Supersymmetric Standard Model Spectra
from
Orientifolds of Gepner Models

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• Features searched for

— Supersymmetric
— Tadpole-Free
— Unbroken Chan-Paton group

$$SU(3) \times SU(2) \times U(1) \times \text{“hidden”}$$

— Three chiral standard model families
— Nothing else that is chiral

• Rational CFTs[1] searched:

Orientifolds of simple current MIPFs[2] of four-dimensional type-IIB tensor products of N=2 minimal models (“Gepner models”)

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[1] Conformal Field Theory
[2] Modular Invariant Partition Function

- Milestones

Angelantonj, Bianchi, Pradisi, Sagnotti, Stanev (1996)
Chiral spectra from Orbifold-Orientifolds

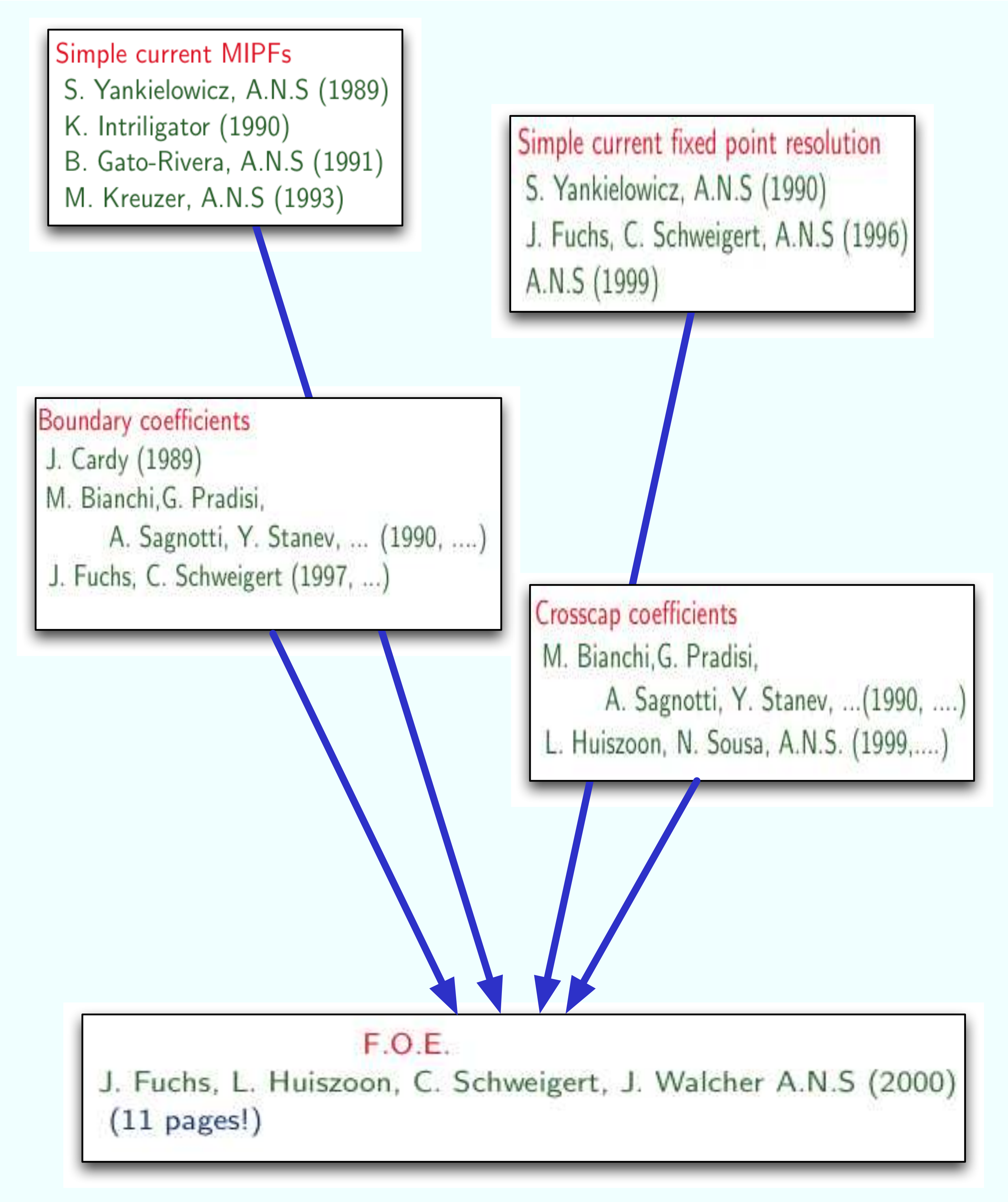
Blumenhagen, Wisskirchen (1998)
Gepner Orientifolds

Aldazabal, Franco, Ibanez, Rabadan, Uranga (2000)
Blumenhagen, Görlich,Körs,Lüst (2000)
Ibanez, Marchesano, Rabadan (2001)
Non-supersymmetric SM-Spectra with RR tadpole cancellation

Cvetic, Shiu, Uranga (2001)
Cvetic, Papadimitriou (2003)
Supersymmetric SM-Spectra with chiral exotics

Blumenhagen, Görlic, Ott (2002)
Honecker (2003)
Supersymmetric Pati-Salam Spectra with brane recombination

Brunner, Hori, Hosomichi, Walcher (2004)
Chiral spectrum from Gepner Orientifolds



Simple current MIPFs
S. Yankielowicz, A.N.S (1989)
K. Intriligator (1990)
B. Gato-Rivera, A.N.S (1991)
M. Kreuzer, A.N.S (1993)
Simple current fixed point resolution
S. Yankielowicz, A.N.S (1990)
J. Fuchs, C. Schweigert, A.N.S (1996)
A.N.S (1999)
Boundary coefficients
J. Cardy (1989)
M. Bianchi, G. Pradisi,
A. Sagnotti, Y. Stanev, ... (1990, ...)
J. Fuchs, C. Schweigert (1997, ...)
Crosscap coefficients
M. Bianchi, G. Pradisi,
A. Sagnotti, Y. Stanev, ...(1990, ...)
L. Huiszoon, N. Sousa, A.N.S. (1999,....)
F.O.E.
J. Fuchs, L. Huiszoon, C. Schweigert, J. Walcher A.N.S (2000)
(11 pages!)

- Simple currents

Fusion rules for primaries ϕ_i and ϕ_j:

$$\phi_i \cdot \phi_j = N_{ij}{}^k \phi_k$$

The primary J is a simple current if, for all a

$$J \cdot a = b$$

The primary a is a fixed point of J if

$$J \cdot a = a$$

Simple current MIPFs are specified by

- A group $\mathcal{H}$ that consists of simple currents[3]
  $\mathcal{H} = \prod_\alpha \mathbb{Z}_{N_\alpha}$.
  The generator of the $\mathbb{Z}_{N_\alpha}$ will be denoted as J_α;
  Then $J = \prod_\alpha J_\alpha^{n_\alpha}$

- A symmetric matrix $X_{\alpha\beta}$ that obeys

$$
\begin{aligned}
2X_{\alpha\beta} &= Q_{J_\alpha}(J_\beta) \text{ mod } 1, \alpha \neq \beta \\
X_{\alpha\alpha} &= -h_{J_\alpha} \\
N_\alpha X_{\alpha\beta} &\in \mathbb{Z} \text{ for all } \alpha, \beta
\end{aligned}
$$

  Here $Q_J(a) = h(a) + h(J) - h(Ja)$, h is the conformal weight.

Then Z_{ij} is the number of currents $L \in \mathcal{H}$ such that

$$
\begin{aligned}
j &= Li \\
Q_M(i) + X(M, L) &= 0 \text{ mod } 1
\end{aligned}
$$

for all $M \in \mathcal{H}$. ($X(J, J') = \prod_{\alpha,\beta} n_\alpha m_\beta X_{\alpha\beta}$)

[3] Satisfying Order x Weight = Integer

Orientifold specification

- A *Klein bottle current* K. This can be any simple current that obeys

$$Q_I(K) = 0 \text{ mod } 1 \text{ for all } I \in \mathcal{H}, I^2 = 0.$$

- A set of phases $\beta_K(J)$ for all $J \in \mathcal{H}$ that satisfy

$$\beta_K(J)\beta_K(J') = \beta_K(JJ')e^{2\pi i X(J,J')} \quad , J, J' \in \mathcal{H}$$

  with $\beta_K(J) = e^{i\pi(h_{KL}-h_K)}\eta(K,L)$, $\eta(K,L) = \pm 1$.
  if $\mathcal{H}$ has N even factors, there are 2^N free signs in the solution of this equation.
  These are called the *crosscap signs*

— This includes all know RCFT orientifold choices.
— Not all choices are inequivalent.

Boundaries and crosscaps

- Boundary coefficients

$$R_{[a,\psi_a](m,J)} = \sqrt{\frac{|\mathcal{H}|}{|\mathcal{C}_a||\mathcal{S}_a|}}\psi_a^*(J)S^J_{am}$$

- Crosscap coefficients

$$U_{(m,J)} = \frac{1}{\sqrt{|\mathcal{H}|}}\sum_{L\in\mathcal{H}}\eta(K,L)P_{LK,m}\delta_{J,0}$$

S^J is the *fixed point resolution matrix*
$\mathcal{S}_a$ is the *Stabilizer* of a
$\mathcal{C}_a$ is the *Central Stabilizer* ($\mathcal{C}_a \subset \mathcal{S}_a \subset \mathcal{H}$)
ψ_a is a discrete group character of cC_a
$P = \sqrt{T}ST^2S\sqrt{T}$

Partition functions

— Klein bottle:

$$K^i = \sum_{m,J,J'} \frac{S^i{}_m U_{(m,J)} g^{\Omega,m}_{J,J'} U_{(m,J')}}{S_{0m}}$$

— Unoriented Annulus:

$$A^i_{[a,\psi_a][b,\psi_b]} = \sum_{m,J,J'} \frac{S^i{}_m R_{[a,\psi_a](m,J)} g^{\Omega,m}_{J,J'} R_{[b,\psi_b](m,J')}}{S_{0m}}$$

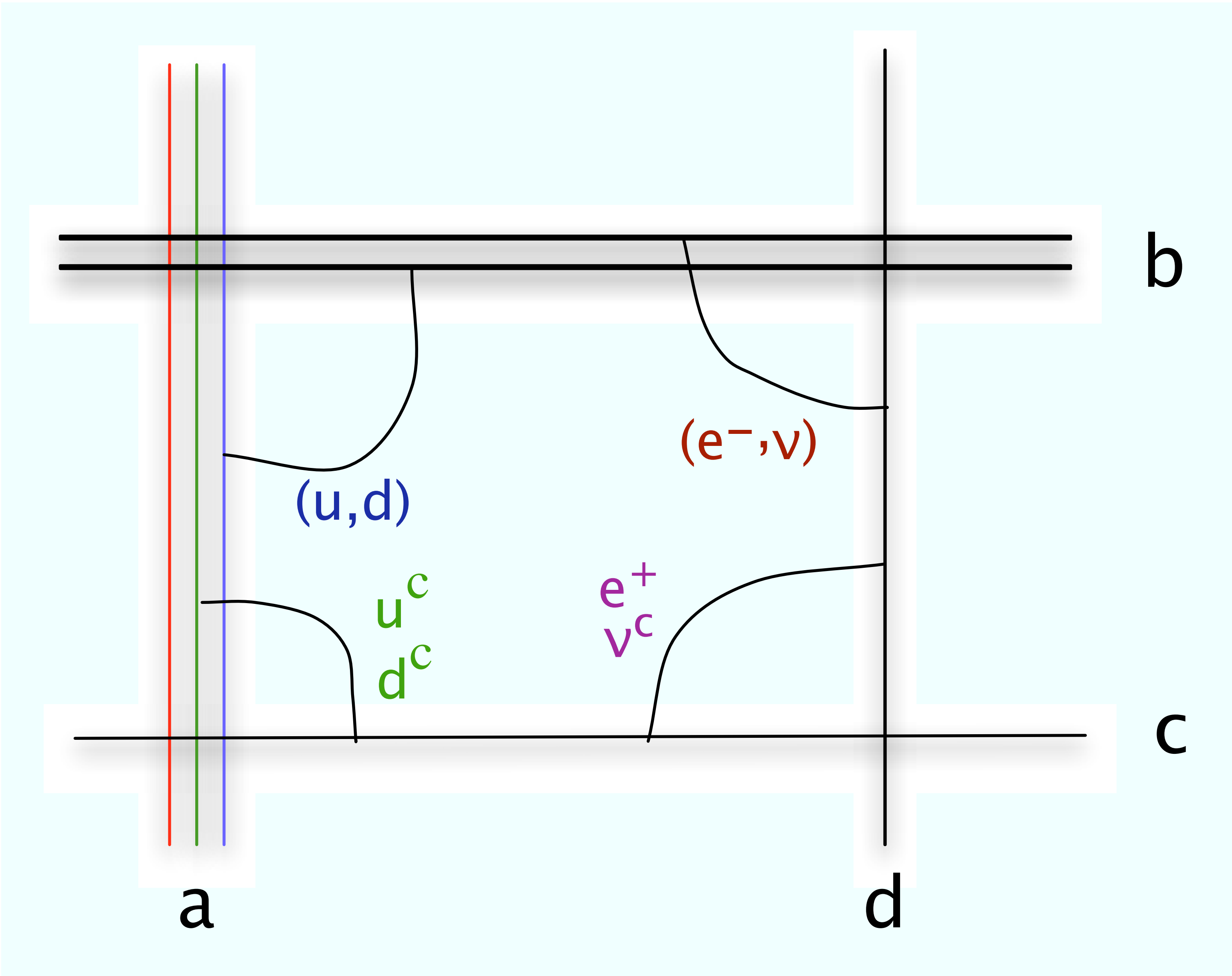
— Moebius:

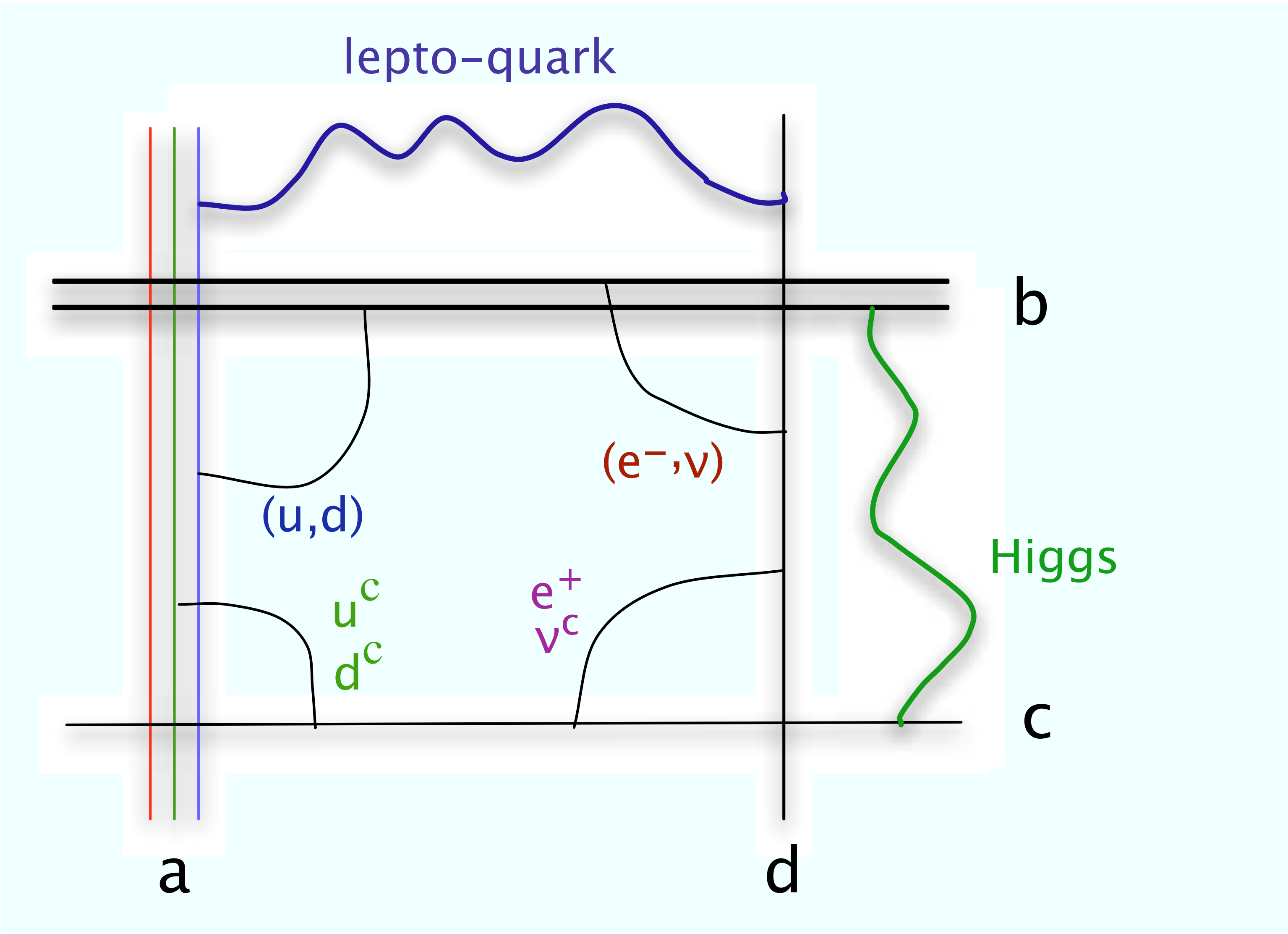
$$M^i_{[a,\psi_a]} = \sum_{m,J,J'} \frac{P^i{}_m R_{[a,\psi_a](m,J)} g^{\Omega,m}_{J,J'} U_{(m,J')}}{S_{0m}}$$

Here $g^{\Omega,m}$ is the *Ishibashi metric*

$$g^{\Omega,m}_{J,J'} = \frac{S_{m0}}{S_{mK}} \beta_K(J) \delta_{J',J^c} \quad .$$

The Four-stack Standard Model




lepto-quark
b
(e⁻,ν)
(u,d)
Higgs
u^c
d^c
e^+
ν^c
c
a
d

• Tadpoles and Anomalies

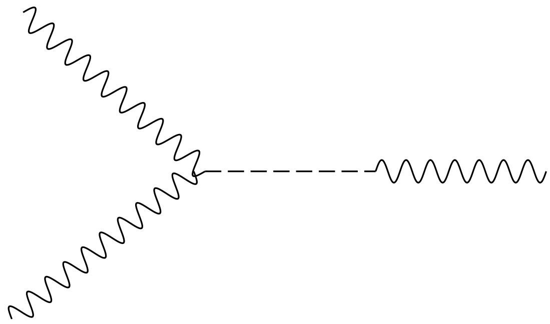
Cancellation of massless tadpoles between disk and crosscap

$$\sum_b N_b R_{b(m,J)} = 4\eta_m U_{(m,J)} \ ,$$

Determines Chan-Paton multiplicities N_b

Then: purely cubic $\mathrm{Tr}\ F^3$ anomalies cancel

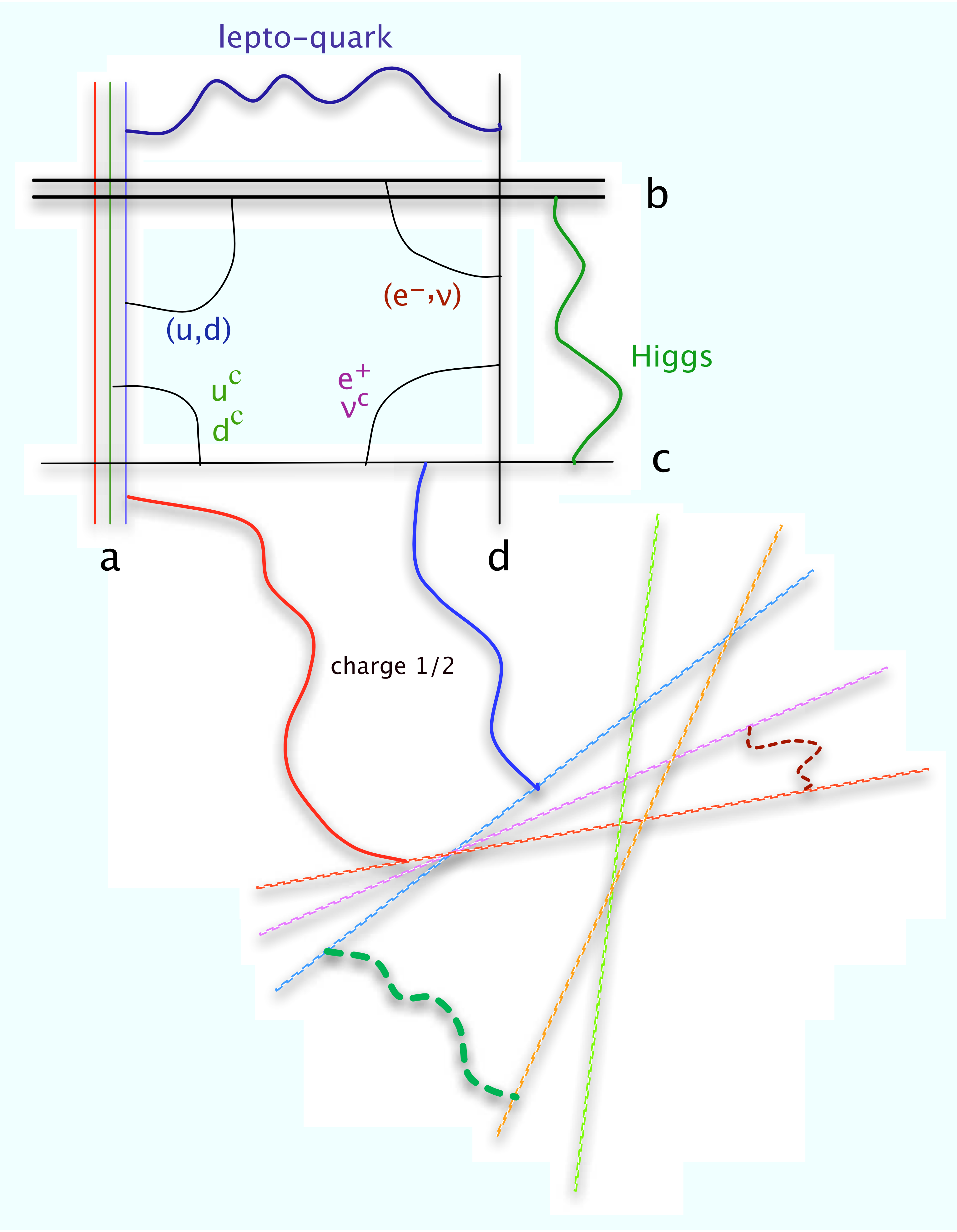
Remaining ones cancelled by Green-Schwarz terms



Two-point RR-twoform/gauge boson vertices generate masses for anomalous $U(1)$ and some non-anomalous ones

In these models: B+L massive, Y massless (required), B-L massive or massless

Baryon and Lepton number remain as perturbative symmetries


lepto-quark
b
(e⁻,ν)
(u,d)
Higgs
u^c
d^c
e^+
ν^c
c
a
d
charge 1/2

• Spectrum Types

Type 0: $U(3) \times Sp(2) \times U(1) \times U(1)$
Type 1: $U(3) \times U(2) \times U(1) \times U(1)$
Type 2: $U(3) \times Sp(2) \times O(2) \times U(1)$
Type 3: $U(3) \times U(2) \times O(2) \times U(1)$
Type 4: $U(3) \times Sp(2) \times Sp(2) \times U(1)$
Type 5: $U(3) \times U(2) \times Sp(2) \times U(1)$

SM $U(1)$ Generators

$Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c - \frac{1}{2}Q_d$ (types 0,1)

$Y = \frac{1}{6}Q_a - T_c - \frac{1}{2}Q_d$ (types 2,3,4,5)

Types 2,3,4,5:

$M_Y = M_{B-L} = 0$

• Chirality

Chiral with respect to $SU(3) \times SU(2) \times U(1)$

$$3(u,d)_L + 3u^c_L + 3d^c_L + 3(e^-,\nu)_L + 3e^+_L$$

Chiral with respect to Chan-Paton group but not with respect to $SU(3) \times SU(2) \times U(1)$

- 3 Left-handed anti-neutrinos [100%]
- Higgs (w.r.t. $U(2)_b$) [0.3%]
- Mirrors of (u,d) or (e^-,ν) (w.r.t. $U(2)_b$) [1.5%]
- SM singlets from hidden sector [12.5%]

- Scope

- 168 Gepner models (5 non-chiral, 2 not done)

- 5403 MIPFs ($\rightarrow 873$ Hodge number pairs)
  (after removing equivalences)

- 49322 Orientifolds
  (after removing equivalences)

- $10^8 \ldots 10^{18}$ 4-stack boundary combinations per orientifold

MIPFs can be chiral algebra extensions, automorphisms or combinations. In all cases, we consider the complete set of boundaries. (# boundaries = #Isibashi states)

• Counting

- Distinct moduli space
  Difficult in RCFT

- Distinct SM+Hidden Spectra
  Requires scan of hidden spectra

- Distinct SM Spectra
  — Distinct (non)-chiral SM multiplicities
  — Distinct dilaton couplings
  — Absence/Presence of Hidden sector
  — Not included: # fractional charges or confinement

- Results

— Solutions exist for 44 (out of 168) Gepner models,
333 (out of 5403) MIPFs
— No SM 4-stacks for 4079 MIPFs
— 4-stacks, but no tadpole solutions for 649 MIPFs
— 4-stacks, but undecided[4] 342 MIPFs

— Total number of distinct SM spectra: 179520
(179119 CY inequiv.)

— Total number SM 4-stacks: 45051902
— Total number SM 4-stacks with solutions: 1635985
— Total number of solutions collected: 10526078

- No solutions for the charge conjugation invariant

[4] Need $> 10^{150}$ yrs of CPU time

- Spectrum types

Type	Quark*	Lepton*	Higgs*	Nr.
0	0	0	0	10564
1	-3	3	0	32
1	-9	3	6	1
1	-9	9	0	22
2	0	0	0	49661
3	-3	-1	4	141
3	-3	-3	6	24
3	-3	1	2	240
3	-3	3	0	740
3	-9	-3	12	24
3	-9	3	6	95
3	-9	5	4	1
3	-9	9	0	116
4	0	0	0	116304
5	-3	1	2	2
5	-3	3	0	1507
5	-9	9	0	46

* $U(2)$ anomaly

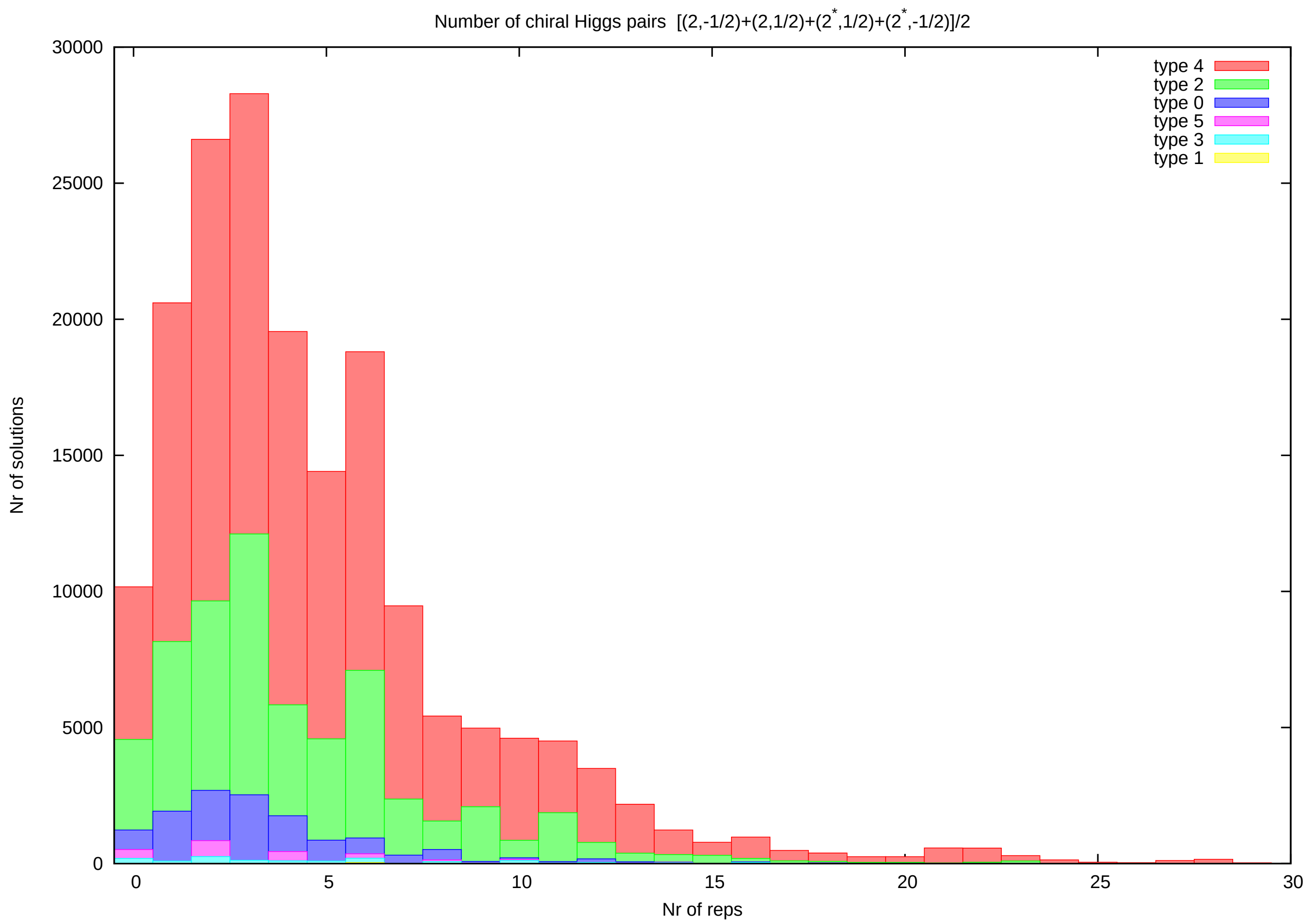
Massive $B-L$, type 0: 403; type 1: 0
Massive $B-L$, no extra branes
(just $SU(3) \times SU(2) \times U(1)$!): 22
No fractional charges: 1680
Confined fractional charges: 91919

nr.	tensor	Nprim	SC	MIPFs	4-stacks	Sol	SM
1	(1,5,41,1804)	28539	1	1,0,0	0	0	0
2	(1,5,42,922)	29772	2	2,-,-	-	-	-
3	(1,5,43,628)	31482	3	2,-,-	-	-	-
4	(1,5,44,481)	9399	1	1,0,0	0	0	0
5	(1,5,46,334) †	35442	6	4,1,0	12	0	0
6	(1,5,47,292) †	37800	7	2(1),1,0	1128	0	0
7	(1,5,49,236)	4575	1	1,0,0	0	0	0
8	(1,5,52,187)	12690	3	2(1),1,0	144	0	0
9	(1,5,54,166) †	48258	14	4,1,0	54	0	0
10	(1,5,58,138)	6156	2	2,0,0	0	0	0
11	(1,5,61,124)	64449	21	4(2),3,0	81044	0	0
12	(1,5,68,103)	20748	7	2(1),1,0	234	0	0
13	(1,5,76,89)	2835	1	1,0,0	0	0	0
14	(1,5,82,82) †	108612	42	8(1),3,0	1744	0	0
15	(1,6,23,598)	12600	2	2,0,0	0	0	0
16	(1,6,24,310) †	13650	4	6(1),1,0	18260	0	0
17	(1,6,25,214)	14742	6	4(2),2,0	212000	0	0
18	(1,6,26,166)	15876	8	10(7),8,0	4939262	0	0
19	(1,6,28,118)	18270	12	12(9),10,2	1289765	217	67

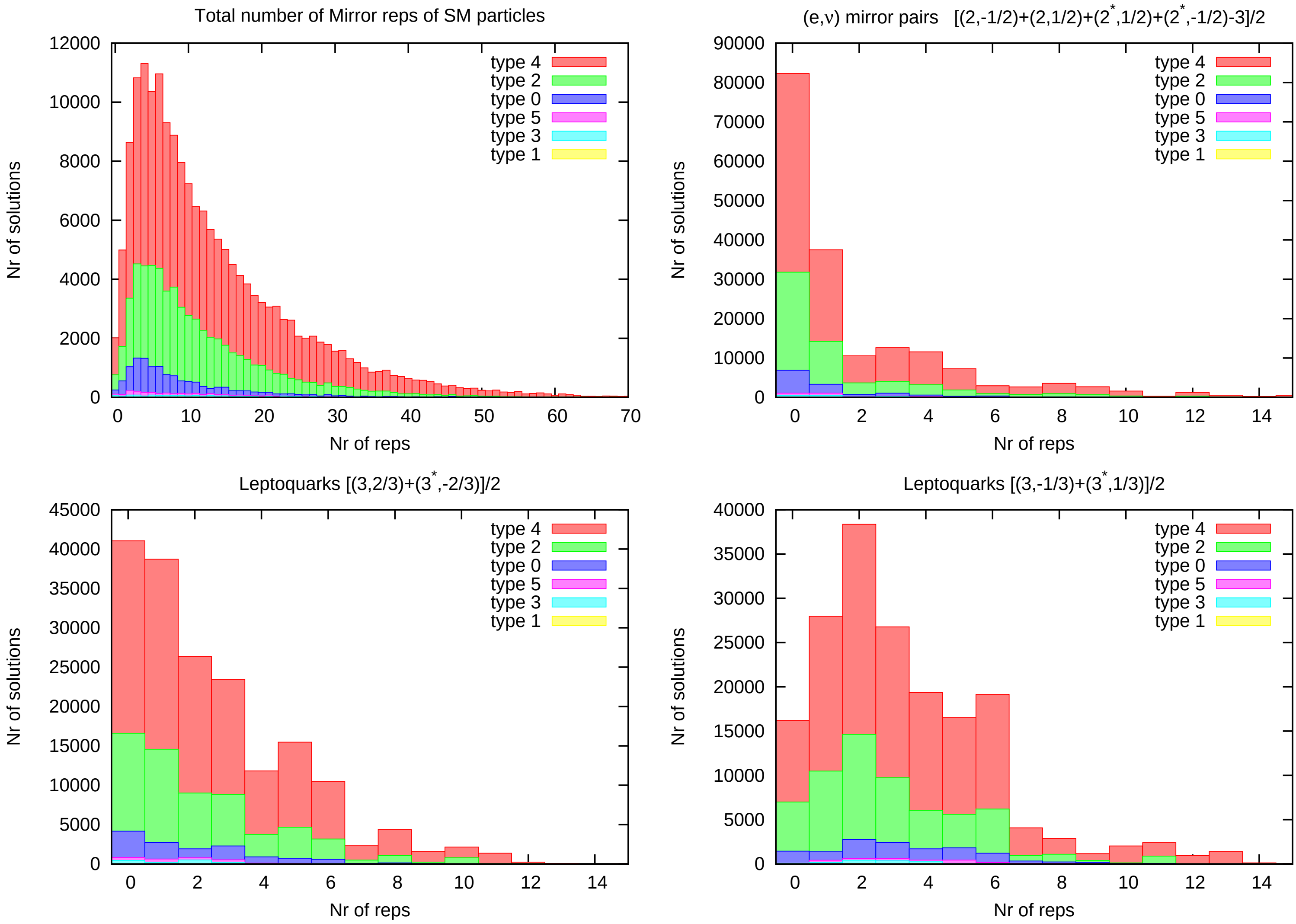
nr.	tensor	Nprim	SC	MIPFs	4-stacks	Sol	SM
66	(2,4,11,154)	3540	4	6(4),4,0	2304160	0	0
67	(2,4,12,82)	4160	8	30(26),26,12	5133558	598	294
68	(2,4,13,58)	4830	12	12(7),9,1	468648	146	69
69	(2,4,14,46)	5340	16	27(24),25,13	1918601	17411	5607
70	(2,4,16,34)	7140	24	60(48),54,23	3700006	218598	45055
71	(2,4,18,28)	2320	8	30(23),25,15	745644	801	360
72	(2,4,19,26)	1100	4	6(2),3,0	23872	0	0
73	(2,4,22,22)	12060	48	54(39),51,25	3403934	423560	43532
74	(2,5,8,138)	2862	4	6(3),4,0	191424	0	0
75	(2,5,10,40)	1230	4	6(1),3,0	5502	0	0
76	(2,5,12,26)	6006	28	12,7,3	34744	2426	431
77	(2,6,7,70)	3024	8	5(1),3,0	28234	0	0
78	(2,6,8,38)	3780	16	27(18),22,7	323662	6313	1368
79	(2,6,10,22)	5544	32	59(20),53,27	361546	18964	3624
80	(2,6,14,14)	9744	64	87(20),71,30	758636	62856	5424
81	(2,7,7,34)	4032	18	6,0,0	0	0	0
82	(2,7,10,16)	2040	12	12,6,1	10504	4	1
83	(2,8,8,18)	6480	40	44(3),32,16	1019592	222006	17311
84	(2,8,10,13)	630	4	6,3,0	1320	0	0
85	(2,10,10,10)	12000	96	92(7),71,34	850844	137472	9878
86	(3,3,9,108)	2900	5	2,0,0	0	0	0
87	(3,3,10,58)	3280	10	4,1,0	124	0	0
88	(3,3,12,33)	1700	5	2,0,0	0	0	0

nr.	tensor	Nprim	SC	MIPFs	4-stacks	Sol	SM
146	(2,2,4,4,4)	1500	48	180,5,0	4640	0	0
147	(3,3,3,3,3)	4000	125	8,0,0	0	0	0
148	(1,1,1,1,5,40)	972	27	8,0,0	0	0	0
149	(1,1,1,1,6,22)	1134	54	16,0,0	0	0	0
150	(1,1,1,1,7,16)	1944	81	34,1,1	6	3	2
151	(1,1,1,1,8,13)	756	27	8,0,0	0	0	0
152	(1,1,1,1,10,10)	2592	162	58,0,0	0	0	0
153	(1,1,1,2,3,18)	96	6	4,0,0	0	0	0
154	(1,1,1,2,4,10)	1350	108	72,0,0	0	0	0
155	(1,1,1,2,6,6)	252	24	10,0,0	0	0	0
156	(1,1,1,3,3,8)	240	15	4,0,0	0	0	0
157	(1,1,1,4,4,4)	2673	324	142,0,0	0	0	0
158	(1,1,2,2,2,10)	912	96	52,0,0	0	0	0
159	(1,1,2,2,4,4)	900	72	110,0,0	0	0	0
160	(1,2,2,2,2,4)	440	64	138,0,0	0	0	0
161	(2,2,2,2,2,2)	2944	512	1031,10,0	448	0	0
162	(1,1,1,1,1,2,10)	810	162	34,0,0	0	0	0
163	(1,1,1,1,1,4,4)	1944	486	156,0,0	0	0	0
164	(1,1,1,1,2,2,4)	540	108	48,0,0	0	0	0
165	(1,1,1,2,2,2,2)	264	96	46,0,0	0	0	0
166	(1,1,1,1,1,1,1,4)	2187	729	124,0,0	0	0	0
167	(1,1,1,1,1,1,2,2)	324	162	24,0,0	0	0	0
168	(1,1,1,1,1,1,1,1,1)	2187	2187	152,0,0	0	0	0

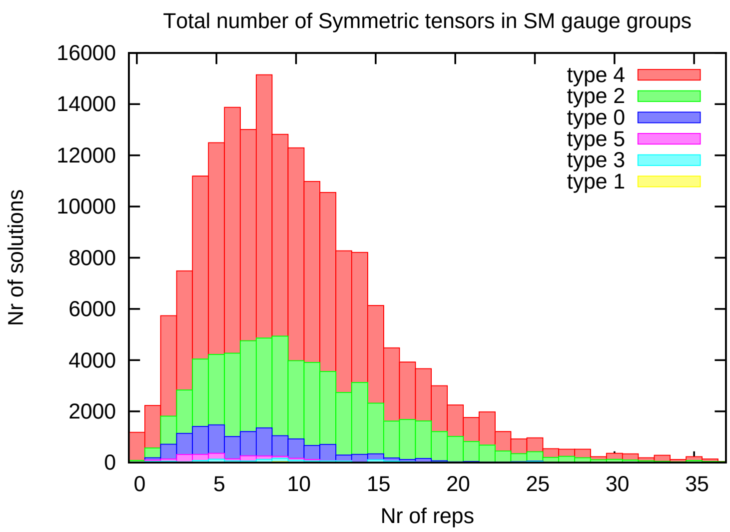
- Higgs distribution

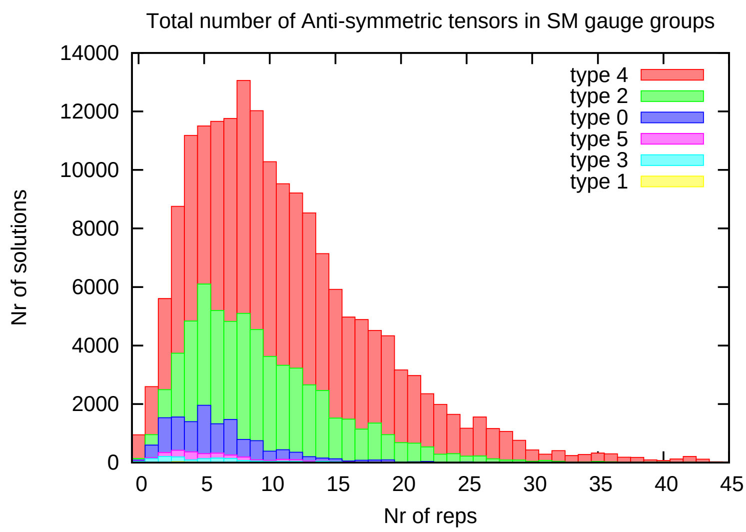


- Mirror, leptoquark distributions



• Adjoint, tensor distributions





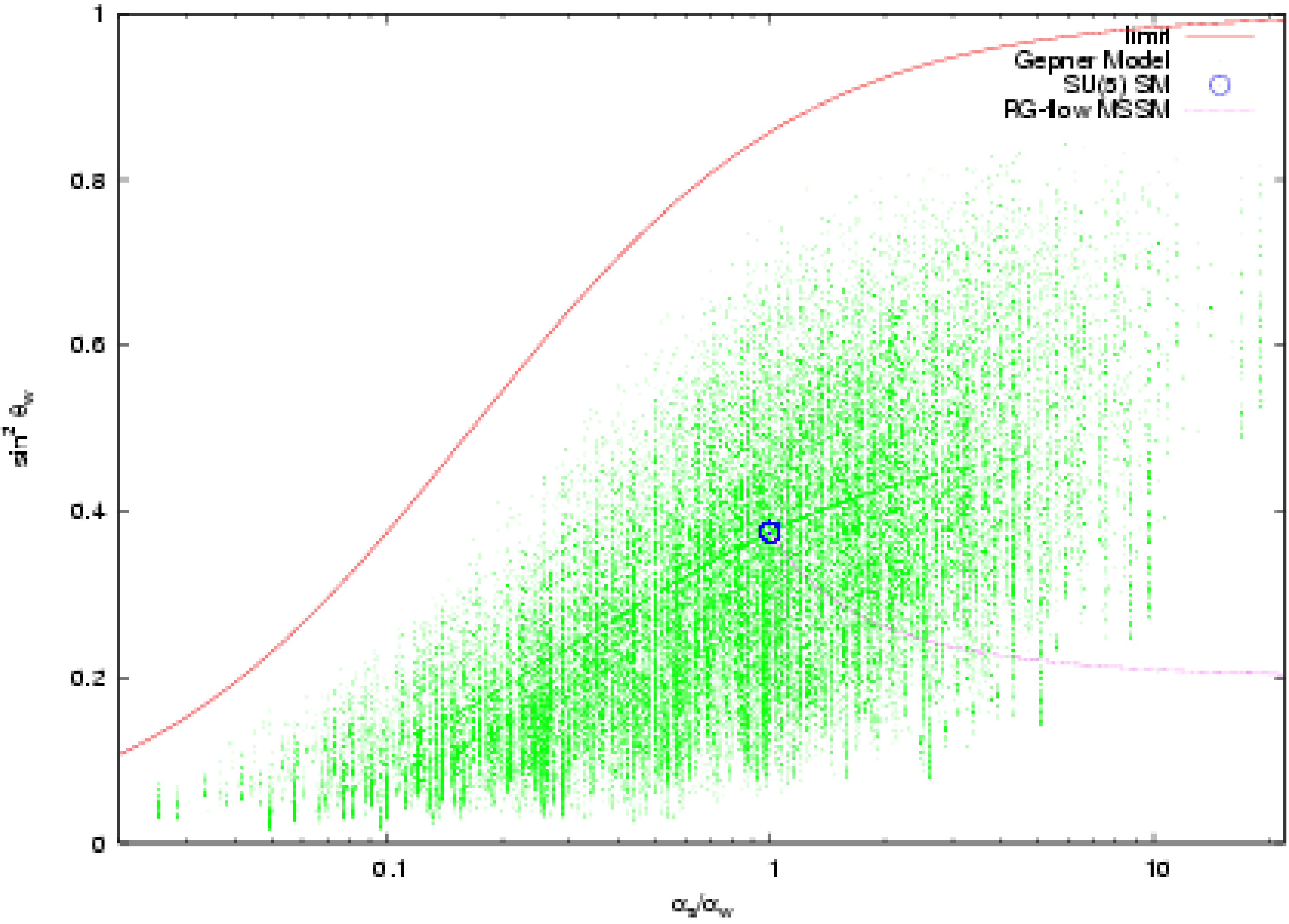
- Gauge couplings

Relations at the string scale:

$$\frac{g_2^2}{g_3^2} = \frac{R_{0\mathrm{a}}}{\kappa R_{0\mathrm{b}}}$$

$$\sin^2\theta_W = \frac{\kappa R_{0\mathrm{b}}}{\kappa R_{0\mathrm{b}} + \frac{1}{6}R_{0\mathrm{a}} + \frac{1}{2}R_{0\mathrm{c}} + \frac{1}{2}R_{0\mathrm{d}}}$$

$\kappa = 1$ for spectra of types 1,3 and 5
$\kappa = \frac{1}{2}$ for spectra of type 0,2 and 4.

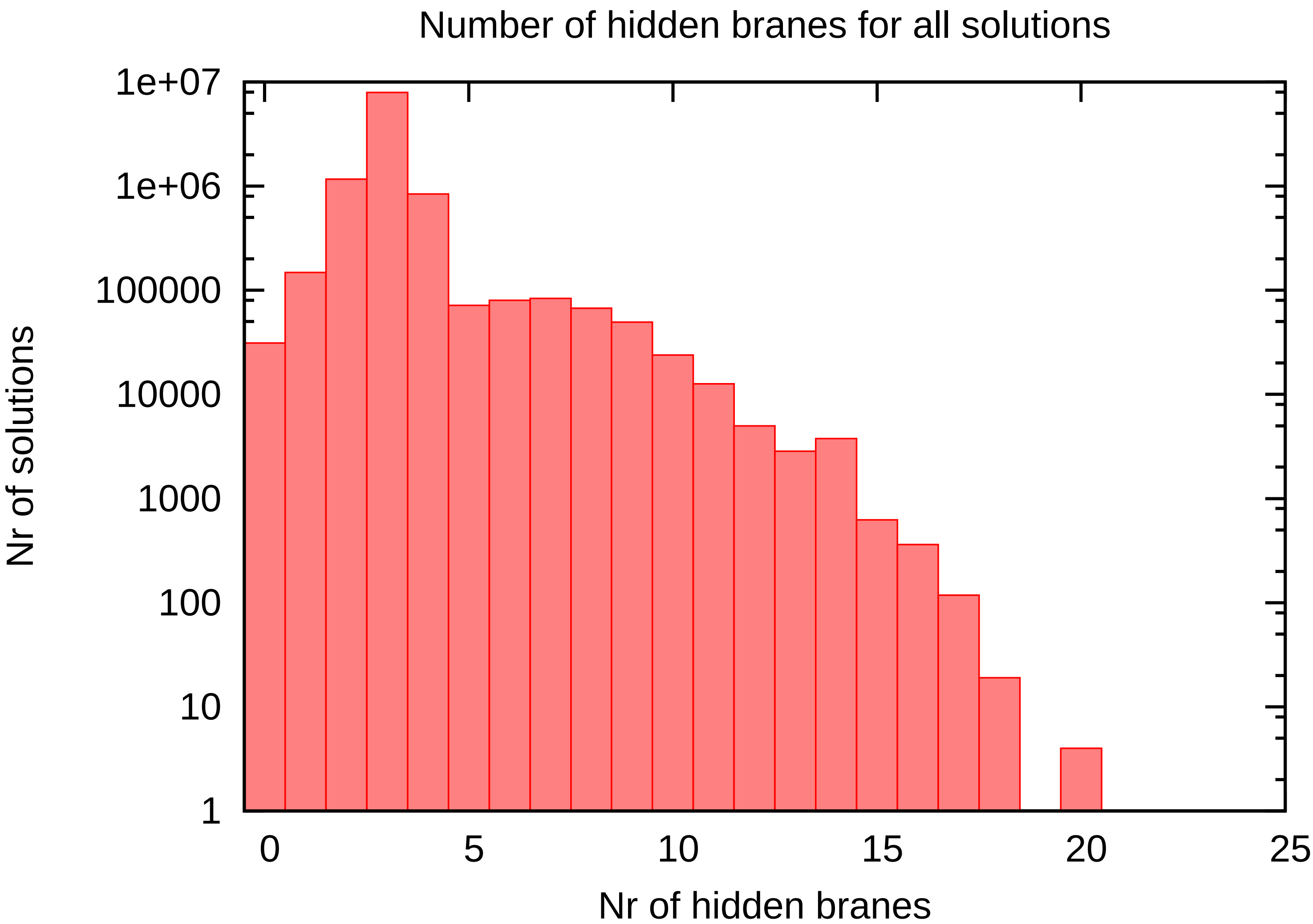


limit
Gepner Model
SU(5) SM
RG-flow MSSM
1
0.8
0.6
0.4
0.2
0
0.1
1
10
$\sin^2\theta_w$
α_s/α_w

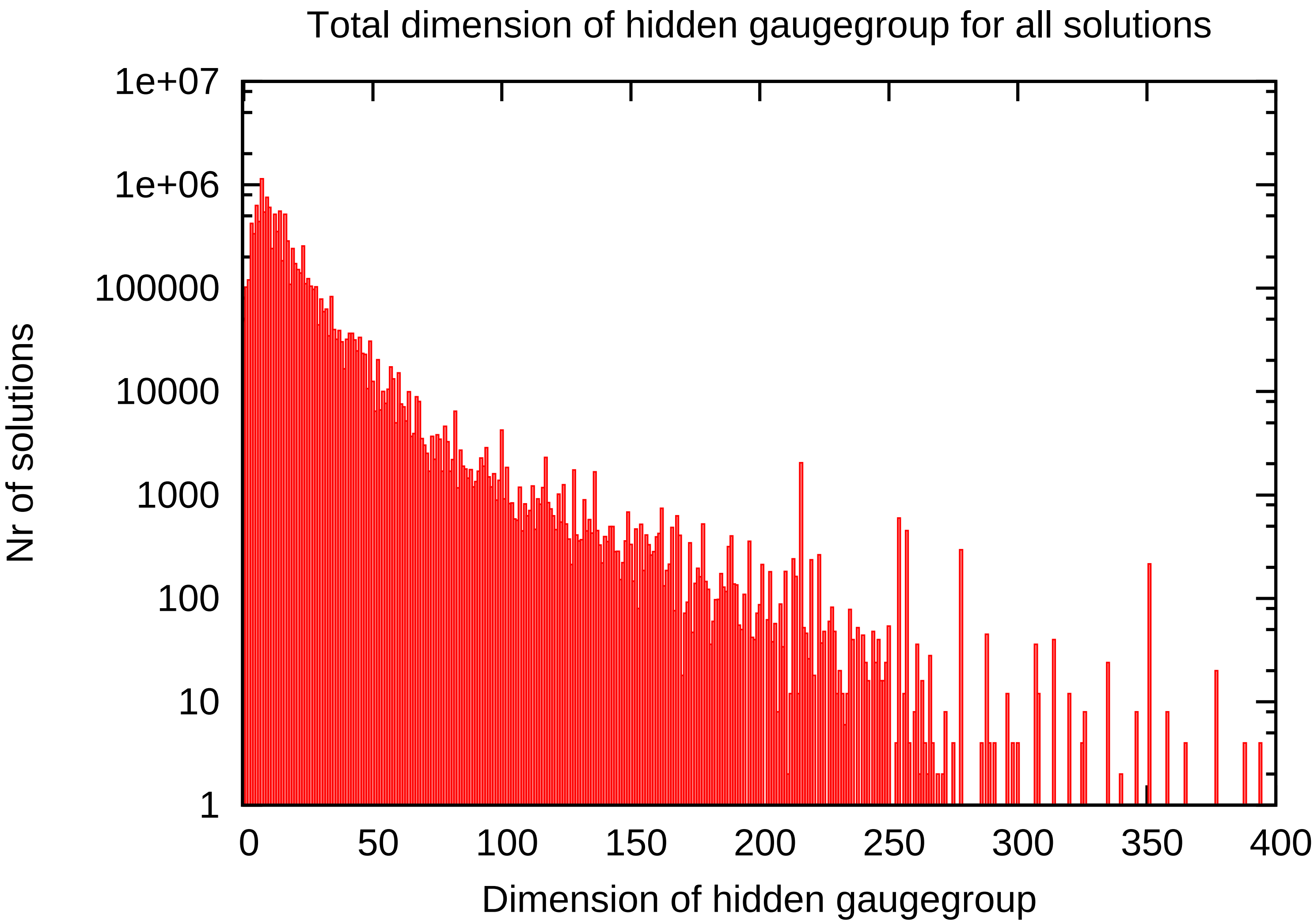
Blumenhagen,
Lüst,
Stieberger:

$$\frac{1}{\alpha_Y} = \frac{2}{3}\frac{1}{\alpha_s} + \frac{1}{\alpha_w}$$

- The hidden sector(1)

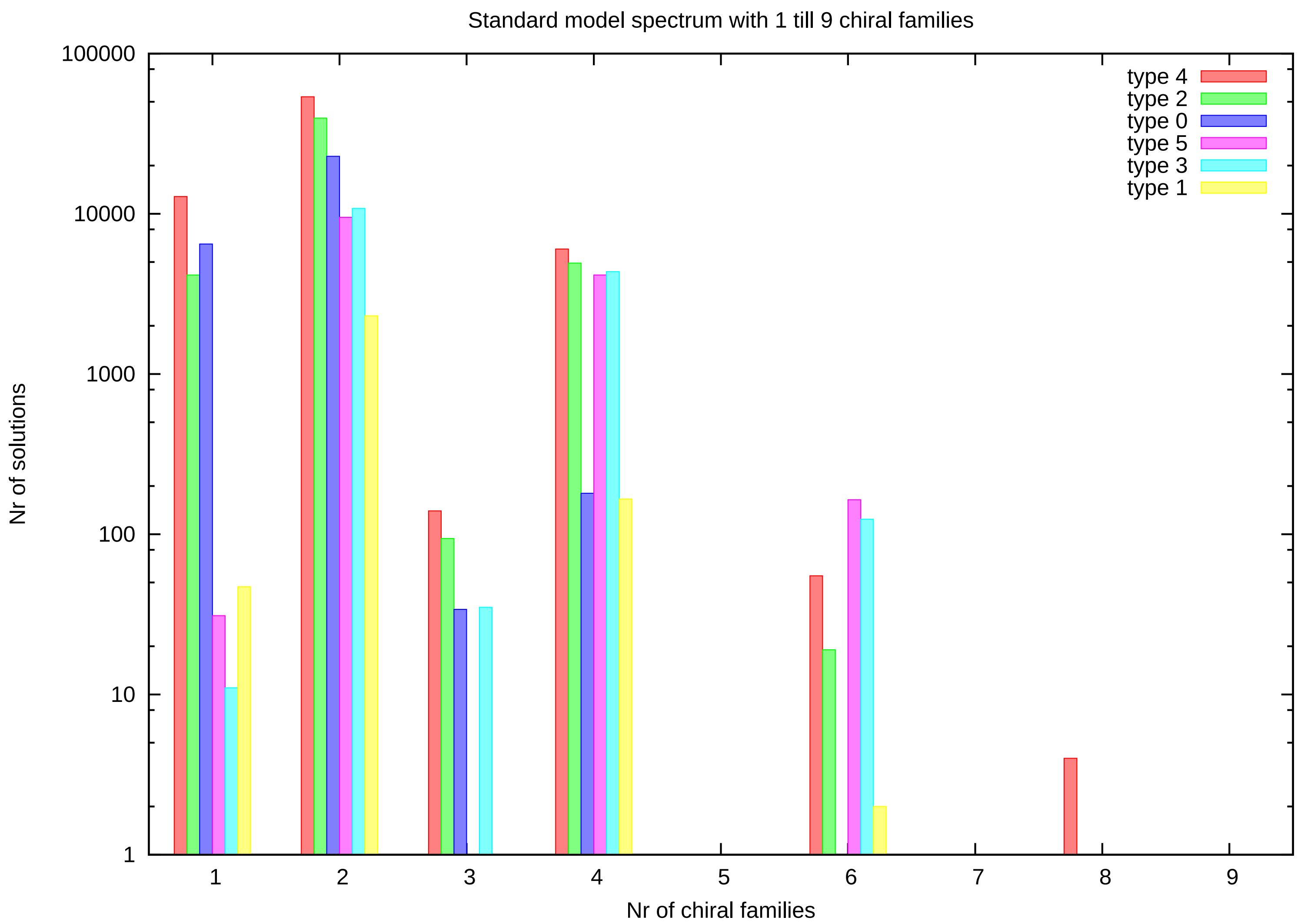


- The hidden sector(2)

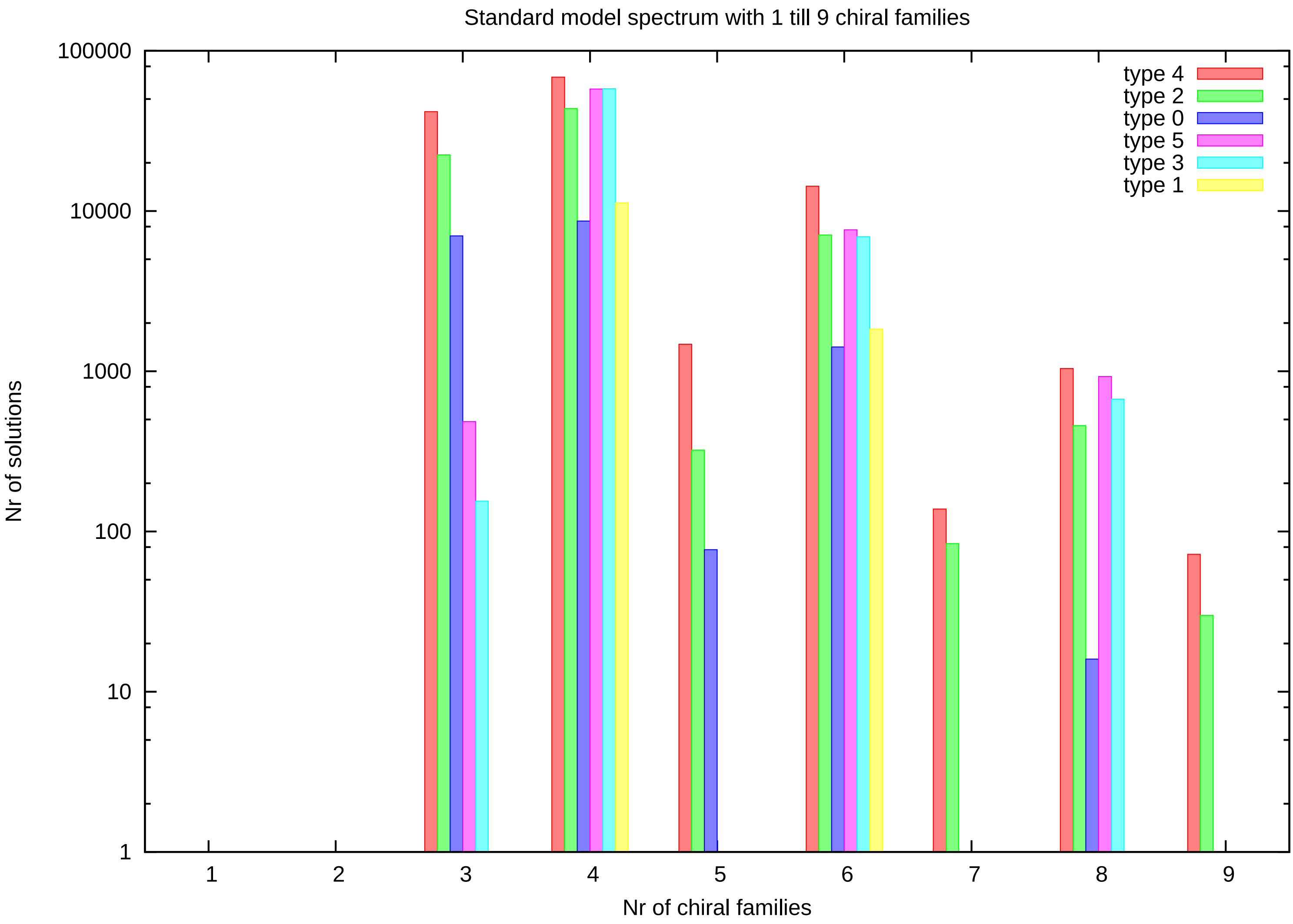


Largest factors encountered:
SO(18), Sp(24) and U(18)

- Number of families



Note: includes type-1 spectra with massive B-L
(for 1,2 and 4 families; not found with 3 families)



Standard model spectrum with 1 till 9 chiral families
100000
10000
1000
100
10
1
Nr of solutions
1
2
3
4
5
6
7
8
9
Nr of chiral families
type 4
type 2
type 0
type 5
type 3
type 1

- Possible future directions

— Other RCFT's

e.g. Kazama-Suzuki, permutation orbifolds

Note: so far only 827 of Kreuzer's 30000 Hodge numbers covered

— Other SM realizations

e.g. $SU(5)$, *Pati-Salam, anti-symmetric tensors*

— Other physical quantities

e.g. Yukawa couplings,

closed string moduli couplings

- Conclusions

Life outside orbifolds is hard, but abundant!