# Heterotic Weight Lifting

String phenomenology
Paris, 6 july 2010

Bert Schellekens
NIKHEF

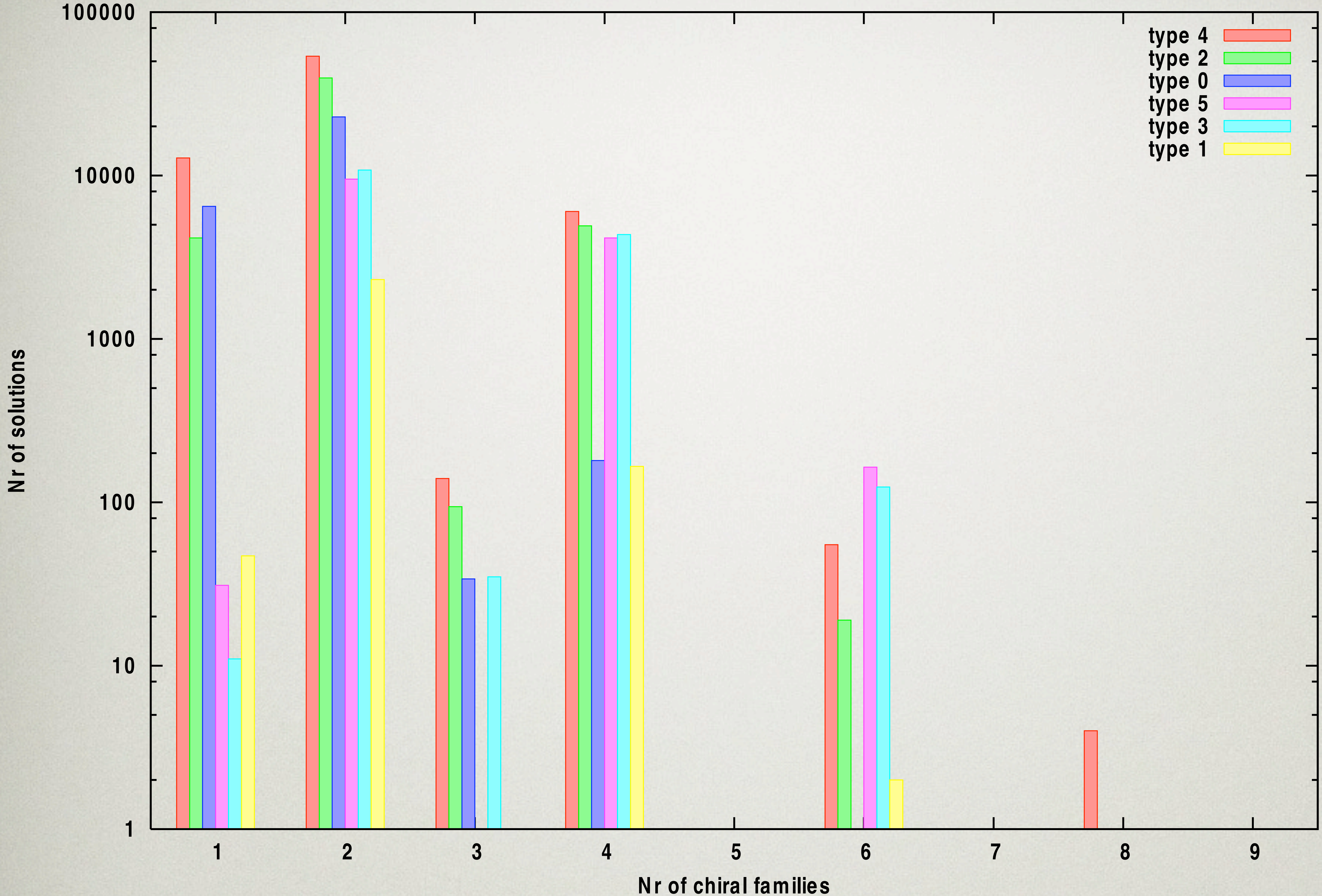
# Based on:

**B. Gato-Rivera and A.N. Schellekens**

**Nucl.Phys.B828:375-389,2010;**

**arXiv:1003.6075;**

**and to appear**

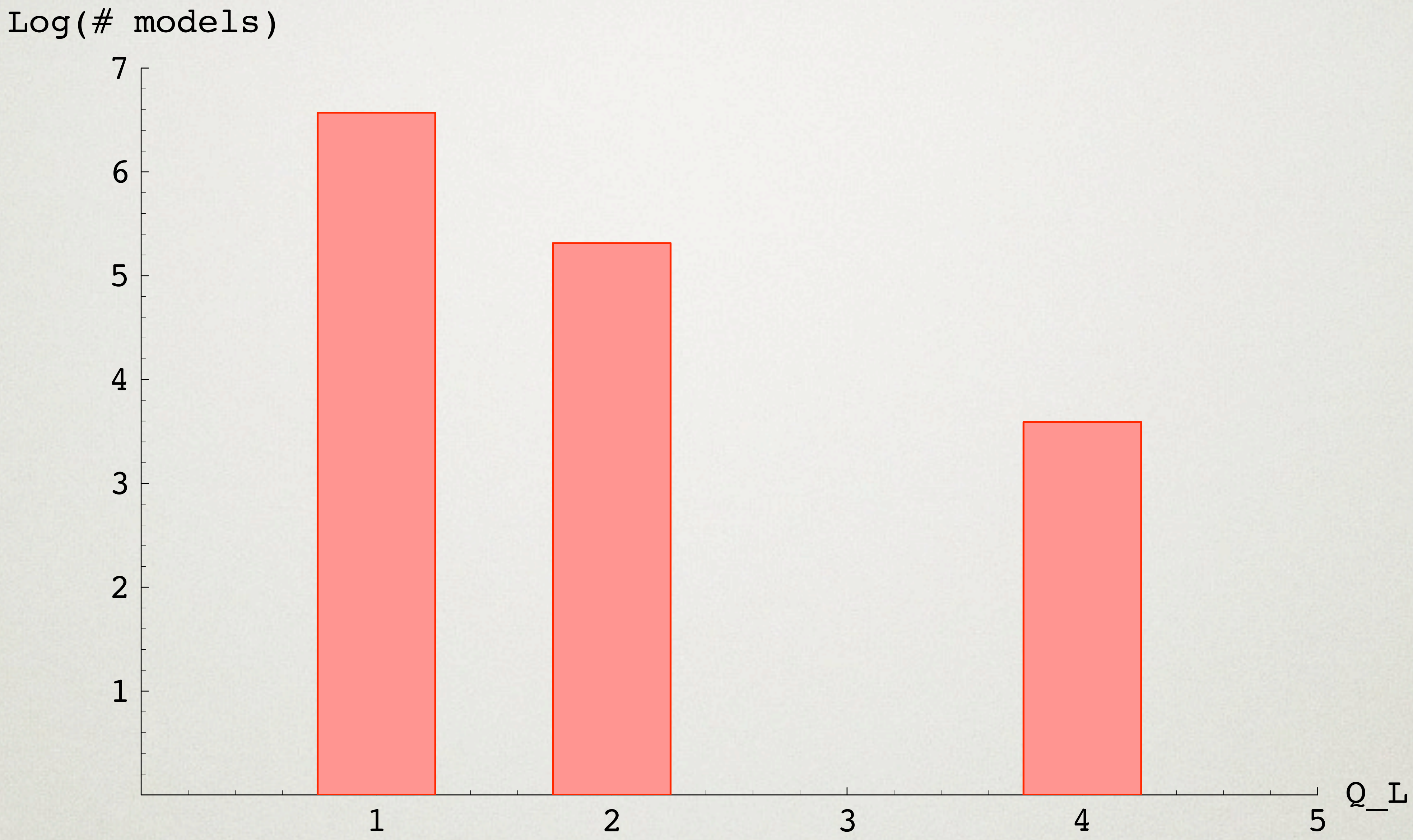



*Dijkstra, Huiszoon, Schellekens (2004)*
*See also Gmeiner et. al. "One in a billion"*

**Orientifolds of Gepner Models**

## *Possible ways out:*

- Artefact of method?
  (Orientifold, RCFT, Susy+moduli)
- Anthropic argument?
- Bad luck?




**One in a Billion: MSSM-like D-Brane Statistics** (JHEP 0601:004,2006)
Florian Gmeiner, Ralph Blumenhagen, Gabriele Honecker, Dieter Lüst, Timo Weigand

What is the analogous distribution in heterotic strings?

CERN–TH.5440/89

# NEW MODULAR INVARIANTS FOR N=2 TENSOR PRODUCTS AND FOUR-DIMENSIONAL STRINGS

A. N. Schellekens

and

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*CERN, 1211 Geneva 23, Switzerland*

## ABSTRACT

The construction of modular invariant partition functions of tensor products of $N = 2$ superconformal field theories is clarified and extended by means of a recently proposed method using simple currents, *i.e.* primary fields with simple fusion rules. Apart from providing a conceptually much simpler way of understanding space-time and world-sheet supersymmetry projections in modular invariant string theories, this makes a large class of modular invariant partition functions accessible for investigation. We demonstrate this by constructing thousands of (2,2), (1,2) and (0,2) string theories in four dimensions, including more than 40 new three generation models.

# CFT landscape scans

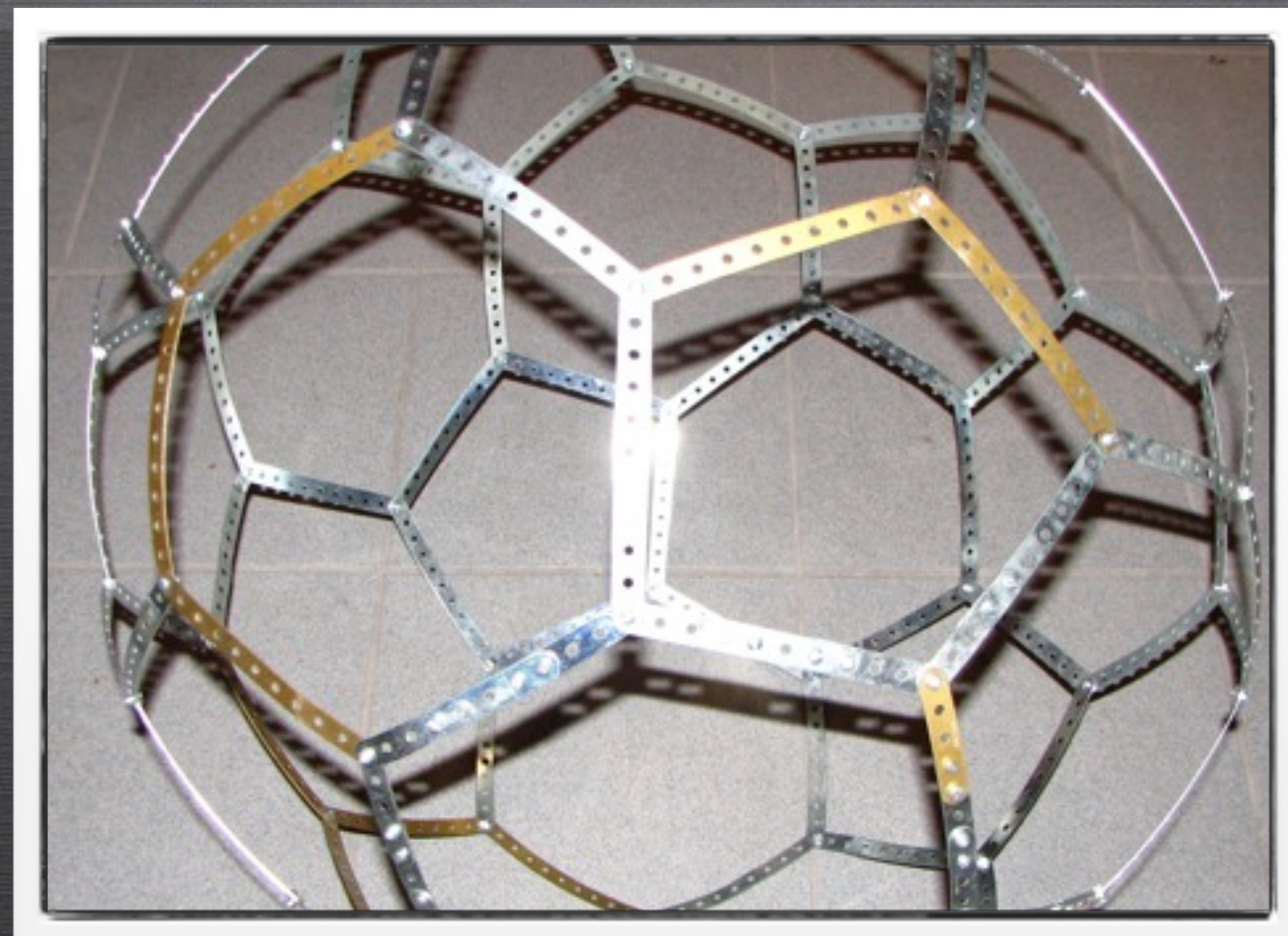
Orientifolds:

*Dijkstra, Huiszoon, Schellekens*
*Gmeiner, Blumenhagen, Honecker, Lüst, T. Weigand*
*Anastasopoulos, Dijkstra, Kiritsis, Schellekens*
*Douglas, Taylor*
*Kiritsis, Lennek, Schellekens*
*Gmeiner, Honecker*

Heterotic:

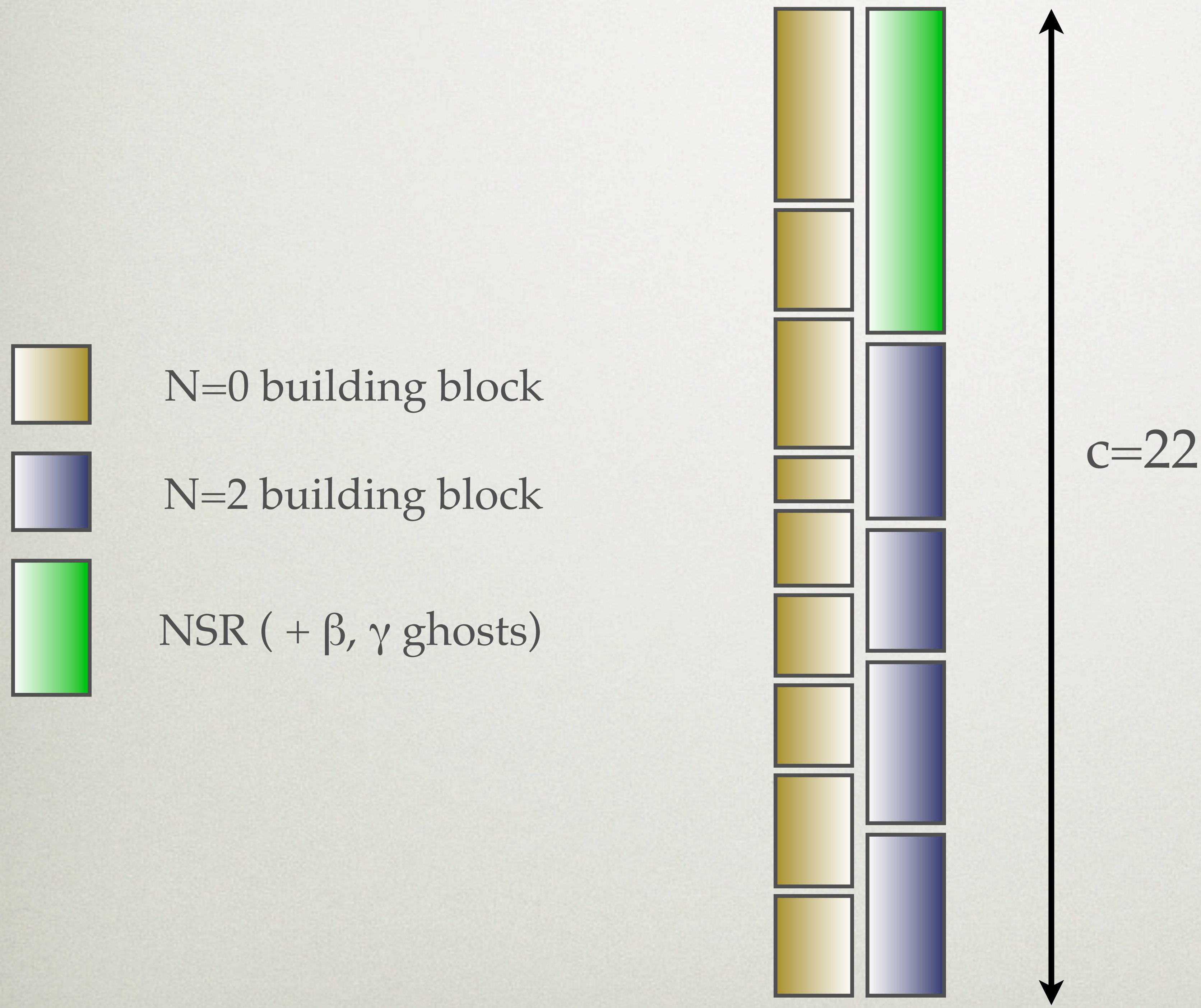
*Lutken, Ross (1988)*
*Schellekens, Yankielowicz (1989)*
*Fuchs, Klemm, Scheich, Schmidt (1989)*
*Donagi, Faraggi (2004),*
*Ploger, Ramos-Sanchez, Ratz, Vaudrevange (2007)*
*Donagi, Wendland (2008)*
*Kiritsis, Lennek, Schellekens (2008)*

*Dienes, Senechal (2007)*
*Assel, Christodoulides, Faraggi, Kounnas, Rizos (2009)*

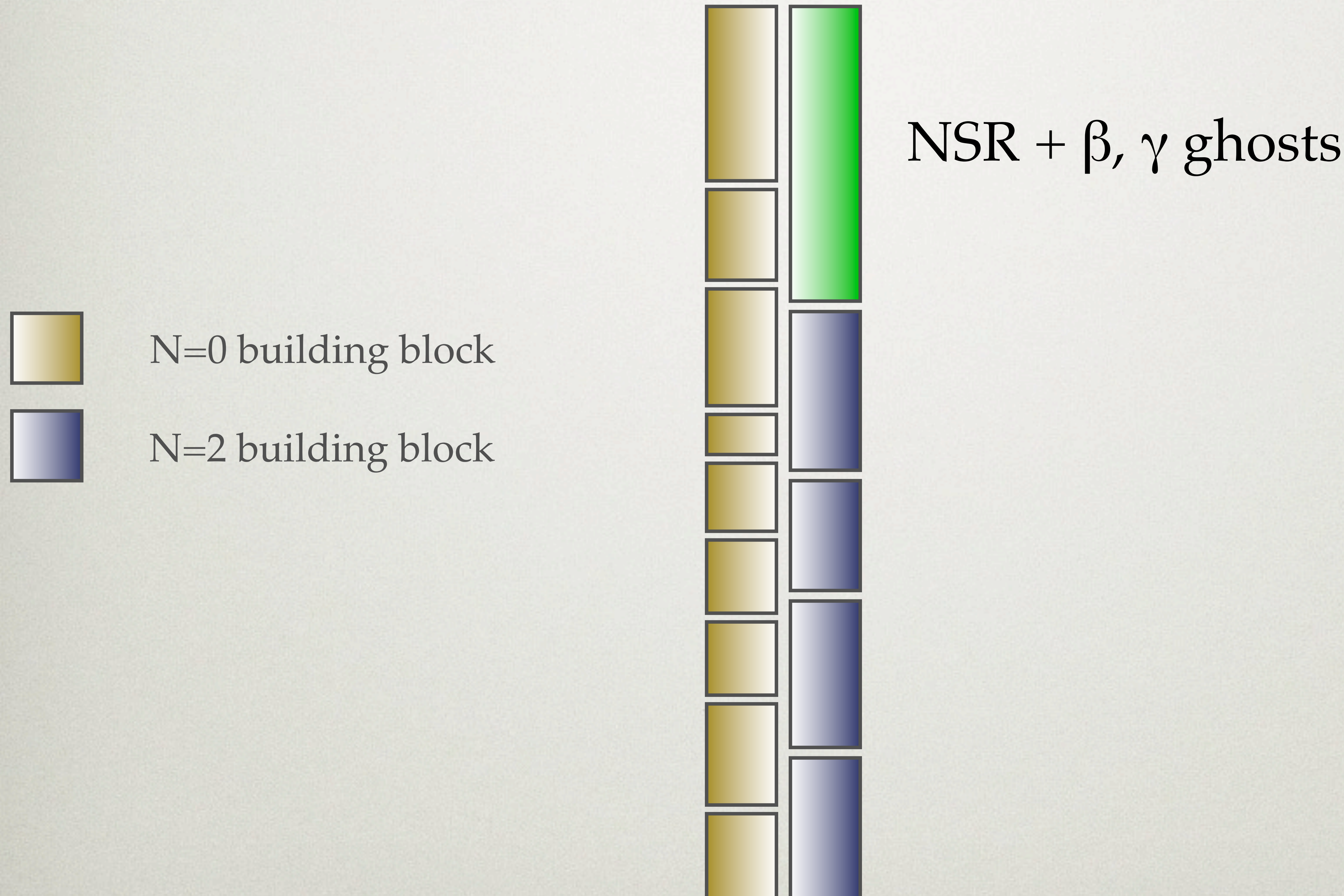


# Modular Engineering

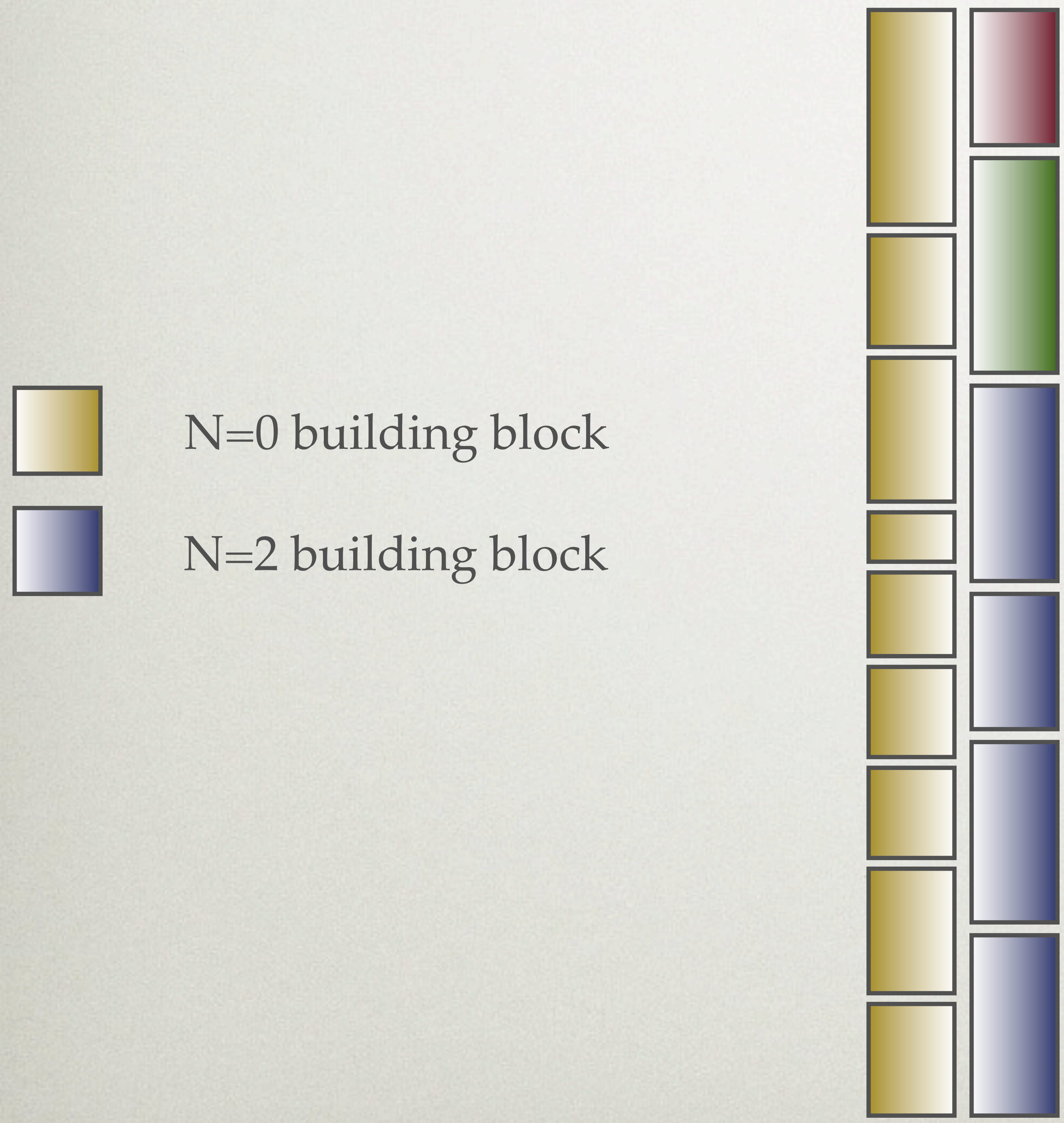
# General (0,2) model in RCFT



# The Bosonic String Map



# The Bosonic String Map

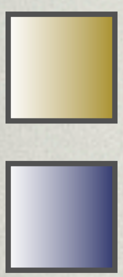



$SO(10) \times E_8$

"Bosonic string map"

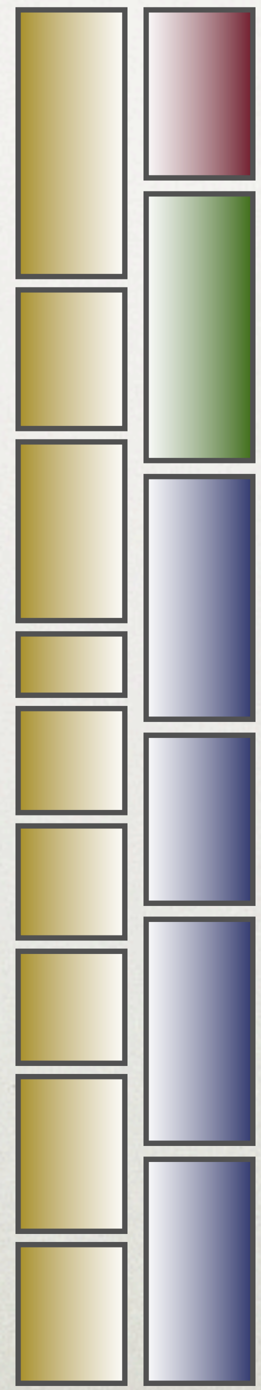
*(Lerche, Lüst, Schellekens, 1986)*

# The Bosonic String Map



N=0 building block

N=2 building block



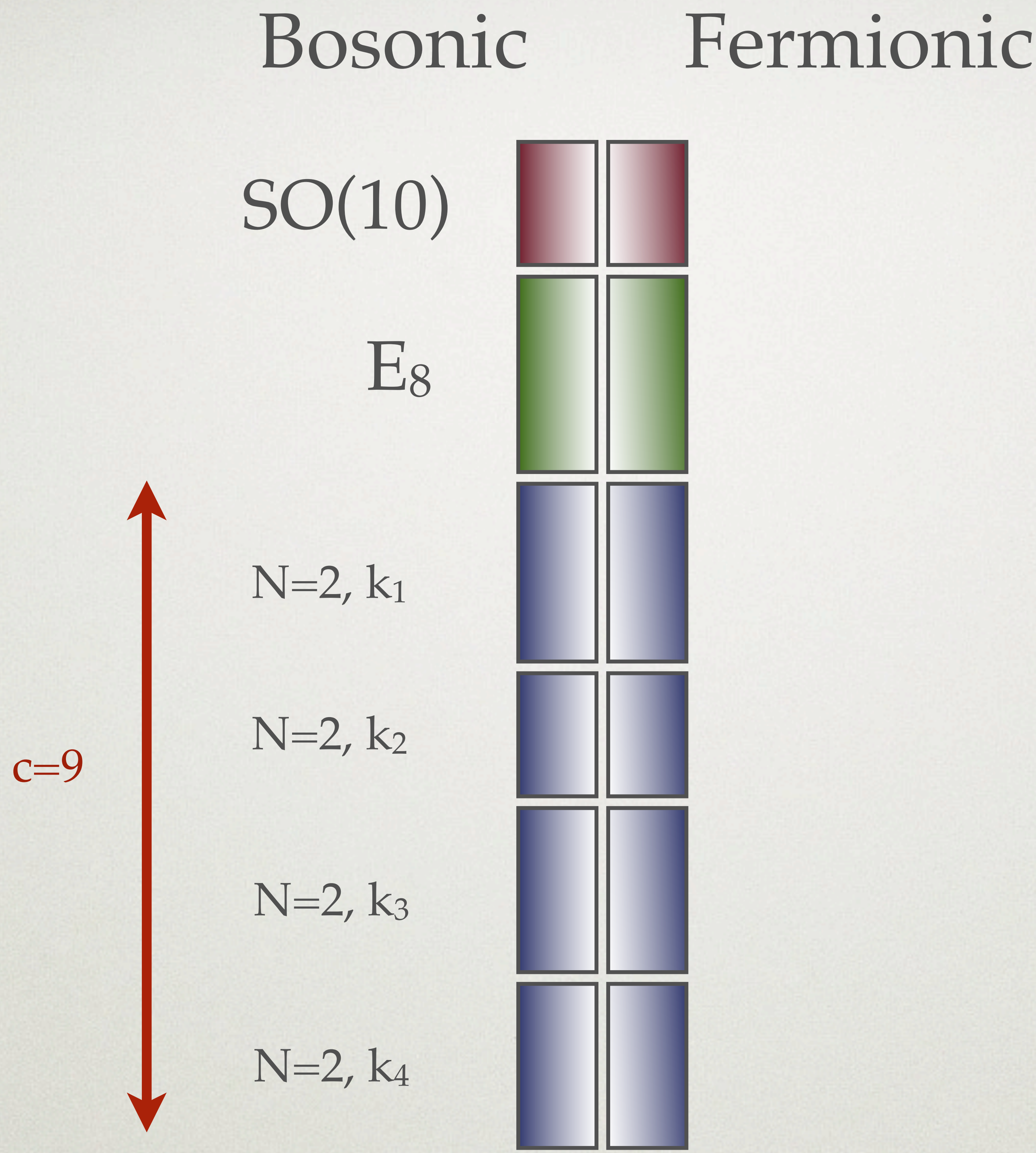
$$SO(10) \times E_8$$

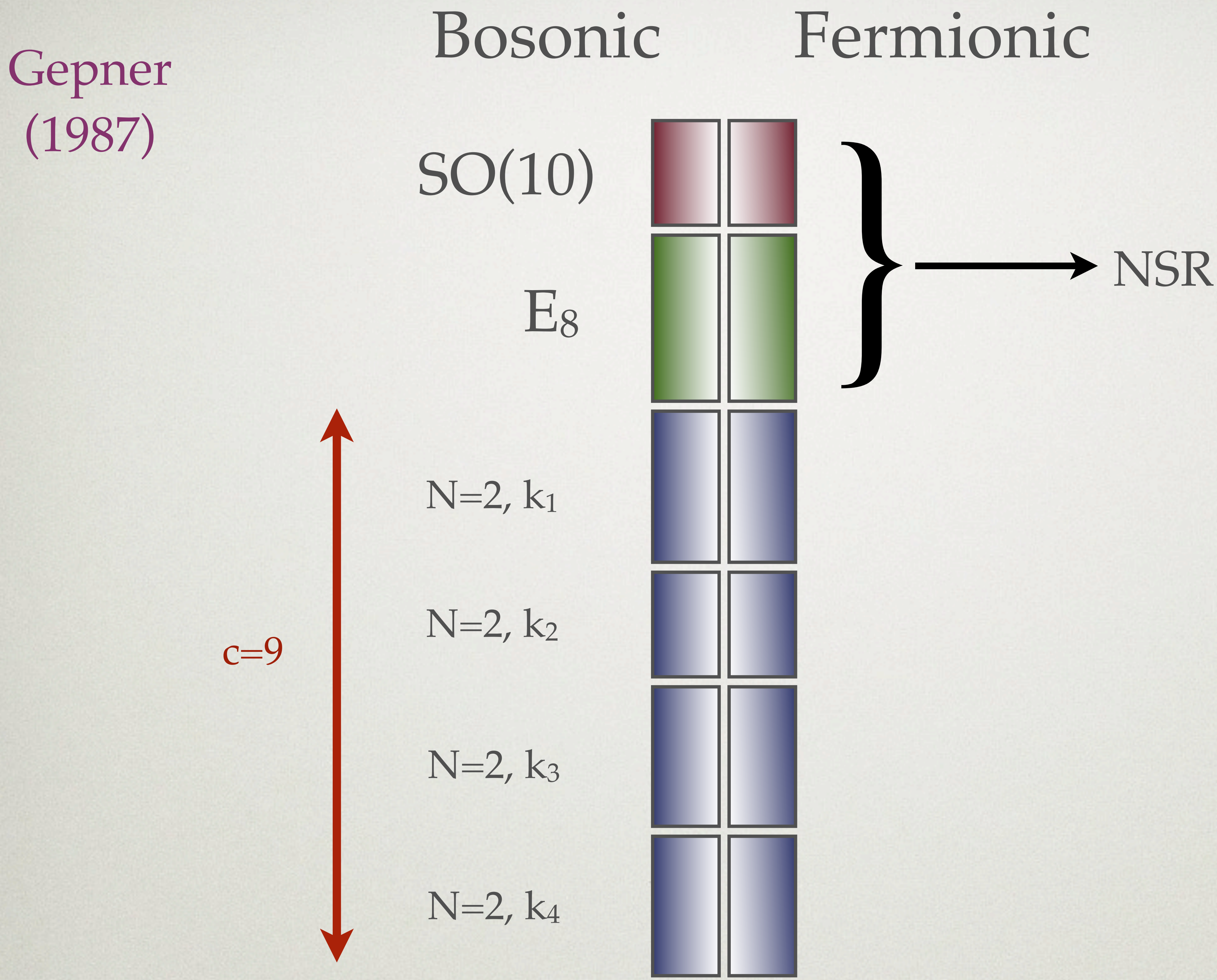
"Bosonic string map"

*(Lerche, Lüst, Schellekens, 1986)*

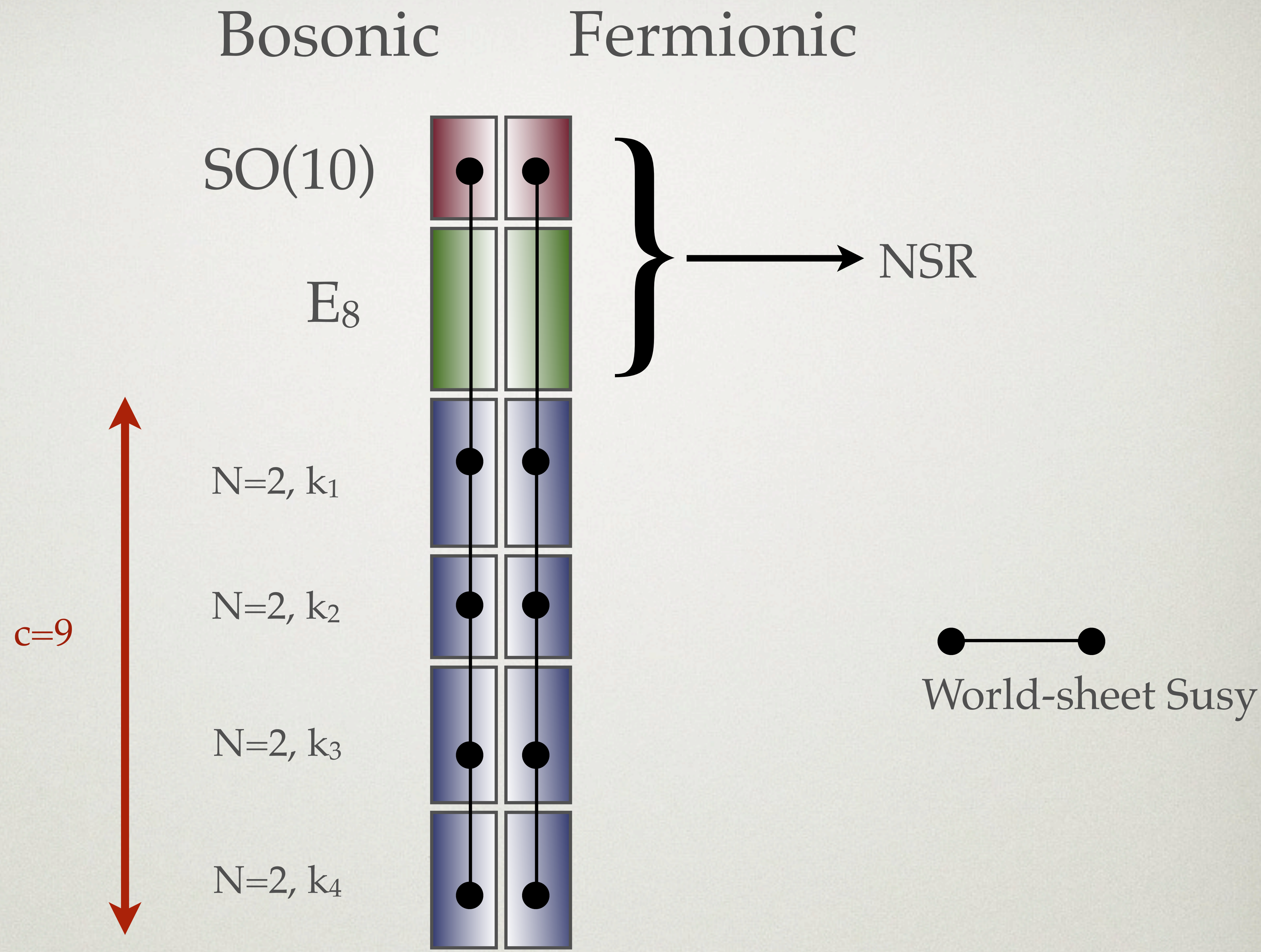
If the other building blocks are made out of free bosons, Narain lattices can be used to get modular invariance.

Gepner (1987)

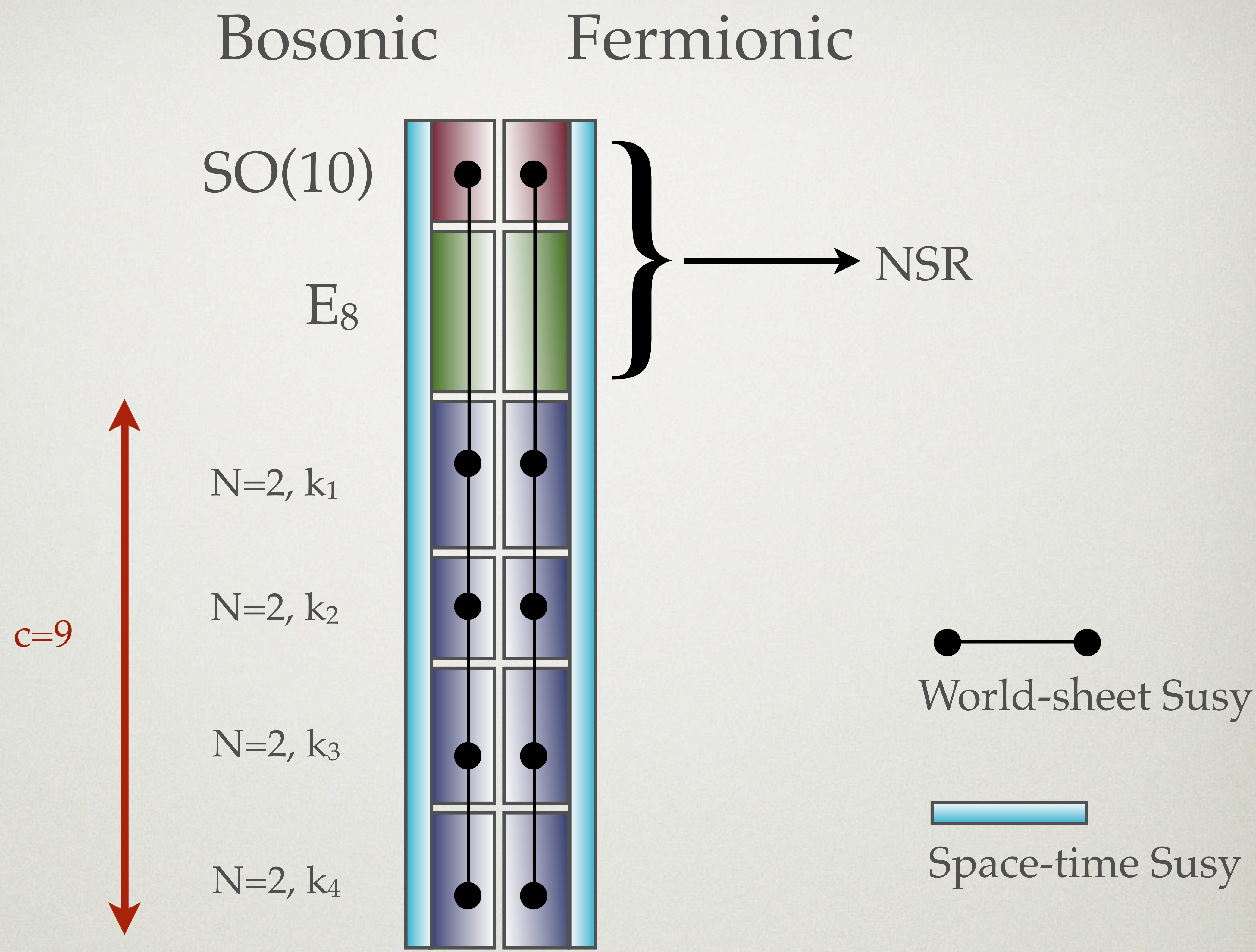



Gepner
(1987)
Bosonic
Fermionic
SO(10)
E8
NSR
c=9
N=2, k1
N=2, k2
N=2, k3
N=2, k4

Gepner
(1987)


Bosonic
Fermionic
SO(10)
$E_8$
NSR
$N=2, k_1$
$N=2, k_2$
$N=2, k_3$
$N=2, k_4$
c=9
World-sheet Susy

Gepner (1987)


Bosonic
Fermionic
SO(10)
E8
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N=2, k1
N=2, k2
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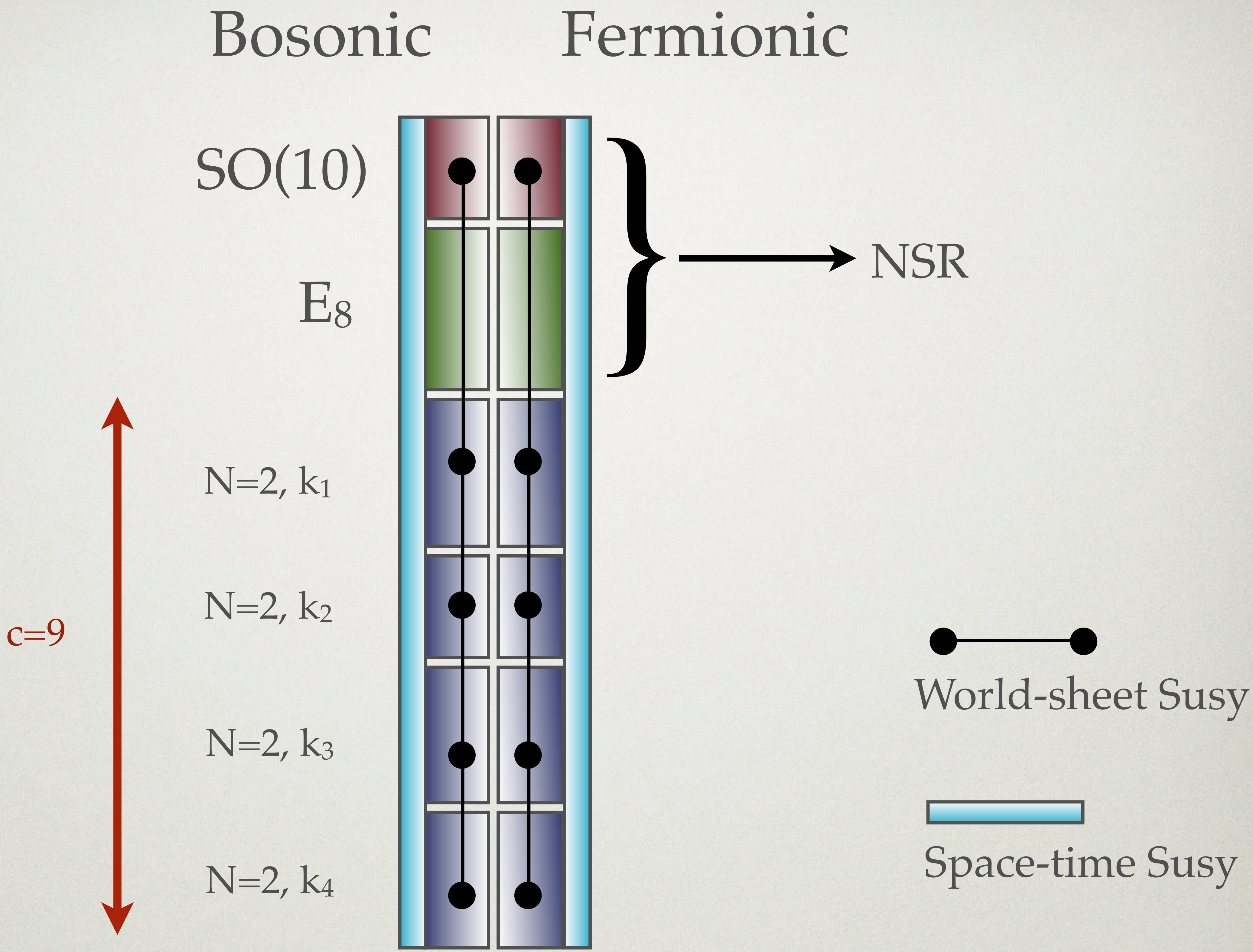
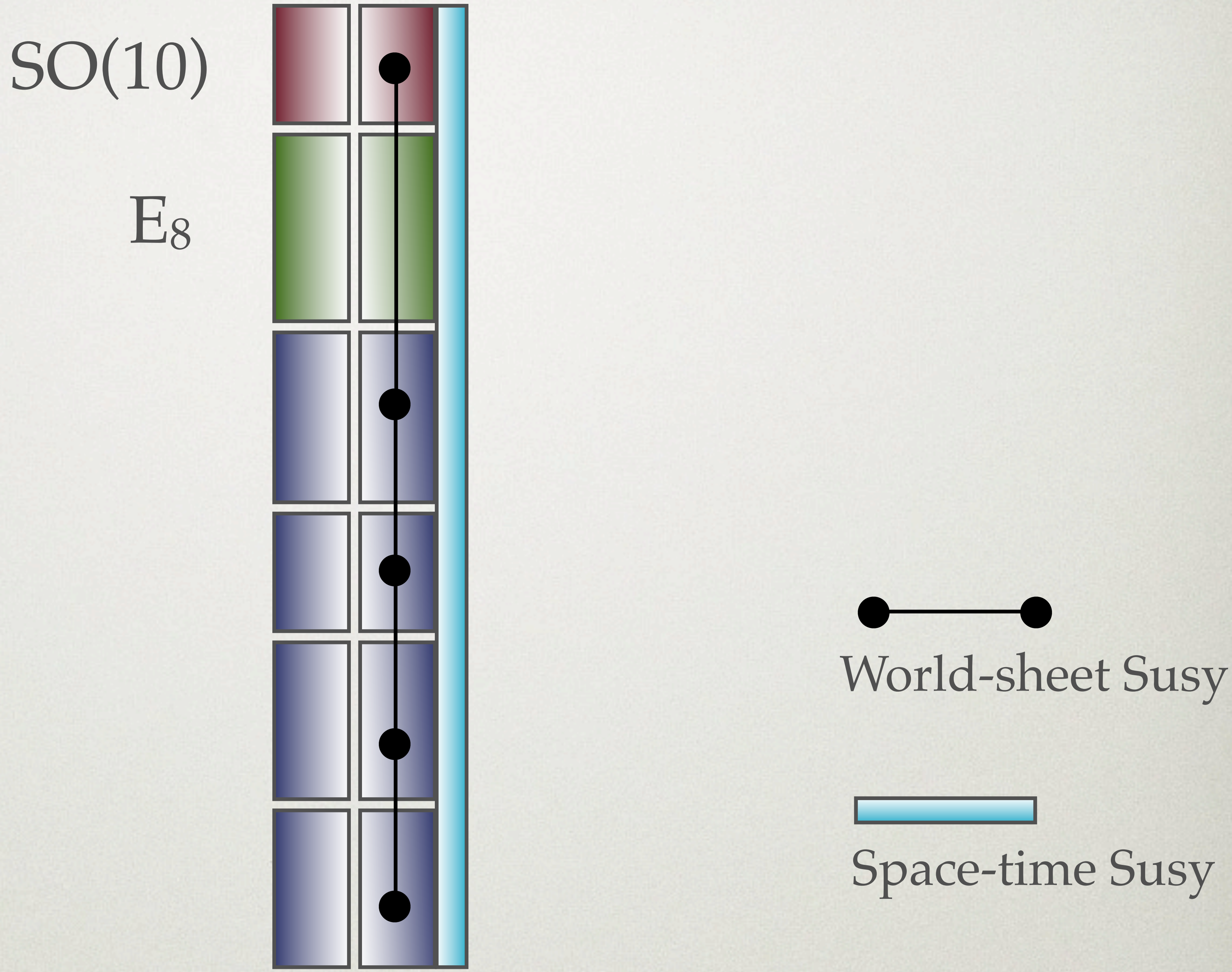
N=2, k4
World-sheet Susy
Space-time Susy

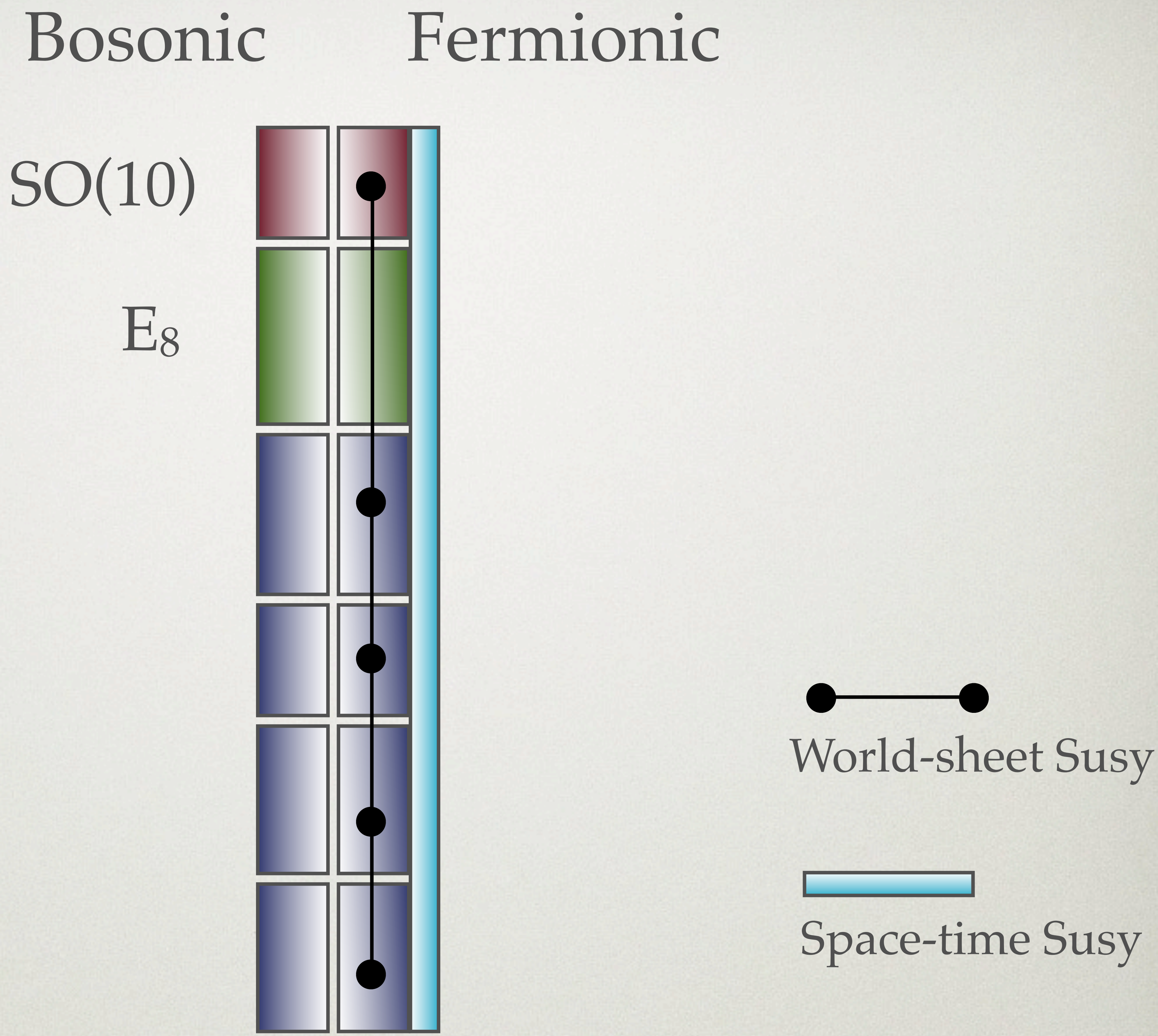
Gepner (1987)



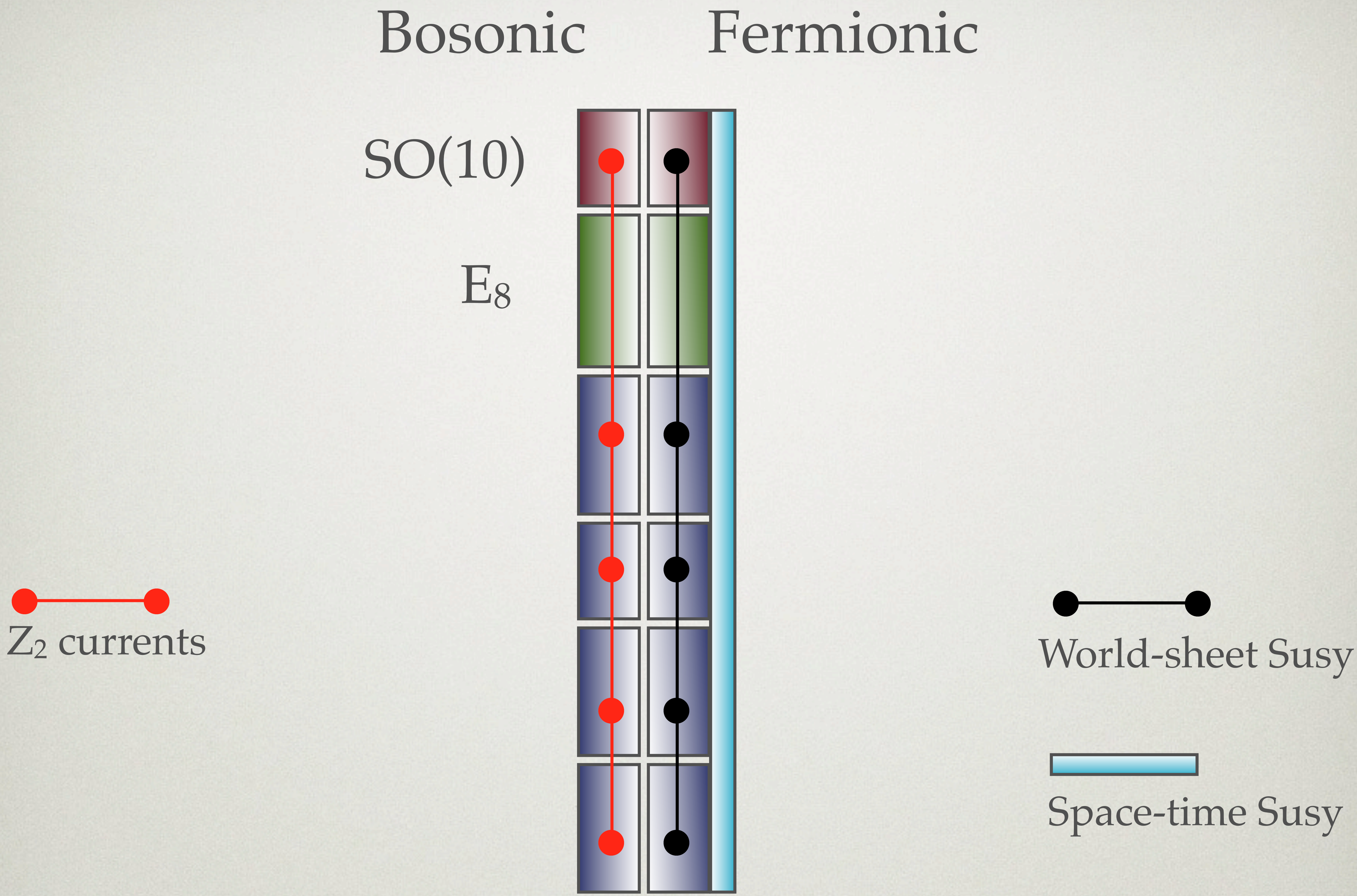

(2,2) model. Gauge group $E_6$ ($\times E_8 \times$ ....)



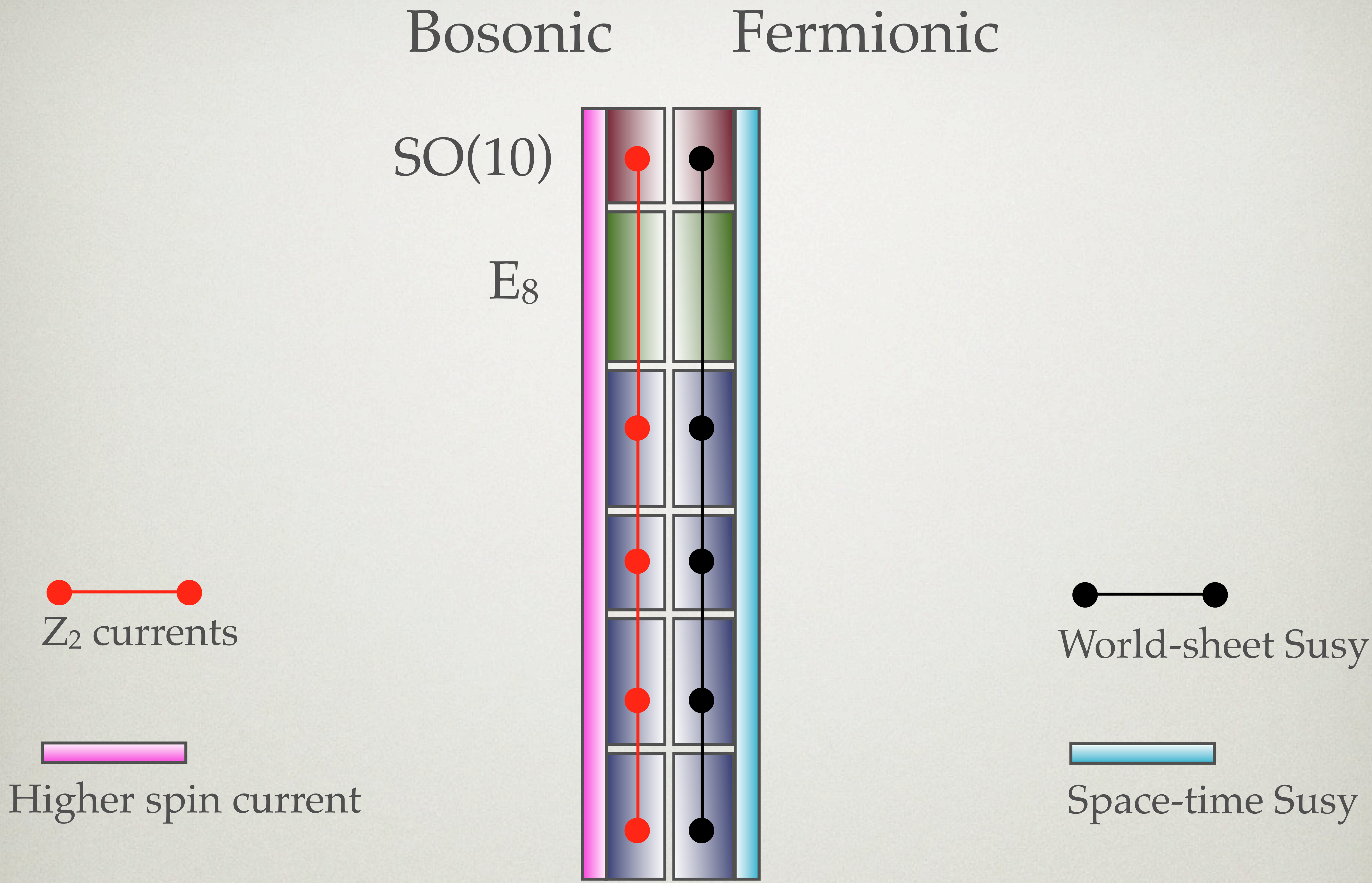
SO(10)
E8
World-sheet Susy
Space-time Susy

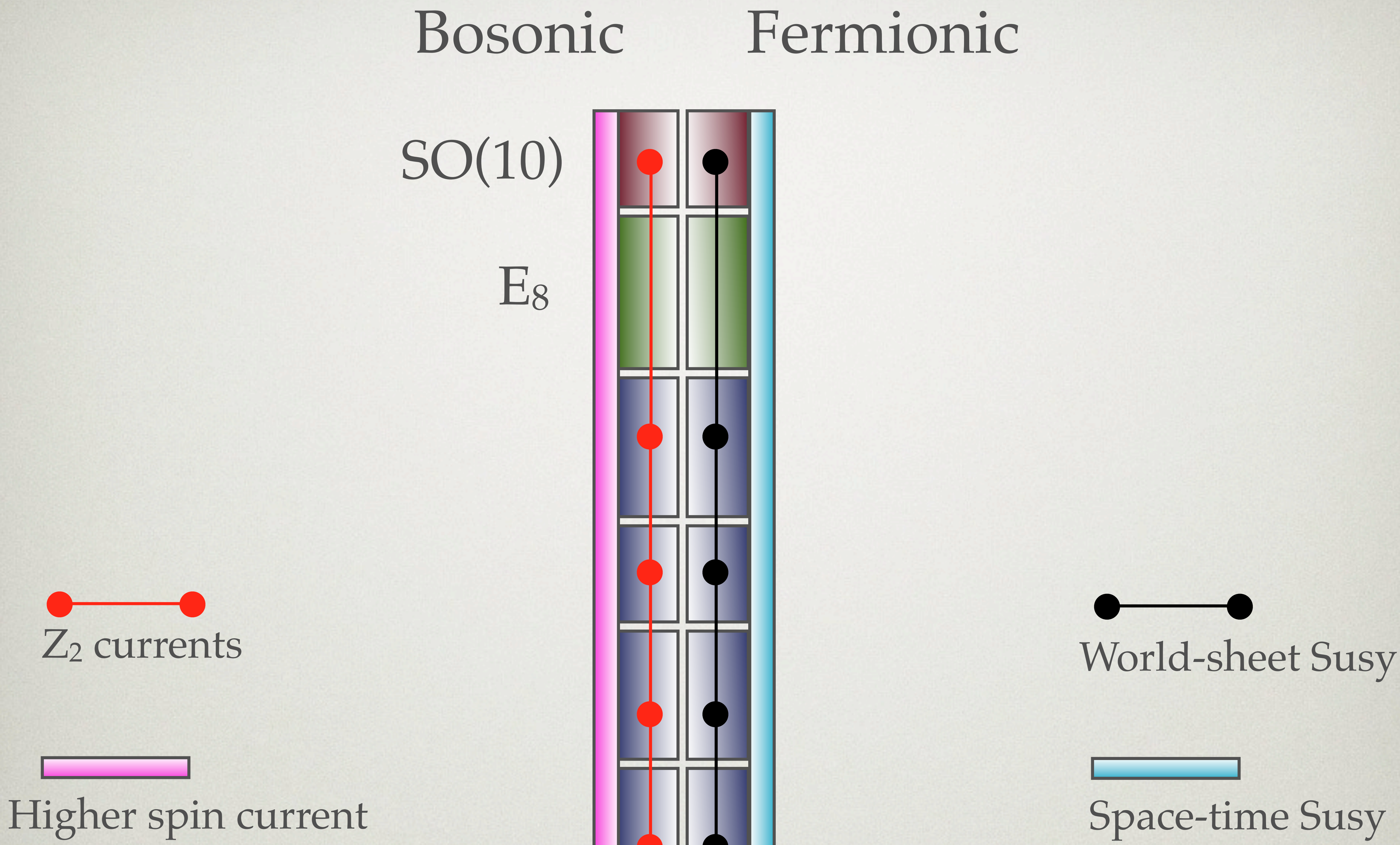


Bosonic
Fermionic
SO(10)
E8
World-sheet Susy
Space-time Susy

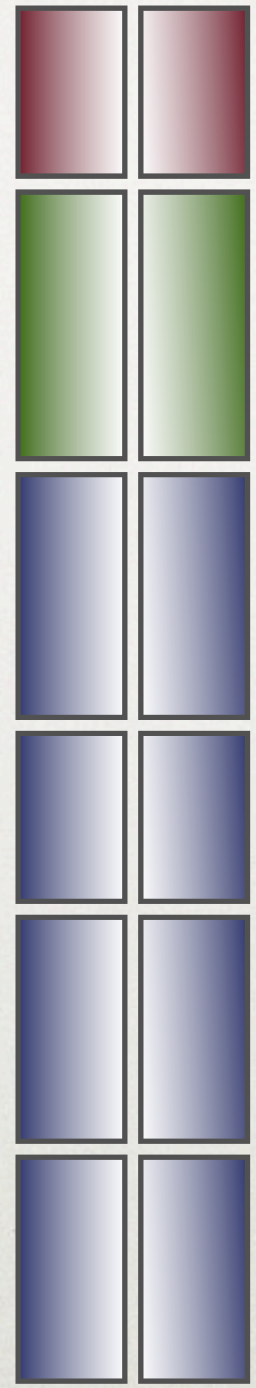


Bosonic
Fermionic
SO(10)
$E_8$
$Z_2$ currents
World-sheet Susy
Space-time Susy

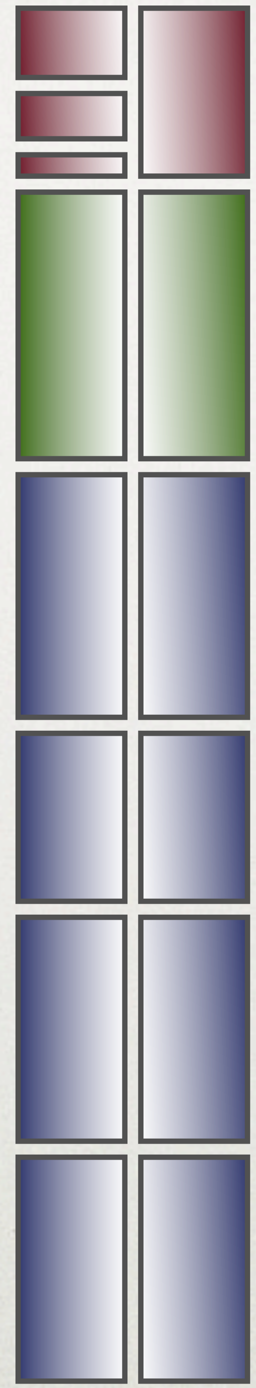

Bosonic
Fermionic
SO(10)
E8
Z2 currents
World-sheet Susy
Higher spin current
Space-time Susy




(0,2) model.  Gauge  group SO(10) (× $E_8$ × ....)



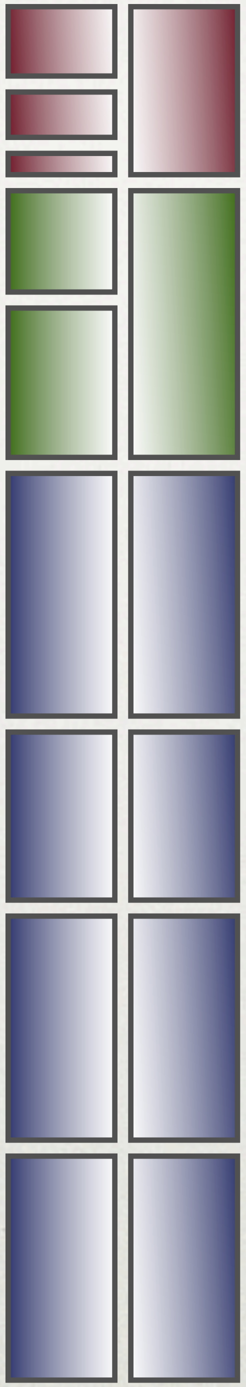
SO(10) currents replaced by operators of higher weight

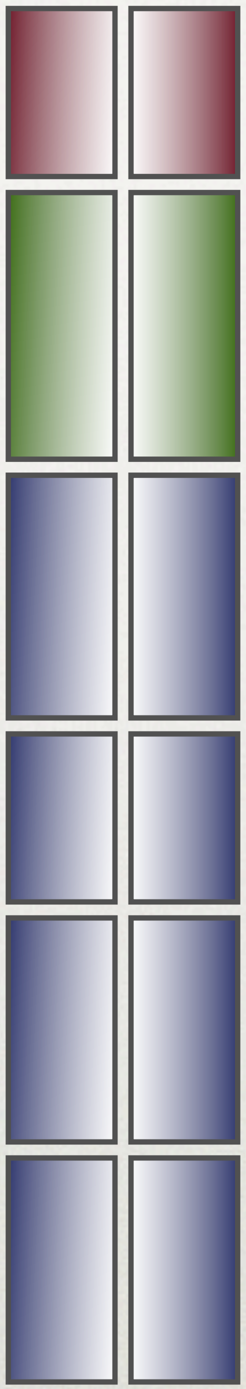


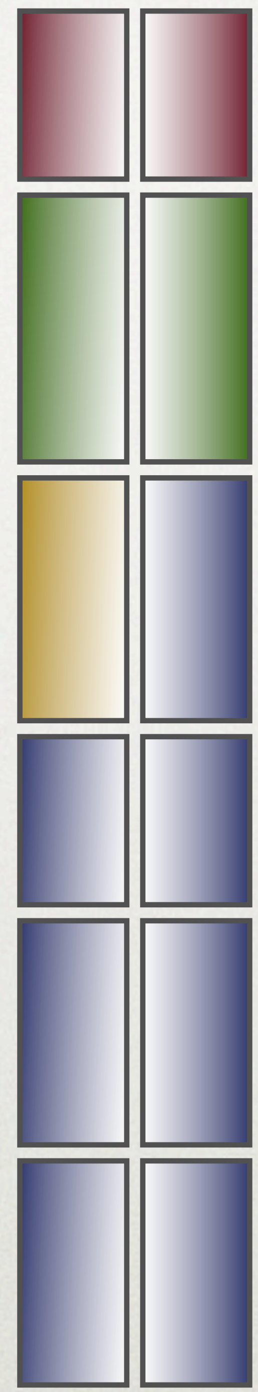
(0,2) model. Gauge group $H \subset SO(10)$ ($\times$ $H' \subset E_8 \times$ ....)

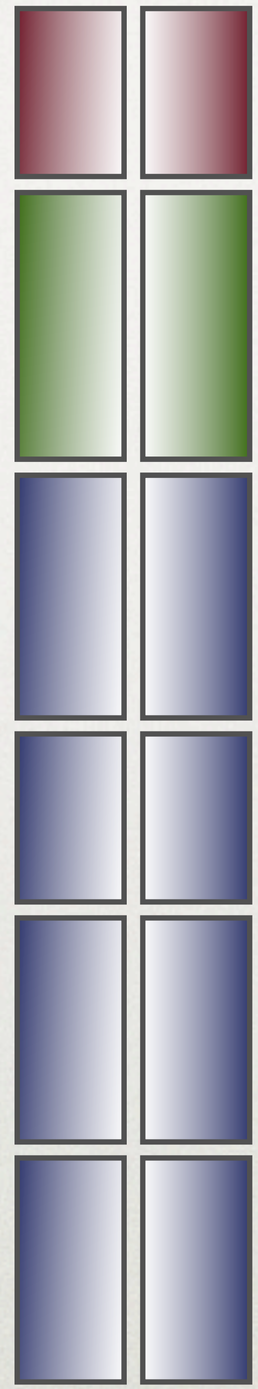
SO(10) currents replaced by operators of higher weight

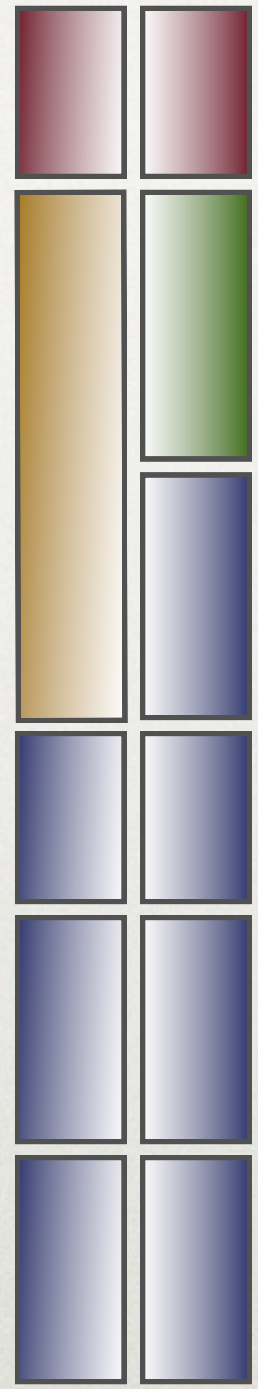
$E_8$ currents replaced by operators of higher weight

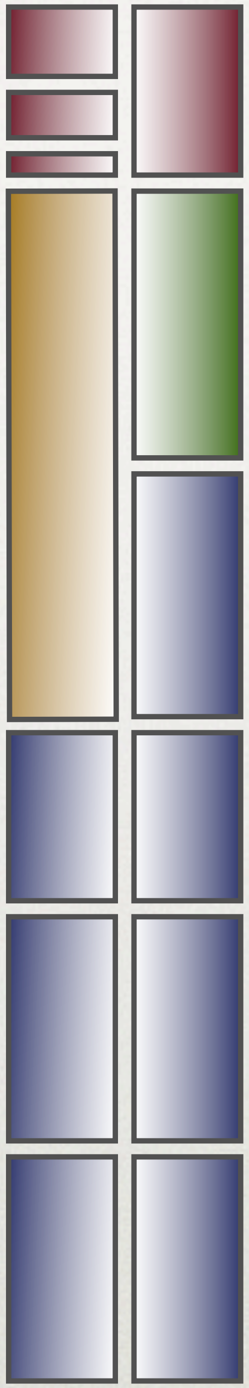


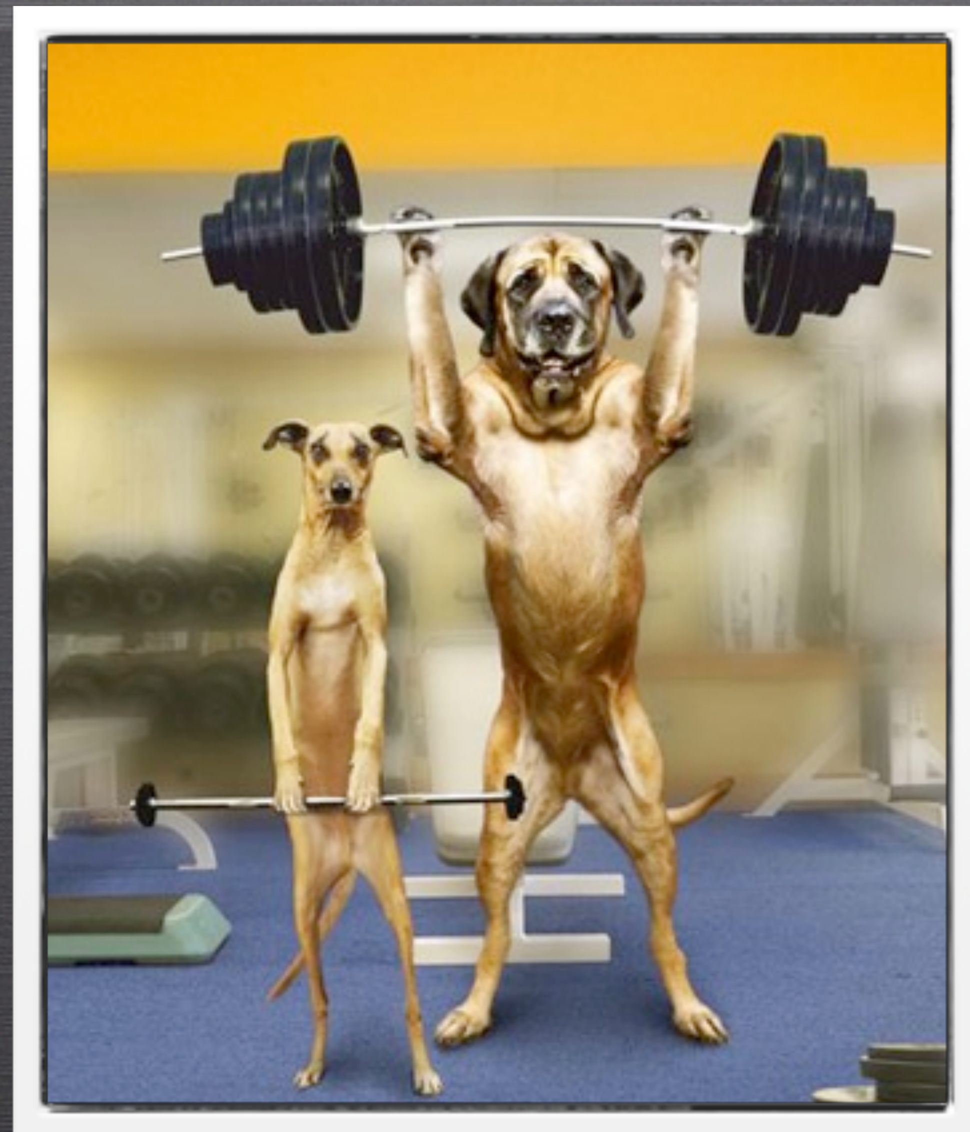












# Heterotic Weight Lifting

Our goal is to find, for some minimal N=2 model with central charge c, a replacement that has central charge c+8, and exactly the same S and T matrices.

Hence it must have the same number of primaries, and the same spectrum, up to integers.

Minimal N=2 model at level k: $c = \frac{3k}{k+2}$

Coset description:

$$\frac{SU(2)_k \times SO(2)}{U_{k+2}}$$

Plus "field identification"

*(Gepner; Schellekens and Yankielowicz, 1989)*

Now we remove the field identification extension, and consider

$$SU(2)_{k+2} \times SO(2) \times \frac{E_8}{U_{k+2}}$$

In other words, we embed the U(1) in $E_8$ instead of SU(2) x SO(2).

Next we identify a CFT $X_7$ which can be combined with $U_{k+2}$ to $E_8$, so that

$$E_8 = [U_{k+2} \times X_7]_{\rm ext}$$

Then we can write the CFT as

$$SU(2)_{k+2} \times SO(2) \times X_7$$

And finally we re-establish the equivalent of the field identification, as a standard, higher spin extension

The result is guaranteed, by construction, to have the same S and T matrices as the original minimal model.

But the spectrum is different

Standard coset field $h_i^G - h_j^H \qquad (j \in i)$

Replacement $h_i^G + h_j^{H^c}$

$$h_j^{H^c} = -h_j^H \mod 1$$

All weight of H and $H^c$ are positive
Therefore standard weights are lifted:

$$h_i^G + h_j^{H^c} > h_i^G - h_J^H$$

(but equal mod 1)

| $k$ | Lift | Lifted | Lowered | Unchanged |
|---|---|---|---|---|
| 1 | $E_6 \times A_1$ | 4 | 1 | 4 |
| 2 | $A_7$ | 7 | 1 | 12 |
| 3 | $[D_6 \times U_{10}]_{\text{ext}}$ | 10 | 3 | 22 |
| 4 | $D_5 \times A_2$ | 21 | 4 | 23 |
| 5 | $A_6 \times A_1$ | 32 | 8 | 29 |
| 5 | $[E_6 \times U_{42}]_{\text{ext}}$ | 24 | 11 | 37 |
| 6 | $[A_6 \times U_{112}]_{\text{ext}}$ | 33 | 15 | 39 |
| 8 | $A_4 \times A_3$ | 65 | 29 | 37 |
| 9 | $[A_6 \times U_{154}]_{\text{ext}}$ | 76 | 41 | 39 |
| 11 | $[E_6 \times U_{78}]_{\text{ext}}$ | 104 | 61 | 39 |
| 11 | $[D_6 \times U_{26}]_{\text{ext}}$ | 98 | 60 | 45 |
| 12 | $A_6 \times U_4$ | 125 | 66 | 39 |
| 13 | $A_4 \times A_2 \times A_1$ | 136 | 81 | 37 |
| 14 | $[A_4 \times A_2 \times U_{480}]_{\text{ext}}$ | 147 | 105 | 47 |
| 14 | $[A_6 \times U_{224}]_{\text{ext}}$ | 153 | 95 | 41 |
| 17 | $[E_6 \times U_{114}]_{\text{ext}}$ | 202 | 105 | 37 |
| 17 | $[A_4 \times A_2 \times U_{570}]_{\text{ext}}$ | 198 | 133 | 41 |
| 19 | $E_6 \times U_{14}$ | 228 | 119 | 42 |
| 20 | $[A_6 \times U_{308}]_{\text{ext}}$ | 243 | 143 | 42 |
| 23 | $[D_6 \times U_{50}]_{\text{ext}}$ | 300 | 161 | 41 |
| 26 | $A_6 \times U_8$ | 349 | 199 | 39 |
| 30 | $[A_6 \times U_{448}]_{\text{ext}}$ | 417 | 235 | 46 |
| 41 | $[E_6 \times U_{258}]_{\text{ext}}$ | 610 | 297 | 44 |
| 41 | $[A_6 \times U_{602}]_{\text{ext}}$ | 606 | 325 | 48 |
| 42 | $[A_6 \times U_{616}]_{\text{ext}}$ | 627 | 337 | 46 |
| 44 | $[A_6 \times U_{644}]_{\text{ext}}$ | 673 | 361 | 42 |
| 44 | $[A_4 \times A_2 \times U_{1380}]_{\text{ext}}$ | 659 | 465 | 56 |
| 47 | $[E_6 \times U_{294}]_{\text{ext}}$ | 728 | 367 | 46 |
| 54 | $A_6 \times U_{16}$ | 857 | 455 | 51 |
| 58 | $A_4 \times A_2 \times U_8$ | 923 | 611 | 56 |
| 86 | $[A_6 \times U_{1232}]_{\text{ext}}$ | 1501 | 741 | 52 |
| 89 | $[E_6 \times U_{546}]_{\text{ext}}$ | 1556 | 705 | 49 |
| 238 | $A_4 \times A_2 \times U_{32}$ | 4959 | 2729 | 73 |
| 1,1 | $A_2 \times A_1 \times A_2 \times A_1$ | 16 | 1 | 14 |

# Computing the spectrum

Very easy: start with the **full** spectrum of a standard Gepner model.
For example, all states associated with a massless space-time spinor in the fermionic sector

$$\sum_j M_{ij}(\mathrm{dim}_1, h_1, \ldots, \mathrm{dim}_n, h_n)_j$$

To compute the consequences of "lifting" factor k, just replace $\mathrm{dim}_k$ and $h_k$ by the corresponding values in the lift CFT.

Powerful check: anomaly factorization $\propto (\mathrm{Tr}\ F^2 - \mathrm{Tr}\ R^2)$

*(Schellekens, Warner, 1986)*

# Chiral spectra?

All R ground states are lifted.
Hence no extension SO(10) → $E_6$
But also all chiral families are removed.

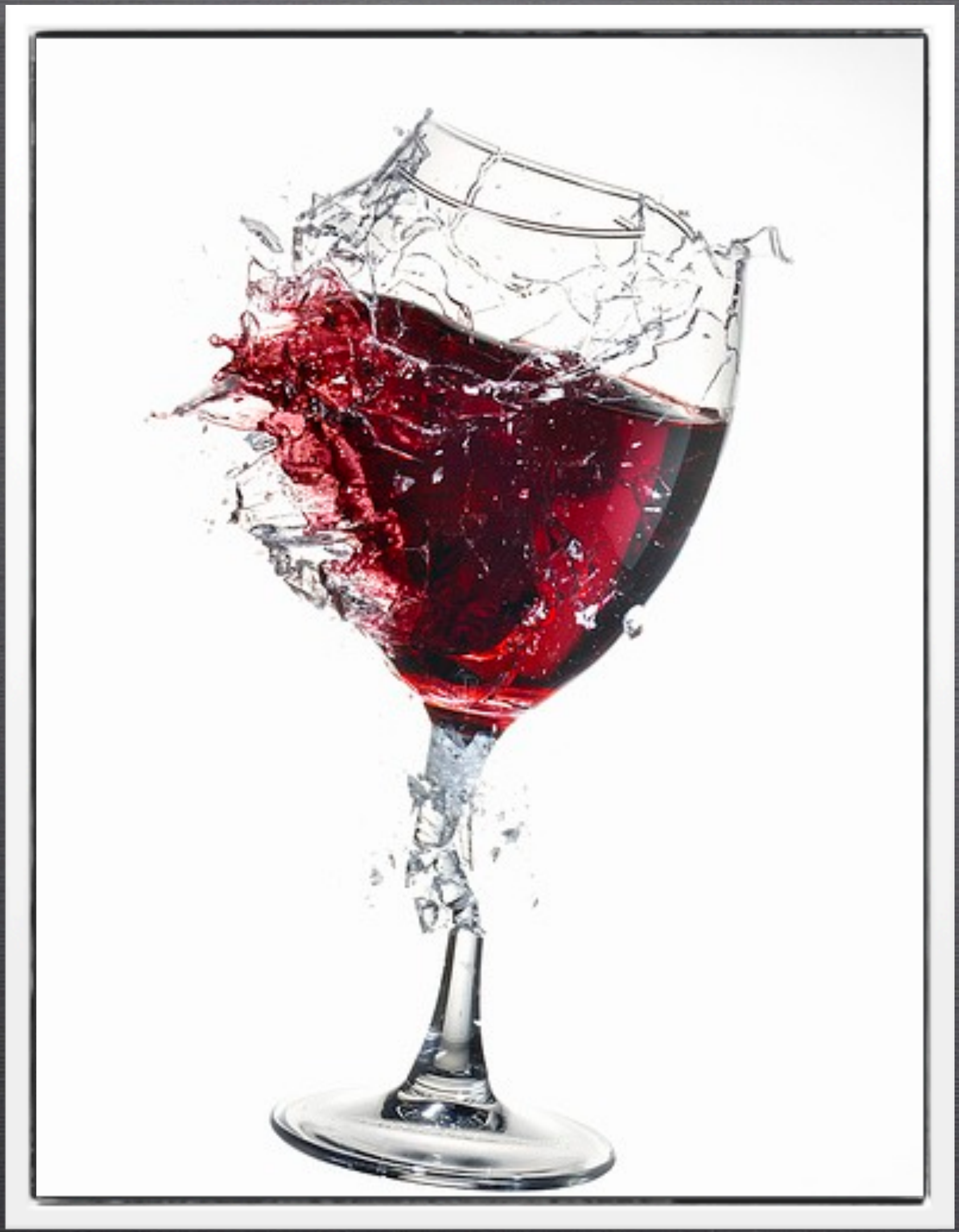
The diagonal MIPF yields, for $(4,4,8,\widehat{13})$

Before lifting:

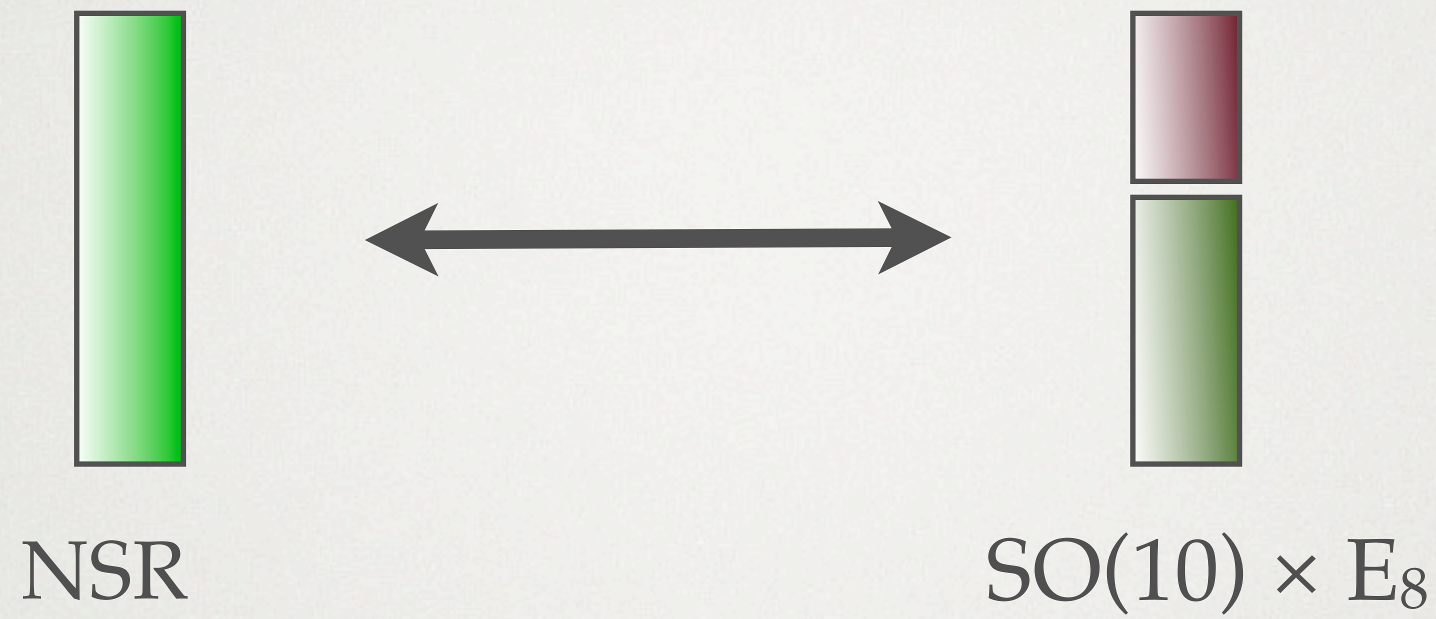
$$75(27) + 3(\overline{27}) + 450(1) \quad \text{of } E_6$$

After lifting:

$$20 \times (10) + 1088(1) \text{ of } SO(10)$$



# SO(10) Breaking




In “classic” heterotic strings SO(10) emerges naturally...

...but without a Higgs to break it to the Standard model.

If we try to do the breaking in string theory, one of the nicest features of GUTs is lost: an elegant explanation for the absence of color singlets with fractional electric charge.

# Standard Model Embedding

Consider* $SU(3) \times SU(2) \times U(1)_{30} \times U(1)_{20} \subset SO(10)$

This should give chiral families of $SU(3) \times SU(2) \times U(1)$
with standard gauge coupling unification.

Indeed, it does, but there is a major disappointment:
**All these spectra contain fractionally charged particles.**

This was easily seen to be a very general result.
*(A.N. Schellekens, Phys. Lett. B237, 363, 1990; see also Wen and Witten, Nucl. Phys. B261, 651, 1985).*

But there are ways out: they can be massive, vector-like
(or confined by another gauge group)

*(*) A.N. Schellekens and S. Yankielowicz (1989)*
*Other subgroups were considered by Blumenhagen, Wisskirchen, Schimmrigk (1995, 1996)*

# SO(10) sub-algebras

| Nr. | Name | Current | Order | Gauge group | Grp. | CFT |
|---|---|---|---|---|---|---|
| 0 | SM, Q=1/6 | $(1,1,0,0)$ | 1 | $SU(3)\times SU(2)\times U(1)\times U(1)$ | $\frac{1}{6}$ | $\frac{1}{6}$ |
| 1 | SM, Q=1/3 | $(1,2,15,0)$ | 2 | $SU(3)\times SU(2)\times U(1)\times U(1)$ | $\frac{1}{6}$ | $\frac{1}{3}$ |
| 2 | SM, Q=1/2 | $(3,1,10,0)$ | 3 | $SU(3)\times SU(2)\times U(1)\times U(1)$ | $\frac{1}{6}$ | $\frac{1}{2}$ |
| 3 | LR, Q=1/6 | $(1,1,6,4)$ | 5 | $SU(3)\times SU(2)_L\times SU(2)_R\times U(1)$ | $\frac{1}{6}$ | $\frac{1}{6}$ |
| 4 | SU(5) GUT | $(\bar{3},2,5,0)$ | 6 | $SU(5)\times U(1)$ | 1 | 1 |
| 5 | LR, Q=1/3 | $(1,2,3,-8)$ | 10 | $SU(3)\times SU(2)_L\times SU(2)_R\times U(1)$ | $\frac{1}{6}$ | $\frac{1}{3}$ |
| 6 | Pati-Salam | $(\bar{3},0,2,8)$ | 15 | $SU(4)\times SU(2)_L\times SU(2)_R$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 7 | SO(10) GUT | $(3,2,1,4)$ | 30 | $SO(10)$ | 1 | 1 |

# SO(10) SUB-ALGEBRAS

| Nr. | Name | Current | Order | Gauge group | Grp. | CFT |
|---|---|---|---|---|---|---|
| | SM, Q=1/6 | $(1,1,0,0)$ | 1 | $SU(3)\times SU(2)\times U(1)\times U(1)$ | $\frac{1}{6}$ | $\frac{1}{6}$ |
| | SM, Q=1/3 | $(1,2,15,0)$ | 2 | $SU(3)\times SU(2)\times U(1)\times U(1)$ | $\frac{1}{6}$ | $\frac{1}{3}$ |
| | SM, Q=1/2 | $(3,1,10,0)$ | 3 | $SU(3)\times SU(2)\times U(1)\times U(1)$ | $\frac{1}{6}$ | $\frac{1}{2}$ |
| | LR, Q=1/6 | $(1,1,6,4)$ | 5 | $SU(3)\times SU(2)_L\times SU(2)_R\times U(1)$ | $\frac{1}{6}$ | $\frac{1}{6}$ |
| | SU(5) GUT | $(\bar{3},2,5,0)$ | 6 | $SU(5)\times U(1)$ | 1 | 1 |
| | LR, Q=1/3 | $(1,2,3,-8)$ | 10 | $SU(3)\times SU(2)_L\times SU(2)_R\times U(1)$ | $\frac{1}{6}$ | $\frac{1}{3}$ |
| | Pati-Salam | $(\bar{3},0,2,8)$ | 15 | $SU(4)\times SU(2)_L\times SU(2)_R$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| | SO(10) GUT | $(3,2,1,4)$ | 30 | $SO(10)$ | 1 | 1 |

# RESULTS

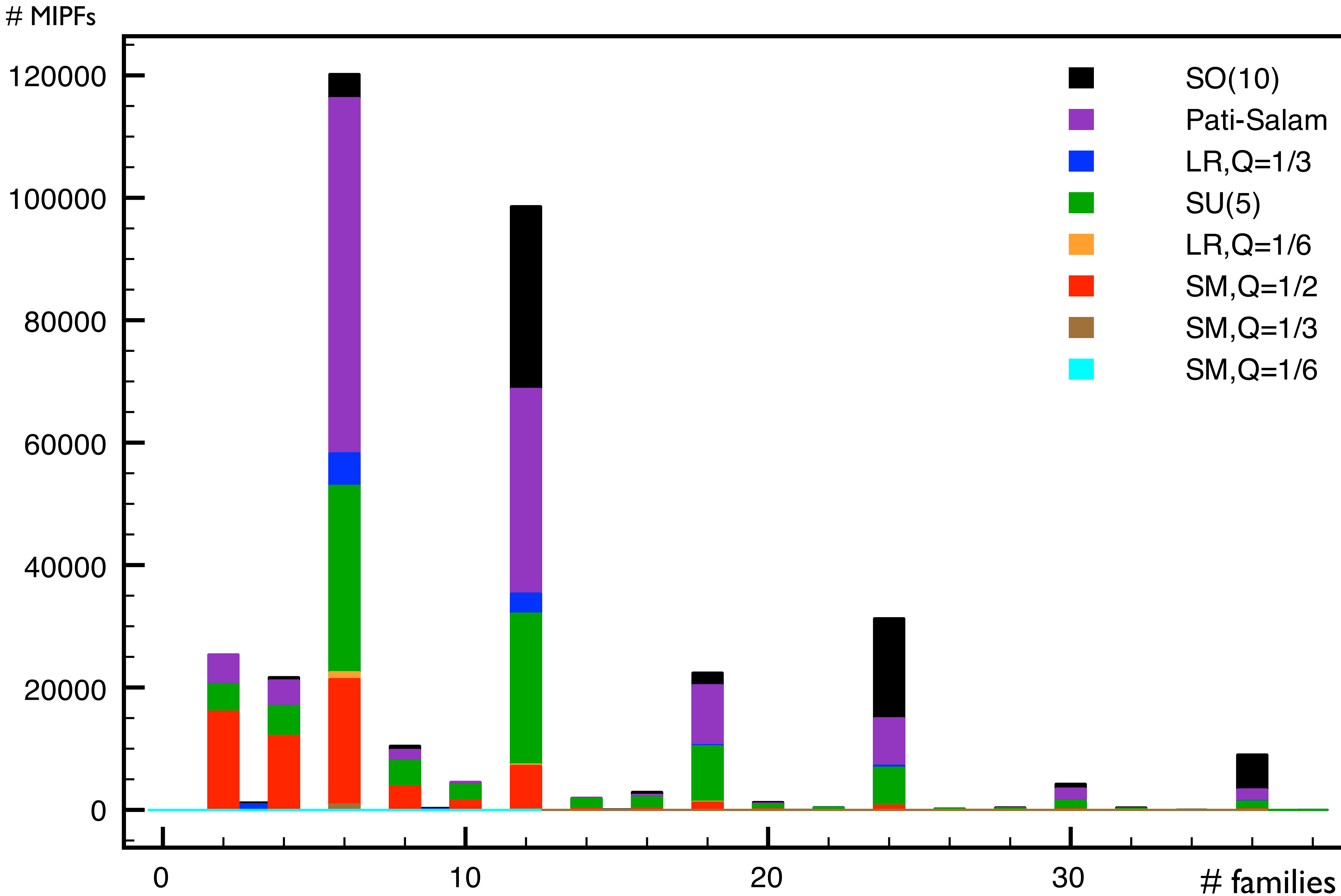
- Spectra distinguished on the basis of:

  Group type (0...7)
  Observable gauge group
  Size of hidden sector gauge group
  Nr. of families
  Nr. of mirrors (Q,U,D,L,E)
  Nr. of SM singlets
  Total nr. of fractional charge exotics

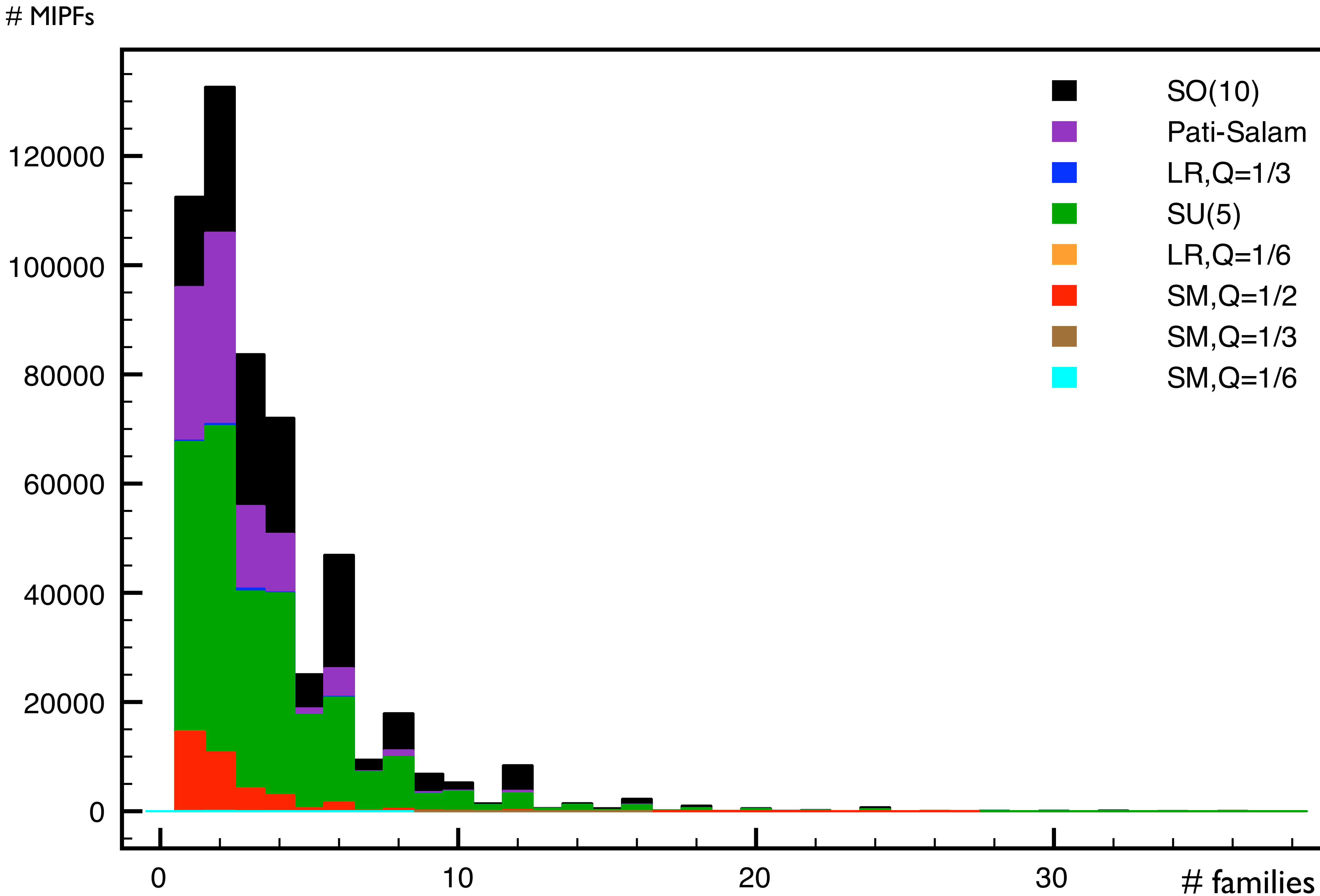
- Rejected:

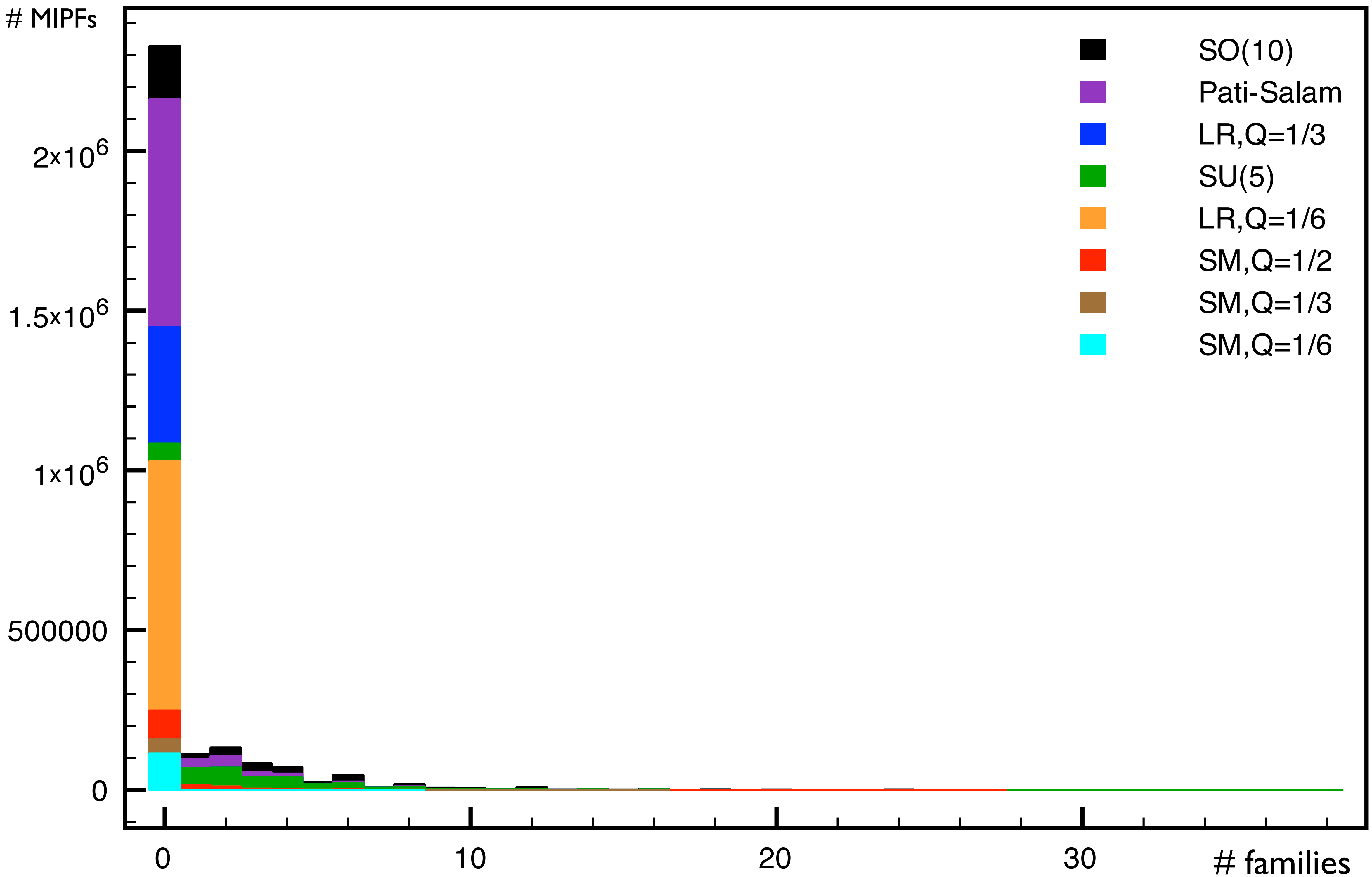
  Chiral exotics
  Anomalous Y-charge (possible for lifted case only)

# Standard Gepner



# Lifted Gepner





# MIPFs
2x10^6
1.5x10^6
1x10^6
500000
0
0
10
20
30
# families
SO(10)
Pati-Salam
LR,Q=1/3
SU(5)
LR,Q=1/6
SM,Q=1/2
SM,Q=1/3
SM,Q=1/6

# An example from $(3, \widehat{8}, 8, 8)$

## Gauge group:

$$SU(3) \times SU(2) \times U(1) \times [SU(2)_8 \times SO(2) \times SU(4) \times SU(5)] \times U(1)^3$$

(anomalous "B-L")

## Spectrum:

$$3 \times (Q + U^c + D^c + L + E^c) + 3 \times (D + D^c) + 3 \times (H_1 + H_2)$$

$$+ \text{ 250 singlets}$$

$$+ \text{ 172 fractionally charged particles}$$

**Fractional charges:**

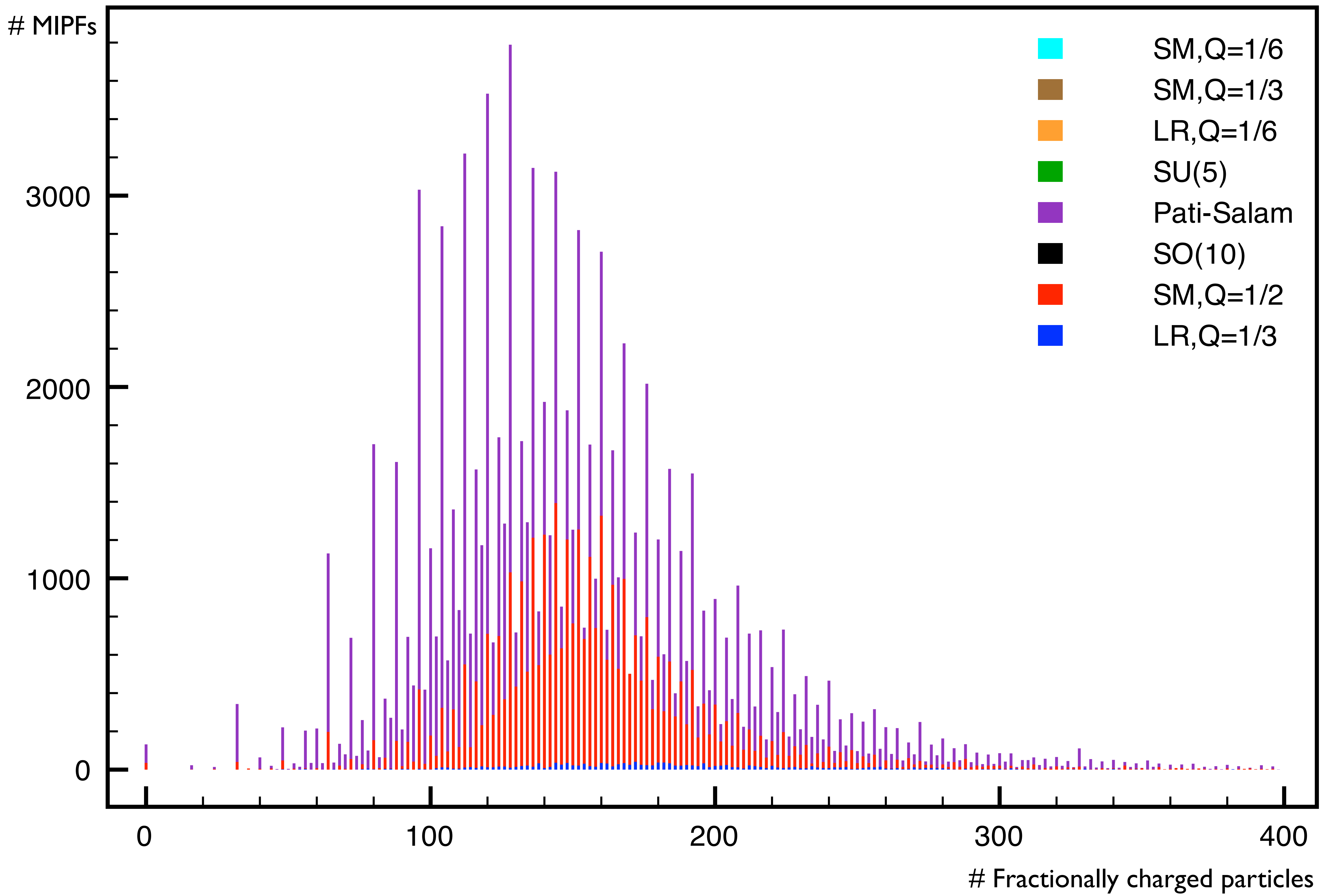
Non-chiral.
Only half-integer (no sixth or third).
Confined by $SU(2)_8$

**Singlets**: (of $SU(3) \times SU(2) \times U(1)$)

Only three are absolute singlets of the full gauge group.
Many are in nontrivial SU(4) and SU(5) reps.

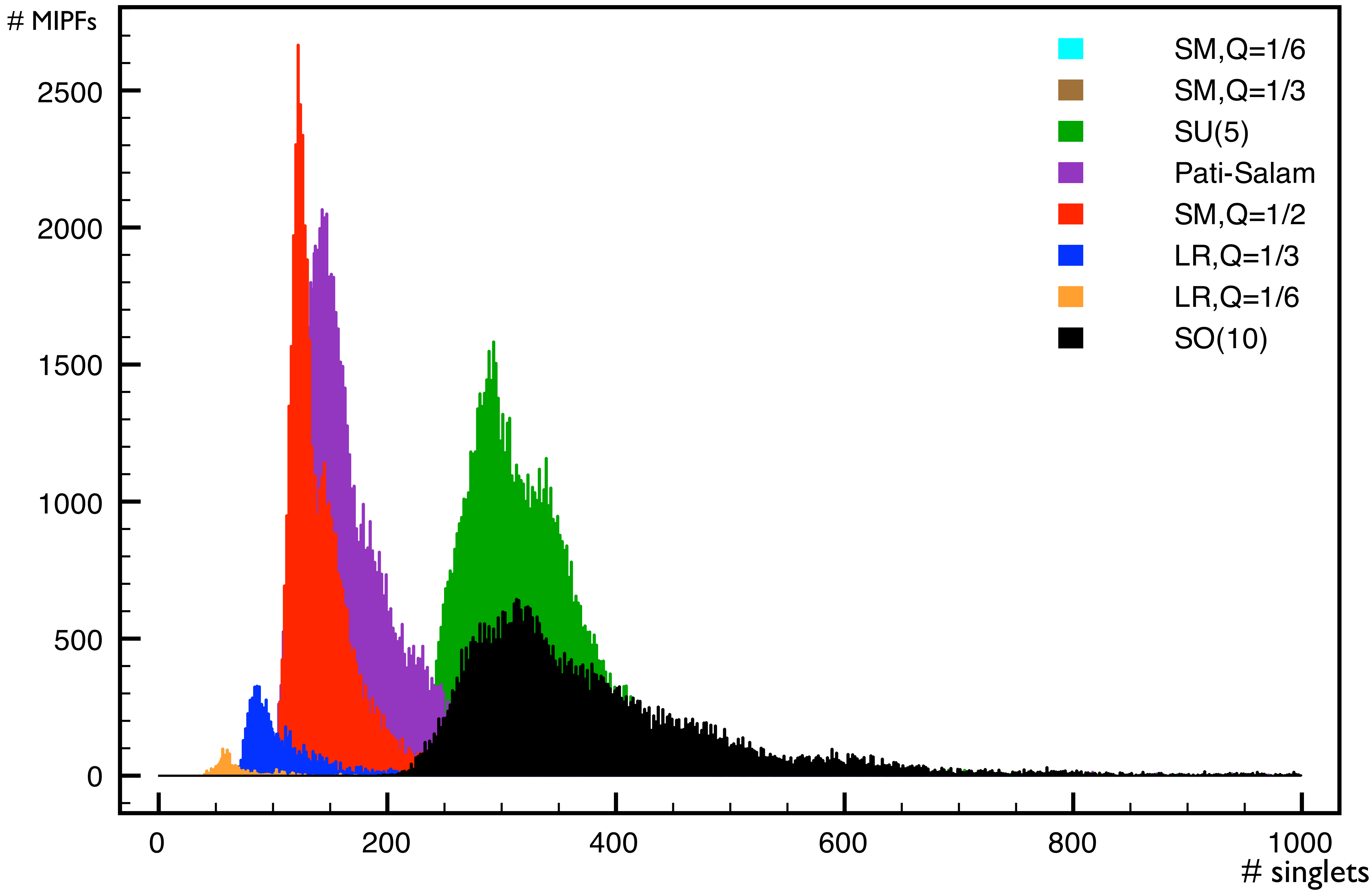
**The bad news:**

$U^c$, $D^c$ and $E^c$ are in the triplet representation of $SU(2)_8$;
Higgs candidates and weak doublets are $SU(2)_8$ singlets.

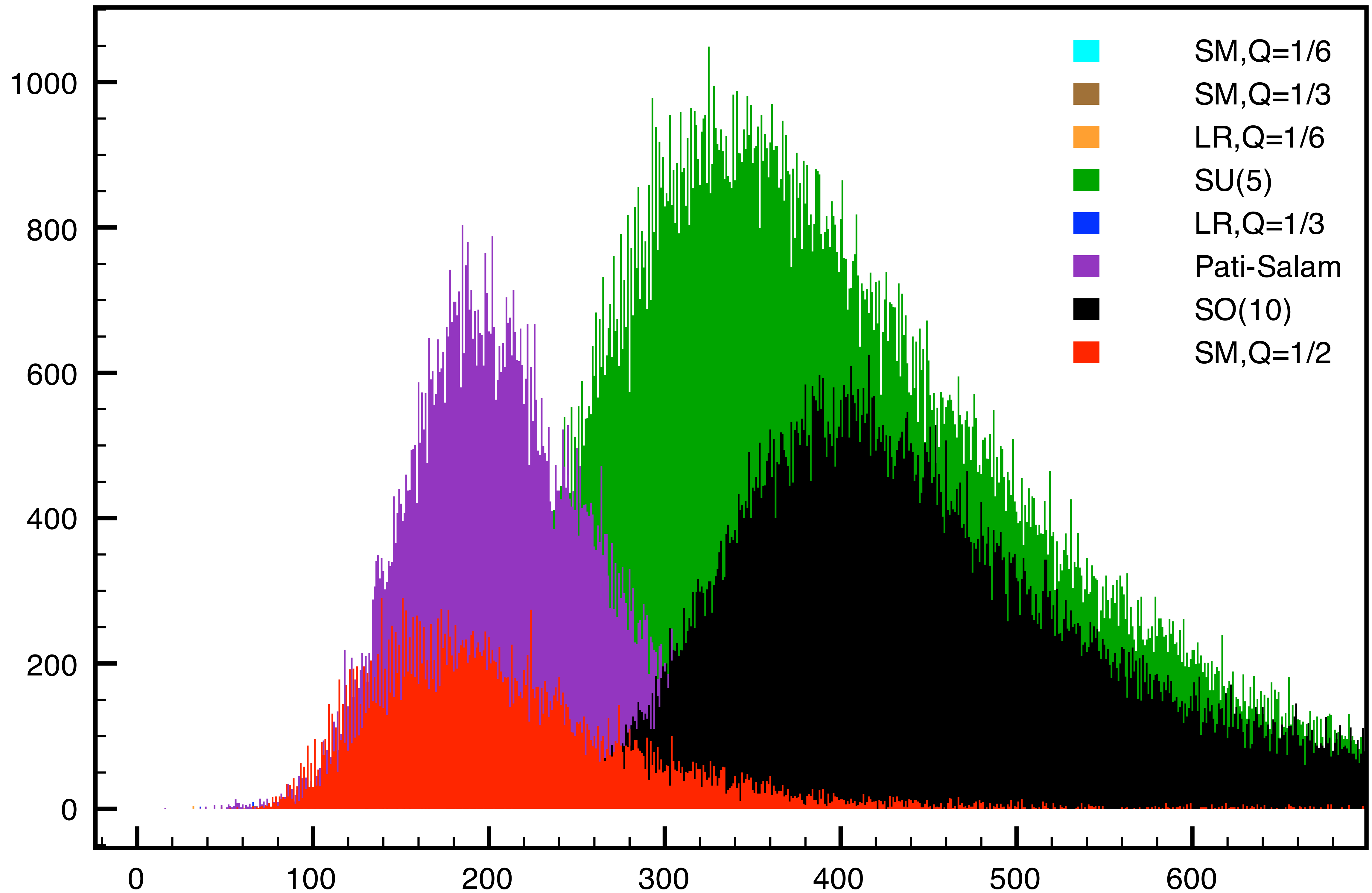


# MIPFs
3000
2000
1000
0
0
100
200
300
400
# Fractionally charged particles
SM,Q=1/6
SM,Q=1/3
LR,Q=1/6
SU(5)
Pati-Salam
SO(10)
SM,Q=1/2
LR,Q=1/3

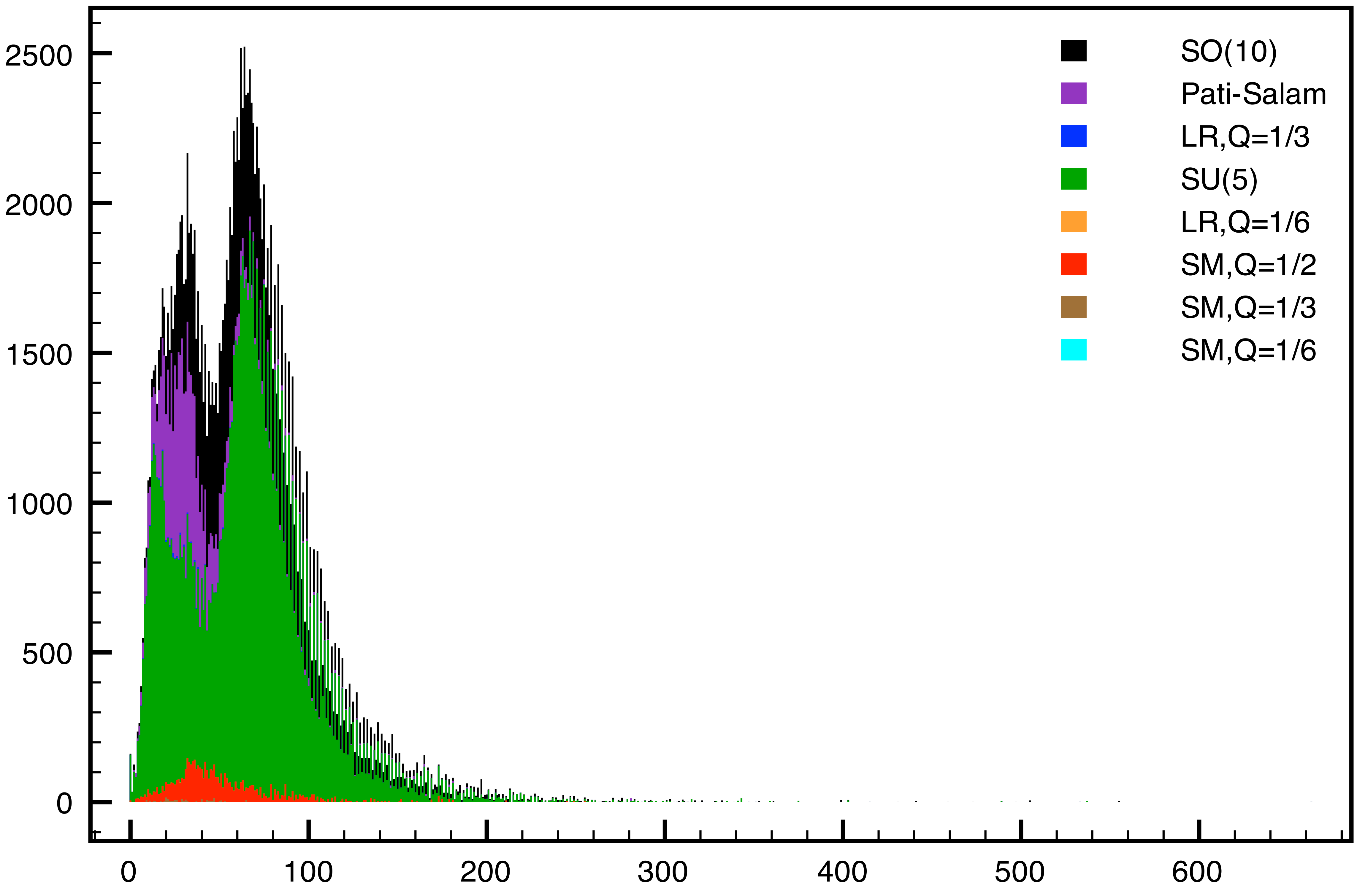
# Singlets (Standard Gepner)

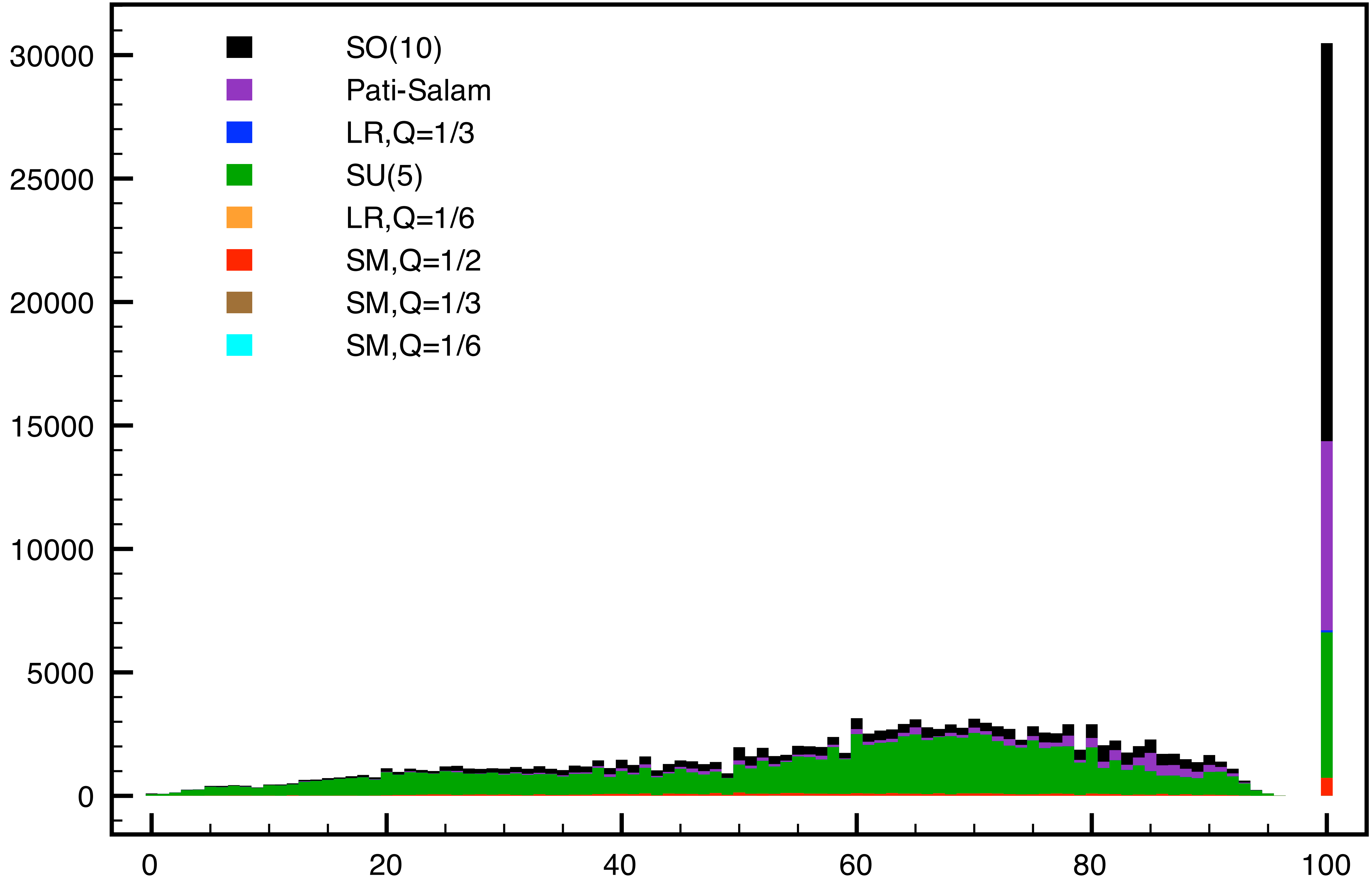


# Singlets (Lifted Gepner)



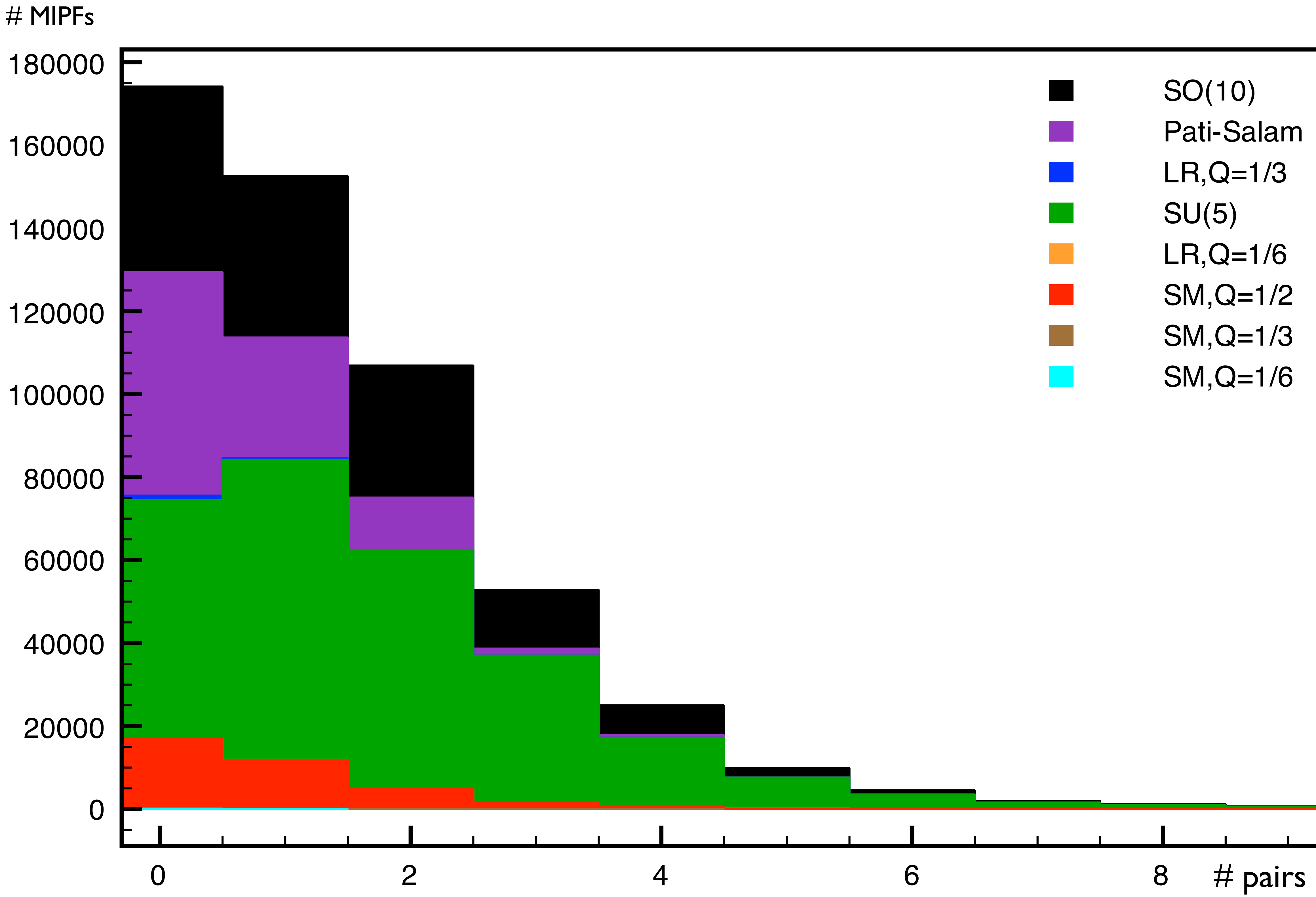
# Abelian singlets (Lifted Gepner)





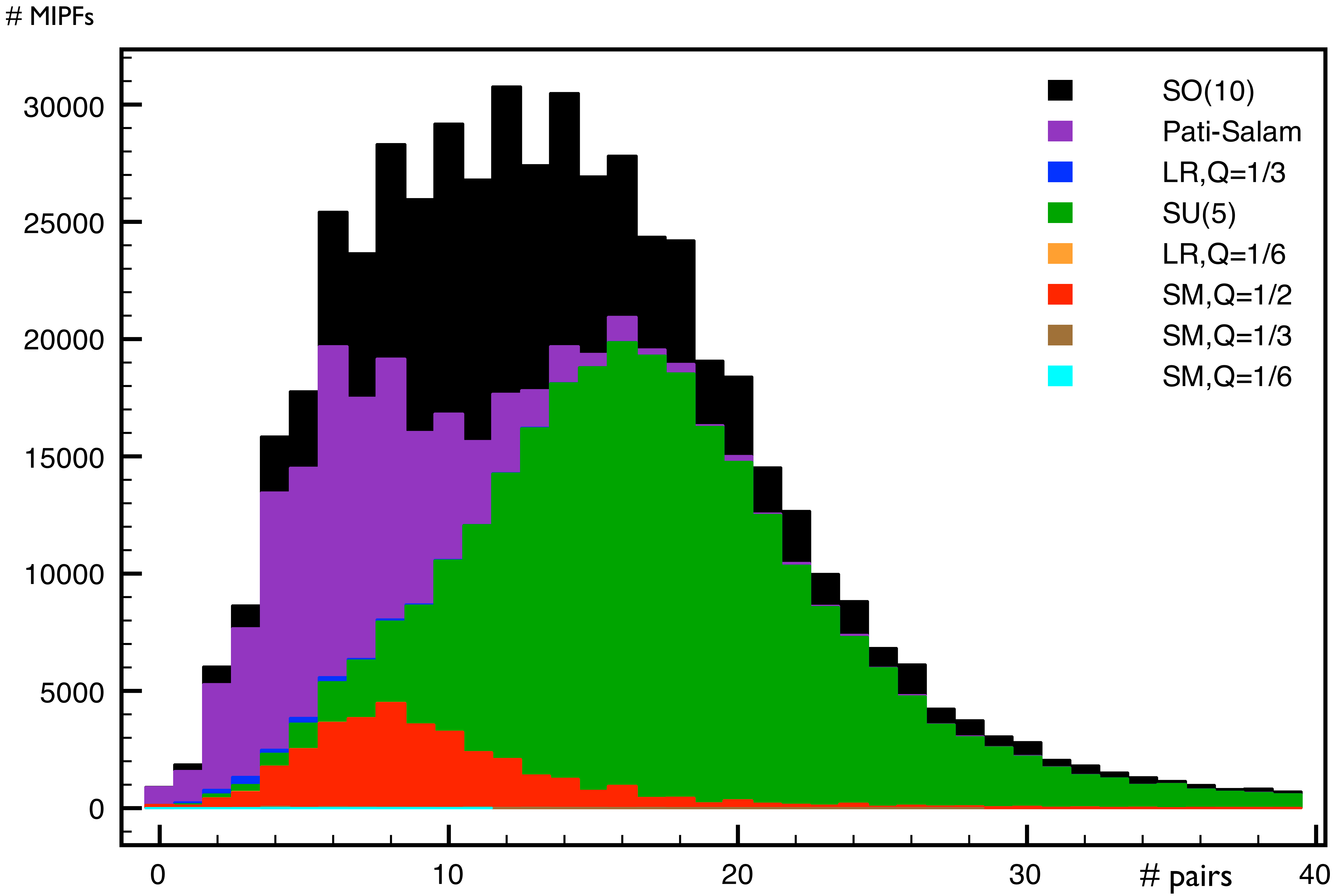
SO(10)
Pati-Salam
LR,Q=1/3
SU(5)
LR,Q=1/6
SM,Q=1/2
SM,Q=1/3
SM,Q=1/6
30000
25000
20000
15000
10000
5000
0
0
20
40
60
80
100

# Q-mirrors (~ E, U)



# MIPFs
180000
160000
140000
120000
100000
80000
60000
40000
20000
0
0
2
4
6
8
# pairs
SO(10)
Pati-Salam
LR,Q=1/3
SU(5)
LR,Q=1/6
SM,Q=1/2
SM,Q=1/3
SM,Q=1/6

# L Mirrors (~ D)


# MIPFs
30000
25000
20000
15000
10000
5000
0
0
10
20
30
40
# pairs
SO(10)
Pati-Salam
LR,Q=1/3
SU(5)
LR,Q=1/6
SM,Q=1/2
SM,Q=1/3
SM,Q=1/6

# Conclusions

- Away from the (2,2) lamppost, three families occur abundantly.
- Lots of directions still unexplored.
- Hundreds of thousands of three-family models.
- Hundreds of novel heterotic "mini-landscapes".
- Absence of fractional charges nearly impossible in RCFT(*).
  Just vector-like fractional charges very common.
  ((*) some examples with even nr. of chiral families)
  [See also *Blaszczyk, Groot-Nibbelink, Ratz, Ruehle, Trapletti, Vaudrevange*;
  *Assel, Christodoulides, Faraggi, Kounnas, Rizos*.]
- Explicit examples with combinations of good features, such as:
  - Three families, no mirrors, one Higgs, just the SM gauge group (plus hidden sector), vector-like half-integer charged exotics.
  - Three families, no gauge singlets (all SM singlets couple to non-abelian gauge group)