Discrete Symmetries in Discrete Orientifolds

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S.N.S. Pisa, 23 may 2013.

Discrete symmetries
in BSM model building

- May prevent fast proton decay and/or lepton number violation due to dimension 4 operators in the MSSM (R-parity, baryon triality...)
- May prevent proton decay due to dimension 5 operators in the MSSM ($QQQL$, $U^cU^cD^cE^c$).
- But they may also forbid operators that are desirable (Yukawa couplings, Majorana masses for neutrinos,....)
- Many other uses in a variety of BSM ideas.
- But: so far, however, nature does not seem to use them (except CPT).

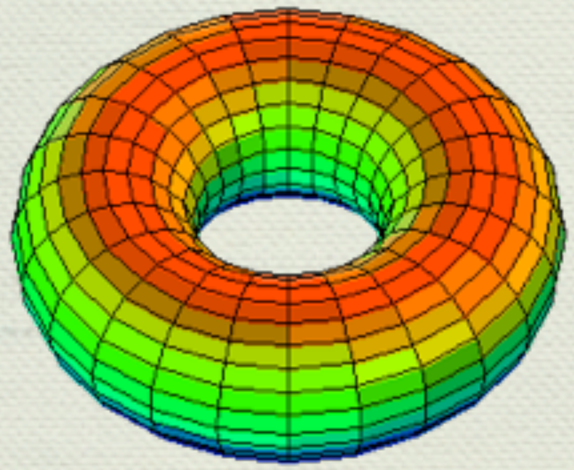
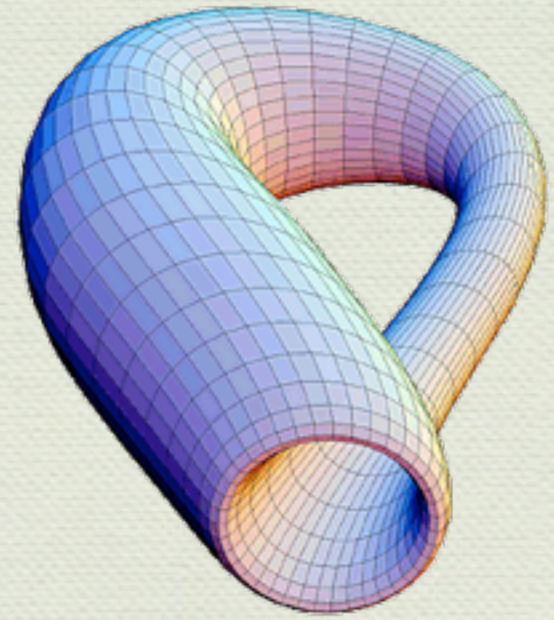
Discrete symmetries

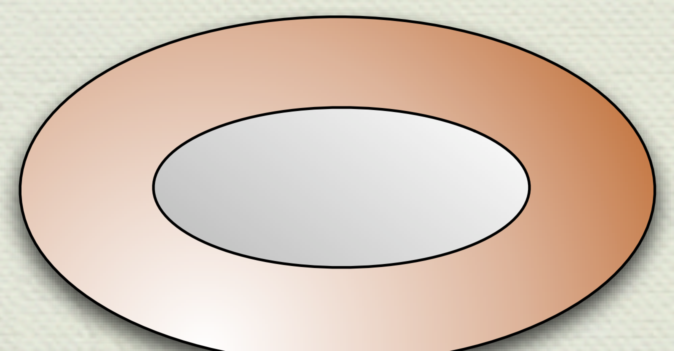
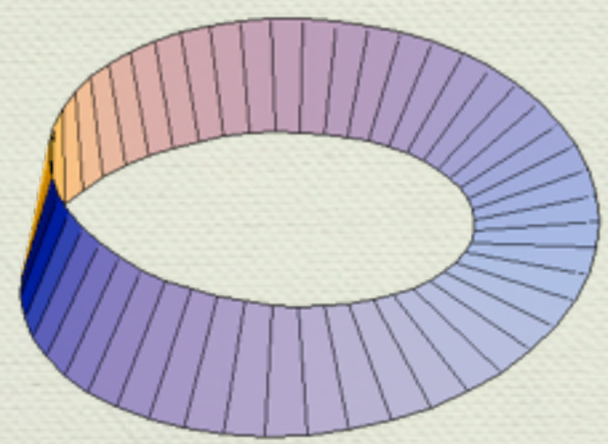
in quantum gravity and string theory

- Folk theorems against existence of ungauged symmetries (continuous or discrete).
- Gauged discrete symmetries are allowed.
  (Kraus, Wilczek,...,1989)
- In string theory, specific "gauged, anomaly free" discrete symmetries are possible (Z_N subgroups of $U(1)$'s).
  (Ibanez, Ross, 1991).
- Are they "generic" in the string theory landscape?
  Here we will focus on orientifolds

Orientifolds

Orientifold Partition Functions

$\frac{1}{2}\Big\{$  $+$  $\Big\}$

$\frac{1}{2}\Big\{$  $+$  $\Big\}$

Orientifold Partition Functions

- Closed $$\frac{1}{2}\left[\sum_{ij}\chi_i(\tau)Z_{ij}\chi_i(\bar{\tau})+\sum_i K_i\chi_i(2\tau)\right]$$

- Open $$\frac{1}{2}\left[\sum_{i,a,n}N_aN_bA^i{}_{ab}\chi_i(\frac{\tau}{2})+\sum_{i,a}N_aM^i{}_a\hat{\chi}_i(\frac{\tau}{2}+\frac{1}{2})\right]$$

i : Primary field label (finite range)

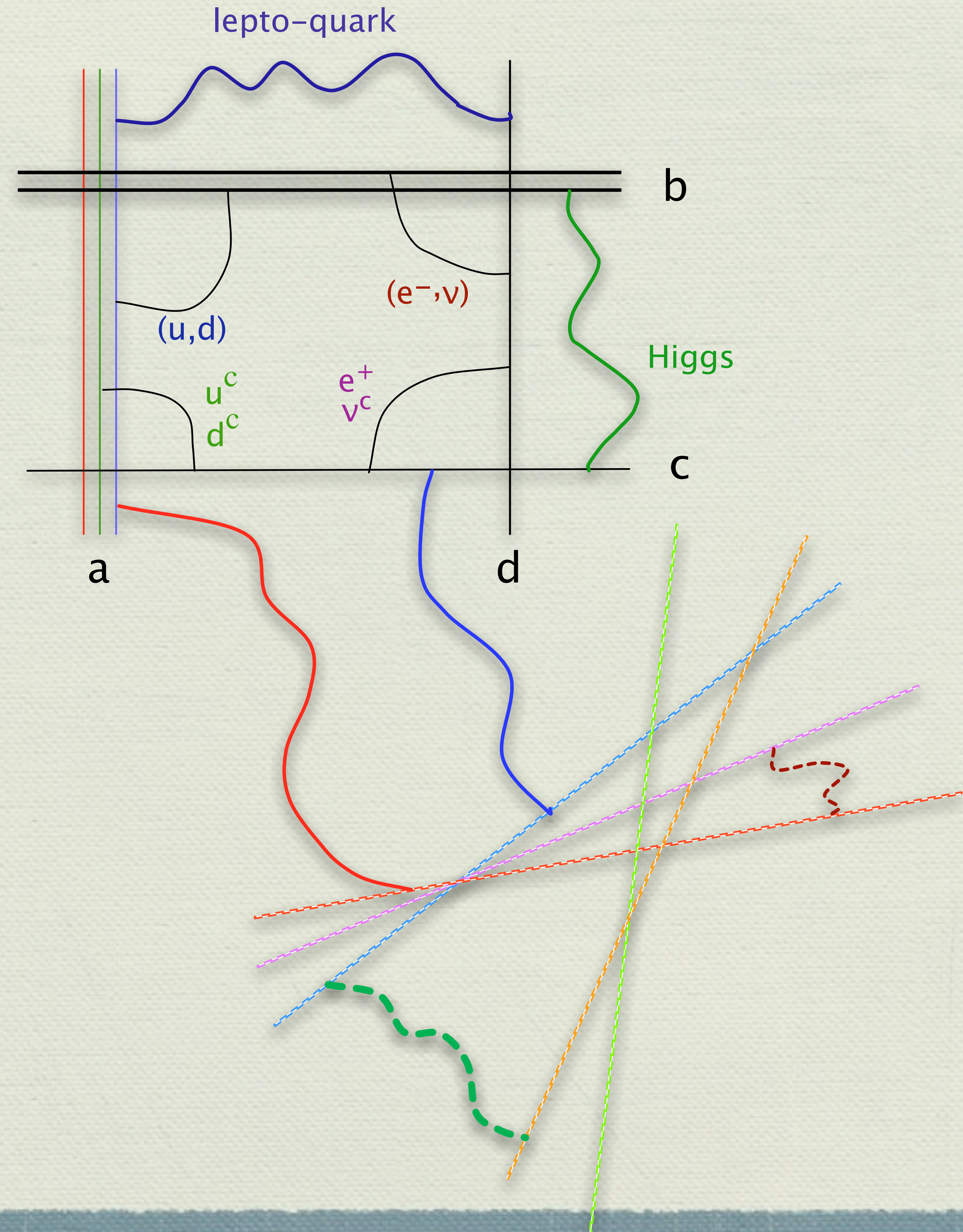
a : Boundary label (finite range)

χ_i : Character

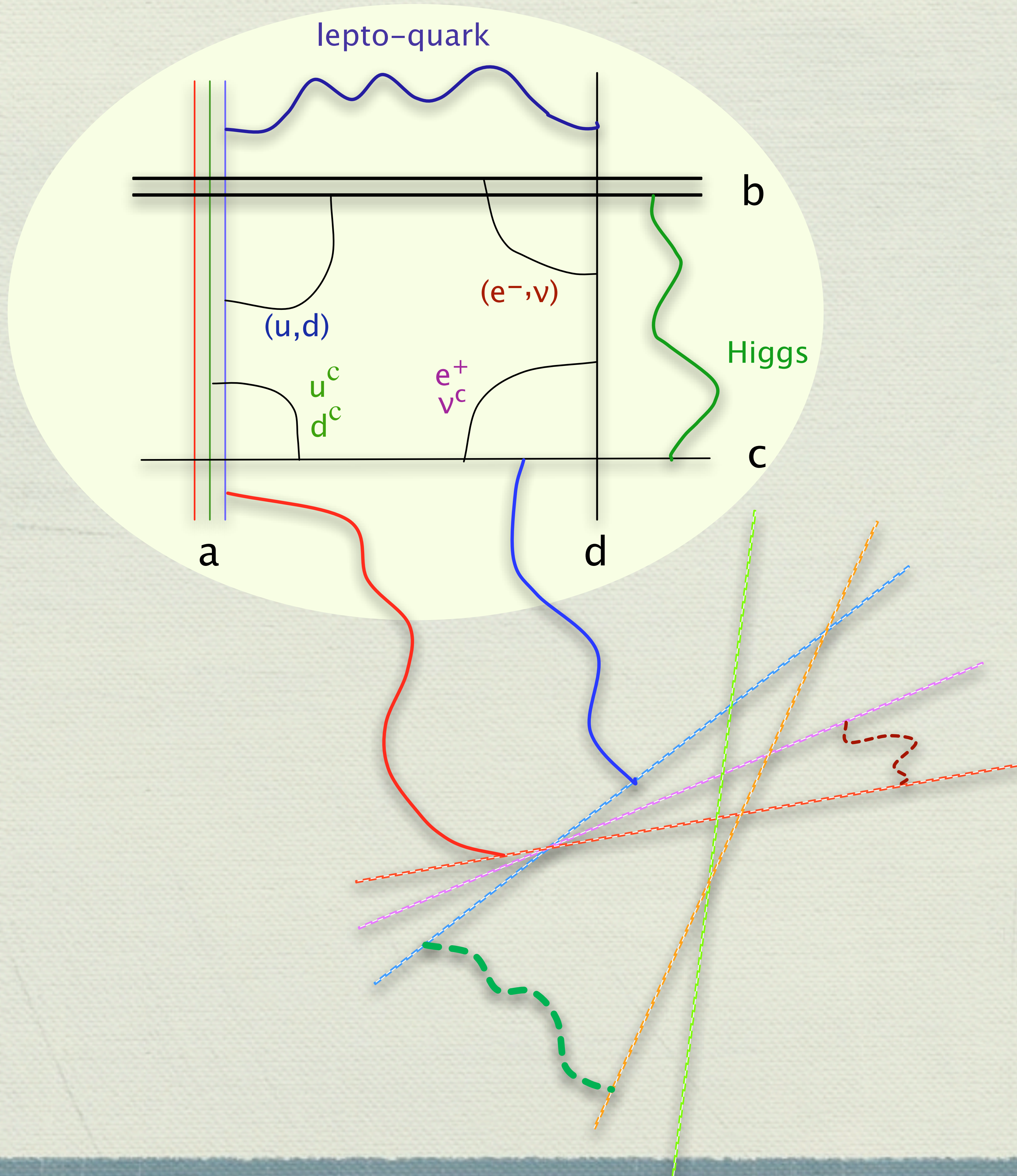
N_a : Chan-Paton (CP) Multiplicity

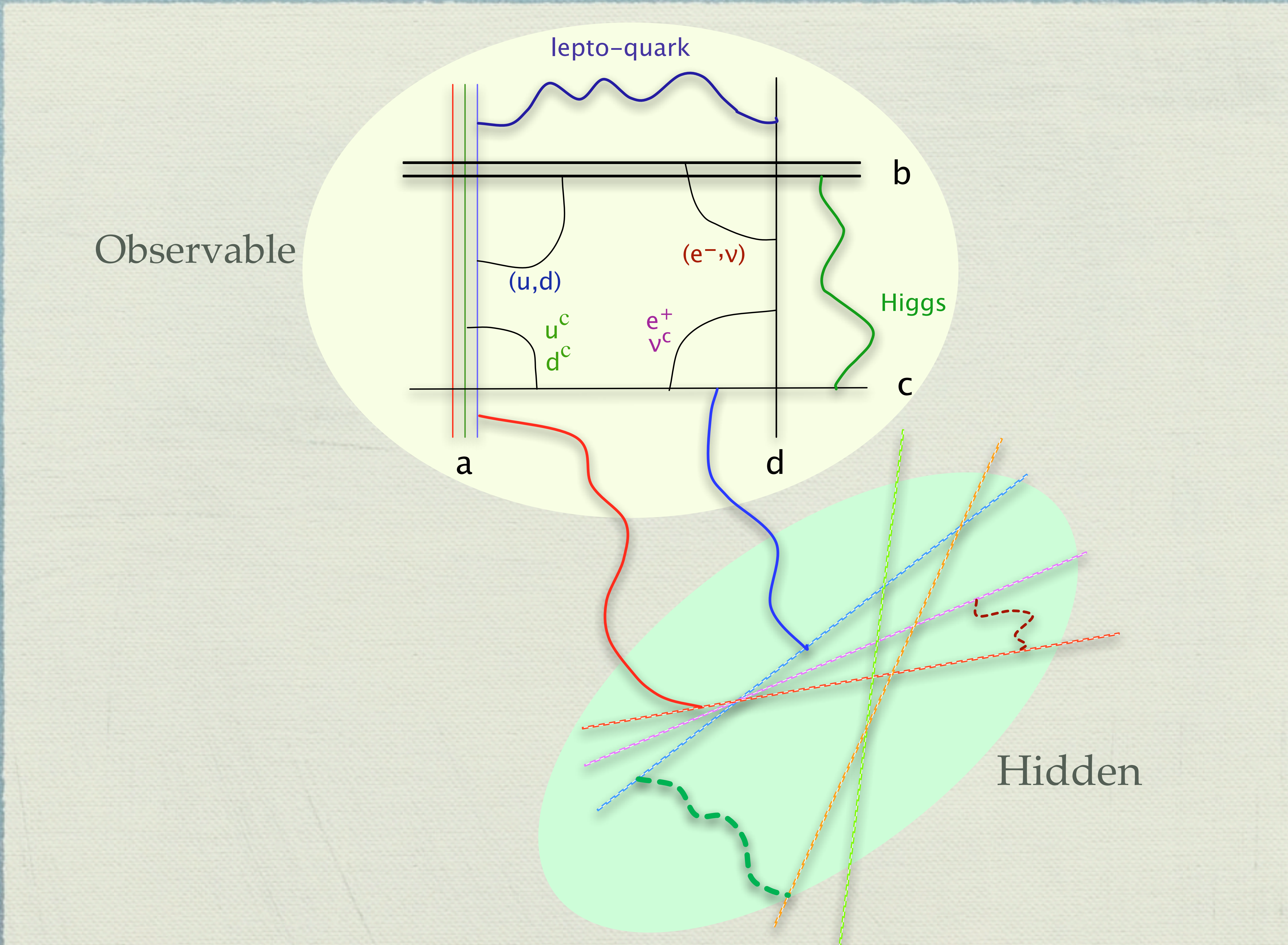
Chan-Paton Gauge Group

- CFT boundaries can be identified with branes.
- Chan-Paton multiplicities N_a can be identified with brane multiplicities in a stack of branes.
- These multiplicities give rise to gauge groups.
- Possible groups: *U(N)*, *O(N)*, *Sp(N)*
  (Marcus and Sagnotti, 1987)
- Can be used to build the Standard Model


lepto-quark
b
(e⁻,ν)
(u,d)
Higgs
u^c
d^c
e^+
ν^c
c
a
d

Observable



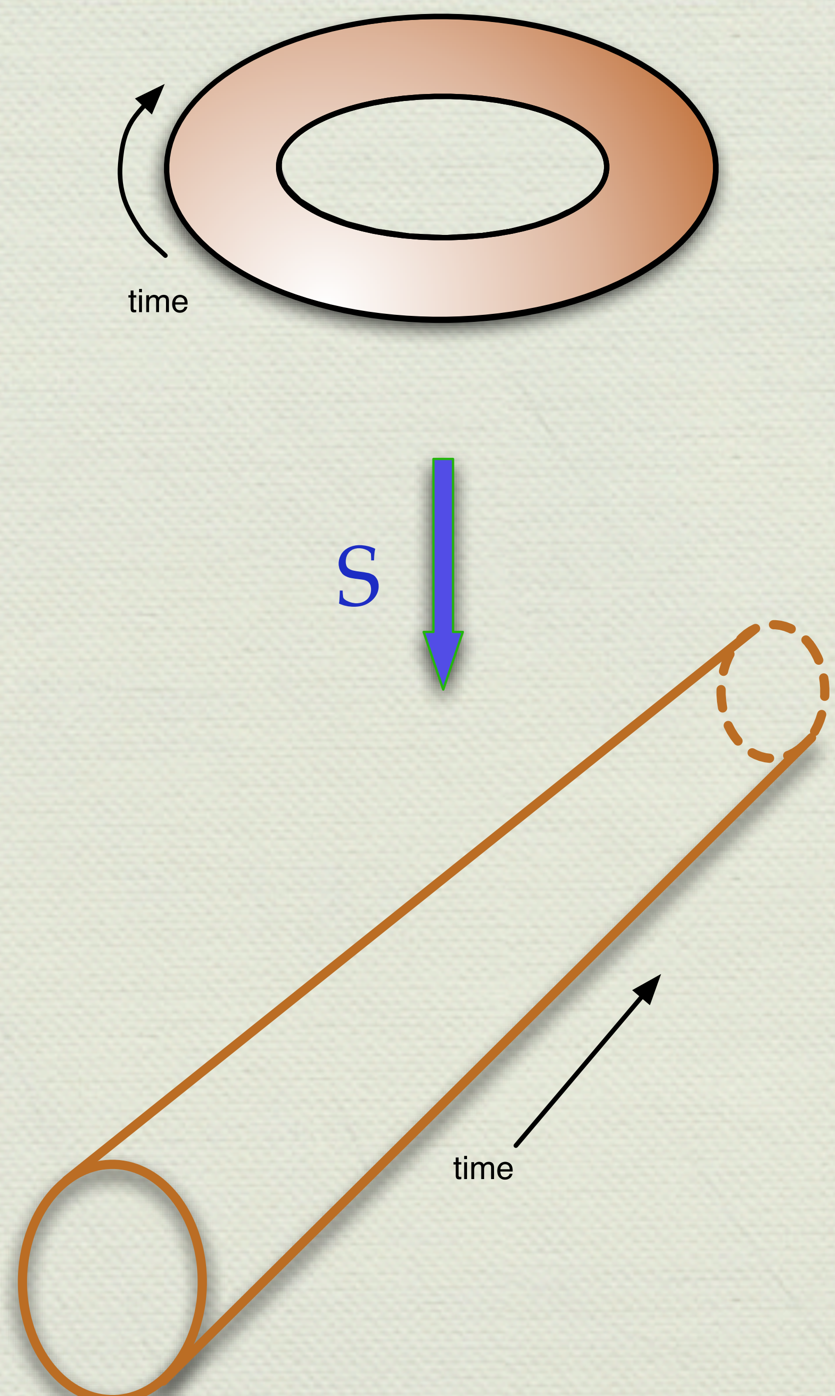


Thursday, 23 May, 2013

Annulus Coefficients

$$A^i_{ab} = \sum_m \frac{S^i{}_m R_{am} R_{bm}}{S_{0m}}$$

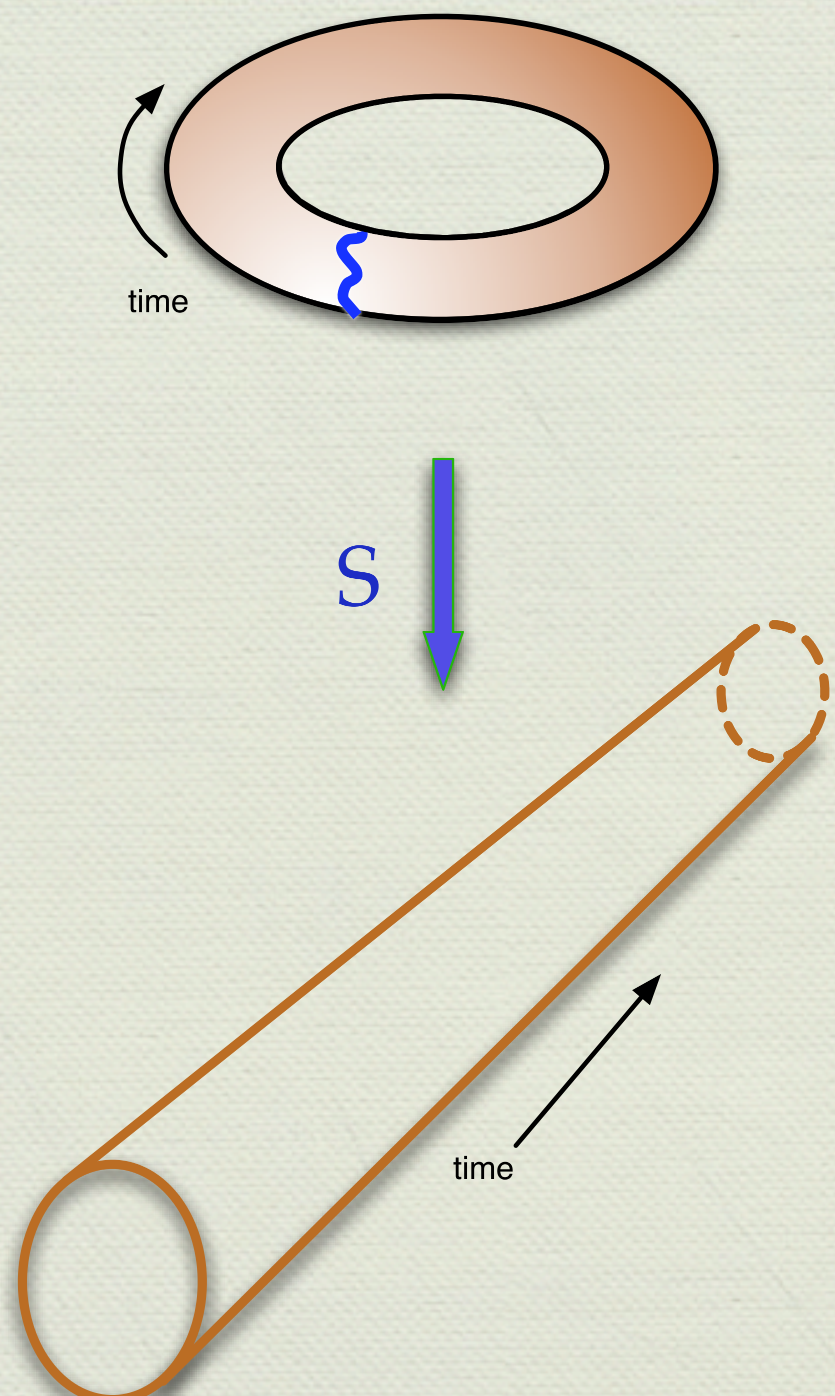
Transverse Channel



boundary state

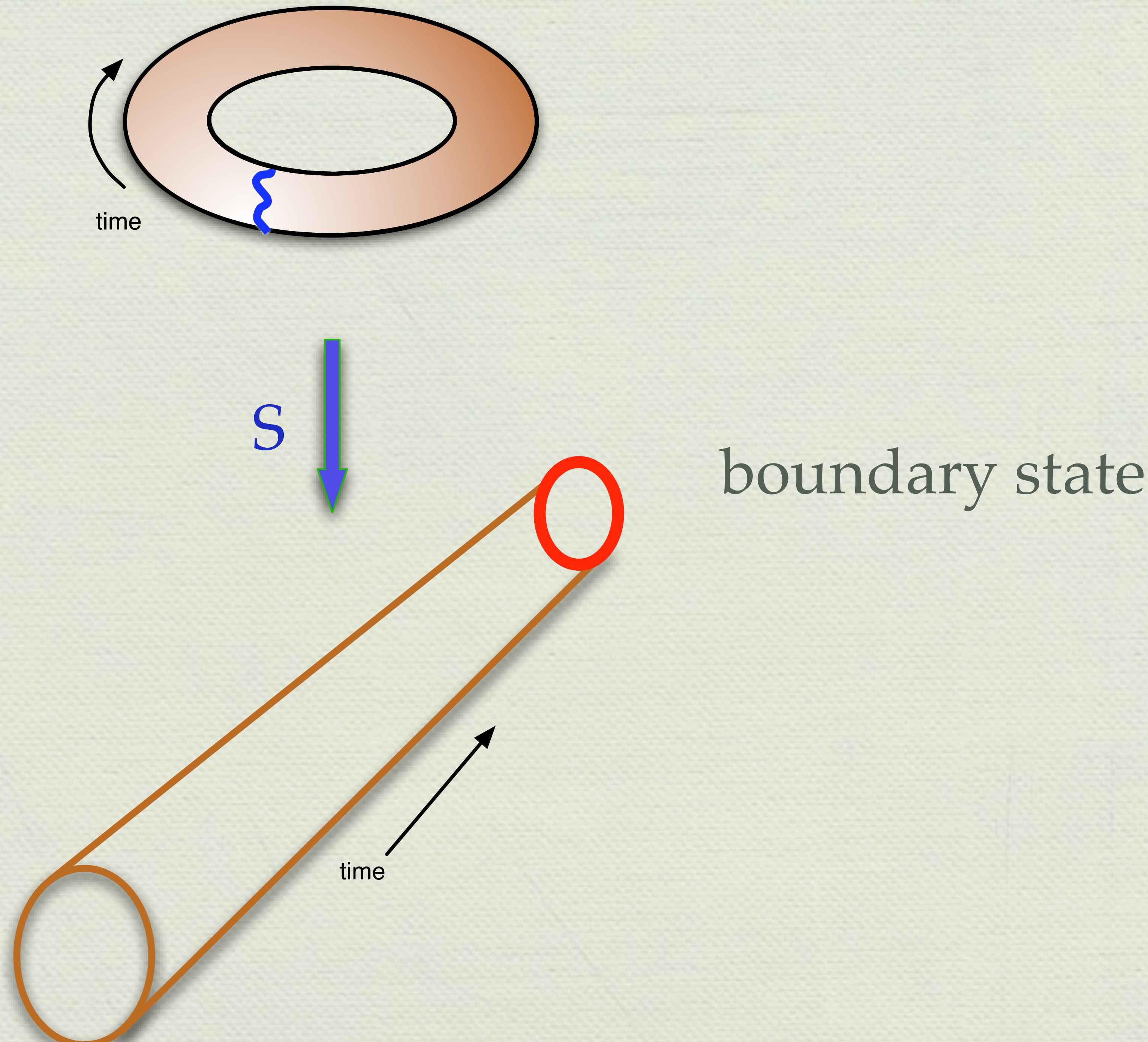
Thursday, 23 May, 2013

Transverse Channel

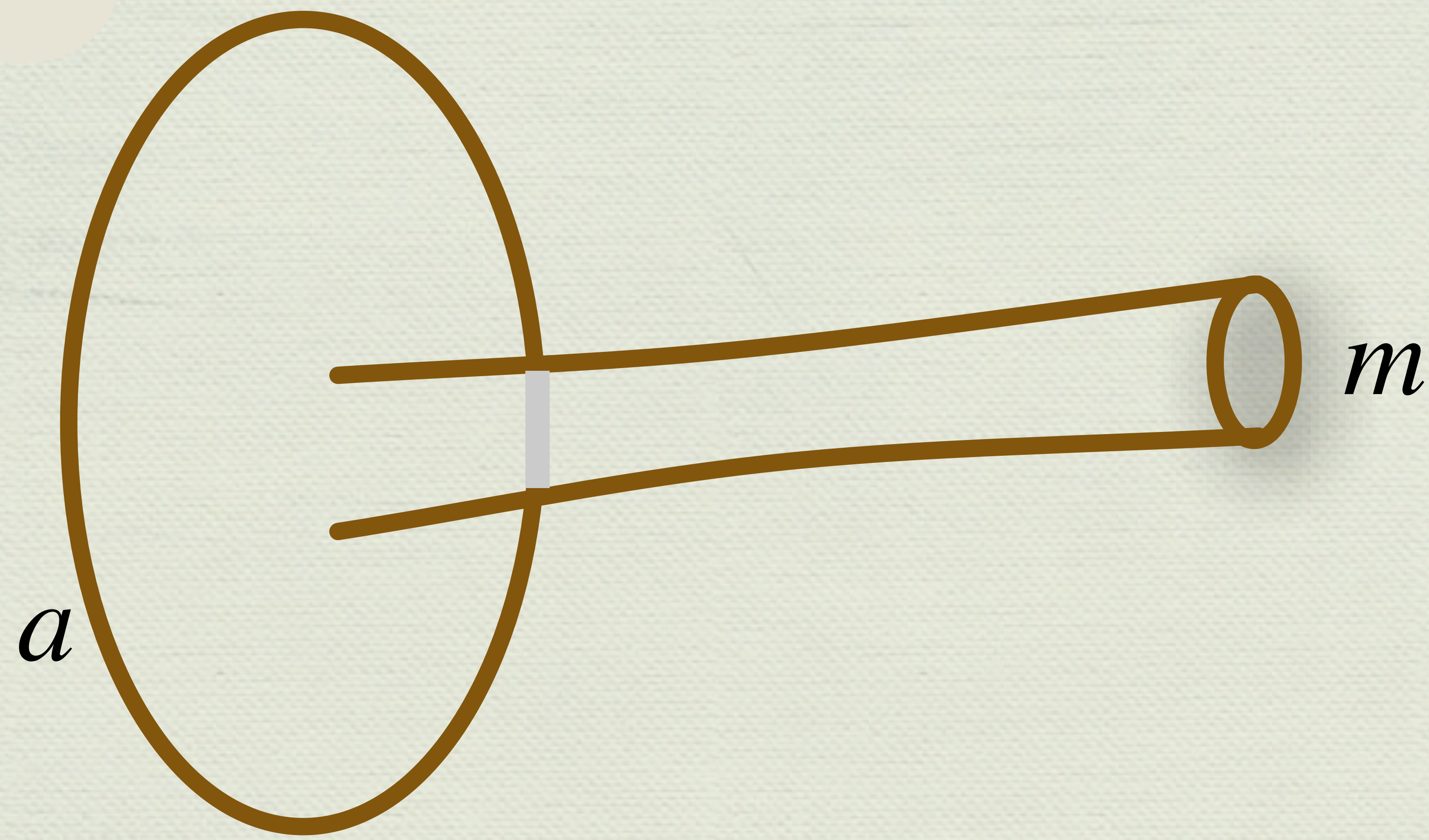


boundary state

Transverse Channel



Coupling R_{am}



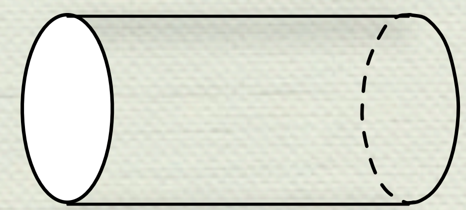
In general: a complex number

Annulus Coefficients

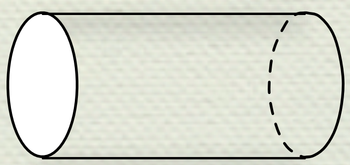
$$A^i_{ab} = \sum_m \frac{S^i{}_m R_{am} R_{bm}}{S_{0m}}$$

Annulus Coefficients

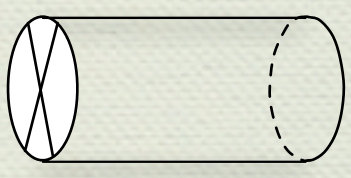
Channel Transformation



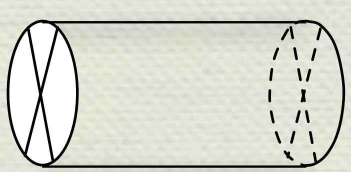
$$A^i_{ab} = \sum_m \frac{S^i{}_m R_{am} R_{bm}}{S_{0m}}$$



$$A^i_{[a,\psi_a][b,\psi_b]} = \sum_{m,J,J'} \frac{{S^i}_m R_{[a,\psi_a](m,J)} g^{\Omega,m}_{J,J'} R_{[b,\psi_b](m,J')}}{S_{0m}}$$



$$M^i_{[a,\psi_a]} = \sum_{m,J,J'} \frac{{P^i}_m R_{[a,\psi_a](m,J)} g^{\Omega,m}_{J,J'} U_{(m,J')}}{S_{0m}}$$



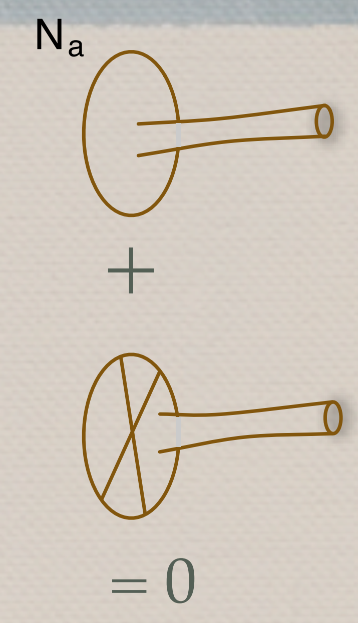
$$K^i = \sum_{m,J,J'} \frac{{S^i}_m U_{(m,J)} g^{\Omega,m}_{J,J'} U_{(m,J')}}{S_{0m}}$$

$$g^{\Omega,m}_{J,J'} = \frac{S_{m0}}{S_{mK}} \beta_K(J) \delta_{J',J^c}$$

Tadpoles & Anomalies

- Tadpole cancellation condition:

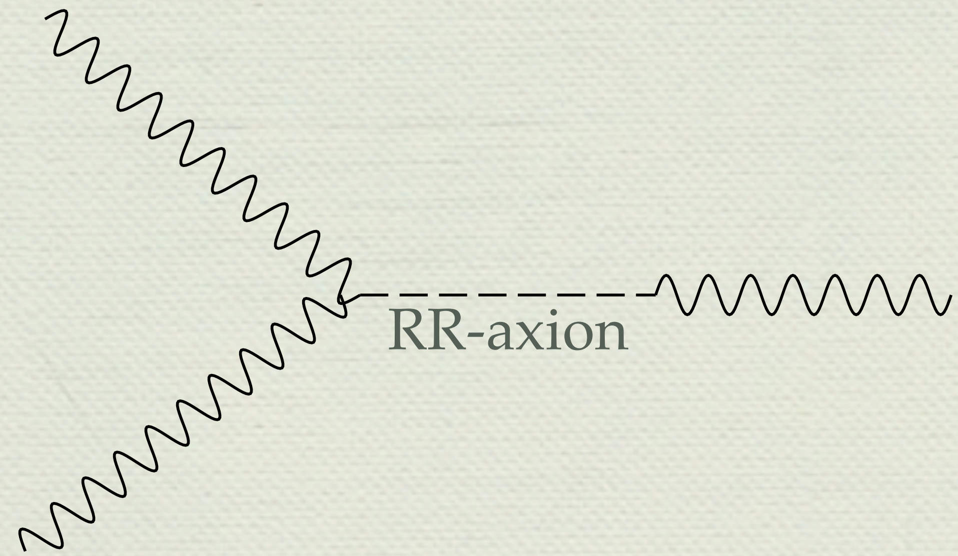
$$\sum_b N_b R_{bm} = 4U_m$$



- Implies that cubic anomalies cancel.
- Remaining, factorizable anomalies cancelled by a multi-axion Green-Schwarz mechanism.
- This breaks some of the U(1)'s and produces a mass for their gauge bosons.

Abelian Masses

Green-Schwarz mechanism



Axion-Vector boson vertex

Generates mass vector bosons of anomalous symmetries $(e.g.\ B+L)$

But may also generate mass for non-anomalous ones $(Y, B-L)$

Discrete symmetries in orientifolds

- An obvious way to get an anomaly free discrete symmetry is to break a *U(1)* to $\mathbb{Z}_N$.
- Orientifolds have lots of *U(1)*'s, one for every complex brane stack. A good place to look for discrete symmetries!
- These *U(1)*'s are often broken due to axion mixing. This happens always if the *U(1)* is anomalous, and sometimes (usually?) if it is not.
- We need to determine if, and how often an unbroken discrete abelian symmetry remains.

Axion Couplings

$$\sum_{a,m} N_a \, V_{am} \, B_m \wedge F_a$$

B_m: axions, typically ~ 10 ... 100

F_a: *U(1)* gauge field strength.

N_a: Chan-Paton multiplicity of stack *a*

in CFT: $$V_{am} = R_{am} - R_{a^c m}$$

R_{am} Coupling strength of bulk mode *m* ("Ishibashi state") to boundary *a*

Consider a linear combination of *U(1)*'s

$$\sum_a x_a Y_a$$

Y_a *U(1)* generator of brane *a*

This remains massless if and only if

$$\sum_a x_a N_a (R_{am} - R_{a^c m}) = 0 \text{ for all } m.$$

If Y_a acquires a mass, the *U(1)* is not always completely broken.
A discrete subgroup may remain.
How can we detect this?

Instantons

- Brane stack U(1)'s broken by axion mixing are respected by all perturbative amplitudes.

- Instanton amplitudes may break these symmetries. These can be gauge instantons or "exotic", "stringy" instantons from stacks without a gauge group.

  Blumenhagen, Cvetic, Weigand
  Ibáñez,Uranga
  Florea, Kachru, McGreevy, Saulina
  Billo, Frau, Pesando, Fucito, Lerda, Liccardo

- If there is a $\mathbb{Z}_N$ discrete symmetry, any instanton amplitude can only violate the corresponding charge only by multiples of N.

Dual description of the axion couplings in terms of RR scalars ϕ_m (with $\phi_m \sim \phi_m+1$)

$$\sum_m \left[\partial_\mu \phi_m \; - \; \left(\textstyle\sum_a x_a N_a V_{am}\right) A_\mu \right]^2$$

U(1) gauge transformation

$$A_\mu \to A_\mu + \partial_\mu \lambda \quad ; \quad \phi_m \to \phi_m + \left(\textstyle\sum_a x_a N_a V_{am}\right)\lambda$$

Then an instanton amplitude transforms as

$$e^{-2\pi i \phi_m} \to e^{-2\pi i \phi_m} \exp\left[-2\pi i \left(\textstyle\sum_a x_a N_a V_{am}\right)\lambda\right]$$

$$e^{-2\pi i\phi_m} \to e^{-2\pi i\phi_m}\exp[-2\pi i(\textstyle\sum_a x_a N_a V_{am})\lambda]$$

Which is gauge invariant only if an operator is inserted that violates the charge by $(\sum_a x_a N_a V_{am})$

This suggests the following characterization of a discrete symmetry

$$\sum_a x_a N_a (R_{am} - R_{a^c m}) = 0 \bmod N \quad \text{for all } m.$$

Then all instantons can only violate the charge by multiples of N.
This would imply a $\mathbb{Z}_N$ symmetry.

But this condition makes little sense, as it stands, because the coefficients R are complex numbers.

Condition for continuous U(1)

$$\sum_a x_a N_a (R_{am} - R_{a^c m}) = 0 \text{ for all } m.$$

Condition for $\mathbb{Z}_N$

$$\sum_a x_a N_a (R_{am} - R_{a^c m}) = 0 \bmod N \quad \text{for all } m.$$

In a geometric setting (type-IIA on CY) one can define these numbers in terms of a basis of 3-cycles on the manifold. Then one can write the condition for discrete symmetries entirely in terms of integers, and one can use this to construct explicit examples.

Berasaluce González, Ibáñez, Soler, Uranga, 2011

Discrete Orientifolds

Start with a $c=9$, $N=2$ rational conformal field theory, used as an "internal" sector of a type-II compactification.

Define the corresponding boundary CFT on surfaces with boundaries and crosscaps, by adding boundary and crosscap states consistent with the RCFT symmetries.

This allows the explicit construction of Annulus amplitudes, yielding exact open string partition functions, and Möbius and Klein bottle amplitudes defining the orientifold projections.

This gives rise to exact perturbative string spectra, with all massless and massive states explicitly known.

Cardy (1989); Sagnotti, Pradisi, Stanev (1995); ....

Discrete Orientifolds

In principle, one expects a huge number of such RCFTs to exist.

In practice, we are limited to tensor products of N=2 minimal models[(1)] (in total 168 c=9 combinations) and some permutation orbifolds[(2)] (modding out the exchange of two identical factors).

The first steps towards realistic spectra started in 2003[(3)], and led to chirally exact MSSM spectra in 2004[(4)].

(1) Gepner, 1987 (heterotic)
Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Y. S. Stanev, 1996 (Orientifolds)
(2) Maio, Schellekens (2011)
(3) Aldazabal, Andres, Leston, Nunez; Blumenhagen, Weigand
(4) Dijkstra, Huiszoon, Schellekens

Discrete Orientifolds

The resulting spectra are presumably best though of as discrete points in an open and closed string moduli space, hence the term "discrete orientifold".

Most features of geometric orientifolds can be analysed in this context: tadpole cancellation, hidden sectors, axion-vector boson mixing, absence of global anomalies(1), stringy instantons(2). We would like to extend that to discrete symmetries.

(Note that orientifold discreteness has no relation to the discrete symmetries.)

To investigate the presence of discrete symmetries we need to know the boundary coefficients R_{ma}

(1) Gato-Rivera and Schellekens, 2006
(2) Ibáñez, Schellekens, Uranga, 2007
Kiritsis, Lennek, Schellekens, 2010
Anastasopoulos, Leontaris, Richter, Schellekens, 2011

Discrete Orientifolds

We use the FHSSW[1] formalism, applied to the 168 Gepner models and all their simple current partition functions[2].

This gives us a total of 32990 non-zero tension orientifolds (for 5392 MIPFs). The MSSM spectra appear for a large subset of these.

Geometric intuition suggests that we should try to find a suitable integral basis for the boundary coefficients.

Something similar was done by *Brunner, Hori, Hosomichi and Walcher (2004)* for the (3,3,3,3,3) Gepner model realization of the quintic Calabi-Yau, by explicit construction. But that is just one of the 32990 cases.

(1) Fuchs, Huiszoon, Schellekens, Schweigert, Walcher (2000)
(2) Gato-Rivera, Schellekens (1991); Kreuzer, Schellekens (1993)

Boundary coefficients

$$R_{[a,\psi_a](m,J)} = \sqrt{\frac{|\mathcal{H}|}{|\mathcal{C}_a||\mathcal{S}_a|}}\psi_a^*(J)S_{am}^J$$

$(m,J): \quad J \in \mathcal{S}_m$

with $Q_L(m) + X(L,J) = 0$ mod 1 for all $L \in \mathcal{H}$

$\mathcal{S}_m : J \in \mathcal{H}$ with $J \cdot m = m$

(Stabilizer of m)

S_{am}^J : matrix element of the modular transformation matrix of the fixed point CFT

$[a,\psi_a]$, ψ_a is a character of the group $\mathcal{C}_a$

$\mathcal{C}_a$ is the Central Stabilizer of a

$$\mathcal{C}_i := \{J \in \mathcal{S}_i \,|\, F_i^X(K,J) = 1 \text{ for all } K \in \mathcal{S}_i\}$$

$$F_i^X(K,J) := \mathrm{e}^{2\pi \mathrm{i} X(K,J)} F_i(K,J)^*$$

$$S_{Ki,j}^J = F_i(K,J)\,\mathrm{e}^{2\pi \mathrm{i} Q_K(j)}\,S_{i,j}^J\,.$$

In general, a complex number

Finding an integral basis

Axion couplings

$$V_{am} = R_{am} - R_{a^c m} \qquad a = 1, \ldots N_{\text{bound}}, \quad m = 1, \ldots N_{\text{Ishibashi}}$$

Remove vanishing and identical columns

$V_{a\mu}, \quad a = 1, \ldots N_{\text{bound}}, \quad \mu = 1, \ldots N_{\text{axion}}$

$N_{\text{axion}} = \mathcal{O}(10, \ldots 100)$ (maximally 480),

$N_{\text{bound}} = \mathcal{O}(100 \ldots 1000)$

Try to find a subset c of N_{axion} "basic" boundaries so that

$$V_{a\mu} = \sum_{\mu=1}^{N_{\text{axion}}} Q_{a\mu} V_{c(\mu)\nu}, \quad Q_{a\mu} \in \mathbb{Z}$$

This assumes that the basis can be related to RCFT boundary states.

The instanton charge violation for a *U(1)* associated with brane a due to an instanton on brane b is given by the chiral zero mode count

$$I_b(a) = N_a \sum_i w_i ({A^i}_{ba} - {A^i}_{ba^c})$$

Here w_i is the Witten index of representation *i*, and ${A^i}_{ab}$ are Annulus coefficients. The latter can be expressed in terms of boundary coefficients as

$$I_b(a) = N_a \sum_i w_i \sum_{m,J',J} \left[\frac{S_{im} R_{b(m,J')} g^{\Omega,m}_{J'J}}{S_{0m}} \right] (R_{a(m,J)} - R_{a^c(m,J)})$$

If we have an integral basis, we can express this in terms of that basis

$$I_b(a) = \sum_\mu N_a Q_{a\mu} I_b(c(\mu))$$

For a $U(1)$ $\quad Y = \sum_a x_a Y_a \quad$ (choose x_a integer)

$$I_b(x) = \sum_a x_a I_b(a) = \sum_\mu \Big(\sum_a x_a N_a Q_{a\mu} \Big) I_b(c(\mu))$$

$$I_b(x) = \sum_a x_a I_b(a) = \sum_\mu \Big(\underbrace{\sum_a x_a N_a Q_{a\mu}} \Big) \underbrace{I_b(c(\mu))}$$

Manifestly integer in the new basis
(if it exists...)

Instanton intersection number: Integer

If all basis coefficients $\sum_a x_a N_a Q_{a\mu}$ are a multiple of *N*, we have a $\mathbb{Z}_N$ discrete symmetry

Finding an integral basis

Choose a suitable normalization for the columns of the matrix $V_{a\mu}$: $V_{a\mu} \rightarrow X(\mu)\, V_{a\mu}$

$$N_{ab} = \sum_{\mu} V_{a\mu} V_{b\mu} \equiv V_a \cdot V_b$$

For a suitable choice, all N_{ab} are rational numbers, in all 33290 cases.

Now choose a set of independent vectors $V_{c(\mu)\nu}$

Finding an integral basis

The "charges" with respect to this basis are defined as

$$V_{a\nu} = \sum_{\mu} Q_{a\mu} V_{c(\mu)\nu}$$

and can be computed by contracting both sides with the basis vectors

$$N_{ac(\nu)} = \sum_{\mu} Q_{a\mu} N_{c(\mu)c(\nu)}$$

Here N_{ab} are the numbers which we just found to be rational. We can compute $Q_{a\mu}$ by inverting the rational matrix $N_{c(\mu)c(\nu)}$

-2356527325219910903428901754662427149894 / 4206361037817712426172307166805027949946515
2784948741071505418128346476378730597441 / 2804240691878474950781538111203351966631010
-2585499736215948357280656786524657232 2 / 221387423043037496114331956147633049997185
6898072845027098208081359744435277277501 / 8412722075635424852344614333610055899893030
1089767156814089868909643376718230779 77 / 2804240691878474950781538111203351966631010
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-7302743703051896141872125832386047219 79 / 280424069187847495078153811120335196663101
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611318690656773378790218435058802631 89 / 737958076810124987047773187158776833323 95
-374532049778615855527085083530427594312 1 / 8412722075635424852344614333610055899893030
1693796173771342973378581388458204267177 / 2804240691878474950781538111203351966631010
1205444211082390872412284617701674410251 / 2804240691878474950781538111203351966631010
2221438778472648889039857348099343644511 / 4206361037817712426172307166805027949946515
2778141893267937173717166855104761029721 / 1201817439376489264620659190515722271413290
-328790319741952612198224637596271270733 / 572294018750709173628885328817010605434 90
-1069694589484159743500618889634159465640 9 / 1682544415127084970468922866722011179978606
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1338855860925514201916068360184844342233 9 / 1682544415127084970468922866722011179978606 0
-1300537957404161190372106954643781901 33 / 1121696276751389980312615244481340786652404
-1875021717318049489809407814891893702 83 / 1201817439376489264620659190515722271413 29
-6198670319599937925646262302209652096 83 / 2804240691878474950781538111203351966631010
-1925028850509606135456711776999153695741 / 1402120345939237475390769055601675983315505
-5533393457226609011652599227355348627 99 / 841272207563542485234461433361005589989303
3622588600596306878973447873960345776869 / 8412722075635424852344614333610055899893030

Finding an integral basis

...but this gives us only rational charges. This is not good enough.
Now consider a boundary that has a rational charge

$$W_\nu = \sum_\mu Q_\mu V_{c(\mu)\nu} = \sum_\mu \frac{p_\mu}{q_\mu} V_{c(\mu)\nu}$$

Suppose for one value of μ (denoted $\mu = \hat{\mu}$), $p_{\hat{\mu}} = 1.$
Then we replace the corresponding basis vectors by W_ν. In terms of the new basis, the old basis vector in terms of the new basis has an expansion

$$V_{c(\hat{\mu})\nu} = \sum_{\mu,\mu\neq\hat{\mu}} -\frac{p_\mu q_{\hat{\mu}}}{q_\mu} V_{c(\mu)\nu} + q_{\hat{\mu}} W_\nu$$

This is "more integral" than the previous basis, and the volume spanned by the basis decreases by q.

Finding an integral basis

This process converges in a maximum of 19 steps.
In 3 out of the 32990 cases it did not converge to pure integers.

These cases could be dealt with by choosing a different starting point.

In the end we did indeed find an integer basis for all 32990 Orientifolds.

This gives a “charge lattice” for axion charges.

(But: there must be a better way of doing this...)

Examples

We have a database of ~19000 chirally distinct standard model realizations. *(Anastasopoulos, Dijkstra, Kiritsis, Schellekens, 2006)*.

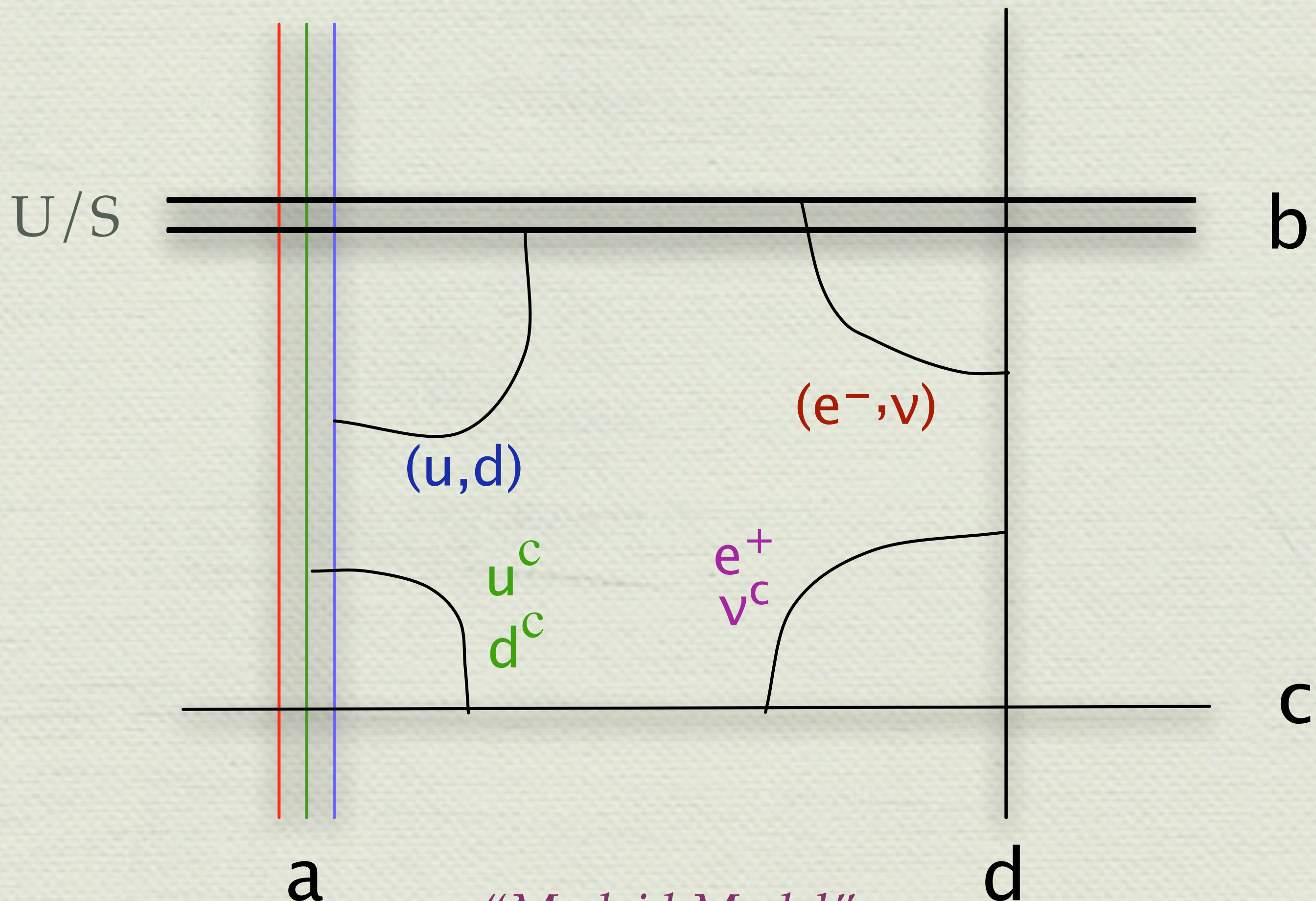
This contains more or less anything that can be realized with orientifolds (Madrid-type models, SU(5) GUTs, Pati-Salam, trinification,....)

In each class there may be up to 10^7 brane configurations.
(tadpole cancellation not imposed).

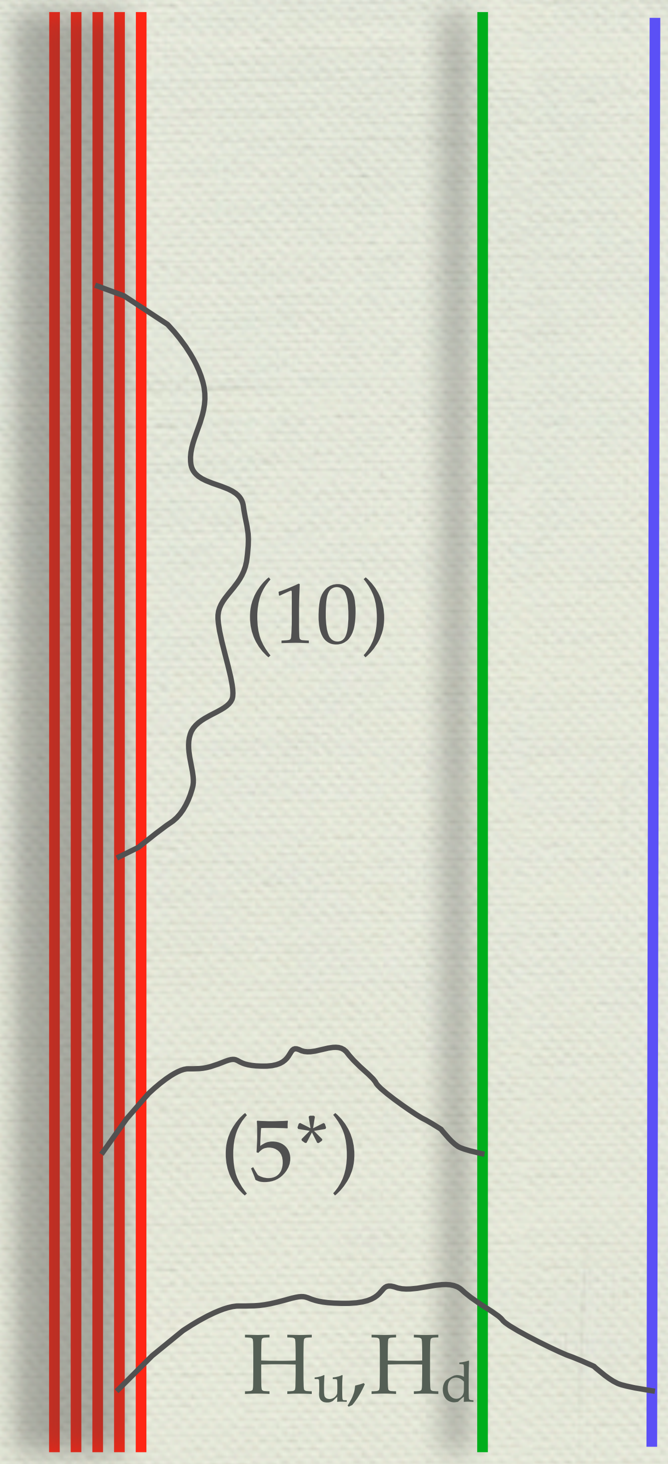
We have checked a small subset of the 19000 models for discrete symmetries (all Madrid models, 12 of the 700 SU(5) models).

We found $\mathbb{Z}_2$ symmetries for about 0.2% of all cases, and $\mathbb{Z}_3$ for about 6.4%.

A remarkably large percentage of the latter allow a tadpole cancelling hidden sector (65%, usually around 1%).
Presumably this is due to a large degeneracy, but in principle discrete symmetries may enhance the chance of cancelling tadpoles.



"Madrid Model"



Nr	U/S	$U(1)$	**ab**	**ab****	**a*****c**	**a*****c***	**a*****d**	**a*****d***	**bd***	**b*****d***	**c*****d**	**cd**	**bc**	**bc***	Total	$\mathbb{Z}_2$	$\mathbb{Z}_3$	Tadp.
			Q	Q	U^c	D^c	D^c	U^c	L	L	E^c	N^c	H_d/L	H_u				
7506	S	1	3	–	3	3	0	0	3	–	3	3	0	0	40590	2152	16	320 ($\mathbb{Z}_2$)
2751	S	2	3	–	3	3	0	0	3	–	3	3	0	0	869428	0	59808	41136
14704	S	1	3	–	1	2	1	2	0	–	3	0	3	0	380	0	0	0
14062	S	1	3	–	2	2	1	1	2	–	3	1	1	0	304	0	0	0
8745	S	1	3	–	3	2	1	0	4	–	3	2	0	1	92	0	0	0
11196	S	1	3	–	3	4	-1	0	2	–	3	4	1	0	40	0	0	0
10551	U	1	1	2	3	3	0	0	3	0	3	3	0	0	116	0	0	0
1352	U	2	1	2	3	3	0	0	3	0	3	3	0	0	20176	0	1472	0
13058	U	1	1	2	3	3	0	0	1	2	3	3	2	2	68	0	0	0
7573	U	2	1	2	3	3	0	0	1	2	3	3	2	2	14744	0	0	0
16074	U	1	0	3	3	3	0	0	3	0	3	3	3	3	128	0	0	0
7967	U	2	0	3	3	3	0	0	3	0	3	3	3	3	5856	0	0	0
12106	U	1	1	2	3	3	0	0	2	1	3	3	1	1	32	0	0	0
7976	U	2	1	2	3	3	0	0	2	1	3	3	1	1	5764	0	192	0
13844	U	2	1	2	3	3	0	0	0	3	3	3	3	3	1096	0	0	0
14793	U	2	2	1	3	3	0	0	4	-1	3	3	-1	-1	400	0	0	0
13762	U	2	0	3	3	3	0	0	6	-3	3	3	0	0	320	0	0	0
14850	U	2	0	3	3	3	0	0	4	-1	3	3	2	2	96	0	0	0
14792	U	2	0	3	3	3	0	0	0	3	3	3	6	6	32	0	32	0
7488	U	1	1	2	1	2	1	2	0	0	3	0	3	0	2864	0	144	0
13015	U	2	1	2	1	2	1	2	0	0	3	0	3	0	352	0	0	0
18086	U	1	2	1	2	4	-1	1	0	0	3	3	3	0	68	0	0	0
13644	U	1	0	3	1	3	0	2	1	-2	3	1	5	1	8	0	0	0
653	U	1	0	3	0	3	0	3	0	-3	3	0	6	0	4	0	0	0

Non-Madrid

U(5) models

Nr	Type	$U(1)$	$A_{\mathbf{a}}$	$\mathbf{a}^*\mathbf{b}$	$\mathbf{a}^*\mathbf{b}^*$	$\mathbf{ac}$	$\mathbf{bc}$	$\mathbf{bc}^*$	A_2	S_2	A_3	S_3	Total	$\mathbb{Z}_2$	$\mathbb{Z}_3$	Tadp.
7	UO	1	3	3	–	–	–	–	–	–	–	–	16845	0	0	0
218	UU	2	3	3	0	–	–	–	0	-3	–	–	1049	0	0	0
345	UU	1	3	3	0	–	–	–	0	-3	–	–	1136	18	0	0
742	UU	1	3	2	1	–	–	–	0	-1	–	–	146	0	0	0
18371	UU	1	3	6	-3	–	–	–	0	-9	–	–	12	0	0	0
57	UUO	1	3	3	3	3	0	0	0	0	–	–	13402	552	0	0
998	UUO	2	3	3	3	3	0	0	0	0	–	–	18890	0	0	0
1000	UUU	3	3	3	3	3	-3	3	0	0	3	0	7276	0	0	0
4004	UUU	2	3	3	3	3	-3	3	0	0	3	0	1706	4	0	0
4316	UUU	2	3	3	3	3	-3	3	0	0	3	0	5236	180	120	0
4324	UUU	1	3	3	3	3	-3	3	0	0	3	0	1278	8	0	0
4325	UUU	1	3	3	3	3	-4	4	0	0	4	1	96	48	0	0

Example 1

$\mathbb{Z}_2$ in $U(3)\times Sp(2)\times U(1)\times U(1)$ with broken $B-L$

Class 7506, Tensor (2,4,14,46), MIPF 10, Orientifold 2, boundaries (630,41,1070,631)

Axion Charges (including Chan-Paton multiplicity factor)

a:	0	-3	0	0	-3	-3	-3	3	0	-3	0	3	3	6	3	6
c:	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0
d:	0	1	0	0	1	1	-1	-1	0	1	0	-1	-1	-2	-1	-2

Null vectors

$$3Q_{\mathbf{d}} - 3Q_{\mathbf{c}} + Q_{\mathbf{a}} = 0 \;\rightarrow\; Y \text{ unbroken}$$

$$3Q_{\mathbf{d}} + Q_{\mathbf{a}} \neq 0 \;\rightarrow\; B-L \text{ broken}$$

$$Q_{\mathbf{a}} = 0 \bmod 3 \;\rightarrow\; \text{Conservation of color}$$

$$Q_{\mathbf{a}} + Q_{\mathbf{d}} = 0 \bmod 2 \;\rightarrow\; \text{R-parity}$$

```
Type:        U   S   U   U
Dimension:   3   2   1   1
      5 x ( V ,0 ,V*,0 ) chirality -3
      5 x ( 0 ,0 ,V ,V ) chirality -3
      3 x ( V ,0 ,V ,0 ) chirality -3
      3 x ( 0 ,0 ,V ,V*) chirality 3
      5 x ( V ,V ,0 ,0 ) chirality 3
      5 x ( 0 ,V ,0 ,V ) chirality 3
      6 x ( V ,0 ,0 ,V*) chirality 0
      2 x ( 0 ,S ,0 ,0 ) chirality 0
     12 x ( 0 ,0 ,S ,0 ) chirality 0
      4 x ( A ,0 ,0 ,0 ) chirality 0
      6 x ( V ,0 ,0 ,V ) chirality 0
      4 x ( 0 ,0 ,0 ,A ) chirality 0
      4 x ( S ,0 ,0 ,0 ) chirality 0
      4 x ( 0 ,0 ,0 ,S ) chirality 0
      1 x ( Ad,0 ,0 ,0 ) chirality 0
      2 x ( 0 ,A ,0 ,0 ) chirality 0
      6 x ( 0 ,0 ,Ad,0 ) chirality 0
      1 x ( 0 ,0 ,0 ,Ad) chirality 0
      6 x ( 0 ,0 ,A ,0 ) chirality 0
      8 x ( 0 ,V ,V ,0 ) chirality 0
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Example 1

$\mathbb{Z}_2$ in $U(3) \times Sp(2) \times U(1) \times U(1)$ with broken $B-L$

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Axion Charges (including Chan-Paton multiplicity factor)

a:	0	-3	0	0	-3	-3	-3	3	0	-3	0	3	3	6	3	6
c:	0	0	0	0	0	0	-2	0	0	0	0	0	0	0	0	0
d:	0	1	0	0	1	1	-1	-1	0	1	0	-1	-1	-2	-1	-2

Null vectors

$3Q_{\mathbf{d}} - 3Q_{\mathbf{c}} + Q_{\mathbf{a}} = 0 \;\rightarrow\; Y$ unbroken

$3Q_{\mathbf{d}} + Q_{\mathbf{a}} \neq 0 \;\rightarrow\; B-L$ broken

$Q_{\mathbf{a}} = 0 \bmod 3 \;\rightarrow\;$ Conservation of color

$Q_{\mathbf{a}} + Q_{\mathbf{d}} = 0 \bmod 2 \;\rightarrow\;$ R-parity

Example 2

$\mathbb{Z}_3$ in $U(3)\times Sp(2)\times U(1)\times U(1)$ with broken $B-L$

Class 7506, Tensor (2,10,10,10), MIPF 63, Orientifold 0, boundaries (192,503,227,237)

Axion Charges (including Chan-Paton multiplicity factor)

a:	0	0	-6	0	3
c:	-6	6	5	-3	-4
d:	-6	6	7	-3	-5

Null vectors

$$3Q_{\mathbf{d}} - 3Q_{\mathbf{c}} + Q_{\mathbf{a}} = 0 \;\rightarrow\; Y \text{ unbroken}$$

$$3Q_{\mathbf{d}} + Q_{\mathbf{a}} \neq 0 \;\rightarrow\; B-L \text{ broken}$$

$$Q_{\mathbf{a}} = 0 \bmod 3 \;\rightarrow\; \text{Conservation of color}$$

$$Q_{\mathbf{d}} + Q_{\mathbf{c}} = 0 \bmod 3 \rightarrow \mathbb{Z}_3$$

Forbids UUD, QDL, LLE, LH_u, QQQL, UUDE, ν Majorana mass

Allows all Yukawa's, μ-term

Example 3

$\mathbb{Z}_3$ in $U(3) \times Sp(2) \times U(1) \times U(1)$ with unbroken $B-L$

Class 2751, Tensor (2,10,10,10), MIPF 64, Orientifold 0, boundaries (46,5,48,415)

Axion Charges (including Chan-Paton multiplicity factor)

a:	9	0	0	0	0
c:	0	0	0	0	0
d:	3	0	0	0	0

Null vectors

$$3Q_{\mathbf{d}} - 3Q_{\mathbf{c}} + Q_{\mathbf{a}} = 0 \;\rightarrow\; Y \text{ unbroken}$$

$$3Q_{\mathbf{d}} + Q_{\mathbf{a}} = 0 \;\rightarrow\; B-L \text{ unbroken}$$

$$Q_{\mathbf{a}} = 0 \bmod 3 \;\rightarrow\; \text{Conservation of color}$$

$$Q_{\mathbf{d}} = 0 \bmod 3 \;\rightarrow\; \mathbb{Z}_3$$

Forbids QQQL, UUDE (not forbidden by B-L)

Example 12

$\mathbb{Z}_2$ in $U(5) \times U(1) \times U(1)$

Class 4324, Tensor (1,10,22,22), MIPF 27, Orientifold 0, boundaries (365,365,1393,572)

Axion Charges (including Chan-Paton multiplicity factor)

a:	0	5	-5	-10	-5	0	-10	0	5
b:	2	0	0	0	0	0	0	0	0
c:	0	0	-1	-2	-1	0	-2	0	1

Null vectors

$Q_{\mathbf{a}} = 0 \text{ mod } 5 \;\rightarrow\; SU(5)$ pentality

$Q_{\mathbf{b}} = 0 \text{ mod } 2 \;\rightarrow\; \mathbb{Z}_2$

All brane charge *U(1)*'s broken

This class was discussed in *(Anastasopoulos, Leontaris, Richter, Schellekens, 2010)* because it avoids the problem that top-quark Yukawa generating instantons in two-stack U(5) model also generate dimensions 5 operators that violate baryon number. *(Kiritsis, Lennek, Schellekens, 2009)*.

In this models the two contributes come from distinct instantons with unrelated strength.

Here the $\mathbb{Z}_2$ symmetry forbids those dimensions five operators.

But: plenty of problems left (dimension 4, doublet-triplet splitting,....)

Example 13

$\mathbb{Z}_2$ in $U(5) \times U(1) \times U(1)$

Class 4325, Tensor (1,10,22,22), MIPF 27, Orientifold 0, boundaries (365,365,1506,818)

Axion Charges (including Chan-Paton multiplicity factor)

a:	0	5	-5	-10	-5	0	-10	0	5
b:	1	0	0	0	0	0	0	0	0
c:	-1	1	-3	-2	-1	0	-2	0	1

Null vectors

$Q_{\mathbf{a}} = 0 \text{ mod } 5 \ \rightarrow \ SU(5)$ pentality

$Q_{\mathbf{a}} + Q_{\mathbf{b}} + Q_{\mathbf{c}} = 0 \text{ mod } 2 \ \rightarrow$ Matter Parity

Matter sector strings have parity 0, Matter/Hidden sector strings have parity 1

Conclusions

- We know how to determine discrete symmetries in Gepner orientifolds.
- Room for improvement in underlying formalism.
- How does this work in generic RCFT?
- Many known field theory examples can indeed be found.
- Discrete symmetries do not seem to be very common in this class (a few percent).
  But:
- More than random.
- Tadpole cancellation, massless Y and discrete symmetries appear to have a positive correlation.
  (all three favoured by small h_{21}).