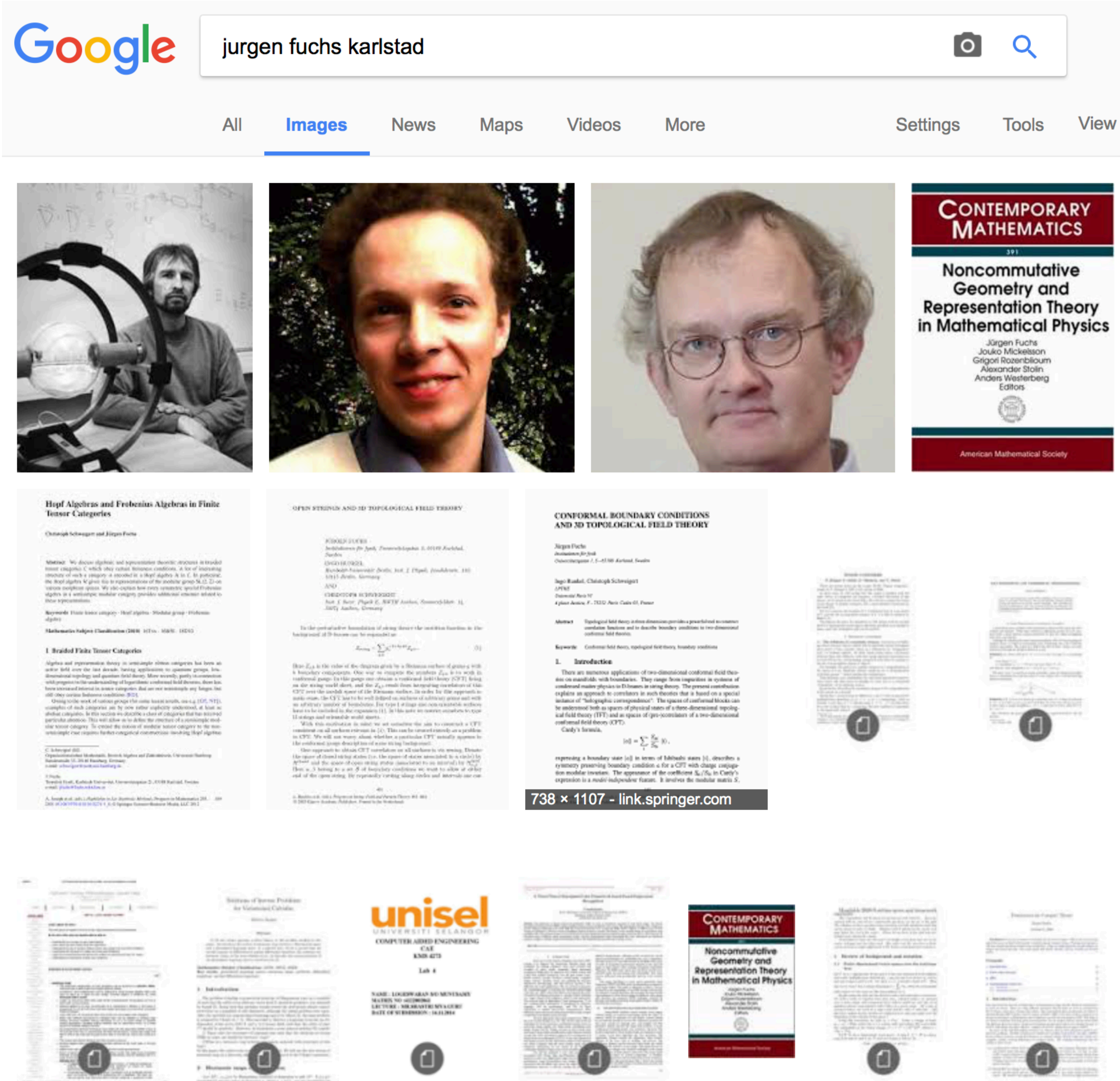
The Joy of Discreteness

"JürgenFest"
Hamburg 9 June 2017

Discrete Symmetries
in
Discrete Orientifolds

Nucl.Phys. B865 (2012) 509-540 with Luis Ibáñez and Angel Uranga

Some pictures from my collection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EXTENDED CHIRAL ALGEBRAS AND MODULAR INVARIANT PARTITION FUNCTIONS

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Received 12 April 1989

We show how the fusion rules can be used to associate with every rational conformal field theory a discrete group, the center. The center is generated by primary fields having unique fusion rules with any other field. The existence of a non-trivial center implies the existence of non-diagonal modular invariants, which are related to extended integer or fractional spin algebras. Applied to Kac–Moody algebras this method yields all known as well as many new infinite series of modular invariants. Some results on exceptional invariants are also presented, including an example of an exceptional integer spin invariant that does not correspond to a conformal embedding.

ON THE CONNECTION BETWEEN WZW AND FREE FIELD THEORIES

Jürgen FUCHS[1] and Doron GEPNER[2]

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A large class of primary fields which appear at any level of the WZW theories (of types A_N, B_N, C_N, D_N, E_6, and E_7) are shown to possess simple power-like four-point functions. As a consequence, these fields, which are in 1-1 correspondence with the center of the covering group, may be written as symmetrized products of level one fields. The latter are known to be related to free fermions (A_N, B_N, D_N) or free bosons (A_N, D_N, E_6, E_7). Our results indicate that a relation to free field theory exists also for the case of C_N.

BONUS SYMMETRY IN CONFORMAL FIELD THEORY

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Received 9 June 1989

Conformal field theories typically have an enlarged symmetry over that of the chiral algebra. These enlarged symmetries simplify the analysis of a theory by linking representations that would appear independent based on considerations of the smaller symmetry of the chiral algebra. It will be shown that this bonus symmetry occurs whenever a primary field g has a fusion rule with only the identity on the r.h.s. It will be seen that the additional symmetry generated by such a field g will be reflected in the fusion rules and in the modular transformation properties of the chiral characters. The way in which this enlarged symmetry may be exploited is illustrated in some simple examples. When the field g is of integer conformal dimension, g can be incorporated into an extended chiral algebra; the resulting extended, modular invariant partition function will be constructed. It will also be seen that especially strong simplifications arise when the field g with the mentioned fusion rule is of neither integer nor half-integer conformal dimension.

Simple Currents and Field Identification

- “Simple Currents” allowed the construction of large sets of chiral algebra extensions and automorphism MIPFs for many CFT’s (mainly WZW-based).
- They also offered an elegant solution to a problem in coset CFT’s first pointed out by Gepner: Field Identification
- But this solution leads to another problem: field identification fixed points (self-identified fields)

$$|\chi_0 + \chi_1|^2 + |\chi_2 + \chi_3|^2 + 2|\chi_4|^2$$

Fixed point resolution

With S. Yankie

- Fixed Poi
- Character

$$\mathfrak{g} = B^{(1)}_{n+1}$$

$$\breve{\mathfrak{g}} = \tilde{B}^{(2)}_{n}$$

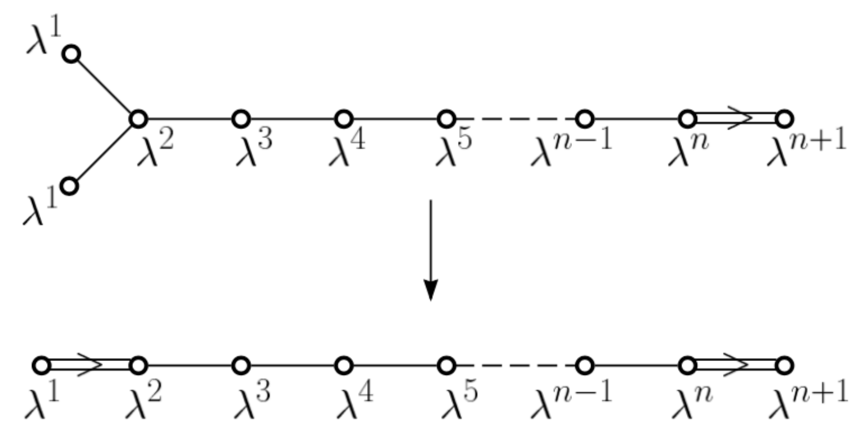


Figure 1a: Relation between symmetric weights ('fixed points') of $B^{(1)}_{n+1}$ and weights of the orbit Lie algebra $\tilde{B}^{(2)}_{n}$.

A formula for S

$$\tilde{S}_{(a,i),(b,j)} = \frac{|\mathcal{G}|}{\sqrt{|\mathcal{U}_a|\,|\mathcal{S}_a|\,|\mathcal{U}_b|\,|\mathcal{S}_b|}} \sum_{J\in\mathcal{G}} \Psi_i^a(J)\, S^J_{a,b}\, \Psi_j^b(J)^*$$

Fuchs, Schellekens, Schweigert (1996)

Boundaries and Crosscaps

$$R_{[a,\psi_a](m,J)} = \sqrt{\frac{|\mathcal{H}|}{|\mathcal{C}_a||\mathcal{S}_a|}}\psi_a^*(J)S_{am}^J$$

$$\Gamma_{(i,J)} = \frac{1}{|\mathcal{G}|}\sum_{K\in\mathcal{G}}\eta(K)\,\frac{P_{K,i}}{\sqrt{S_{0,i}}}\,\delta^{J,0}$$

Fuchs, Huiszoon, Schellekens, Schweigert, Walcher (2000)

$(m, J): \quad J \in \mathcal{S}_m$

with $Q_L(m) + X(L, J) = 0 \bmod 1$ for all $L \in \mathcal{H}$

$\mathcal{S}_m : J \in \mathcal{H}$ with $J \cdot m = m$

(Stabilizer of m)

$[a, \psi_a], \quad \psi_a$ is a character of the group $\mathcal{C}_a$

$\mathcal{C}_a$ is the Central Stabilizer of a

$$\mathcal{C}_i := \{ J \in \mathcal{S}_i \mid F_i^X(K, J) = 1 \text{ for all } K \in \mathcal{S}_i \}$$

$$F_i^X(K, J) := \mathrm{e}^{2\pi \mathrm{i} X(K,J)} \, F_i(K, J)^*$$

$$S^J_{Ki,j} = F_i(K, J) \, \mathrm{e}^{2\pi \mathrm{i} Q_K(j)} \, S^J_{i,j} \, .$$

S^J_{am} : matrix element of the modular transformation
matrix of the fixed point CFT

Discrete string constructions

- MIPFs of Heterotic Gepner models
  Jürgen Fuchs, Albrecht Klemm, Christoph Scheich, Michael G. Schmidt (1989)
  A.N. Schellekens, S. Yankielowicz (1989)

  Based on a complete classification of N=2 minimal model tensor product MIPFs
  B. Gato-Rivera, A.N. Schellekens (1991); M. Kreuzer, A.N. Schellekens (1993)

- Gepner Orientifolds
  Dijkstra, Huiszoon, Schellekens (2004)

  Based on the aforementioned MIPFs plus a classification of all boundaries and crosscaps
  (Cardy, Ishibashi, Sagnotti, Pradisi, Stanev, Bianchi, Behrend, Pearce, Petkova, Zuber, Fuchs, Schweigert, Birke, Walcher, Huiszoon, Sousa, Schellekens, ….. 1989-2000)

Discrete Orientifolds

Start with a $c=9$, $N=2$ rational conformal field theory, used as an "internal" sector of a type-II compactification.

Define the corresponding boundary CFT on surfaces with boundaries and crosscaps, by adding boundary and crosscap states consistent with the RCFT symmetries.

This allows the explicit construction of Annulus amplitudes, yielding exact open string partition functions, and Möbius and Klein bottle amplitudes defining the orientifold projections.

This gives rise to exact perturbative string spectra, with all massless and massive states explicitly known.

Discrete Orientifolds

In principle, one expects a huge number of such RCFTs to exist.

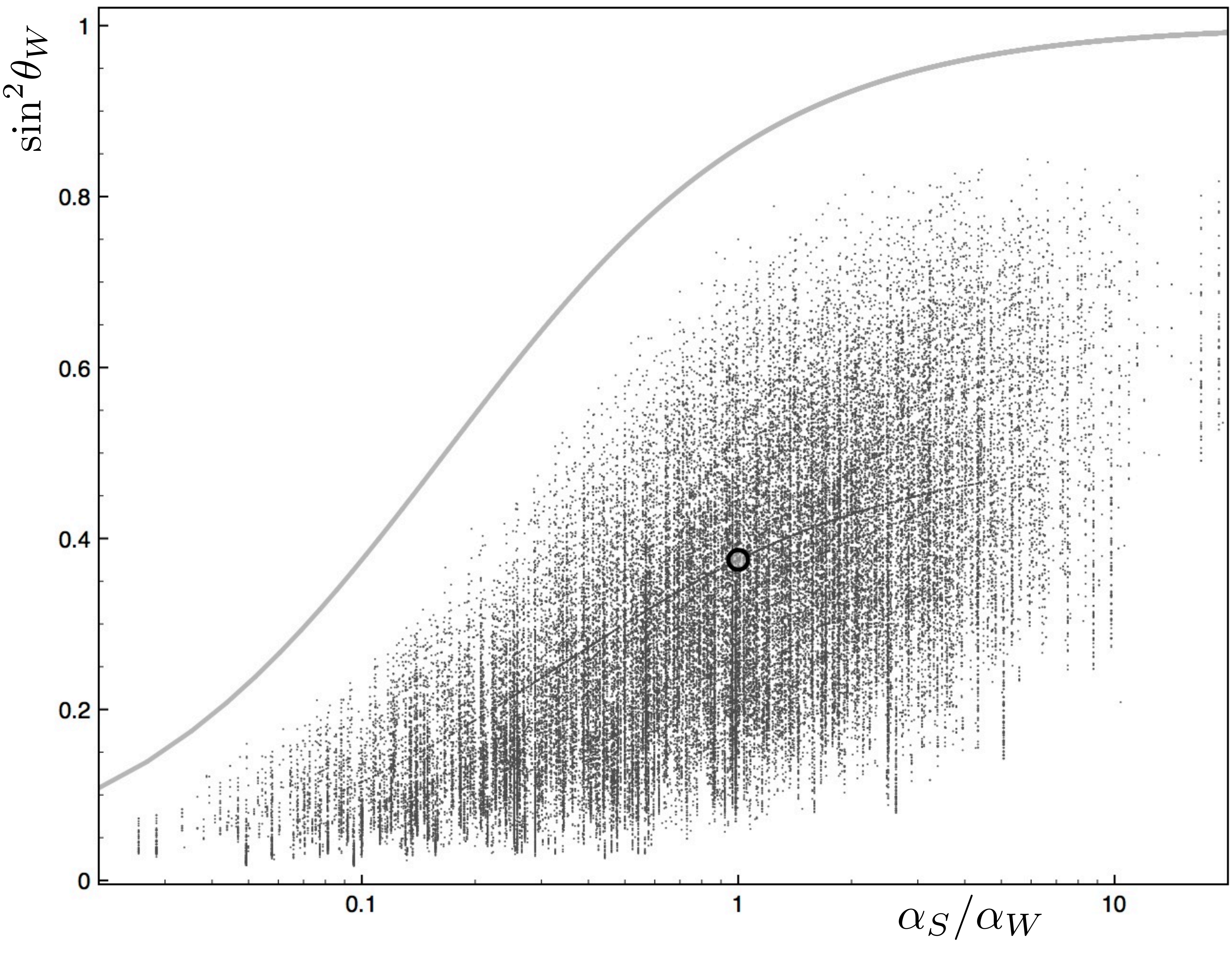
In practice, we are limited to tensor products of N=2 minimal models.

We have at our disposal:

- 168 c=9 combinations
- 5403 MIPFs
- 32990 orientifolds
- About 10^{20} 4-boundary combinations

We found 200.000 chirally exact MSSM spectra in this set.

Dijkstra, Huiszoon, Schellekens (2004)



$\sin^2\theta_W$
1
0.8
0.6
0.4
0.2
0
0.1
1
10
α_S/α_W

Discrete Orientifolds

The resulting spectra are presumably best though of as discrete points in an open and closed string moduli space, hence the term "discrete orientifold".

Most features of geometric orientifolds can be analysed in this context: tadpole cancellation, hidden sectors, axion-vector boson mixing, absence of global anomalies, stringy instantons. We would like to extend that to discrete symmetries.

The two concepts of discreteness are unrelated.

Discrete symmetries

- May prevent fast proton decay and/or lepton number violation due to dimension 4 operators in the MSSM (and it may forbid other undesirable operators)
- So far, however, nature does not seem to use them (except CPT).
- How generic are discrete symmetries in the string landscape?
- Quantum gravity: folk theorems against existence of ungauged symmetries (continuous or discrete).
- Gauged discrete symmetries are allowed. (Kraus, Wilczek,…,1989)
- In string theory, specific "gauged, anomaly free" discrete symmetries are possible. (Ibanez, Ross, 1991).

Discrete symmetries in string theory

- An obvious way to get an anomaly free discrete symmetry is to break a $U(1)$ to $\mathbb{Z}_N$.
- Orientifolds have lots of $U(1)$'s, one for every complex brane stack. A good place to look for discrete symmetries!
- These $U(1)$'s are often broken due to axion mixing. This happens always if the $U(1)$ is anomalous, and sometimes if it is not.

Axion couplings

$$\sum_{a,m} N_a V_{am} \xi_m \wedge F_a$$

ξ_m: axions, typically ~ 10 ... 100

F_a: *U(1)* gauge field strength.

N_a: Chan-Paton multiplicity of stack *a*

in CFT: $V_{am} = R_{am} - R_{a^c m}$

R_{am} Coupling strength of bulk mode *m* ("Ishibashi state") to boundary *a*

$$R_{[a,\psi_a](m,J)} = \sqrt{\frac{|\mathcal{H}|}{|\mathcal{C}_a||\mathcal{S}_a|}} \psi_a^*(J) S_{am}^J$$

Complex numbers!

Consider a linear combination of *U(1)*'s

$$\sum_a x_a Y_a$$

Y_a : *U(1)* generator of brane *a*

This remains massless if and only if

$$\sum_a x_a N_a (R_{am} - R_{a^c m}) = 0 \text{ for all } m.$$

If Y_a acquires a mass, the *U(1)* is not always completely broken.
A discrete subgroup may remain.
How can we detect this?

Geometric constructions

Condition for continuous U(1)

$$\sum_a x_a N_a (R_{am} - R_{a^c m}) = 0 \text{ for all } m.$$

Condition for $\mathbb{Z}_N$

$$\sum_a x_a N_a (R_{am} - R_{a^c m}) = 0 \bmod N \quad \text{for all } m.$$

In a geometric setting (type-IIA on CY) one can define these numbers in terms of a basis of 3-cycles on the manifold. Then one can write the condition for discrete symmetries entirely in terms of integers, and one can use this to construct explicit examples.

Berasaluce González, Ibáñez, Soler, Uranga, 2011

Instantons

- Brane stack U(1)'s broken by axion mixing are respected by all perturbative amplitudes.
- Instanton amplitudes may break these symmetries. These can be gauge instantons or "exotic", "stringy" instantons from stacks without a gauge group.

  Blumenhagen, Cvetic, Weigand
  Ibáñez,Uranga
  Florea, Kachru, McGreevy, Saulina

- If there is a $\mathbb{Z}_N$ discrete symmetry, any instanton amplitude can violate the corresponding charge only by multiples of N.

Instantons in discrete CFT

The instanton charge violation for a *U(1)* associated with brane a due to an instanton on brane b is given by the chiral zero mode count

$$I_b(a) \;=\; N_a \sum_i w_i ({A^i}_{ba} - {A^i}_{ba^c})$$

Here w_i is the Witten index of representation *i*, and ${A^i}_{ab}$ are Annulus coefficients.The latter can be expressed in terms of boundary coefficients as

$$I_b(a) = N_a \sum_i w_i \sum_{m,J',J} \left[\frac{S_{im} R_{b(m,J')} g^{\Omega,m}_{J'J}}{S_{0m}} \right] (R_{a(m,J)} - R_{a^c(m,J)})$$

Is there an integral basis?

Axion couplings

$$V_{am} = R_{am} - R_{a^c m} \qquad a = 1, \dots N_{\text{bound}}, \quad m = 1, \dots N_{\text{Ishibashi}}$$

Remove vanishing and identical columns

$$V_{a\mu}, \quad a = 1, \dots N_{\text{bound}}, \quad \mu = 1, \dots N_{\text{axion}}$$

$$N_{\text{axion}} = \mathcal{O}(10, \dots, 100) \quad (\text{maximally } 480)$$

$$N_{\text{bound}} = \mathcal{O}(100, \dots, 100000) \quad (\text{maximally } 108612)$$

Try to find a subset c of N_{axion} "basic" boundaries so that

$$V_{a\nu} = \sum_{\mu=1}^{N_{\text{axion}}} Q_{a\mu} V_{c(\mu)\nu} \; , \quad Q_{a\mu} \in \mathbb{Z}$$

$$I_b(a) = N_a \sum_i w_i \sum_{m,J',J} \left[\frac{S_{im} R_{b(m,J')} g^{\Omega,m}_{J'J}}{S_{0m}} \right] (R_{a(m,J)} - R_{a^c(m,J)})$$

If we have an integral basis, we can express this in terms of that basis

$$I_b(a) = \sum_\mu N_a Q_{a\mu} I_b(c(\mu))$$

For a $U(1)$ $\quad Y = \sum_a x_a Y_a$ $\quad$ (choose x_a integer)

$$I_b(x) = \sum_a x_a I_b(a) = \sum_\mu \Big(\sum_a x_a N_a Q_{a\mu} \Big) I_b(c(\mu))$$

$$I_b(x) = \sum_a x_a I_b(a) = \sum_\mu \Big(\underbrace{\sum_a x_a N_a Q_{a\mu}} \Big) \underbrace{I_b(c(\mu))}$$

Manifestly integer in the new basis
(if it exists...)

Instanton intersection number: Integer

If all basis coefficients $\sum_a x_a N_a Q_{a\mu}$ are a multiple of N, we have a $\mathbb{Z}_N$ discrete symmetry

Finding an integral basis

Choose a suitable normalization for the columns of the matrix $V_{a\mu}$: $V_{a\mu} \rightarrow Z(\mu)\, V_{a\mu}$

$$X_{ab} = \sum_{\mu} V_{a\mu} V_{b\mu} \equiv V_a \cdot V_b$$

For a suitable choice, all X_{ab} are rational numbers, in all 33290 cases.

Now choose a set of independent vectors $V_{c(\mu)\nu}$

Finding an integral basis

The "charges" with respect to this basis are defined as

$$V_{a\nu} = \sum_{\mu} Q_{a\mu} V_{c(\mu)\nu}$$

and can be computed by contracting both sides with the basis vectors

$$X_{ac(\nu)} = \sum_{\mu} Q_{a\mu} X_{c(\mu)c(\nu)}$$

Here X_{ab} are the numbers which we just found to be rational.
We can compute $Q_{a\mu}$ by inverting the rational matrix $X_{c(\mu)c(\nu)}$

-3487599604882512548405449298226026691619 / 841272207563542485234461433361005589989303
-2356527325219910903428901754662427149894 / 4206361037817712426172307166805027949946515
2784948741071505418128346476378730597441 / 2804240691878474950781538111203351966631010
-2585499736215948357280656786524657232 2 / 2213874230430374961143319561476330499971 85
6898072845027098208081359744435277277501 / 8412722075635424852344614333610055899893030
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-7302743703051896141872125832386047219 79 / 280424069187847495078153811120335196663101
-1470314626408978969502185087675203236204 3 / 8412722075635424852344614333610055899893030
-9664090016347793236032782991128845807 63 / 6009087196882446323103295952578611357066 45
-9830945987763481134300870031400680853 83 / 8412722075635424852344614333610055899893030
6113186906567733787902184350588026318 9 / 7379580768101249870477731871587768333239 5
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-3743804873812056516625539569089761533 43 / 7379580768101249870477731871587768333239 5
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-1925028850509606135456711776999153695741 / 14021203459392374753907690556016759833155 05
-5533393457226609011652599227355348627 99 / 841272207563542485234461433361005589989303
3622588600596306878973447873960345776869 / 8412722075635424852344614333610055899893030

Finding an integral basis

...but this gives us only rational charges. This is not good enough. Now consider a boundary that has a rational charge

$$W_\nu = \sum_\mu Q_\mu V_{c(\mu)\nu} = \sum_\mu \frac{p_\mu}{q_\mu} V_{c(\mu)\nu}$$

Suppose for one value of μ (denoted $\mu = \hat{\mu}$), $p_{\hat{\mu}} = 1$.

Then we replace the corresponding basis vectors by W_ν. In terms of the new basis, the old basis vector in terms of the new basis has an expansion

$$V_{c(\hat{\mu})\nu} = \sum_{\mu,\mu\neq\hat{\mu}} -\frac{p_\mu q_{\hat{\mu}}}{q_\mu} V_{c(\mu)\nu} + q_{\hat{\mu}} W_\nu$$

This is "more integral" than the previous basis, and the volume spanned by the basis decreases by q_μ.

Finding an integral basis

This process converges in a maximum of 19 steps.
In 3 out of the 32990 cases it did not converge to pure integers.

These cases could be dealt with by choosing a different starting point.

In the end we did indeed find an integer basis for all 32990 Orientifolds.

This gives a “charge lattice” for axion charges.

(But: there must be a better way of doing this…)

Discrete physics is fun

Many more years of discreteness, Jürgen!