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# The Joy of Discreteness

“JürgenFest”  
Hamburg 9 June 2017

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Discrete Symmetries  
in  
Discrete Orientifolds



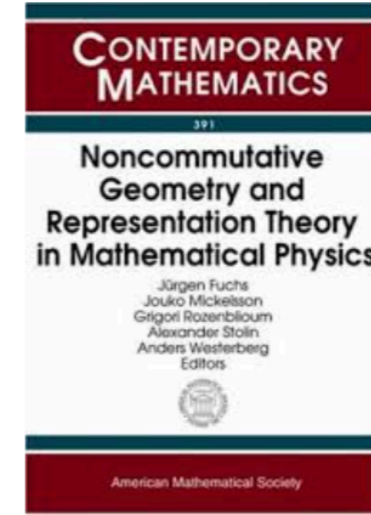
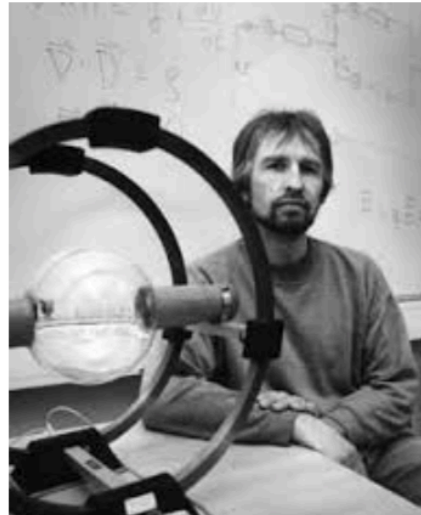
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# Some pictures from my collection

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**Hopf Algebras and Frobenius Algebras in Finite Tensor Categories**

Christoph Schweigert and Jürgen Fuchs

**Abstract.** We discuss algebraic and representation-theoretic aspects in braided tensor categories  $\mathcal{C}$  which obey certain finiteness conditions. A key of increasing interest in such a category is associated to a Hopf algebra  $H$  in  $\mathcal{C}$ . In particular, the Hopf algebra  $H$  gives rise to representations of the modular group  $SL_2(\mathbb{Z})$  on various coefficient spaces. We also explain how every connected, graded Frobenius algebra in a semisimple modular category provides additional structure related to these representations.

**Keywords.** Finite tensor category · Hopf algebra · Modular group · Frobenius algebra

**Mathematics Subject Classification (2010).** 16W30 · 18D10

**1. Braided Finite Tensor Categories**

Algebra and representation theory in one-object ribbon categories has been an active field over the last decade, having applications to quantum groups, low-dimensional topology and quantum field theory. More recently, partly in connection with progress in the understanding of logarithmic conformal field theories, there has been increased interest in tensor categories that are not necessarily any longer, but still obey certain finiteness conditions [20].

Going back to work of various groups that only began to appear, see e.g. [17, 18], examples of such categories are, by now rather explicitly understood, at least in abstract categories. In this section we describe a class of categories that has received particular attention. This will allow us to define the structure of a symmetric modular tensor category. To extend the notion of modular tensor category to the non-semisimple case requires further topological constructions involving Hopf algebras.

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OPEN STRING AND 3D TOPOLOGICAL FIELD THEORY

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To the conformal or formulation of string theory the condition function in the background of 3D theories can be regarded as

$$Z_{\text{string}} = \sum_{\text{topology}} \int_{\text{moduli}} \mathcal{Z}_{\text{string}} \cdot \mathcal{Z}_{\text{topology}} \quad (1)$$

Here  $Z_{\text{string}}$  is the value of the string partition function on a Riemann surface of genus  $g$  with  $k$  boundary components. One may also interpret the partition  $Z_{\text{string}}$  in its own right as a modular invariant in the modular group. In this paper we obtain a modular invariant (CFT) living on the string world sheet, and the  $Z_{\text{string}}$  condition then, regarding the relation of this CFT to the modular group of the Riemann surface. In order for the approach to work, the CFT has to be well defined on surfaces of arbitrary genus and with an arbitrary number of boundaries. The type I string case can be extended to type II string and M-theory world sheets.

With this motivation in mind we set ourselves the aim to construct a CFT consistent on all surfaces relevant in (1). This can be viewed either as a problem in CFT. We will not worry about whether a particular CFT actually applies to the worldsheet group description of a given string background.

Our approach to obtain CFT conformal on all surfaces in this setting. Consider the space of closed string states  $\mathcal{H}$  (the space of states associated to a circle) by  $\mathcal{H}^{\text{open}}$  and the space of open string states associated to an interval by  $\mathcal{H}^{\text{open}}$ . Then  $\mathcal{H} = \mathcal{H}^{\text{open}} \oplus \mathcal{H}^{\text{open}}$  is an  $\mathcal{H}$  of boundary conditions we want to allow at either end of the open string. We repeatedly verify along circles and intervals our condition.

**J. Fuchs** & **C. Schweigert** · 100 · Progress in Mathematics 281, 2010, pp. 107–170, doi:10.1007/978-1-4939-9132-1\_5, © Springer Science+Business Media LLC 2011

CONFORMAL BOUNDARY CONDITIONS AND 3D TOPOLOGICAL FIELD THEORY

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**Abstract.** Topological field theory in three dimensions provides a powerful tool to construct conformal boundary and to describe boundary conditions in two-dimensional conformal field theories.

**Keywords.** Conformal field theory, topological field theory, boundary conditions

**1. Introduction**

There are numerous applications of two-dimensional conformal field theories on manifolds with boundaries. They range from inquiries in systems of condensed matter physics to D-branes in string theory. The present contribution explains an approach to conformal boundary conditions that is based on a special instance of “logarithmic conformal field theory”. The space of conformal blocks can be understood both as space of physical states of a three-dimensional topological field theory (TFT) and as space of operators of a two-dimensional conformal field theory (CFT).

Cady’s formula,

$$|h\rangle = \sum_{\mu} \frac{S_{\mu}}{S_0} |\mu\rangle,$$

representing a boundary state  $|h\rangle$  in terms of Ishibashi states  $|\mu\rangle$ , describes a symmetry preserving boundary condition  $\sigma$  for a CFT with charge conjugation modular invariance. The appearance of the coefficient  $S_{\mu}/S_0$  in Cady’s expression is a result independent feature. It involves the modular matrix  $S$ .

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# EXTENDED CHIRAL ALGEBRAS AND MODULAR INVARIANT PARTITION FUNCTIONS

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We show how the fusion rules can be used to associate with every rational conformal field theory a discrete group, the center. The center is generated by primary fields having unique fusion rules with any other field. The existence of a non-trivial center implies the existence of non-diagonal modular invariants, which are related to extended integer or fractional spin algebras. Applied to Kac–Moody algebras this method yields all known as well as many new infinite series of modular invariants. Some results on exceptional invariants are also presented, including an example of an exceptional integer spin invariant that does not correspond to a conformal embedding.



# ON THE CONNECTION BETWEEN WZW AND FREE FIELD THEORIES

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A large class of primary fields which appear at any level of the WZW theories (of types  $A_N$ ,  $B_N$ ,  $C_N$ ,  $D_N$ ,  $E_6$ , and  $E_7$ ) are shown to possess simple power-like four-point functions. As a consequence, these fields, which are in 1-1 correspondence with the center of the covering group, may be written as symmetrized products of level one fields. The latter are known to be related to free fermions ( $A_N, B_N, D_N$ ) or free bosons ( $A_N, D_N, E_6, E_7$ ). Our results indicate that a relation to free field theory exists also for the case of  $C_N$ .



# **BONUS SYMMETRY IN CONFORMAL FIELD THEORY**

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Conformal field theories typically have an enlarged symmetry over that of the chiral algebra. These enlarged symmetries simplify the analysis of a theory by linking representations that would appear independent based on considerations of the smaller symmetry of the chiral algebra. It will be shown that this bonus symmetry occurs whenever a primary field  $g$  has a fusion rule with only the identity on the r.h.s. It will be seen that the additional symmetry generated by such a field  $g$  will be reflected in the fusion rules and in the modular transformation properties of the chiral characters. The way in which this enlarged symmetry may be exploited is illustrated in some simple examples. When the field  $g$  is of integer conformal dimension,  $g$  can be incorporated into an extended chiral algebra; the resulting extended, modular invariant partition function will be constructed. It will also be seen that especially strong simplifications arise when the field  $g$  with the mentioned fusion rule is of neither integer nor half-integer conformal dimension.



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# Simple Currents and Field Identification

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- ❖ “Simple Currents” allowed the construction of large sets of chiral algebra extensions and automorphism MIPFs for many CFT’s (mainly WZW-based).
- ❖ They also offered an elegant solution to a problem in coset CFT’s first pointed out by Gepner: Field Identification
- ❖ But this solution leads to another problem: field identification fixed points (self-identified fields)

$$|\chi_0 + \chi_1|^2 + |\chi_2 + \chi_3|^2 + 2|\chi_4|^2$$

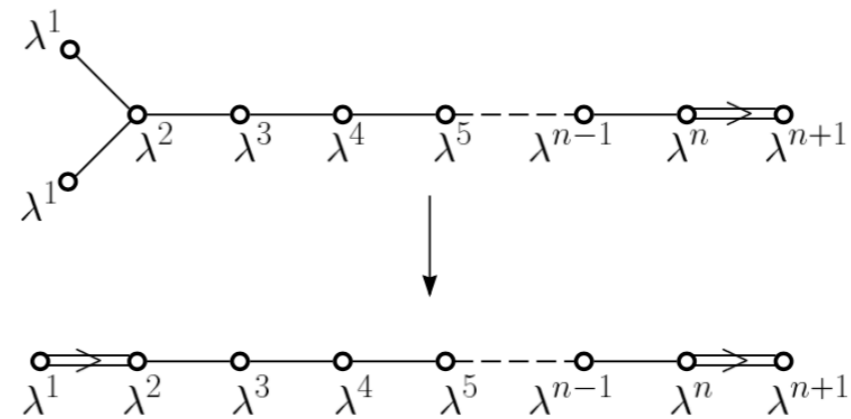


# Fixed point resolution

With S. Yankie

- Fixed Point
- Character

$$\mathfrak{g} = B_{n+1}^{(1)}$$



$$\mathfrak{g} = \tilde{B}_n^{(2)}$$

Figure 1a: Relation between symmetric weights ('fixed points') of  $B_{n+1}^{(1)}$  and weights of the orbit Lie algebra  $\tilde{B}_n^{(2)}$ .





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# A formula for $S$

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$$\tilde{S}_{(a,i),(b,j)} = \frac{|\mathcal{G}|}{\sqrt{|\mathcal{U}_a| |\mathcal{S}_a| |\mathcal{U}_b| |\mathcal{S}_b|}} \sum_{J \in \mathcal{G}} \Psi_i^a(J) S_{a,b}^J \Psi_j^b(J)^*$$

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# Boundaries and Crosscaps

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$$R_{[a, \psi_a]}(m, J) = \sqrt{\frac{|\mathcal{H}|}{|\mathcal{C}_a| |\mathcal{S}_a|}} \psi_a^*(J) S_{am}^J$$

$$\Gamma_{(i, J)} = \frac{1}{|\mathcal{G}|} \sum_{K \in \mathcal{G}} \eta(K) \frac{P_{K, i}}{\sqrt{S_{0, i}}} \delta^{J, 0}$$



$$(m, J) : J \in \mathcal{S}_m$$

with  $Q_L(m) + X(L, J) = 0 \pmod{1}$  for all  $L \in \mathcal{H}$

$$\mathcal{S}_m : J \in \mathcal{H} \text{ with } J \cdot m = m$$

(Stabilizer of  $m$ )

$[a, \psi_a]$ ,  $\psi_a$  is a character of the group  $\mathcal{C}_a$

$\mathcal{C}_a$  is the Central Stabilizer of  $a$

$$\mathcal{C}_i := \{J \in \mathcal{S}_i \mid F_i^X(K, J) = 1 \text{ for all } K \in \mathcal{S}_i\}$$

$$F_i^X(K, J) := e^{2\pi i X(K, J)} F_i(K, J)^*$$

$$S_{Ki,j}^J = F_i(K, J) e^{2\pi i Q_K(j)} S_{i,j}^J.$$

$S_{am}^J$  : matrix element of the modular transformation  
matrix of the fixed point CFT

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# Discrete string constructions

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## ❖ MIPFs of Heterotic Gepner models

Jürgen Fuchs, Albrecht Klemm, Christoph Scheich, Michael G. Schmidt (1989)

A.N. Schellekens, S. Yankielowicz (1989)

Based on a complete classification of  $N=2$  minimal model tensor product MIPFs

B. Gato-Rivera, A.N. Schellekens (1991); M. Kreuzer, A.N. Schellekens (1993)

## ❖ Gepner Orientifolds

Dijkstra, Huiszoon, Schellekens (2004)

Based on the aforementioned MIPFs plus a classification of all boundaries and crosscaps  
(Cardy, Ishibashi, Sagnotti, Pradisi, Stanev, Bianchi, Behrend, Pearce, Petkova, Zuber, Fuchs, Schweigert, Birke, Walcher, Huiszoon, Sousa, Schellekens, ..... 1989-2000)



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# Discrete Orientifolds

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Start with a  $c=9$ ,  $N=2$  rational conformal field theory, used as an “internal” sector of a type-II compactification.

Define the corresponding boundary CFT on surfaces with boundaries and crosscaps, by adding boundary and crosscap states consistent with the RCFT symmetries.

This allows the explicit construction of Annulus amplitudes, yielding exact open string partition functions, and Möbius and Klein bottle amplitudes defining the orientifold projections.

This gives rise to exact perturbative string spectra, with all massless and massive states explicitly known.



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# Discrete Orientifolds

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In principle, one expects a huge number of such RCFTs to exist.

In practice, we are limited to tensor products of  $N=2$  minimal models.

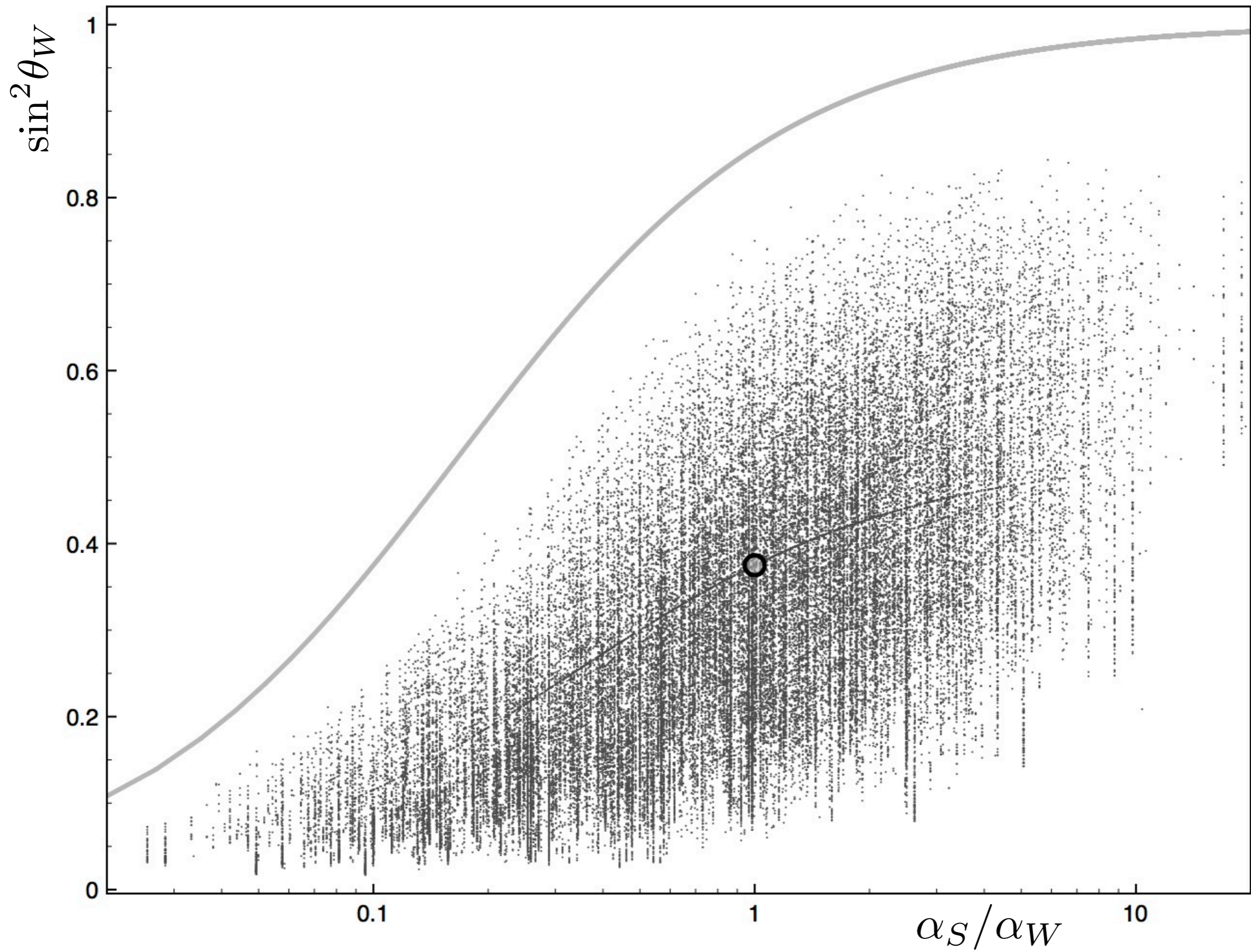
We have at our disposal:

- 168  $c=9$  combinations
- 5403 MIPFs
- 32990 orientifolds
- About  $10^{20}$  4-boundary combinations

We found 200.000 chirally exact MSSM spectra in this set.

*Dijkstra, Huiszoon, Schellekens (2004)*







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# Discrete Orientifolds

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The resulting spectra are presumably best thought of as discrete points in an open and closed string moduli space, hence the term “discrete orientifold”.

Most features of geometric orientifolds can be analysed in this context: tadpole cancellation, hidden sectors, axion-vector boson mixing, absence of global anomalies, stringy instantons. We would like to extend that to discrete symmetries.

**The two concepts of discreteness are unrelated.**



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# Discrete symmetries

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- May prevent fast proton decay and / or lepton number violation due to dimension 4 operators in the MSSM (and it may forbid other undesirable operators)
- So far, however, nature does not seem to use them (except CPT).
- How generic are discrete symmetries in the string landscape?
- Quantum gravity: folk theorems against existence of ungauged symmetries (continuous or discrete).
- Gauged discrete symmetries are allowed. (Kraus, Wilczek,...,1989)
- In string theory, specific “gauged, anomaly free” discrete symmetries are possible. (Ibanez, Ross, 1991).



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# Discrete symmetries in string theory

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- An obvious way to get an anomaly free discrete symmetry is to break a  $U(1)$  to  $\mathbb{Z}_N$ .
- Orientifolds have lots of  $U(1)$ 's, one for every complex brane stack. A good place to look for discrete symmetries!
- These  $U(1)$ 's are often broken due to axion mixing. This happens always if the  $U(1)$  is anomalous, and sometimes if it is not.



## Axion couplings

$$\sum_{a,m} N_a V_{am} \xi_m \wedge F_a$$

$\xi_m$ : axions, typically  $\sim 10 \dots 100$

$F_a$ :  $U(1)$  gauge field strength.

$N_a$ : Chan-Paton multiplicity of stack  $a$

in CFT: 
$$V_{am} = R_{am} - R_{a^c m}$$

$R_{am}$  Coupling strength of bulk mode  $m$  (“Ishibashi state”) to boundary  $a$

$$R_{[a,\psi_a](m,J)} = \sqrt{\frac{|\mathcal{H}|}{|\mathcal{C}_a||\mathcal{S}_a|}} \psi_a^*(J) S_{am}^J$$

**Complex numbers!**



Consider a linear combination of  $U(1)$ 's

$$\sum_a x_a Y_a$$

$Y_a$ :  $U(1)$  generator of brane  $a$

This remains massless if and only if

$$\sum_a x_a N_a (R_{am} - R_{a^c m}) = 0 \text{ for all } m.$$

If  $Y_a$  acquires a mass, the  $U(1)$  is not always completely broken.  
A discrete subgroup may remain.

How can we detect this?



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# Geometric constructions

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Condition for continuous U(1)

$$\sum_a x_a N_a (R_{am} - R_{a^c m}) = 0 \text{ for all } m.$$

Condition for  $\mathbb{Z}_N$

$$\sum_a x_a N_a (R_{am} - R_{a^c m}) = 0 \text{ mod } N \text{ for all } m.$$

In a geometric setting (type-IIA on CY) one can define these numbers in terms of a basis of 3-cycles on the manifold. Then one can write the condition for discrete symmetries entirely in terms of integers, and one can use this to construct explicit examples.



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# Instantons

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- Brane stack  $U(1)$ 's broken by axion mixing are respected by all perturbative amplitudes.
- Instanton amplitudes may break these symmetries. These can be gauge instantons or “exotic”, “stringy” instantons from stacks without a gauge group.

*Blumenhagen, Cvetič, Weigand*

*Ibáñez, Uranga*

*Florea, Kachru, McGreevy, Saulina*

- If there is a  $\mathbb{Z}_N$  discrete symmetry, any instanton amplitude can violate the corresponding charge only by multiples of  $N$ .



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# Instantons in discrete CFT

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The instanton charge violation for a  $U(1)$  associated with brane  $a$  due to an instanton on brane  $b$  is given by the chiral zero mode count

$$I_b(a) = N_a \sum_i w_i (A_{ba}^i - A_{ba^c}^i)$$

Here  $w_i$  is the Witten index of representation  $i$ , and  $A_{ab}^i$  are Annulus coefficients. The latter can be expressed in terms of boundary coefficients as

$$I_b(a) = N_a \sum_i w_i \sum_{m, J', J} \left[ \frac{S_{im} R_{b(m, J')} g_{J'J}^{\Omega, m}}{S_{0m}} \right] (R_{a(m, J)} - R_{a^c(m, J)})$$



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# Is there an integral basis?

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Axion couplings

$$V_{am} = R_{am} - R_{a^c m} \quad a = 1, \dots, N_{\text{bound}}, \quad m = 1, \dots, N_{\text{Ishibashi}}$$

Remove vanishing and identical columns

$$V_{a\mu}, \quad a = 1, \dots, N_{\text{bound}}, \quad \mu = 1, \dots, N_{\text{axion}}$$

$$N_{\text{axion}} = \mathcal{O}(10, \dots, 100) \quad (\text{maximally } 480)$$

$$N_{\text{bound}} = \mathcal{O}(100, \dots, 100000) \quad (\text{maximally } 108612)$$

Try to find a subset  $c$  of  $N_{\text{axion}}$  “basic” boundaries so that

$$V_{a\nu} = \sum_{\mu=1}^{N_{\text{axion}}} Q_{a\mu} V_{c(\mu)\nu}, \quad Q_{a\mu} \in \mathbb{Z}$$



$$I_b(a) = N_a \sum_i w_i \sum_{m, J', J} \left[ \frac{S_{im} R_{b(m, J')} g_{J' J}^{\Omega, m}}{S_{0m}} \right] (R_{a(m, J)} - R_{a^c(m, J)})$$

If we have an integral basis, we can express this in terms of that basis

$$I_b(a) = \sum_{\mu} N_a Q_{a\mu} I_b(c(\mu))$$

For a  $U(1)$   $Y = \sum_a x_a Y_a$  (choose  $x_a$  integer)

$$I_b(x) = \sum_a x_a I_b(a) = \sum_{\mu} \left( \sum_a x_a N_a Q_{a\mu} \right) I_b(c(\mu))$$



$$I_b(x) = \sum_a x_a I_b(a) = \sum_\mu \left( \underbrace{\sum_a x_a N_a Q_{a\mu}}_{\text{Manifestly integer in the new basis (if it exists...)}} \right) \underbrace{I_b(c(\mu))}_{\text{Instanton intersection number: Integer}}$$

Manifestly integer in the new basis  
(if it exists...)

Instanton intersection number: Integer

If all basis coefficients  $\sum_a x_a N_a Q_{a\mu}$  are a

multiple of  $N$ , we have a  $\mathbb{Z}_N$  discrete symmetry



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# Finding an integral basis

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Choose a suitable normalization for the columns of the matrix

$$V_{a\mu}: V_{a\mu} \rightarrow Z(\mu) V_{a\mu}$$

$$X_{ab} = \sum_{\mu} V_{a\mu} V_{b\mu} \equiv V_a \cdot V_b$$

For a suitable choice, all  $X_{ab}$  are rational numbers, in all 33290 cases.

Now choose a set of independent vectors  $V_{c(\mu)v}$



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# Finding an integral basis

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The “charges” with respect to this basis are defined as

$$V_{a\nu} = \sum_{\mu} Q_{a\mu} V_{c(\mu)\nu}$$

and can be computed by contracting both sides with the basis vectors

$$X_{ac(\nu)} = \sum_{\mu} Q_{a\mu} X_{c(\mu)c(\nu)}$$

Here  $X_{ab}$  are the numbers which we just found to be rational.

We can compute  $Q_{a\mu}$  by inverting the rational matrix  $X_{c(\mu)c(\nu)}$



5487599604882512345465449298220020091019 / 8412722075635424852344614333610055899893030  
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# Finding an integral basis

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...but this gives us only rational charges. This is not good enough.  
Now consider a boundary that has a rational charge

$$W_\nu = \sum_{\mu} Q_{\mu} V_{c(\mu)\nu} = \sum_{\mu} \frac{p_{\mu}}{q_{\mu}} V_{c(\mu)\nu}$$

Suppose for one value of  $\mu$  (denoted  $\mu = \hat{\mu}$ ),  $p_{\hat{\mu}} = 1$ .

Then we replace the corresponding basis vectors by  $W_\nu$ . In terms of the new basis, the old basis vector in terms of the new basis has an expansion

$$V_{c(\hat{\mu})\nu} = \sum_{\mu, \mu \neq \hat{\mu}} -\frac{p_{\mu} q_{\hat{\mu}}}{q_{\mu}} V_{c(\mu)\nu} + q_{\hat{\mu}} W_\nu$$

This is “more integral” than the previous basis, and the volume spanned by the basis decreases by  $q_{\hat{\mu}}$ .



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# Finding an integral basis

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This process converges in a maximum of 19 steps.

In 3 out of the 32990 cases it did not converge to pure integers.

These cases could be dealt with by choosing a different starting point.

In the end we did indeed find an integer basis for all 32990 Orientifolds.

This gives a “charge lattice” for axion charges.

(But: there must be a better way of doing this...)





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*Discrete physics is fun*

Many more years  
of discreteness,  
Jürgen!