

HETEROTIC WEIGHT LIFTING

B. Gato-Rivera and A.N Schellekens

(Nucl.Phys.B828:375-389,2010; arXiv:1003.6075) and to appear

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GEPNER MODELS

Tensor product of an NSR model in 4 space time dimensions with a number of N=2 minimal CFT's with total central charge 9.

Partition function $\sum_{i,j} \chi_i(\tau) M_{ij} \chi_j(\bar{\tau})$

Heterotic:

Map the NSR model to $SO(10) \times E_8$ in the bosonic sector.

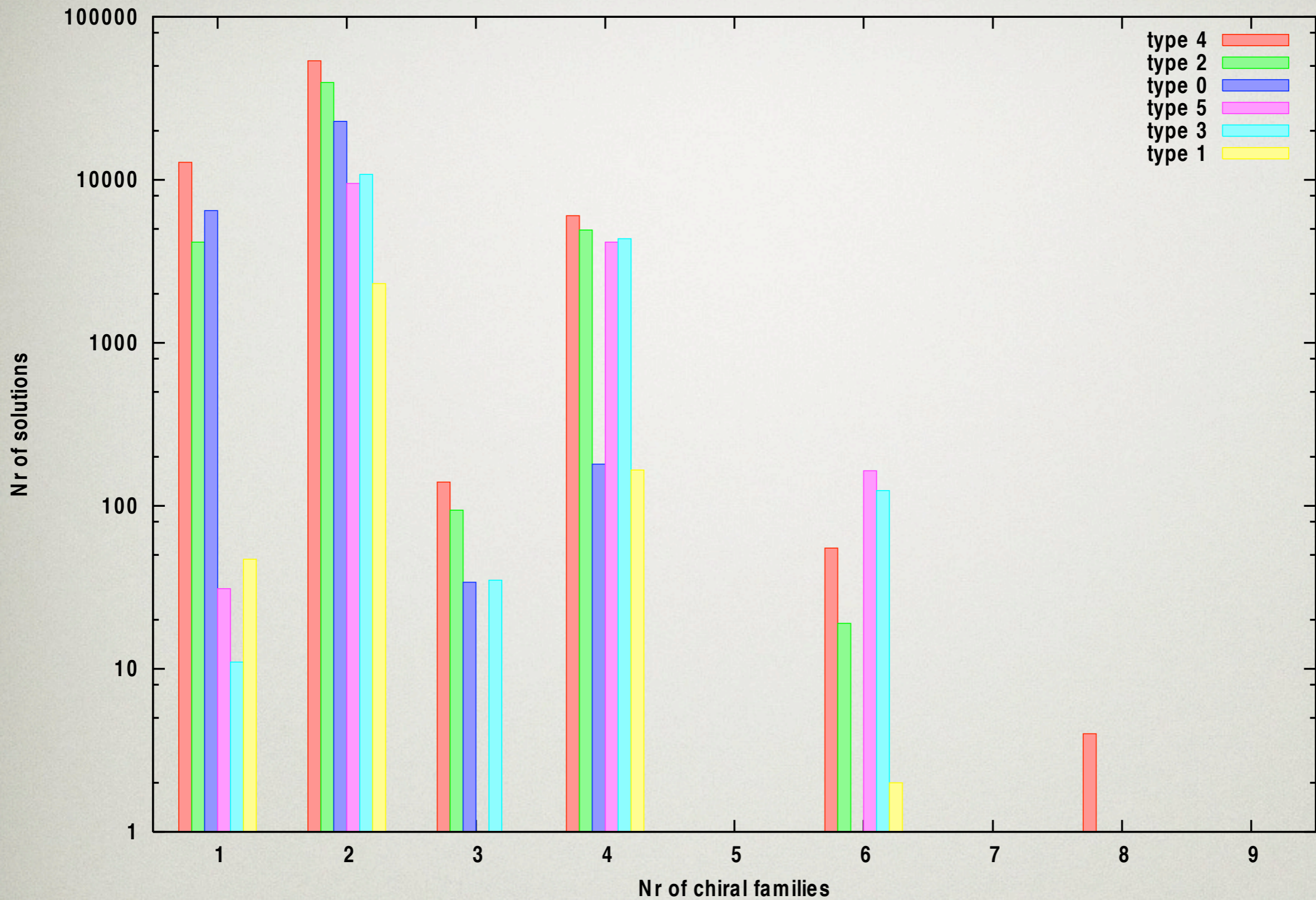
M not necessarily symmetric; Standard model embedded in $SO(10)$

Orientifold:

Symmetric matrix M (type-II)

Mod out world-sheet orientation.

Add boundary and crosscap states, Standard Model from intersecting branes.



Dijkstra, Huiszoon, Schellekens (2004)
See also Gmeiner et. al. "One in a billion"

RCFT: HETEROTIC VS ORIENTIFOLD

During the last five years, orientifolds were scanned systematically for Standard Model spectra

Dijkstra, Huiszoon, Schellekens

Gmeiner, Blumenhagen, Honecker, Lust, T. Weigand

Anastasopoulos, Dijkstra, Kiritsis, Schellekens

Douglas, Taylor

Kiritsis, Lennek, Schellekens

Gmeiner, Honecker

Few comparable results exist for heterotic strings. All we have are Hodge number scans¹, and fermionic construction scans²

- (1) *Lutken, Ross (1988)*
Schellekens, Yankielowicz (1989)
Fuchs, Klemm, Scheich, Schmidt (1989)
Kreuzer, Skarke (1992)
Donagi, Faraggi (2004),
Ploger, Ramos-Sanchez, Ratz, Vaudrevange (2007)
Donagi, Wendland (2008)
Kiritsis, Lennek, Schellekens (2008)
- (2) *Dienes, Senechal (2007)*
Assel, Christodoulides, Faraggi, Kounnas, Rizos (2009)



CERN-TH.5440/89

NEW MODULAR INVARIANTS FOR $N=2$ TENSOR PRODUCTS AND FOUR-DIMENSIONAL STRINGS

A. N. Schellekens

and

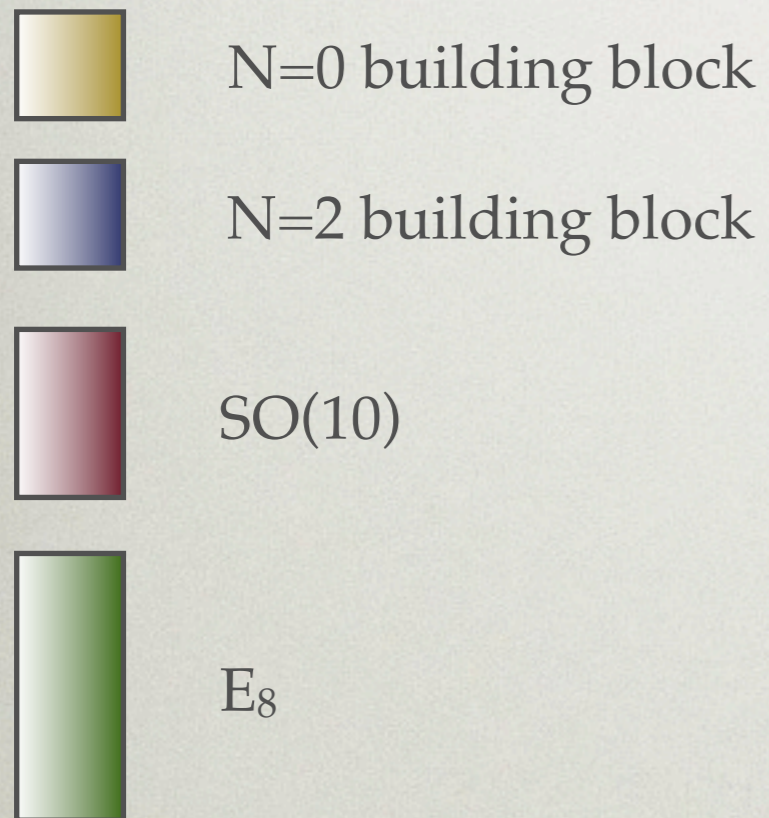
S. Yankielowicz^{*†}

CERN, 1211 Geneva 23, Switzerland

ABSTRACT

The construction of modular invariant partition functions of tensor products of $N = 2$ superconformal field theories is clarified and extended by means of a recently proposed method using simple currents, *i.e.* primary fields with simple fusion rules. Apart from providing a conceptually much simpler way of understanding space-time and world-sheet supersymmetry projections in modular invariant string theories, this makes a large class of modular invariant partition functions accessible for investigation. We demonstrate this by constructing thousands of (2,2), (1,2) and (0,2) string theories in four dimensions, including more than 40 new three generation models.

General (0,2) model in RCFT



Modular invariance makes this very hard

$$P(\tau, \bar{\tau}) = \sum_{ij} \chi_i(\tau) M_{ij} \xi_j(\bar{\tau})$$

$$P\left(-\frac{1}{\tau}, -\frac{1}{\bar{\tau}}\right) = P(\tau, \bar{\tau})$$

S

$$P(\tau + 1, \bar{\tau} + 1) = P(\tau, \bar{\tau})$$

T

Has a canonical solution, $M_{ij} = \delta_{ij}$, if the left and the right CFT are identical, so that $\chi = \xi$.

But they do not have to be identical, only isomorphic as representations of S and T.

In particular, this allows certain integer shifts of the eigenvalues of T, the conformal weights.

Left

Right

SO(10)

E_8

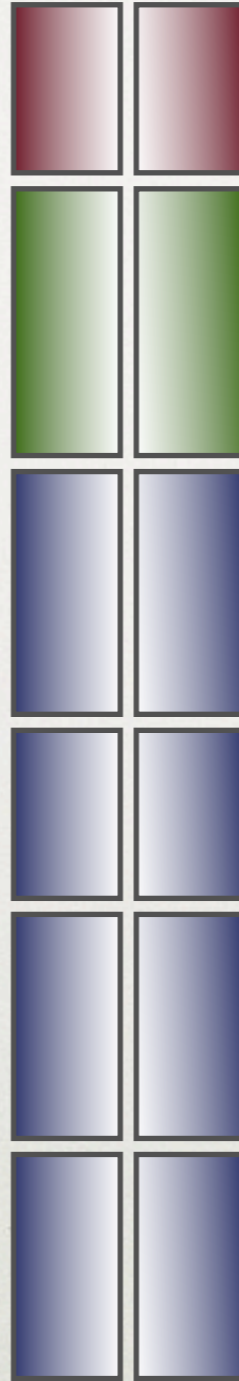
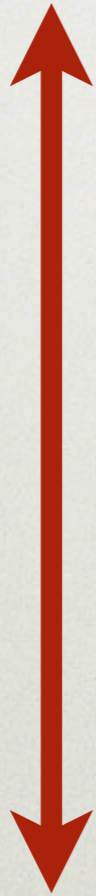
$N=2, k_1$

$N=2, k_2$

$N=2, k_3$

$N=2, k_4$

$c=9$



Bosonic

Fermionic

SO(10)

E_8

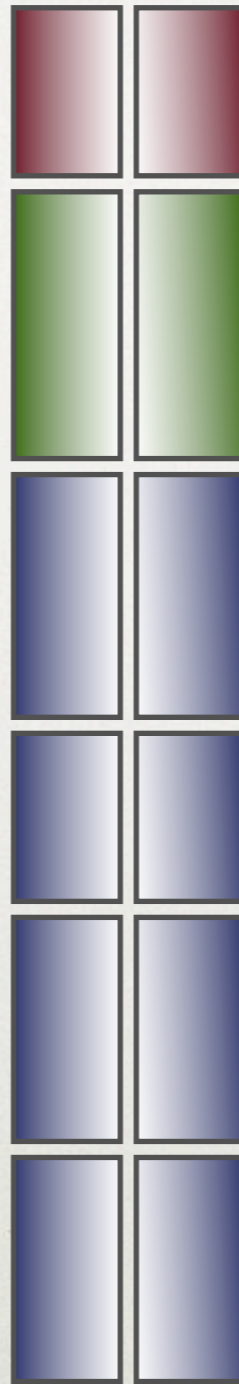
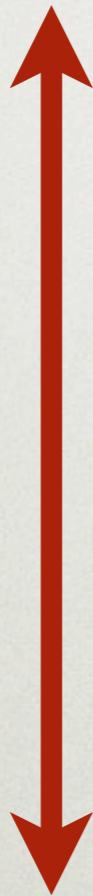
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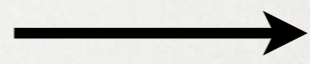
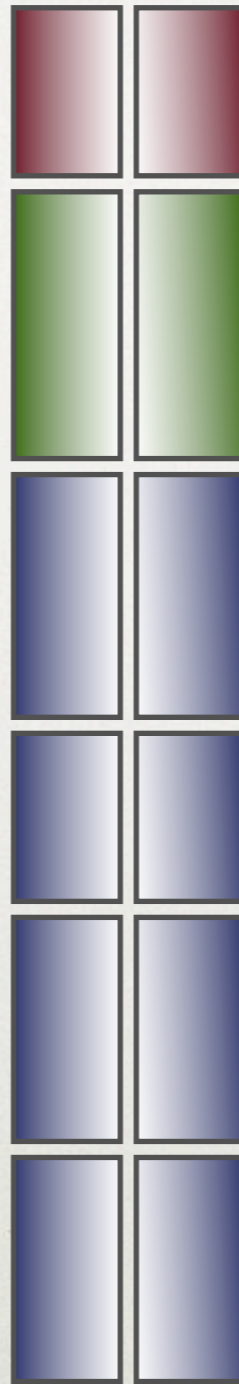
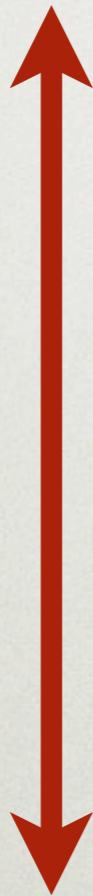
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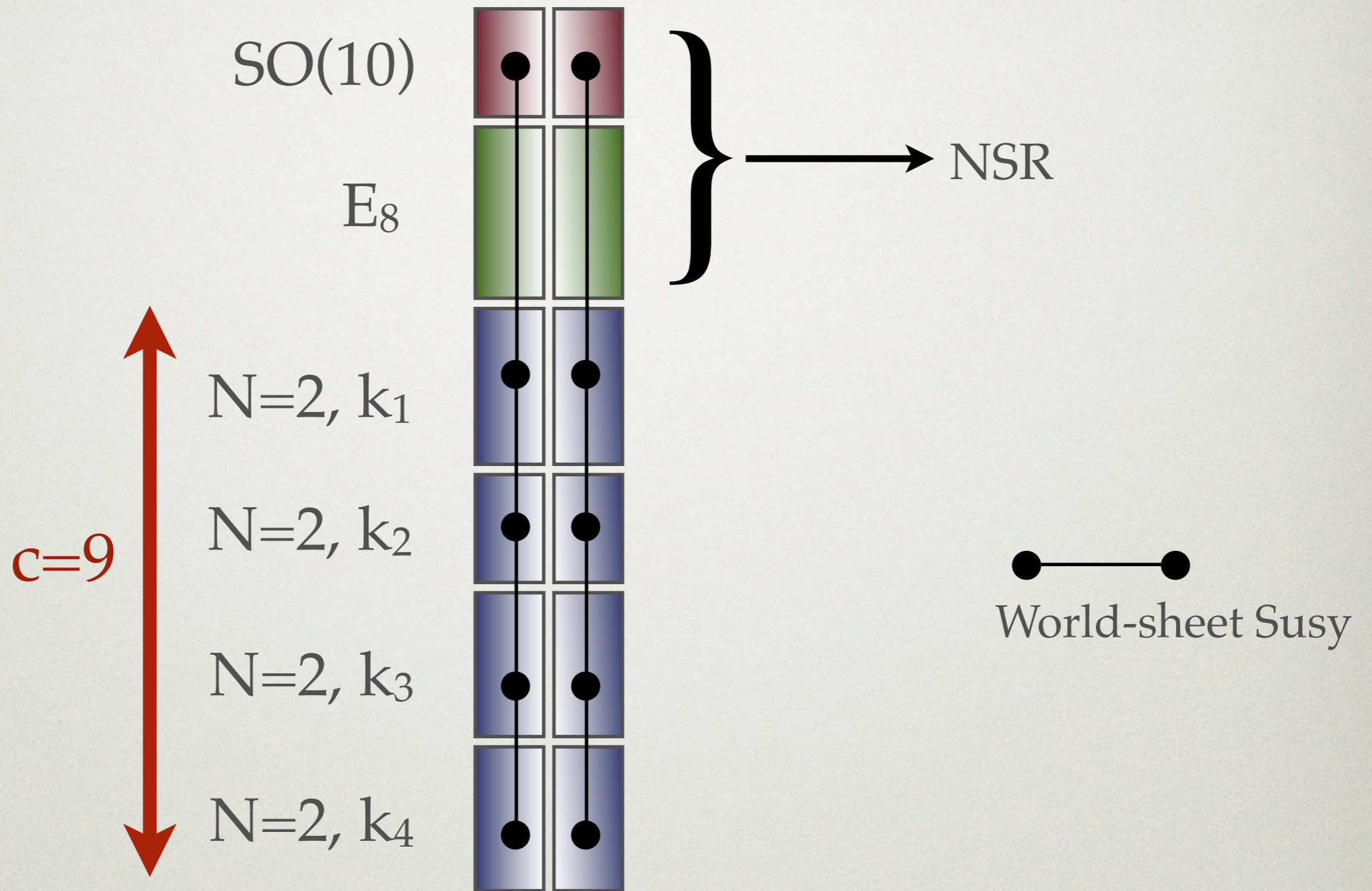
$c=9$



NSR

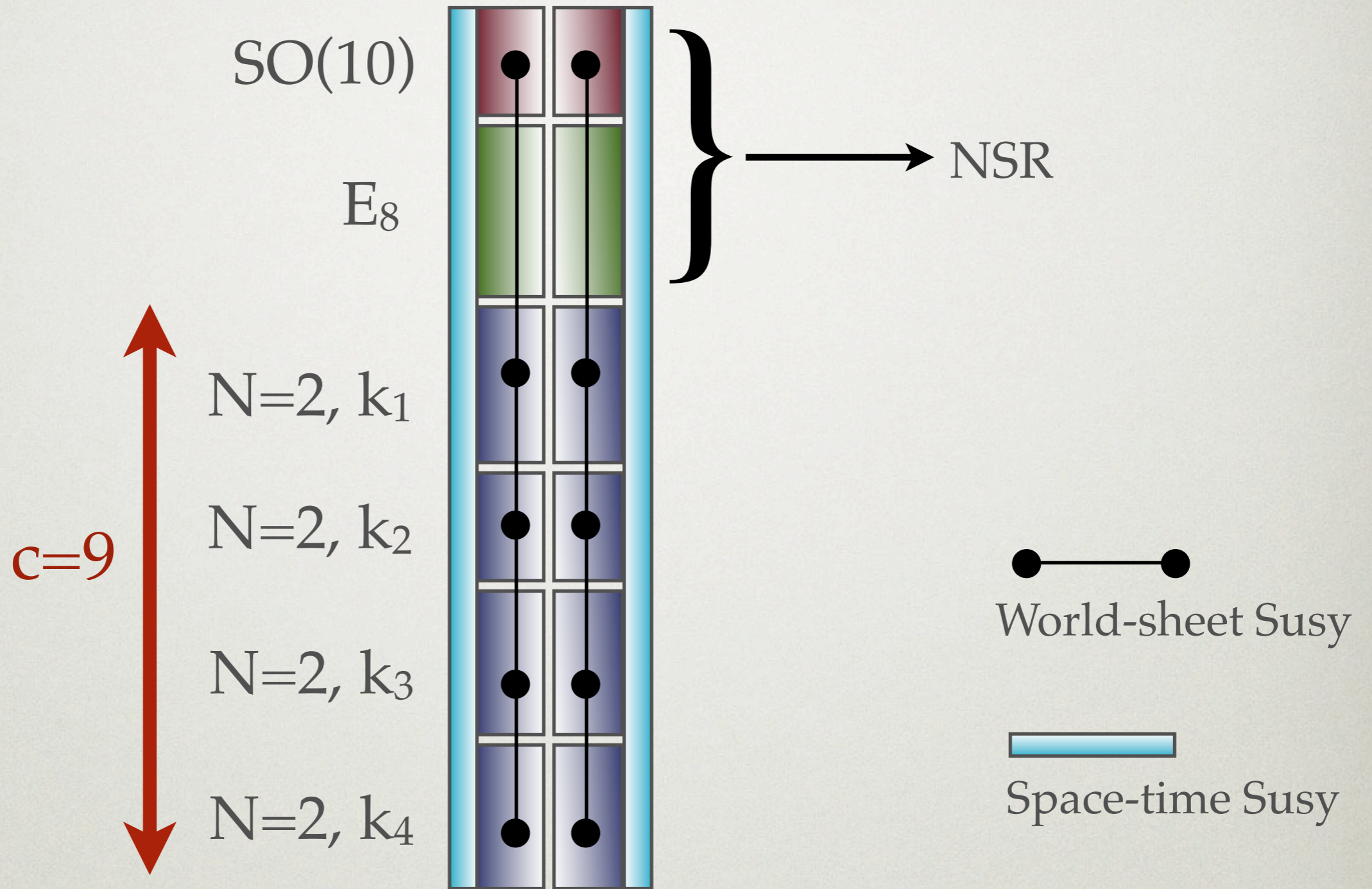
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Fermionic



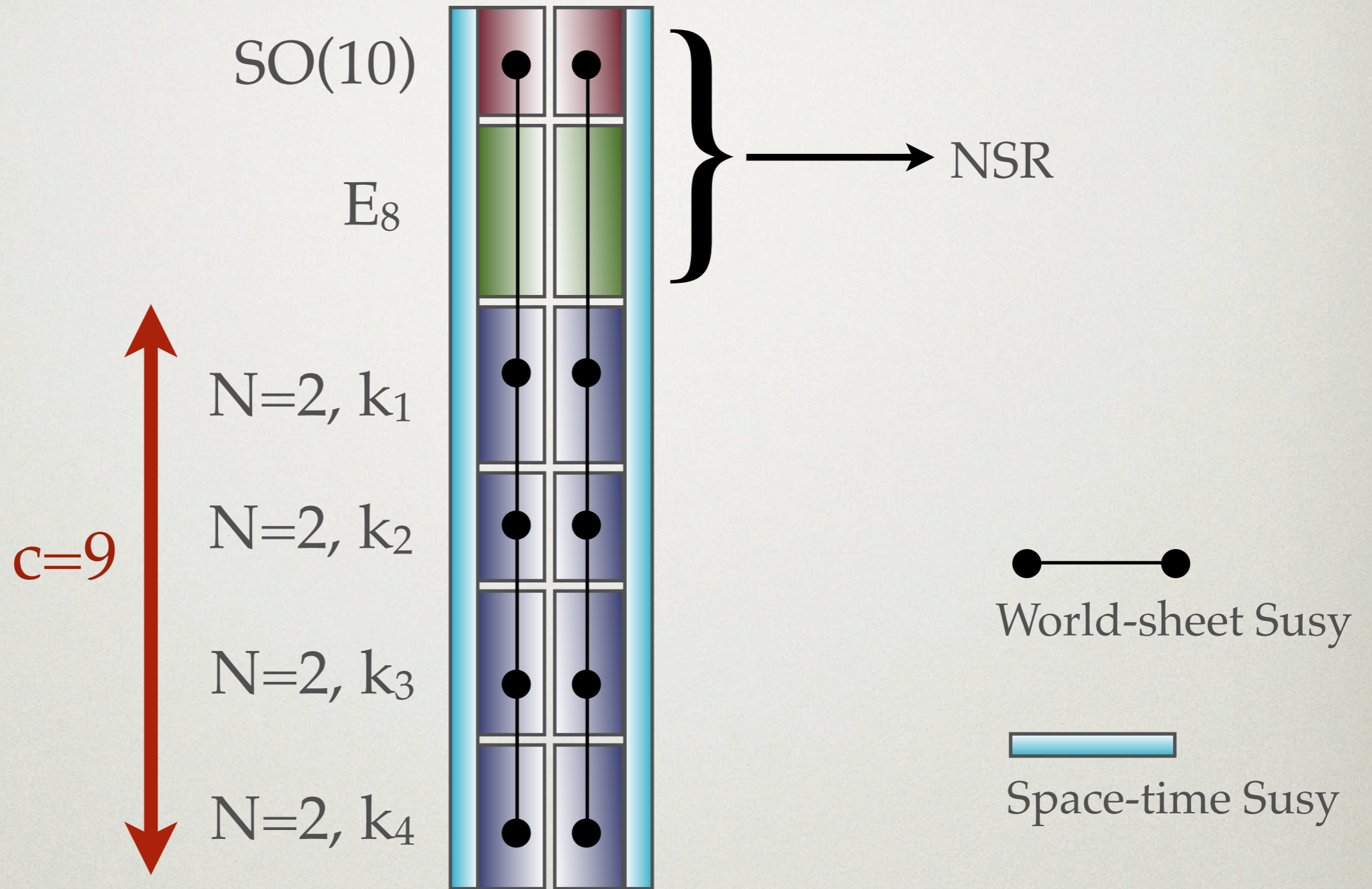
Bosonic

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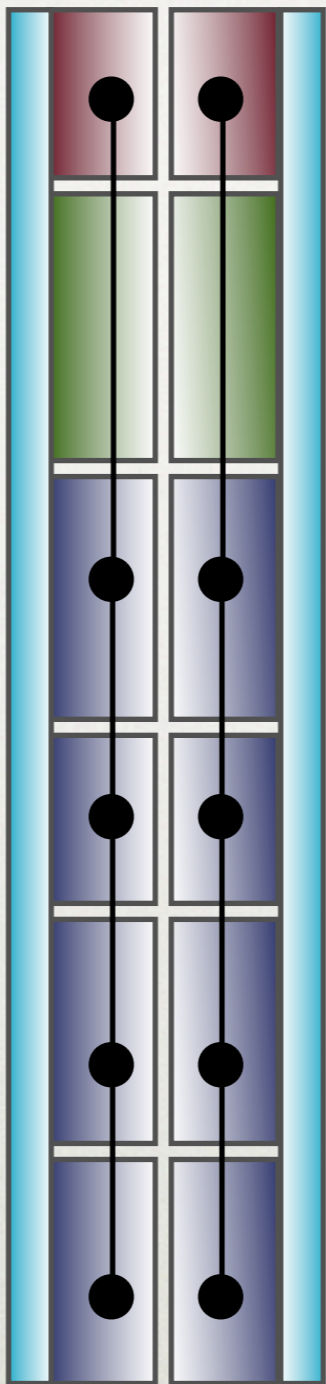


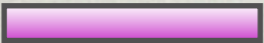
Bosonic

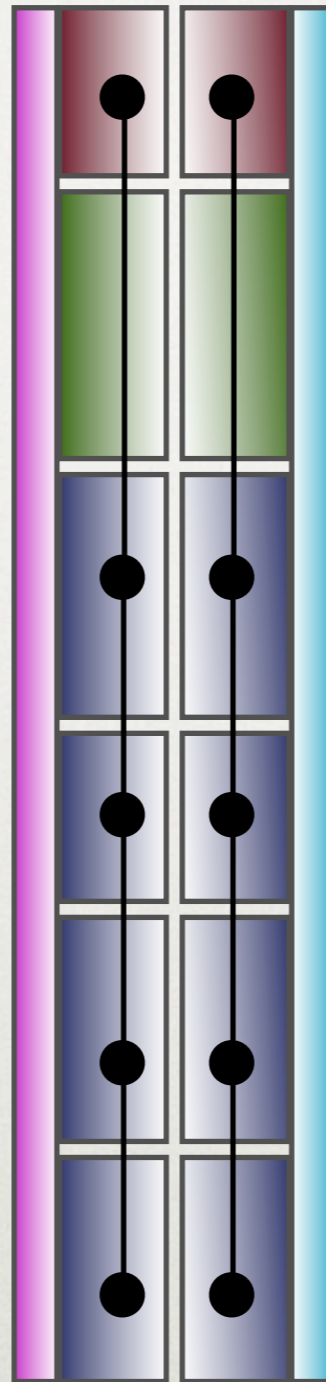
Fermionic

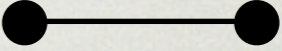



(2,2) model. Gauge group $E_6 (\times E_8 \times \dots)$



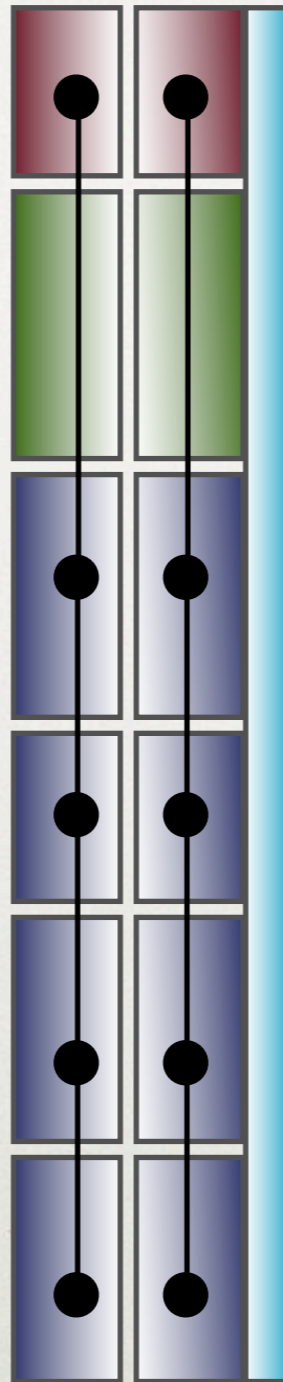

Higher spin algebra

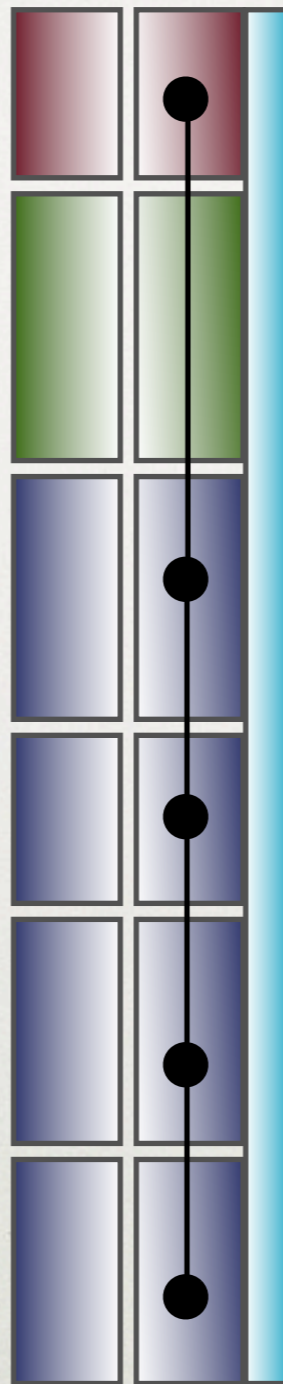



World-sheet Susy


Space-time Susy

(1,2) model. Gauge group $SO(10) (\times E_8 \times \dots)$





(0,2) model. Gauge group $SO(10) (\times E_8 \times \dots)$

Old results on Gepner model simple current MIPFs

Schellekens, Yankielowicz (1989): (2,2) , (1,2)

Fuchs, Klemm, Scheich, Schmidt (1989) (2,2)

Number of families:

Define Δ : the greatest common divisor of the number of families for a given CFT

The following values of Δ occur for the 168 minimal model combinations and 58 of the 59 exceptional ones: 120, 96, 72, 60, 48, 40, 36, 32, 24, 12, 8, 6, 4 and 0.

There is one case with $\Delta=3$: (1,16*,16*,16*) (Gepner, unpublished).

This allowed us to get 3-family (2,2), (1,2) and (0,2) models with gauge groups E_6 or $SO(10)$ (44 distinct ones)

[(0,2) was only tried for the (1,16,16*,16*) combination]*

6. Outlook and conclusions

Clearly the method we have advocated in this paper greatly extends the list of four-dimensional string theories accessible to exploration. However, this is by no means all one can do. Up to now we have always kept an unbroken $SO(10) \times E_8$ Kac-Moody algebra on the left. However, just as one can break the left-moving “space-time” and world-sheet supersymmetries, one can break this KM-algebra as well. To do so, one simply starts with characters of some conformal sub-algebra of $SO(10) \times E_8$. Of course one wants to get the full $SO(10) \times E_8$ algebra on the *right*, in order to be able to map this sector to a fermionic one. But this can always be achieved by putting some projection matrices in front of the right-moving characters to add the missing $SO(10) \times E_8$ roots.

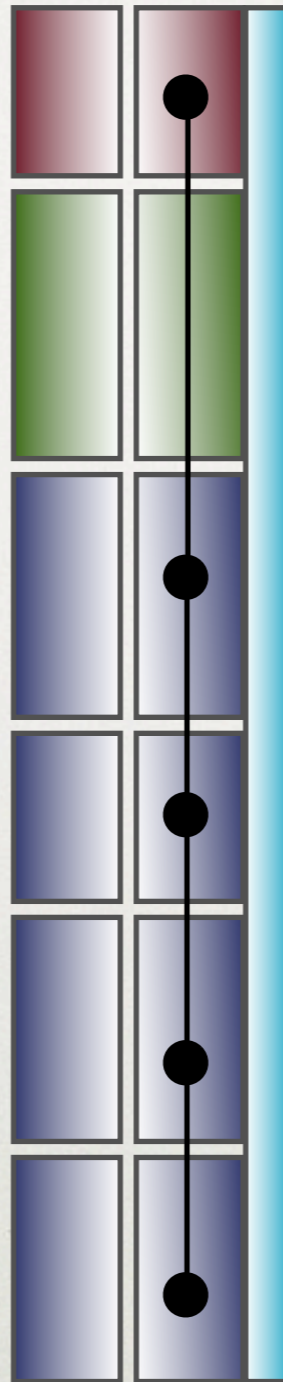
This opens the way to constructing string theories whose gauge group is something a bit closer to the standard model than $SO(10)$, perhaps even $SU(3) \times SU(2) \times U(1)^n$ (where n is almost inevitably larger than 1). There is no reason why one could not get 3 generations in such a model, and in fact there could well be many more models than those listed in table III, since the center of the conformal field theory one starts with is even larger. We hope to come back to this in the future.

6. Outlook and conclusions

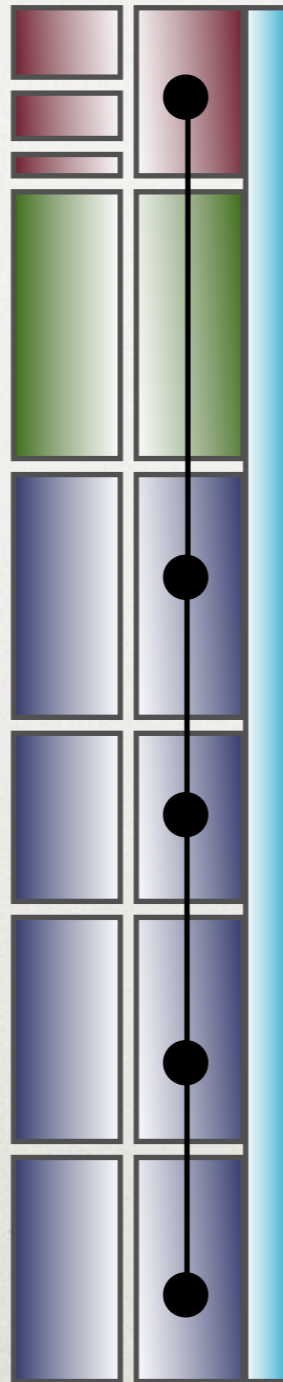
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SO(10) BREAKING



SO(10) currents replaced by
operators of higher weight



(0,2) model. Gauge group $H \subset SO(10) (\times H' \subset E_8 \times \dots)$

BREAKING SO(10)

Consider* $SU(3) \times SU(2) \times U(1)_{30} \times U(1)_{20} \subset SO(10)$

This should give chiral families of $SU(3) \times SU(2) \times U(1)$ with standard gauge coupling unification.

Indeed, it does, but there was a major disappointment:

All these spectra contain fractionally charged particles.

This was easily seen to be a very general result.

(A.N. Schellekens, Phys. Lett. B237, 363, 1990).

But there are ways out: they can be massive, vector-like (or confined by another gauge group)

(* *A.N. Schellekens and S. Yankielowicz (1989)*

Other subgroups were considered by Blumenhagen, Wisskirchen, Schimmrigk (1995, 1996)

Modular Invariant Partition Function:

$$\sum_{ij} \chi_i(\bar{\tau}) M_{il}^{\text{proj}} M_{lk}(J_1, \dots, J_n) \chi_k(\tau)$$

Worldsheet susy
Space-time susy
SO(10) projection

$N \times N$ matrix
for n simple currents

For K minimal models:

$$N = 3 \times 2 \times 60 \times 20 \times \prod_i^K N_i$$

(3,3,3,3,3)

368.640.000.000

Simple current MIPFs

CFT factors contribute:

SU(3)	\mathbf{Z}_3
SU(2)	\mathbf{Z}_2
U ₃₀	\mathbf{Z}_{30}
U ₂₀	\mathbf{Z}_{20}
Minimal N=2, k even	$\mathbf{Z}_{4k+2} \times \mathbf{Z}_2$
Minimal N=2, k odd	\mathbf{Z}_{8k+4}

Choose a subgroup, plus a matrix of rational numbers on that subgroup

Potentially a huge landscape:

For K currents of order p (prime)

(B. Gato-Rivera, A.N. Schellekens, Comm. Math. Phys. 145, 85 (1992))

$$N_{\text{MIPF}} = \prod_{l=0}^{K-1} (1 + p^l)$$

The seven \mathbf{Z}_5 factors in $SU(3) \times SU(2) \times U_{30} \times U_{20} \times (\mathbf{k}=3)^5$ contribute a factor

1.202.088.011.709.312

This is reduced by at most $5! \times 2^8$ (permutations, outer automorphisms),
and enhanced by a factor 8 for $(\mathbf{Z}_3)^2$ and an unknown, huge factor for $(\mathbf{Z}_2)^2 \times (\mathbf{Z}_4)^6$

SO(10) SUB-ALGEBRAS

Nr.	Name	Current	Order	Gauge group	Grp.	CFT
0	SM, Q=1/6	(1, 1, 0, 0)	1	$SU(3) \times SU(2) \times U(1) \times U(1)$	$\frac{1}{6}$	$\frac{1}{6}$
1	SM, Q=1/3	(1, 2, 15, 0)	2	$SU(3) \times SU(2) \times U(1) \times U(1)$	$\frac{1}{6}$	$\frac{1}{3}$
2	SM, Q=1/2	(3, 1, 10, 0)	3	$SU(3) \times SU(2) \times U(1) \times U(1)$	$\frac{1}{6}$	$\frac{1}{2}$
3	LR, Q=1/6	(1, 1, 6, 4)	5	$SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$	$\frac{1}{6}$	$\frac{1}{6}$
4	SU(5) GUT	($\bar{3}$, 2, 5, 0)	6	$SU(5) \times U(1)$	1	1
5	LR, Q=1/3	(1, 2, 3, -8)	10	$SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$	$\frac{1}{6}$	$\frac{1}{3}$
6	Pati-Salam	($\bar{3}$, 0, 2, 8)	15	$SU(4) \times SU(2)_L \times SU(2)_R$	$\frac{1}{2}$	$\frac{1}{2}$
7	SO(10) GUT	(3, 2, 1, 4)	30	$SO(10)$	1	1

Internal CFT restrictions:

To remove any of these sub-algebras we must be able to map these currents to a different current in the left sector.

This imposes constraints on the internal sector.

To project out the $SU(2)_R$ extension we need a simple current of order 5 (k_i+2 divisible by 5 for at least one i).

This extension is undesirable

To project out the half-integer charge constraint, we need one i with k_i+2 divisible by 3.

This extension is desirable.

New results on Gepner model simple current MIPFs

Gato-Rivera, Schellekens (2010): $(2,2)$, $(1,2)$, $(0,2)$, broken $SO(10)$

Number of families:

The following values of Δ occur for the 168 minimal model combinations and 58 of the 59 exceptional ones: 120, 96, 72, 60, 48, 40, 36, 32, 24, 12, 8, 6, 4 and 0.

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$\Delta=2$: $(6,6,6,6)$
 $(3,3,3,3,3)$
 $(3,6,6,18)$
 $(3,3,18,18)$
 $(3,3,12,33)$
 $(3,3,9,108)$

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Obvious pattern.

Appears to extend to other cases
(Free fermions, Kazama-Suzuki*, in the latter
there are a few cases with $\Delta=3$)

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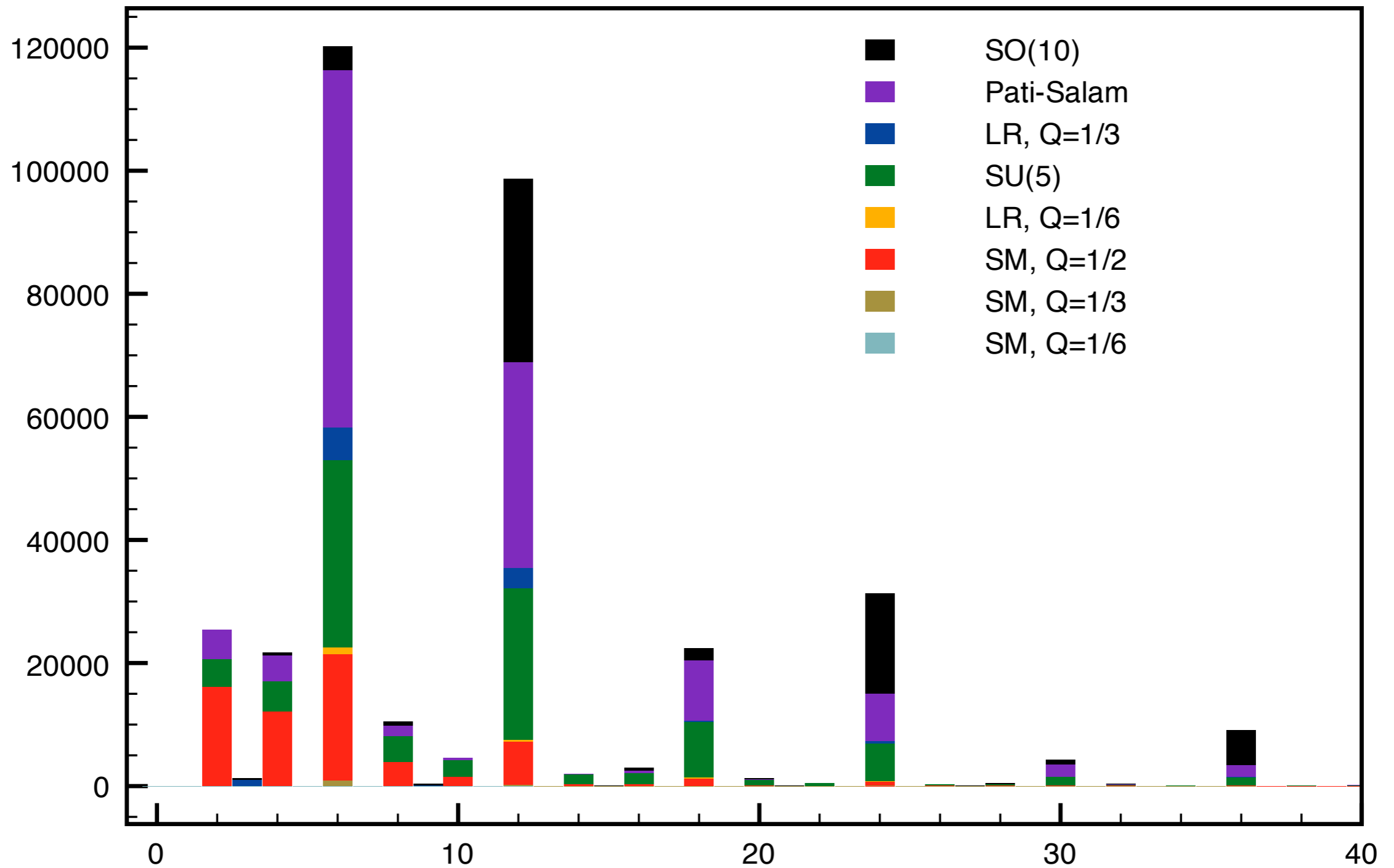
() Ibanez, Font, Quevedo (1989)*

Schellekens (1991)

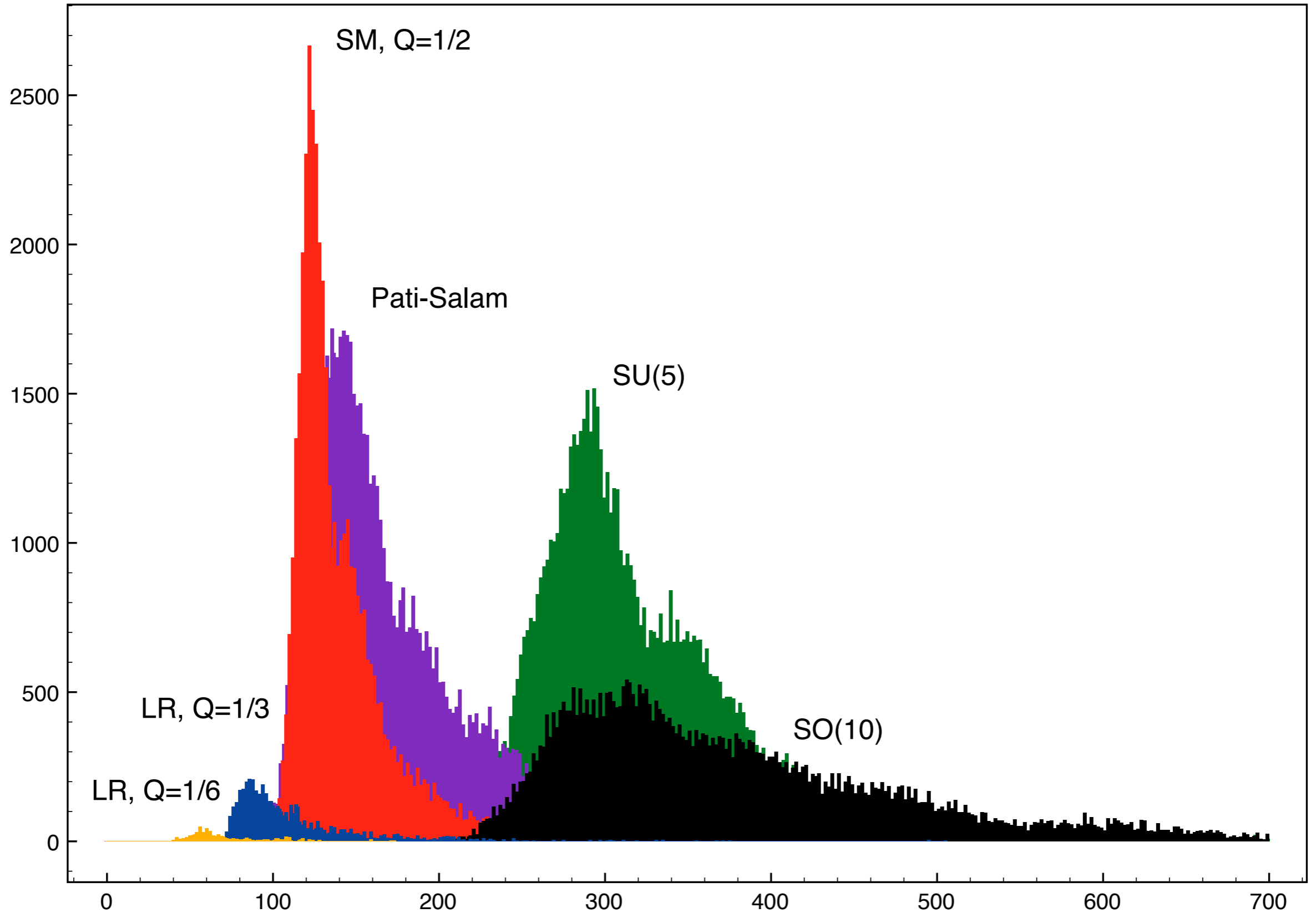
[only $(2,2)$ diagonal known]

Family Distribution

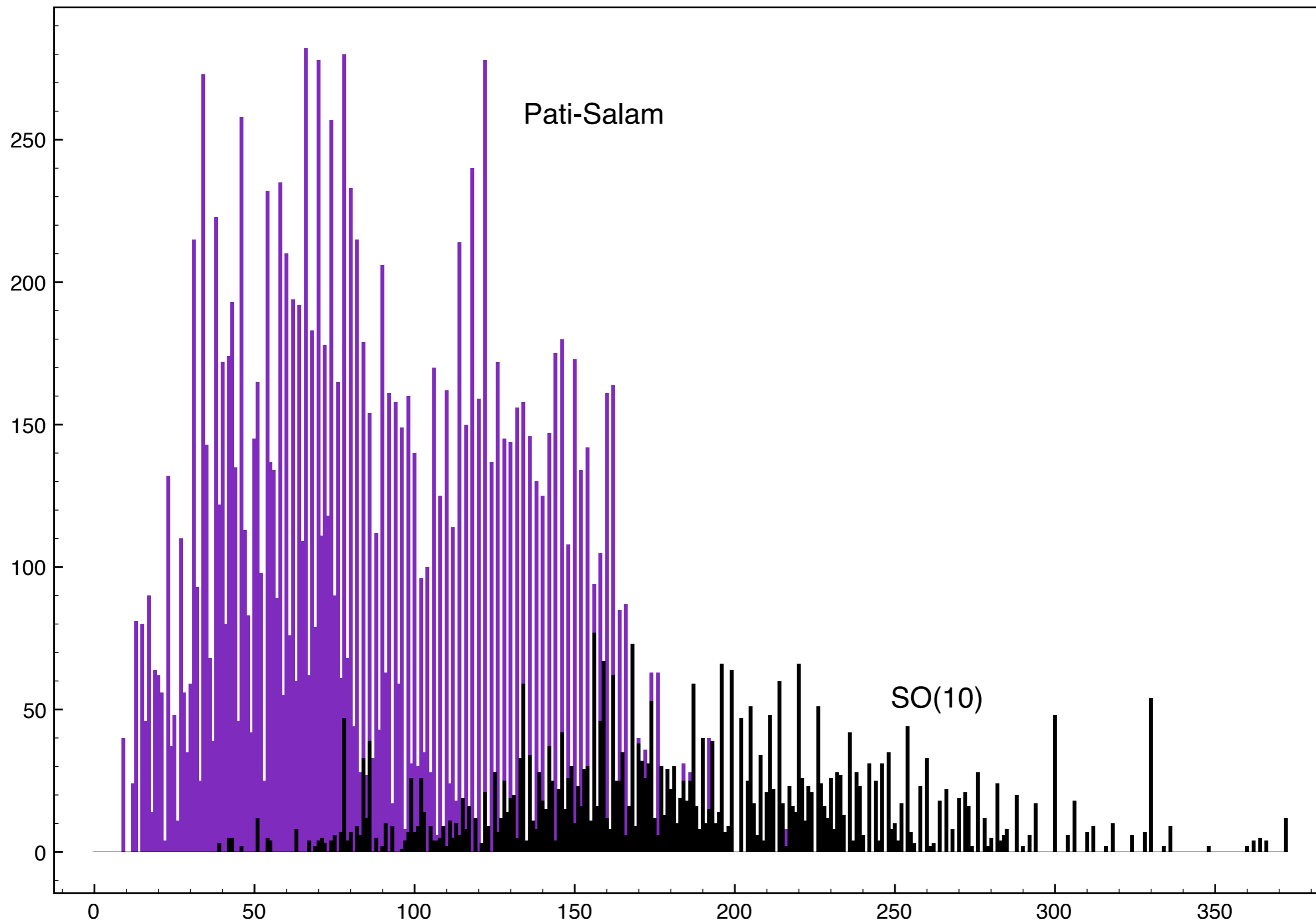
Nr. of MIPFs



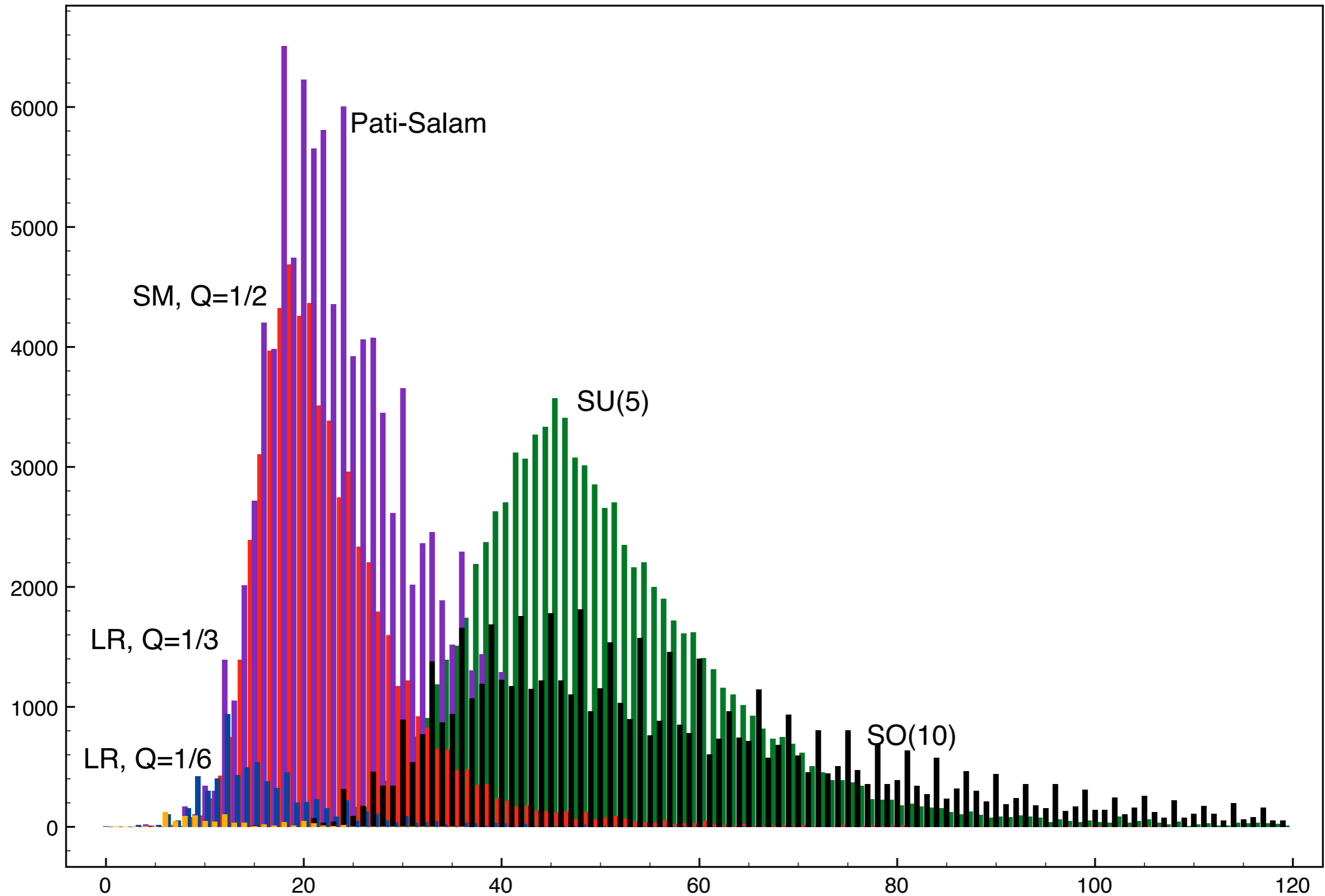
Singlet distribution



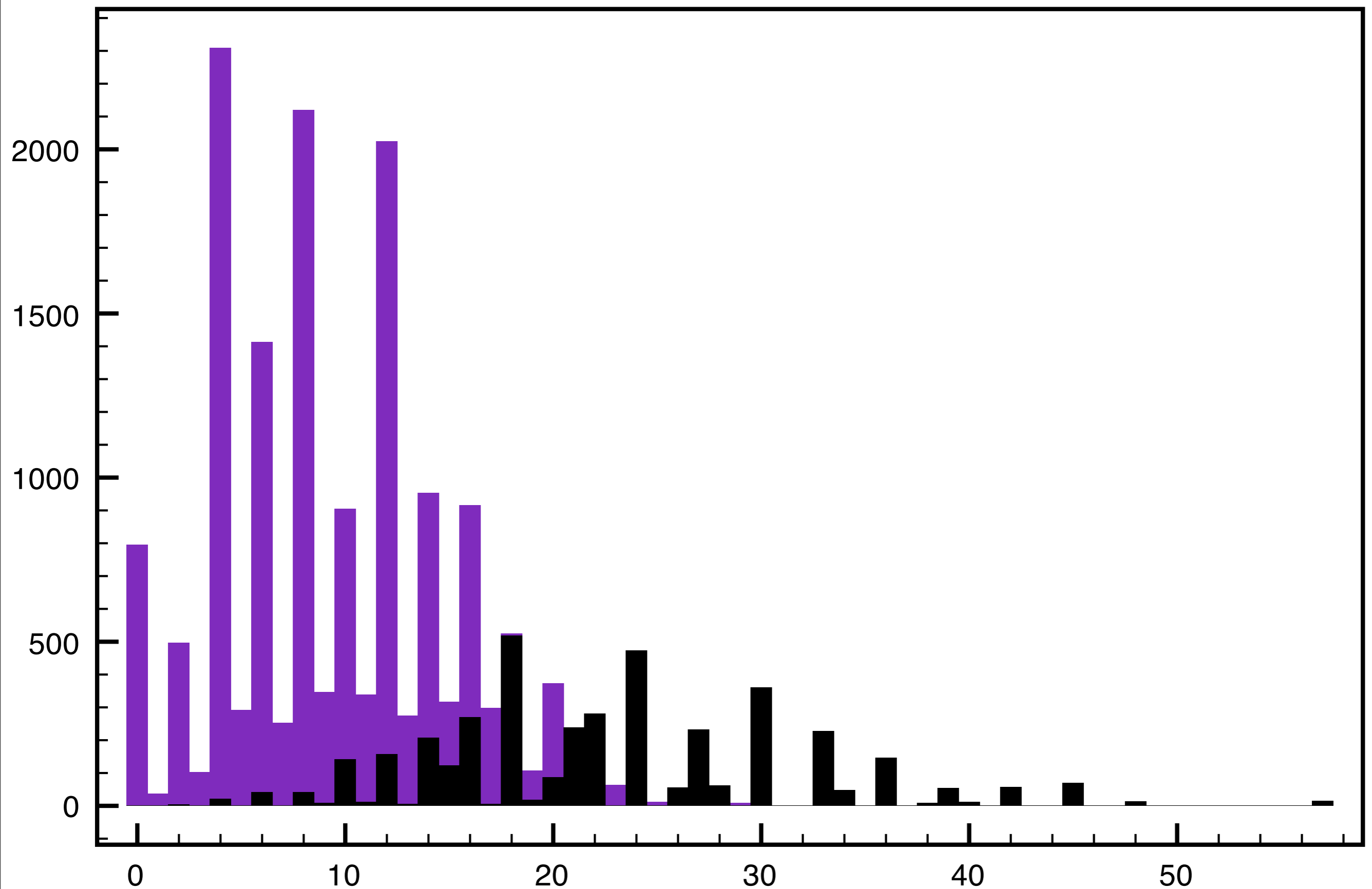
Singlet distribution (Free Fermions)



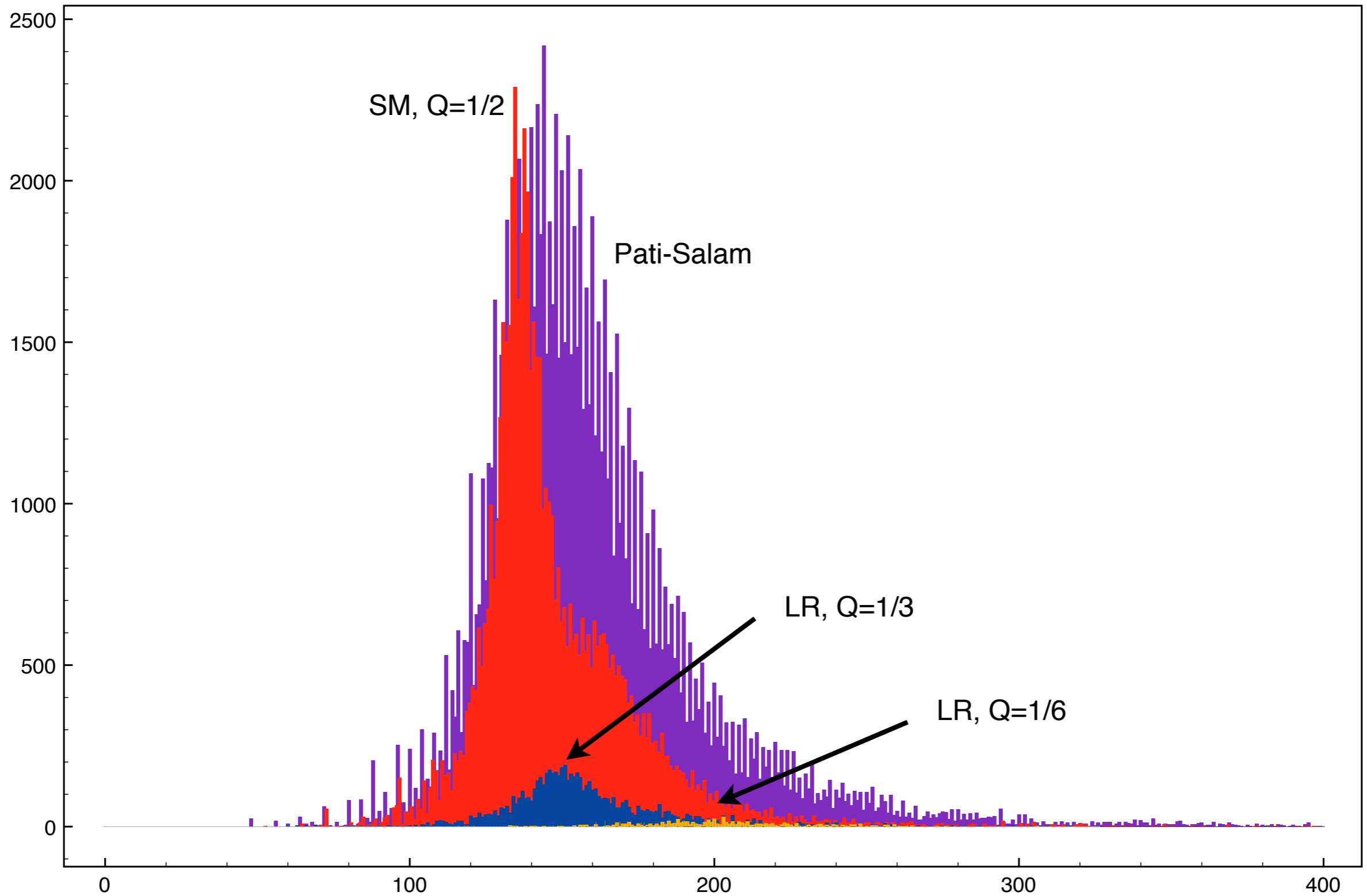
L-type mirror distributions



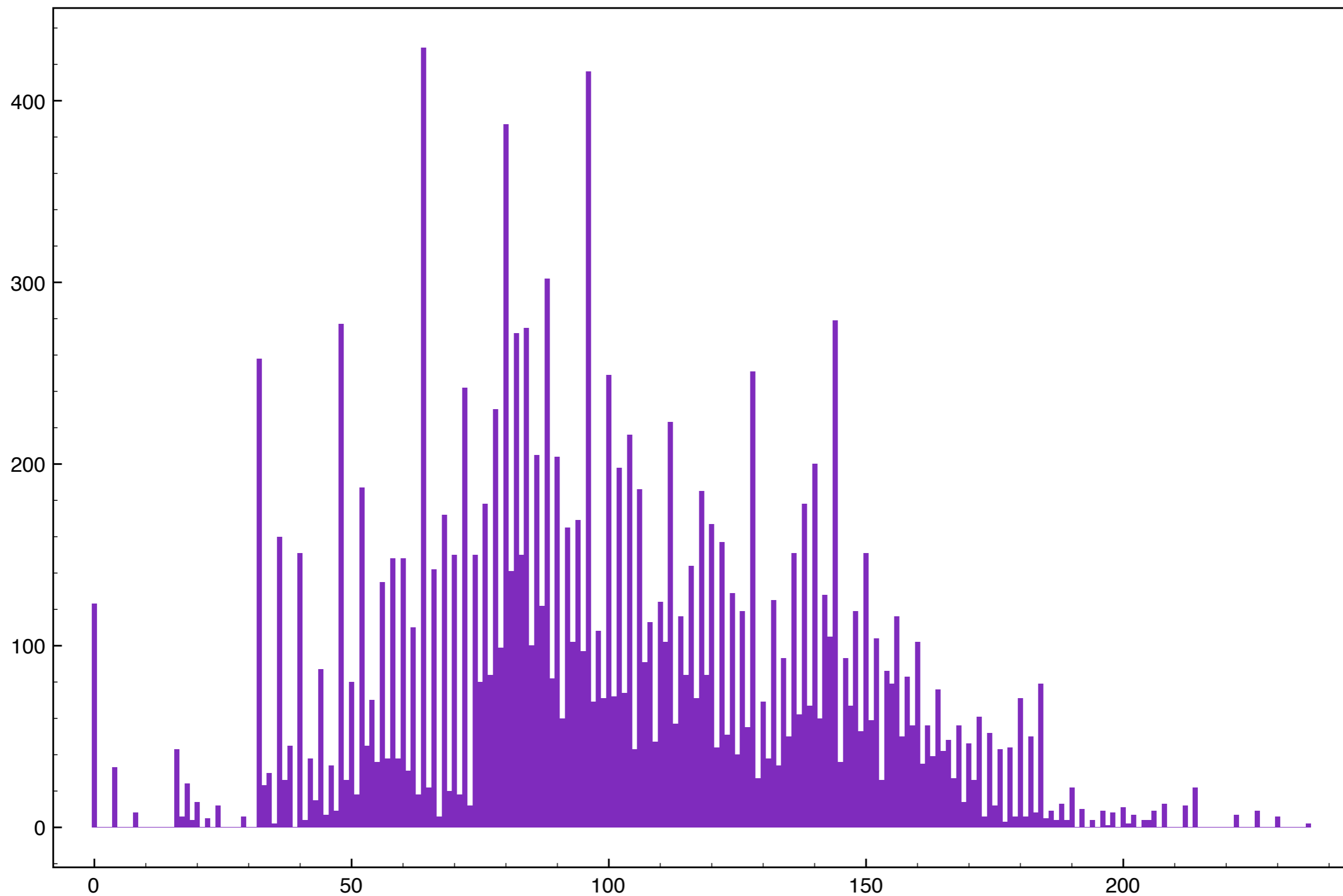
L-type mirror distribution (Free Fermions)



Fractional charge distribution (non-chiral pairs)



Fractional charge distribution: Free fermions



See also: Assel, Christodoulides, Farraggi, Kounnas, Rizos (2009)

Fractional Charges

Type	Fractional charges		CFT type		
	Massless	Chiral?	Gepner	Gepner(exc.)	Free Fermion
SM, Q=1/2	$\frac{1}{2}$	NO	0.97%	0.30%	-
SM, Q=1/3	$\frac{1}{3}$	NO	0.42%	0.76%	-
SM, Q=1/6	$\frac{1}{3}$	NO	0.000023%	-	-
SM, Q=1/6	$\frac{1}{6}$	NO	0.23%	0.28%	-
LR, Q=1/3	NO	NO	0.0000031%	-	-
LR, Q=1/3	$\frac{1}{3}$	NO	3.18%	10.15%	-
LR, Q=1/6	$\frac{1}{3}$	NO	0.0013%	0.010%	-
LR, Q=1/6	$\frac{1}{2}$	NO	0.000029%	0.0000126%	-
LR, Q=1/6	$\frac{1}{6}$	NO	1.06%	2.86%	-
Pati-Salam	NO	NO	0.001%	0.016%	8.43%
Pati-Salam	$\frac{1}{2}$	NO	15.53%	9.41%	68.42%
SU(5) GUT	NO	NO	13.06%	4.67%	-
SO(10) GUT	NO	NO	32.7%	32.99%	21.63%
SM, Q=1/2	$\frac{1}{2}$	YES	1.09%	0.09%	-
SM, Q=1/3	$\frac{1}{3}$	YES	1.63%	0.82%	-
SM, Q=1/6	$\frac{1}{6}$	YES	0.66%	0.25%	-
LR, Q=1/6	$\frac{1}{6}$	YES	4.98%	2.86%	-
LR, Q=1/3	$\frac{1}{3}$	YES	22.88%	33.89%	-
Pati-Salam	$\frac{1}{2}$	YES	1.65%	0.78%	1.5%

The three-family case

There is one case with $\Delta=3$: $(1,16^*,16^*,16^*)$

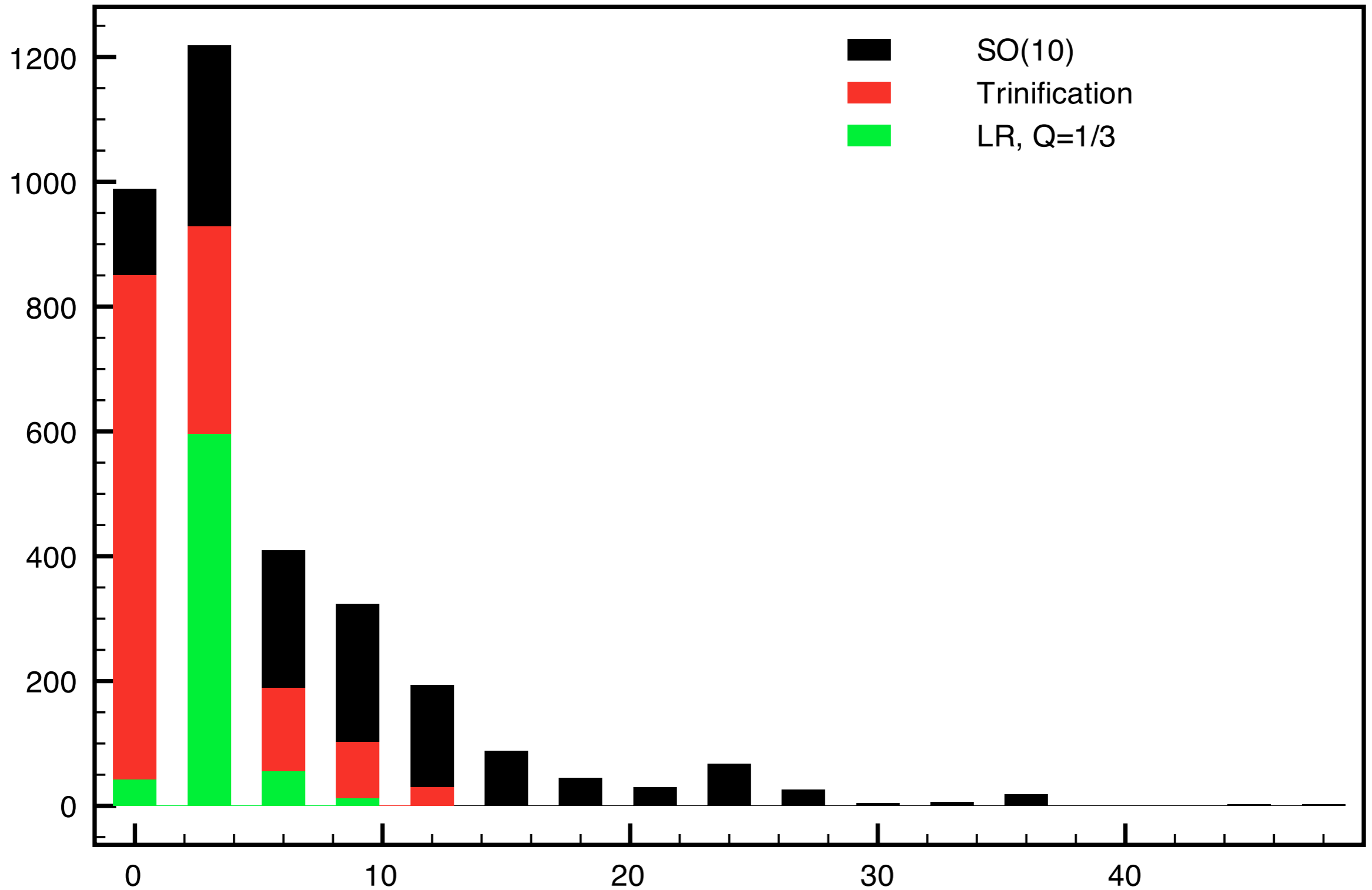
This yields 3-family $(2,2)$, $(1,2)$ and $(0,2)$ models with gauge groups
 $SU(3) \times SU(2) \times SU(2) \times U(1)$ or $SO(10)$
(also $SU(3) \times SU(3) \times SU(3)$ or E_6)

1220 distinct 3-family spectra (610 mirror pairs); All mirror pairs are complete.

$SU(2)_R$ remains always unbroken (hence no $SU(5)$ models)

Fractional charges (if any) are always third-integer (hence no Pati-Salam models).

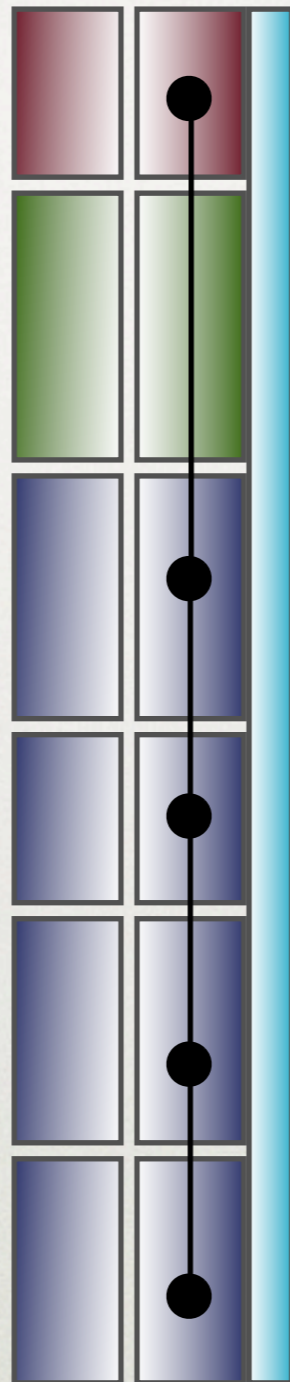
Family distribution

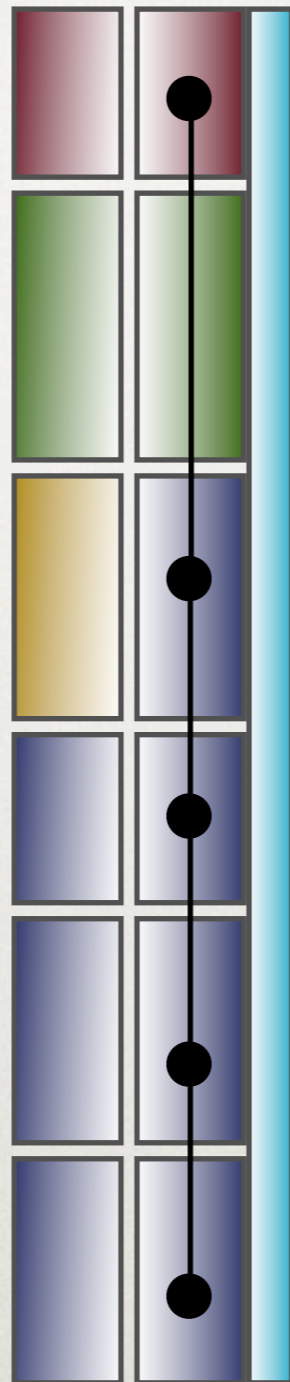


$$SU(3) \times SU(2) \times SU(2) \times U(1)$$

Representation	Particles	Multiplicity
$(3,2,1,\frac{1}{6})$	Q	3
$(3^*,1,2,-\frac{1}{6})$	$U^* + D^*$	$4+1^*$
$(1,2,1,-\frac{1}{2})$	L	$5+2^*$
$(1,1,2,\frac{1}{2})$	$E^* + N^*$	$5+2^*$
$(3^*,1,1,\frac{1}{3})$	D^*	$5+5^*$
$(1,2,2,0)$	$H_1 + H_2$	9
$(1,1,0,0)$	singlets	80
$(1,1,1,\frac{1}{3})$	<div style="border: 2px solid black; border-radius: 15px; padding: 20px; width: fit-content; margin: 0 auto;"> <p style="font-size: 2em; margin: 0;">Charge</p> <p style="font-size: 3em; margin: 0;">1/3</p> </div>	$41+41^*$
$(1,1,2,-\frac{1}{6})$		$20+20^*$
$(1,2,1,-\frac{1}{6})$		$19+19^*$
$(3,1,1,0)$		$17+17^*$
$(3,1,1,\frac{1}{3})$		$8+8^*$
$(3,2,1,-\frac{1}{6})$		$3+3^*$
$(3^*,1,2,\frac{1}{6})$		$3+3^*$
$(1,2,2,\frac{1}{3})$		$2+2^*$
$(1,1,1,-\frac{2}{3})$		$2+2^*$

HETEROTIC WEIGHT LIFTING

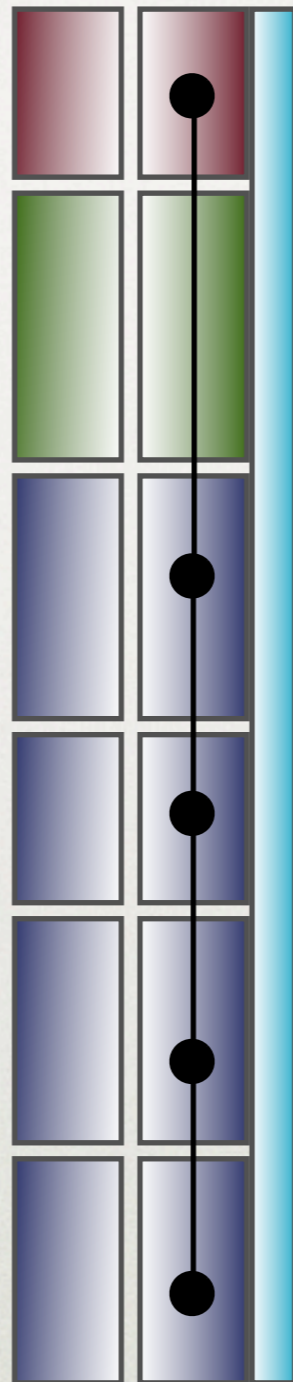


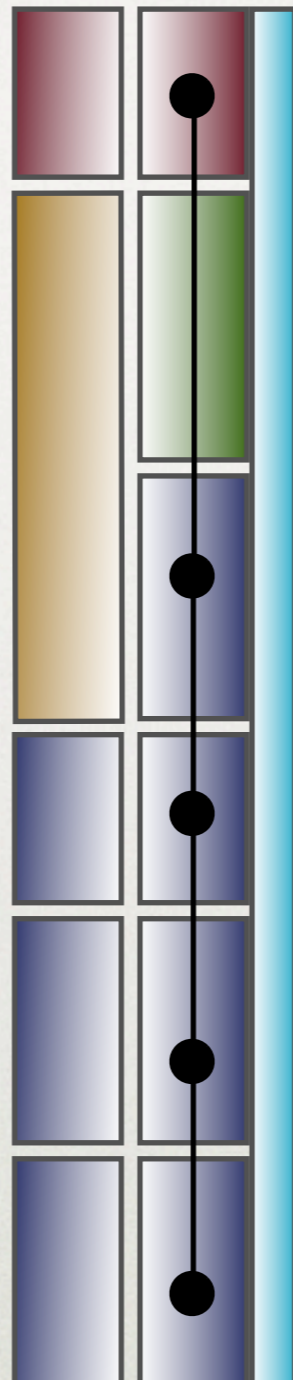


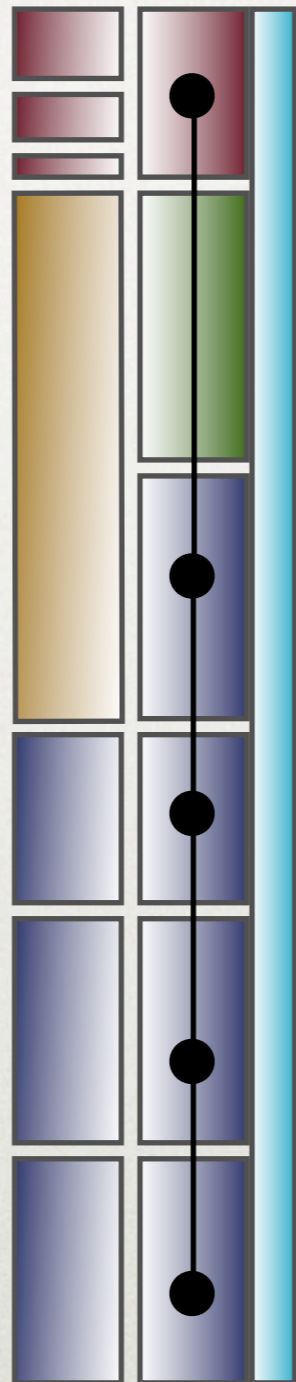
... but we have to find a $N=0$ CFT with the same S , T , and central charge as some $N=2$ model, without being identical to it.

This looks difficult.

But there is something else we could try:







So our goal is to find, for some minimal $N=2$ model with central charge c , a replacement that has central charge $c+8$, and exactly the same S and T matrices.

Hence it must have the same number of primaries, and the same spectrum, up to integers.

Minimal N=2 model at level k:

$$c = \frac{3k}{k+2}$$

Coset description:

$$\frac{SU(2)_k \times SO(2)}{U_{k+2}}$$

Plus “field identification”

(Gepner; Schellekens and Yankielowicz, 1989)

Field identification is a formal simple current extension of the coset CFT by a current of spin 0. This relates multiple vacua.

This “extends” the chiral algebra so that the identity representation is doubled, and roughly half the states (that do not satisfy the G/H selection rules) are removed.

The coset CFT may be thought of as tensor product

$$SU(2)_k \times SO(2) \times U(1)_{k+2}^c$$

Where $U(1)^c$ is the “complement”: an auxiliary representation of the modular group with complex conjugate S and T matrices, and $c = -1 + 8N$

Now we remove the field identification extension, and consider

$$SU(2)_{k+2} \times SO(2) \times \frac{E_8}{U_{k+2}}$$

In other words, we embed the $U(1)$ in E_8 instead of $SU(2) \times SO(2)$.

Next we identify a CFT X_7 which can be combined with U_{k+2} to E_8 , so that

$$E_8 = [U_{k+2} \times X_7]_{\text{ext}}$$

Then we can write the CFT as

$$SU(2)_{k+2} \times SO(2) \times X_7$$

And finally we re-establish the equivalent of the field identification, as a standard, higher spin extension

The result is guaranteed, by construction, to have the same S and T matrices as the original minimal model.

But the spectrum is different

Standard coset field $h_i^G - h_j^H \quad (j \in i)$

Replacement $h_i^G + h_j^{H^c}$

$$h_j^{H^c} = -h_j^H \pmod{1}$$

All weight of H and H^c are positive

Therefore standard weights are lifted:

$$h_i^G + h_j^{H^c} > h_i^G - h_j^H$$

(but equal mod 1)

The simplest class of examples: find a $U(1)$ in E_8 through subgroup embeddings:

For example the Standard Model $U(1), Y$

$$\begin{aligned}SU(3) \times SU(2) \times U(1)_{30} \times U(1)_{20} &\subset SO(10) \\SO(10) \times SO(6) &\subset E_8\end{aligned}$$

This implies

$$\frac{E_8}{U_{30}} = A_{2,1} A_{1,1} A_{4,1}$$

And hence

$$(N = 2, k = 13) \sim A_{1,13} U_4 A_{2,1} A_{1,1} A_{4,1}$$

Extended by the current $(J, v, 0, J, 0)$

The minimal $N=2$, $k=13$ model has 420 primaries.
We have compared the S and T matrices explicitly,
and they are identical.

But many states in the spectrum are shifted:

136 massless ($h \leq 1$) are lifted(*)
81 massive ones become massless
37 are massless before and after
166 are massive before and after

(*) Including all Ramond ground states

OTHER LIFTS

- So far we found 30; there may be more.
- For several values of k there is more than one.
- There are also double lifts. Perhaps also triple and quadruple lifts.
- Single lifts give rise to 435 lifted Gepner models.

k	Lift	Lifted	Lowered	Unchanged
1	$E_6 \times A_1$	4	1	4
2	A_7	7	1	12
3	$[D_6 \times U_{10}]_{\text{ext}}$	10	3	22
4	$D_5 \times A_2$	21	4	23
5	$A_6 \times A_1$	32	8	29
5	$[E_6 \times U_{42}]_{\text{ext}}$	24	11	37
6	$[A_6 \times U_{112}]_{\text{ext}}$	33	15	39
8	$A_4 \times A_3$	65	29	37
9	$[A_6 \times U_{154}]_{\text{ext}}$	76	41	39
11	$[E_6 \times U_{78}]_{\text{ext}}$	104	61	39
11	$[D_6 \times U_{26}]_{\text{ext}}$	98	60	45
12	$A_6 \times U_4$	125	66	39
13	$A_4 \times A_2 \times A_1$	136	81	37
14	$[A_4 \times A_2 \times U_{480}]_{\text{ext}}$	147	105	47
14	$[A_6 \times U_{224}]_{\text{ext}}$	153	95	41
17	$[E_6 \times U_{114}]_{\text{ext}}$	202	105	37
17	$[A_4 \times A_2 \times U_{570}]_{\text{ext}}$	198	133	41
19	$E_6 \times U_{14}$	228	119	42
20	$[A_6 \times U_{308}]_{\text{ext}}$	243	143	42
23	$[D_6 \times U_{50}]_{\text{ext}}$	300	161	41
26	$A_6 \times U_8$	349	199	39
30	$[A_6 \times U_{448}]_{\text{ext}}$	417	235	46
41	$[E_6 \times U_{258}]_{\text{ext}}$	610	297	44
41	$[A_6 \times U_{602}]_{\text{ext}}$	606	325	48
42	$[A_6 \times U_{616}]_{\text{ext}}$	627	337	46
44	$[A_6 \times U_{644}]_{\text{ext}}$	673	361	42
44	$[A_4 \times A_2 \times U_{1380}]_{\text{ext}}$	659	465	56
47	$[E_6 \times U_{294}]_{\text{ext}}$	728	367	46
54	$A_6 \times U_{16}$	857	455	51
58	$A_4 \times A_2 \times U_8$	923	611	56
86	$[A_6 \times U_{1232}]_{\text{ext}}$	1501	741	52
89	$[E_6 \times U_{546}]_{\text{ext}}$	1556	705	49
238	$A_4 \times A_2 \times U_{32}$	4959	2729	73
1,1	$A_2 \times A_1 \times A_2 \times A_1$	16	1	14

COMPUTING THE SPECTRUM

Very easy: start with the **full** spectrum of a standard Gepner model. For example, all states associated with a massless space-time spinor in the fermionic sector

$$\sum_j M_{ij}(\dim_1, h_1, \dots, \dim_n, h_n)_j$$

To compute the consequences of “lifting” factor k , just replace \dim_k and h_k by the corresponding values in the lift CFT

CHIRAL SPECTRA?

All R ground states are lifted.

Hence no extension $SO(10) \rightarrow E_6$

But also all chiral families are removed.

The diagonal MIPF yields, for (4,4,8,13)

Before lifting:

$$75(27) + 3(\overline{27}) + 450(1) \text{ of } E_6$$

After lifting:

$$20 \times (10) + 1088(1) \text{ of } SO(10)$$

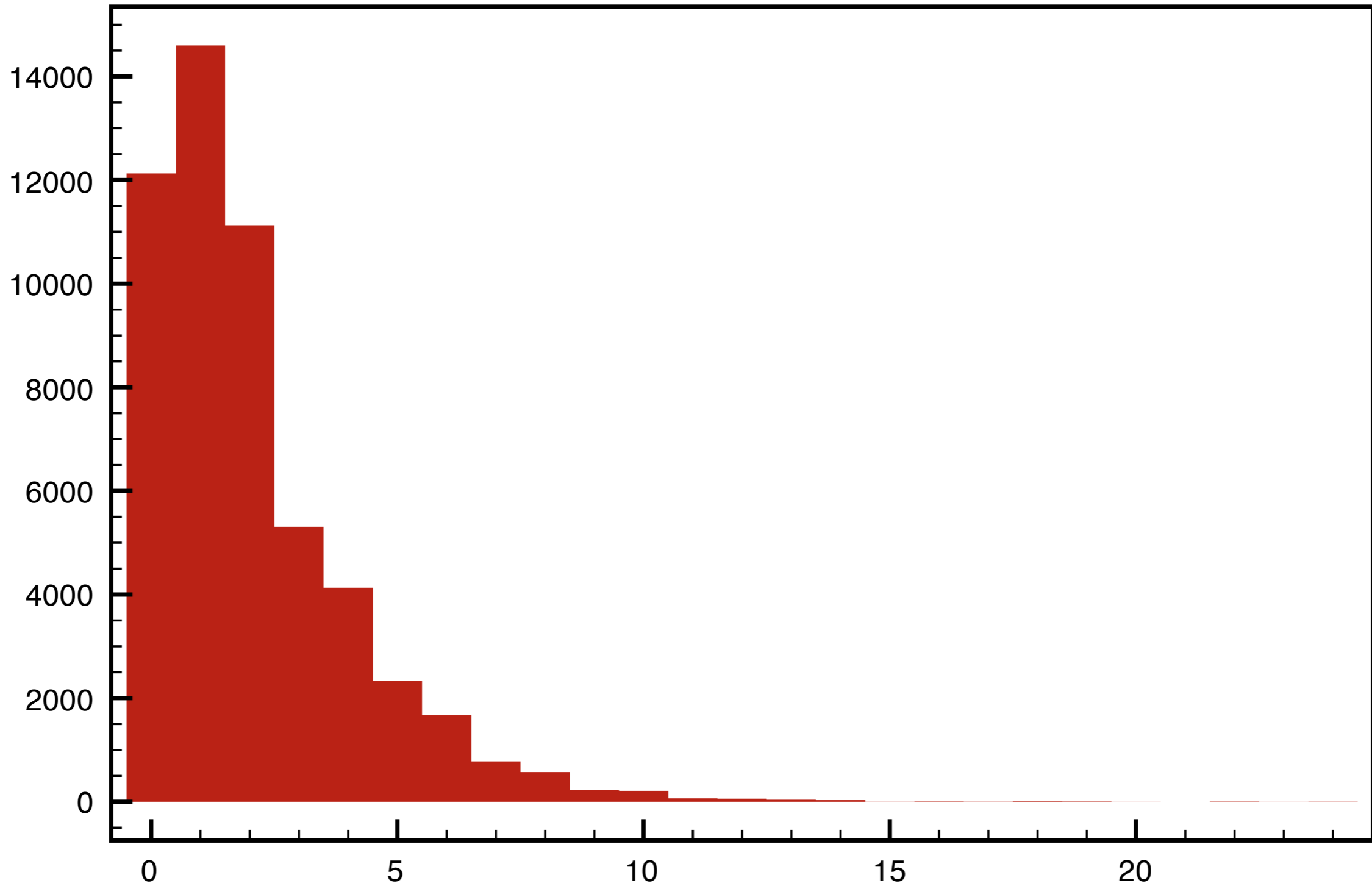
CHIRAL SPECTRA?

But now we can break all non-essential symmetries in the bosonic sector. In particular world-sheet susy.

So we do not need Ramond to get massless fermions!

And this is what came out:

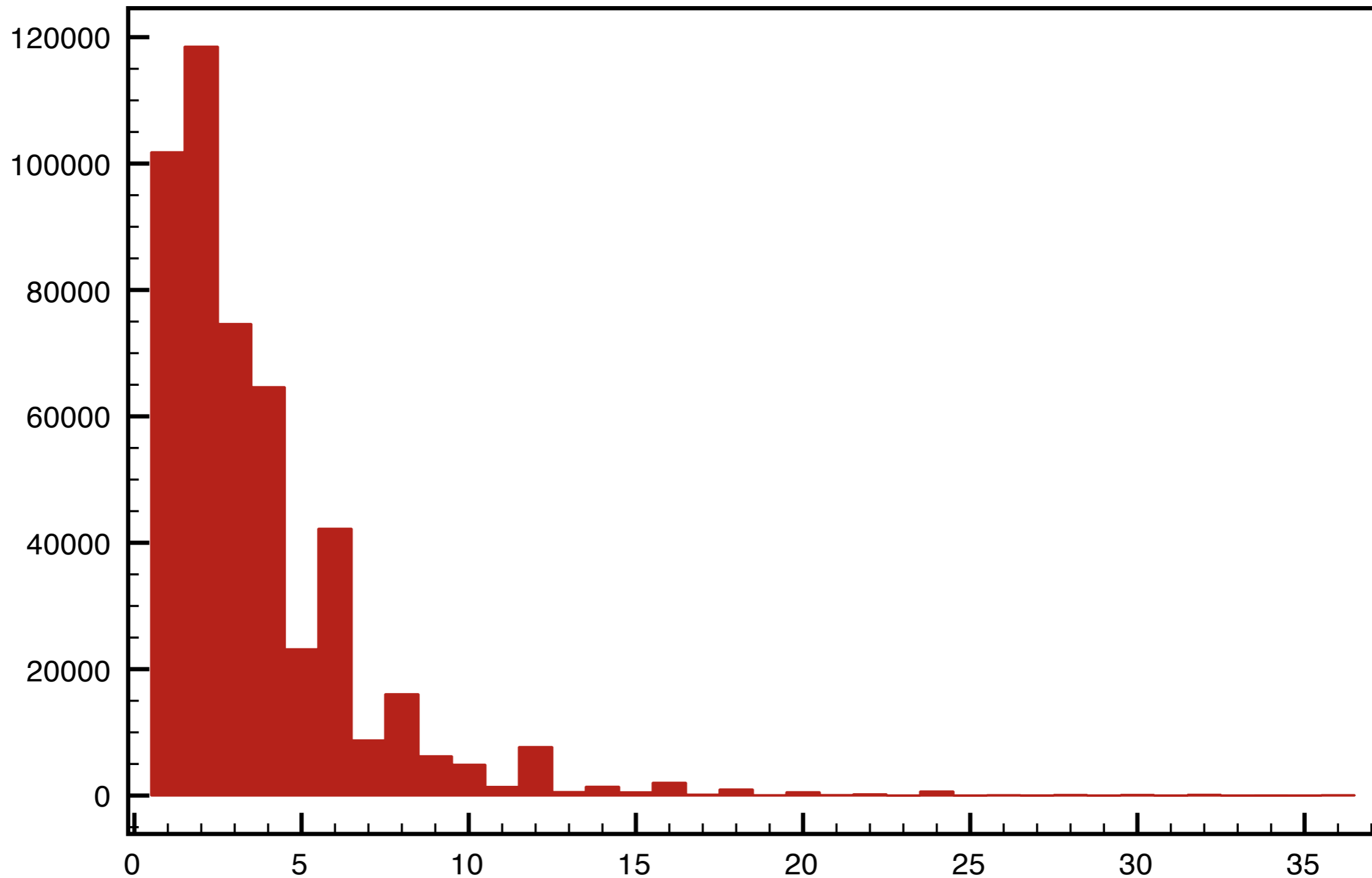
Distinct spectra



Families

$$(\hat{3}, 3, 3, 3, 3)$$

Distinct
Spectra



Family distribution for 435 lifted Gepner models

- For the first time in Heterotic Gepner models, family quantization in units of 1!
- Many cases with three families.
- Roughly exponential fall-off with the number of families
- Not as steep as in orientifolds.
- Three families relatively much more common.

Δ -Distribution

Δ	
0	94
1	198
2	57
3	60
4	8
5	0
6	18

Distinct 3-family: 75000 (out of 480000 N-family, $N \geq 1$)
(Modulo mirror symmetry: 41000)

SOME FEATURES

- Many different gauge groups, from just $SU(3) \times SU(2) \times U(1)$ to $SO(10)$.
- Additional non-abelian gauge group from the lift CFT.
- Distribution of number of mirrors from a few tens to zero.
- Examples with no mirror fermions at all
- Examples with $3 \times (16) + (10)$ of $SO(10)$
(exactly the minimal $SO(10)$ susy-GUT).
- Examples with just* $SU(3) \times SU(2) \times U(1)$ and B-L broken by anomalies.

(* from $SO(10)$)

APPROACHING THE SM

An example from $(3, \hat{8}, 8, 8)$

Gauge group:

$$SU(3) \times SU(2) \times U(1) \times [SU(2)_8 \times SO(2) \times SU(4) \times SU(5)] \times U(1)^3$$

(anomalous "B-L")

Spectrum:

$$3 \times (Q + U^c + D^c + L + E^c) + 3 \times (D + D^c) + 3 \times (H_1 + H_2)$$

+ 250 singlets

+ 172 fractionally charged particles

Fractional charges:

Non-chiral.

Only half-integer (no sixth or third).

Confined by $SU(2)_8$

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Only half-integer (no sixth or third).

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Singlets: (of $SU(3) \times SU(2) \times U(1)$)

Only three are absolute singlets of the full gauge group.

Many are in nontrivial $SU(4)$ and $SU(5)$ reps.

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Many are in nontrivial $SU(4)$ and $SU(5)$ reps.

The bad news:

U^c , D^c and E^c are in the triplet representation of $SU(2)_8$;

Higgs candidates and weak doublets are $SU(2)_8$ singlets.

QUESTIONS

- What, if any, is the geometric interpretation of these models?
- Are they related to other constructions, and how?
- Is there a related Landau-Ginzburg description?
- What are their strong coupling duals?
- Is there an exact mirror symmetry?
- Is it possible to classify all the lifts?
- Are there any generic bad features that rule out this entire class phenomenologically?
- What can be said in general about charge quantization and confinement?
- Is there a simple rule for family number quantization?
- How close can we get to the MSSM spectrum?
- Without supersymmetry, how close can we get to the SM spectrum?