Instanton induced neutrino masses in RCFT orientifolds

“Open Descendant” Partition Functions

- Closed

$$\frac{1}{2}\left[\sum_{ij}\chi_i(\tau)Z_{ij}\chi_i(\bar{\tau})+\sum_i K_i\chi_i(2\tau)\right]$$

- Open

$$\frac{1}{2}\left[\sum_{i,a,n}N_aN_bA^i{}_{ab}\chi_i(\frac{\tau}{2})+\sum_{i,a}N_aM^i{}_a\hat{\chi}_i(\frac{\tau}{2}+\frac{1}{2})\right]$$

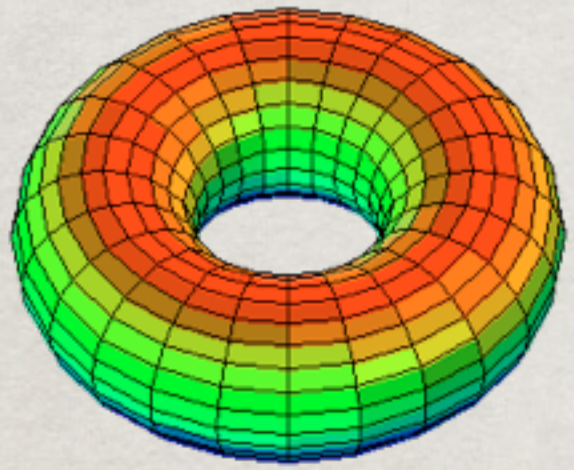
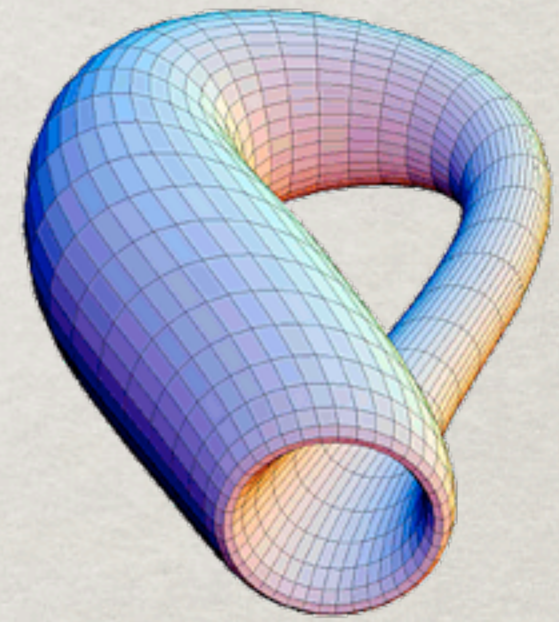
i : Primary field label (finite range)

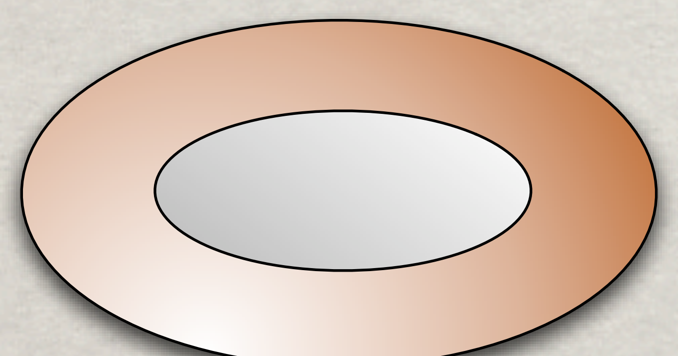
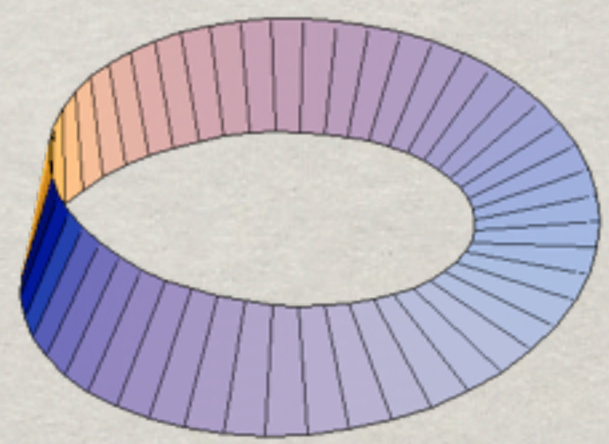
a : Boundary label (finite range)

χ_i : Character

N_a : Chan-Paton (CP) Multiplicity

Orientifold Partition Functions

$\frac{1}{2}\Big\{$  $+$  $\Big\}$

$\frac{1}{2}\Big\{$  $+$  $\Big\}$

Boundaries and Crosscaps*

- Boundary coefficients

$$R_{[a,\psi_a](m,J)} = \sqrt{\frac{|\mathcal{H}|}{|\mathcal{C}_a||\mathcal{S}_a|}}\psi_a^*(J)S_{am}^J$$

- Crosscap coefficients

$$U_{(m,J)} = \frac{1}{\sqrt{|\mathcal{H}|}}\sum_{L\in\mathcal{H}} e^{\pi i(h_K - h_{KL})}\beta_K(L)P_{LK,m}\delta_{J,0}$$

**Huiszoon, Fuchs, Schellekens, Schweigert, Walcher (2000)*

Coefficients

- Klein bottle

$$K^i = \sum_{m,J,J'} \frac{S^i{}_m U_{(m,J)} g^{\Omega,m}_{J,J'} U_{(m,J')}}{S_{0m}}$$

- Annulus

$$A^i_{[a,\psi_a][b,\psi_b]} = \sum_{m,J,J'} \frac{S^i{}_m R_{[a,\psi_a](m,J)} g^{\Omega,m}_{J,J'} R_{[b,\psi_b](m,J')}}{S_{0m}}$$

- Moebius

$$M^i_{[a,\psi_a]} = \sum_{m,J,J'} \frac{P^i{}_m R_{[a,\psi_a](m,J)} g^{\Omega,m}_{J,J'} U_{(m,J')}}{S_{0m}}$$

$$g^{\Omega,m}_{J,J'} = \frac{S_{m0}}{S_{mK}} \beta_K(J) \delta_{J',J^c}$$

TADPOLES & ANOMALIES

- Tadpole cancellation condition:

$$\sum_b N_b R_{b(m,J)} = 4\eta_m U_{(m,J)}$$

- Cubic $\mathrm{Tr}F^3$ anomalies cancel
- Remaining anomalies by Green-Schwarz mechanism
- In rare cases, additional conditions for global anomaly cancellation*

**Gato-Rivera, Schellekens (2005)*

Formalism can be applied to:

- "Gepner Models"
  (minimal N=2 tensor products)
- Free fermions (4n real + (9-2n) complex)
- Kazama-Suzuki models
  (requires exact spectrum computation)
- Permutation orbifolds
- ...

Gepner Orientifolds

C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Y. S. Stanev, Phys. Lett. B **387** (1996) 743 [arXiv:hep-th/9607229].

R. Blumenhagen and A. Wisskirchen, Phys. Lett. B **438**, 52 (1998) [arXiv:hep-th/9806131].

G. Aldazabal, E. C. Andres, M. Leston and C. Nunez, JHEP **0309**, 067 (2003) [arXiv:hep-th/0307183].

I. Brunner, K. Hori, K. Hosomichi and J. Walcher, arXiv:hep-th/0401137.

R. Blumenhagen and T. Weigand, JHEP **0402** (2004) 041 [arXiv:hep-th/0401148].

G. Aldazabal, E. C. Andres and J. E. Juknevich, JHEP **0405**, 054 (2004) [arXiv:hep-th/0403262].

The SM spectrum

Current experimental information:

3 chiral families + vector-like states

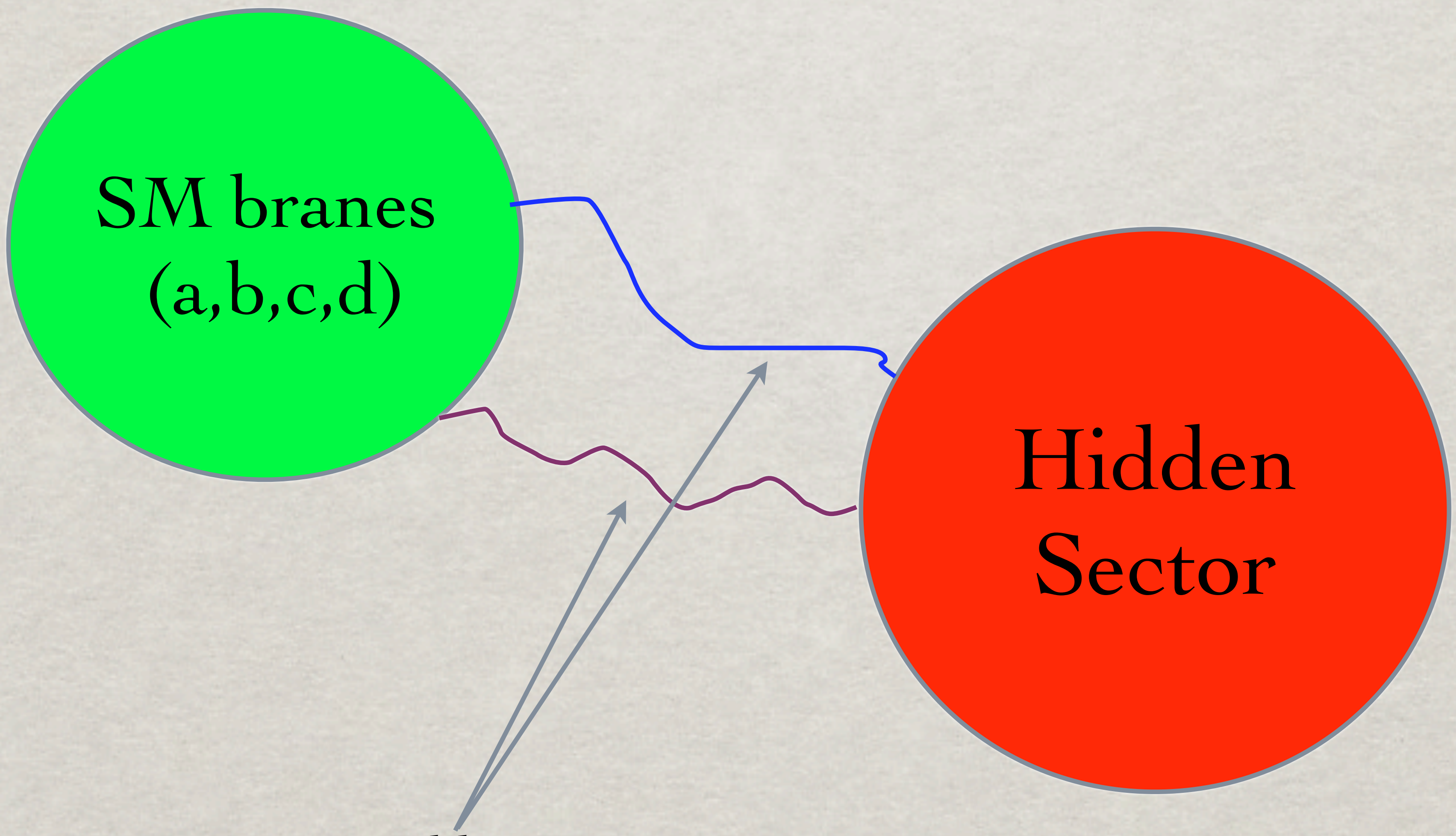
Possible vector-like states:

Higgs?
right-handed neutrinos?
squarks, sleptons?
gluinos?
who knows what else?

(Some constraints from unification, if you believe it)

Models

3 families
+ anything vector-like



Vector-like
(not always present)

Anything that cancels the
tadpoles (not always needed)

De-confusion

- Space-time susy imposed (not necessary).
- No moduli stabilization.
- Boundary state ≈ brane
- Complete set of CFT boundary states (in the sense of Sagnotti, Pradisi, Stanev)
- But: not the complete set of geometric branes.

DATA

	2004-2005*	2005-2006†
Trigger	"Madrid"	All 3 family models
Chiral types	19	19345
Tadpole-free(per type)	18	1900
Total configs	45×10^6	145×10^6
Tadpole free, distinct	210.000	1900
Max. primaries	∞	1750

(*) Huiszoon, Dijkstra, Schellekens
Phys.Lett.B609:408-417,2005, Nucl.Phys.B710:3-57,2005

(†) Anastasopoulos, Dijkstra, Kiritsis, Schellekens
Nucl.Phys.B759:83-146,2006

SU(5)

```
Type:          U   O   O
Dimension      5   1   1
          3 x  (A ,0 ,0 ) chirality 3
         11 x  (V ,V ,0 ) chirality -3
          8 x  (S ,0 ,0 ) chirality 0
          3 x  (Ad,0 ,0 ) chirality 0
          1 x  (0 ,A ,0 ) chirality 0
          3 x  (0 ,V ,V ) chirality 0
          8 x  (V ,0 ,V ) chirality 0
          2 x  (0 ,S ,0 ) chirality 0
          4 x  (0 ,0 ,S ) chirality 0
          4 x  (0 ,0 ,A ) chirality 0
```

Note: gauge group is just SU(5)!

A curiosity

Type:	U	S	U	U	U		
Dimension	3	2	1	1	2		
3 x	V	V	0	0	0	chirality 3	Q
3 x	0	0	V	V	0	chirality -3	E*
1 x	V	0	0	V*	0	chirality -1	U*
2 x	V	0	V	0	0	chirality -2	D*
2 x	0	V	0	V	0	chirality 2	L
3 x	V	0	0	V	0	chirality -1	D*+(D+D*)
3 x	0	V	V	0	0	chirality 1	$L+H_1+H_2$
2 x	V	0	V*	0	0	chirality -2	U*
1 x	0	0	V	V*	0	chirality 1	N*
4 x	A	0	0	0	0	chirality 0	U+U*
2 x	0	0	0	S	0	chirality 0	E+E*

Truly hidden
hidden sector

Goals

Finding the standard model?

Only a small part of the orientifold landscape

(880 out of 30000 hodge numbers....)

Only rational points in moduli space.

No chance unless SM is extremely abundant.

Central question

"Is the standard model a plausible solution to landscape and anthropic constraints?"

Too hard, even for string theorists...
but some simpler sub-questions may be within reach

- Does it exist in the landscape?
- Which BSM versions can be realized?
- Generic features?
- Correlations?
- Are some SM features extremely rare without a potential anthropic explanation?

NEUTRINO MASSES*

- In field theory: easy; several solutions.
- In string theory: non-trivial.
  (String theory is much more falsifiable!).
- Potentially anthropic.

(*) Ibañez, Schellekens, Uranga, **arXiv:0704.1079, JHEP (to appear)**

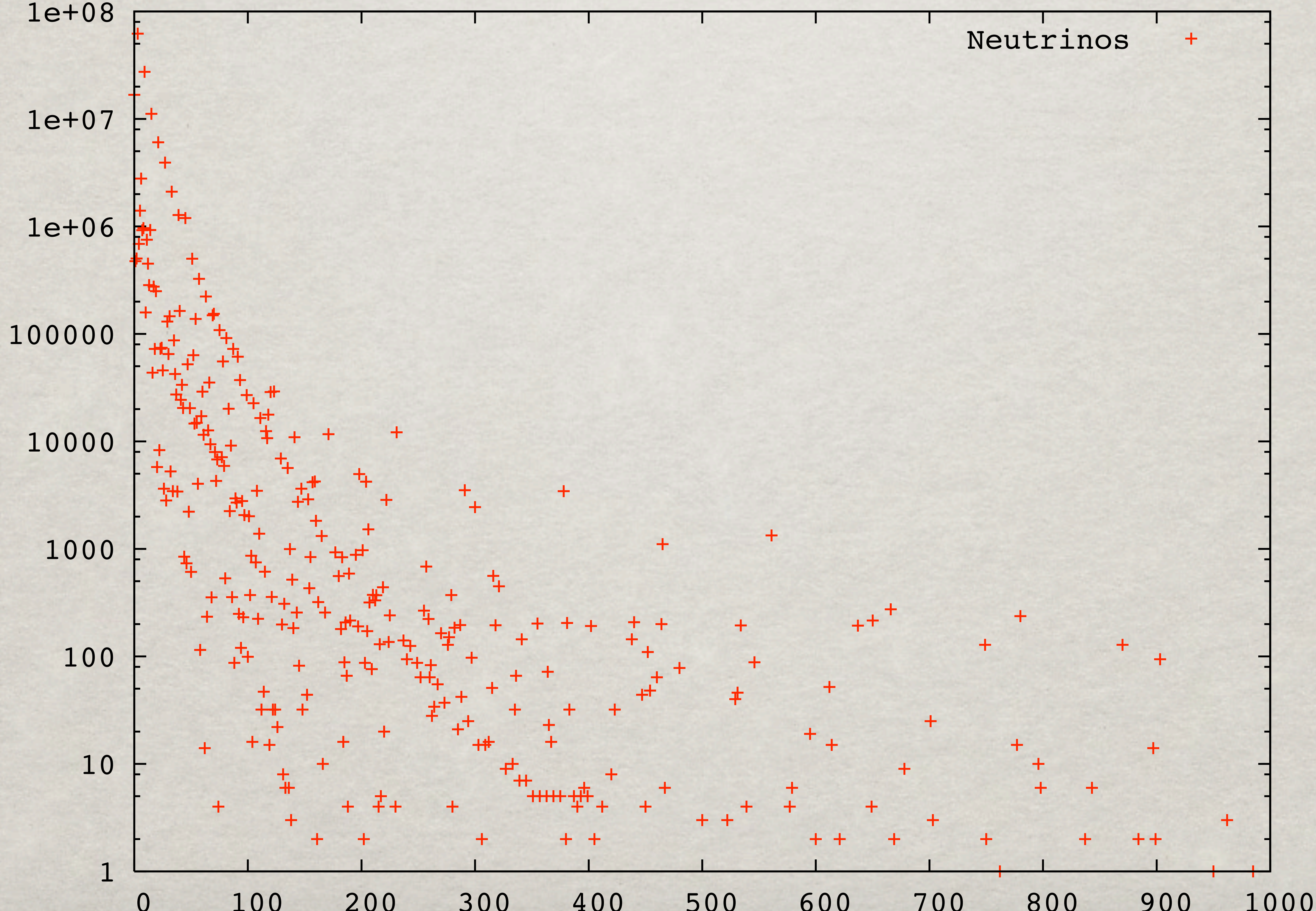
The following ingredients cannot be taken for granted in String Theory:

- Existence of a Weinberg operator.

$$\mathcal{L}_W = \frac{\lambda}{M}(L\overline{H}L\overline{H})$$

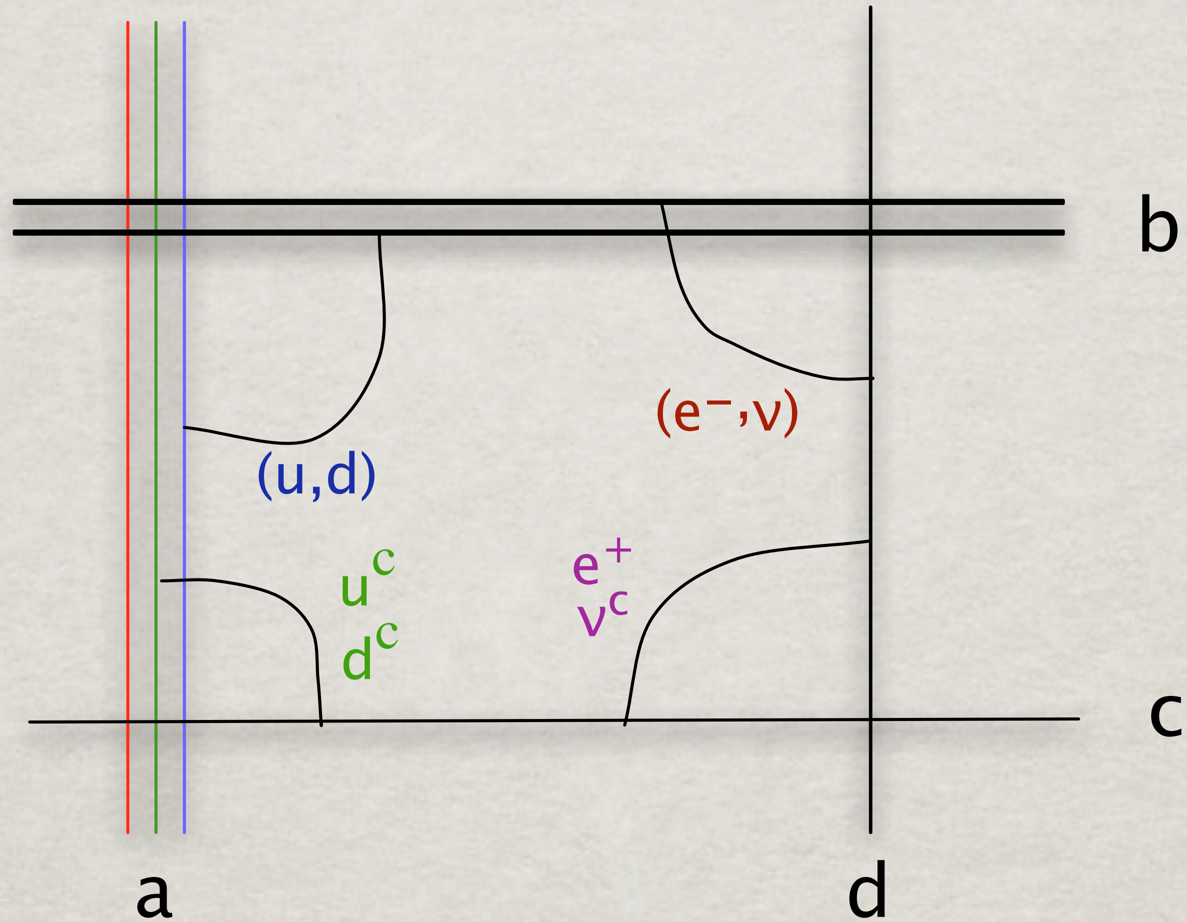
- Existence of right-handed neutrinos.
- Existence of non-zero Dirac masses.
- Absence of massless B-L vector bosons.
- Existence of Majorana masses.

Right-handed neutrinos



	Total number in 2005/2006 database
0	16766656
1	475928
2	502820
3	62149717
4	686961

The Madrid Model



Chiral SU(3) x SU(2) x U(1) spectrum:

$$3(u,d)_L + 3u^c_L + 3d^c_L + 3(e^-,\nu)_L + 3e^+_L$$

Y massless $\qquad Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c - \frac{1}{2}Qd$

Madrid models

All these models have three right-handed neutrinos (required for cubic anomaly cancellation)

In most of these models:
B-L survives as an exact gauge symmetry

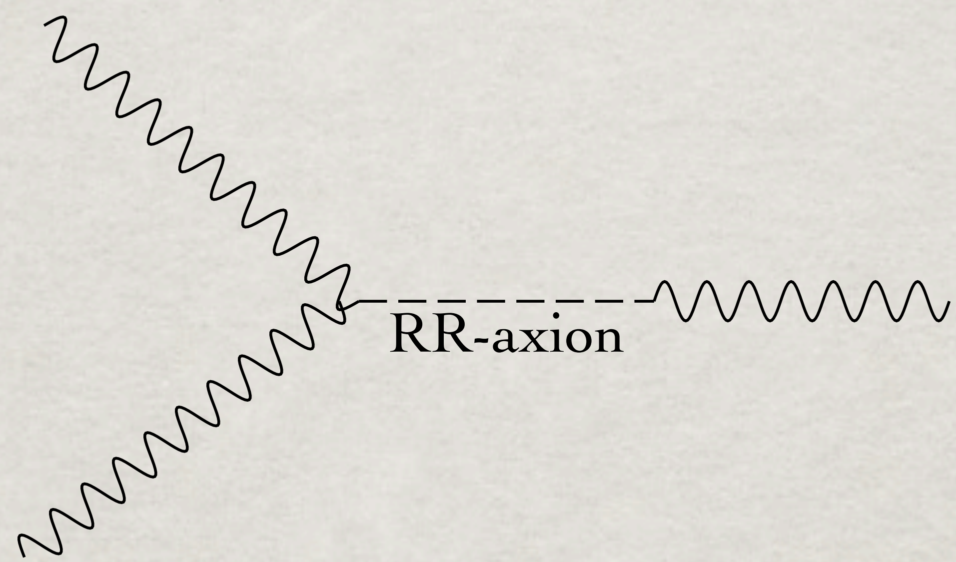
Neutrino's can get Dirac masses, but not Majorana masses (both needed for see-saw mechanism).

In a very small* subset, B-L acquires a mass due to axion couplings.

(*) 391 out of 10000 models with $SU(3)\times Sp(2)\times U(1)\times U(1)$
(out of 211000 in total)

Abelian Masses

Green-Schwarz mechanism



Axion-Vector boson vertex

Generates mass vector bosons of anomalous symmetries

$(e.g.\ B+L)$

But may also generate mass for non-anomalous ones

$(Y, B-L)$

B-L Violation

But even then, B-L still survives as a perturbative symmetry.

It may be broken to a discrete subgroup by instantons.

This possibility can be explored if the instanton is described by a RCFT brane M.
B-L violation manifests itself as:

$$I_{M\mathbf{a}} - I_{M\mathbf{a}'} - I_{M\mathbf{d}} + I_{M\mathbf{d}'} \neq 0$$

I_{Ma} = chiral [# (V,V*) - # (V*,V)]   between branes M and a

a' = boundary conjugate of a

B-L anomalies

$$I_{M\mathbf{a}} - I_{M\mathbf{a}'} - I_{M\mathbf{d}} + I_{M\mathbf{d}'} \neq 0$$

Implies a cubic B-L anomaly if M is a “matter” brane (Chan-Paton multiplicity ≠ 0).

$\Rightarrow$ *M cannot be a matter brane:*
non-gauge-theory instanton
(stringy instanton, exotic instanton)

Implies a (B-L)$(G_M)^2$ anomaly even if we cancel the cubic anomaly

$\Rightarrow$ *B-L must be massive*

(The converse is not true: there are massive B-L models without such instanton branes)

Instantons:

See talks by Lüst,
Plauschinn, Lerda, Morales,
Blumenhagen, Cvetic,
Kiritsis

Required zero-modes

Neutrino mass generation by non-gauge theory instantons*

The desired neutrino mass term $\nu^c\nu^c$
violates c and d brane charge by two units.
To compensate this, we must have

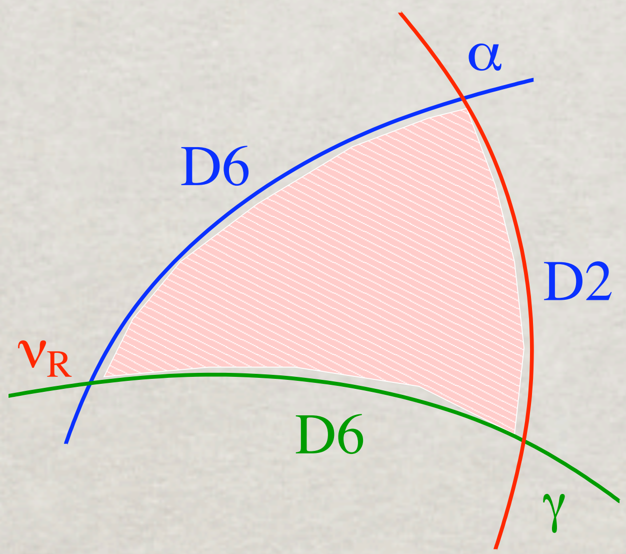
$$I_{M\mathbf{c}} = 2\ ;\ I_{M\mathbf{d}} = -2 \quad or \quad I_{M\mathbf{d}'} = 2\ ;\ I_{M\mathbf{c}'} = -2$$

and all other intersections 0.
(d' is the boundary conjugate of d)

(*) Blumenhagen, Cvetic, Weigand, hep-th/0609191
Ibañez, Uranga, hep-th/0609213

Neutrino-zero mode coupling

The following world-sheet disk is allowed by all symmetries



$$L_{cubic} \propto d_a^{ij} \left(\alpha_i \, \nu^a \gamma_j\right) \quad , a = 1, 2, 3$$

Zero-mode integrals

$$\int d^2\alpha\, d^2\gamma\; e^{-d_a^{ij}\;(\alpha_i \nu^a \gamma_j)} \;=\; \nu_a \nu_b\, (\, \epsilon_{ij}\epsilon_{kl} d_a^{ik} d_b^{jl}\,)$$

Additional zero modes yield additional fermionic integrals and hence no contribution

Therefore $I_{Ma}=I_{Mb}=I_{Mx}=0$ (x = Hidden sector), and there should be no vector-like zero modes.

There should also be no instanton-instanton zero-modes except 2 required by susy.

INSTANTON TYPES

In orientifold models we can have complex and real branes

Matter brane M	Instanton brane M
U(N)	U(k)
O(N)	Sp(2k)
Sp(2N)	O(k)

$$I_{M\mathbf{c}} = 2\ ;\ I_{M\mathbf{d}} = -2 \quad or \quad I_{M\mathbf{d}'} = 2\ ;\ I_{M\mathbf{c}'} = -2$$

Possible for:

- U, k=1 or 2
- Sp, k=1
- O, k=1,2

Universal instanton-instanton zero-modes

- U(k): 4 Adj
- Sp(2k): 2 A + 2 S
- O(k): 2 S + 2 A

Only O(1) has the required 2 zero modes

(See also: Argurio, Bertolini, Ferretti, Lerda,Peterson, arXiv:0704.0262)

Instanton scan

Can we find such branes M in the 391 models with massive B-L?

- About 30.000 “instanton branes”
  (violations of the sum rule, i.e. $I_{M\mathbf{a}} - I_{M\mathbf{a}'} - I_{M\mathbf{d}} + I_{M\mathbf{d}'} \neq 0$)
- Violations between -8 and +8
- Quantized in units of 1,2 or 4
  (1 may give R-parity violation, 4 means no Majorana mass)
- Some models have no RCFT instantons
- 1315 instantons have the right number of zero modes, counted chirally.
- None of these models has R-parity violating instantons.

Tensor	MIPF	Orientifold	Instanton	Solution
(2,4,18,28)	17	0		
(2,4,22,22)	13	3	$S2^+!$, $S2^-!$	Yes!
(2,4,22,22)	13	2	$S2^+!$, $S2^-!$	Yes
(2,4,22,22)	13	1	$S2^+$, $S2^-$	No
(2,4,22,22)	13	0	$S2^+$, $S2^-$	Yes
(2,4,22,22)	31	1	$U1^+, U1^-$	No
(2,4,22,22)	20	0		
(2,4,22,22)	46	0		
(2,4,22,22)	49	1	$O2^+, O2^-, O1^+, O1^-$	Yes
(2,6,14,14)	1	1	$U1^+$	No
(2,6,14,14)	22	2		
(2,6,14,14)	60	2		
(2,6,14,14)	64	0		
(2,6,14,14)	65	0		
(2,6,10,22)	22	2		
(2,6,8,38)	16	0		
(2,8,8,18)	14	2	$S2^+!$, $S2^-!$	Yes
(2,8,8,18)	14	0	$S2^+!$, $S2^-!$	No
(2,10,10,10)	52	0	$U1^+, U1^-$	No
(4,6,6,10)	41	0		
(4,4,6,22)	43	0		
(6,6,6,6)	18	0		

A model with S2 instantons

```
 5 x (V ,V ,0 ,0 ) chirality 3
 3 x (V ,0 ,V ,0 ) chirality -3
 3 x (V ,0 ,V*,0 ) chirality -3
 3 x (0 ,V ,0 ,V ) chirality 3
 5 x (0 ,0 ,V ,V ) chirality -3
 3 x (0 ,0 ,V ,V*) chirality 3
 6 x (V ,0 ,0 ,V )
18 x (0 ,V ,V ,0 )
 2 x (Ad,0 ,0 ,0 )
 2 x (A ,0 ,0 ,0 )
 2 x (S ,0 ,0 ,0 )
14 x (0 ,A ,0 ,0 )
 6 x (0 ,S ,0 ,0 )
 9 x (0 ,0 ,Ad,0 )
 6 x (0 ,0 ,A ,0 )
14 x (0 ,0 ,S ,0 )
 3 x (0 ,0 ,0 ,Ad)
 4 x (0 ,0 ,0 ,A )
 6 x (0 ,0 ,0 ,S )
```

Gauge group: SU(3) × SU(2) × U(1) × Nothing.
Exactly the correct number of instanton zero modes
(except for 2 universal symmetric tensors)

A model with S2 instantons

```
 5 x (V ,V ,0 ,0 ) chirality 3
 3 x (V ,0 ,V ,0 ) chirality -3
 3 x (V ,0 ,V*,0 ) chirality -3
 3 x (0 ,V ,0 ,V ) chirality 3
 5 x (0 ,0 ,V ,V ) chirality -3
 3 x (0 ,0 ,V ,V*) chirality 3
 6 x (V ,0 ,0 ,V )
18 x (0 ,V ,V ,0 )
 2 x (Ad,0 ,0 ,0 )
 2 x (A ,0 ,0 ,0 )
 2 x (S ,0 ,0 ,0 )
14 x (0 ,A ,0 ,0 )
 6 x (0 ,S ,0 ,0 )
 9 x (0 ,0 ,Ad,0 )
 6 x (0 ,0 ,A ,0 )
14 x (0 ,0 ,S ,0 )
 3 x (0 ,0 ,0 ,Ad)
 4 x (0 ,0 ,0 ,A )
 6 x (0 ,0 ,0 ,S )
```

Gauge group: SU(3) × SU(2) × U(1) × Nothing.
Exactly the correct number of instanton zero modes
(except for 2 universal symmetric tensors)

$$\sin^2(\theta_w) = .5271853$$

$$\frac{\alpha_3}{\alpha_2} = 3.2320501$$

The O1 instanton

Type:		U	S	U	U	U	O	O	U	O	O	O	U	S	S	O	S	
Dimension		3	2	1	1	1	2	2	3	1	2	3	1	2	2	2	--	
2	x	0	0	0	V	0	0	0	0	0	0	0	0	0	0	0	V	chirality 2
5	x	V	0	V	0	0	0	0	0	0	0	0	0	0	0	0	0	chirality -3
5	x	0	0	V	V*	0	0	0	0	0	0	0	0	0	0	0	0	chirality 3
12	x	0	0	V	0	0	0	0	0	0	0	0	0	0	0	0	V	chirality -2
3	x	V	0	V*	0	0	0	0	0	0	0	0	0	0	0	0	0	chirality -3
3	x	0	0	V	V	0	0	0	0	0	0	0	0	0	0	0	0	chirality -3
3	x	V	V	0	0	0	0	0	0	0	0	0	0	0	0	0	0	chirality 3
3	x	0	V	0	V	0	0	0	0	0	0	0	0	0	0	0	0	chirality 3
25	x	0	0	Ad	0	0	0	0	0	0	0	0	0	0	0	0	0	chirality 0
2	x	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	chirality 0
4	x	V	0	0	V	0	0	0	0	0	0	0	0	0	0	0	0	chirality 0
2	x	0	0	0	A	0	0	0	0	0	0	0	0	0	0	0	0	chirality 0
34	x	0	0	A	0	0	0	0	0	0	0	0	0	0	0	0	0	chirality 0
14	x	0	0	S	0	0	0	0	0	0	0	0	0	0	0	0	0	chirality 0
2	x	V	0	0	0	0	0	0	0	0	0	0	0	0	0	0	V	chirality 0
2	x	0	0	0	V	0	0	0	0	0	0	0	0	0	V	0	0	chirality 0
1	x	Ad	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	chirality 0
2	x	0	S	0	0	0	0	0	0	0	0	0	0	0	0	0	0	chirality 0
1	x	0	0	0	Ad	0	0	0	0	0	0	0	0	0	0	0	0	chirality 0
6	x	0	0	V	0	0	0	0	0	0	0	0	0	0	V	0	0	chirality 0
2	x	S	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	chirality 0
2	x	0	0	0	S	0	0	0	0	0	0	0	0	0	0	0	0	chirality 0
2	x	0	V	0	0	0	0	0	0	0	0	0	V	0	0	0	0	chirality 0
1	x	0	V	0	0	0	0	0	0	V	0	0	0	0	0	0	0	chirality 0
1	x	0	V	0	0	0	0	0	0	0	V	0	0	0	0	0	0	chirality 0
1	x	0	V	0	0	0	0	0	0	0	0	V	0	0	0	0	0	chirality 0
2	x	V	0	0	V*	0	0	0	0	0	0	0	0	0	0	0	0	chirality 0
2	x	V	0	0	0	0	0	0	V	0	0	0	0	0	0	0	0	chirality 0
2	x	0	0	0	V	0	0	0	V	0	0	0	0	0	0	0	0	chirality 0
2	x	0	0	V	0	0	0	0	0	0	0	0	0	0	0	V	0	chirality 0
6	x	0	V	V	0	0	0	0	0	0	0	0	0	0	0	0	0	chirality 0
6	x	0	0	V	0	0	0	0	0	0	0	0	0	V	0	0	0	chirality 0
2	x	V	0	0	0	V	0	0	0	0	0	0	0	0	0	0	0	chirality 0
2	x	0	0	0	V	V	0	0	0	0	0	0	0	0	0	0	0	chirality 0
3	x	0	0	0	0	S	0	0	0	0	0	0	0	0	0	0	0	chirality -1
3	x	0	0	0	0	0	V	0	0	0	0	0	V	0	0	0	0	chirality 1
1	x	0	0	0	0	A	0	0	0	0	0	0	0	0	0	0	0	chirality -1
2	x	0	0	0	0	V	0	V	0	0	0	0	0	0	0	0	0	chirality 2

Conclusions

- Many desirable SM features can be realized in the RCFT orientifold landscape...
  - Chiral SM spectrum
  - No mirrors
  - No adjoints, rank-2 tensors
  - No hidden sector
  - No hidden-observable massless matter
  - Matter free hidden sector
  - Exact SU(3)× SU(2) ×U(1)
  - O1 instantons

  but not all at the same time.
- Neutrino masses:
  "an incomplete success."
  With sufficient statistics, O1 instantons without superfluous zero-modes will be found.
- Boundary state statistics:
  12 million Unitary
  3 million Orthogonal
  2 million Symplectic ➡ 270000 O1
- But what is the real reason why neutrino masses are small?