Heterotic Strings Revisited

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Madrid

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Early String Theory Expectations: (≈ 1985)

"*The hope is that the constraints imposed on such theories solely by the need for mathematical consistency are so strong that they essentially determine a single possible theory uniquely, and that by working out the consequences of the theory in detail one might eventually be able to show that there must be particles with precisely the masses, interactions, and so on, of the known elementary particles: in other words, that the world we live in is the only possible one.*"

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From "The Problems of Physics" by Antony Legget (1987)

A.N. Schellekens,
Contribution to the proceedings of the EPS conference, Uppsala, June 1987

The prevailing attitude seems to be that "non-perturbative string effects" will somehow select a unique vacuum. This is unreasonable and unnecessary wishful thinking. We do not know at present how to discuss such effects, and have no idea whether they impose any restrictions at all. One cannot reasonably expect that a mathematical condition will have a unique solution corresponding to the standard model with three generations and a bizarre mass matrix. It is important to realize that this quest for uniqueness is based on philosophy, not on physics. There is no logical reason why the "theory of everything" should have a unique vacuum.

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Lerche, Lüst, Schellekens (1986)

$(\Gamma_{22}\times D_3\times(D_7)^9)_L$, a Euclidean lattice of dimension 88. A lower limit on the total number of such lattices is provided by the Siegel mass formula [21] [22]

... this number is of order 10^{1500} !

Possible attitudes:

- Huge landscape, so string theory must be wrong.
  (‘t Hooft)
- String theory is correct, but landscape must be wrong.
  (Gross)
- The string theory landscape is correct, and there is nothing left to do.
  (Susskind?)

But:

There is plenty of structure in the standard model that seems neither “random” nor required for the existence of life.

So we should be able to extract more information from the landscape.

This will require less focus on finding “the” SM and more focus on the way features are distributed.

In doing so, we cannot avoid the “a-word”.

Ironically, the basic SM structure fits so easily in string theory that it is hard to decide where to start looking...

Embedding the Standard Model in string theory

Or: how long did it take to find it?

Heterotic strings:	November 1984 - December 1984
Open Strings:	1975-2000
F-theory:	1996-2008

SM structure

- SU(3) × SU(2) × U(1)

  Partly “anthropic”; Fundamental insight from string theory not likely.

- Anomaly cancellation

  String theory explains this very well.

- Small representations

  String theory (almost) explains this.

- Absence of fractional electric charge

  To be discussed.

- Families fit nicely in (16) of SO(10)

  Almost inevitable consequence of the foregoing.

- Three families

  To be discussed.

- Coupling unification

  To be discussed.

Open Strings

The ends of open strings give rise to U(N), O(N) or Sp(2N) gauge groups.

Since each open string has two ends, matter must be in bi-fundamentals (or rank-two tensors).

One may think of the endpoints as open strings ending on a membrane or a stack of N membranes.

By considering suitable combinations of stacks of branes one may obtain the standard model.

Classes of open String Models

Non-orientable

Non-orientable

Orientable

The different models are distinguished by the realization of Y:

(assuming at most four participating branes)

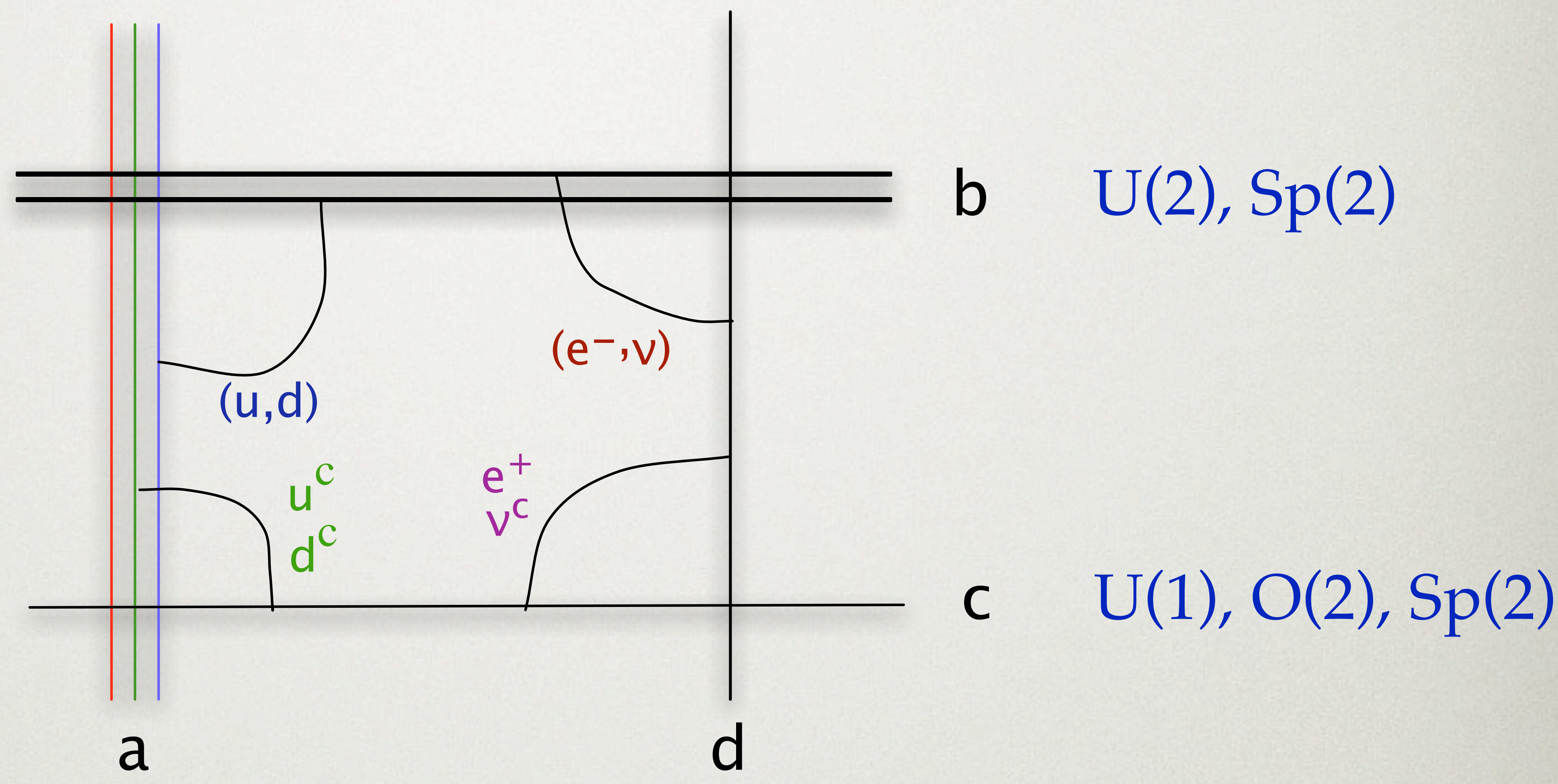
$$Y = \overbrace{(x-\tfrac{1}{3})Q_{\mathbf{a}}}^{\text{U(3)}} + \overbrace{(x-\tfrac{1}{2})Q_{\mathbf{b}}}^{\text{U(2) or Sp(2)}} + \overbrace{xQ_{\mathbf{c}} + (x-1)Q_{\mathbf{d}}}^{\text{extra branes}}$$

The following three possibilities exist*

1. x=½ (Madrid model, Pati-Salam model, ...)
2. x=0 (SU(5), ...)
3. x not quantized, strings orientable. (Trinification, ...)

(*)Anastasopoulos, Dijkstra, Kiritsis, Schellekens (2006)

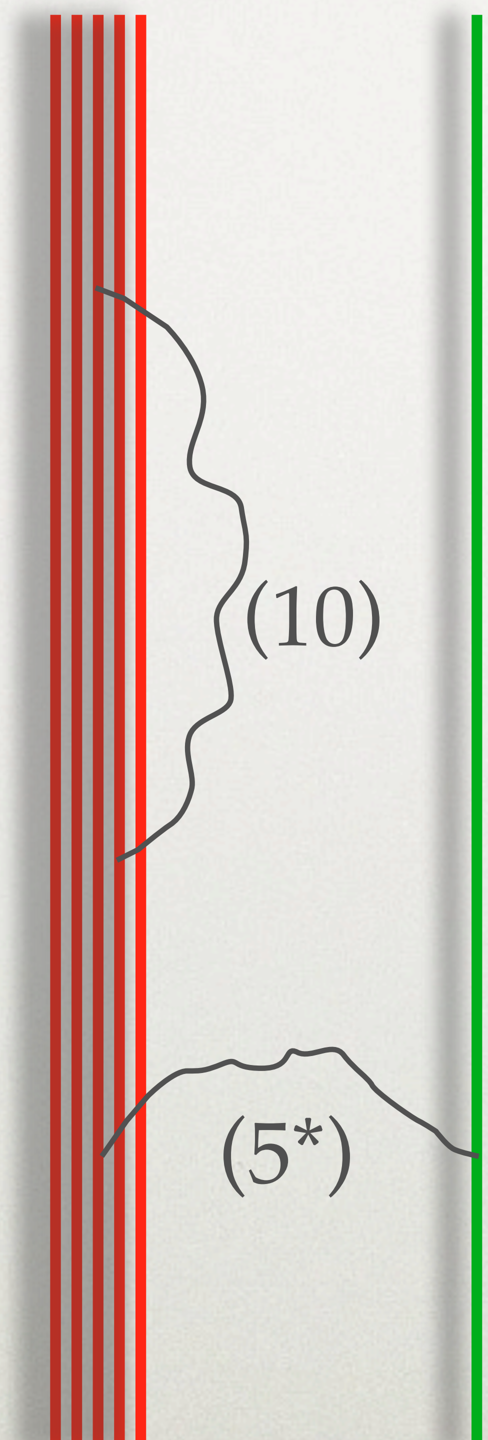
The Madrid Model*



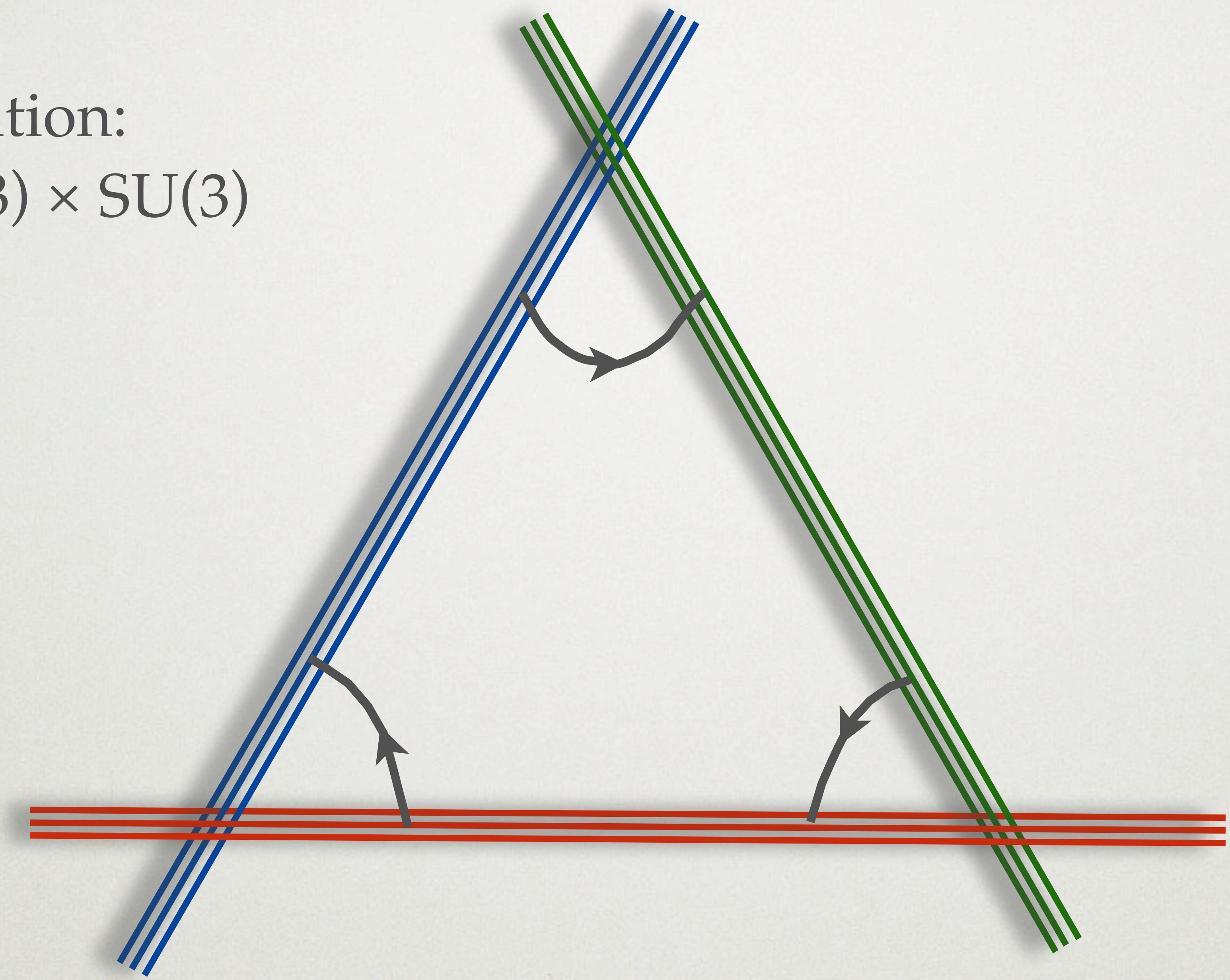
$$Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c - \frac{1}{2}Qd$$

(*) Ibanez, Marchesano, Rabadan (2000)

SU(5)



Trinification:
SU(3) × SU(3) × SU(3)



$$(3, 3^*, 1) + (3^*, 1, 3) + (1, 3, 3^*)$$

Open Strings: Fractional charge

All matter from intersections of standard model branes has integral charge.
But often there are additional "hidden sector" branes, intersecting the SM.
These have fractional charges x:

- x=½ class: Half-integer fractional charges.
- x=0 class: Only integer charges:
- orientable class: Fractional charge x.
  (e.g. third-integer for trinification)

Note: fractionally charged matter **must** couple to the hidden sector, and may be confined by it

Open Strings: Small representations

Limited to fundamental representations and rank-2 tensors.
Still allows some wrong representations
(6 of SU(3), adjoints, charge-2 particles)

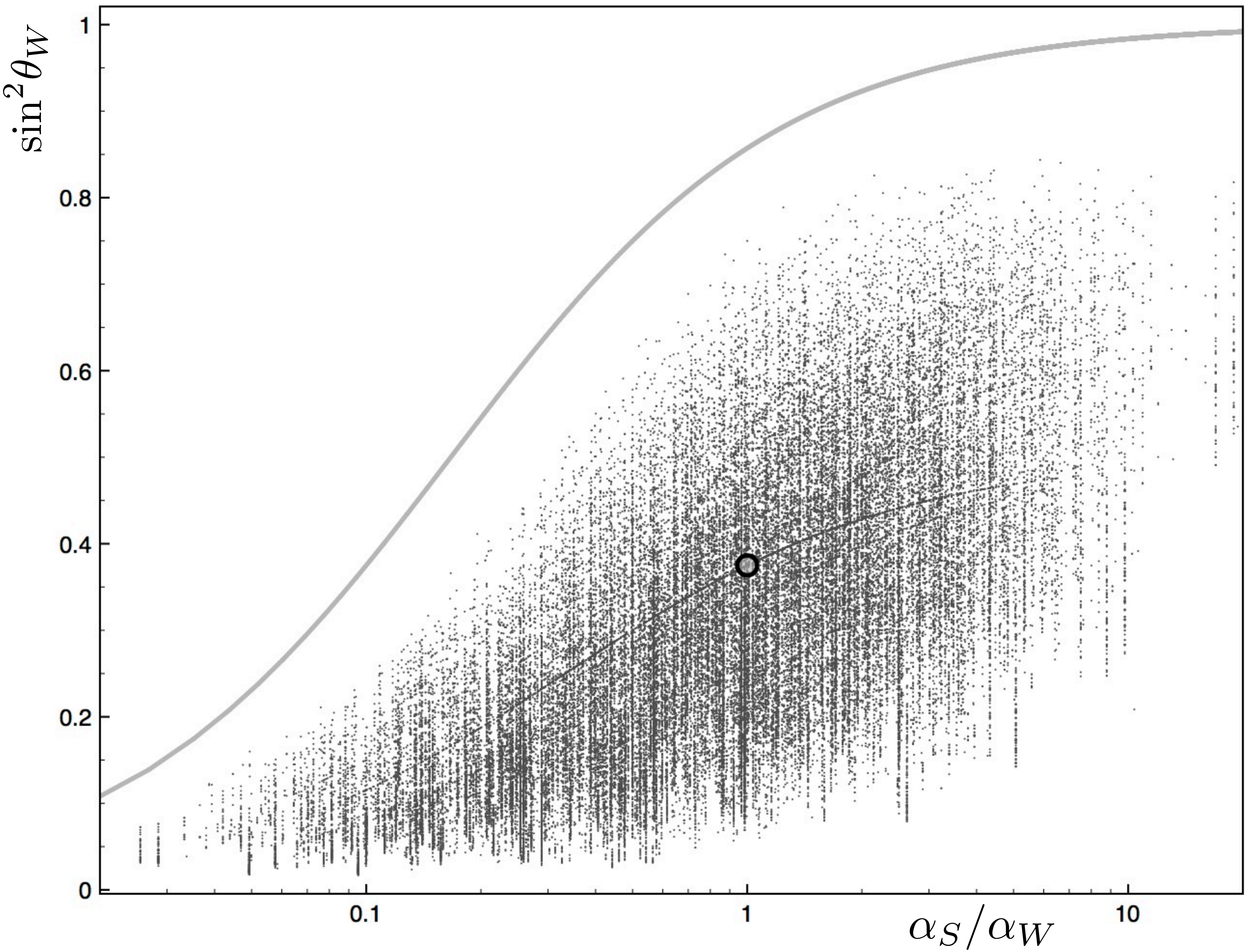
Open Strings: Coupling Unification

In all classes there are at least three in principle unrelated brane stacks. Each stack gives rise to its own gauge coupling.

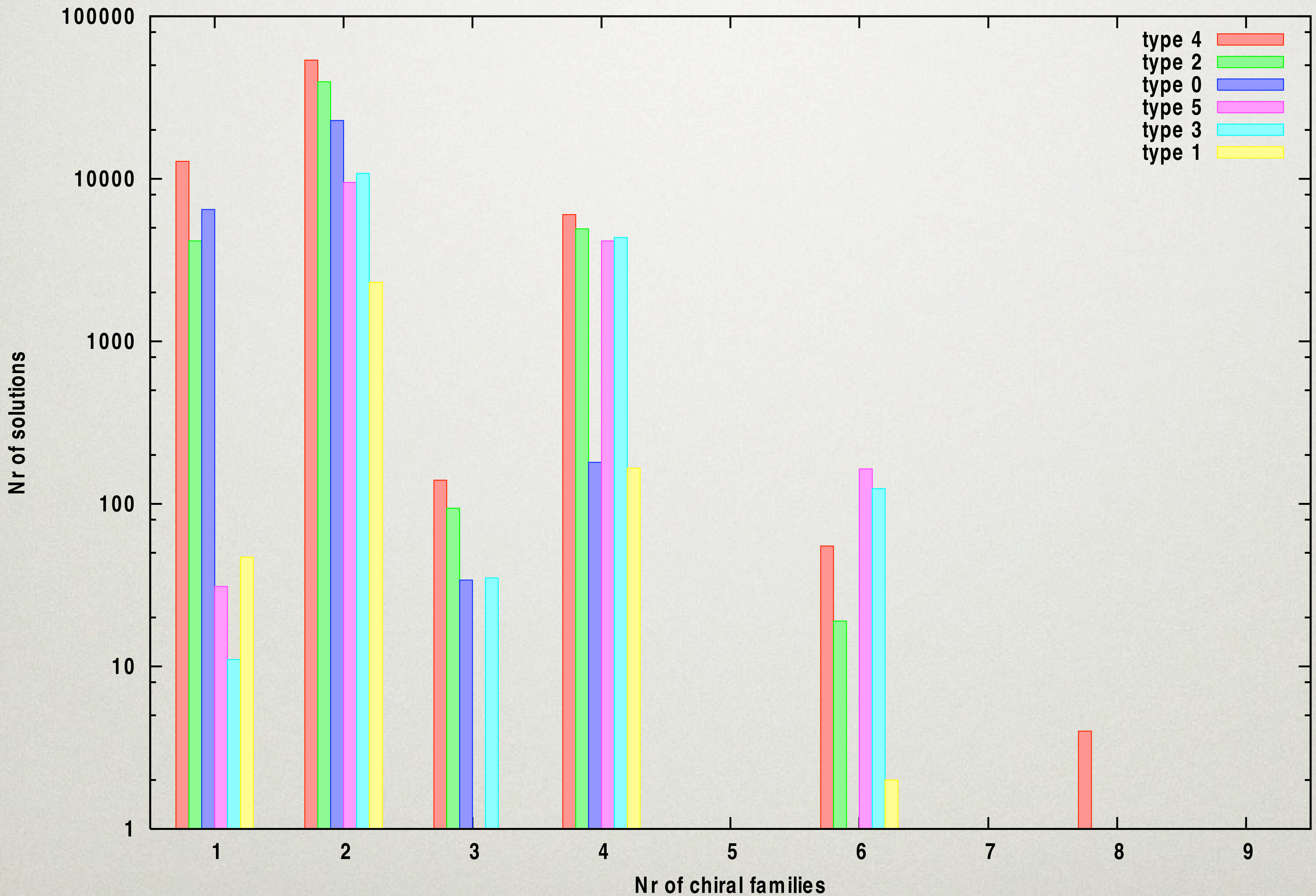
These are, in principle, unrelated. Therefore no coupling unification is expected.

In special cases, some stacks may coincide, and yield for example Pati-Salam or SU(5) models.

If we plot the coupling ratios for the entire class of $x=\frac{1}{2}$ models, we get the expected result:


$\sin^2\theta_W$
1
0.8
0.6
0.4
0.2
0
0.1
1
10
α_S/α_W

Open Strings: Number of families



(x=½ models)

Dijkstra, Huiszoon, Schellekens (2004)
See also Gmeiner et. al. "One in a billion"

Explicit realizations

It's easy enough to draw these pictures.
But finding an explicit example is another matter.
This involves:

- Finding a suitable CFT.
- Finding a type-IIB modular invariant partition function.
- Computing the "boundary coefficients" and the "crosscap coefficients"(*)
- Computing the Annulus, Klein bottle and Moebius coefficients.
- Checking if the massless spectrum matches the Standard Model.
- Checking if Y remains massless
- Cancelling the disk and crosscap tadpoles

(*) Cardy (1989), Sagnotti, Pradisi, Stanev, Bianchi (1990-1996), Fuchs, Schweigert, Huiszoon, Sousa, Walcher (1995-2000), ...

Gauge group: Exactly SU(3) × SU(2) × U(1)!

[U(3)×Sp(2)×U(1)×U(1), Massive B-L, No hidden sector]

3 x (V ,V ,0 ,0)	chirality 3	Q
3 x (V ,0 ,V ,0)	chirality -3	U*
3 x (V ,0 ,V* ,0)	chirality -3	D*
3 x (0 ,V ,0 ,V)	chirality 3	L
5 x (0 ,0 ,V ,V)	chirality -3	E*+(E+E*)
3 x (0 ,0 ,V ,V*)	chirality 3	N*
18 x (0 ,V ,V ,0)		Higgs
2 x (V ,0 ,0 ,V)		
2 x (Ad ,0 ,0 ,0)		
2 x (A ,0 ,0 ,0)		
6 x (S ,0 ,0 ,0)		
14 x (0 ,A ,0 ,0)		
6 x (0 ,S ,0 ,0)		
9 x (0 ,0 ,Ad ,0)		
6 x (0 ,0 ,A ,0)		
14 x (0 ,0 ,S ,0)		
3 x (0 ,0 ,0 ,Ad)		
4 x (0 ,0 ,0 ,A)		
6 x (0 ,0 ,0 ,S)		

Dijkstra, Huiszoon, Schellekens (2004)

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3 x (V	,0	,V*	,0)	chirality -3	D*
3 x (0	,V	,0	,V)	chirality 3	L
5 x (0	,0	,V	,V)	chirality -3	E*+(E+E*)
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18 x (0	,V	,V	,0)		Higgs
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2 x (Ad	,0	,0	,0)		
2 x (A	,0	,0	,0)		
6 x (S	,0	,0	,0)		
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9 x (0	,0	,Ad	,0)		
6 x (0	,0	,A	,0)		
14 x (0	,0	,S	,0)		
3 x (0	,0	,0	,Ad)		
4 x (0	,0	,0	,A)		
6 x (0	,0	,0	,S)		

Vector-like matter

V=vector

A=Anti-symm. tensor

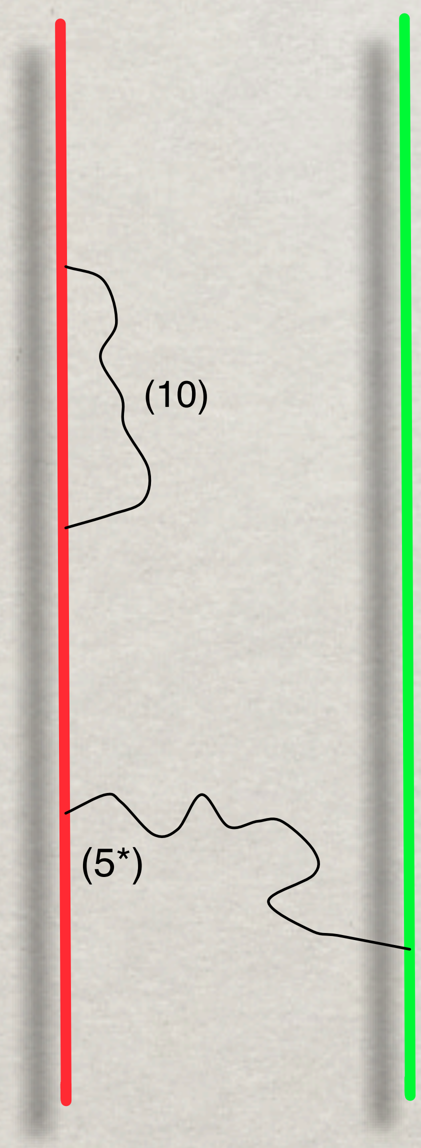
S=Symmetric tensor

Ad=Adjoint

Dijkstra, Huiszoon, Schellekens (2004)

An SU(5) model

Gauge group is just SU(5)!



U5 O1 O1

3 x	(A	,0	,0)	chirality	3
11 x	(V	,V	,0)	chirality	-3
8 x	(S	,0	,0)		
3 x	(Ad	,0	,0)		
1 x	(0	,A	,0)		
3 x	(0	,V	,V)		
8 x	(V	,0	,V)		
2 x	(0	,S	,0)		
4 x	(0	,0	,S)		
4 x	(0	,0	,A)		

(*)Anastasopoulos, Dijkstra, Kiritsis, Schellekens (2006)

Heterotic Strings

Polyakov action:

$$S[X,\gamma] = -\frac{1}{4\pi\alpha'}\int d\sigma d\tau \sqrt{-\det\gamma}\sum_{\alpha\beta}\gamma^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu$$

$X^\mu(\sigma,\tau)$ defines the embedding of the string in space-time.

$(\mu = 0, \ldots, D-1)$ Only consistent if D=26.

This can be overcome by replacing part of the action by a more general conformal field theory (CFT).

Virasoro algebra:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{1}{12}c(m^3 - m)\delta_{m+n}$$

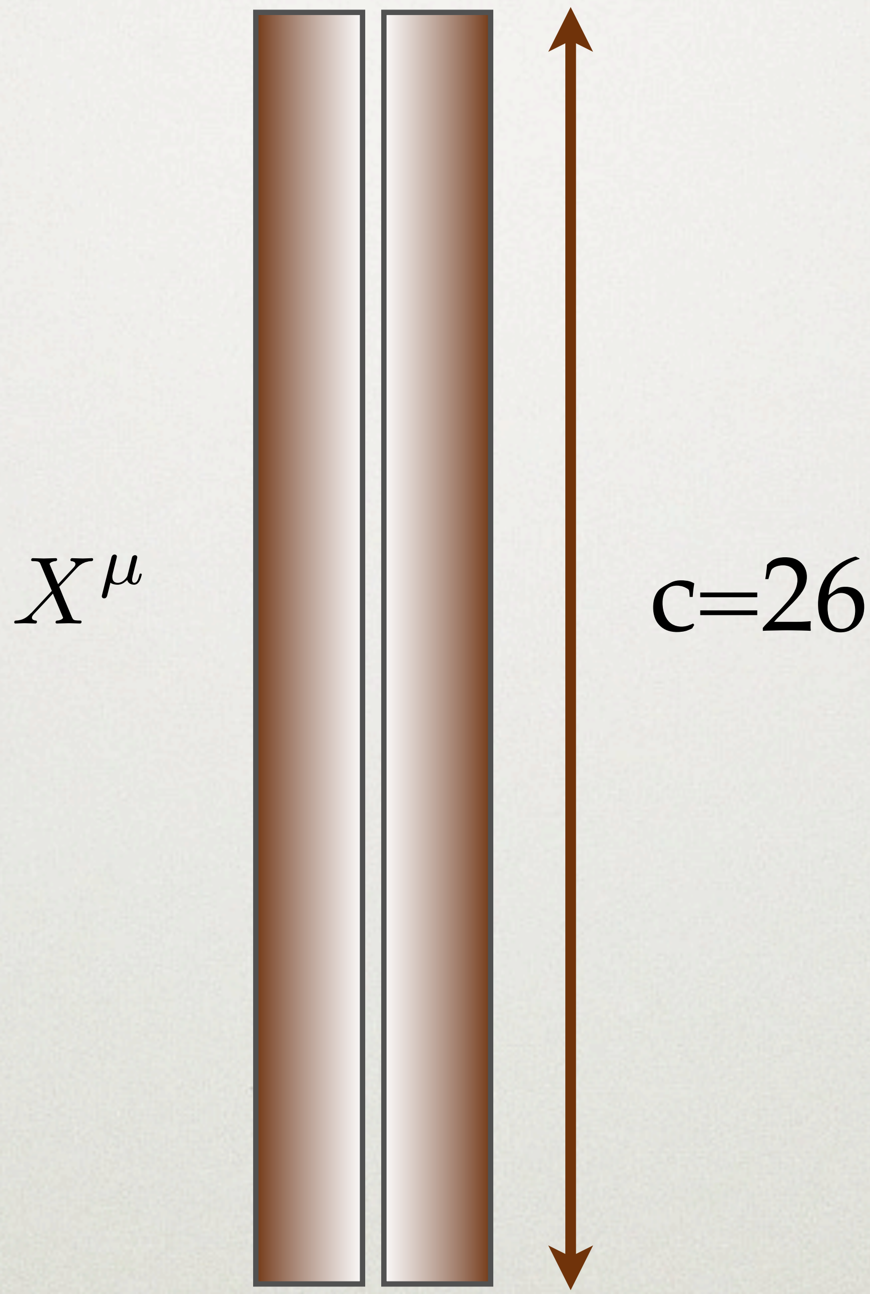
The constant c measures the contribution of a term in the action. It is additive, and has to add up to 26.

Typically, the theory is build out of some simple building blocks, in order to get some computational control.

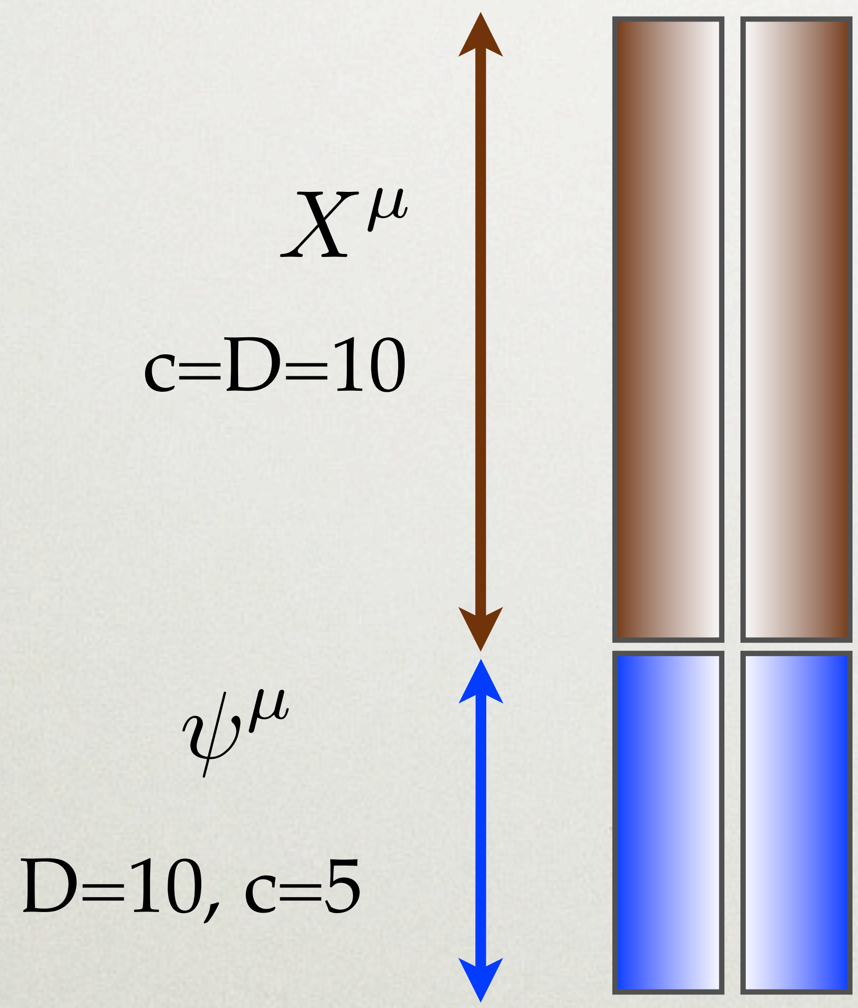
In closed strings, there are separate algebras for left-moving and right-moving modes.

One may build the left-moving sector and the right-moving separately out of different building blocks.

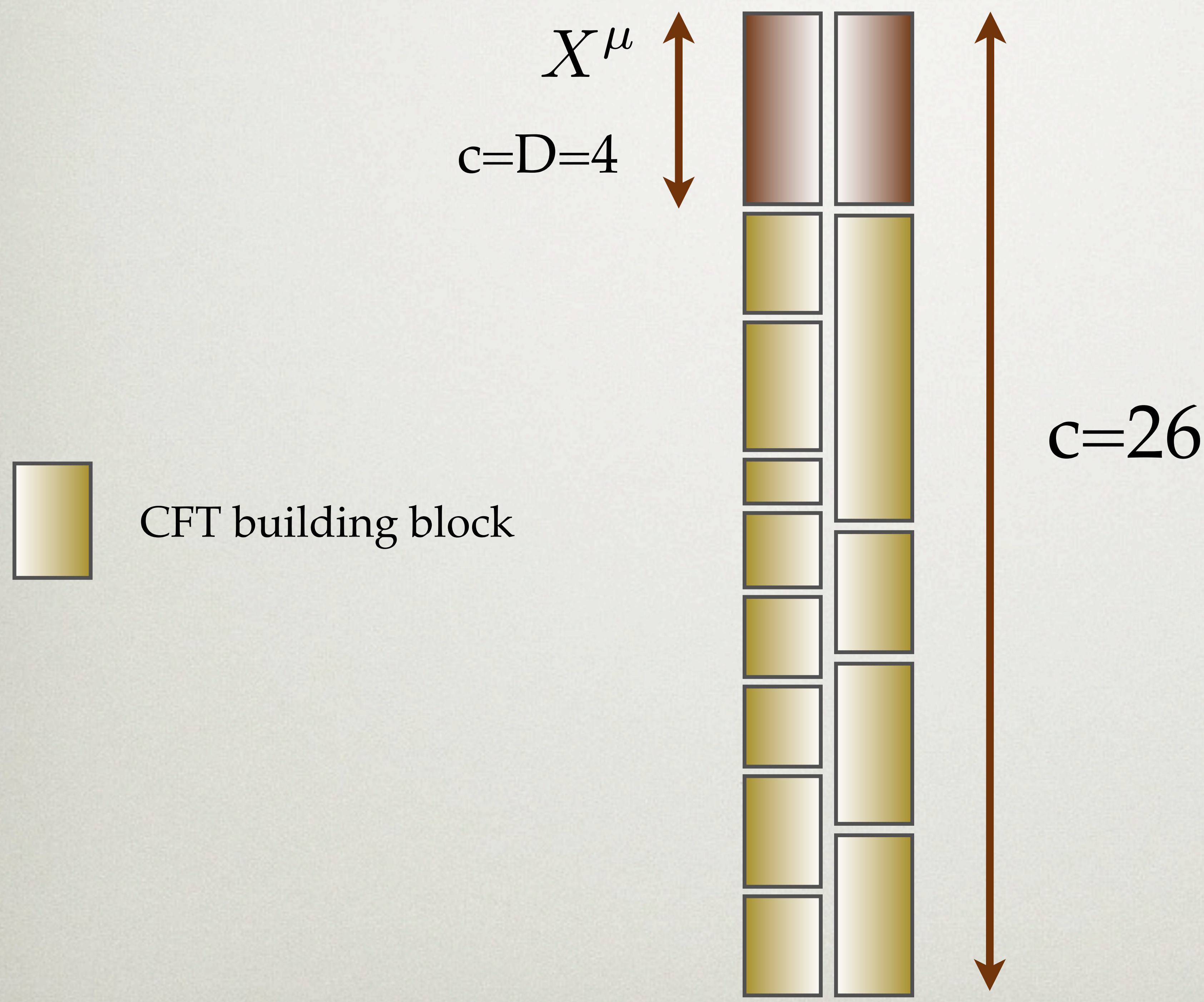
Basic Bosonic String



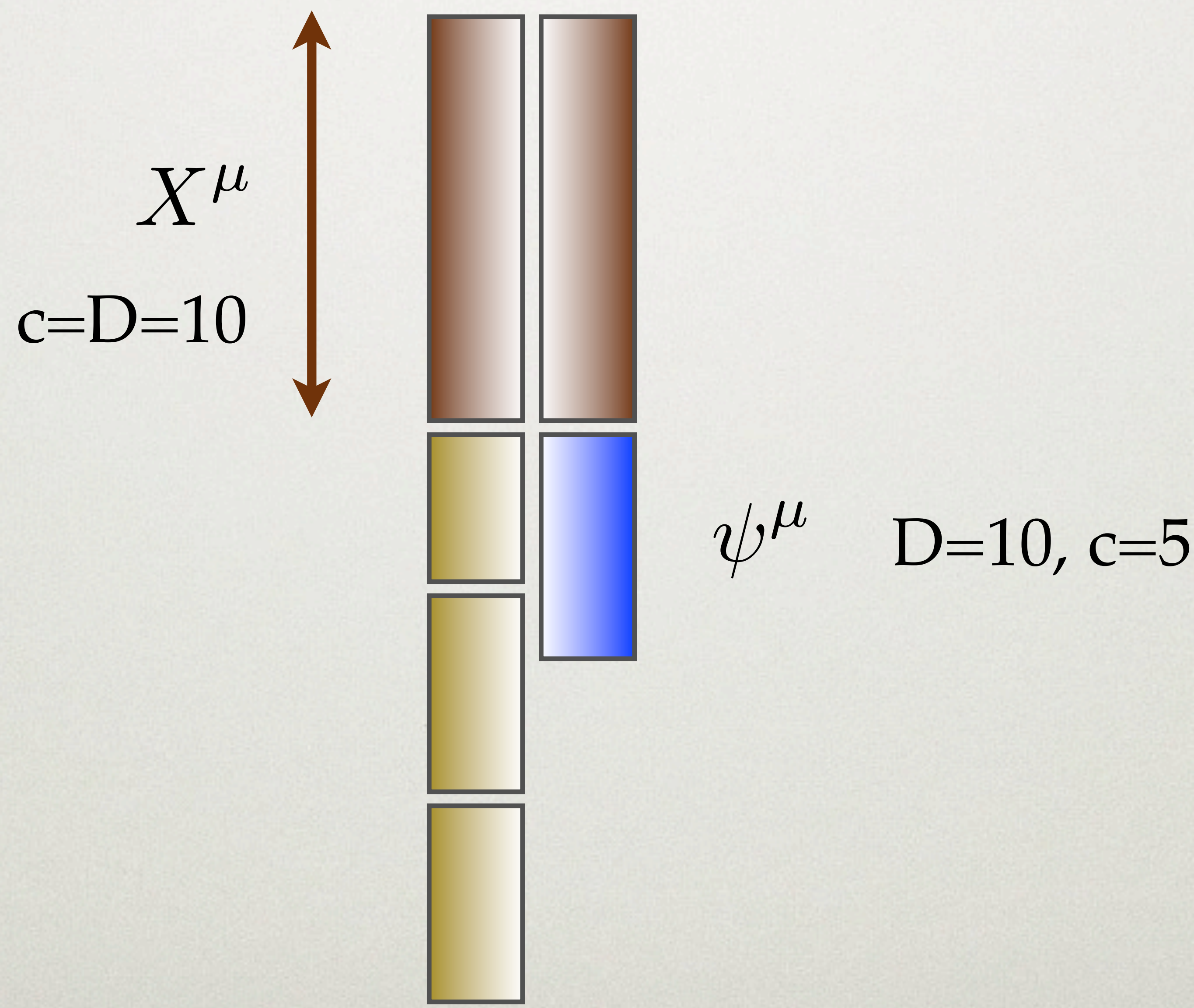
Fermionic Strings



Compactified Bosonic String

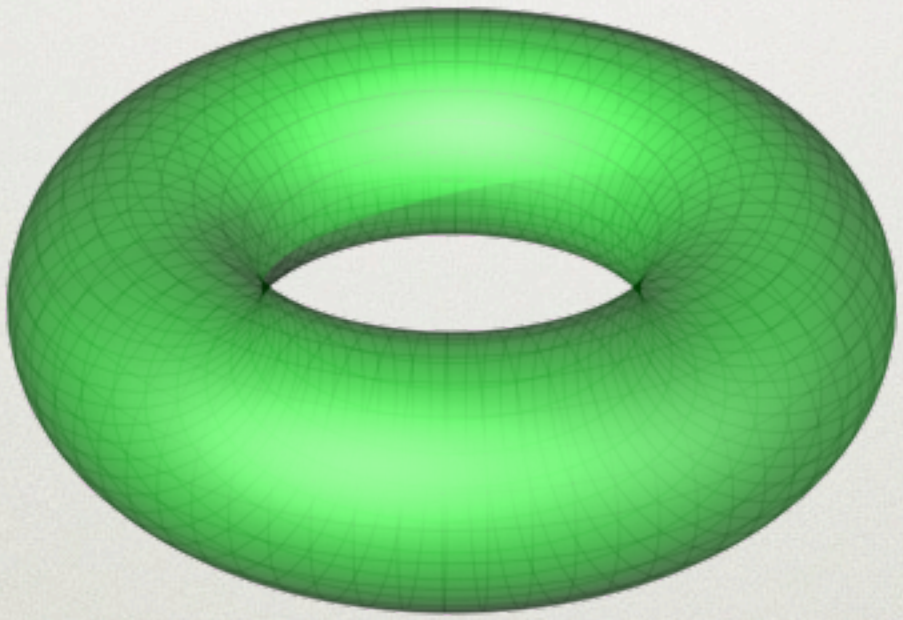


Heterotic strings



Modular invariance

The freedom of associating left and right building blocks is severely limited by a constraint arising from the consistency of the simplest one-loop diagram, the torus.



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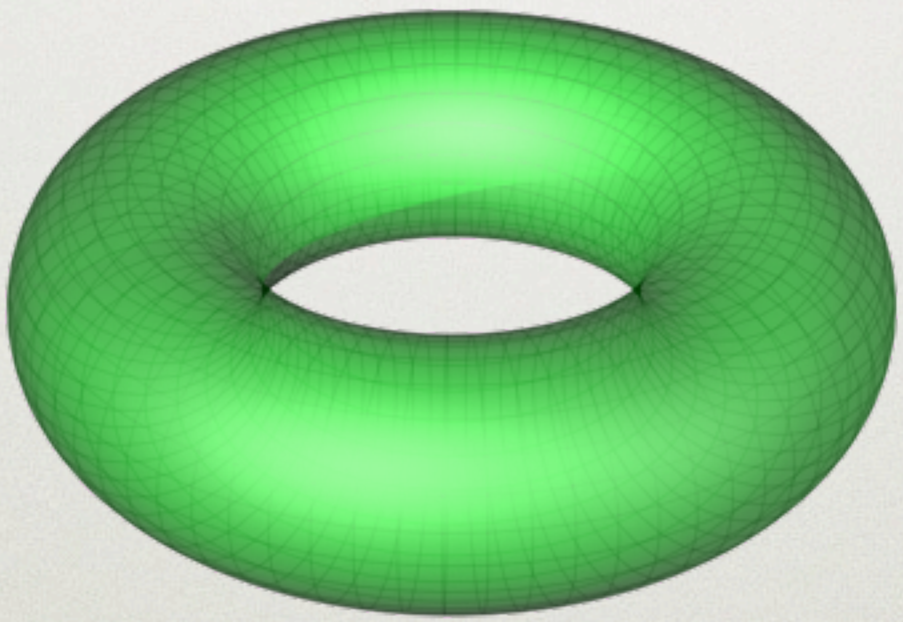
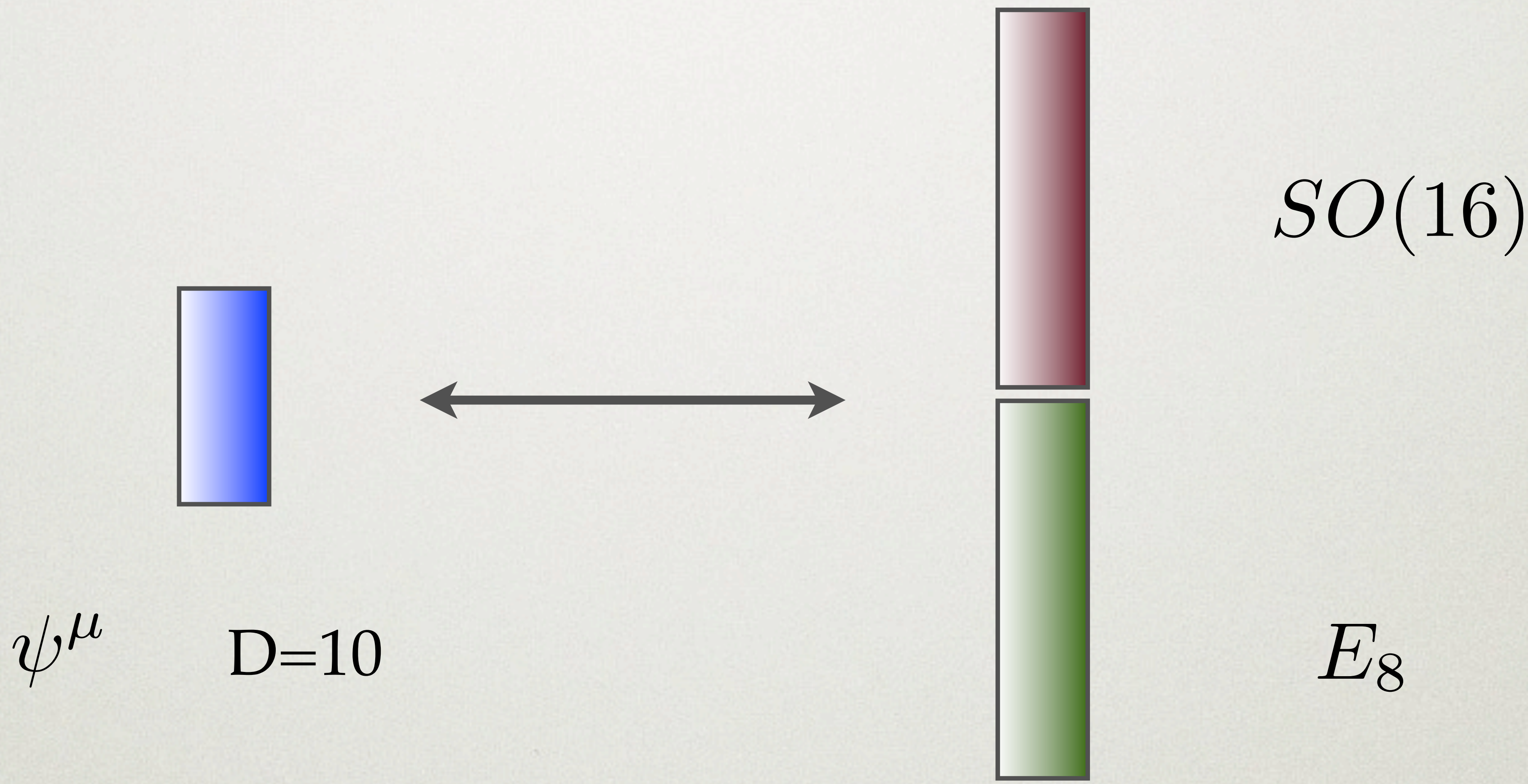


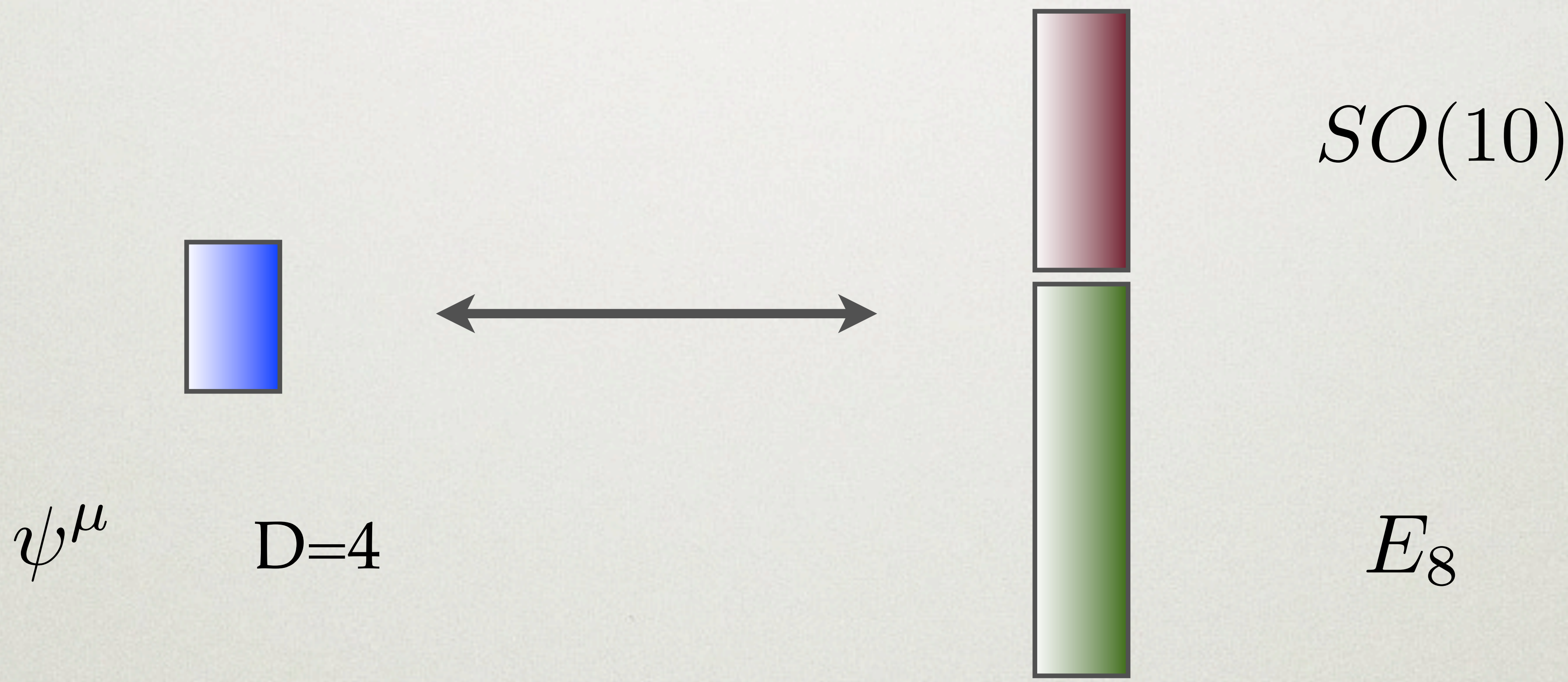
Modular invariance restricts this severely. Solutions exist because of isomorphisms between modular group representations.



$SO(16)$, E_8 are special CFT building blocks called affine Lie algebras. They appear in the spectrum as gauge symmetries

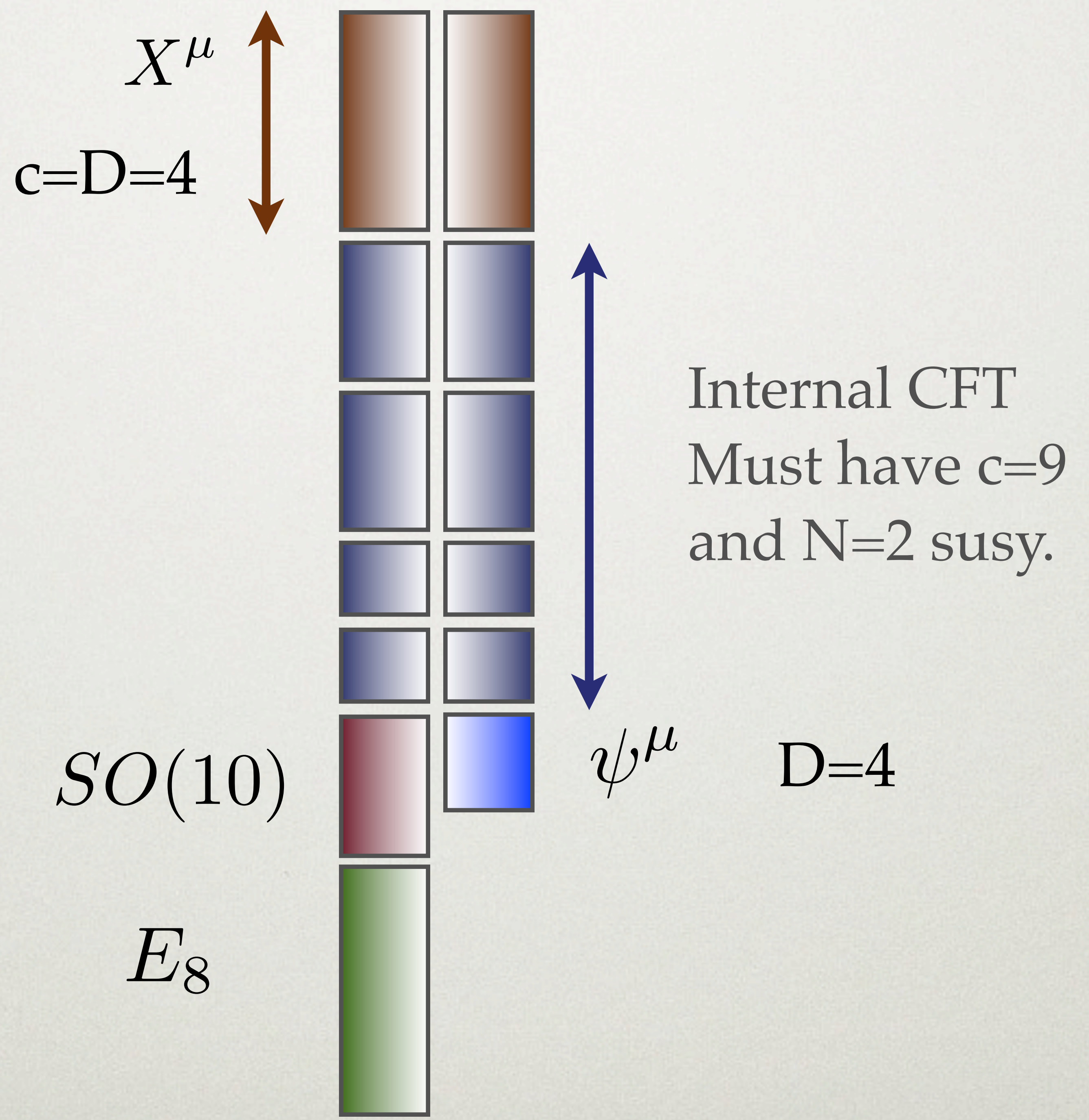
The Bosonic String Map

This also works in 4 dimensions:



Lerche, Lüst, Schellekens (1986)

Now we can build 4-dimensional strings



Electric Charge Quantization

- All color singlets in the Standard Model have integer charges.
- This can be most easily understood by assuming an embedding in SU(5) (or SO(10)).
- But how does this work in string theory?

CERN–TH.5440/89

New Modular Invariants for N=2 Tensor Products and Four-Dimensional Strings

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and

S. Yankielowicz* †

CERN, 1211 Geneva 23, Switzerland

ABSTRACT

The construction of modular invariant partition functions of tensor products of $N = 2$ superconformal field theories is clarified and extended by means of a recently proposed method using simple currents, *i.e.* primary fields with simple fusion rules. Apart from providing a conceptually much simpler way of understanding space-time and world-sheet supersymmetry projections in modular invariant string theories, this makes a large class of modular invariant partition functions accessible for investigation. We demonstrate this by constructing thousands of (2,2), (1,2) and (0,2) string theories in four dimensions, including more than 40 new three generation models.

6. Outlook and conclusions

Clearly the method we have advocated in this paper greatly extends the list of four-dimensional string theories accessible to exploration. However, this is by no means all one can do. Up to now we have always kept an unbroken $SO(10) \times E_8$ Kac-Moody algebra on the left. However, just as one can break the left-moving "space-time" and world-sheet supersymmetries, one can break this KM-algebra as well. To do so, one simply starts with characters of some conformal sub-algebra of $SO(10) \times E_8$. Of course one wants to get the full $SO(10) \times E_8$ algebra on the *right*, in order to be able to map this sector to a fermionic one. But this can always be achieved by putting some projection matrices in front of the right-moving characters to add the missing $SO(10) \times E_8$ roots.

This opens the way to constructing string theories whose gauge group is something a bit closer to the standard model than $SO(10)$, perhaps even $SU(3) \times SU(2) \times U(1)^n$ (where n is almost inevitably larger than 1). There is no reason why one could not get 3 generations in such a model, and in fact there could well be many more models than those listed in table III, since the center of the conformal field theory one starts with is even larger. We hope to come back to this in the future.

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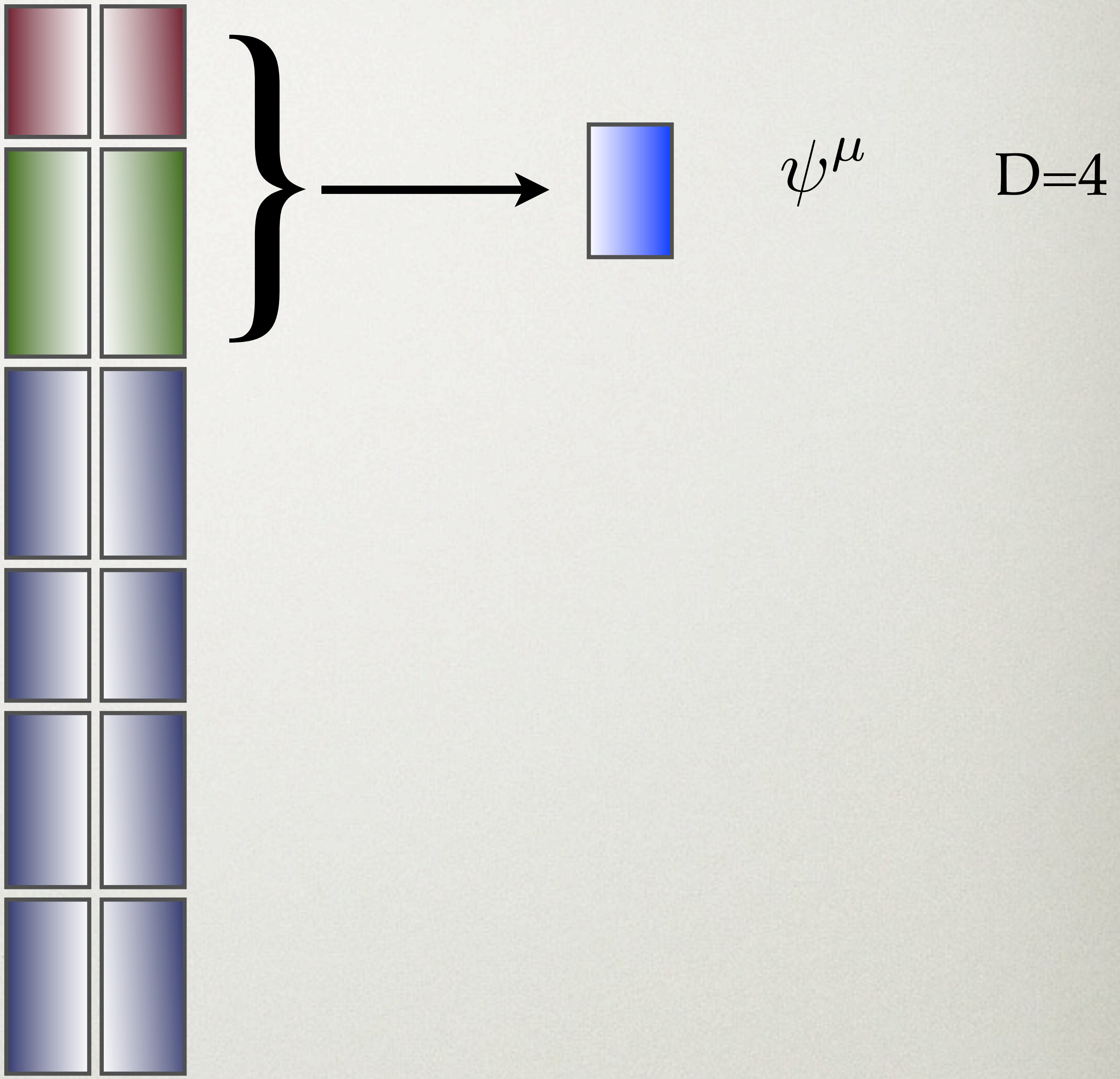
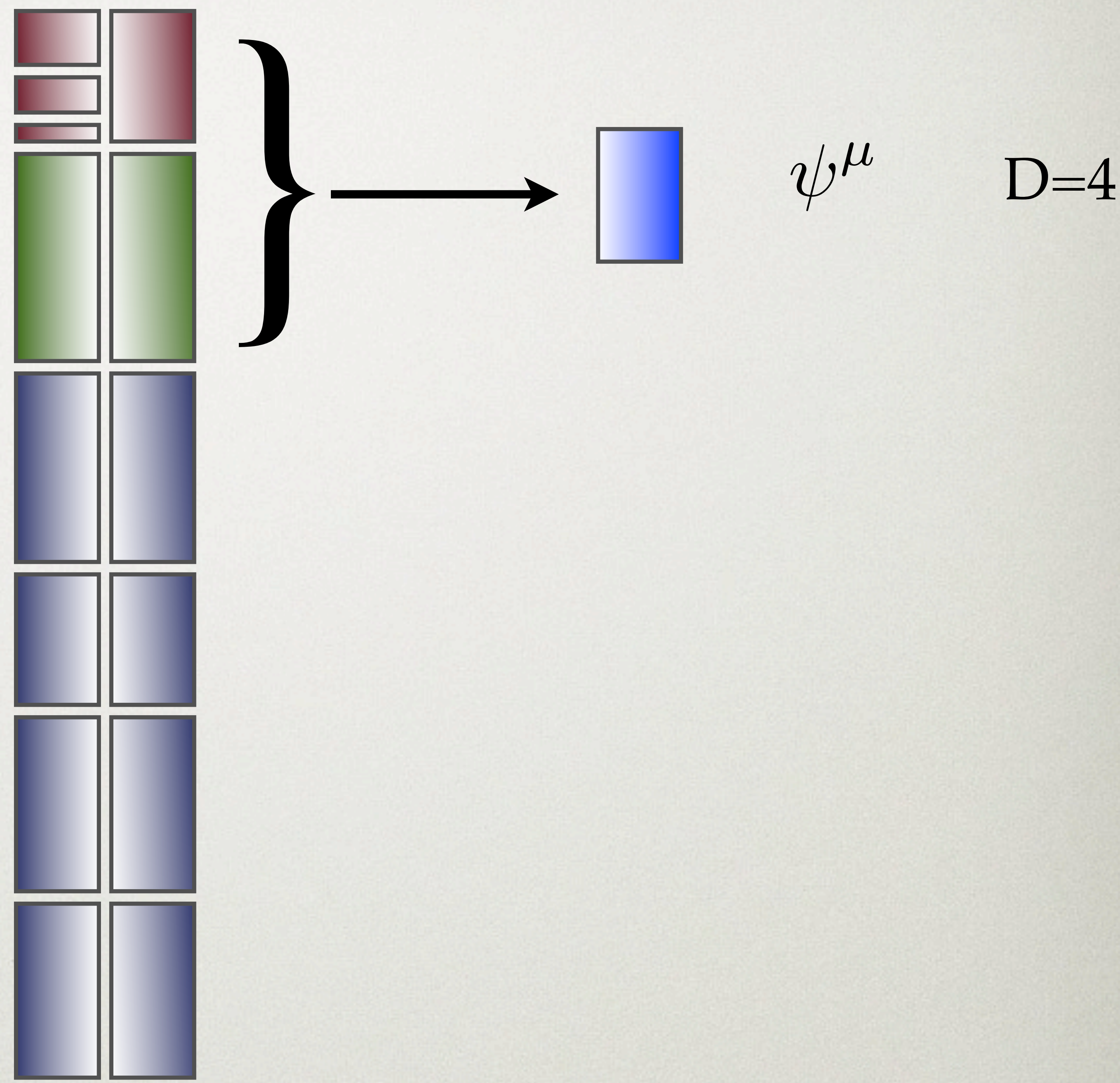
The future has finally arrived (Gato-Rivera, Schellekens, 2010)


ψ^μ
D=4

SO(10) currents replaced by operators of higher weight



Gauge group H ⊂ SO(10) (× H′ ⊂ E_8 × ....)

Breaking SO(10)

Consider* $SU(3) \times SU(2) \times U(1)_{30} \times U(1)_{20} \subset SO(10)$

This should give chiral families of $SU(3) \times SU(2) \times U(1)$ with standard gauge coupling unification.

Indeed, it does, but there was a major disappointment:
All these spectra contain fractionally charged particles.

This was easily seen to be a very general result.
(A.N. Schellekens, Phys. Lett. B237, 363, 1990).

But there are ways out: they can be massive, vector-like (or confined by another gauge group)

(*) A.N. Schellekens and S. Yankielowicz (1989)
Other subgroups were considered by Blumenhagen, Wisskirchen, Schimmrigk (1995, 1996)

SO(10) sub-algebras

Nr.	Name	Current	Order	Gauge group	Grp.	CFT
0	SM, Q=1/6	$(1,1,0,0)$	1	$SU(3)\times SU(2)\times U(1)\times U(1)$	$\frac{1}{6}$	$\frac{1}{6}$
1	SM, Q=1/3	$(1,2,15,0)$	2	$SU(3)\times SU(2)\times U(1)\times U(1)$	$\frac{1}{6}$	$\frac{1}{3}$
2	SM, Q=1/2	$(3,1,10,0)$	3	$SU(3)\times SU(2)\times U(1)\times U(1)$	$\frac{1}{6}$	$\frac{1}{2}$
3	LR, Q=1/6	$(1,1,6,4)$	5	$SU(3)\times SU(2)_L\times SU(2)_R\times U(1)$	$\frac{1}{6}$	$\frac{1}{6}$
4	SU(5) GUT	$(\bar{3},2,5,0)$	6	$SU(5)\times U(1)$	1	1
5	LR, Q=1/3	$(1,2,3,-8)$	10	$SU(3)\times SU(2)_L\times SU(2)_R\times U(1)$	$\frac{1}{6}$	$\frac{1}{3}$
6	Pati-Salam	$(\bar{3},0,2,8)$	15	$SU(4)\times SU(2)_L\times SU(2)_R$	$\frac{1}{2}$	$\frac{1}{2}$
7	SO(10) GUT	$(3,2,1,4)$	30	$SO(10)$	1	1

Results:

- Half-integer or third-integer charges can be avoided by clever choices of the CFT, but not simultaneously.
- In about half of the cases the fractional charges are present, but at least they are vector-like: they can get masses under perturbations

A return to the heterotic string

II The number of families

Schellekens, Yankielowicz (1989): (2,2) , (1,2) unbroken SO(10)

Gato-Rivera, Schellekens (2010): (2,2) , (1,2), (0,2), broken SO(10)

Number of families:

Turned out to be quantized in terms of a quantity Δ for each class of CFT's (there are 168+59 classes, each containing thousands of distinct spectra)

The following values of Δ occur for the 168 minimal model combinations and 58 of the 59 exceptional ones: 120, 96, 72, 60, 48, 40, 36, 32, 24, 12, 8, 6, 4 and 0.

There is one class with Δ=3, which indeed does contain 3-family models (Gepner, 1987)

Schellekens, Yankielowicz (1989): *(2,2) , (1,2) unbroken SO(10)*

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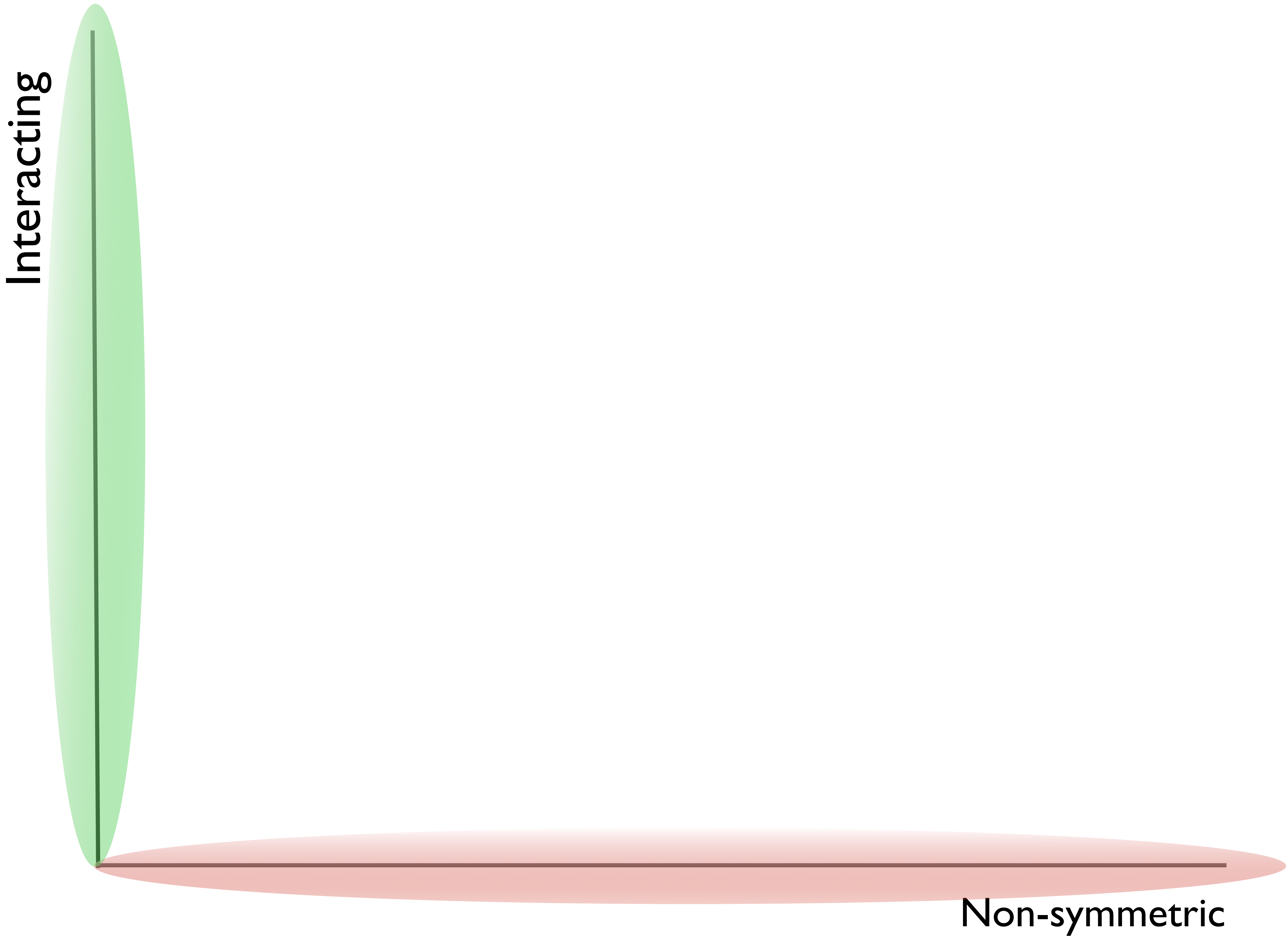
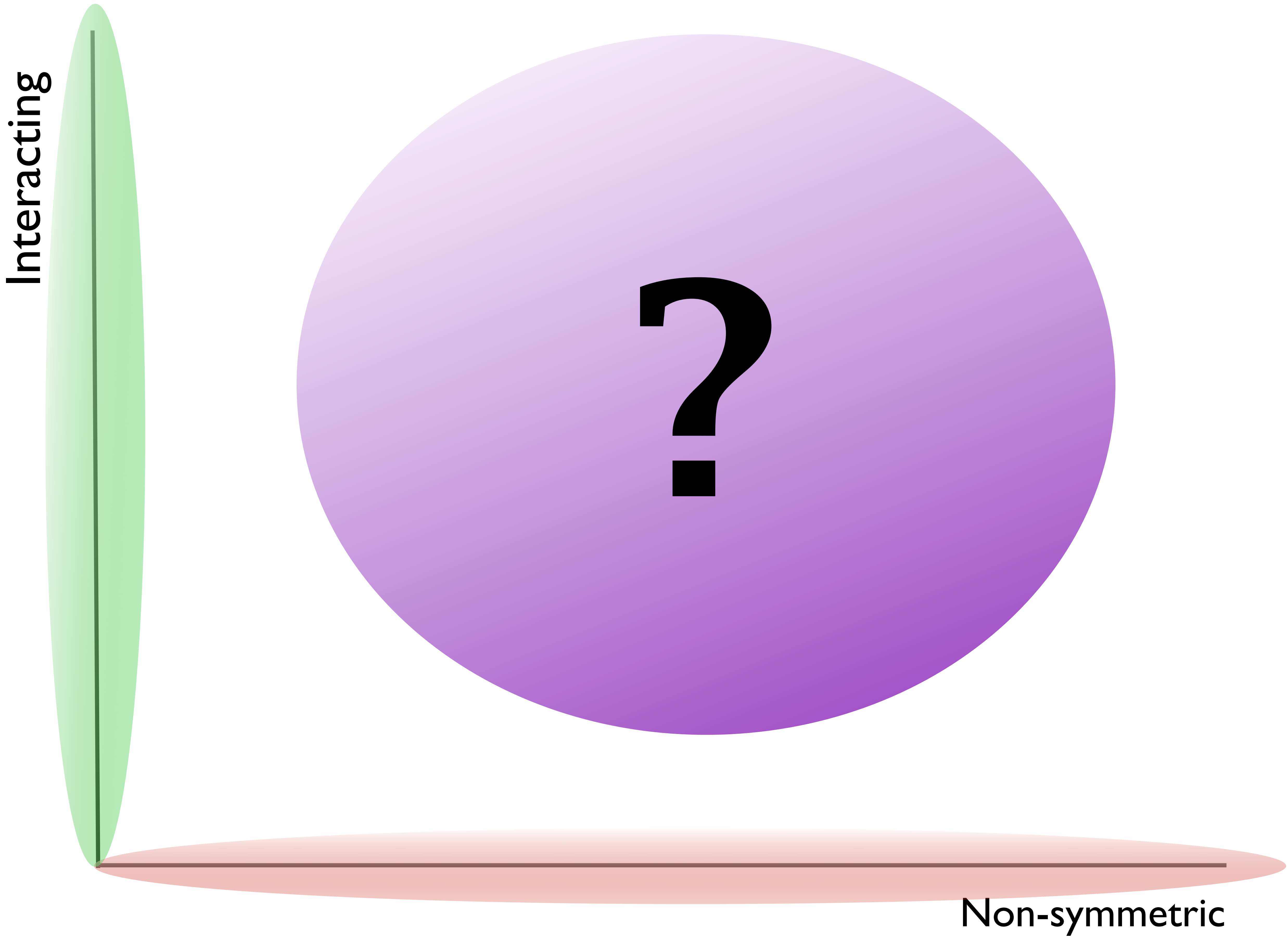
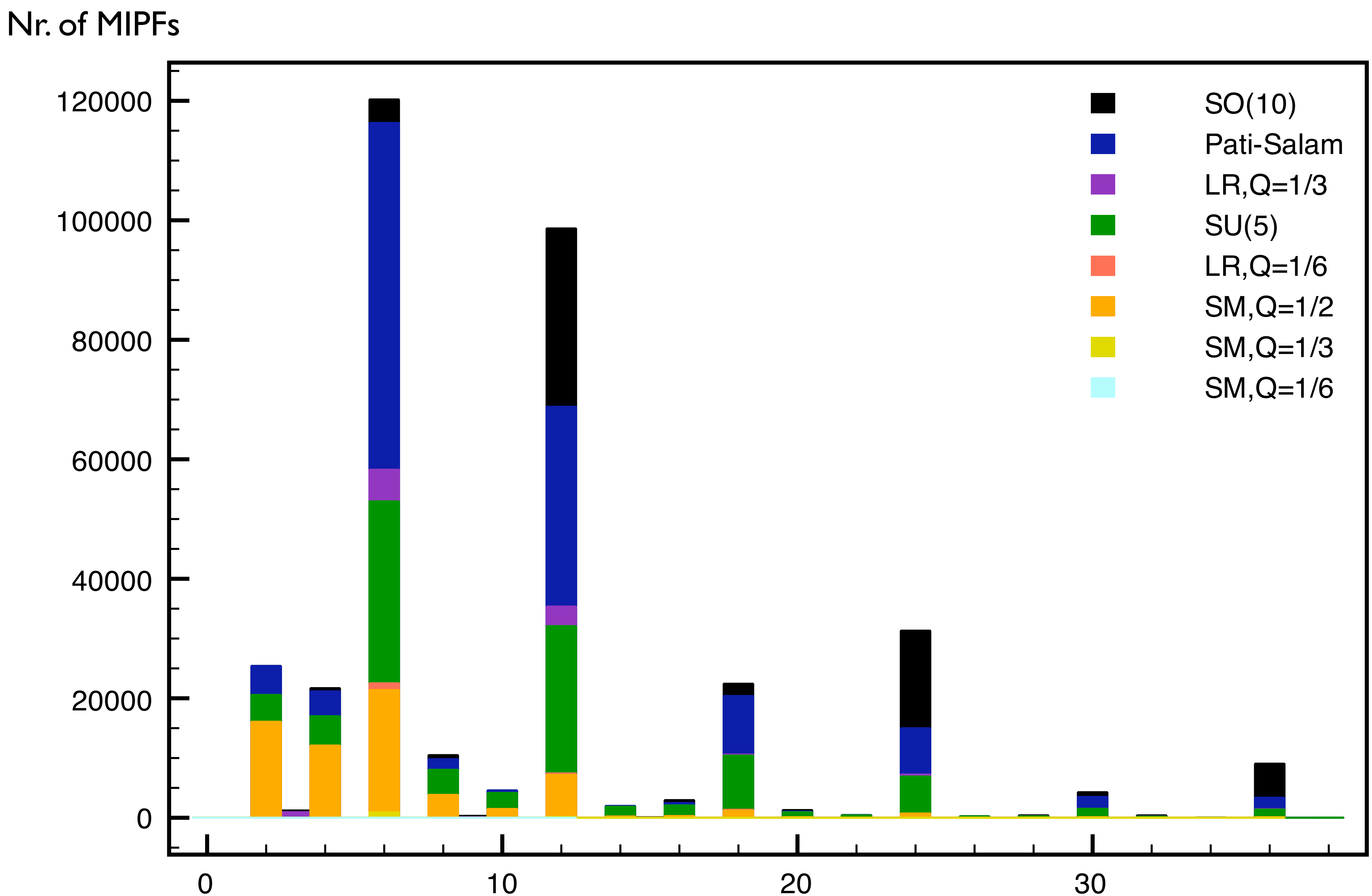


Interacting
Non-symmetric

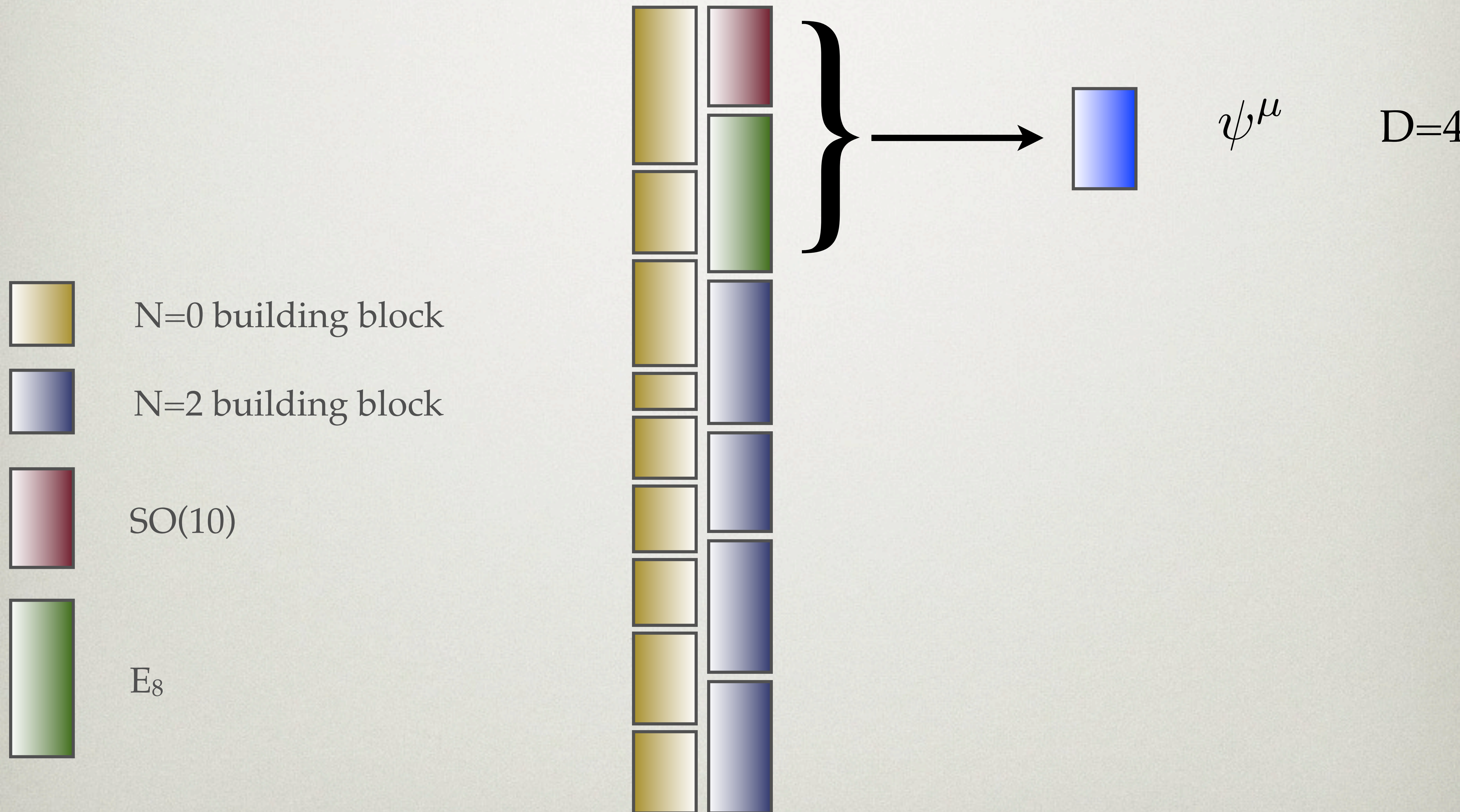

Interacting
?
Non-symmetric

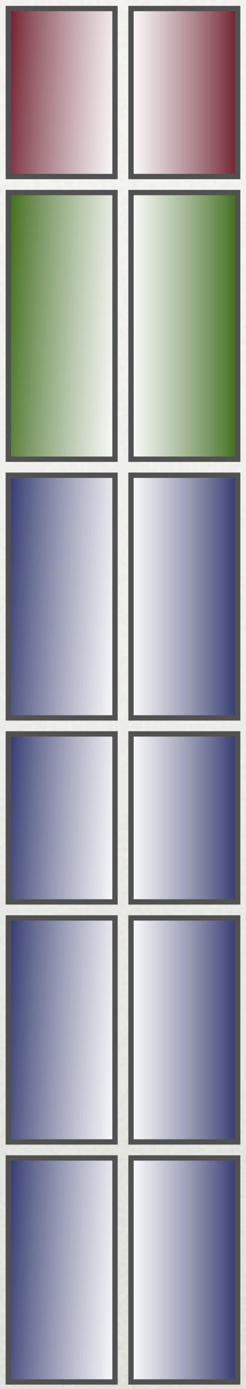
Family Distribution

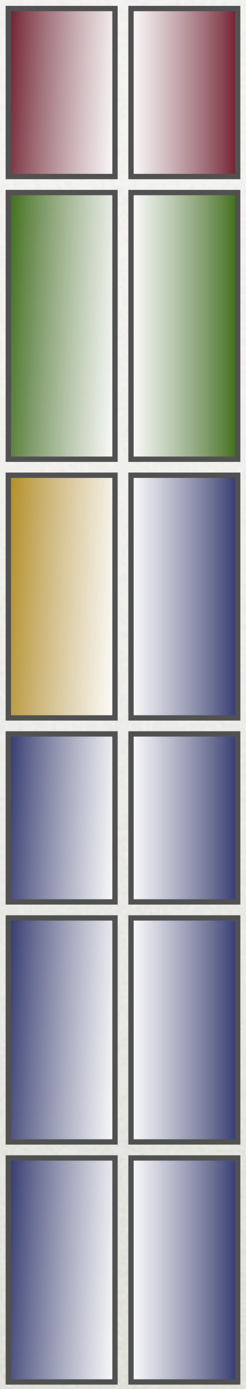


Heterotic weight lifting

General Heterotic String



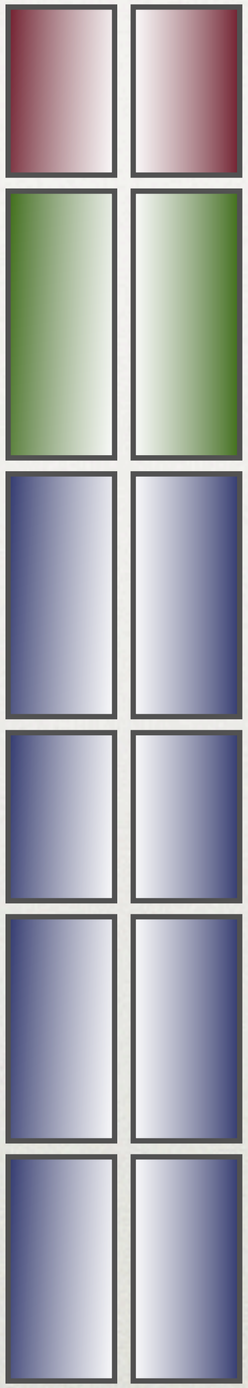




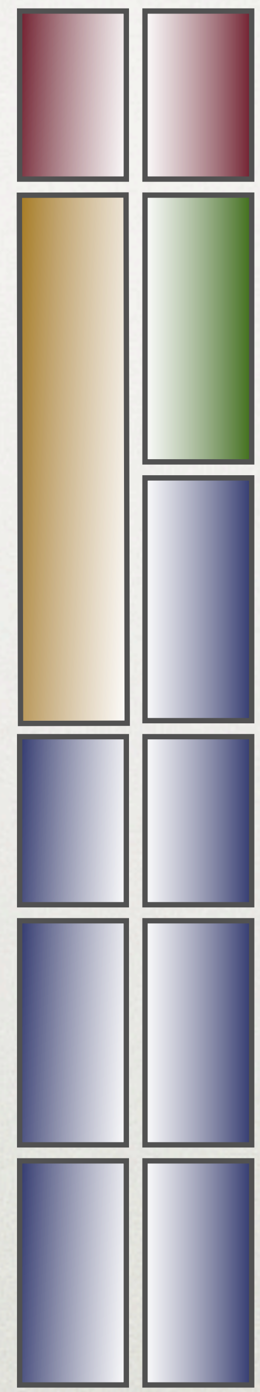
... but we have to find a N=0 CFT with the same S, T, and central charge as some N=2 model, without being identical to it.

This looks difficult.

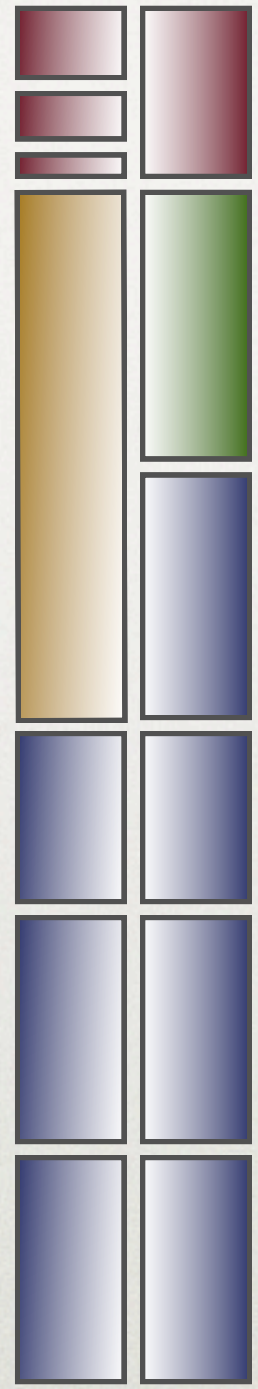
But there is something else we could try:



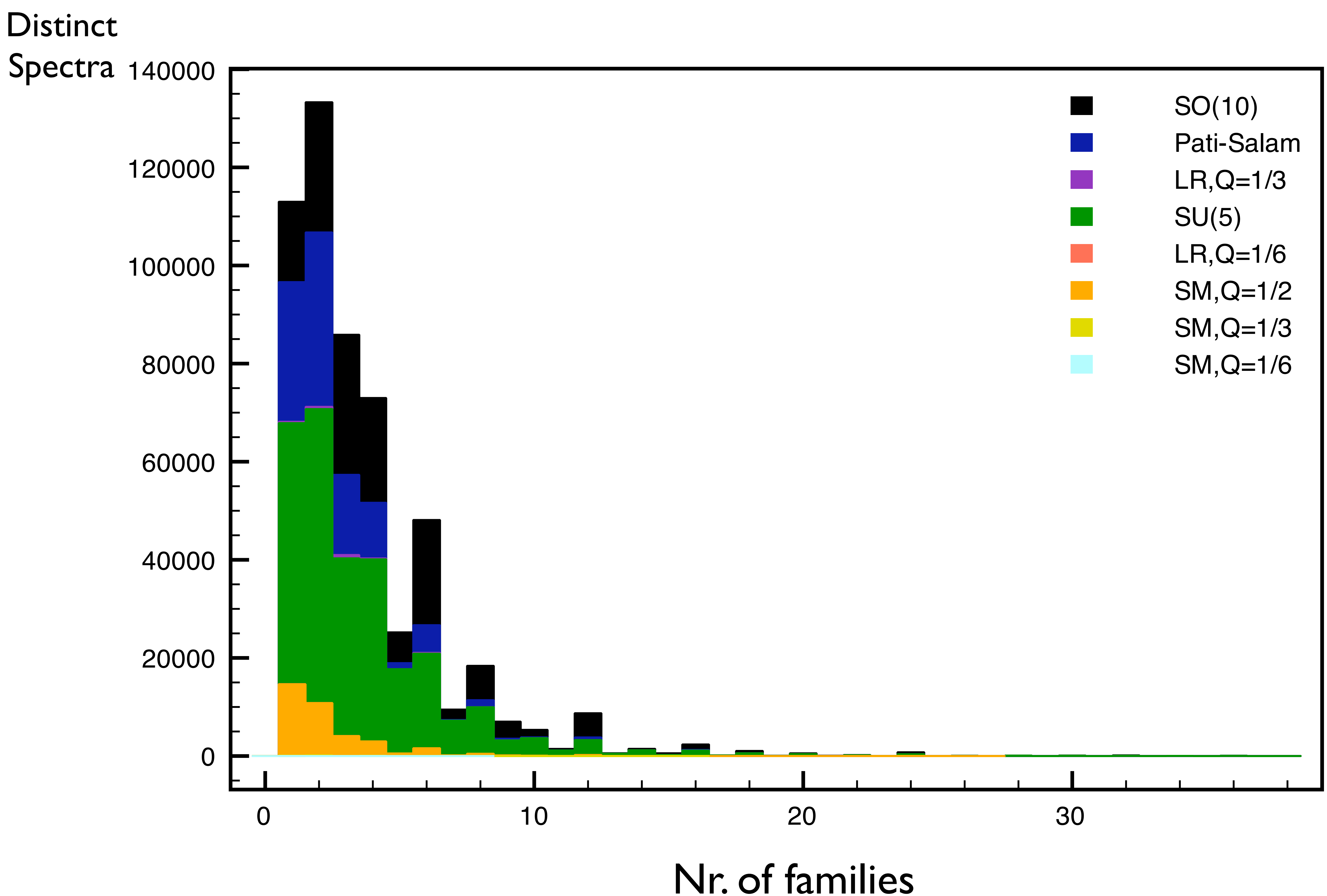
Gato-Rivera, Schellekens, 2009



Gato-Rivera, Schellekens, 2009



Gato-Rivera, Schellekens, 2009



Distinct Spectra
140000
120000
100000
80000
60000
40000
20000
0
0
10
20
30
Nr. of families
SO(10)
Pati-Salam
LR,Q=1/3
SU(5)
LR,Q=1/6
SM,Q=1/2
SM,Q=1/3
SM,Q=1/6

Conclusions

- The rough features of the Standard Model come out very easily and in several ways in string theory.
- But there is a problem with GUTs: either they don't arise naturally, or they don't work as they should.
- The number of families is another worry.
- But on closer inspection, for heterotic strings both worries are reduced.