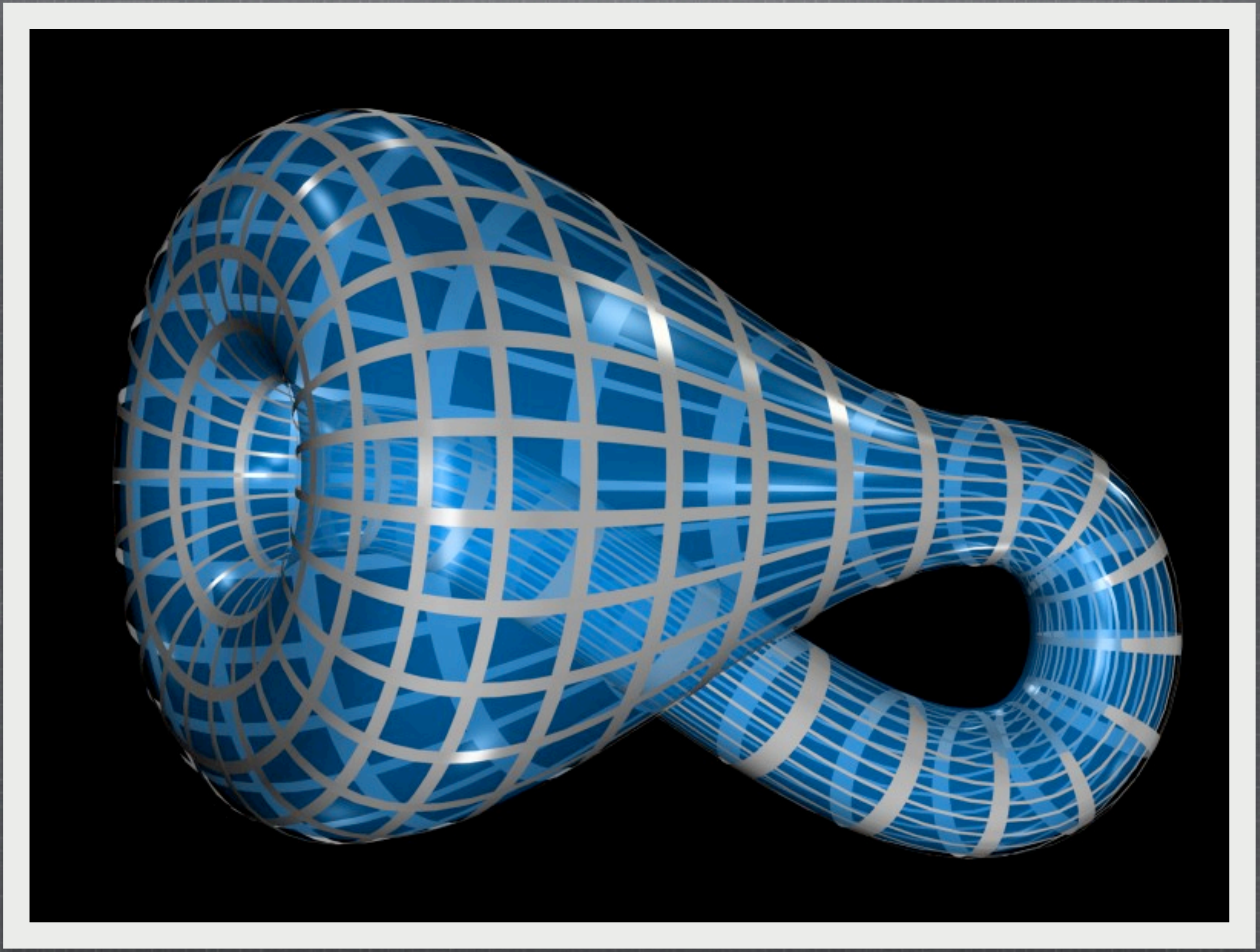
FREE FERMION ORIENTIFOLDS

A. N. Schellekens, Liverpool, 28-4-2008



WHAT WE DID NOT FIND, AND WHERE WE DID NOT FIND IT.

ORIENTIFOLD PARTITION FUNCTIONS

- Closed $$\frac{1}{2}\left[\sum_{ij}\chi_i(\tau)Z_{ij}\chi_i(\bar{\tau})+\sum_{i}K_i\chi_i(2\tau)\right]$$

- Open $$\frac{1}{2}\left[\sum_{i,a,n}N_aN_bA^i{}_{ab}\chi_i(\frac{\tau}{2})+\sum_{i,a}N_aM^i{}_a\hat{\chi}_i(\frac{\tau}{2}+\frac{1}{2})\right]$$

i : Primary field label (finite range)

a : Boundary label (finite range)

χ_i : Character

N_a : Chan-Paton (CP) Multiplicity

ALGEBRAIC CHOICES

- Basic CFT (N=2 tensor, free fermions...)
  (Type IIB closed string theory)

- Chiral algebra extension(*)
  May imply space-time symmetry (e.g. Susy: GSO projection).
  Reduces number of characters.

- Modular Invariant Partition Function (MIPF)(*)
  May imply bulk symmetry (e.g Susy), not respected by all boundaries.
  Defines the set of boundary states
  (Sagnotti-Pradisi-Stanev completeness condition)

- Orientifold choice(*)

(*) all these choices are simple current related

BOUNDARIES AND CROSSCAPS

(For simple current MIPFs and Orientifolds)

- Boundary coefficients

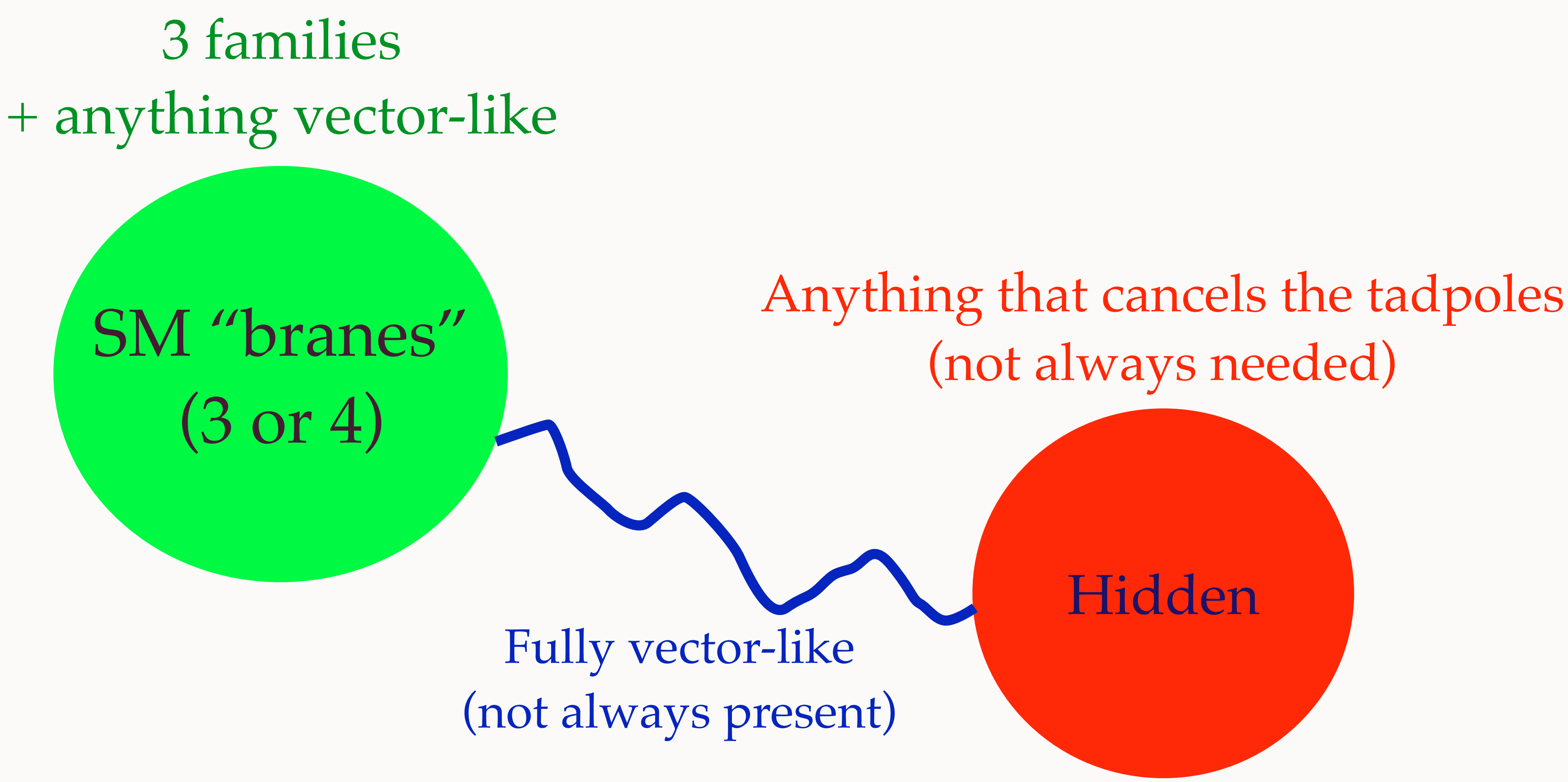
$$R_{[a,\psi_a](m,J)} = \sqrt{\frac{|\mathcal{H}|}{|\mathcal{C}_a||\mathcal{S}_a|}}\psi_a^*(J)S^J_{am}$$

- Crosscap coefficients

$$U_{(m,J)} = \frac{1}{\sqrt{|\mathcal{H}|}}\sum_{L\in\mathcal{H}} e^{\pi i(h_K - h_{KL})}\beta_K(L)P_{LK,m}\delta_{J,0}$$

Cardy (1989)
Sagnotti, Pradisi, Stanev (~1995)
Huiszoon, Fuchs, Schellekens, Schweigert, Walcher (2000)

MODELS



Vector-like: mass allowed by SU(3) × SU(2) × U(1)
(Higgs, right-handed neutrino, gauginos, sparticles....)

CHAN-PATON GROUP

$$G_{CP} = U(3)_a \times \left\{ \begin{matrix} U(2)_b \\ Sp(2)_b \end{matrix} \right\} \times G_c \quad (\times G_d)$$

Embedding of Y:

$$Y = \alpha Q_a + \beta Q_b + \gamma Q_c + \delta Q_d + W_c + W_d$$

Q: Brane charges (for unitary branes)

W: Traceless generators

CLASSIFICATION

$$Y = (x - \frac{1}{3})Q_a + (x - \frac{1}{2})Q_b + \underbrace{xQ_C + (x-1)Q_D}$$

Distributed over
c and *d*

Allowed values for *x*

1/2	Madrid model, Pati-Salam, Flipped SU(5)
0	(broken) SU(5)
1	Antoniadis, Kiritsis, Tomaras model -1/2, 3/2
any	Trinification ($x = 1/3$) (orientable)

DATA

	2004-2005*	2005-2006†
Trigger	"Madrid"	All 3 family models
Chiral types	19	19345
Tadpole-free (per type)	18	1900
Total configs	45×10^6	145×10^6
Tadpole free, distinct	210.000	1900
Max. primaries	∞	1750

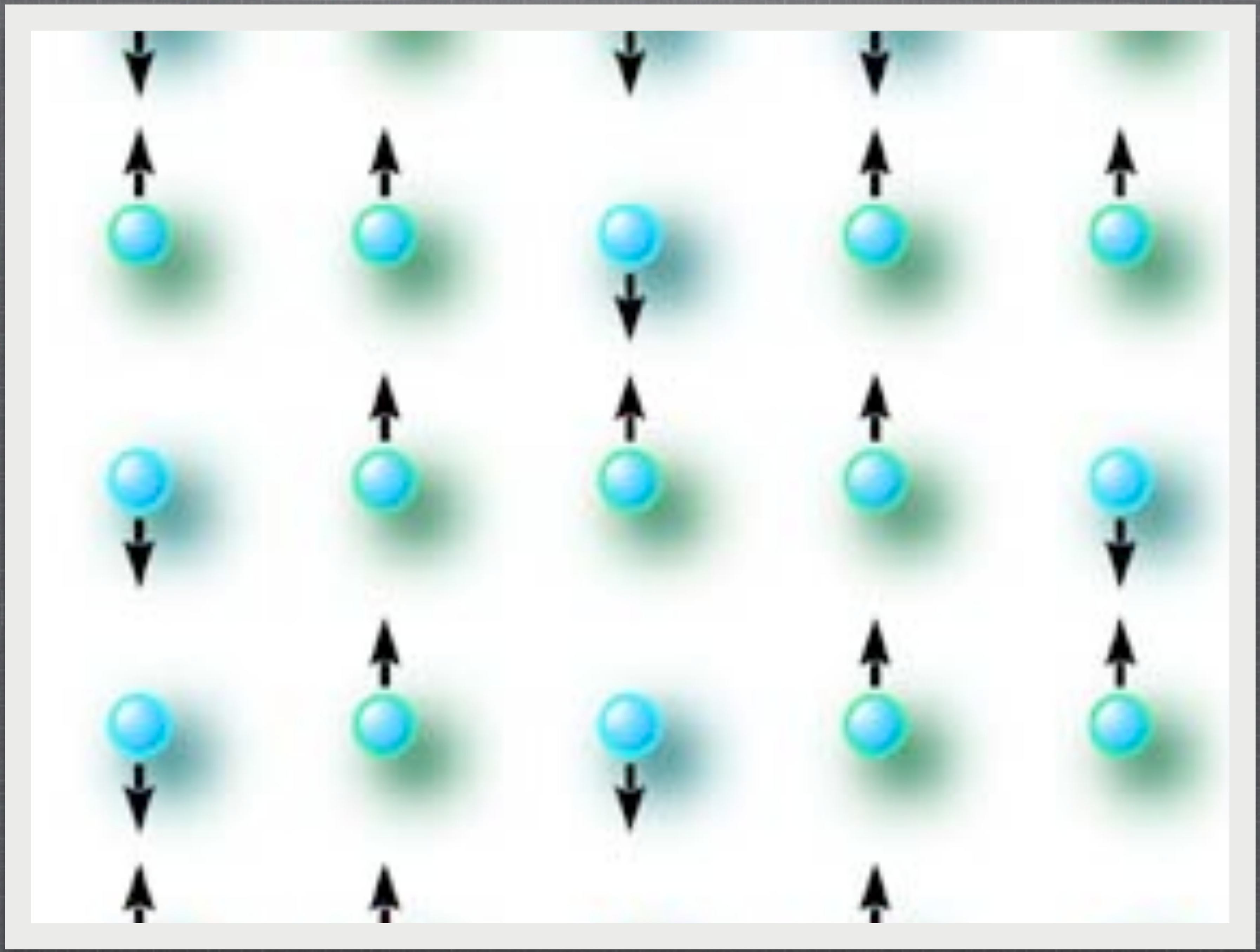
(*) Huiszoon, Dijkstra, Schellekens

(†) Anastasopoulos, Dijkstra, Kiritsis, Schellekens

COUPLINGS

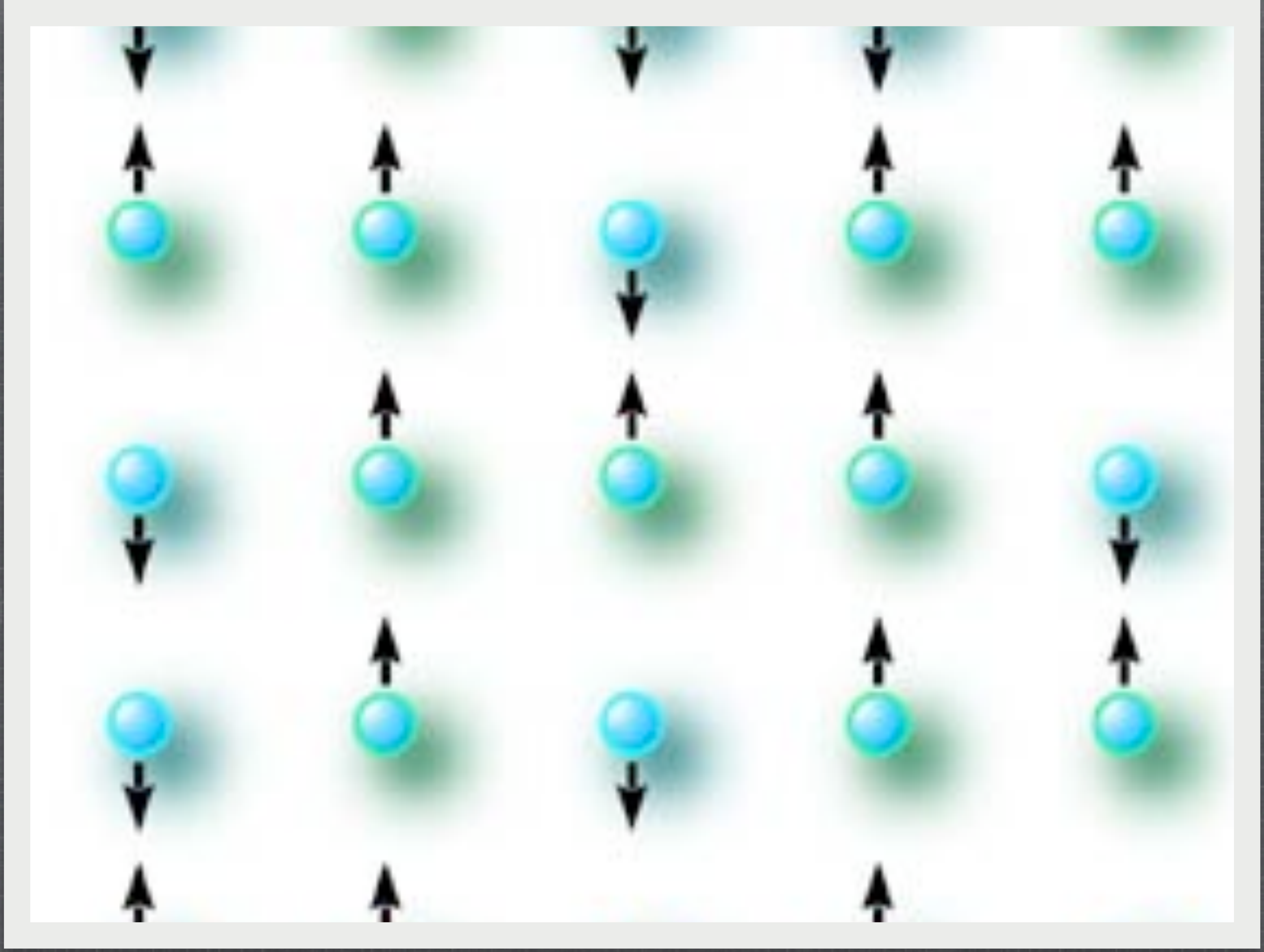
- Three-point couplings are computable ``in principle"
- But formalism not yet available
- Try something simpler: the Ising model?

FREE FERMIONS



FREE FERMIONS

M. Lennek, E. Kiritsis, A.N. Schellekens



ISING MODEL

RCFT with just three primary fields

$$\begin{aligned} 0 &: \quad h = 0 \\ \psi &: \quad h = \frac{1}{2} \\ \sigma &: \quad h = \frac{1}{16} \end{aligned}$$

Fusion rules:

$$[\psi] \times [\psi] = [0]$$

Simple current

$$[\sigma] \times [\sigma] = [0] + [\psi]$$

TENSORING

Central charge: $c = 1/2$

To get c=9 we tensor 18 copies.
But: the Ising model has no supersymmetry.

This can be overcome by imposing it on the tensor product by means of a chiral algebra extension:

KLT/ABK Triplet constraint (1986)

Current $\psi^\mu \partial X_\mu \psi_i \psi_j \psi_k$

This is a simple current, so the FHSSW formalism applies

SPACE-TIME SUSY

This requires another chiral algebra extension

Current $S_\alpha \sigma_1 \sigma_4 \sigma_7 \sigma_{10} \sigma_{13} \sigma_{16}$

SPACE-TIME SUSY

This requires another chiral algebra extension

Current $S_\alpha \sigma_1 \sigma_4 \sigma_7 \sigma_{10} \sigma_{13} \sigma_{16}$

But this is *not* a simple current;
we do not have a boundary state
formalism for such an extension.

Solution: pair two Ising models into a real boson.

$$|\chi_0\chi_0 + \chi_\psi\chi_\psi|^2 + |\chi_0\chi_\psi + \chi_\psi\chi_0|^2 + 2|\chi_\sigma|^2$$

0 $\qquad$ ψ $\qquad$ σ_1 $\qquad$ σ_2

Simple Currents

This yields the D_1 free boson CFT

ACCESSIBLE MIPFS

6 of the 18 fermions *must* be paired into bosons to get a susy simple current.

The other fermions *may* be paired into bosons in a definite way.

Such a pairing produces a new class of models because the spinor currents are now available as simple currents.

COMBINATIONS

$(NSR)\ (D_1)^9$

$(NSR)\ (D_1)^7\ (Ising)^4$

$(NSR)\ (D_1)^5\ (Ising)^8$

$(NSR)\ (D_1)^3\ (Ising)^{12}$

DEGENERACIES

The number of simple current MIPFs is extremely large ($> 10^{28}$ for (NSR) $(D_1)^3$ $(\text{Ising})^{12}$).

But there are many degeneracies

- Permutations of identical factors.
  [occurs also in Gepner models with identical factors]

- Ising degeneracy $[\psi] \times [\sigma] = \sigma$
  (some generically distinct MIPFs are identical)
  [occurs also in Gepner models with k=2 factors]

- Non-trivial free field theory relations.
  [occurs also in Gepner models with k=1 factors]

NUMBER OF MIPFS

(NSR) $(D_1)^9$	685 MIPFs
(NSR) $(D_1)^7$ $(\text{Ising})^4$	5084 MIPFs
(NSR) $(D_1)^5$ $(\text{Ising})^8$	57474 MIPFs
(NSR) $(D_1)^3$ $(\text{Ising})^{12}$	1570138 MIPFs

This is modulo all permutations, except the last case, which is modulo $S_3 \times S_8 \times S_4$.

The other degeneracies are still present

HODGE NUMBERS

To get an idea about the number of really distinct MIPFs we can compute the Hodge numbers.

The following values occur

51, 3	3, 51
31, 7	7, 31
27, 3	3, 27
25, 1	1, 25
21, 9	9, 21
19, 7	7, 19
17, 5	5, 17
15, 3	3, 15
12, 6	6, 12

21, 21 (N=2)	7, 7
19, 19	5, 5 (N=2)
15, 15	3, 3
13, 13	1, 1 (N=2)
13, 13 (N=2)	
11, 11	
9, 9	
9, 9 (N=2)	
9, 9 (N=4)	

HODGE NUMBERS

In total: 31 distinct Hodge pairs
(cf. 880 for Gepner models, 30.000 on the Kreuzer-Skarke list)

Some MIPFs with identical Hodge numbers may still be distinct.

They may be distinguished further by computing the number of Heterotic singlets, and the number of boundary states. For Gepner models, this is usually sufficient.

The former distribution is exactly mirror symmetric, the latter not.

DISTINCT MIPS

Case	MIPFs	Distinct (Singlets)	Distinct (Singlets + Boundaries)
(NSR) $(D_1)^9$	685	34	121
(NSR) $(D_1)^7$ $(\text{Ising})^4$	5084	55	325
(NSR) $(D_1)^5$ $(\text{Ising})^8$	57474	135	973
(NSR) $(D_1)^3$ $(\text{Ising})^{12}$	1570138	181	1356

MIRROR SYMMETRY

e.g. for Euler number 12 in (NSR) $(D_1)^3$ $(\text{Ising})^{12}$

544 x	(12,6,129)
544 x	(6,12,129)
7728 x	(12,6,126)
7728 x	(6,12,126)
52384 x	(12,6,123)
52384 x	(6,12,123)
133408 x	(12,6,120)
133408 x	(6,12,120)

SEARCH RESULTS

(NSR) $(D_1)^9$	SM configuration, no tadpole cancellation
(NSR) $(D_1)^7$ $(\text{Ising})^4$	Nothing
(NSR) $(D_1)^5$ $(\text{Ising})^8$	Nothing
(NSR) $(D_1)^3$ $(\text{Ising})^{12}$	Nothing (using random MIPF selection)

SM CONFIGURATIONS

- All occur for just one of the 685 MIPFs
- In total: 512 times ADKS spectrum 800,
  512 times ADKS spectrum 101 (*)

(*) Nrs. assigned as in Anastasopoulos, Dijkstra, Kiritsis, Schellekens

SPECTRA

U(4)	U(2)	U(2)	mult.
0	V*	V	2
V*	0	V	1
V	V	0	2
V*	0	V*	2
V	V*	0	1

Exact! No non-chiral states!

Also a U(3)×U(1) version

CONCLUSION

- We are all just scratching the surface...