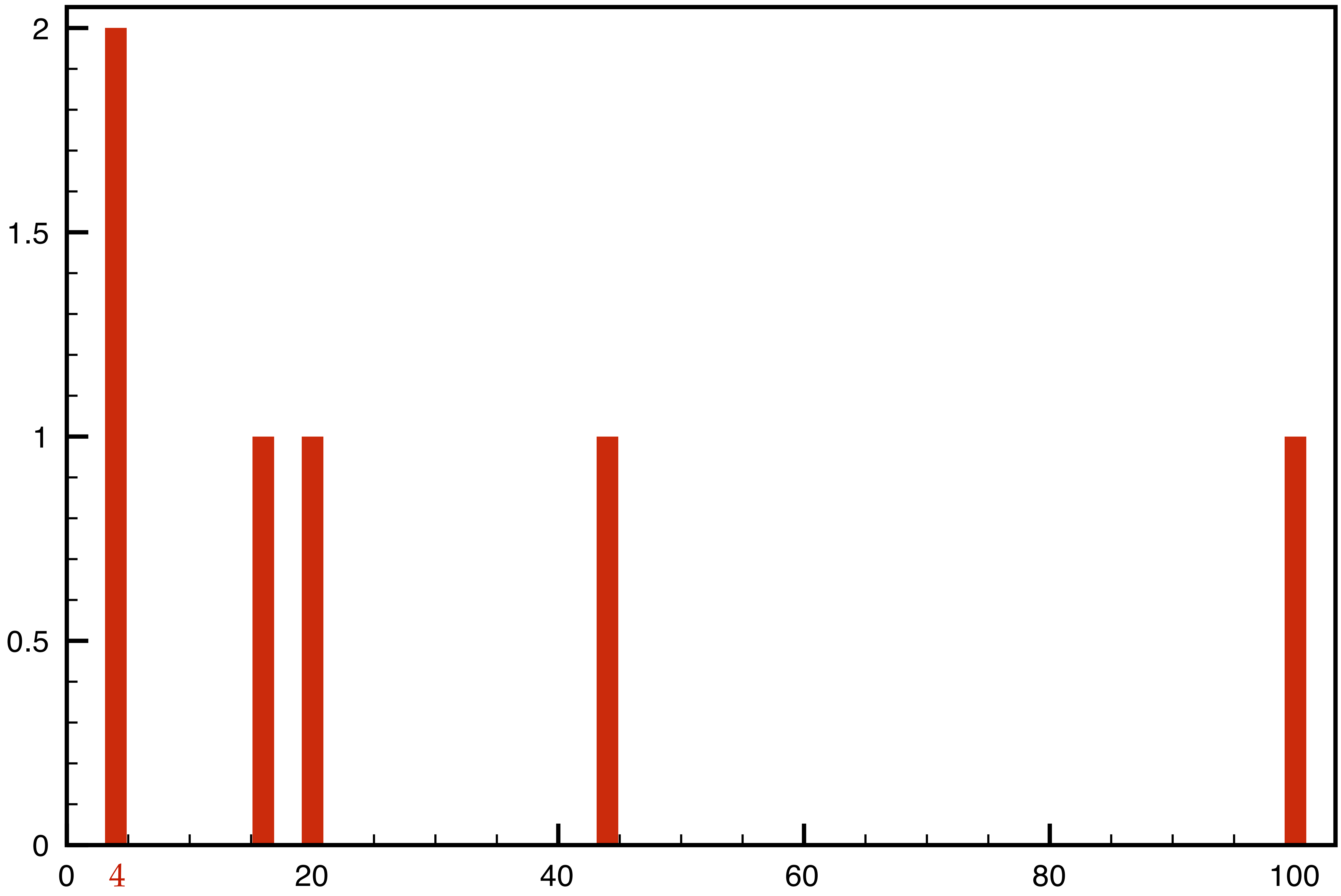
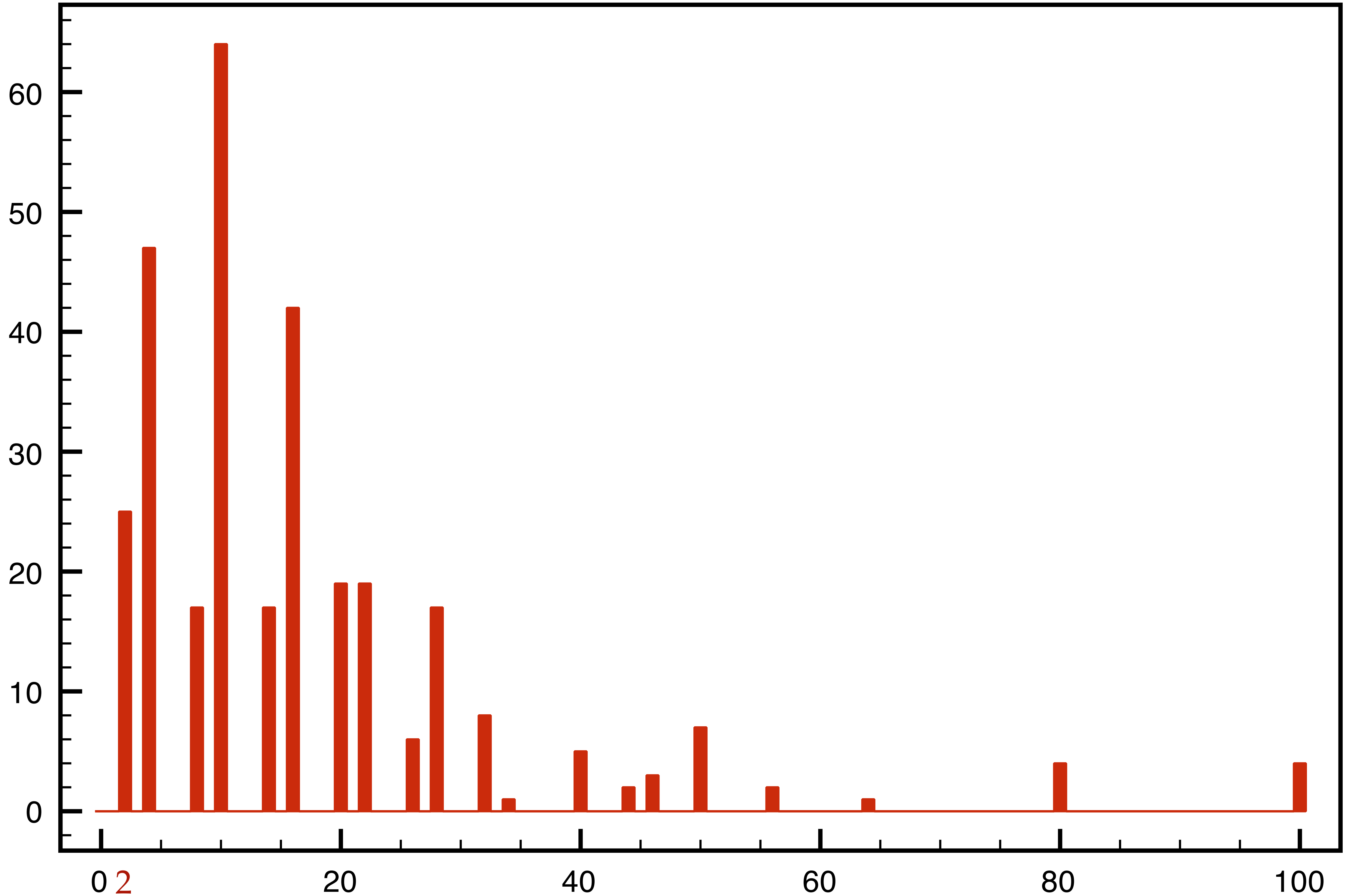
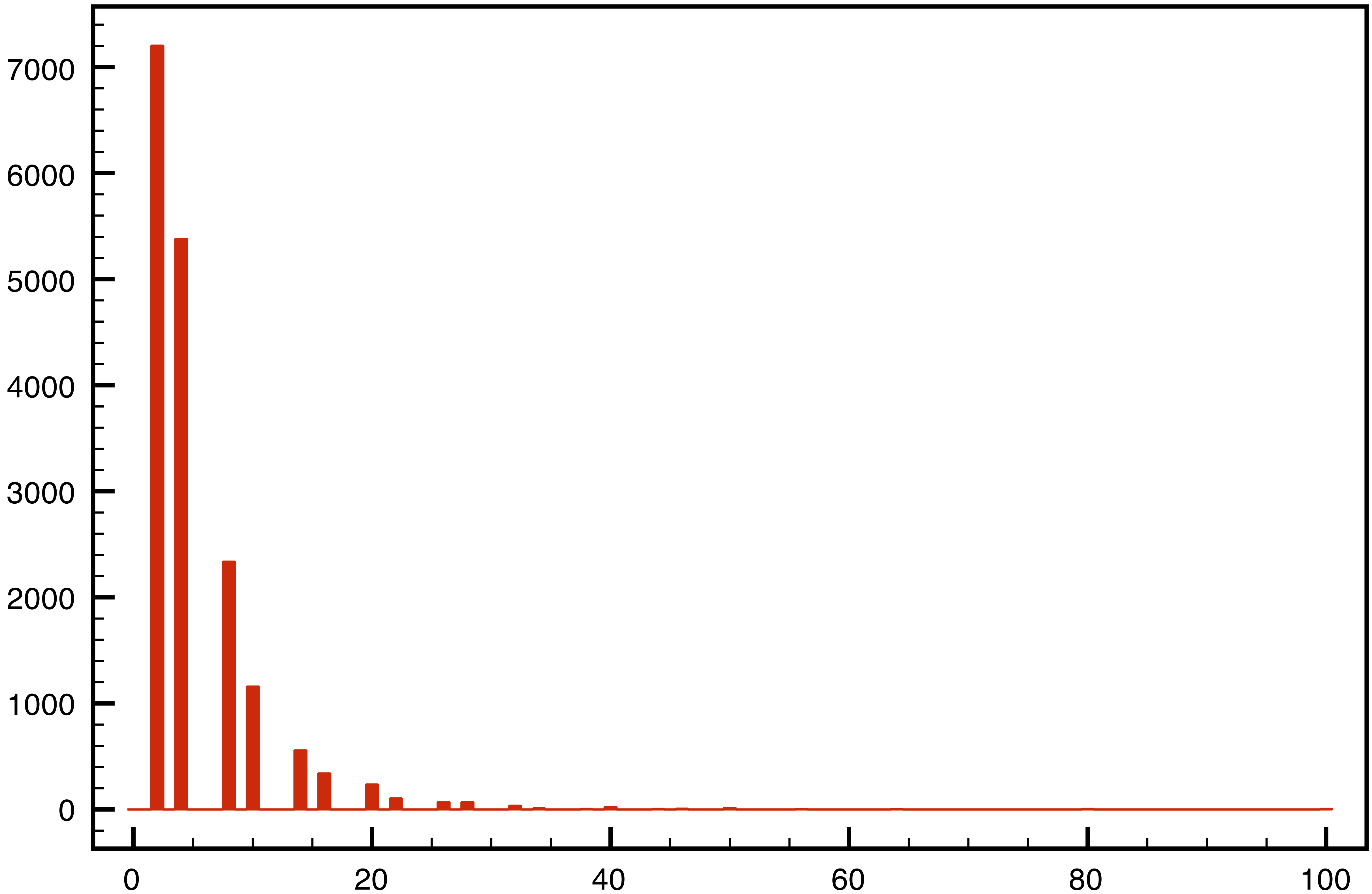
Tensor product (3,3,3,3,3)



(2,2) models: gauge group E_6



(2,0) models: various gauge groups; using one simple current

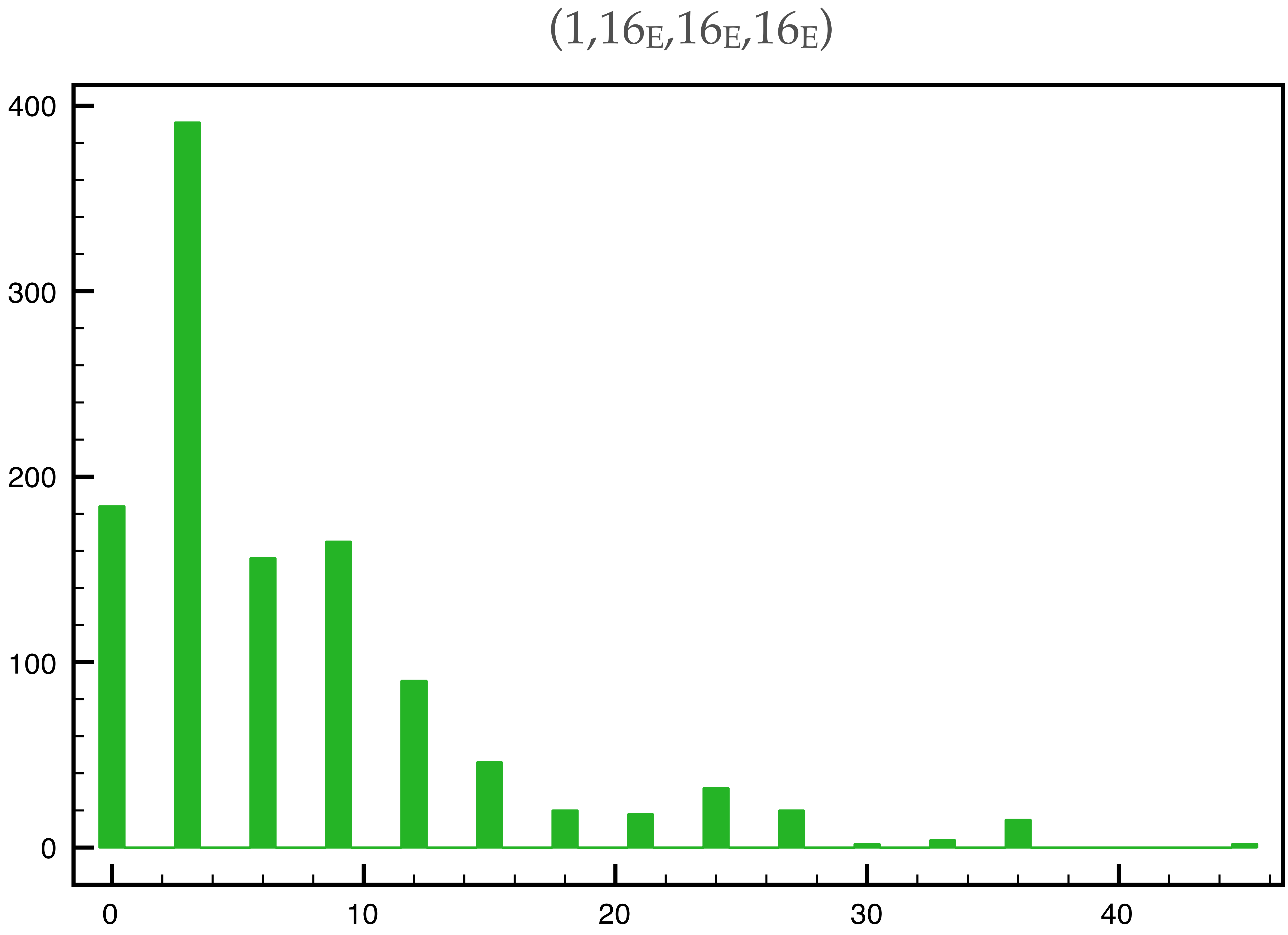


(2,0) models: various gauge groups; using two simple currents

Δ is reduced by a factor two in this case; but multiples of 3 do not occur.

In most other cases we have considered so far (about 15), Δ remains unchanged.

Three family models


$(1, 16_E, 16_E, 16_E)$
400
300
200
100
0
0
10
20
30
40

$SU(3) \times SU(2) \times SU(2) \times U(1)$

Representation	Particles	Multiplicity
$(3,2,1,\frac{1}{6})$	Q	3
$(3^*,1,2,-\frac{1}{6})$	U* + D*	4+1*
$(1,2,1,-\frac{1}{2})$	L	5+2*
$(1,1,2,\frac{1}{2})$	E* + N*	5+2*
$(3^*,1,1,\frac{1}{3})$	D*	5+5*
$(1,2,2,0)$	$H_1 + H_2$	9
$(1,1,0,0)$	singlets	80
$(1,1,1,\frac{1}{3})$		41+41*
$(1,1,2,-\frac{1}{6})$		20+20*
$(1,2,1,-\frac{1}{6})$	Charge	19+19*
$(3,1,1,0)$		17+17*
$(3,1,1,\frac{1}{3})$		8+8*
$(3,2,1,-\frac{1}{6})$	1/3	3+3*
$(3^*,1,2,\frac{1}{6})$		3+3*
$(1,2,2,\frac{1}{3})$		2+2*
$(1,1,1,-\frac{2}{3})$		2+2*

Fractional Charges

(1,4,4,4,4)

Minimal charge	Chiral	Non-chiral
$\frac{1}{6}$	1048538	16614
$\frac{1}{3}$	709334	65809
$\frac{1}{2}$	12037	228183
1	0	219493

23% non-chiral

(6,6,6,6)

Minimal charge	Chiral	Non-chiral
$\frac{1}{6}$	0	0
$\frac{1}{3}$	0	0
$\frac{1}{2}$	41240	1076404
1	0	973604

98.5% non-chiral

(Always at least a Pati-Salam extension)

(3,3,3,3,3)

Minimal charge	Chiral	Non-chiral
$\frac{1}{6}$	0	0
$\frac{1}{3}$	0	0
$\frac{1}{2}$	853368	401795(*)
1	0	2409517

76% non-chiral

(*) includes cases with just $SU(3) \times SU(2) \times U(1) \times U(1)^6$

(5,5,5,12)

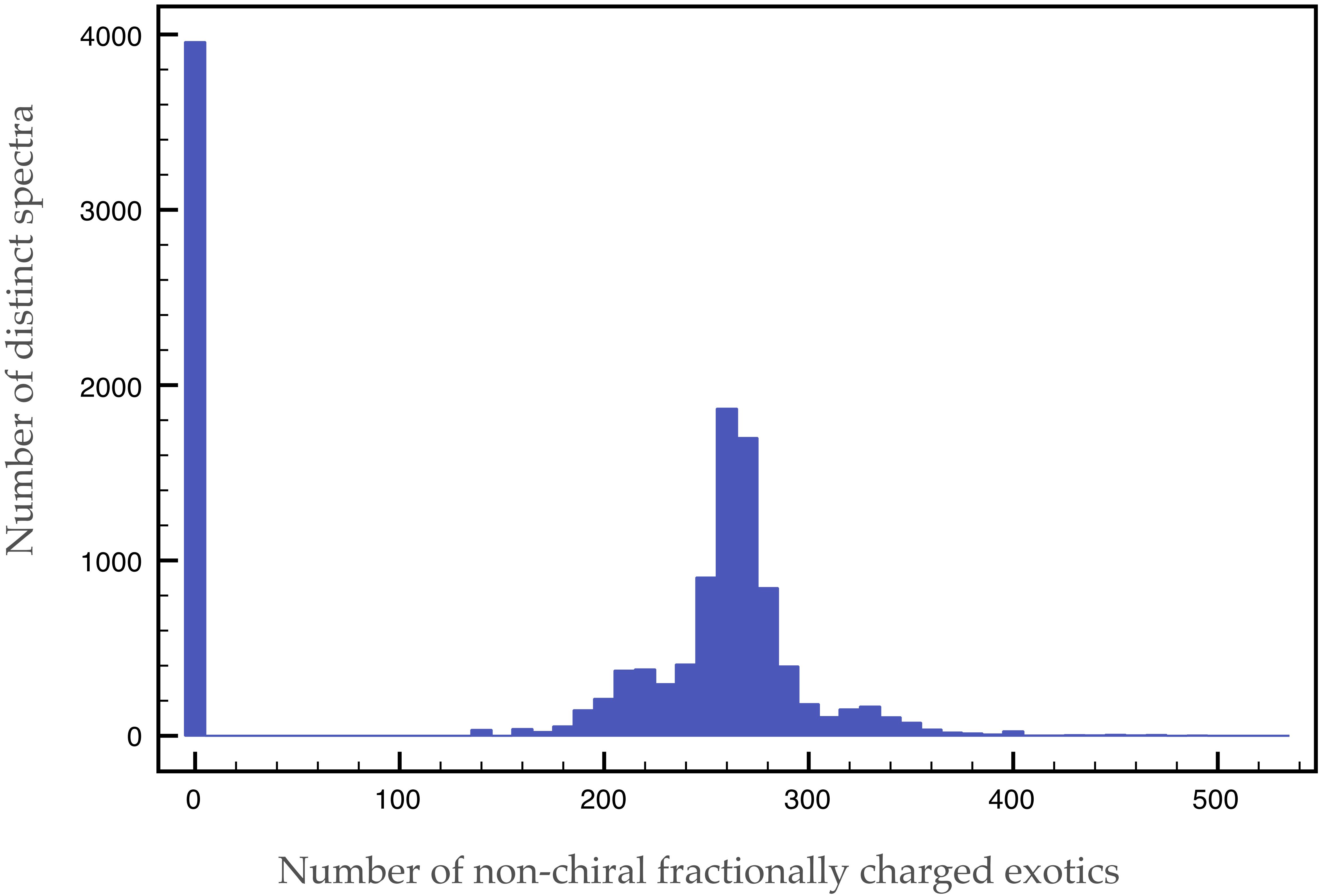
Minimal charge	Chiral	Non-chiral
$\frac{1}{6}$	0	0
$\frac{1}{3}$	0	0
$\frac{1}{2}$	0	262987
1	0	755413

100% non-chiral

It seems to be easy to get only *non-chiral* fractional charges.

Any chance of getting only *massive* fractional charges?

(3,3,3,3,3)



Number of distinct spectra
4000
3000
2000
1000
0
0
100
200
300
400
500
Number of non-chiral fractionally charged exotics

Conclusions

- Asymmetric Gepner models provide a huge and largely unexplored part of the landscape.
- Family distributions peak at small values.
- Three families still hard to get.
- Fractional charges occur, but are reasonably often non-chiral.
- Many other possibilities exist.