Heterotic Weight Lifting

B. Gato-Rivera and A.N Schellekens

(Nucl.Phys.B828:375-389,2010) and to appear

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Februari 2010

1984

Successes:

Small representations.

Anomaly cancellation.

GUT unification (if observed)

2010

Successes:

Small representations.

Anomaly cancellation.

Worries:

Non-chiral particles

Number of families

Fractional charges

Massless B-L (and other extra gauge bosons)

GUT unification (if observed)

RCFT: Heterotic vs Orientifold

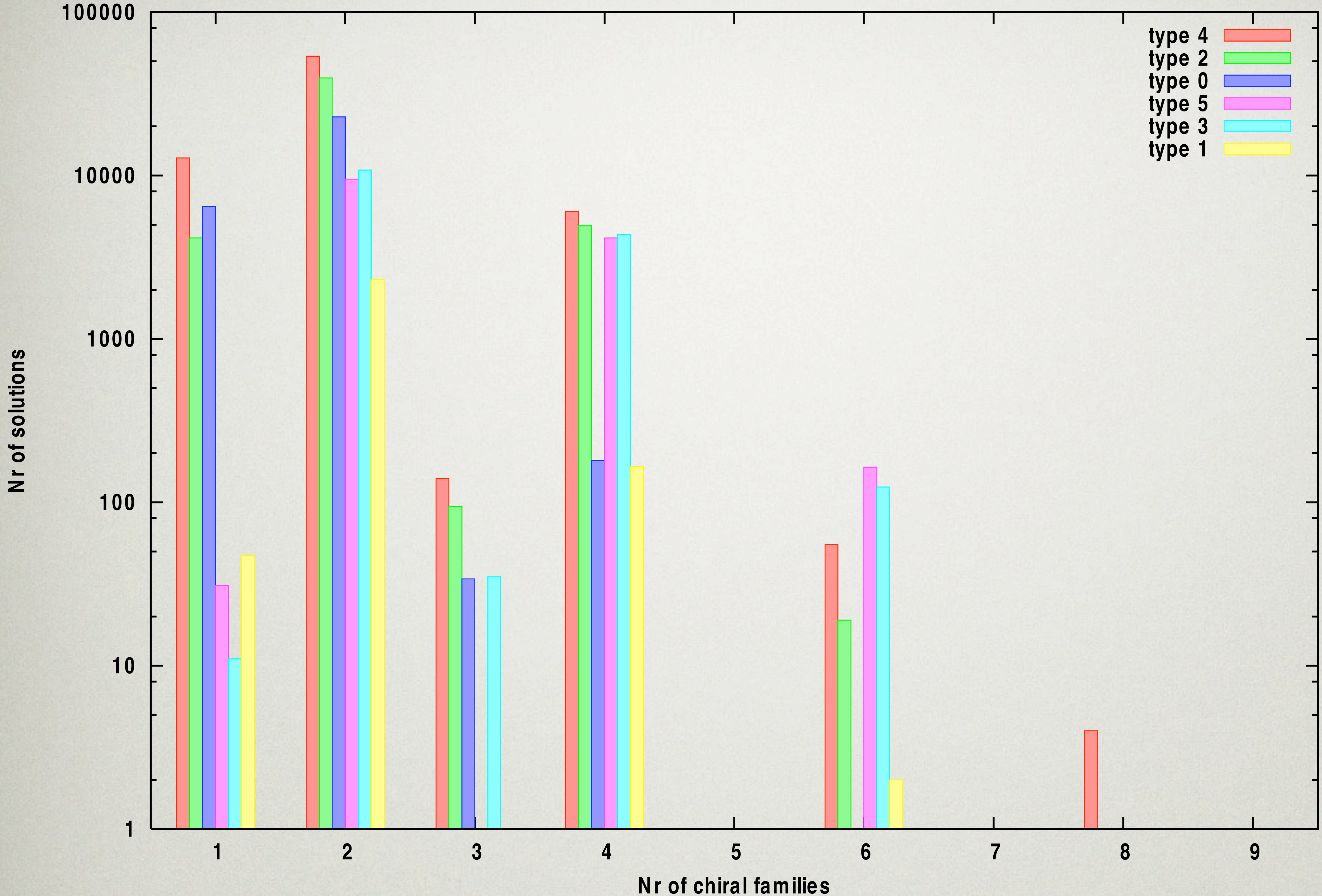
During the last five years, orientifolds were scanned systematically for Standard Model spectra

Dijkstra, Huiszoon, Schellekens
Gmeiner, Blumenhagen, Honecker, Lust, T. Weigand
Anastasopoulos, Dijkstra, Kiritsis, Schellekens
Douglas, Taylor
Kiritsis, Lennek, Schellekens
Gmeiner, Honecker

Few comparable results exist for heterotic strings. All we have are Hodge number scans[1], and fermionic construction scans[2]

(1)
Lutken, Ross (1988)
Schellekens, Yankielowicz (1989)
Fuchs, Klemm, Scheich, Schmidt (1989)
Kreuzer, Skarke (1992)
Donagi, Faraggi (2004),
Ploger, Ramos-Sanchez, Ratz, Vaudrevange (2007)
Donagi, Wendland (2008)
Kiritsis, Lennek, Schellekens (2008)

(2)
Dienes, Senechal (2007)
Assel, Christodoulides, Faraggi, Kounnas, Rizos (2009)



Dijkstra, Huiszoon, Schellekens (2004)
See also Gmeiner et. al. "One in a billion"

Gepner Models

Tensor product of an NSR model in 4 space time dimensions with a number of N=2 minimal CFT's with total central charge 9.

Partition function $\sum_{i,j} \chi_i(\tau) M_{ij} \chi_j(\bar{\tau})$

Heterotic:

Map the NSR model to SO(10) x E_8 in the bosonic sector.

M not necessarily symmetric;   Standard model in SO(10)

Orientifold:

Symmetric matrix M (type-II)

Mod out world-sheet orientation.

Add boundary states, Standard Model from intersecting branes.

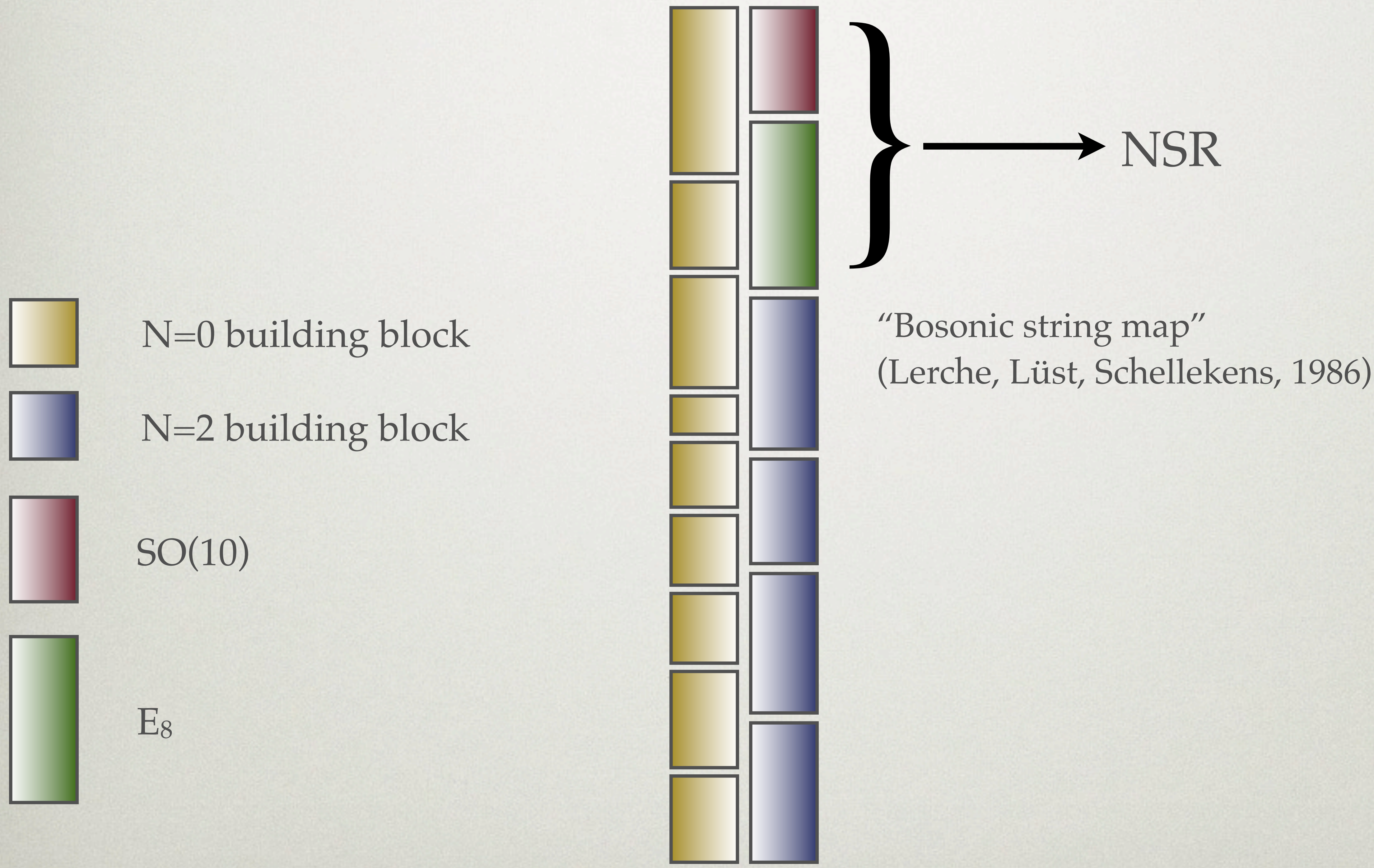
Heterotic (0,2) strings in RCFT

Two methods

- Build a (2,2) theory and map the fermionic sector to a bosonic one using the bosonic string map.
  Disadvantage: misses a lot of the (0,2) landscape.

- Build a (0,0) theory and impose susy on the fermionic sector.
  Only known way: free fermions or bosons.
  Disadvantage: misses a lot of the interacting CFT landscape.

General (0,2) model in RCFT



Modular invariance makes this very hard

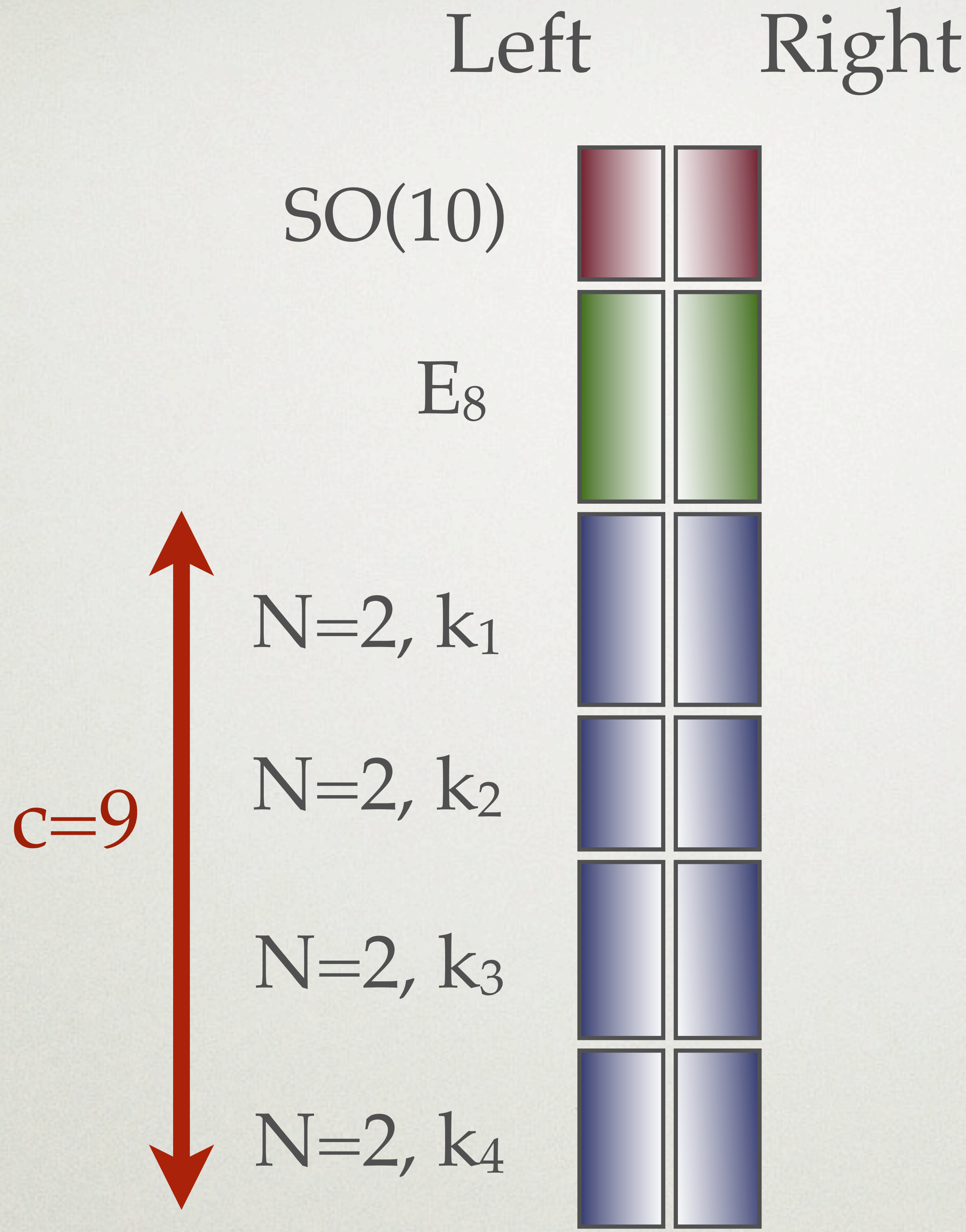
$$P(\tau,\bar\tau)=\sum_{ij}\chi_i(\tau)M_{ij}\xi_j(\bar\tau)$$

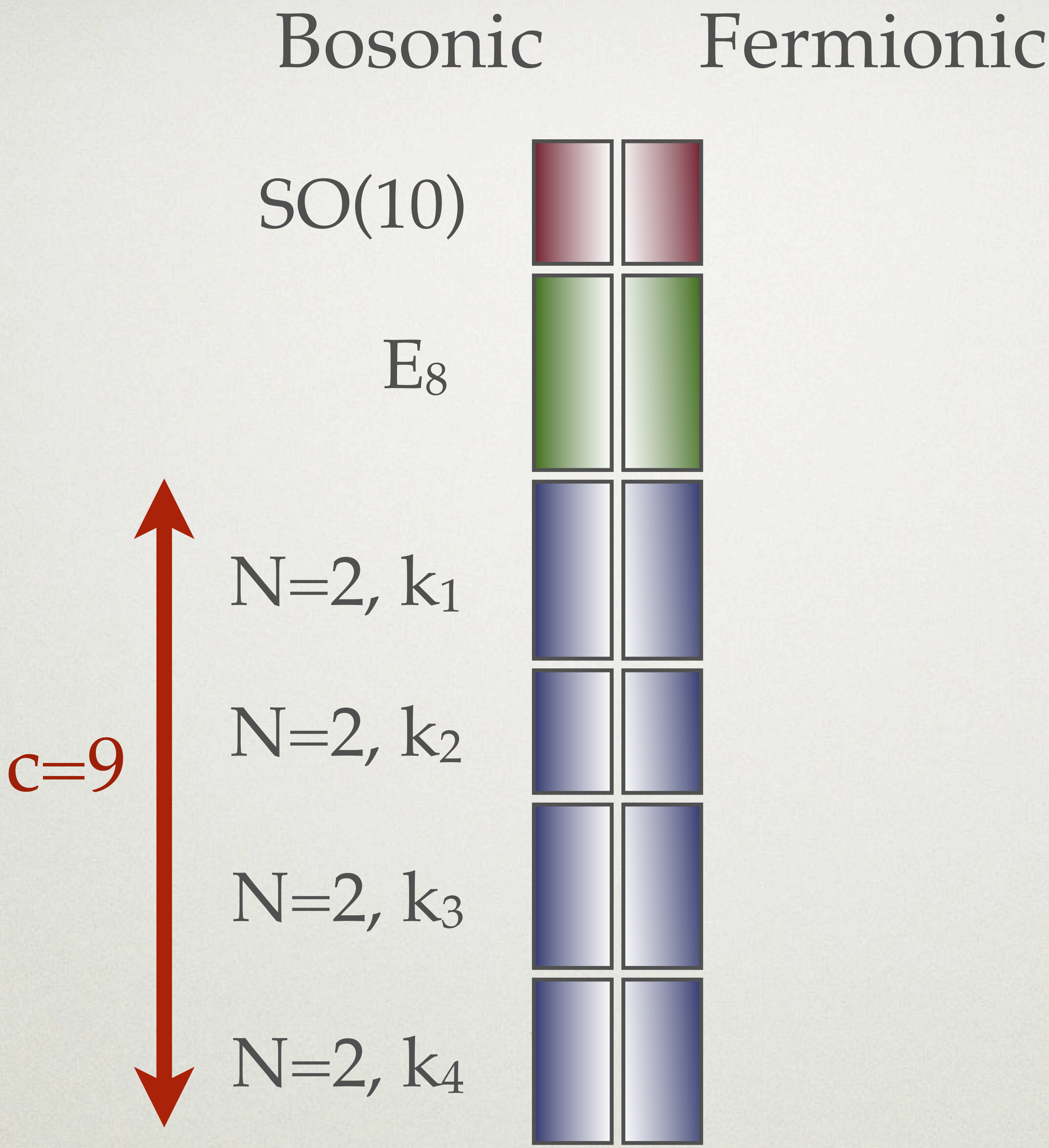
$$P(-\frac{1}{\tau},-\frac{1}{\bar\tau})=P(\tau,\bar\tau) \qquad \text{S}$$

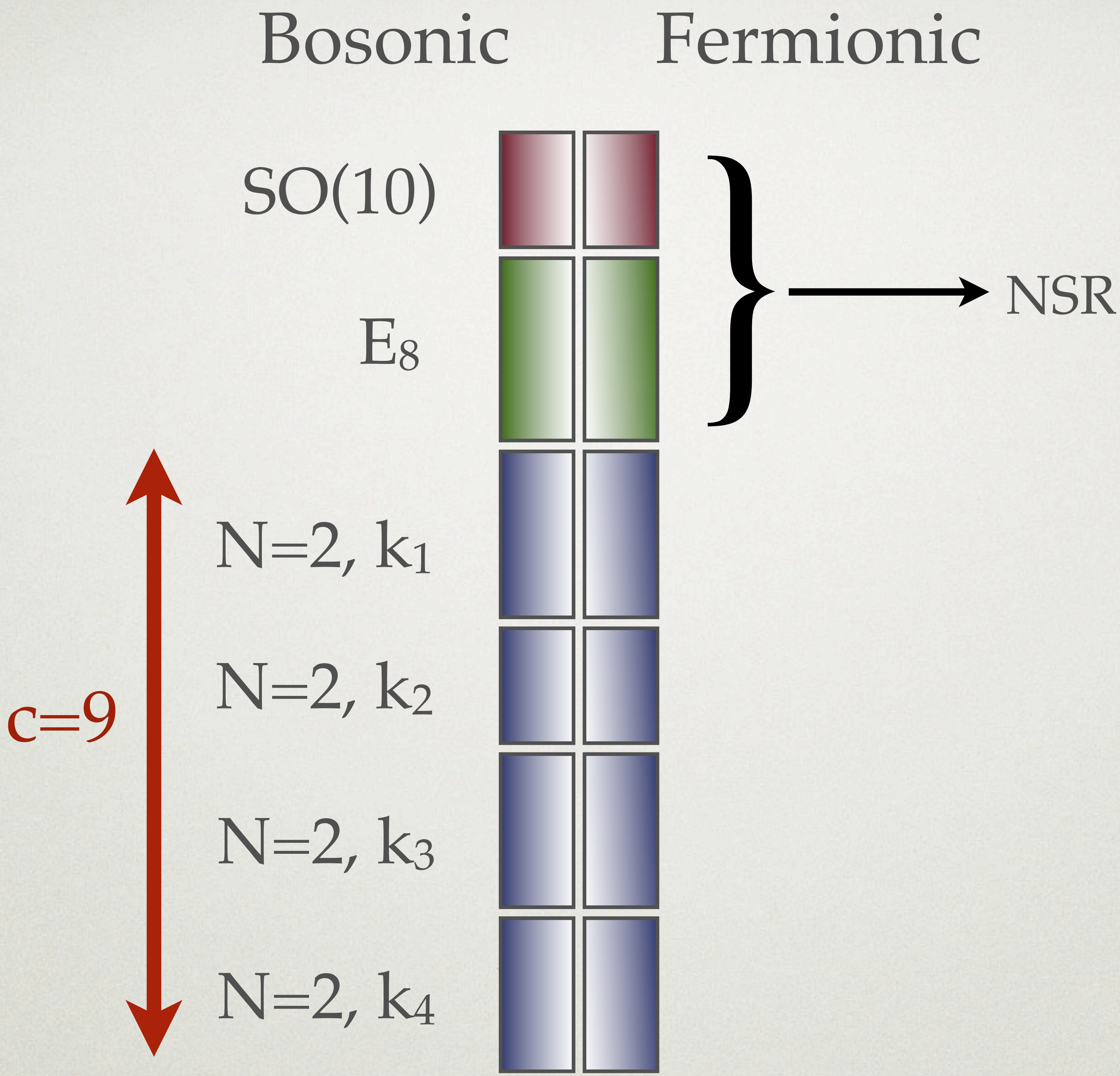
$$P(\tau+1,\bar\tau+1)=P(\tau,\bar\tau) \qquad \text{T}$$

Has a canonical solution, $M_{ij}=\delta_{ij}$, if the left and the right CFT are identical, so that $\chi=\xi$.

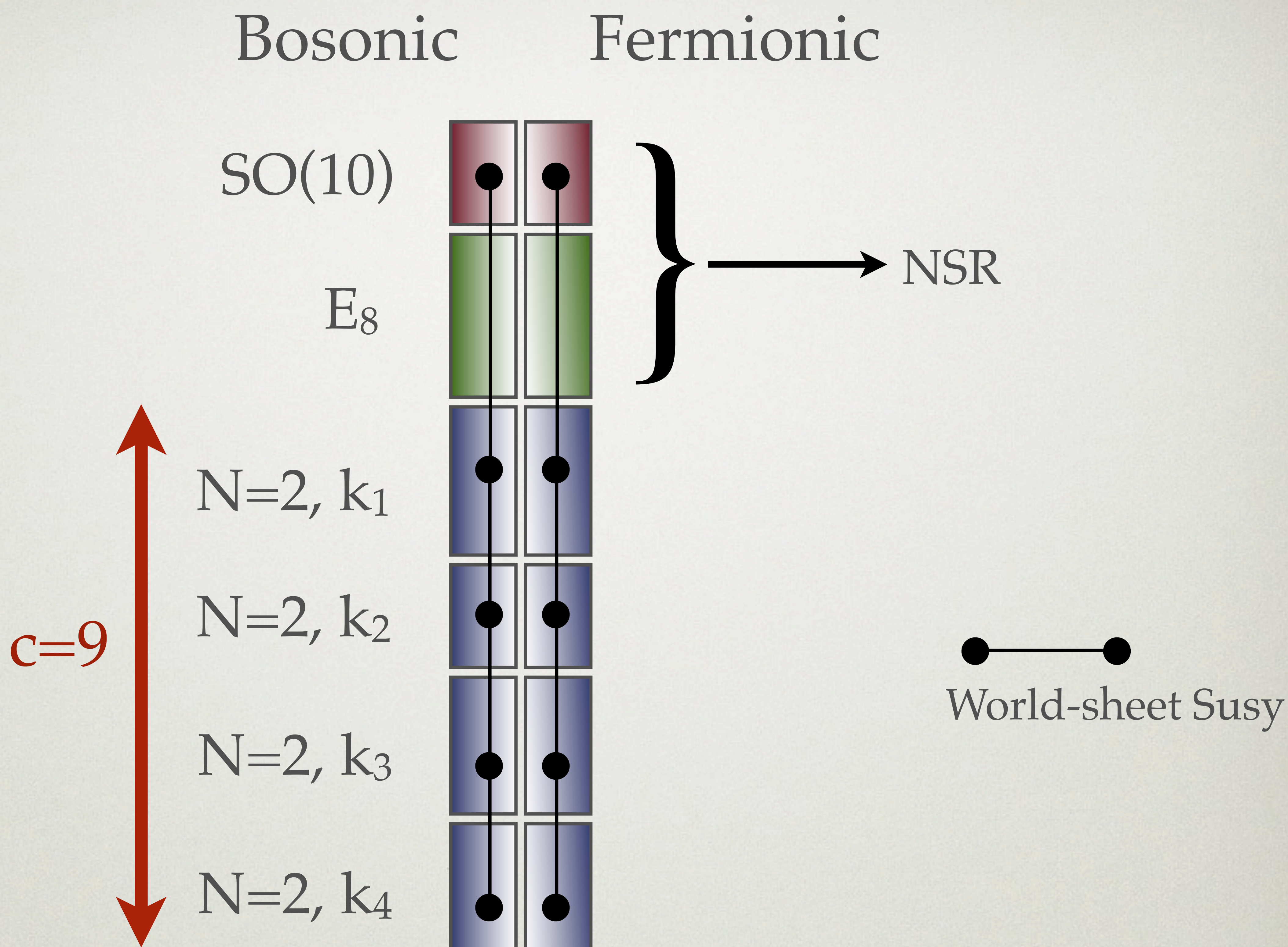
But they do not have to be identical, only isomorphic as representations of S and T.
In particular, this allows certain integer shifts of the eigenvalues of T, the conformal weights.


Left
Right
SO(10)
E_8
$N=2,\ k_1$
$N=2,\ k_2$
$N=2,\ k_3$
$N=2,\ k_4$
$c=9$


Bosonic
Fermionic
SO(10)
E8
N=2, k1
N=2, k2
N=2, k3
N=2, k4
c=9



Bosonic
Fermionic
SO(10)
E8
NSR
c=9
N=2, k1
N=2, k2
N=2, k3
N=2, k4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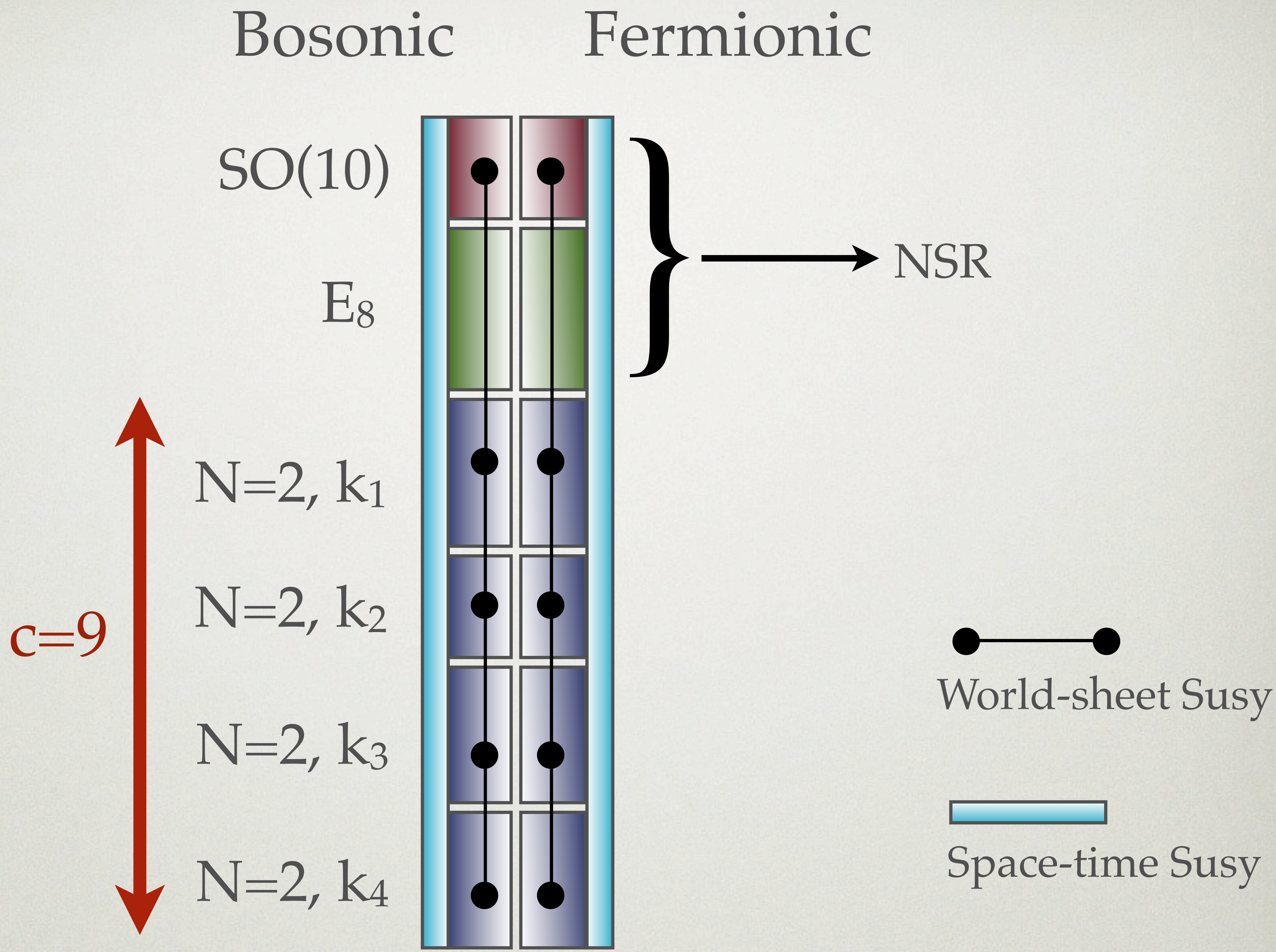
World-sheet Susy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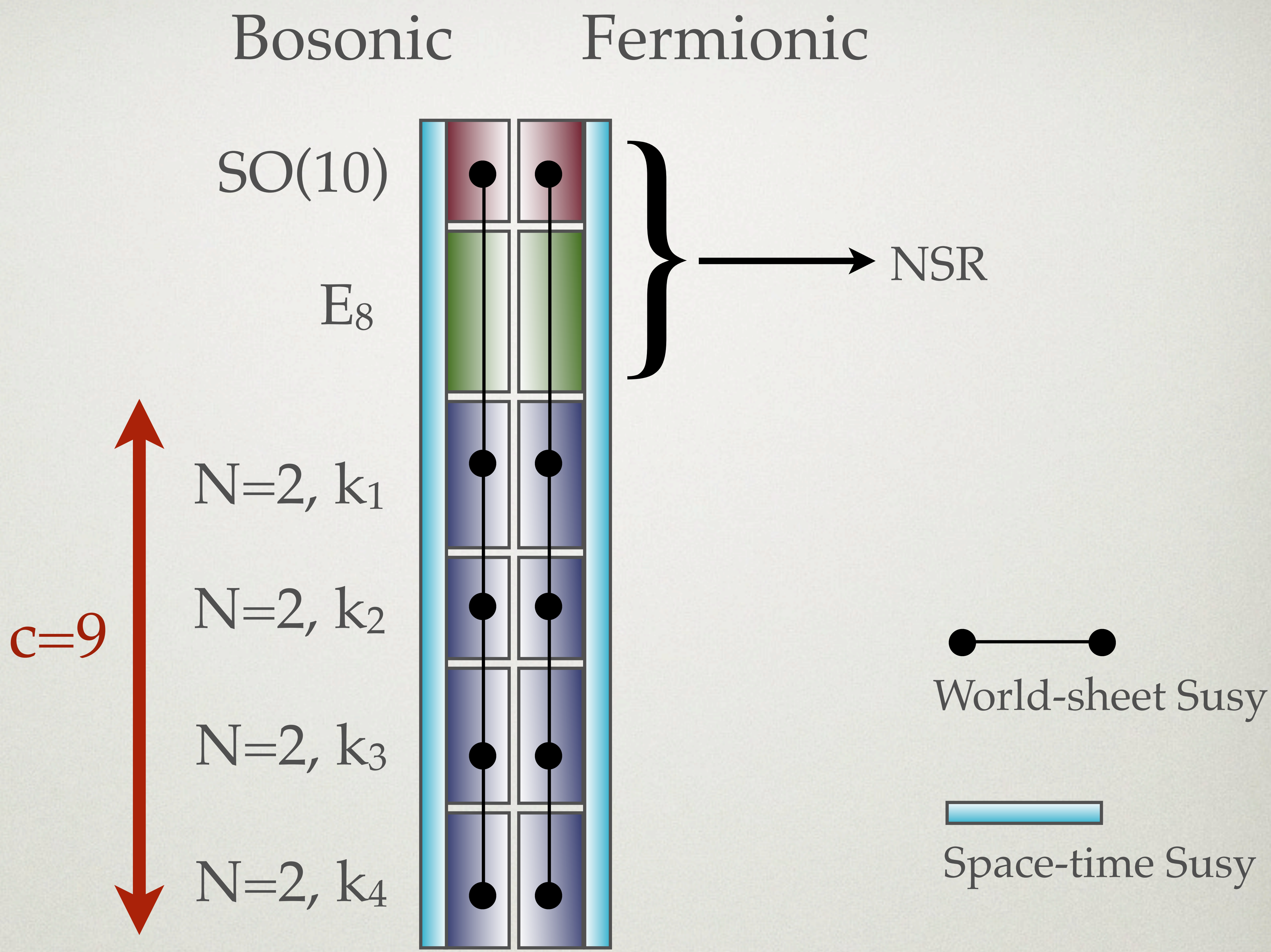
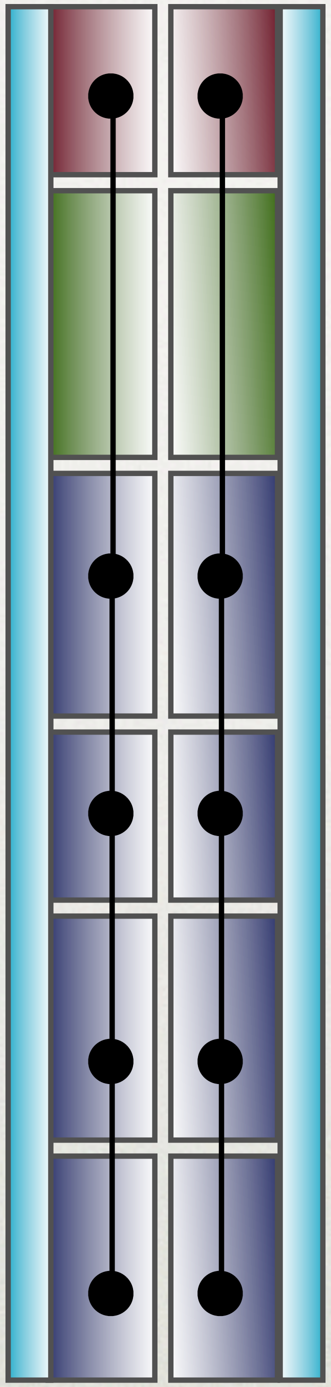
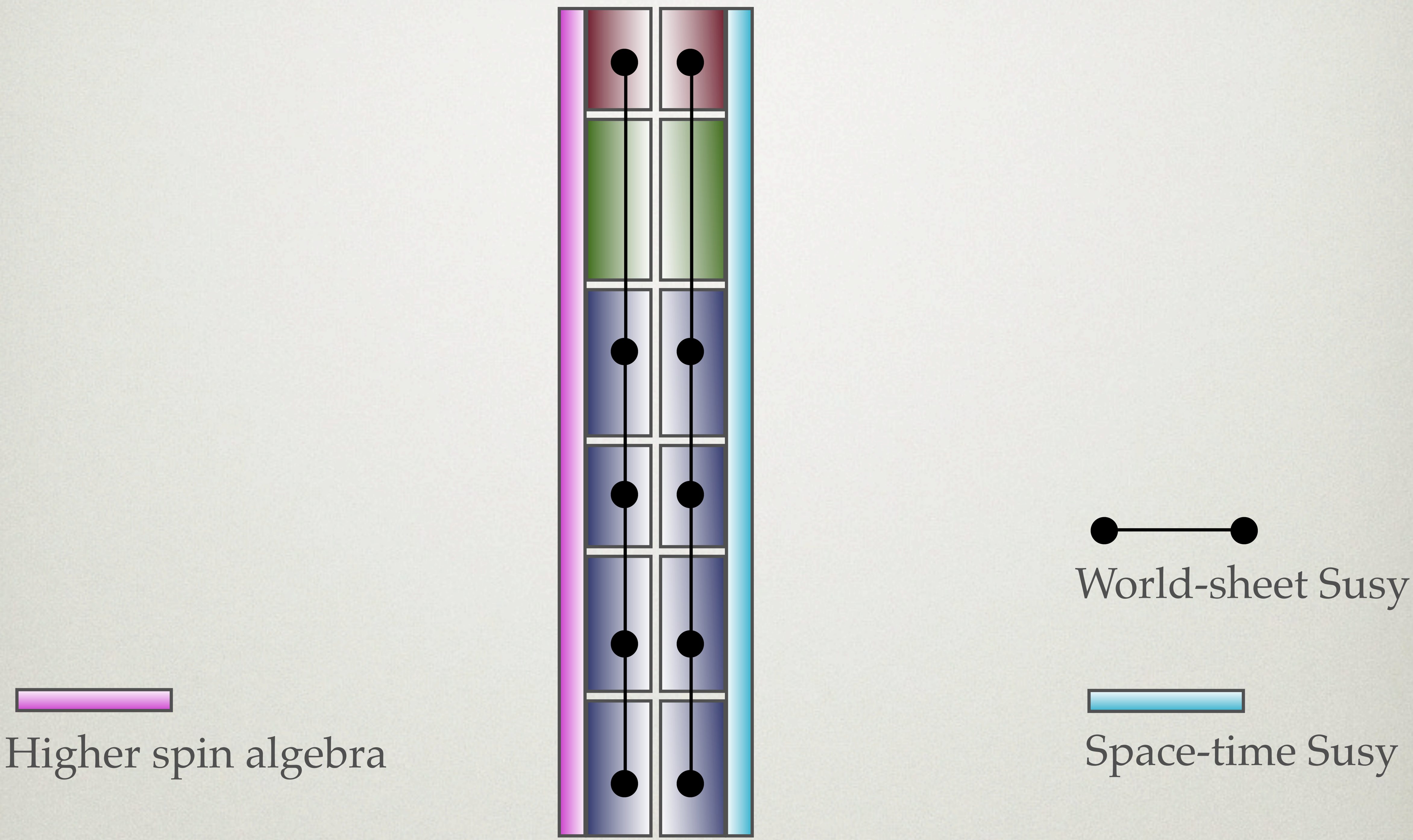
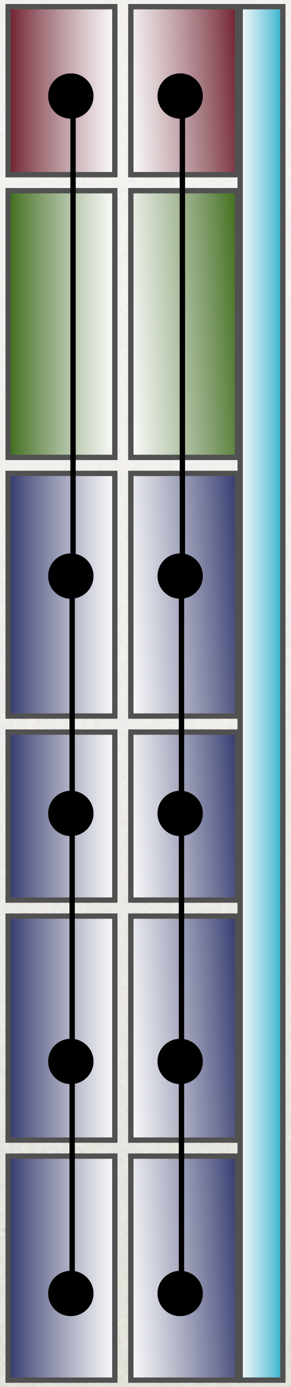


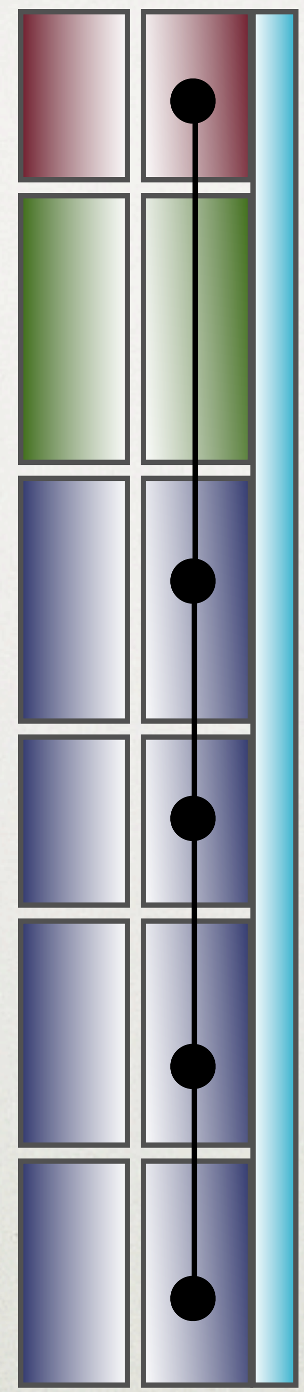
(2,2) model. Gauge group E_6 ($\times$ E_8 $\times$ ....)





(1,2) model. Gauge group SO(10) (× E_8 × ....)





(0,2) model. Gauge group SO(10) (× E_8 × ....)

Old results on Gepner model simple current MIPFs

Schellekens, Yankielowicz (1989): (2,2) , (1,2)
Fuchs, Klemm, Scheich, Schmidt (1989) (2,2)

Number of families:

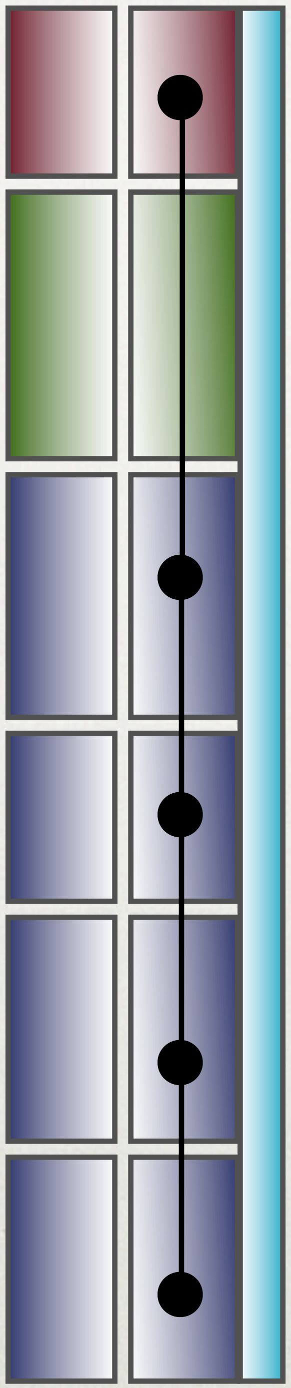
Define Δ: the greatest common divisor of the number of families for a given CFT

The following values of Δ occur for the 168 minimal model combinations and 58 of the 59 exceptional ones: 120, 96, 72, 60, 48, 40, 36, 32, 24, 12, 8, 6, 4 and 0.

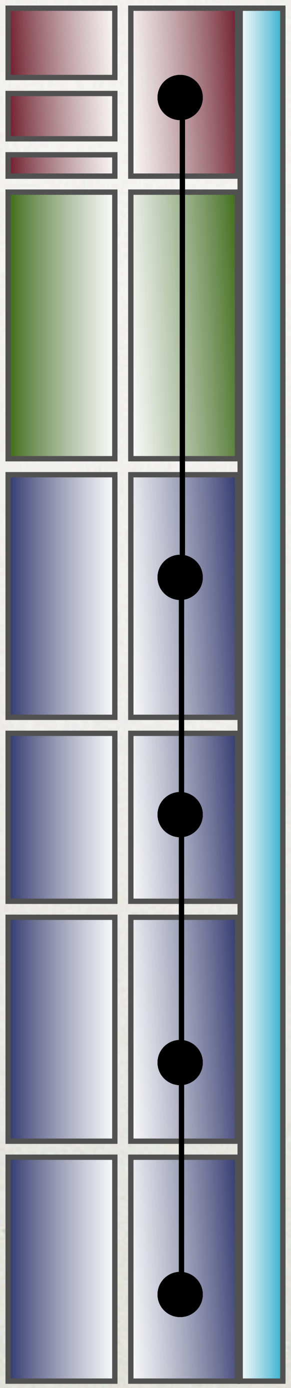
There is one case with Δ=3: (1,16*,16*,16*) (Gepner, unpublished).

This allowed us to get 3-family (2,2), (1,2) and (0,2) models with gauge groups E_6 or SO(10) (44 distinct ones)
[(0,2) was only tried for the (1,16*,16*,16*) combination]

SO(10) breaking



SO(10) currents replaced by operators of higher weight



(0,2) model. Gauge group $H \subset SO(10)$ ($\times$ $H' \subset E_8 \times$ ....)

Breaking SO(10)

 Y $\quad$ B-L

Consider* $SU(3) \times SU(2) \times U(1)_{30} \times U(1)_{20} \subset SO(10)$

This should give chiral families of $SU(3) \times SU(2) \times U(1)$
with standard gauge coupling unification.

Indeed, it does, but there was a major disappointment:
All these spectra contain fractionally charged particles.

This was easily seen to be a very general result.
(A.N. Schellekens, Phys. Lett. B237, 363, 1990).

(*) A.N. Schellekens and S. Yankielowicz (1989)
Other subgroups were considered by Blumenhagen, Wisskirchen, Schimmrigk (1995, 1996)

Sub-algebras

Current	Order	Gauge group	Group-Q	CFT-Q
1	30	$SO(10)$	1	1
2	15	$SU(4) \times SU(2) \times SU(2)$	$\frac{1}{2}$	$\frac{1}{2}$
3	10	$SU(3) \times SU(2) \times SU(2) \times U(1)$	$\frac{1}{6}$	$\frac{1}{3}$
5	6	$SU(5) \times U(1)$	1	1
6	5	$SU(3) \times SU(2) \times SU(2) \times U(1)$	$\frac{1}{6}$	$\frac{1}{6}$
10	3	-	$\frac{1}{6}$	$\frac{1}{2}$
15	2	-	$\frac{1}{6}$	$\frac{1}{3}$
30	1	-	$\frac{1}{6}$	$\frac{1}{6}$

To remove any of these sub-algebras we must be able to map these currents to a different current in the left sector.

This imposes constraints on the internal sector.

To project out the $SU(2)_R$ extension we need a simple current of order 5 (k_i+2 divisible by 5 for at least one *i*).
This extension is undesirable

To project out the half-integer charge constraint, we need one *i* with k_i+2 divisible by 3.
This extension is desirable.

New results on Gepner model simple current MIPFs

Gato-Rivera, Schellekens (2010): *(2,2) , (1,2), (0,2), broken SO(10)*

Number of families:

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The cases with Δ=2 exclude all numbers of families divisible by 3 (0,6,12,....)

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Δ=2: (6,6,6,6)
(3,3,3,3,3)
(3,6,6,18)
(3,3,18,18)
(3,3,12,33)
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Obvious pattern.

Appears to extend to other cases
(Free fermions, Kazama-Suzuki*, in the latter there are a few cases with Δ=3)

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(*) Ibanez, Font, Quevedo (1989)
Schellekens (1991)
[only (2,2) diagonal known]

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Number of families:

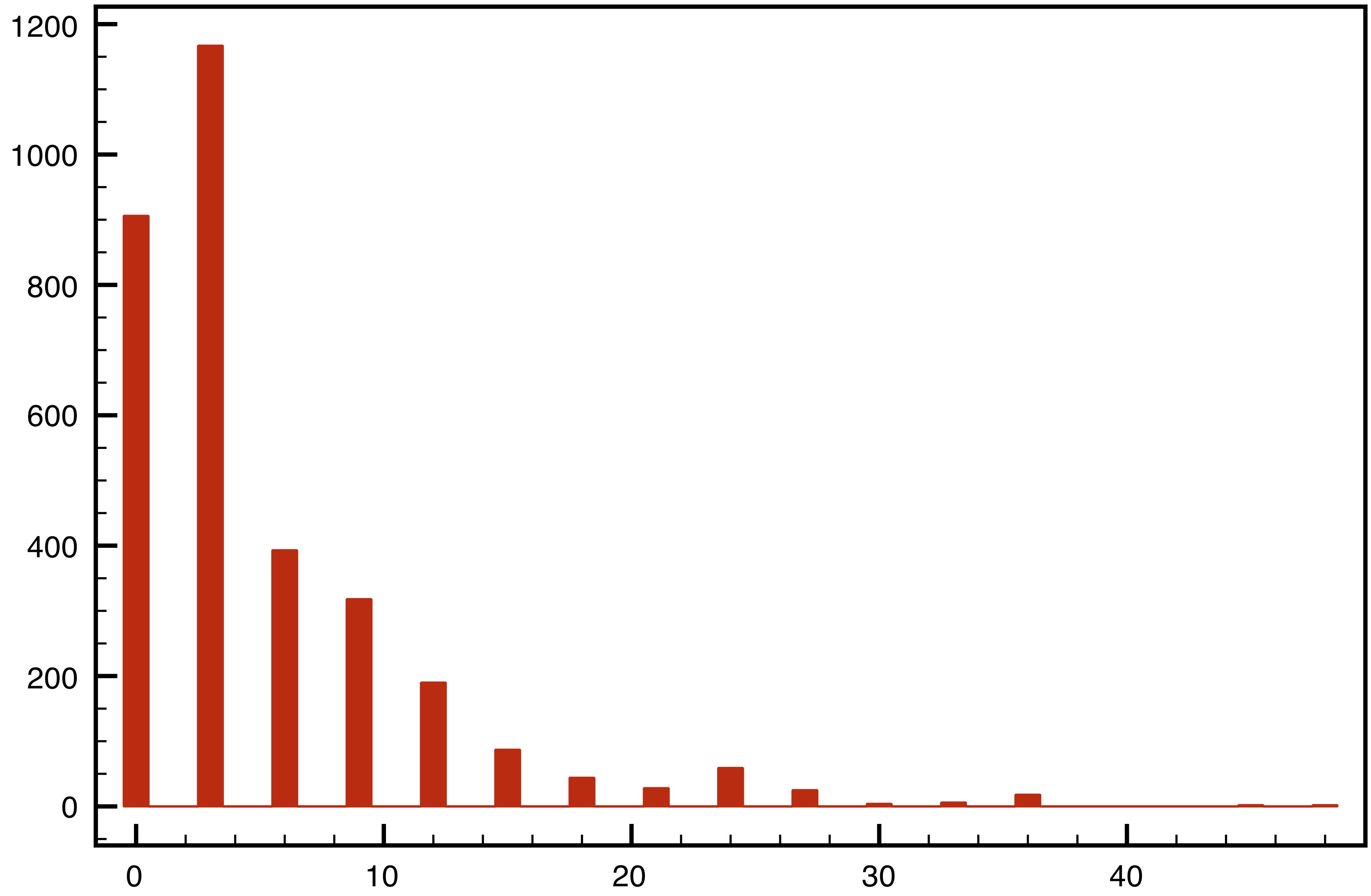
There is one case with $\Delta=3$: $(1,16^*,16^*,16^*)$

This yields 3-family (2,2), (1,2) and (0,2) models with gauge groups
$SU(3) \times SU(2) \times SU(2) \times U(1)$ or $SO(10)$
(also $SU(3) \times SU(3) \times SU(3)$ or E_6)

$>$ 1100 distinct 3-family spectra

$SU(2)_R$ remains always unbroken (hence no SU(5) models)
Fractional charges (if any) are always third-integer (hence no Pati-Salam models).

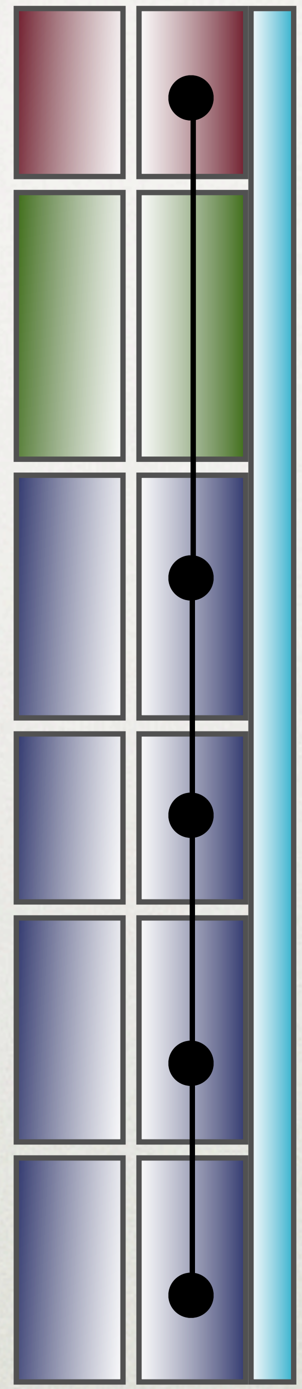
$(1, 16_E, 16_E, 16_E)$

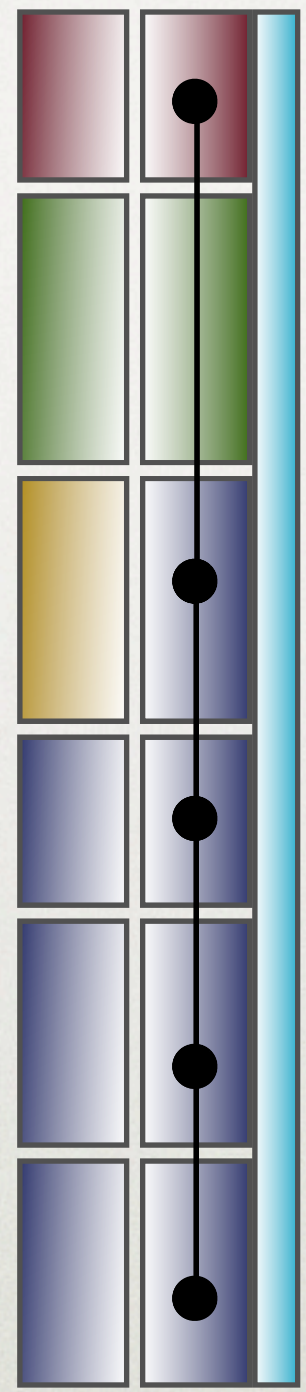


$SU(3) \times SU(2) \times SU(2) \times U(1)$

Representation	Particles	Multiplicity
$(3,2,1,\frac{1}{6})$	Q	3
$(3^*,1,2,-\frac{1}{6})$	$U^* + D^*$	4+1*
$(1,2,1,-\frac{1}{2})$	L	5+2*
$(1,1,2,\frac{1}{2})$	$E^* + N^*$	5+2*
$(3^*,1,1,\frac{1}{3})$	D^*	5+5*
$(1,2,2,0)$	$H_1 + H_2$	9
$(1,1,0,0)$	singlets	80
$(1,1,1,\frac{1}{3})$		41+41*
$(1,1,2,-\frac{1}{6})$		20+20*
$(1,2,1,-\frac{1}{6})$	Charge	19+19*
$(3,1,1,0)$		17+17*
$(3,1,1,\frac{1}{3})$		8+8*
$(3,2,1,-\frac{1}{6})$	1/3	3+3*
$(3^*,1,2,\frac{1}{6})$		3+3*
$(1,2,2,\frac{1}{3})$		2+2*
$(1,1,1,-\frac{2}{3})$		2+2*

Heterotic weight lifting

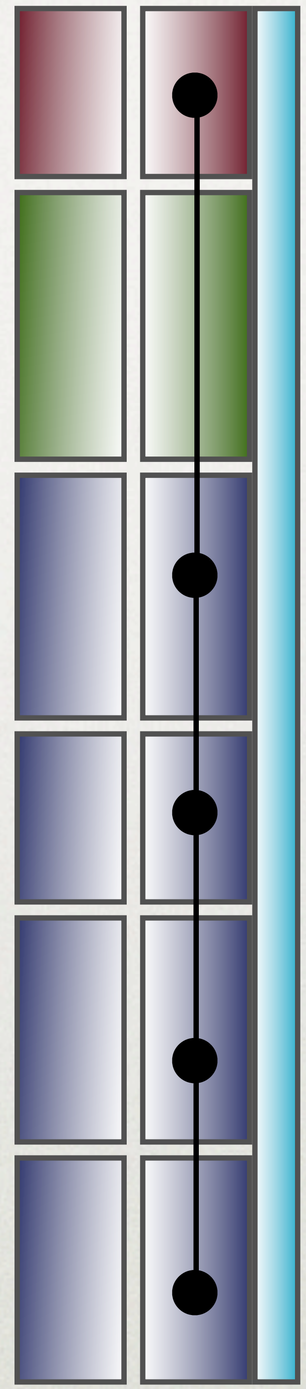


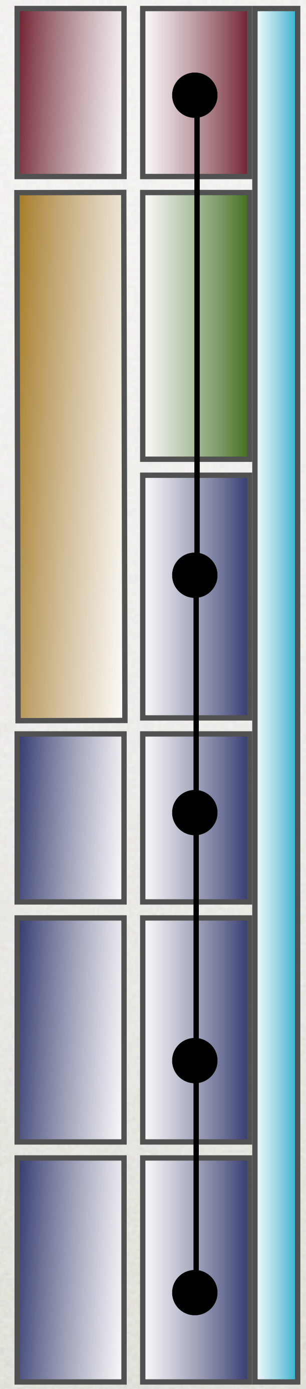


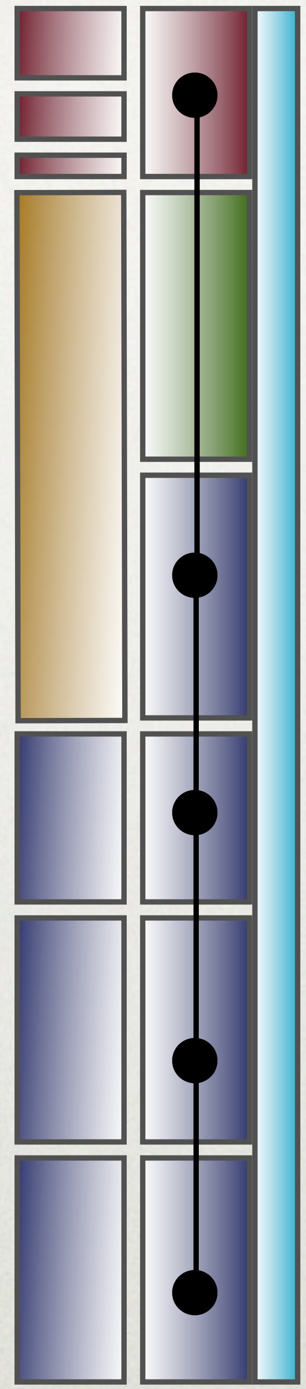
... but we have to find a N=0 CFT with the same S, T, and central charge as some N=2 model, without being identical to it.

This looks difficult.

But there is something else we could try:







So our goal is to find, for some minimal N=2 model with central charge c, a replacement that has central charge c+8, and exactly the same S and T matrices.

Hence it must have the same number of primaries, and the same spectrum, up to integers.

Minimal N=2 model at level k: $c = \frac{3k}{k+2}$

Coset description:

$$\frac{SU(2)_k \times SO(2)}{U_{k+2}}$$

Plus “field identification”

(Gepner; Schellekens and Yankielowicz, 1989)

Field identification is a formal simple current extension of the coset CFT by a current of spin 0. This relates multiple vacua.

This "extends" the chiral algebra so that the identity representation is doubled, and roughly half the states (that do not satisfy the G/H selection rules) are removed.

The coset CFT may be thought of as tensor product

$$SU(2)_k \times SO(2) \times U(1)^c_{k+2}$$

Where $U(1)^c$ is the "complement": an auxillary representation of the modular group with complex conjugate S and T matrices, and c=−1+8N

Now we remove the field identification extension, and consider

$$SU(2)_{k+2} \times SO(2) \times \frac{E_8}{U_{k+2}}$$

In other words, we embed the U(1) in E_8 instead of SU(2) x SO(2).

Next we identify a CFT X_7 which can be combined with U_{k+2} to E_8, so that

$$E_8 = [U_{k+2} \times X_7]_{\text{ext}}$$

Then we can write the CFT as

$$SU(2)_{k+2} \times SO(2) \times X_7$$

And finally we re-establish the equivalent of the field identification, as a standard, higher spin extension

The result is guaranteed, by construction, to have the same S and T matrices as the original minimal model.

But the spectrum is different

Standard coset field $h_i^G - h_j^H \qquad (j \in i)$

Replacement $h_i^G + h_j^{H^c}$

$$h_j^{H^c} = -h_j^H \bmod 1$$

All weight of H and H^c are positive
Therefore standard weights are lifted:

$$h_i^G + h_j^{H^c} > h_i^G - h_J^H$$

(but equal mod 1)

The simplest class of examples: find a U(1) in E_8 through subgroup embeddings:

For example the Standard Model U(1), Y

$$SU(3) \times SU(2) \times U(1)_{30} \times U(1)_{20} \subset SO(10)$$
$$SO(10) \times SO(6) \subset E_8$$

This implies

$$\frac{E_8}{U_{30}} = A_{2,1} A_{1,1} A_{4,1}$$

And hence

$$(N = 2, k = 13) \sim A_{1,13} U_4 A_{2,1} A_{1,1} A_{4,1}$$

Extended by the current (J,v,0,J,0)

The minimal N=2, k=13 model has 420 primaries. We have compared the S and T matrices explicitly, and they are identical.

But many states in the spectrum are shifted:

136 massless ($h \leq 1$) are lifted(*)
81 massive ones become massless
37 are massless before and after
166 are massive before and after

(*) Including all Ramond ground states

Other Lifts

- So far we found 30; there may be more.
- For several values of k there is more than one.
- There are also double lifts. Perhaps also triple and quadruple lifts.
- Single lifts give rise to 435 lifted Gepner models.

k	Lift	Lifted	Lowered	Unchanged
1	$E_6 \times A_1$	4	1	4
2	A_7	7	1	12
3	$[D_6 \times U_{10}]_{\text{ext}}$	10	3	22
4	$D_5 \times A_2$	21	4	23
5	$A_6 \times A_1$	32	8	29
5	$[E_6 \times U_{42}]_{\text{ext}}$	24	11	37
6	$[A_6 \times U_{112}]_{\text{ext}}$	33	15	39
8	$A_4 \times A_3$	65	29	37
9	$[A_6 \times U_{154}]_{\text{ext}}$	76	41	39
11	$[E_6 \times U_{78}]_{\text{ext}}$	104	61	39
11	$[D_6 \times U_{26}]_{\text{ext}}$	98	60	45
12	$A_6 \times U_4$	125	66	39
13	$A_4 \times A_2 \times A_1$	136	81	37
14	$[A_4 \times A_2 \times U_{480}]_{\text{ext}}$	147	105	47
14	$[A_6 \times U_{224}]_{\text{ext}}$	153	95	41
17	$[E_6 \times U_{114}]_{\text{ext}}$	202	105	37
17	$[A_4 \times A_2 \times U_{570}]_{\text{ext}}$	198	133	41
19	$E_6 \times U_{14}$	228	119	42
20	$[A_6 \times U_{308}]_{\text{ext}}$	243	143	42
23	$[D_6 \times U_{50}]_{\text{ext}}$	300	161	41
26	$A_6 \times U_8$	349	199	39
30	$[A_6 \times U_{448}]_{\text{ext}}$	417	235	46
41	$[E_6 \times U_{258}]_{\text{ext}}$	610	297	44
41	$[A_6 \times U_{602}]_{\text{ext}}$	606	325	48
42	$[A_6 \times U_{616}]_{\text{ext}}$	627	337	46
44	$[A_6 \times U_{644}]_{\text{ext}}$	673	361	42
44	$[A_4 \times A_2 \times U_{1380}]_{\text{ext}}$	659	465	56
47	$[E_6 \times U_{294}]_{\text{ext}}$	728	367	46
54	$A_6 \times U_{16}$	857	455	51
58	$A_4 \times A_2 \times U_8$	923	611	56
86	$[A_6 \times U_{1232}]_{\text{ext}}$	1501	741	52
89	$[E_6 \times U_{546}]_{\text{ext}}$	1556	705	49
238	$A_4 \times A_2 \times U_{32}$	4959	2729	73
1,1	$A_2 \times A_1 \times A_2 \times A_1$	16	1	14

Computing the spectrum

Very easy: start with the **full** spectrum of a standard Gepner model. For example, all states associated with a massless space-time spinor in the fermionic sector

$$\sum_j M_{ij}(\dim_1, h_1, \ldots, \dim_n, h_n)_j$$

To compute the consequences of "lifting" factor k, just replace $\dim_k$ and h_k by the corresponding values in the lift CFT

Chiral spectra?

All R ground states are lifted.
Hence no extension SO(10) → E_6
But also all chiral families are removed.

The diagonal MIPF yields, for (4,4,8,13)

Before lifting:

$$75(27)+3(\overline{27})+450(1) \quad \text{of } E_6$$

After lifting:

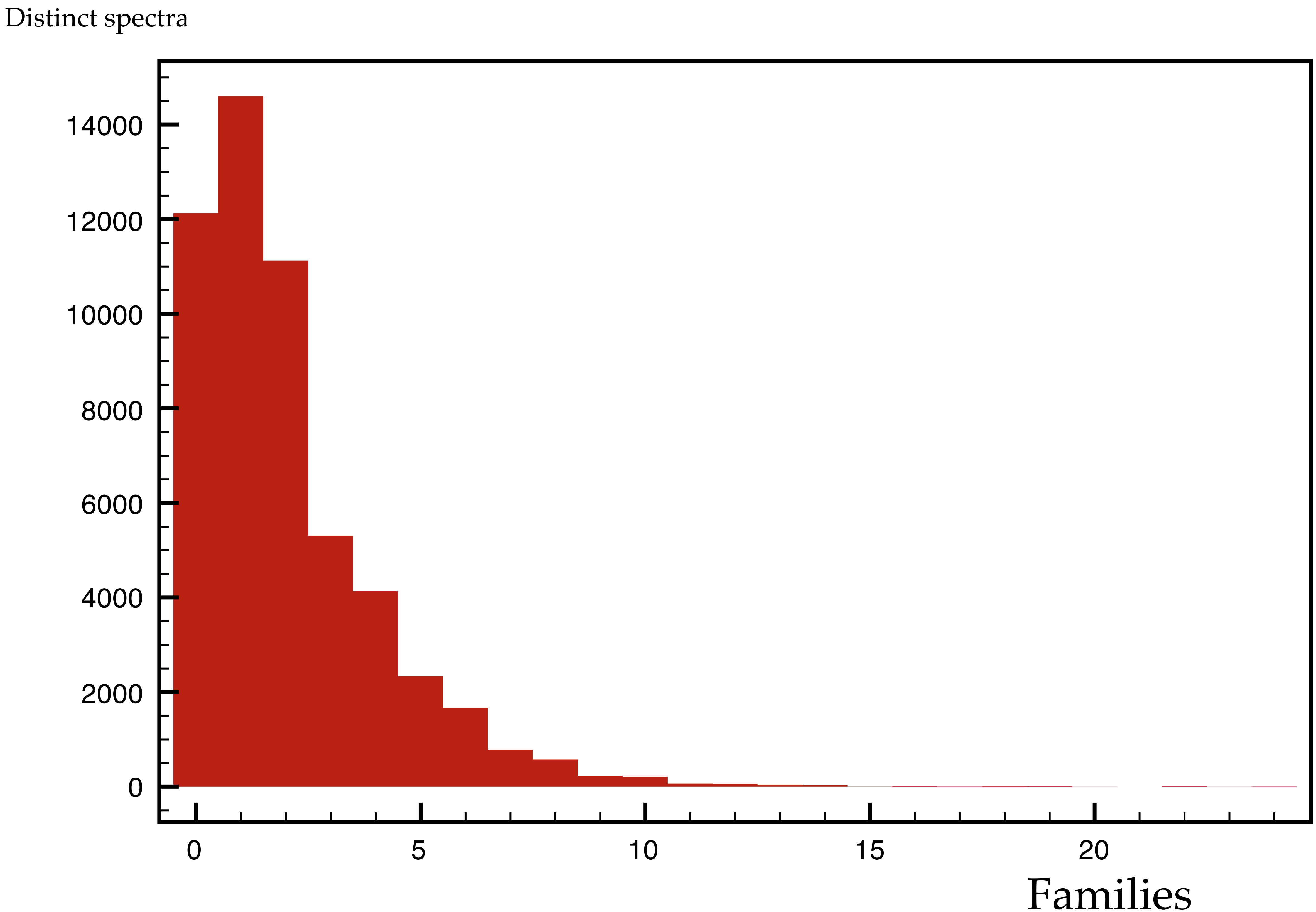
$$20\times(10)+1088(1) \text{ of } SO(10)$$

Chiral spectra?

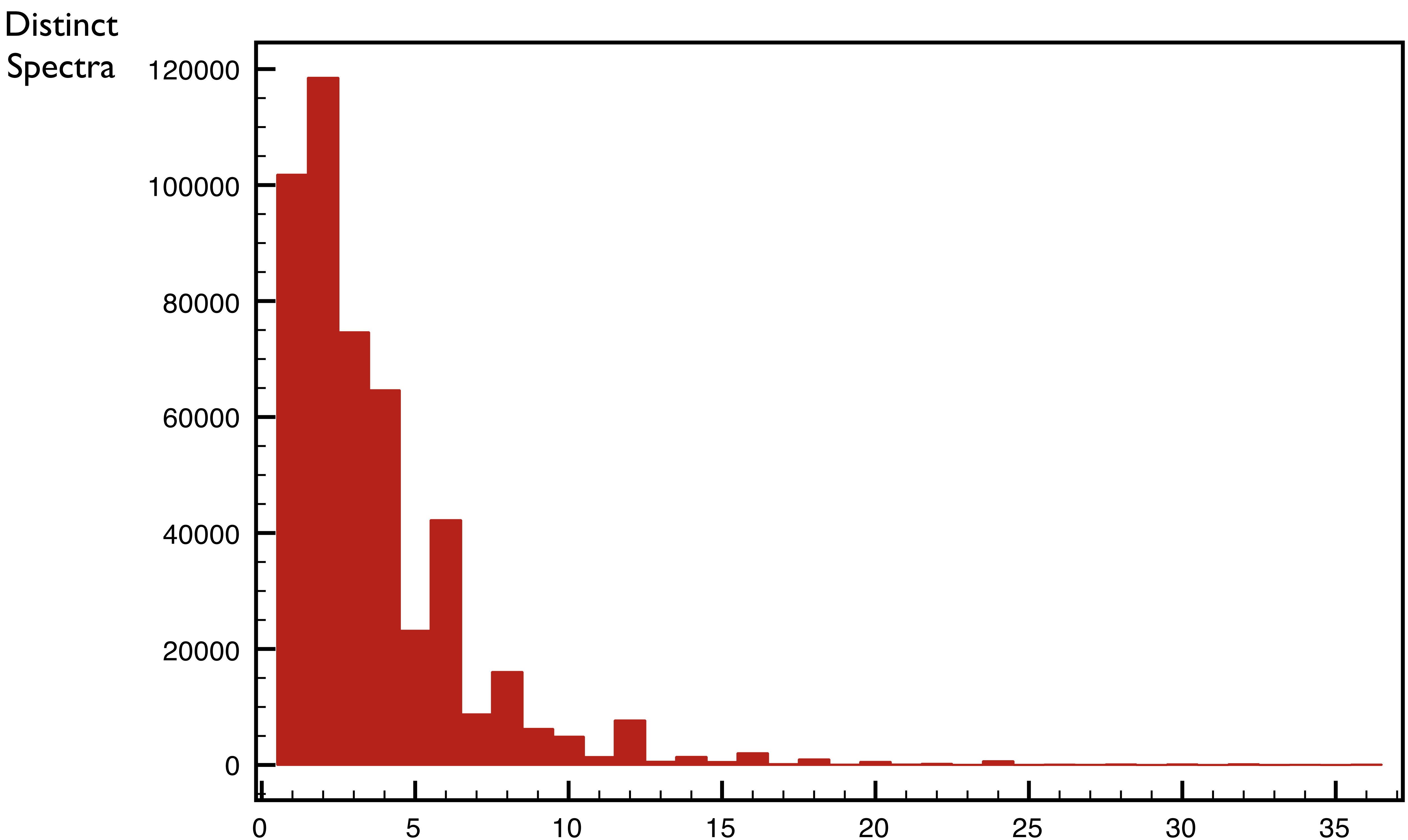
But now we can break all non-essential symmetries in the bosonic sector. In particular world-sheet susy.

So we do not need Ramond to get massless fermions!

And this is what came out:


Distinct spectra
14000
12000
10000
8000
6000
4000
2000
0
0
5
10
15
20
Families

$$(\hat{3}, 3, 3, 3, 3)$$



Family distribution for 435 lifted Gepner models

- For the first time in Heterotic Gepner models, family quantization in units of 1!
- Many cases with three families.
- Roughly exponential fall-off with the number of families
- Not as steep as in orientifolds.
- Three families relatively much more common.

Δ-Distribution

Δ	
0	94
1	198
2	57
3	60
4	8
5	0
6	18

Distinct 3-family: 75000 (out of 480000 N-family, $N \geq 1$)

(Modulo mirror symmetry: 41000)

Fractional Charges

Total MIPFs	620×10^6
Y-anomaly, chiral frac.	151×10^6
SO(10), SU(5) × U(1)	254×10^6
SU(5) (anomalous B-L)	112×10^6
Pati-Salam, frac. 1/2	79×10^6
Pati-Salam, no frac.	3×10^5
SM × U(1)/LRS , frac 1/6	5×10^6
SM × U(1)/LRS , frac 1/3	14×10^6
SM × U(1)/LRS , frac 1/2	2×10^6
SM × U(1)/LRS , no frac	25×10^3
SM (anomalous B-L), frac 1/6	1×10^5
SM (anomalous B-L), frac 1/3	3×10^5
SM (anomalous B-L), frac 1/2	14×10^5
SM (anomalous B-L), no frac	163

Some features

- Many different gauge groups, from just $SU(3) \times SU(2) \times U(1)$ to $SO(10)$.
- Additional non-abelian gauge group from the lift CFT.
- Distribution of number of mirrors from a few tens to zero.
- Examples with no mirror fermions at all
- Examples with $3 \times (16) + (10)$ of $SO(10)$
  (exactly the minimal $SO(10)$ susy-GUT.
- Examples with just* $SU(3) \times SU(2) \times U(1)$ and B-L broken by anomalies.

(*) from SO(10)

Approaching the SM

An example from $(3, \widehat{8}, 8, 8)$

Gauge group:

$$SU(3) \times SU(2) \times U(1) \times [SU(2)_8 \times SO(2) \times SU(4) \times SU(5)] \times U(1)^3$$

(anomalous "B-L")

Spectrum:

$$3 \times (Q + U^c + D^c + L + E^c) + 3 \times (D + D^c) + 3 \times (H_1 + H_2)$$

$$+ \text{ 250 singlets}$$

$$+ \text{ 172 fractionally charged particles}$$

Fractional charges:

Non-chiral.
Only half-integer (no sixth or third).
Confined by $SU(2)_8$

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Singlets: (of SU(3) × SU(2) × U(1))

Only three are absolute singlets of the full gauge group.
Many are in nontrivial SU(4) and SU(5) reps.

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Only three are absolute singlets of the full gauge group.
Many are in nontrivial SU(4) and SU(5) reps.

The bad news:

U^c, D^c and E^c are in the triplet representation of $SU(2)_8$;
Higgs candidates and weak doublets are $SU(2)_8$ singlets.

B-L lifting

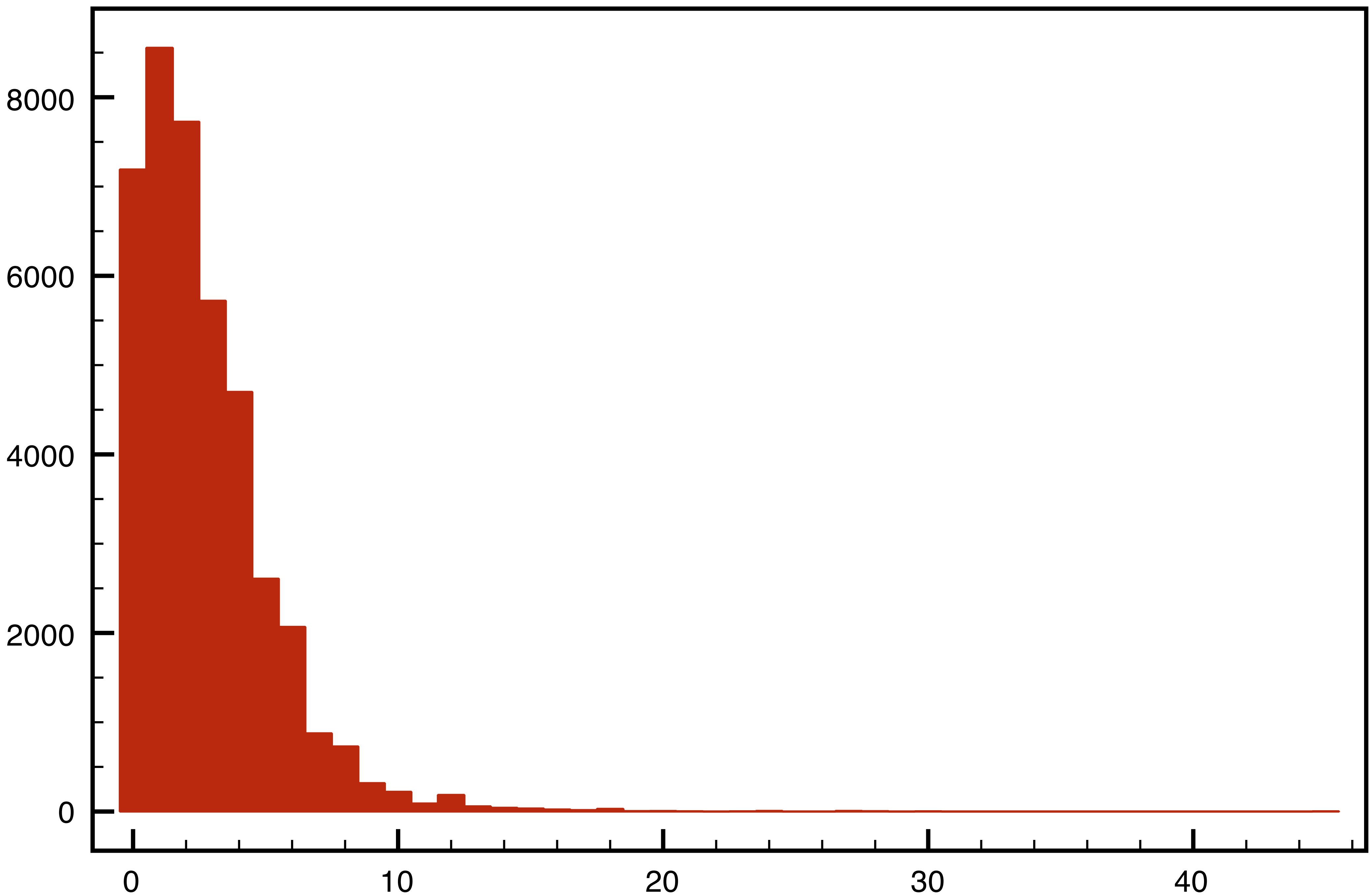
Extra U(1) ("B-L") in SO(10) not needed.

$E_8 \times U(1)_{B-L}$ can be replaced, in the bosonic sector by an isomorphic CFT.

For this purpose we may use SO(10) × SU(5).

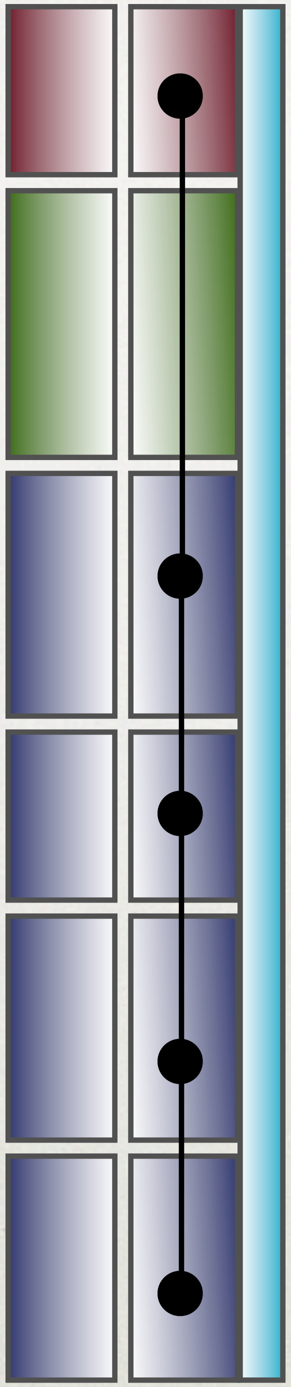
This works for any (2,2)-based construction.

(3,8,8,8)

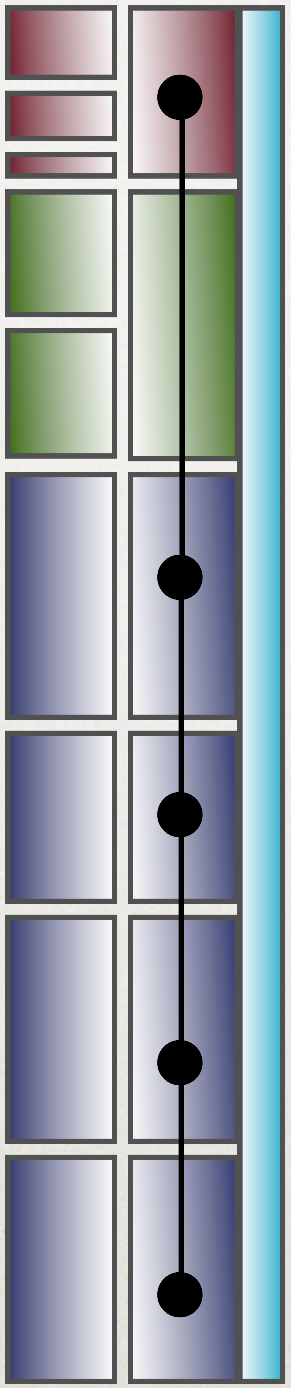


Family distribution (B-L lifting)

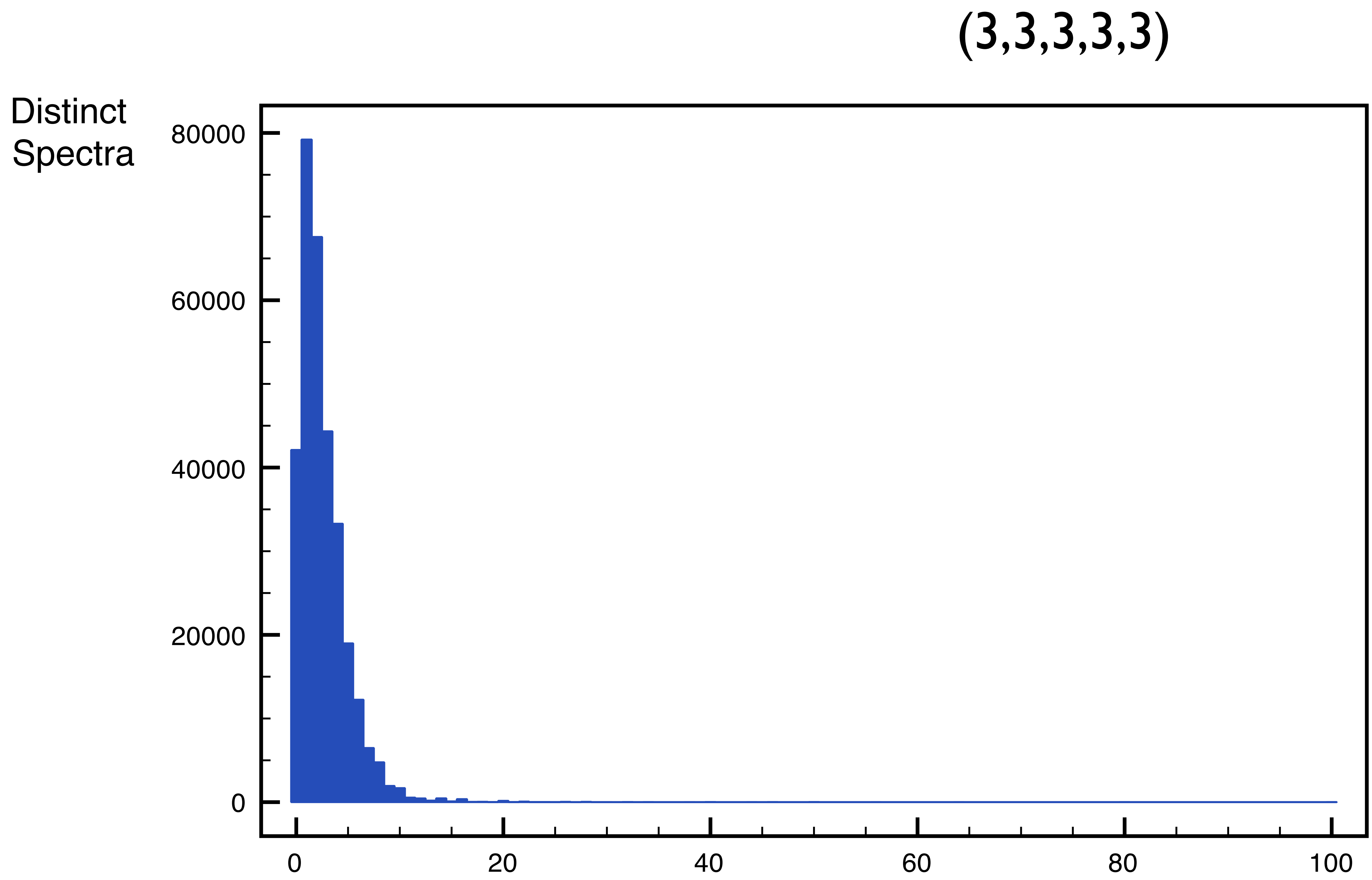
E_8 BREAKING



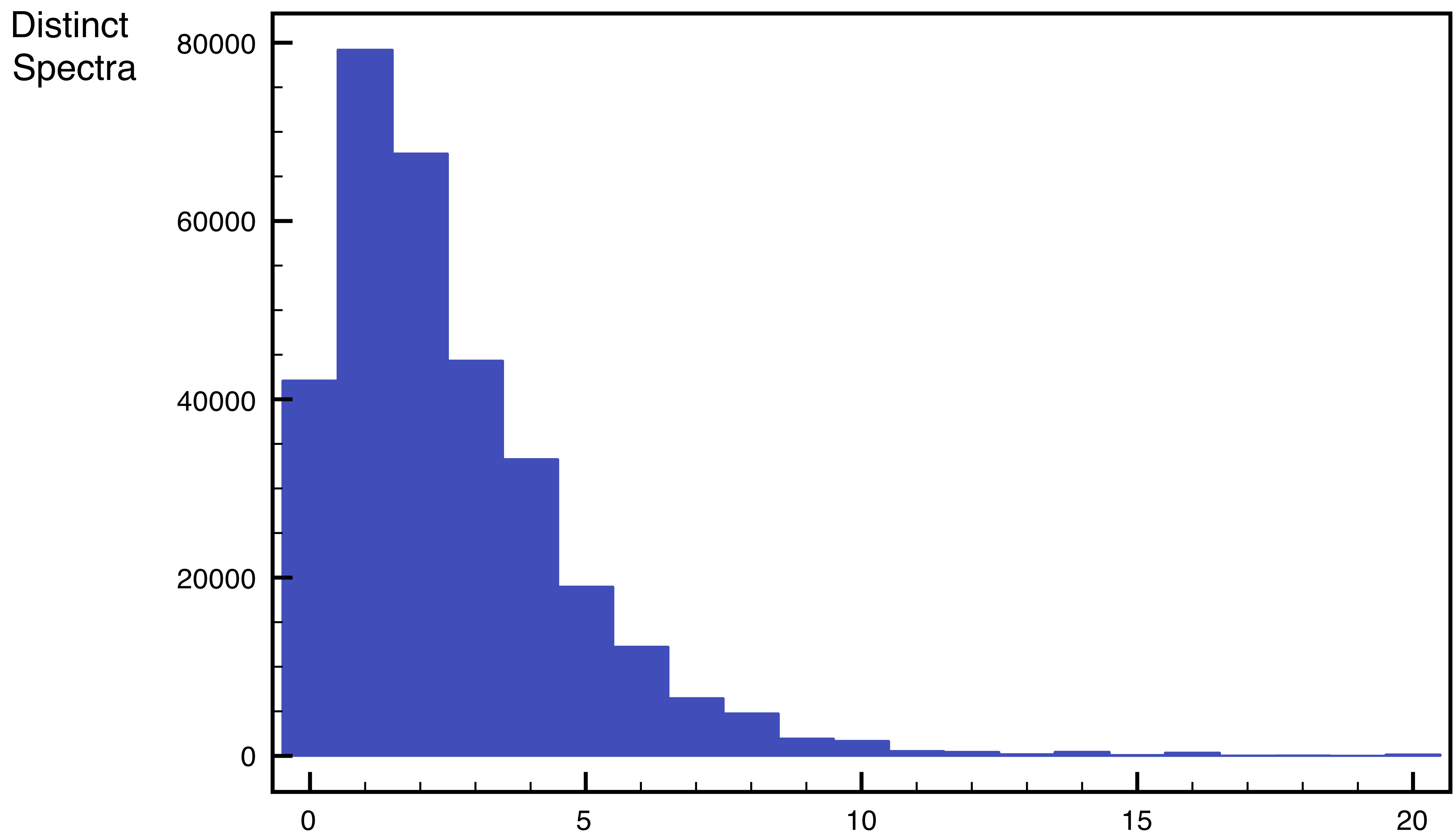
SO(10) and E_8 currents replaced by operators of higher weight



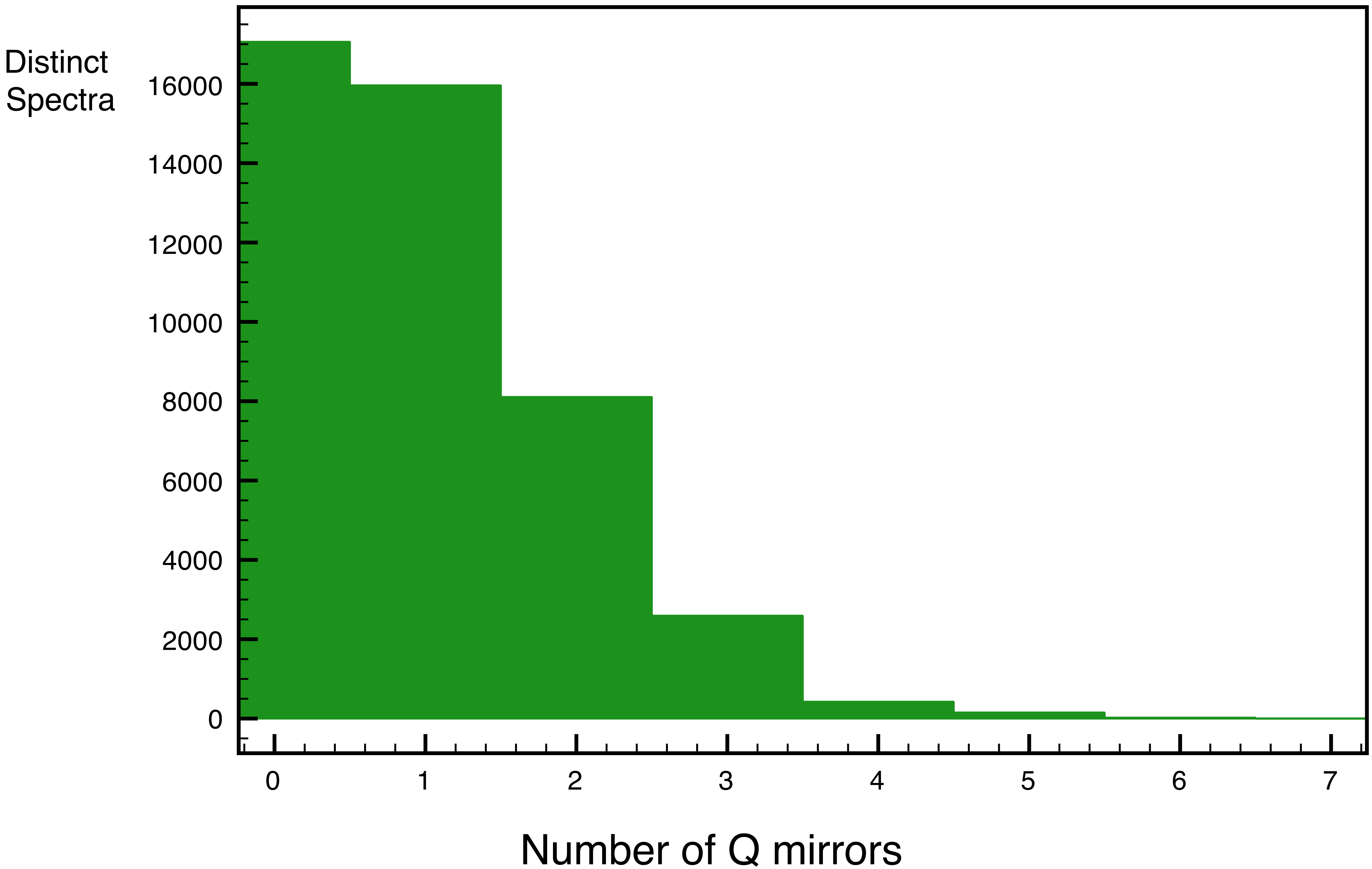
(0,2) model. Gauge group $H \subset SO(10) \times ...$

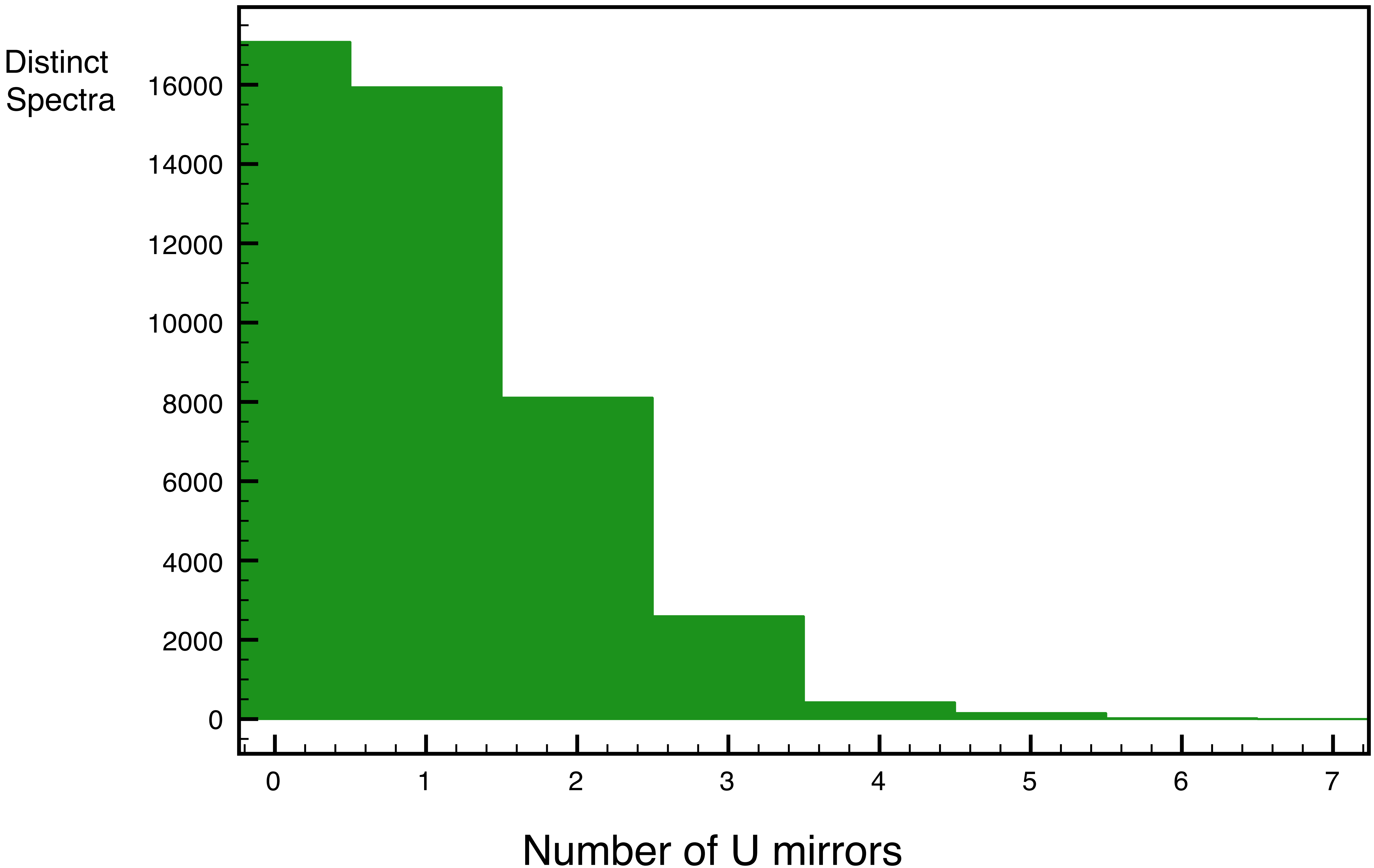


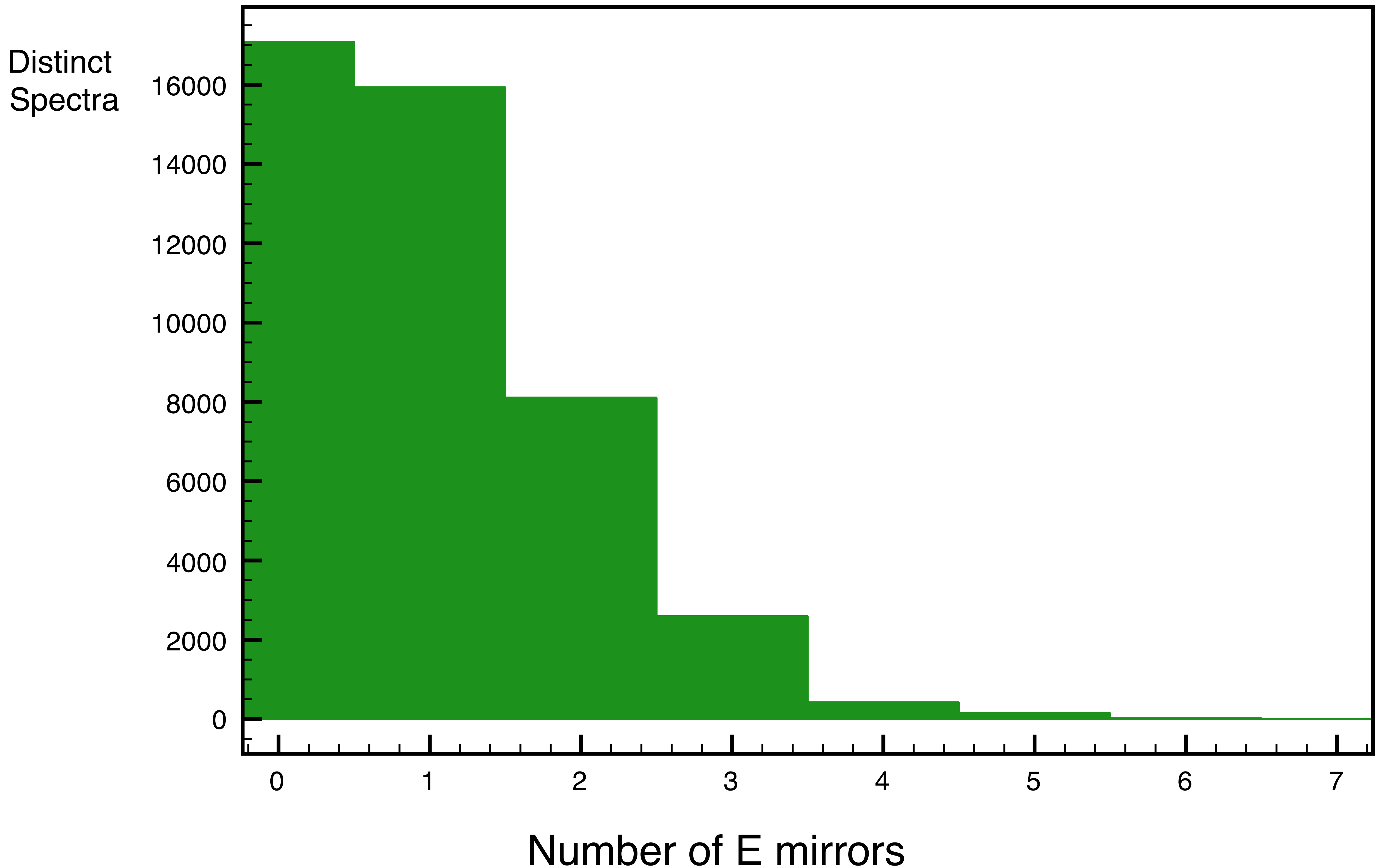
Family distribution for E_8 split to $A_4 \times A_4$

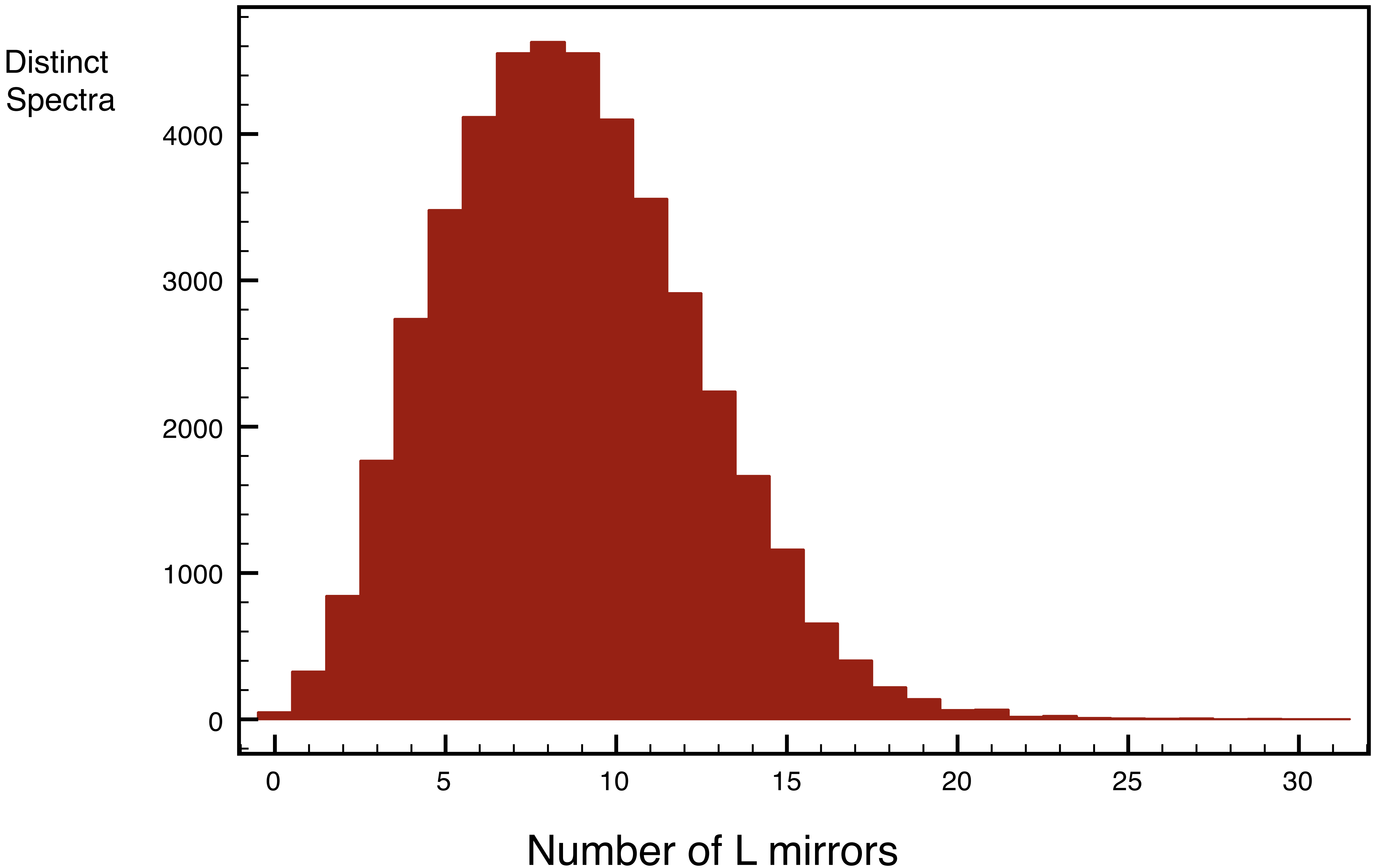


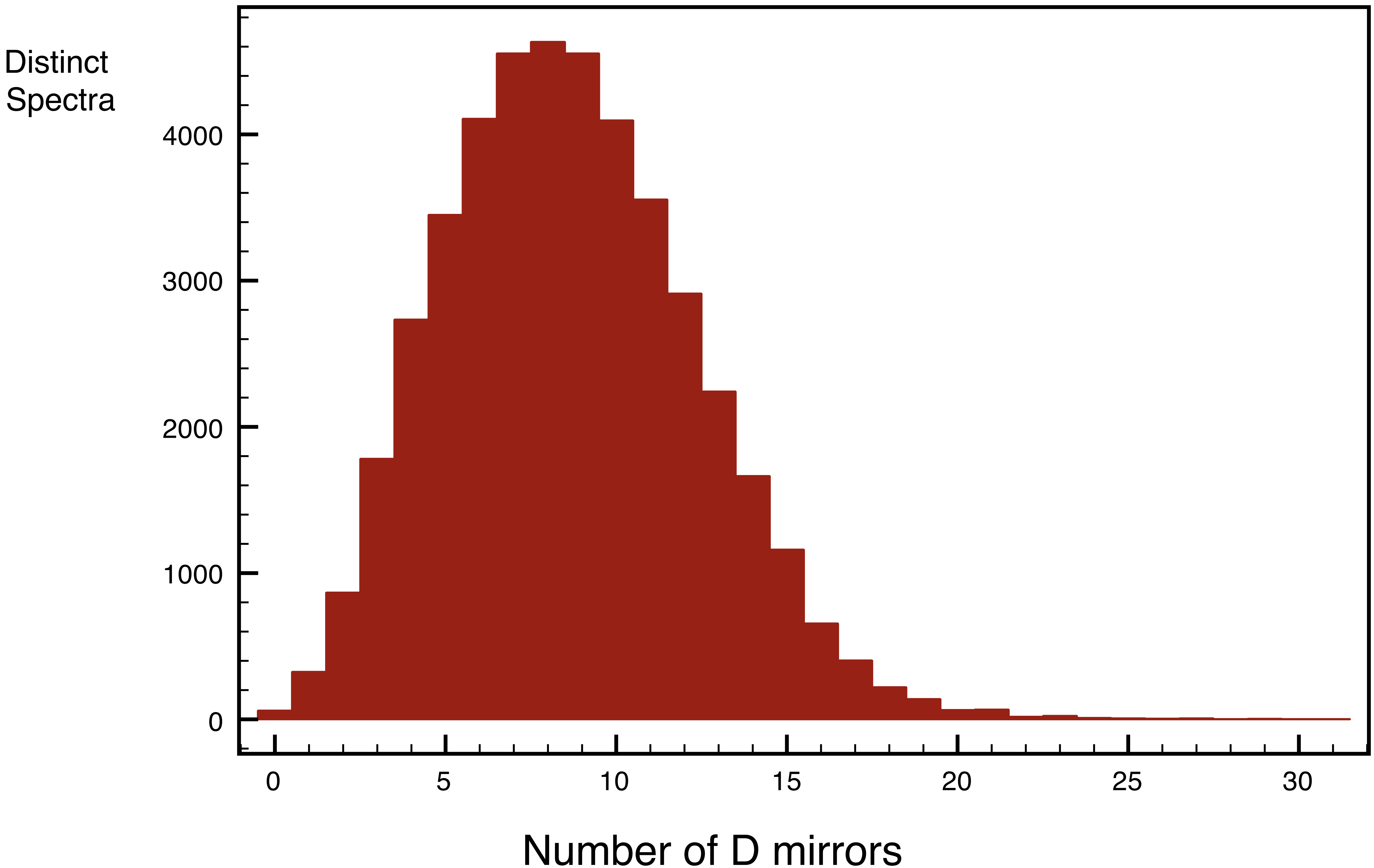
Family distribution for E_8 split to $A_4 \times A_4$ (up to 20 families)


Distinct
Spectra
16000
14000
12000
10000
8000
6000
4000
2000
0
0
1
2
3
4
5
6
7
Number of Q mirrors


Distinct
Spectra
16000
14000
12000
10000
8000
6000
4000
2000
0
0
1
2
3
4
5
6
7
Number of U mirrors

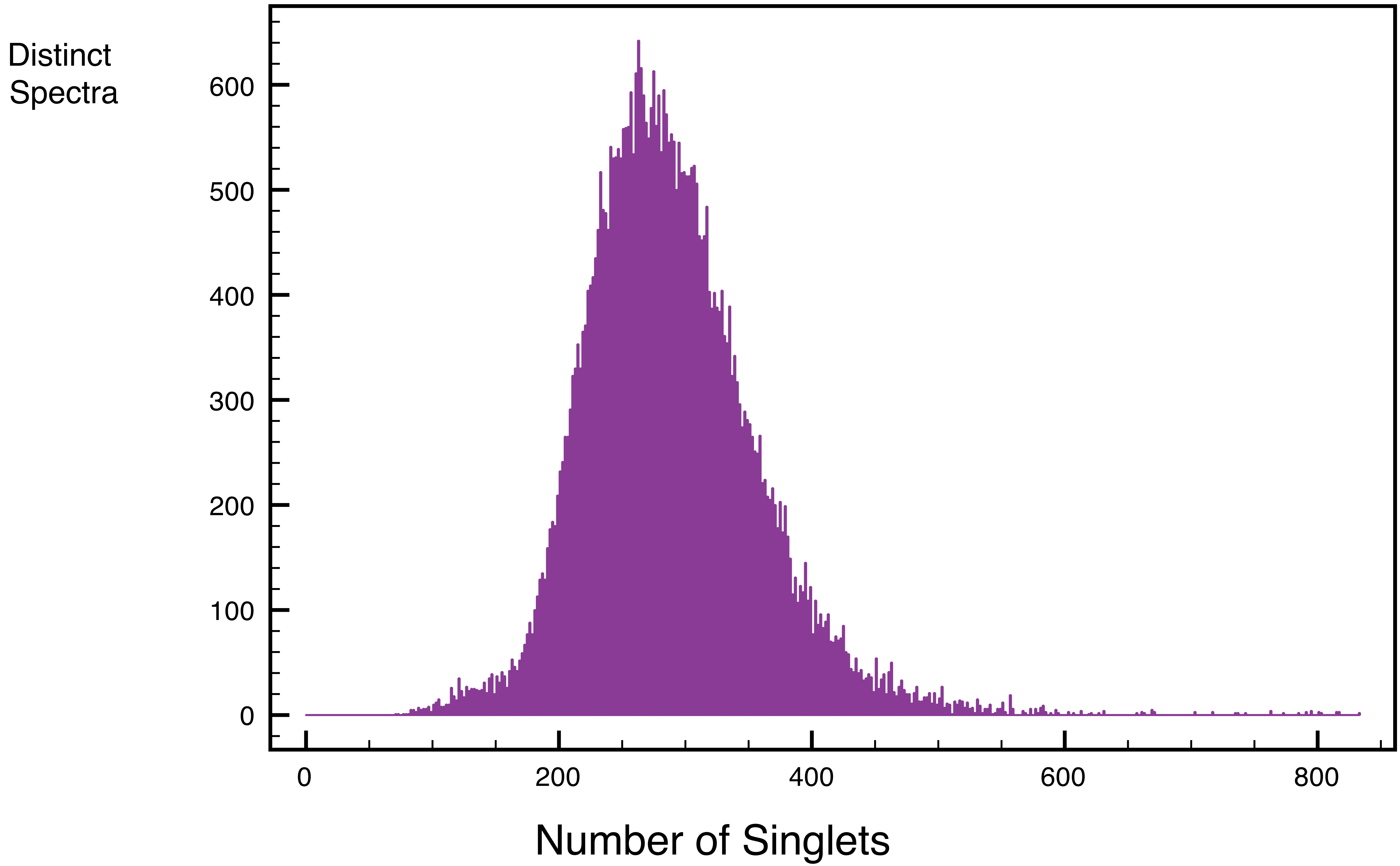

Distinct
Spectra
16000
14000
12000
10000
8000
6000
4000
2000
0
0
1
2
3
4
5
6
7
Number of E mirrors

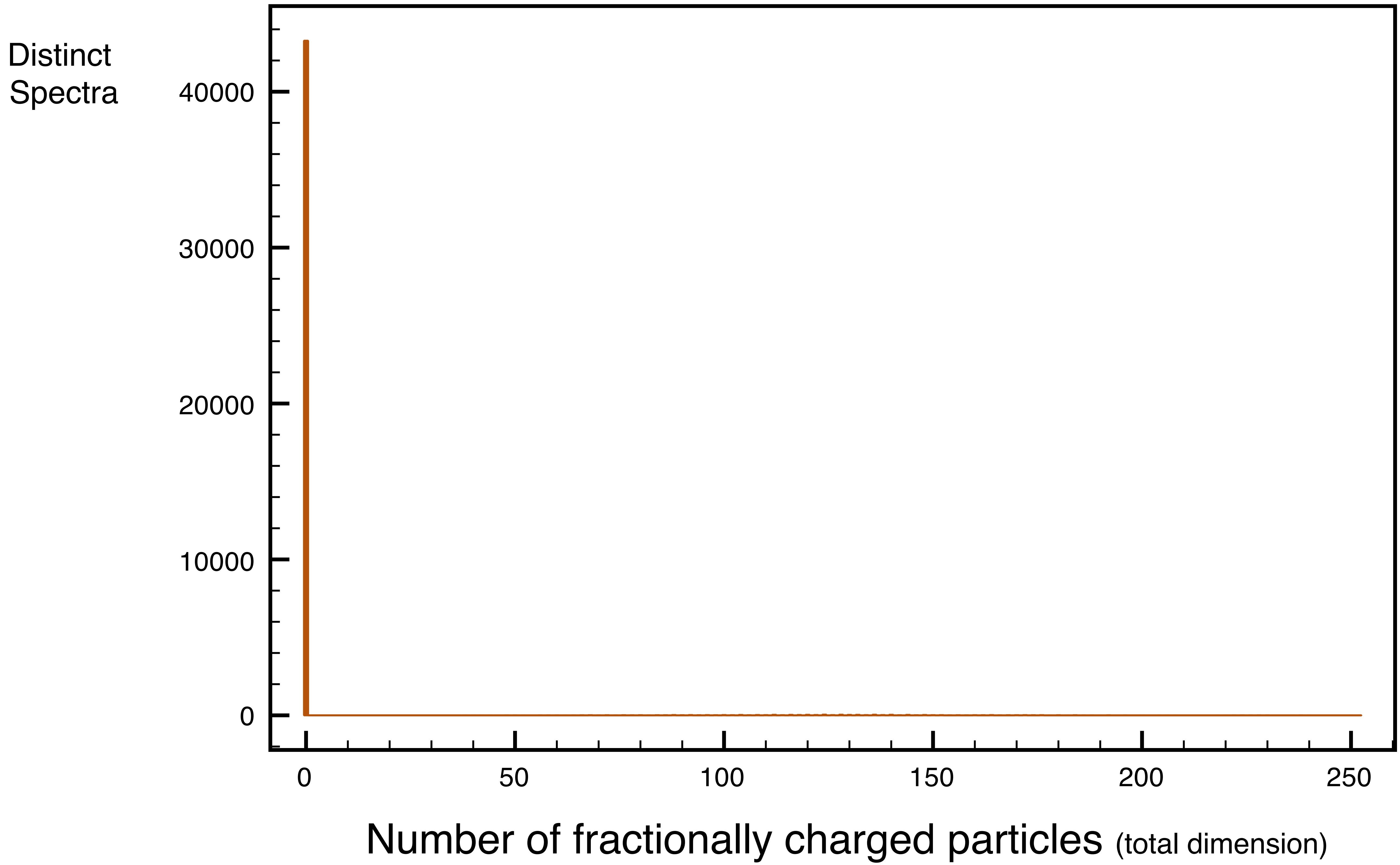

Distinct
Spectra
4000
3000
2000
1000
0
0
5
10
15
20
25
30
Number of L mirrors

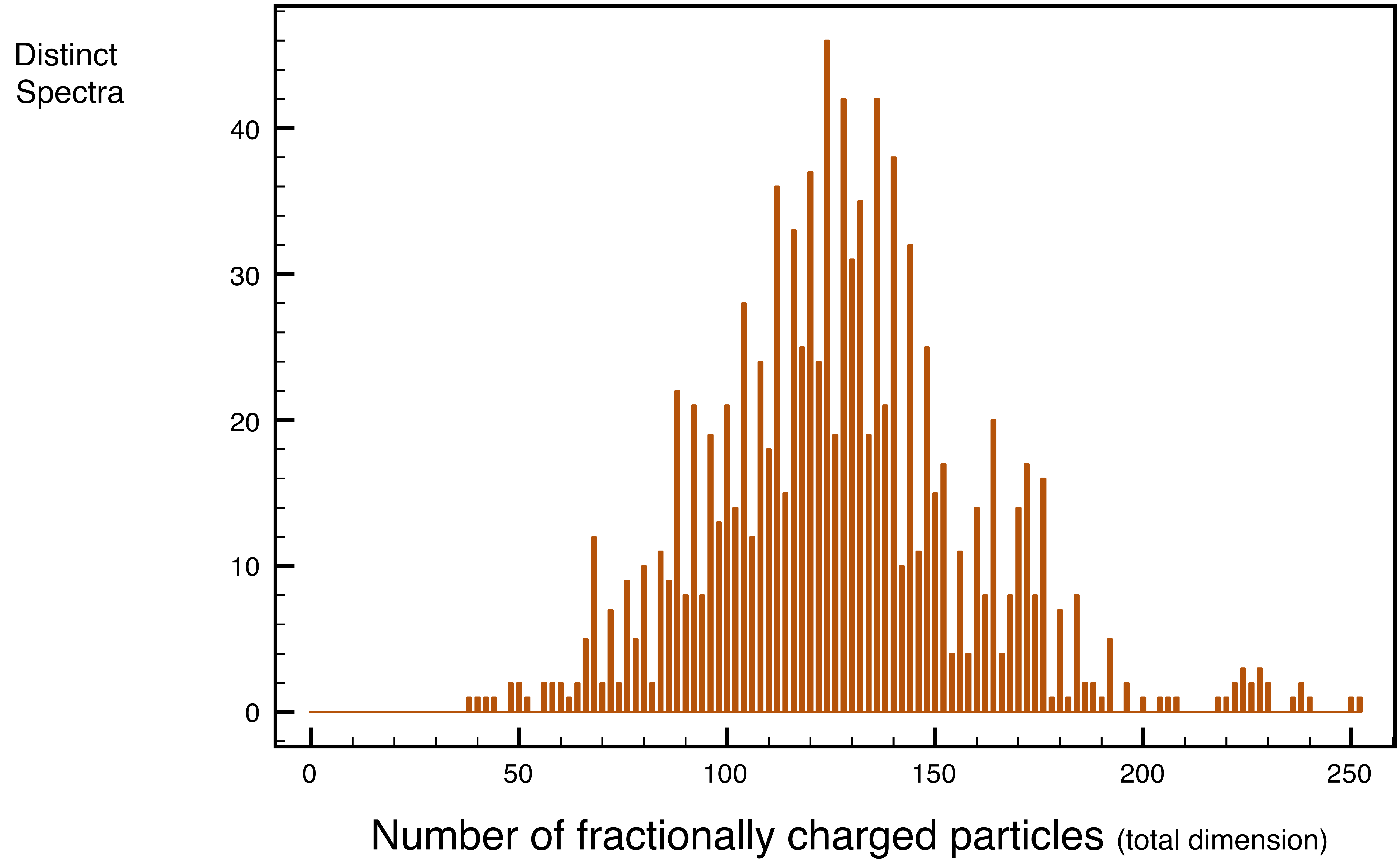

Distinct
Spectra
4000
3000
2000
1000
0
0
5
10
15
20
25
30
Number of D mirrors

GUT symmetry breaking

SU(5) with anomalous $U(1)_{B-L}$	35968
SU(5) × U(1) or SO(10)	7304
Pati-Salam	311
SM with anomalous $U(1)_{B-L}$	680
SM × U(1)	63


Distinct
Spectra
600
500
400
300
200
100
0
0
200
400
600
800
Number of Singlets


Distinct Spectra
40000
30000
20000
10000
0
0
50
100
150
200
250
Number of fractionally charged particles (total dimension)


Distinct
Spectra
40
30
20
10
0
0
50
100
150
200
250
Number of fractionally charged particles (total dimension)

Conclusions

- Asymmetric Gepner models provide a huge and largely unexplored part of the landscape.
- Family distributions peak at small values.
- Three families not strongly suppressed
- Fractional charges occur, but are fairly often non-chiral.
- Can be limited to half-integer charge using higher spin currents.
- Are massive only very rarely.