Neutrino masses from Instantons in RCFT orientifolds

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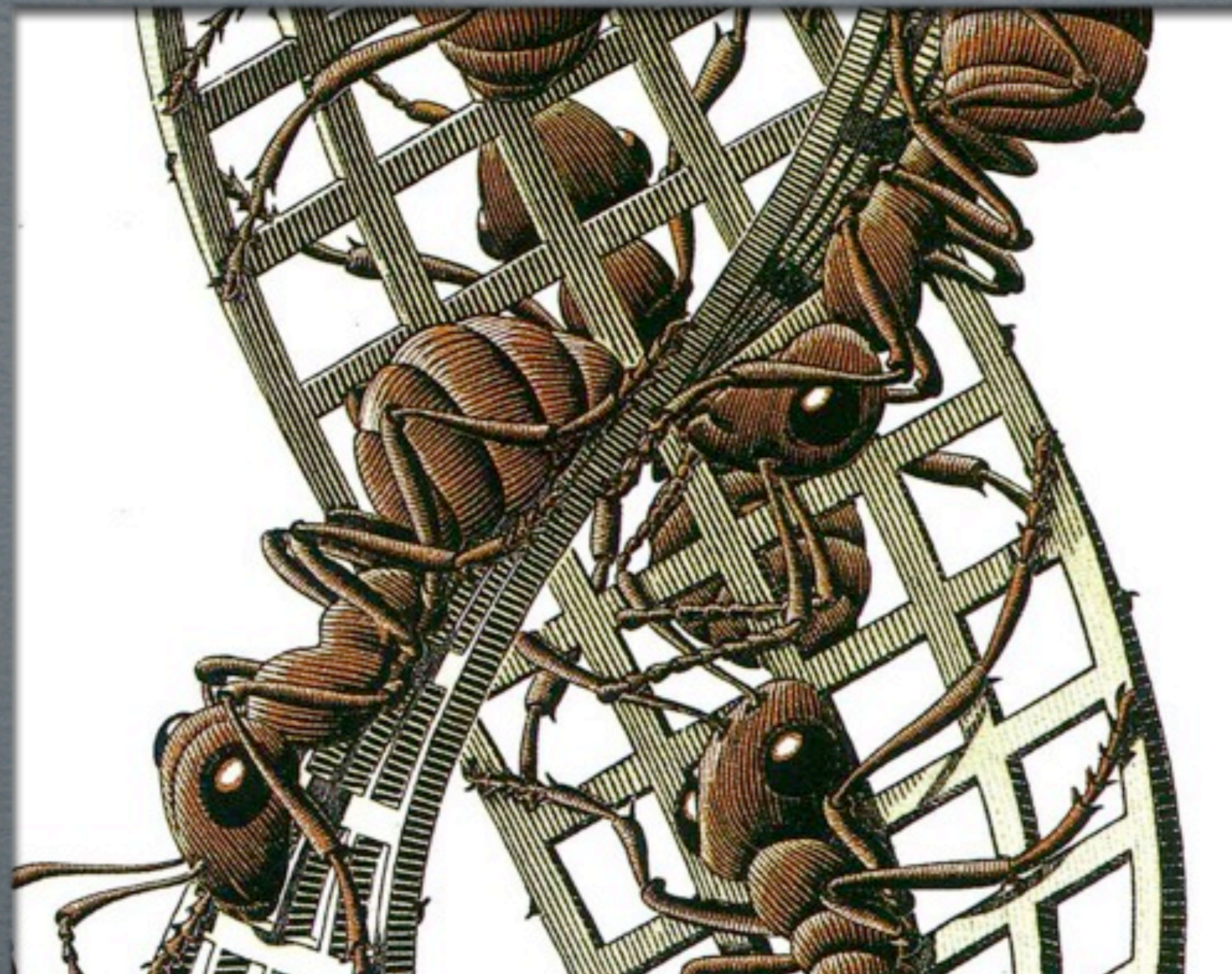
13-11-2007



Orientifolds

Orientifold Partition Functions

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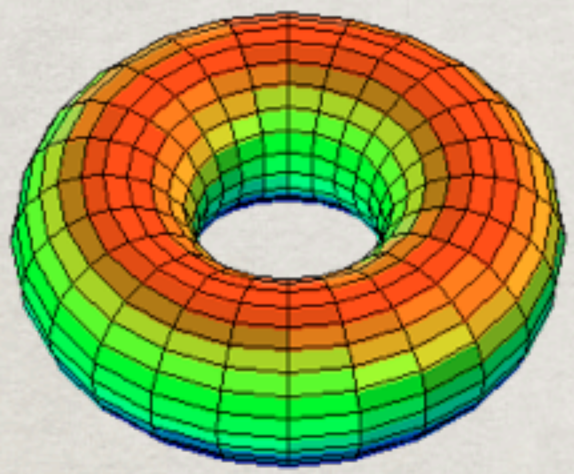

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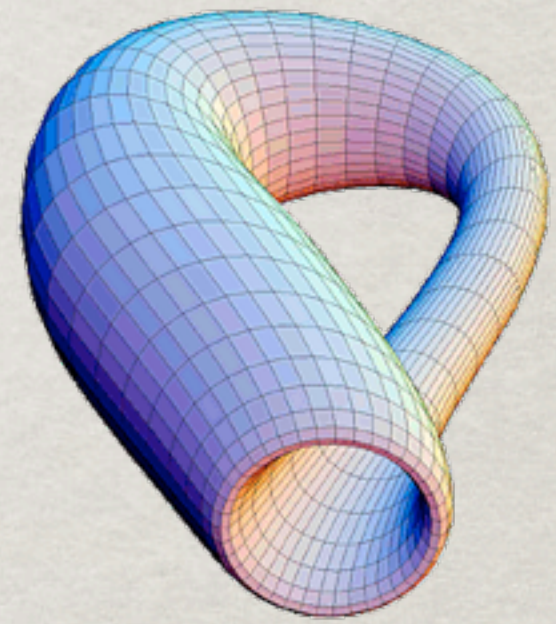

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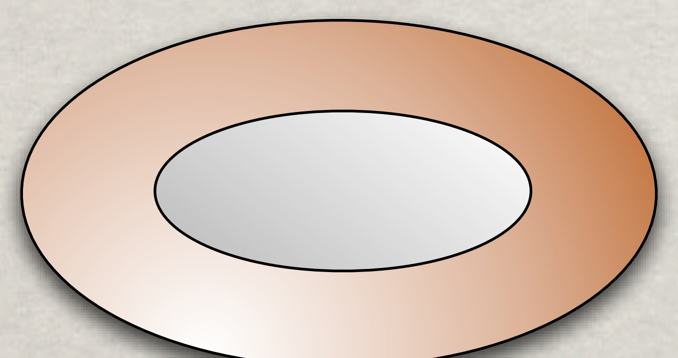

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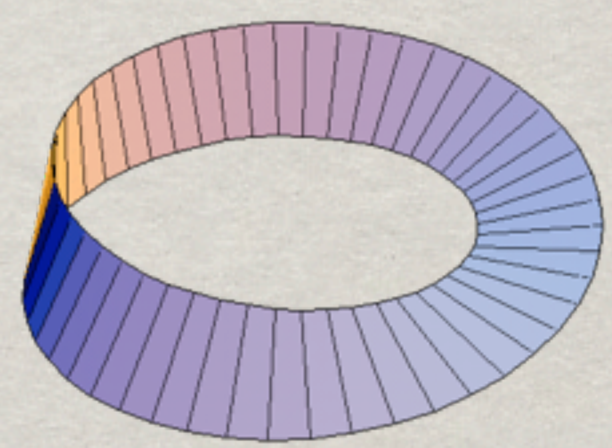


}

Orientifold Partition Functions

- Closed $$\frac{1}{2}\left[\sum_{ij}\chi_i(\tau)Z_{ij}\chi_i(\bar{\tau}) + \sum_i K_i\chi_i(2\tau)\right]$$

- Open $$\frac{1}{2}\left[\sum_{i,a,n} N_a N_b {A^i}_{ab}\chi_i(\frac{\tau}{2}) + \sum_{i,a} N_a {M^i}_a\hat{\chi}_i(\frac{\tau}{2}+\frac{1}{2})\right]$$

i : Primary field label (finite range)

a : Boundary label (finite range)

χ_i : Character

N_a : Chan-Paton (CP) Multiplicity

Algebraic choices

- Basic CFT (N=2 tensor, free fermions...)
  (Type IIB closed string theory)

- Chiral algebra extension(*)
  May imply space-time symmetry (e.g. Susy: GSO projection).
  Reduces number of characters.

- Modular Invariant Partition Function (MIPF)(*)
  May imply bulk symmetry (e.g Susy), not respected by all boundaries.
  Defines the set of boundary states
  (Sagnotti-Pradisi-Stanev completeness condition)

- Orientifold choice(*)

(*) all these choices are simple current related

Accessible RCFT's

- "Gepner Models" (*)
  (minimal N=2 tensor products)

- Free fermions (4n real + (9-2n) complex)

- Kazama-Suzuki models
  (requires exact spectrum computation)

- Permutation orbifolds

- ....

(*) See also: Angelantonj et. al.
Blumenhagen et. al.
Aldazabal et. al.
Brunner et. al.

MIPFs*

- CFT has a discrete "simple current" group $\mathcal{G}$
  Choose a subgroup $\mathcal{H}$ of $\mathcal{G}$

- Choose a rational matrix $X_{\alpha\beta}$ obeying

$$
\begin{aligned}
2X_{\alpha\beta} &= Q_{J_\alpha}(J_\beta) \bmod 1, \alpha \neq \beta \\
X_{\alpha\alpha} &= -h_{J_\alpha} \\
N_\alpha X_{\alpha\beta} &\in \mathbb{Z} \text{ for all } \alpha, \beta
\end{aligned}
$$

$$Q_J(a) = h(a) + h(J) - h(Ja)$$

- This defines the torus partition function as

Z_{ij} is the number of currents $L \in \mathcal{H}$ such that

$$
\begin{aligned}
j &= Li \\
Q_M(i) + X(M,L) &= 0 \bmod 1 \qquad \text{for all } M \in \mathcal{H}.
\end{aligned}
$$

Gato-Rivera, Kreuzer, Schellekens (1991-1993)

ORIENTIFOLD CHOICES*

- "Klein bottle current" K (element of $\mathcal{H}$)
- "Crosscap signs" (signs defined on a subgroup of $\mathcal{H}$), satisfying

$$\beta_K(J)\beta_K(J') = \beta_K(JJ')e^{2\pi i X(J,J')} \quad , J, J' \in \mathcal{H}$$

**Huiszoon, Sousa, Schellekens (1999-2000)*

Boundaries and Crosscaps

- Boundary coefficients

$$R_{[a,\psi_a](m,J)} = \sqrt{\frac{|\mathcal{H}|}{|\mathcal{C}_a||\mathcal{S}_a|}}\psi_a^*(J)S^J_{am}$$

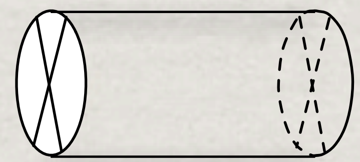
- Crosscap coefficients

$$U_{(m,J)} = \frac{1}{\sqrt{|\mathcal{H}|}}\sum_{L\in\mathcal{H}} e^{\pi i(h_K - h_{KL})}\beta_K(L)P_{LK,m}\delta_{J,0}$$

Cardy (1989)
Sagnotti, Pradisi, Stanev (~1995)
Huiszoon, Fuchs, Schellekens, Schweigert, Walcher (2000)

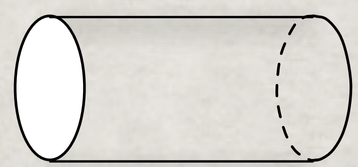
Coefficients

- Klein bottle



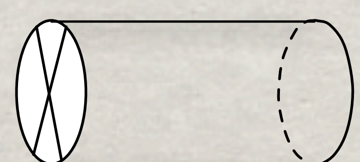
$$K^i = \sum_{m,J,J'} \frac{S^i{}_m U_{(m,J)} g^{\Omega,m}_{J,J'} U_{(m,J')}}{S_{0m}}$$

- Annulus



$$A^i_{[a,\psi_a][b,\psi_b]} = \sum_{m,J,J'} \frac{S^i{}_m R_{[a,\psi_a](m,J)} g^{\Omega,m}_{J,J'} R_{[b,\psi_b](m,J')}}{S_{0m}}$$

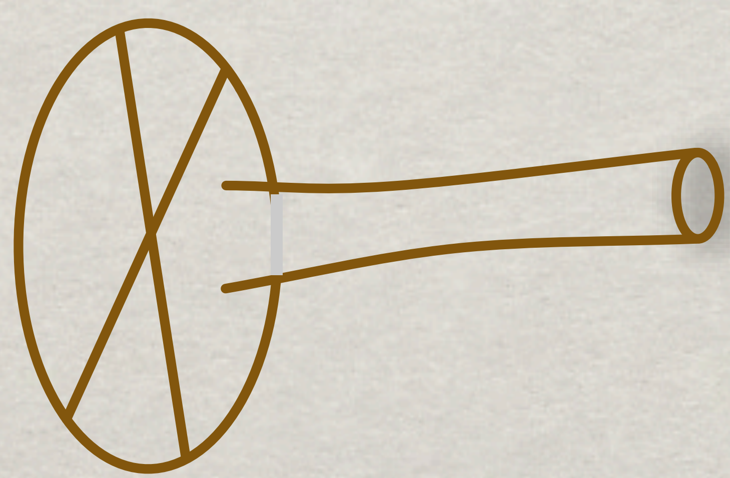
- Moebius



$$M^i_{[a,\psi_a]} = \sum_{m,J,J'} \frac{P^i{}_m R_{[a,\psi_a](m,J)} g^{\Omega,m}_{J,J'} U_{(m,J')}}{S_{0m}}$$

$$g^{\Omega,m}_{J,J'} = \frac{S_{m0}}{S_{mK}} \beta_K(J) \delta_{J',J^c}$$

TADPOLES & ANOMALIES



TADPOLES & ANOMALIES

- Tadpole cancellation condition:

$$\sum_b N_b R_{b(m,J)} = 4\eta_m U_{(m,J)}$$

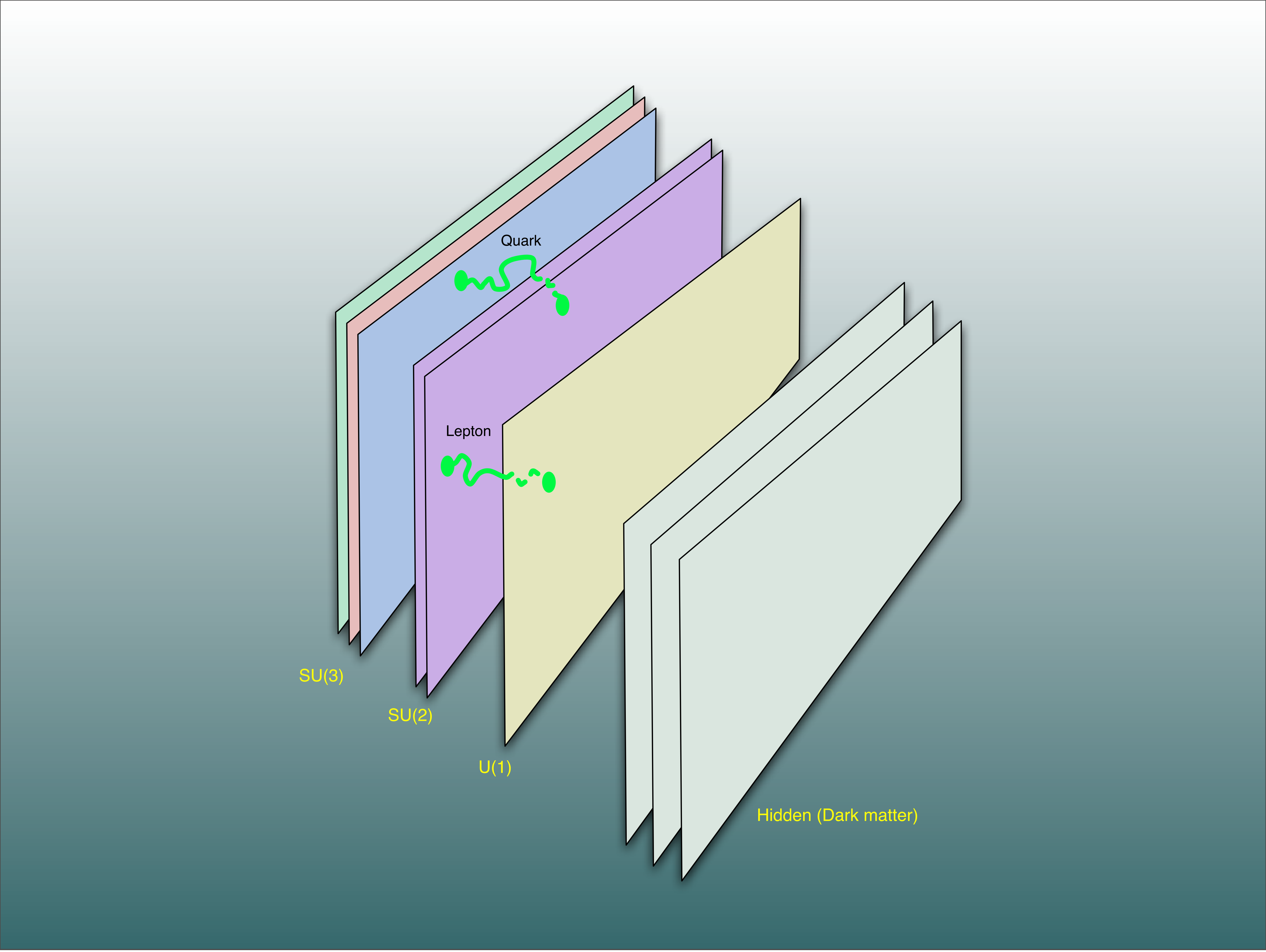
- Cubic $\mathrm{Tr}F^3$ anomalies cancel

- Remaining anomalies by Green-Schwarz mechanism

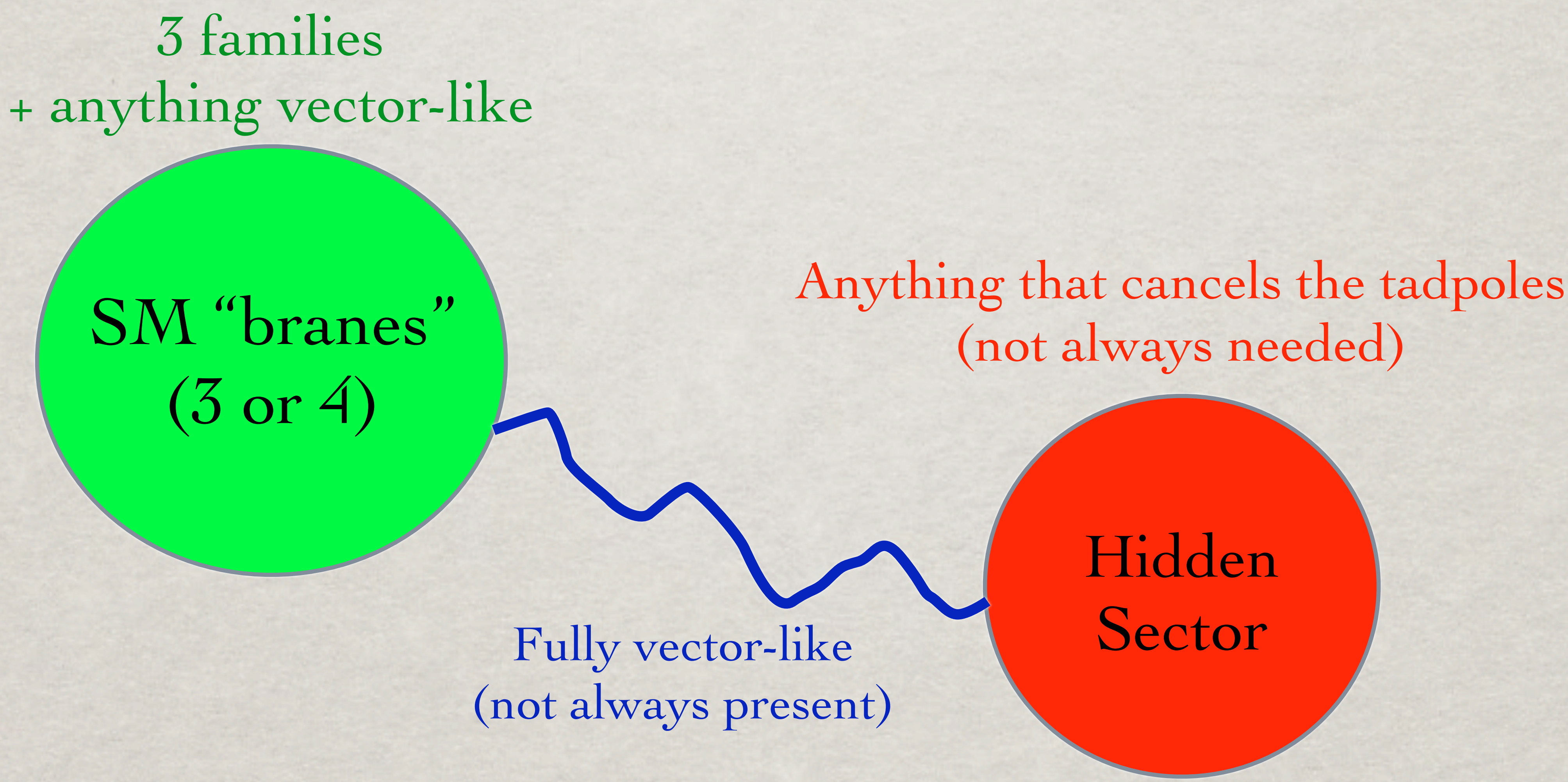
- In rare cases, additional conditions for global anomaly cancellation*

**Gato-Rivera, Schellekens (2005)*

Model Building

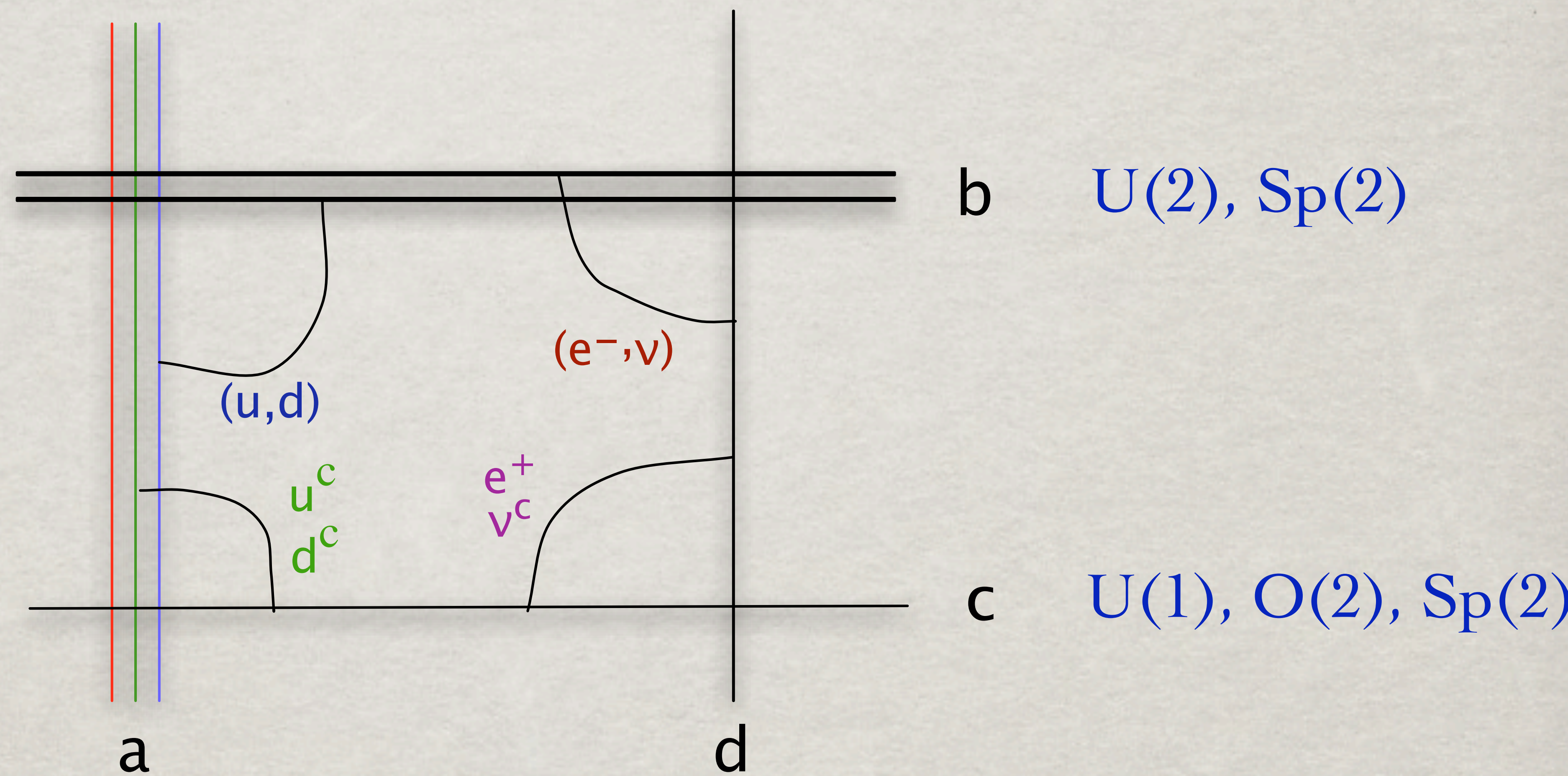

Quark
Lepton
SU(3)
SU(2)
U(1)
Hidden (Dark matter)

Models



Vector-like: mass allowed by SU(3) × SU(2) × U(1)
(Higgs, right-handed neutrino, gauginos, sparticles....)

The Madrid Model*

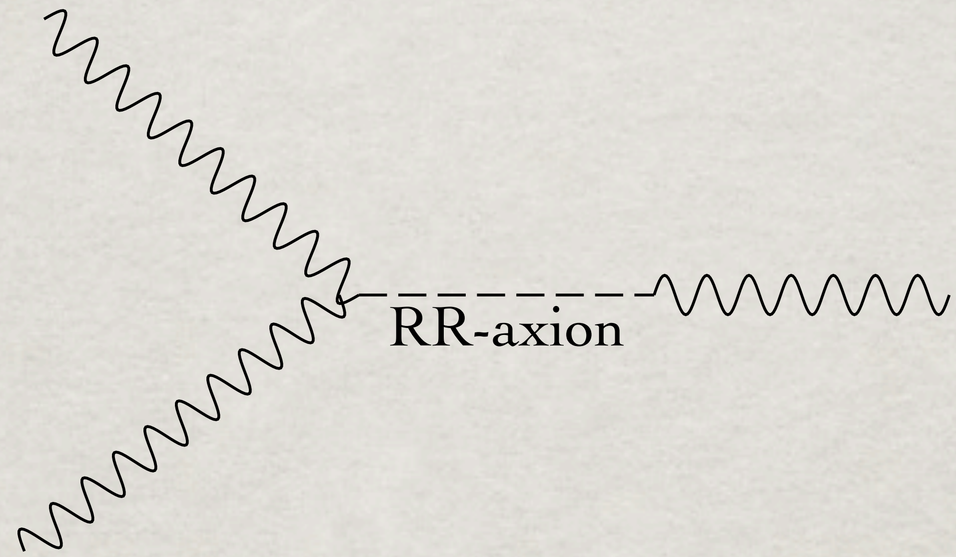


$$Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c - \frac{1}{2}Qd$$

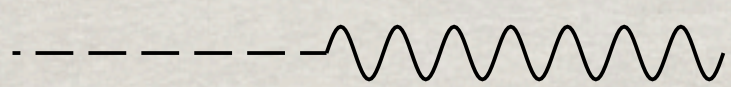
(*) Ibanez, Marchesano, Rabadan

Abelian Masses

Green-Schwarz mechanism



Axion-Vector boson vertex



Generates mass vector bosons of anomalous symmetries

$(e.g.\ B+L)$

But may also generate mass for non-anomalous ones

$(Y, B-L)$

Searches

DATA

	2004-2005*	2005-2006†
Trigger	"Madrid"	All 3 family models
Chiral types	19	19345
Tadpole-free(per type)	18	1900
Total configs	45×10^6	145×10^6
Tadpole free, distinct	210.000	1900
Max. primaries	∞	1750

(*) Huiszoon, Dijkstra, Schellekens

(†) Anastasopoulos, Dijkstra, Kiritsis, Schellekens

A "madrid" Model

Gauge group: Exactly $SU(3) \times SU(2) \times U(1)$!

[$U(3)\times Sp(2)\times U(1)\times U(1)$, Massive B-L, No hidden sector]

3 x (V ,V ,0 ,0)	chirality 3	Q
3 x (V ,0 ,V ,0)	chirality -3	U*
3 x (V ,0 ,V* ,0)	chirality -3	D*
3 x (0 ,V ,0 ,V)	chirality 3	L
5 x (0 ,0 ,V ,V)	chirality -3	E*+(E+E*)
3 x (0 ,0 ,V ,V*)	chirality 3	N*
18 x (0 ,V ,V ,0)		Higgs
2 x (V ,0 ,0 ,V)		
2 x (Ad ,0 ,0 ,0)		
2 x (A ,0 ,0 ,0)		
6 x (S ,0 ,0 ,0)		
14 x (0 ,A ,0 ,0)		
6 x (0 ,S ,0 ,0)		
9 x (0 ,0 ,Ad ,0)		
6 x (0 ,0 ,A ,0)		
14 x (0 ,0 ,S ,0)		
3 x (0 ,0 ,0 ,Ad)		
4 x (0 ,0 ,0 ,A)		
6 x (0 ,0 ,0 ,S)		

A "MADRID" MODEL

Gauge group: Exactly SU(3) × SU(2) × U(1)!

[U(3)×Sp(2)×U(1)×U(1), Massive B-L, No hidden sector]

3 x	(V	,V	,0	,0)	chirality 3	Q
3 x	(V	,0	,V	,0)	chirality -3	U*
3 x	(V	,0	,V*	,0)	chirality -3	D*
3 x	(0	,V	,0	,V)	chirality 3	L
5 x	(0	,0	,V	,V)	chirality -3	E*+(E+E*)
3 x	(0	,0	,V	,V*)	chirality 3	N*
18 x	(0	,V	,V	,0)		Higgs
2 x	(V	,0	,0	,V)		
2 x	(Ad	,0	,0	,0)		
2 x	(A	,0	,0	,0)		
6 x	(S	,0	,0	,0)		
14 x	(0	,A	,0	,0)		
6 x	(0	,S	,0	,0)		
9 x	(0	,0	,Ad	,0)		
6 x	(0	,0	,A	,0)		
14 x	(0	,0	,S	,0)		
3 x	(0	,0	,0	,Ad)		
4 x	(0	,0	,0	,A)		
6 x	(0	,0	,0	,S)		

Vector-like matter

V=vector

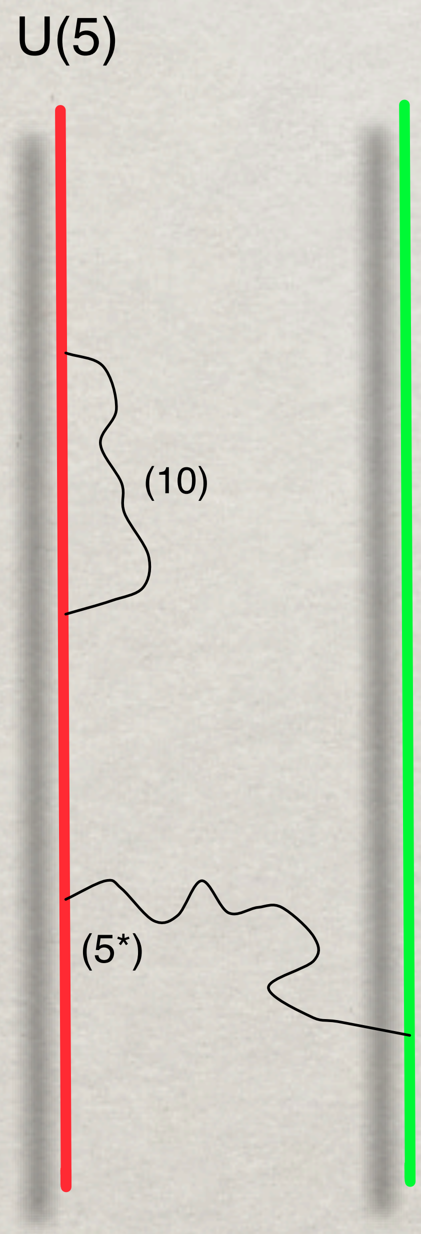
A=Anti-symm. tensor

S=Symmetric tensor

Ad=Adjoint

An SU(5) model

Gauge group is just SU(5)!



U5 O1 O1

3 x	(A	,0	,0)	chirality	3
11 x	(V	,V	,0)	chirality	-3
8 x	(S	,0	,0)		
3 x	(Ad	,0	,0)		
1 x	(0	,A	,0)		
3 x	(0	,V	,V)		
8 x	(V	,0	,V)		
2 x	(0	,S	,0)		
4 x	(0	,0	,S)		
4 x	(0	,0	,A)		

Top quark Yukawa's?

Neutrino masses

NEUTRINO MASSES*

- In field theory: easy; several solutions.
  Most popular:
  add three right-handed neutrinos
  add "natural" Dirac & Majorana masses (see-saw)

$$m_\nu = \frac{(M_D)^2}{M_M}; \quad M_D \approx 100 \text{ MeV}, \qquad M_M \approx 10^{11} \ldots 10^{13} \text{ GeV}$$

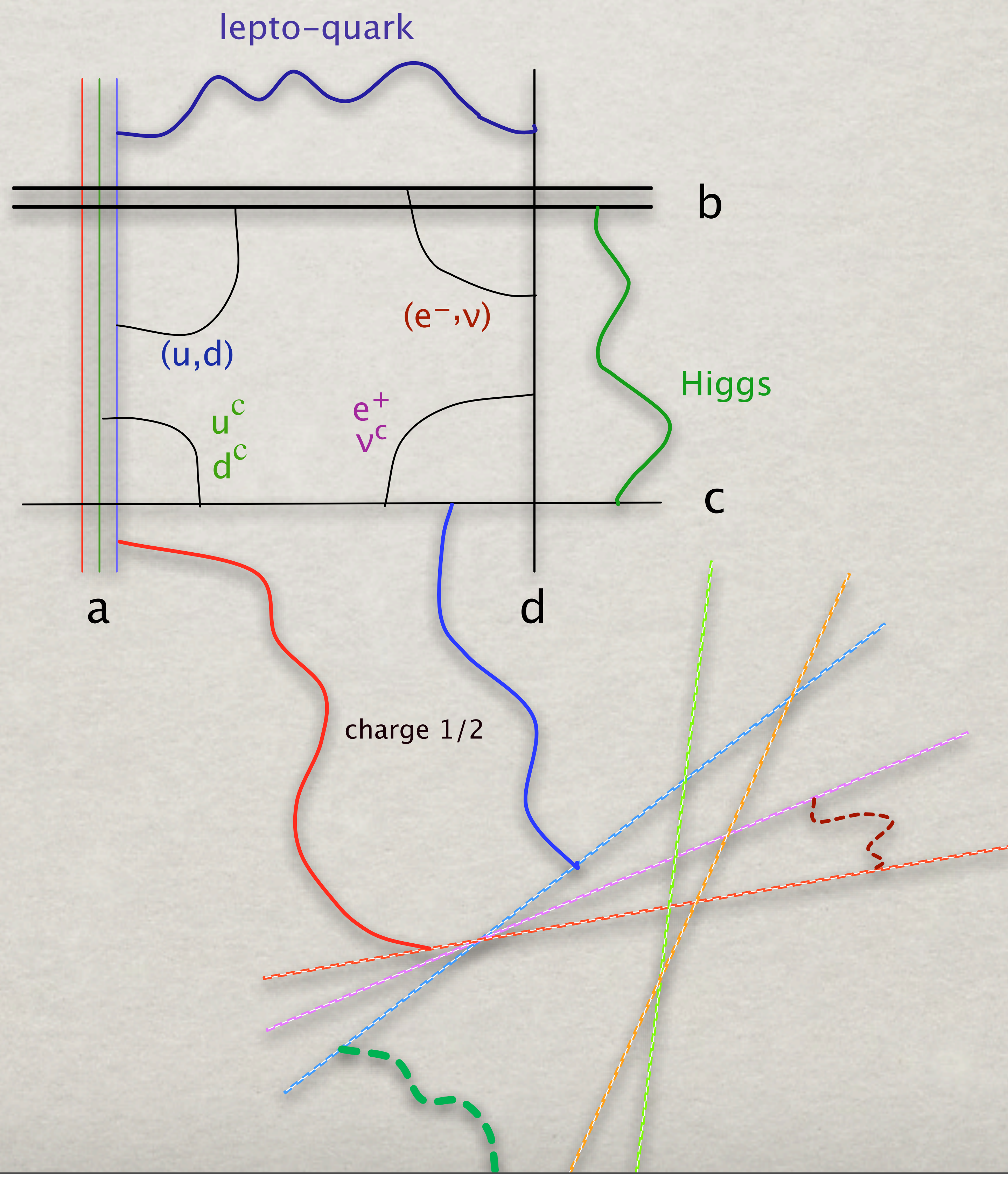
- In string theory: non-trivial.
  (String theory is much more falsifiable!).

- Questions:
  Can we find vacua with small neutrino masses?
  Are they generically small?
  If not, why do we observe small masses?

(*) *Ibañez, Schellekens, Uranga,* arXiv:0704.1079, JHEP (to appear)
Blumenhagen, Cvetic, Weigand, hep-th/0609191
Ibañez, Uranga, hep-th/0609213

Other ideas: see e.g. Conlon, Cremades; Giedt, Kane, Langacker, Nelson; Buchmuller, Hamaguchi, Lebedev, Ratz, ...

Madrid model (with hidden sector)



Neutrinos in Madrid Models

All these models have three right-handed neutrinos (required for cubic anomaly cancellation)

In most of these models:
B-L survives as an exact gauge symmetry

Neutrino's can get Dirac masses, but not Majorana masses (both needed for see-saw mechanism).

In a very small* subset, B-L acquires a mass due to axion couplings.

(*) 391 out of 213000, all with $SU(3)\times Sp(2)\times U(1)\times U(1)$

B-L Violation

But even then, B-L still survives as a perturbative symmetry.

It may be broken to a discrete subgroup by instantons.

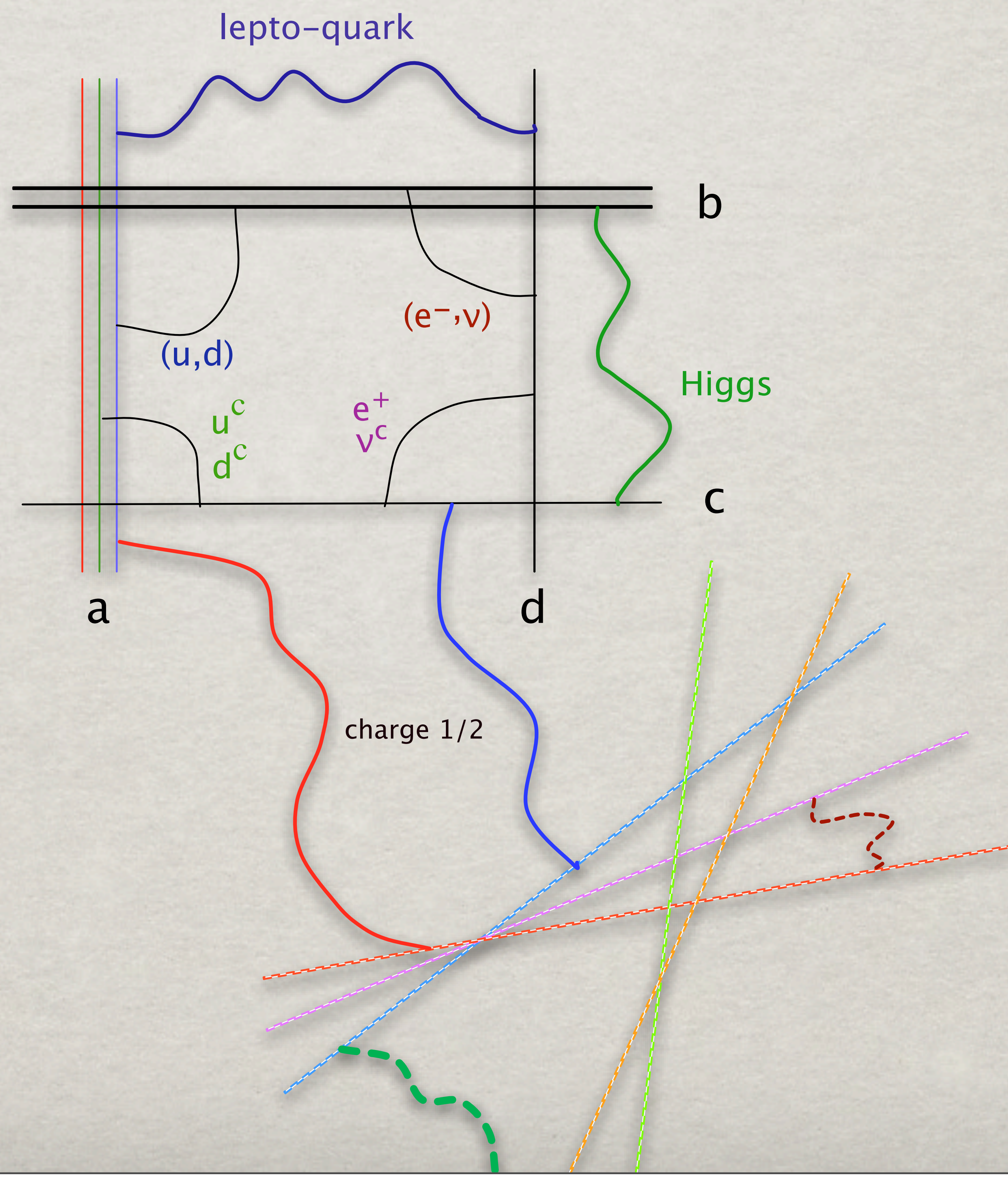
This possibility can be explored if the instanton is described by a RCFT brane M.
B-L violation manifests itself as:

$$I_{M\mathbf{a}} - I_{M\mathbf{a}'} - I_{M\mathbf{d}} + I_{M\mathbf{d}'} \neq 0$$

I_{Ma} = chiral [# (V,V*) - # (V*,V)]   between branes M and a

a' = boundary conjugate of a

Madrid model (with hidden sector)



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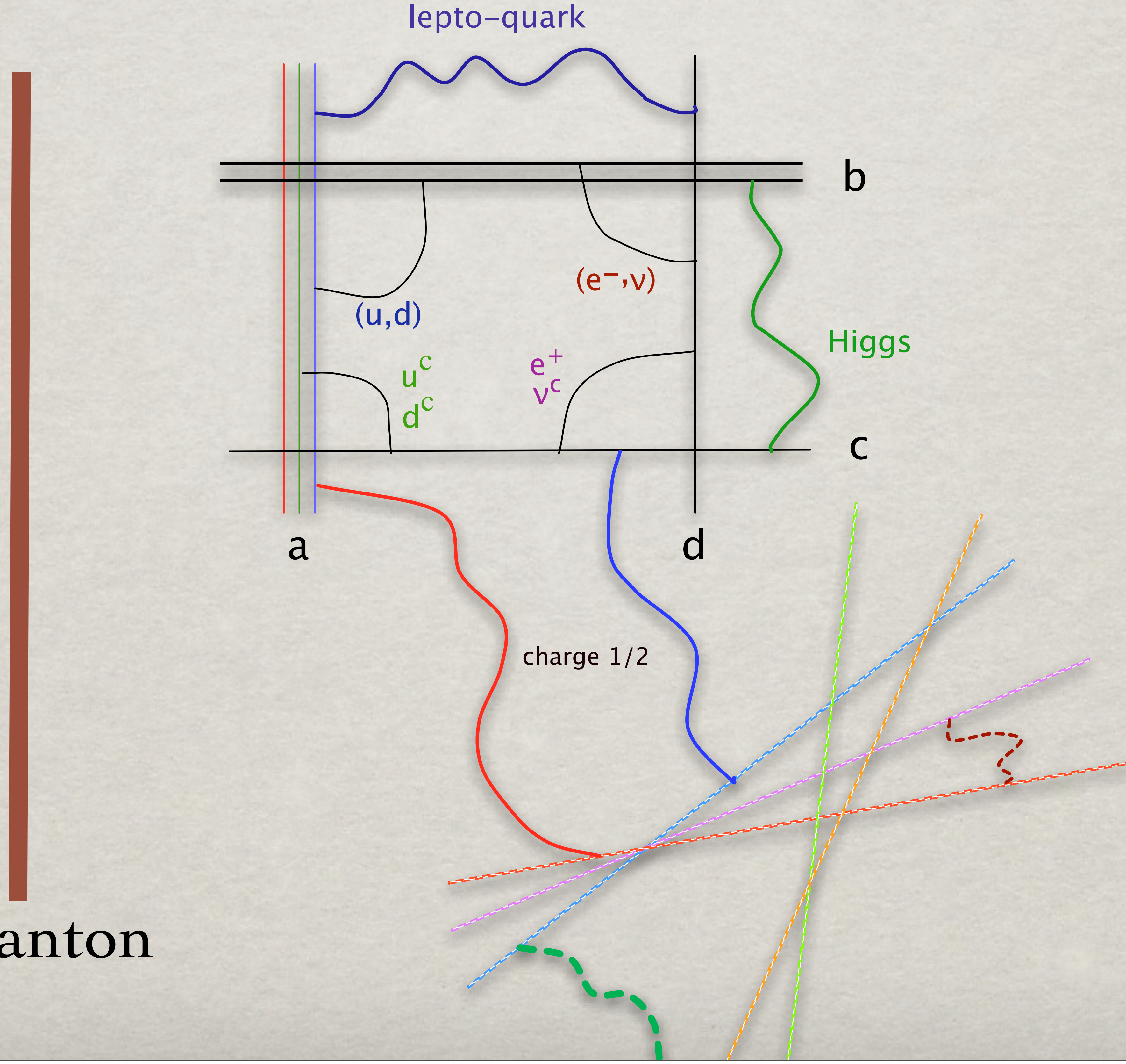
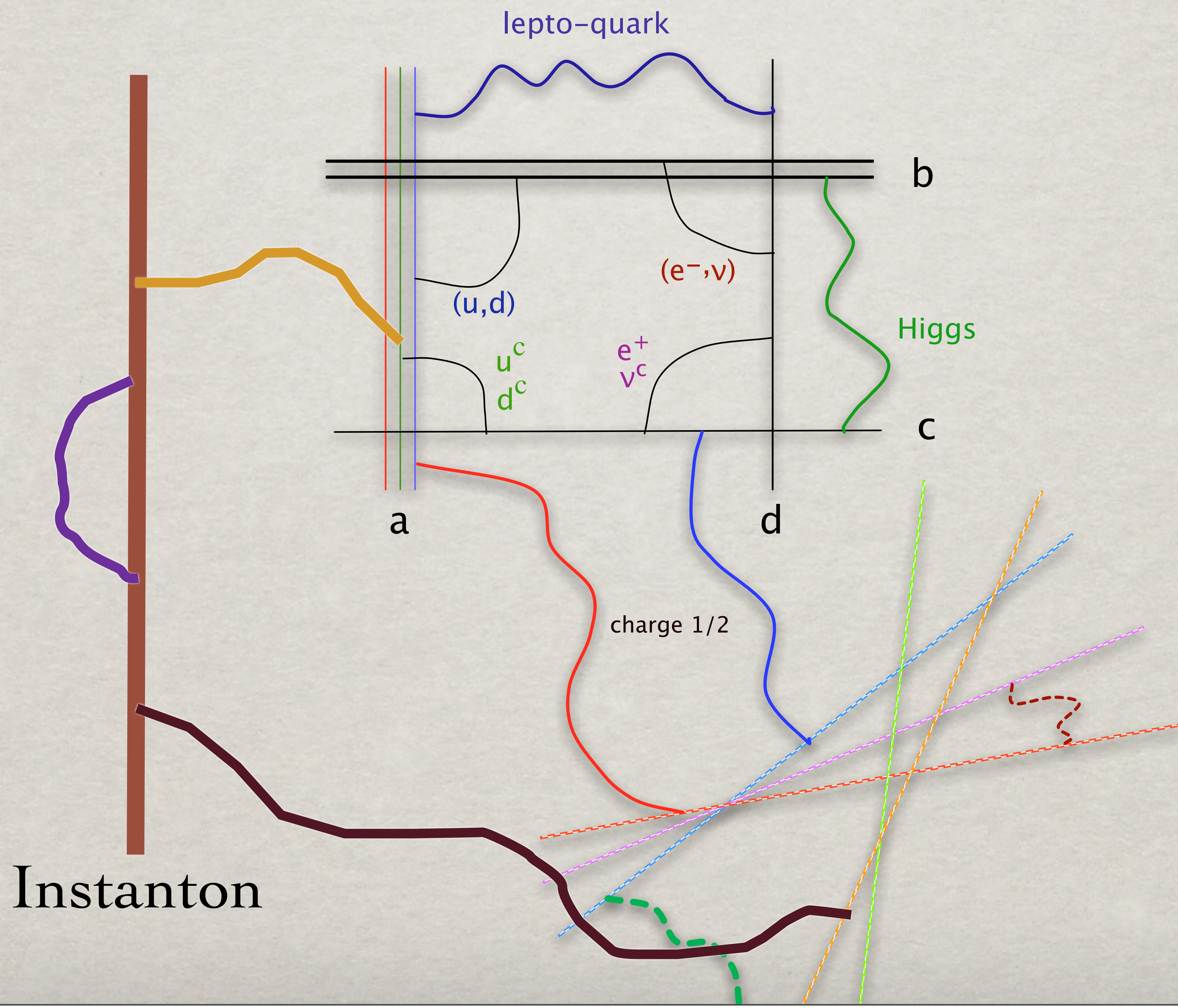


Sunday, 2 May 2010

Madrid model (with hidden sector)



B-L anomalies

$$I_{M\mathbf{a}} - I_{M\mathbf{a}'} - I_{M\mathbf{d}} + I_{M\mathbf{d}'} \neq 0$$

Implies a cubic B-L anomaly if M is a "matter" brane (Chan-Paton multiplicity ≠ 0).

$\Rightarrow$ *M cannot be a matter brane: non-gauge-theory instanton*

Implies a (B-L)$(G_M)^2$ anomaly even if we cancel the cubic anomaly

$\Rightarrow$ *B-L must be massive*

(The converse is not true: there are massive B-L models without such instanton branes)

Required zero-modes

Neutrino mass generation by non-gauge theory instantons*

The desired neutrino mass term $\nu^c \nu^c$
violates c and d brane charge by two units.
To compensate this, we must have

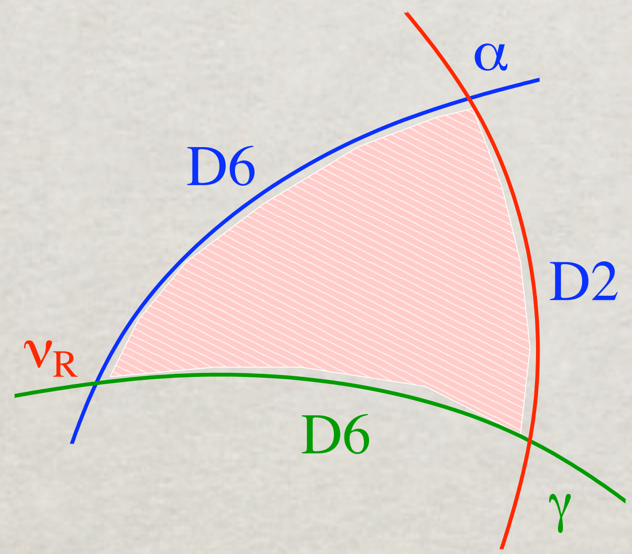
$$I_{M\mathbf{c}} = 2\ ;\ I_{M\mathbf{d}} = -2 \quad or \quad I_{M\mathbf{d}'} = 2\ ;\ I_{M\mathbf{c}'} = -2$$

and all other intersections 0.
(d' is the boundary conjugate of d)

(*) Blumenhagen, Cvetic, Weigand, hep-th/0609191
Ibañez, Uranga, hep-th/0609213

Neutrino-zero mode coupling

The following world-sheet disk is allowed by all symmetries



$$L_{cubic} \propto d_a^{ij} \; (\alpha_i \; \nu^a \gamma_j) \quad , a = 1, 2, 3$$

Zero-mode integrals

$$\int d^2\alpha \, d^2\gamma \; e^{-d_a^{ij} \;\; (\alpha_i \nu^a \gamma_j)} \;=\; \nu_a \nu_b \left(\epsilon_{ij} \epsilon_{kl} d_a^{ik} d_b^{jl} \right)$$

Additional zero modes yield additional fermionic integrals and hence no contribution

Therefore $I_{Ma}=I_{Mb}=I_{Mx}=0$ (x = Hidden sector), and there should be no vector-like zero modes.

There should also be no instanton-instanton zero-modes except 2 required by susy.

INSTANTON TYPES

In orientifold models we can have complex and real branes

Matter brane M	Instanton brane M
U(N)	U(k)
O(N)	Sp(2k)
Sp(2N)	O(k)

$$I_{M\mathbf{c}} = 2\ ;\ I_{M\mathbf{d}} = -2 \quad or \quad I_{M\mathbf{d}'} = 2\ ;\ I_{M\mathbf{c}'} = -2$$

Possible for:

- U, k=1 or 2
- Sp, k=1
- O, k=1,2

Universal instanton-instanton zero-modes

- U(k): 4 Adj
- Sp(2k): 2 A + 2 S
- O(k): 2 A + 2 S

Only O(1) has the required 2 zero modes

Instanton scan

Can we find such branes M in the 391 models with massive B-L?

Instanton scan

Can we find such branes M in the 391 models with massive B-L?

- About 30.000 "instanton branes" ($I_{M\mathbf{a}} - I_{M\mathbf{a}'} - I_{M\mathbf{d}} + I_{M\mathbf{d}'} \neq 0$)
- Quantized in units of 1,2 or 4

  (1 may give R-parity violation, 4 means no Majorana mass)
- Some models have no RCFT instantons
- 1315 instantons with correct ***chiral*** intersections
- None of these models has R-parity violating instantons.
- Most instantons are symplectic in this sample.
- There are examples with exactly the right number, ***non-chirally***, except for the spurious extra susy zero-modes (Sp(2) instantons).

Instanton scan

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...almost

Tensor	MIPF	Orientifold	Instanton	Solution
(1,16,16,16)	12	0	$S2^+$, $S2^-$	Yes
(2,4,12,82)	19	0	$S2^-!$	?
(2,4,12,82)	19	0	$U2^+!, U2^-!$	No
(2,4,12,82)	19	0	$U1^+, U1^-$	No
(2,4,14,46)	10	0		
(2,4,14,46)	16	0		
(2,4,16,34)	15	0		
(2,4,16,34)	15	1		
(2,4,16,34)	16	0	$S2^+$, $S2^-$	Yes
(2,4,16,34)	16	1		
(2,4,16,34)	18	0	$S2^-$	Yes
(2,4,16,34)	18	0	$U1^+, U1^-, U2^+, U2^-$	No
(2,4,16,34)	49	0	$U2^+, S2^-!, U1^+$	Yes

Continued on next page

Tensor	MIPF	Orientifold	Instanton	Solution
(2,4,18,28)	17	0		
(2,4,22,22)	13	3	$S2^+!$, $S2^-!$	Yes!
(2,4,22,22)	13	2	$S2^+!$, $S2^-!$	Yes
(2,4,22,22)	13	1	$S2^+$, $S2^-$	No
(2,4,22,22)	13	0	$S2^+$, $S2^-$	Yes
(2,4,22,22)	31	1	$U1^+, U1^-$	No
(2,4,22,22)	20	0		
(2,4,22,22)	46	0		
(2,4,22,22)	49	1	$O2^+, O2^-, O1^+, O1^-$	Yes
(2,6,14,14)	1	1	$U1^+$	No
(2,6,14,14)	22	2		
(2,6,14,14)	60	2		
(2,6,14,14)	64	0		
(2,6,14,14)	65	0		
(2,6,10,22)	22	2		
(2,6,8,38)	16	0		
(2,8,8,18)	14	2	$S2^+!$, $S2^-!$	Yes
(2,8,8,18)	14	0	$S2^+!$, $S2^-!$	No
(2,10,10,10)	52	0	$U1^+, U1^-$	No
(4,6,6,10)	41	0		
(4,4,6,22)	43	0		
(6,6,6,6)	18	0		

An Sp(2) instanton model

	U3	S2	U1	U1	O	
3 x	V	V	0	0	0	chirality 3
3 x	V	0	V	0	0	chirality -3
3 x	V	0	V*	0	0	chirality -3
3 x	0	V	0	V	0	chirality 3
5 x	0	0	V	V	0	chirality -3
3 x	0	0	V	V*	0	chirality 3
1 x	0	0	V	0	V	chirality -1
1 x	0	0	0	V	V	chirality 1
18 x	0	V	V	0	0	
2 x	V	0	0	V	0	
2 x	Ad	0	0	0	0	
2 x	A	0	0	0	0	
6 x	S	0	0	0	0	
14 x	0	A	0	0	0	
6 x	0	S	0	0	0	
9 x	0	0	Ad	0	0	
6 x	0	0	A	0	0	
14 x	0	0	S	0	0	
3 x	0	0	0	Ad	0	
4 x	0	0	0	A	0	
6 x	0	0	0	S	0	

An Sp(2) instanton model

	U3	S2	U1	U1	O	
3 x	V	V	0	0	0	chirality 3
3 x	V	0	V	0	0	chirality -3
3 x	V	0	V*	0	0	chirality -3
3 x	0	V	0	V	0	chirality 3
5 x	0	0	V	V	0	chirality -3
3 x	0	0	V	V*	0	chirality 3
1 x	0	0	V	0	V	chirality -1
1 x	0	0	0	V	V	chirality 1
18 x	0	V	V	0	0	
2 x	V	0	0	V	0	
2 x	Ad	0	0	0	0	
2 x	A	0	0	0	0	
6 x	S	0	0	0	0	
14 x	0	A	0	0	0	
6 x	0	S	0	0	0	
9 x	0	0	Ad	0	0	
6 x	0	0	A	0	0	
14 x	0	0	S	0	0	
3 x	0	0	0	Ad	0	
4 x	0	0	0	A	0	
6 x	0	0	0	S	0	

The O1 instanton

Type:	U	S	U	U	U	O	O	U	O	O	O	U	S	S	O	S	
Dimension	3	2	1	1	1	2	2	3	1	2	3	1	2	2	2	--	
5 x	V	0	V	0	0	0	0	0	0	0	0	0	0	0	0	0	chirality -3
5 x	0	0	V	V*	0	0	0	0	0	0	0	0	0	0	0	0	chirality 3
3 x	V	0	V*	0	0	0	0	0	0	0	0	0	0	0	0	0	chirality -3
3 x	0	0	V	V	0	0	0	0	0	0	0	0	0	0	0	0	chirality -3
3 x	V	V	0	0	0	0	0	0	0	0	0	0	0	0	0	0	chirality 3
3 x	0	V	0	V	0	0	0	0	0	0	0	0	0	0	0	0	chirality 3
2 x	0	0	0	V	0	0	0	0	0	0	0	0	0	0	0	V	chirality 2
12 x	0	0	V	0	0	0	0	0	0	0	0	0	0	0	0	V	chirality -2
1 x	0	0	0	0	0	0	0	0	0	V	0	0	0	0	0	V	
2 x	0	0	0	0	V	0	0	0	0	0	0	0	0	0	0	V	
1 x	0	0	0	0	0	0	0	0	0	0	0	0	0	0	V	V	
2 x	0	0	0	0	0	0	0	0	0	0	0	0	V	0	0	V	
1 x	0	0	0	0	0	V	0	0	0	0	0	0	0	0	0	V	
3 x	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	S	
4 x	0	0	0	0	0	0	0	0	0	0	0	0	0	V	0	V	
2 x	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	A	
2 x	V	0	0	0	0	0	0	0	0	0	0	0	0	0	0	V	
3 x	0	0	0	0	S	0	0	0	0	0	0	0	0	0	0	0	chirality -1
3 x	0	0	0	0	0	V	0	0	0	0	0	V	0	0	0	0	chirality 1
1 x	0	0	0	0	A	0	0	0	0	0	0	0	0	0	0	0	chirality -1
2 x	0	0	0	0	V	0	V	0	0	0	0	0	0	0	0	0	chirality 2
1 x	0	0	0	0	0	0	0	V	0	0	0	0	0	0	V	0	chirality -1
1 x	0	0	0	0	V	0	0	0	0	V	0	0	0	0	0	0	chirality -1
1 x	0	0	0	0	0	0	0	0	V	0	0	V	0	0	0	0	chirality 1
1 x	0	0	0	0	0	0	0	0	0	0	V	V	0	0	0	0	chirality -1
1 x	0	0	0	0	0	0	V	0	0	0	0	V	0	0	0	0	chirality -1
1 x	0	0	0	0	V	V	0	0	0	0	0	0	0	0	0	0	chirality -1
1 x	0	0	0	0	V	0	0	0	0	0	0	V	0	0	0	0	chirality 1
1 x	0	0	0	0	V	0	0	0	0	0	0	V*	0	0	0	0	chirality -1
3 x	0	0	0	0	V	0	0	0	0	0	0	0	0	0	V	0	chirality 1
1 x	0	0	0	0	0	0	0	V	0	V	0	0	0	0	0	0	chirality 1
2 x	0	0	0	V	0	0	0	0	0	0	0	0	0	V	0	0	
1 x	Ad	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
2 x	0	S	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
1 x	0	0	0	Ad	0	0	0	0	0	0	0	0	0	0	0	0	
6 x	0	0	V	0	0	0	0	0	0	0	0	0	0	V	0	0	
1 x	0	0	0	0	0	0	0	0	0	0	0	0	0	0	A	0	
1 x	0	0	0	0	Ad	0	0	0	0	0	0	0	0	0	0	0	

OTHER OPERATORS

Operator	U	S	O
$\nu_R \nu_R$	3	627	3
LHLH	5	550	3
LH	6	0	4
QDL	12	0	4
UDD	0	0	4
LLE	12	0	4
QQQL	4	0	3892
UUDE	4	0	3880

Conclusions

- Neutrino masses:
  "an incomplete success."
  With sufficient statistics, O1 instantons without superfluous zero-modes will be found.
- Boundary state statistics:
  12 million Unitary
  3 million Orthogonal
  2 million Symplectic ➡ 270000 O1
- But what is the real reason why neutrino masses are small?