The Kreuzer bi-homomorphism

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Simple Currents

EXTENDED CHIRAL ALGEBRAS AND MODULAR INVARIANT PARTITION FUNCTIONS

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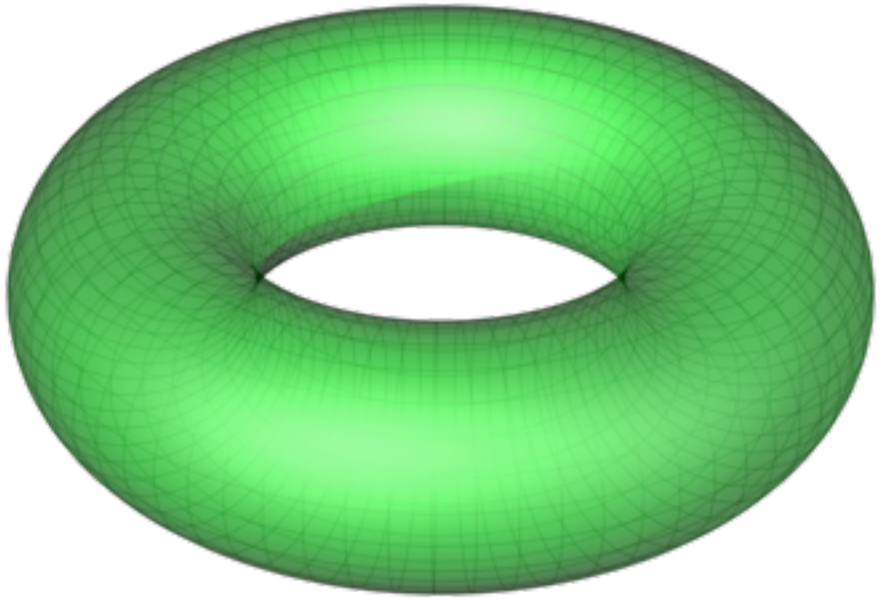
Closed Strings on the Torus



$$P(\tau,\bar\tau) = \mathrm{Tr}\ e^{2\pi i\tau(L_0-c/24)}e^{-2\pi i\bar\tau(\bar L_0-\bar c/24)}$$

Must be invariant under the modular group, $SL(\mathbb{Z},2)/\mathbb{Z}_2$

$$\left.\begin{array}{l}\tau\to-\dfrac{1}{\tau}\\[2em] \tau\to\tau+1\end{array}\right\}\qquad \begin{array}{c}\tau\to\frac{a\tau+b}{c\tau+d}\\ ad-bc=1;\ a,b,c,d\in\mathbb{Z}\end{array}$$

In (rational) CFT we express this in terms of characters of the representations

$$\chi_i(\tau) = \text{Tr}\ e^{2\pi i\tau(L_0 - c/24)}$$

Then

$$P(\tau,\bar\tau) = \sum_i \chi_i(\tau) M_{ij} \chi_j(\bar\tau)$$

$$M_{ij} \in \mathbb{Z},\ M_{ij} \geq 0,\ M_{00} = 1$$

The characters transform as

$$\chi_i(\tau+1) = T_{ij}\chi_j(\tau)$$

$$\chi_i(-\frac{1}{\tau}) = S_{ij}\chi_j(\tau) \qquad \text{with} \quad (ST)^3 = S^2 = C$$

Hence the matrix M must satisfy

$$[S, M] = [T, M] = 0$$

(MIPF: Modular Invariant Partition Function)

Known solutions (1988)

$M_{ij} = \delta_{ij}$ Diagonal invariant

$M_{ij} = C_{ij}$ Charge conjugation invariant

ADE invariants for $SU(2)$ affine Lie algebras at level k

Cappelli, Itzykson, Zuber (1987)

Other affine Lie algebras MIPFs

D. Bernard (1987)
Altschuler, Lacki, Zaugg (1988)
Itzykson (1988)

Fusion rules

Chiral algebra fusion rules

(Belavin, Polyakov, Zamolodchikov (1984))

$$\Phi_i \Phi_j = \sum_k N_{ij}{}^k \Phi_k$$

Verlinde formula *(1988)*

$$N_{ij}{}^k = \sum_m \frac{S_{im} S_{jm} S^*_{km}}{S_{0m}}$$

Simple currents

A representation J is a simple current if

$$\Phi_J \Phi_i = \Phi_{Ji}$$ (precisely one term on the right hand side)

Where "Ji" denotes some representation.
This organizes all representations into orbits.
The orbit of the identity has order N.
The order of all other orbits is a divisor of N.

The fact that the fusion rules are simple allows us to define a "monodromy charge" which is conserved in operator products

$$Q_J(i) = h_i + h_J - h_{Ji} \bmod 1$$

h_i: conformal weight of representation i

Now we can write down a MIPF for any J

$$M_{J^p i, J^q j} = \delta_{ij} \sum_{\ell=1}^{N} \delta^{N_j}_{q,p+\ell} \, \delta^1\Big[Q_J(i) + \left(\frac{2p+\ell}{2N}\right) r\Big]$$

$$h_J = \frac{r(N-1)}{2N} \mod 1 \qquad \text{(defines } r\text{)}$$

This includes all aforementioned MIPFs except “E” of ADE

Related work:

Fuchs, Gepner (1987) (observed simple fusion in 4pt functions)

Intriligator (1989) (“Bonus symmetry”)

Gepner Models

(Gepner, 1987)

"Internal sector" of a type-II or heterotic strings is built out of *N=2* minimal models.

An *N=2* minimal model is characterized by an integer k and has central charge

$$\frac{3k}{k+2}$$

It has a large number of simple currents forming a discrete group

$$\begin{array}{rl} \mathbb{Z}_{4k+8} & \text{for } k \text{ odd} \\ \mathbb{Z}_{2k+4} \times \mathbb{Z}_2 & \text{for } k \text{ even} \end{array}$$

Simple Current Description

All aspects of this construction can be described elegantly in terms of simple currents. Explicit matrices S never needed.
(Schellekens, Yankielowicz (1989))

- Field identification
  (in the coset construction to build the N=2 models)
- World-sheet supersymmetry projections
- Space-time supersymmetry projection ("GSO")
- Non-trivial MIPFs

Use a product formula to build MIPFs

$$P(\tau,\bar{\tau}) = \chi(\tau)M(J_1)M(J_2)\dots M(J_k)\bar{\chi}(\bar{\tau})$$

With implicit summation over $\mathcal{O}(10^{12})$ indices

Results

Large number of heterotic spectra with $SO(10)$ or E_6 gauge group and an *even* number of chiral families.

(List available at www.nikhef.nl/~t58 (page "Hodge numbers"))

Related work

Lutken, Ross (1988)
Lynker, Schimmrigk (1988)
Fuchs, Klemm, Scheich, Schmidt (1989)

Three families (with exceptional MIPF)

Gepner (1987) (see also Schimmrigk, 1987)

Hodge number comparison

- Free Fermions (or $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds): 26

  Donagi, Wendland (2008)
  Kirisis, Lennek, Schellekens (2008)

- Gepner models: 906

  Gato-Rivera, Schellekens (2010)
  (see also 1989 results... if you can find them)

- Calabi-Yau (reflexive polyhedra): 30108

  Kreuzer, Skarke (2000)

Beyond Product MIPFs

Fusion rule automorphisms from integer spin simple currents

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CERN, CH-1211 Geneva 23, Switzerland

Received 26 April 1990

Additional solutions that are not products of single current MIPFs

Complete Classification of Simple Current Modular Invariants for RCFT's with a Center $(Z_p)^k$

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CERN, 1211 Geneva 23, Switzerland

Beyond Product MIPFs

Results for $(\mathbb{Z}_p)^k$

Simple current structure characterized by the current-current monodromies.

$$Q_i(J_j) = Q_j(J_i) \equiv R_{ij} = \frac{1}{p} r_{ij} \quad , \; r_{ij} \in \mathbb{Z}$$

Total number of MIPFs

$$T(r,k,p) = T(k,p) = \prod_{l=0}^{k-1} (1 + p^l).$$

Independent of the matrix *r*

Nuclear Physics B411 (1994) 97–121
North-Holland

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Simple currents versus orbifolds with discrete torsion – a complete classification

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Received 5 July 1993
Accepted for publication 5 August 1993

We give a complete classification of all simple current modular invariants, extending previous results for $(\mathbb{Z}_p)^k$ to arbitrary centers. We obtain a simple explicit formula for the most general case. Using orbifold techniques to this end, we find a one-to-one correspondence between simple current invariants and subgroups of the center with discrete torsions. As a by-product, we prove the conjectured monodromy independence of the total number of such invariants. The orbifold approach works in a straightforward way for symmetries of odd order, but some modifications are required to deal with symmetries of even order. With these modifications the orbifold construction with discrete torsion is complete within the class of simple current invariants. Surprisingly, there are cases where discrete torsion is a necessity rather than a possibility.

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The general formula

Suppose one has a conformal field theory with simple currents generating a center $\mathscr{C}$. Then the complete set of simple current invariants of that theory can be obtained by the following procedure:

(1) Choose any subgroup $\mathscr{H}$ of $\mathscr{C}$.

(2) Choose a basis of currents $J_1, \ldots, J_k$ that generate $\mathscr{H}$.

(3) Compute the current–current monodromies R_{ij} in that basis.

(4) Choose any properly quantized matrix X (see (2.8)) whose symmetric part is $\frac{1}{2}R$ mod 1 (in other words $X + X^{\mathrm{T}} = R$). The modular invariant partition function corresponding to this choice is then given by a matrix whose only non-zero elements are

$$M_{a,[\beta]a} = \mathrm{Mult}(a) \prod_i \delta^1\left[Q_i(a) + X_{ij}\beta_j\right], \tag{2.9}$$

where δ^1 is equal to unity if its argument is an integer, and vanishes otherwise. The factor Mult(a) appears because a may be a fixed point of some currents. In that case the β-sum in (2.7) includes all terms involving a more than once, and Mult(a) is the number of times this happens. This is the generalization of (1.2) to more than one factor.

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The bi-homorphism

Definition 3.17 :
Let H be a subgroup in the effective center $\mathrm{Pic}^\circ(\mathcal{C})$ of a ribbon category. A *Kreuzer–Schellekens bihomomorphism* (or *KSB*, for short) on H is a (not necessarily symmetric) bihomomorphism

$$\Xi: \quad H \times H \to \mathbb{C}^\times \tag{3.31}$$

which on the diagonal coincides with the quadratic form δ introduced in (2.13),

$$\Xi(g,g) = \delta(g) \equiv \theta_g \qquad \text{for all} \;\; g \in H \,. \tag{3.32}$$

Lemma B.3 :
Let H be a subgroup in the effective center $\mathrm{Pic}^\circ(\mathcal{C})$, written in the form (B.1), and X a $k \times k$-matrix. For any two elements g, h of H, written in the form $g = \prod_a (g_a)^{m_a}$ and $h = \prod_a (g_a)^{n_a}$, set

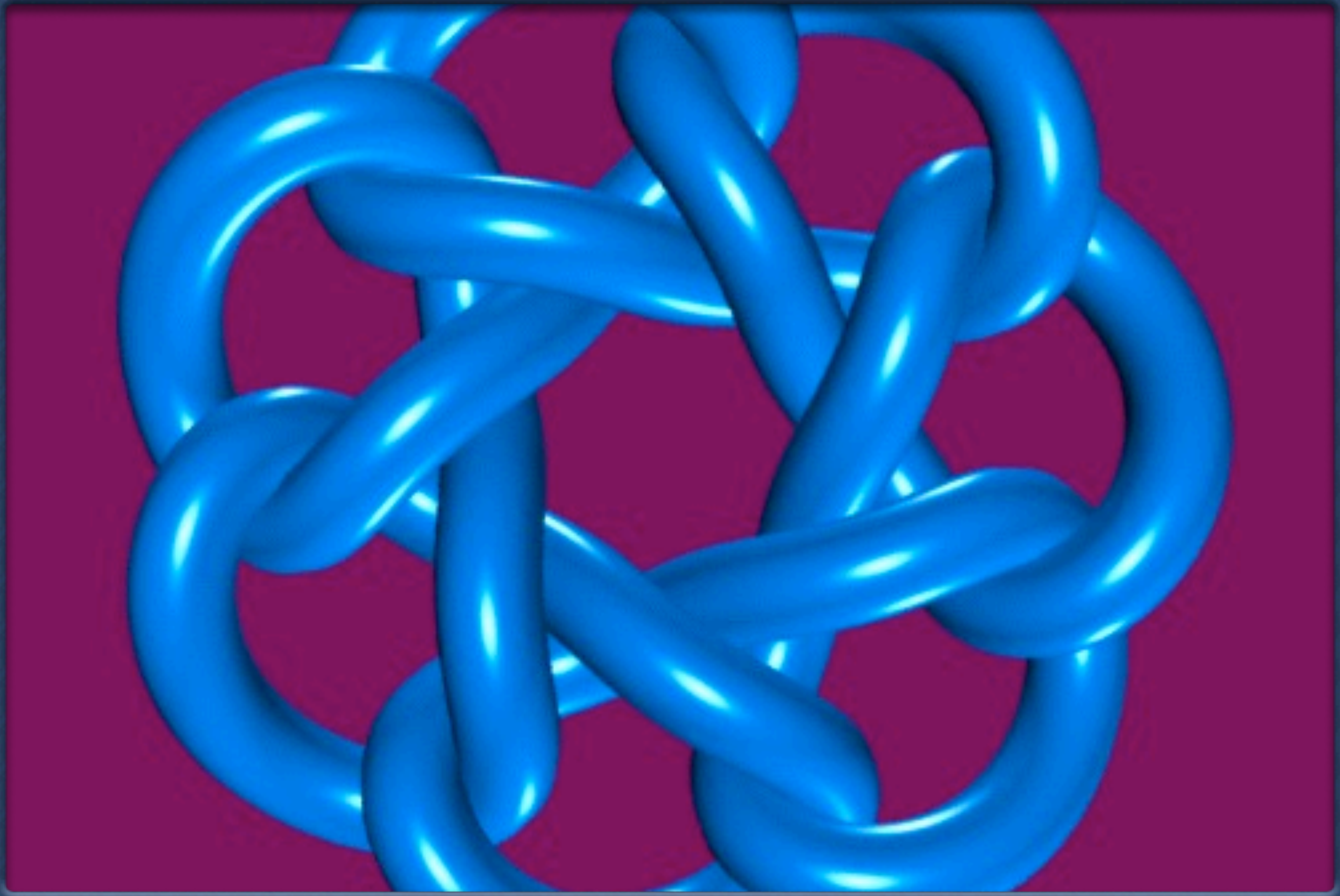
$$\Xi(g,h) := \exp(2\pi\mathrm{i} \sum_{a,b=1}^{k} m_a X_{ab} n_b) \,. \tag{B.7}$$

Then Ξ is a KSB if and only if X is a KS matrix.

From:

J. Fuchs, I. Runkel, C. Schweigert

TFT construction of RCFT correlators. 3. Simple currents. (Nucl.Phys. B694 (2004) 277-353)



Fixed Point Resolution

$$M_{a,[\boldsymbol{\beta}]a} = \text{Mult}(a) \prod_i \delta^1\left[Q_i(a) + X_{ij}\beta_j\right],$$

These multiplicities are due to simple current fixed points.

$$Ja = a$$

Stabilizer $\mathcal{S}_a$ of a:

Subgroup of the simple current group $\mathcal{G}$ that fixes i.

The multiplicity Mult(a) equals $\frac{|\mathcal{G}|}{|\mathcal{S}_a|}$

Fixed point multiplicities (usually) imply that certain representations of the extended chiral algebra implied by the MIPF cannot be distinguished using the original algebra.

The standard example is *SU(2)₄* extended to *SU(3)₁*

$$|\chi_0(\tau)+\chi_4(\tau)|^2+2|\chi_2(\tau)|^2$$

$$|\chi^{SU(3)}_{(\mathbf{1})}(\tau)|^2 \qquad |\chi^{SU(3)}_{(\mathbf{3})}(\tau)|^2 \qquad |\chi^{SU(3)}_{(\bar{\mathbf{3}})}(\tau)|^2$$

Not distinguished in *SU(2)₄*

There are two problems:

How exactly does Mult(a) split?

$$\mathrm{Mult}(a) = \sum_i m_i |n_i|^2$$

Absorbed in character normalization

What is the new matrix S of the extended theory?

$$\chi_i(-\frac{1}{\tau}) = \ldots + \alpha\chi_{f_1}(\tau) + \beta\chi_{f_2}(\tau) + \ldots$$

Both problems where solved for WZW models, in:

From Dynkin diagram symmetries to fixed point structures.
Jurgen Fuchs, Bert Schellekens, Christoph Schweigert (NIKHEF, Amsterdam).
Published in **Commun.Math.Phys. 180 (1996) 39-98**

A Matrix S for all simple current extensions.
J. Fuchs (DESY), A.N. Schellekens (NIKHEF, Amsterdam), C. Schweigert (IHES, Bures-sur-Yvette)
Published in **Nucl.Phys. B473 (1996) 323-366**

(Proving and extending earlier work with S.Yankielowicz)

Can be generalized to coset CFT's and extended WZW models.

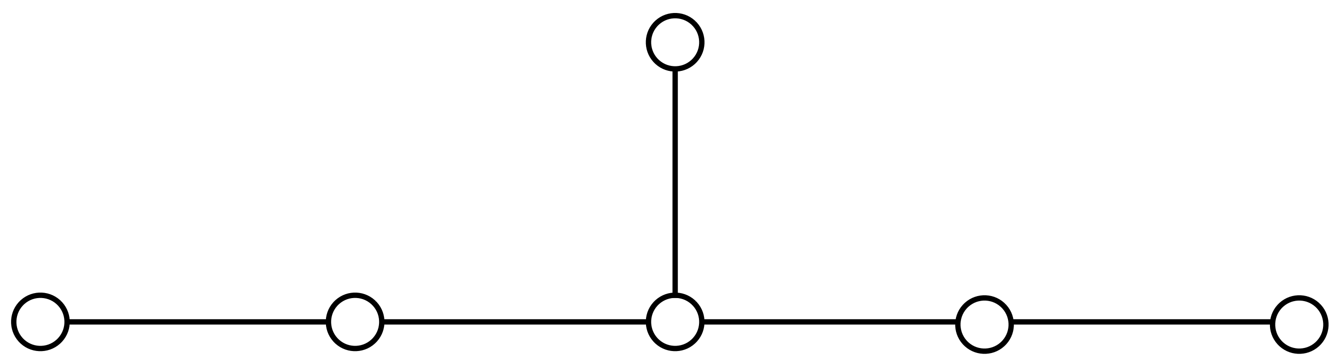
$$\tilde{S}_{(a,i),(b,j)} = \frac{|\mathcal{G}|}{\sqrt{|\mathcal{U}_a|\,|\mathcal{S}_a|\,|\mathcal{U}_b|\,|\mathcal{S}_b|}} \sum_{J\in\mathcal{G}} \Psi_i^a(J)\, S^J_{a,b}\, \Psi_j^b(J)^*$$

$\mathcal{U}_a$: Untwisted stabilizer (subgroup of $\mathcal{S}_a$)

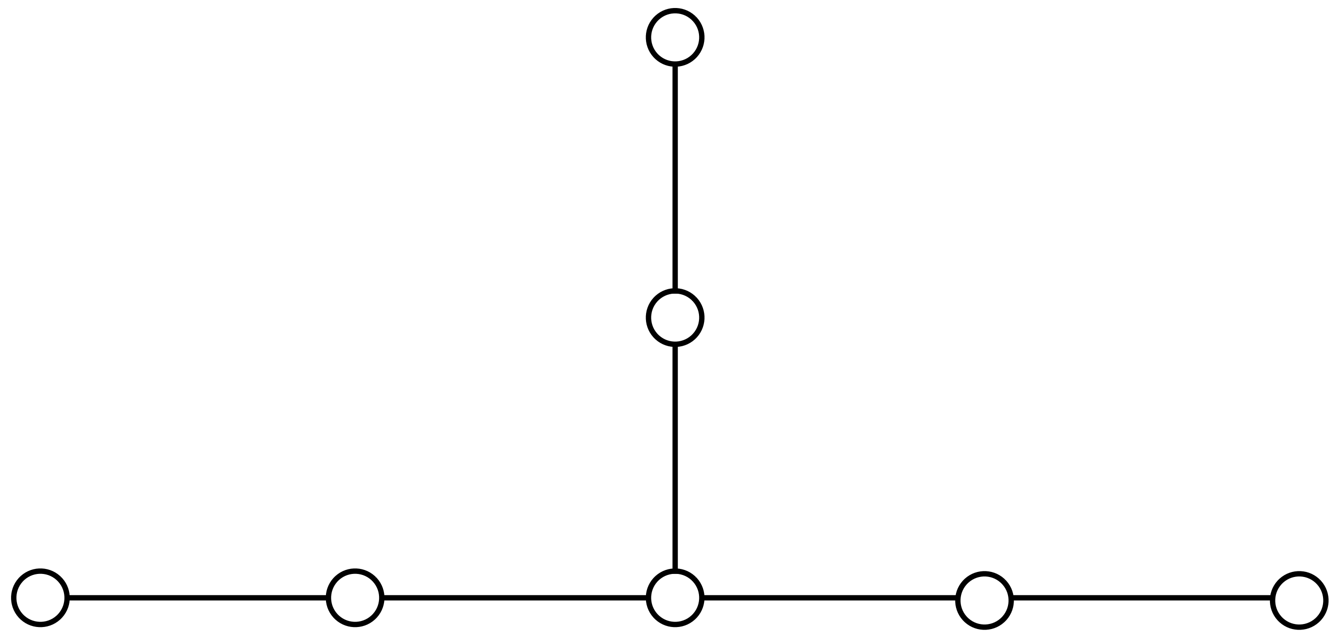
ψ_i^a : Character of $\mathcal{U}_a$

S^J_{ab} Matrix S of the "orbit Lie algebra"

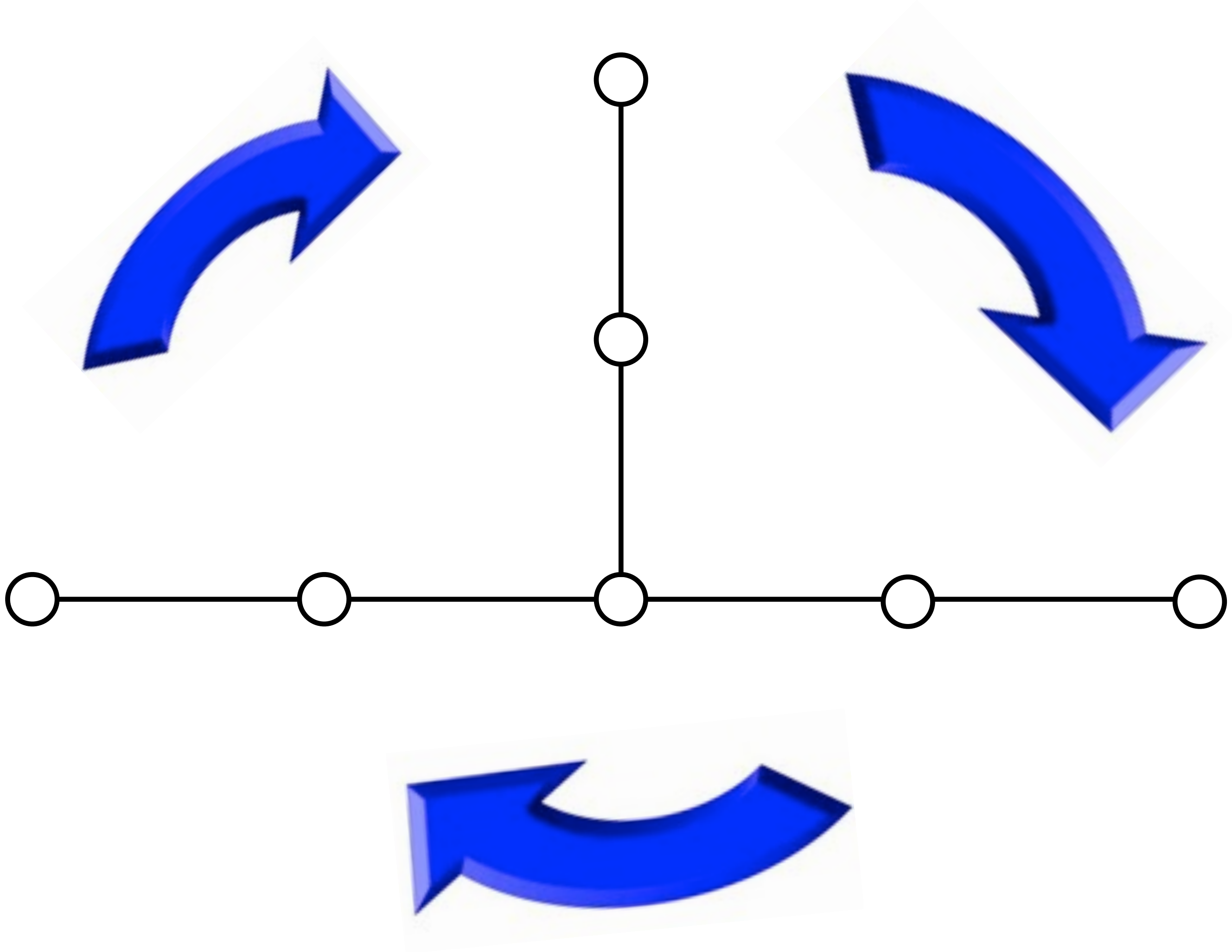
The Dynkin diagram of the orbit Lie algebra is obtained by folding the original Dynkin diagram according to the simple current action.



E_6



$E_6^{(1)}$



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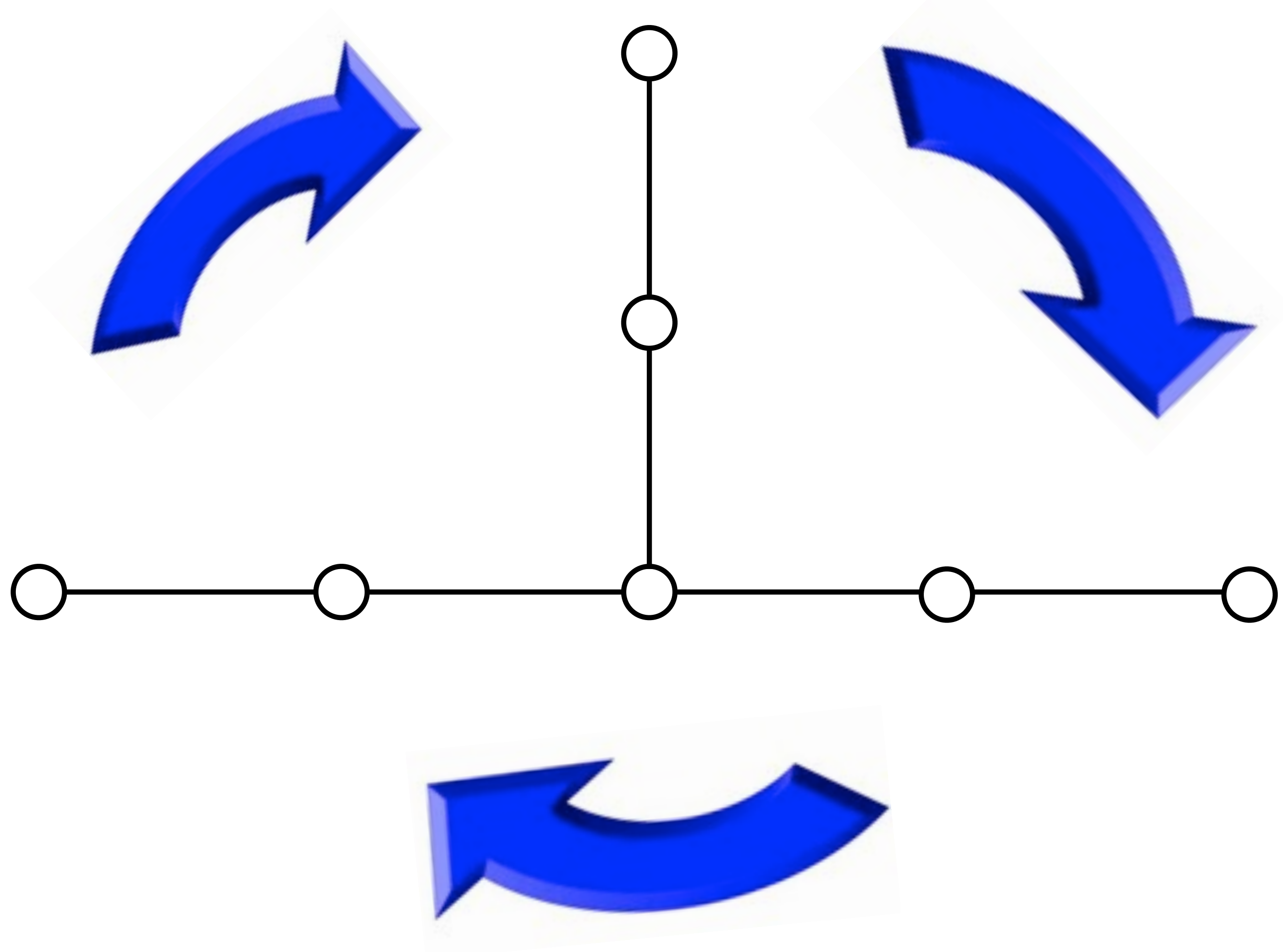
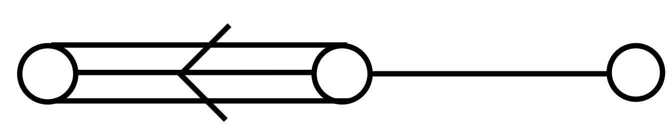
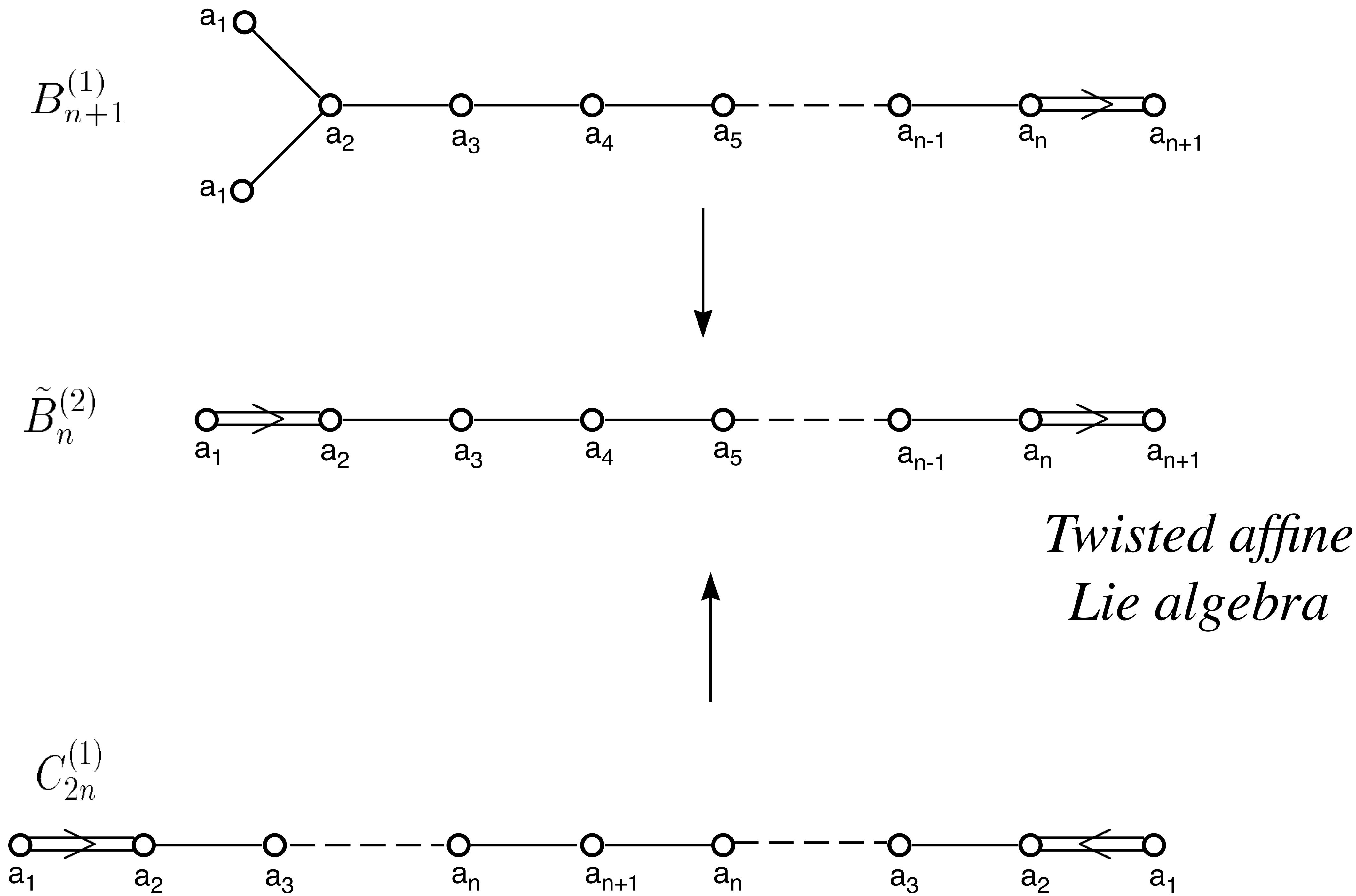


$E_6^{(1)}$



$G_2^{(1)}$



$B_{n+1}^{(1)}$
a1
a1
a2
a3
a4
a5
an-1
an
an+1
$\tilde{B}_n^{(2)}$
a1
a2
a3
a4
a5
an-1
an
an+1
Twisted affine
Lie algebra
$C_{2n}^{(1)}$
a1
a2
a3
an
an+1
an
a3
a2
a1

Twining Characters

" In particular, the generating function for the trace of τ_ω over weight spaces, which we call the 'twining character' of $\mathfrak{g}$ (with respect to the automorphism), is equal to a character of $\breve{\mathfrak{g}}$. "

(from the abstract of ***From Dynkin diagram symmetries to fixed point structures.***)

Other cases

Finding the S^J matrices for arbitrary CFT's is not straightforward
(there is no known algorithm).
Knowing S is not enough.

Apart from WZW models and cosets, we now have a formula for $\mathbb{Z}_2$ permutation orbifolds of CFT's for which S^J is known.

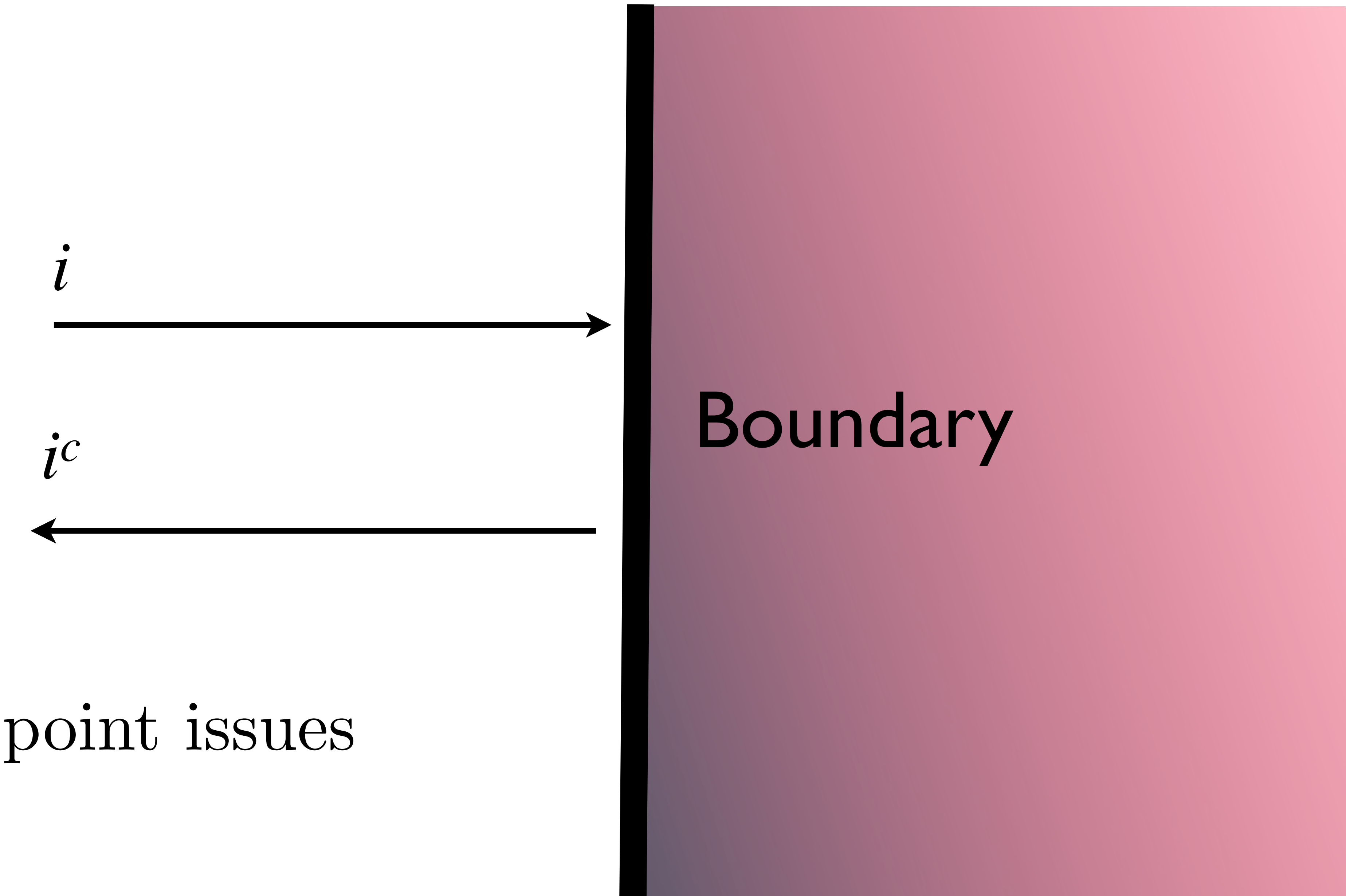
with M. Maio, 2010
Based on Borisov, Halpern, Schweigert (1998) formula for S.

Boundary Conformal Field Theory

Consider RCFT on a Riemann surface with boundaries and crosscaps.

Goal:

Obtain boundary and crosscap coefficients for all simple current MIPFs (i.e. all possible discrete choices in the KS formula).



Number of states coupling to the boundary: M_{ii^c}

$M_{ii^c} > 1:$ fixed point issues

Ishibashi States

Conditions for preservation of symmetries at a boundary (or crosscap)

$$\left[W_n - (-1)^{h_W} \tilde{W}_{-n}\right] |B\rangle = 0 \ ,$$

$$\left[W_n - (-1)^{h_W + n} \tilde{W}_{-n}\right] |C\rangle = 0$$

Formal solution (boundary/crosscap Ishibashi states)

$$|B\rangle = \sum_\ell |\ell\rangle \otimes U_B \, |\ell\rangle \ \ , \qquad |C\rangle = \sum_\ell |\ell\rangle \otimes U_C \, |\ell\rangle$$

$$\tilde{W}_n U_B = (-1)^{h_W} U_B \tilde{W}_n \ ; \qquad \tilde{W}_n U_C = (-1)^{h_W + n} U_C \tilde{W}_n$$

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$$\left[W_n - (-1)^{h_W + n} \tilde{W}_{-n}\right] |C\rangle = 0$$

Formal solution (boundary/crosscap Ishibashi states)

$$|B\rangle_m = \sum_{\ell} |\ell\rangle_m \otimes U_B \, |\ell\rangle_m, \qquad |C\rangle_m = \sum_{\ell} |\ell\rangle_m \otimes U_C \, |\ell\rangle_m$$

$$\tilde{W}_n U_B = (-1)^{h_W} U_B \tilde{W}_n \ ; \qquad \tilde{W}_n U_C = (-1)^{h_W + n} U_C \tilde{W}_n$$

m: *RCFT representation label*

Boundary States

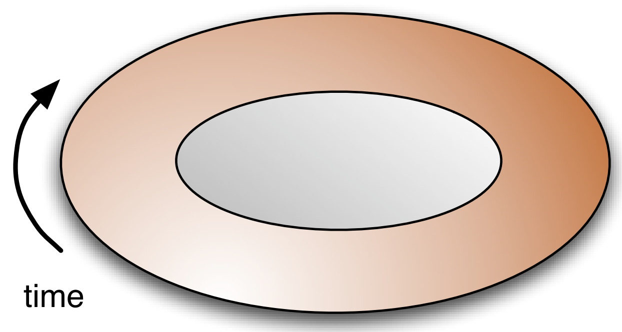
Boundary states are linear combinations of Ishibashi states subject to integrality conditions for the annulus ("Cardy condition" *(Cardy, 1989)*);
Analogously for crosscap states, constrained by Moebius and Klein bottle coefficients.

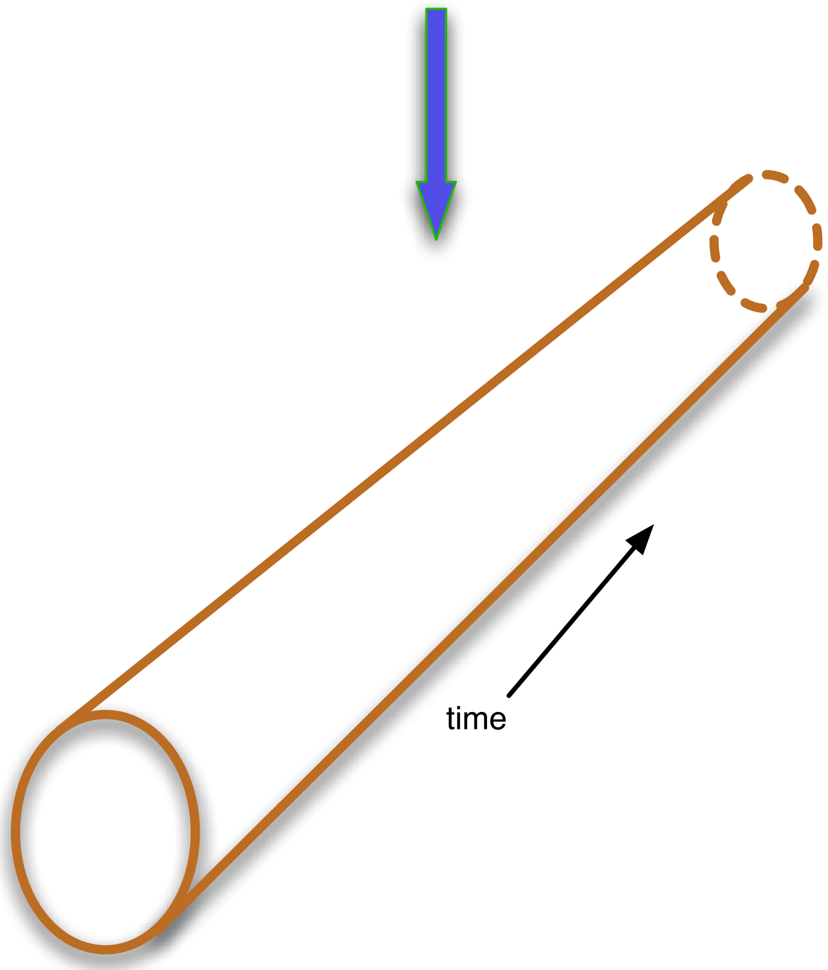
There are as many boundary states as there are Ishibashi states
"completeness condition for boundaries" *(Sagnotti, Pradisi, Stanev 1996)*.

$$|B\rangle_a = \sum_m \frac{B_{am}}{\sqrt{S_{0m}}} |B\rangle_m$$

$$|\Gamma\rangle = \sum_m \frac{\Gamma_m}{\sqrt{S_{0m}}} |C\rangle_m$$

Transverse Channel

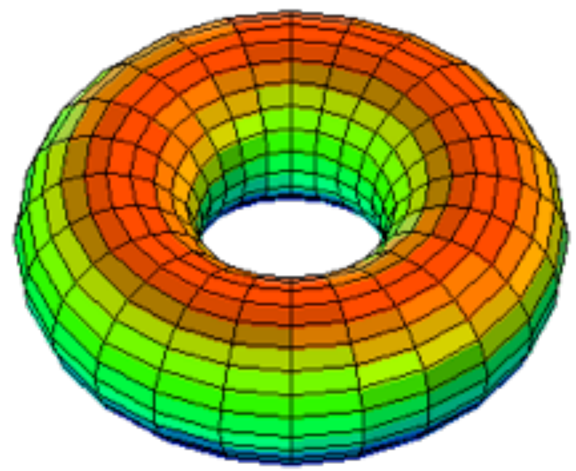
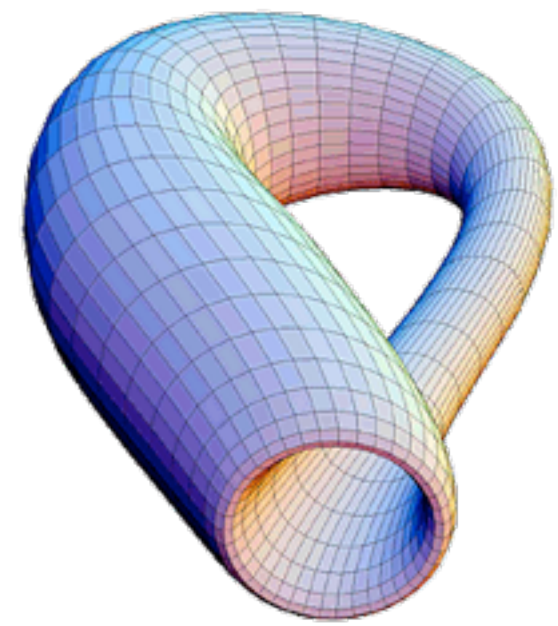




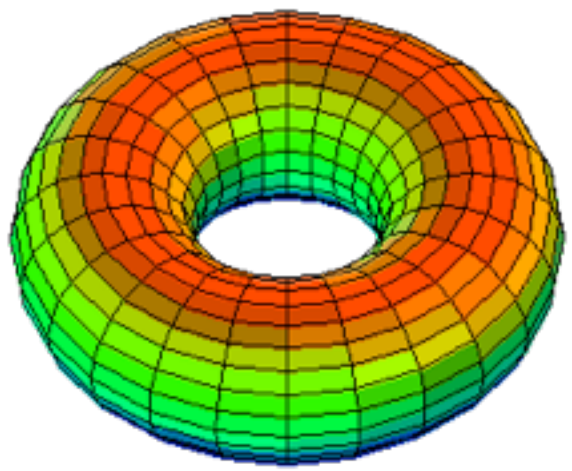
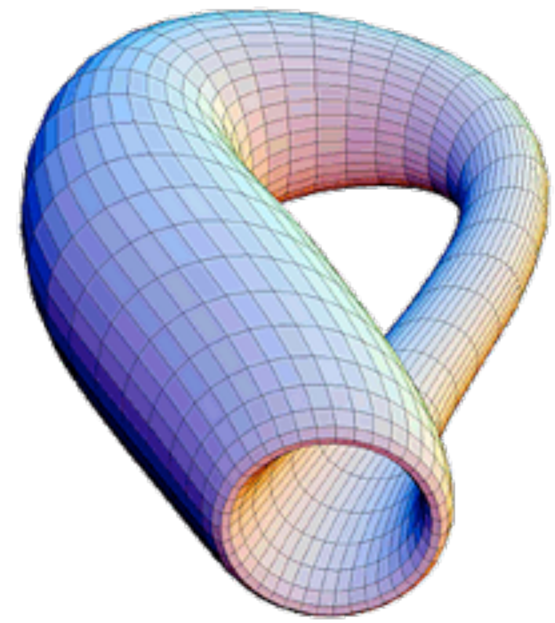
boundary state

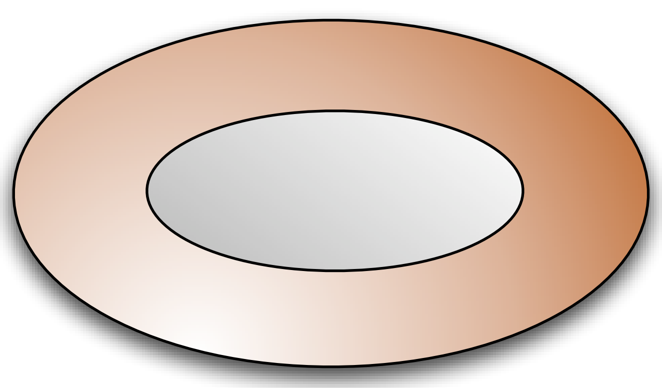
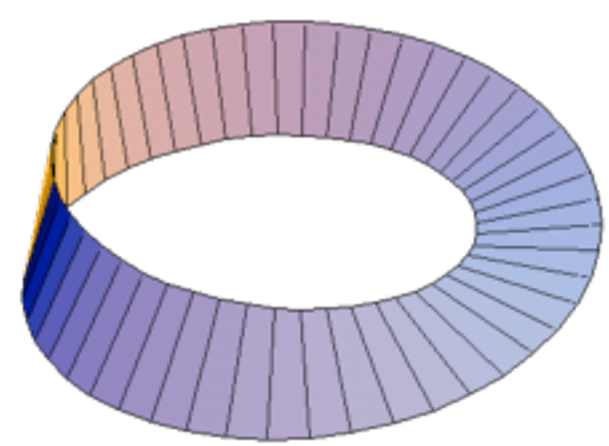
Orientifold Partition Functions

Orientifold Partition Functions

$\frac{1}{2}$ {  +  }

Orientifold Partition Functions

$\frac{1}{2}$ {  +  }

$\frac{1}{2}$ {  +  }

Orientifold Partition Functions

$$\frac{1}{2}\left[\sum_{ij}\chi_i(\tau)M_{ij}\chi_j(\tau)+\sum_i K_i\chi_i(2\tau)\right]$$

$$\frac{1}{2}\left[\sum_{i,a,b}N_aN_bA^i{}_{ab}\chi_i(\frac{\tau}{2})+\sum_{i,a}N_aM^i{}_a\hat{\chi}_i(\frac{\tau}{2}+\frac{1}{2})\right]$$

Coefficients

$$A^i{}_{ab} = \sum_m \frac{S_{im} B_{am} B_{bm}}{S_{0m}}$$

$$M^i{}_a = \sum_m \frac{P_{im} B_{am} \Gamma_m}{S_{0m}}$$

$$K^i = \sum_m \frac{S_{im} \Gamma_m \Gamma_m}{S_{0m}}$$

$$P = \sqrt{T} S T^2 S \sqrt{T}$$

Cardy-Rome Solution

If $M_{ij} = \delta_{j,i^c}$

One Ishibashi states per RFCT representation

One boundary state per CFT representation

$$B_{am} = S_{am}$$

$$\Gamma_m = P_{0m}$$

Cardy (1989)

Tor Vergata group (Sagnotti, Pradisi, Stanev, Bianchi, Fioravante e.a. (1994-1996)

General Solution

For a general MIPF given by the KS formula:

- What are the properly resolved Ishibashi states?
- How does one characterize the boundary states?
- What is the general formula for the boundary states?
- What are the allowed crosscap choices?

Several years of work by several groups(*) finally led to a general formula for **all** MIPFs in the KS formula.

(*) Rome group, Fuchs, Schweigert, Birke, Zuber, Petkova, Behrend, Pearce, Huiszoon, Sousa, Schellekens, (1995-2000)

Ishibashi States

(0+2)^2 + (1+3)^2 + (4+6)*(13+15) + (5+7)*(12+14)
+ (8+10)^2 + (9+11)^2 + (12+14)*(5+7) + (13+15)*(4+6)

.....

+ 2*(2937)*(2939) + 2*(2938)*(2936) + 2*(2939)*(2937)
+ 2*(2940)^2 + 2*(2941)^2 + 2*(2942)^2 + 2*(2943)^2

Ishibashi States

(0+2)^2 + (1+3)^2 + (4+6)*(13+15) + (5+7)*(12+14)
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$\mathcal{C}_a$ is the Central Stabilizer of a

$$\mathcal{C}_i := \{ J \in \mathcal{S}_i \mid F_i^X(K,J) = 1 \text{ for all } K \in \mathcal{S}_i \}$$

$$F_i^X(K,J) := \mathrm{e}^{2\pi \mathrm{i} X(K,J)} \, F_i(K,J)^*$$

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Boundaries and Crosscaps*

"Open string version of the KS formalism for closed strings"

- Boundary coefficients

$$B_{[a,\psi_a](m,J)} = \sqrt{\frac{|\mathcal{H}|}{|\mathcal{C}_a||\mathcal{S}_a|}}\psi_a^*(J)S^J_{am}$$

- Crosscap coefficients

$$\Gamma_{(m,J)} = \frac{1}{\sqrt{|\mathcal{H}|}}\sum_{L\in\mathcal{H}}\eta(K,L)P_{LK,m}\delta_{J,0}$$

**Huiszoon, Fuchs, Schellekens, Schweigert, Walcher (2000)*

Orientifold Choices*

- "Klein bottle current" K (element of $\mathcal{H}$)
- "Crosscap signs" (signs defined on a subgroup of $\mathcal{H}$), satisfying

$$\eta(K,L) = e^{i\pi(h_K - h_{KL})}\beta(L)$$

$$\beta_K(J)\beta_K(J') = \beta_K(JJ')e^{2\pi i X(J,J')} \quad , J, J' \in \mathcal{H}$$

Huiszoon, Sousa, Schellekens (1999-2000)

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Applications

- # Gepner Orientifolds

  Aldazabal, Andres, Juknevitch (2003, 2004)
  Blumenhagen, Weigand (2004)
  Dijkstra, Huiszoon, Schellekens (2004)
  Anastasopoulos, Dijkstra, Kiritsis, Schellekens (2006)

  } *Using the full FHSSW formalism*

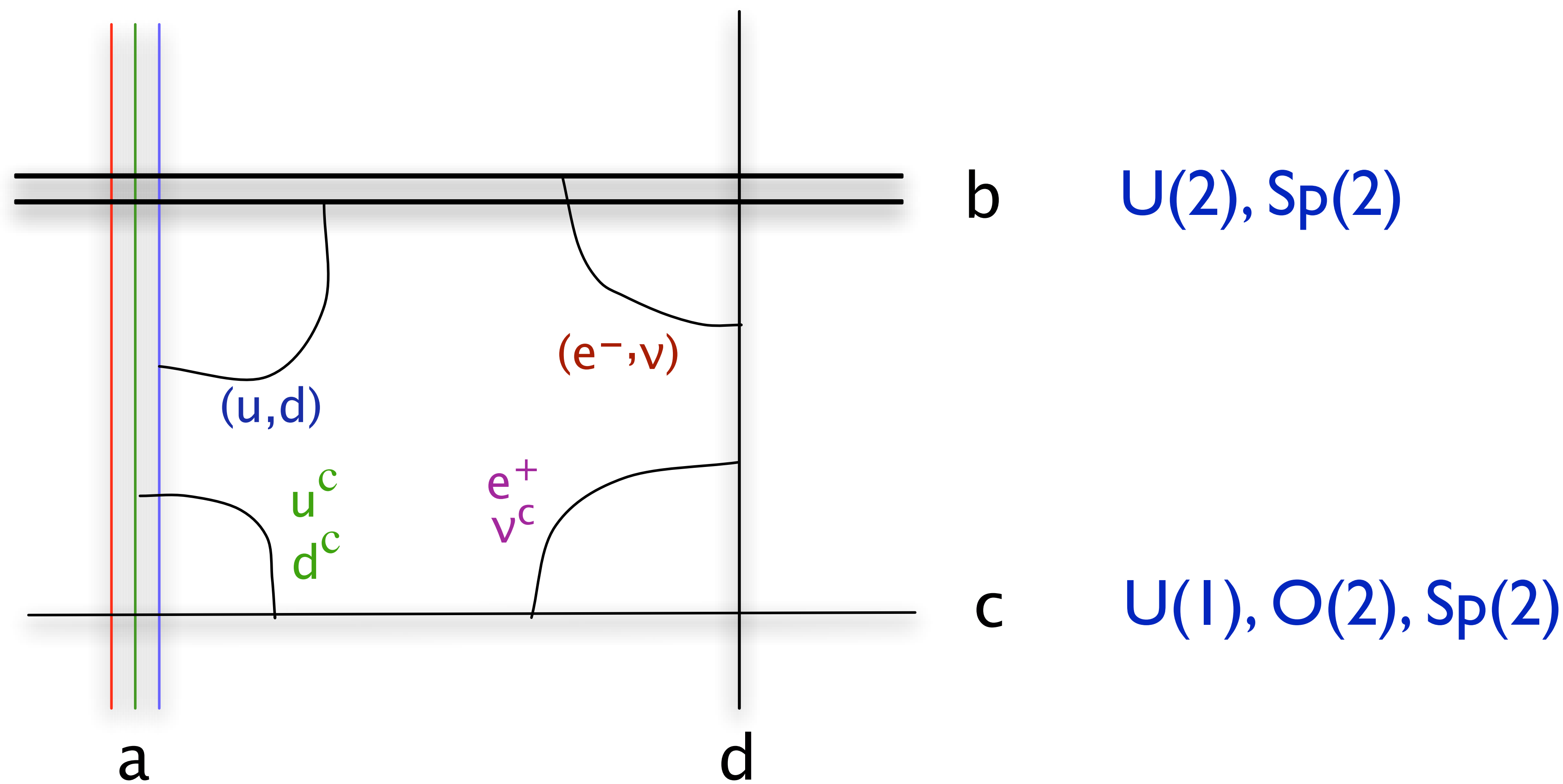
- # Gepner Heterotic Strings

  Gepner, many other authors (late 1980's)
  Blumenhagen, Wisskirchen, Schimmrigk (1995)
  Gato-Rivera, Maio, Schellekens (2010)

  Using the full KS formalism

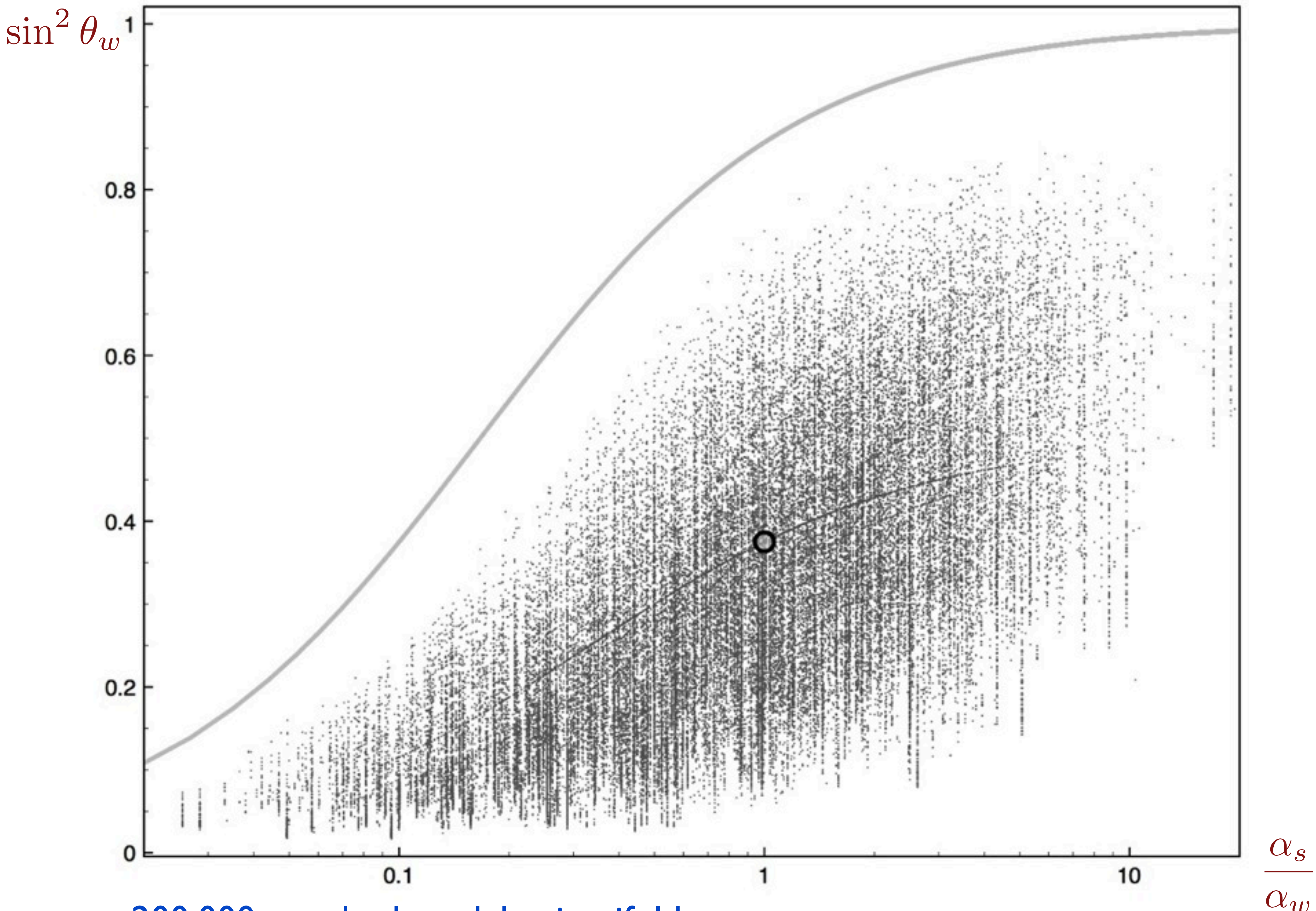
Geometric connections: see (also for further references)
"**Heterotic (0,2) Gepner Models and Related Geometries**"
Maximilian Kreuzer
in *Fundamental Interactions: A Memorial Volume for Wolfgang Kummer*

The Madrid Model*

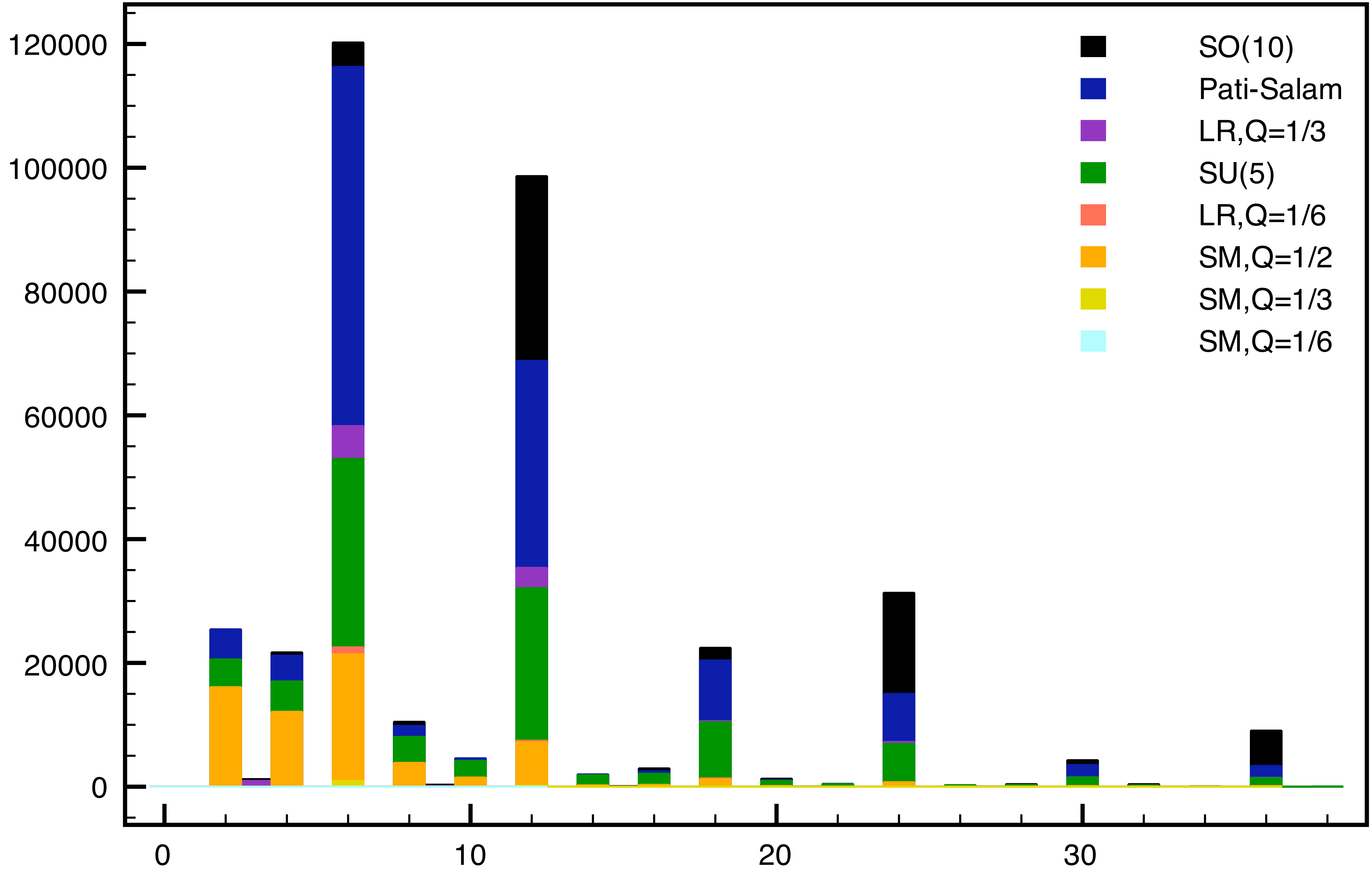


$$Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c - \frac{1}{2}Qd$$

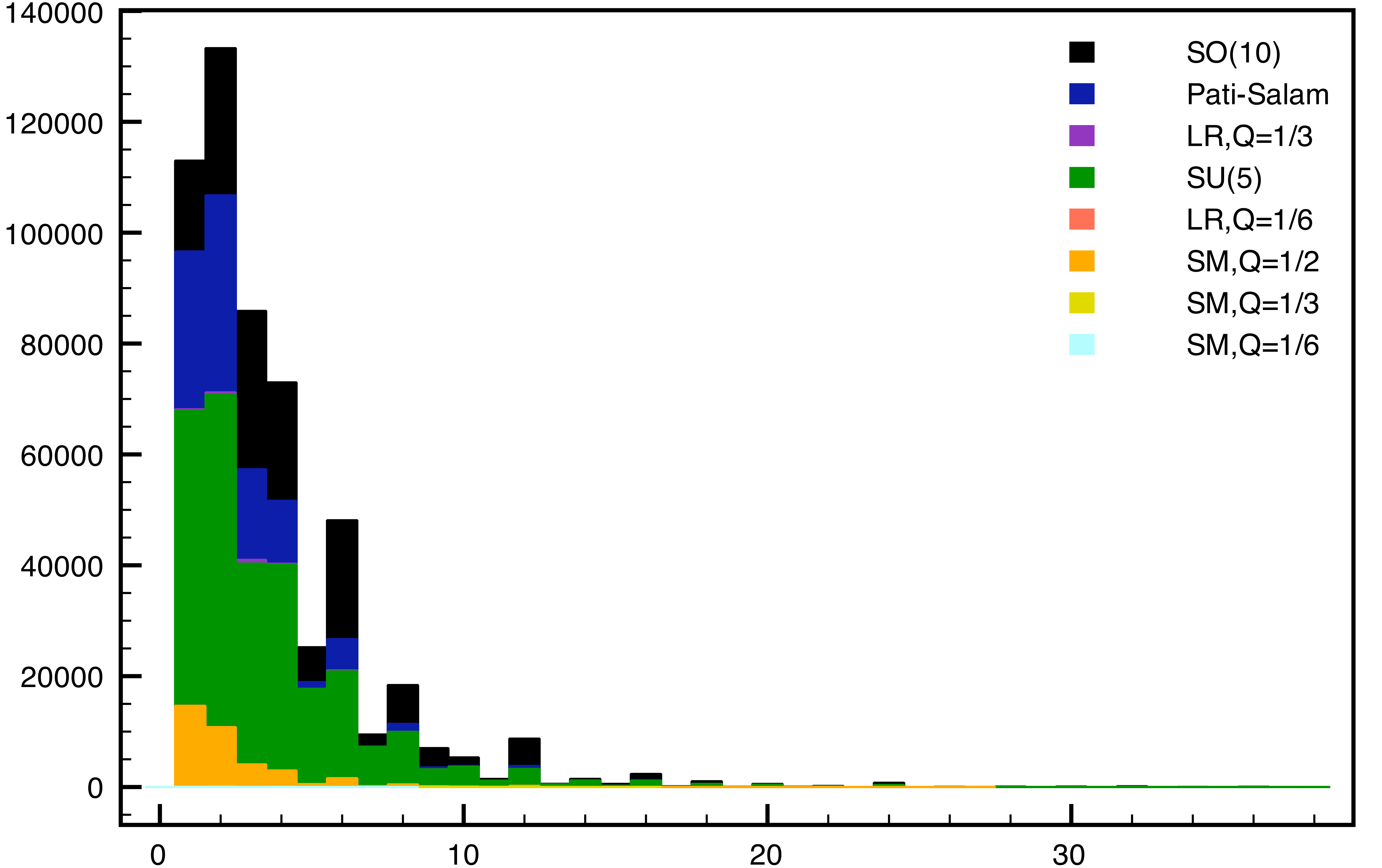
(*) Ibanez, Marchesano, Rabadan (2000)



200,000 standard model orientifold spectra

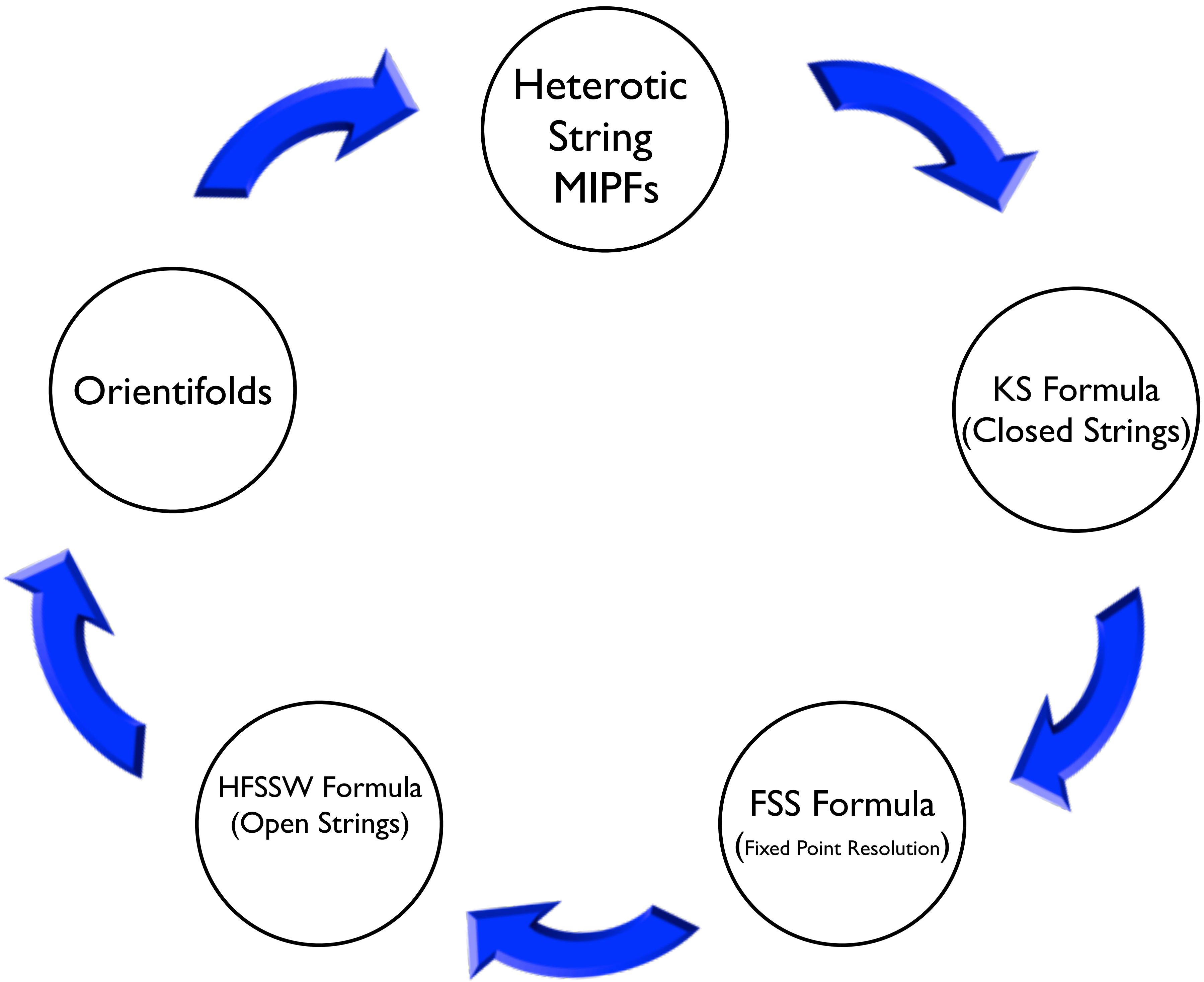


Family distribution for heterotic Gepner models

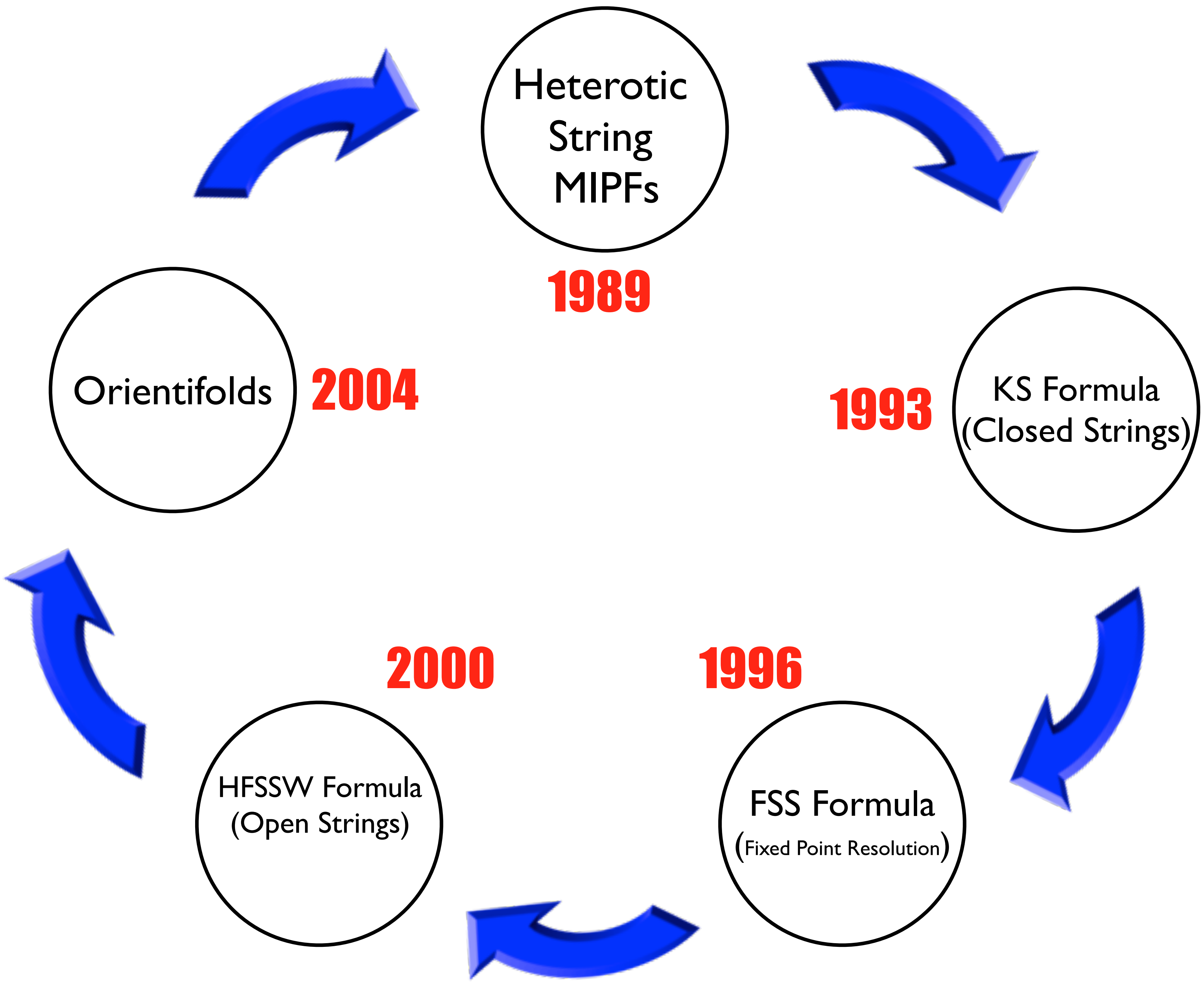


Family distribution for heterotic Gepner models (with weight-lifting)

Summary


Heterotic String MIPFs
KS Formula (Closed Strings)
FSS Formula (Fixed Point Resolution)
HFSSW Formula (Open Strings)
Orientifolds

Summary


Heterotic String MIPFs
1989
KS Formula (Closed Strings)
1993
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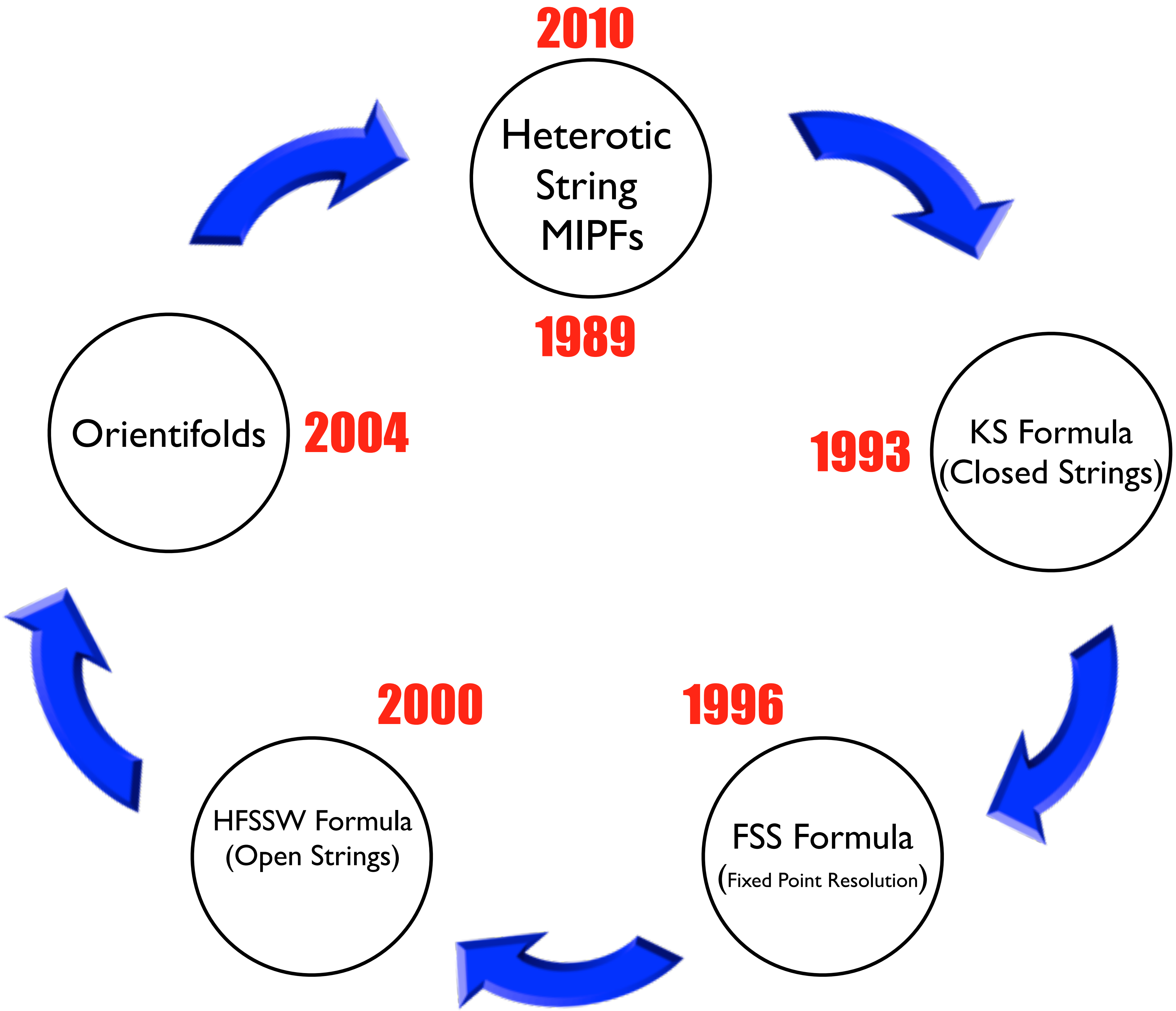
1996
HFSSW Formula (Open Strings)
2000
Orientifolds
2004

Summary


2010
Heterotic String MIPFs
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