Higgs patterns in brane models.

Bert Schellekens

Nikhef

Ioannina, 5-4-2019

Papers

Part 1: GUTs without guts

Nucl.Phys. B883 (2014) 529-580, with B. Gato-Rivera

Part 2: Symmetry breaking by bi-fundamentals

Phys.Rev. D97 (2018) no.5, 056007

A Derivation of the Standard Model

discrete structure of the

A Derivation of the Standard Model

(not including the number of families)

discrete structure of the

A Derivation of the Standard Model

(not including the number of families)

discrete structure of the

A Derivation of the Standard Model

(in a class of intersecting brane models)

Towards

(not including the
number of families)

discrete structure of the

A Derivation of the Standard Model

(in a class of intersecting brane models)

Our goal:

Derive the discrete structure of the Standard Model:
The gauge group and representations.

The standard approach is to use Grand Unification.

Grand Unification

The simplicity is undeniable:

$$SU(3)\times SU(2)\times U(1) \quad \subset \quad SU(5) \quad \subset \quad SO(10)$$

One family matter representation (left-handed)

$$(3,2,\frac{1}{6})+(3^*,1,\frac{1}{3})+(3^*,1,-\frac{2}{3})+(1,2,-\frac{1}{2})+(1,1,1)\ +(1,0,0)$$

Fits beautifully in the $(\mathbf{16})$ of $SO(10)$

And the coupling constants meet if there is low energy supersymmetry.

Plus: automatic explanation of charge quantization

So how could this be wrong?

GUT History

Non-susy GUT era

Januari 1974: Georgi, Glashow — Group Unification
May 1974: Georgi, Quinn, Weinberg — Coupling Unification
April 1982: Cabrera — Magnetic Monopole?
April 1984: Georgi **(5th workshop on Grand Unification)** — Proton decay?

Heterotic GUT era

Januari 1985: Candelas et. al. — E_6 from Heterotic Strings
1986: Kawai et. al, Lerche et. al, Antoniadis et. al. — 4D strings
November 1989: Schellekens — No charge quantization in heterotic GUTS

Susy GUT era

1981: Dimopoulos, Georgi, ….
November 1991: Amaldi et. al. — Susy coupling Unification

LHC $\sim$ 2010: — No evidence for susy

Grand Unification

Even if correct, GUTs do not lead to a derivation of the SM structure:

- Even the smallest group, $SU(5)$, can break in two ways, to $SU(3)\times SU(2)\times U(1)$ or $SU(4)\times U(1)$.
- The Standard Model Higgs is not determined, and does not fit in an $SU(5)$ multiplet.
- In QFT the representations are determined if one assumes some kind of minimality, but what is the motivation for that?
- No top-down arguments selecting $SU(5)$ or $SO(10)$.

Anthropics

vs.

Aesthetics

Anthropics

(concerns existence of observers)

vs.

Aesthetics

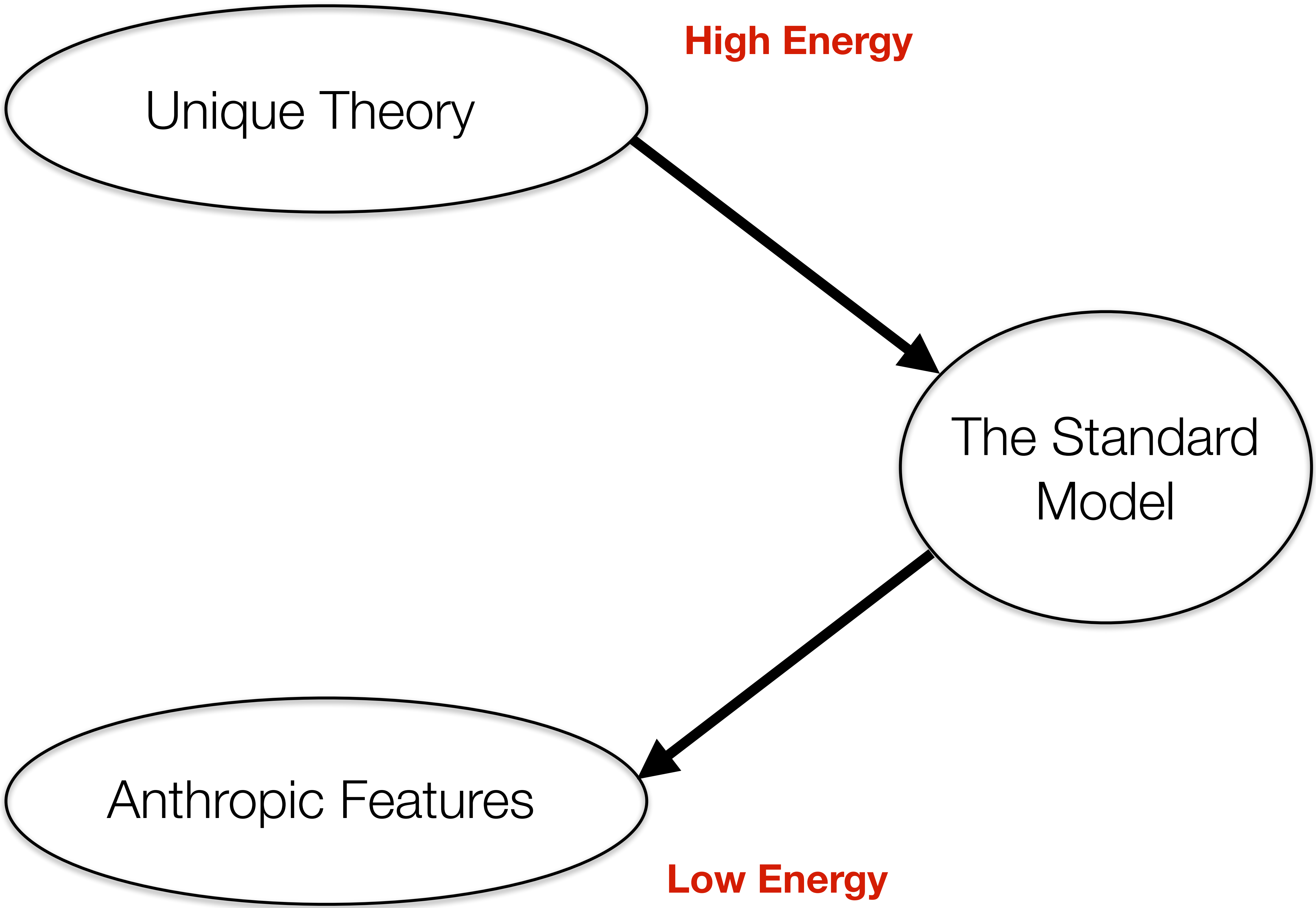
Anthropics

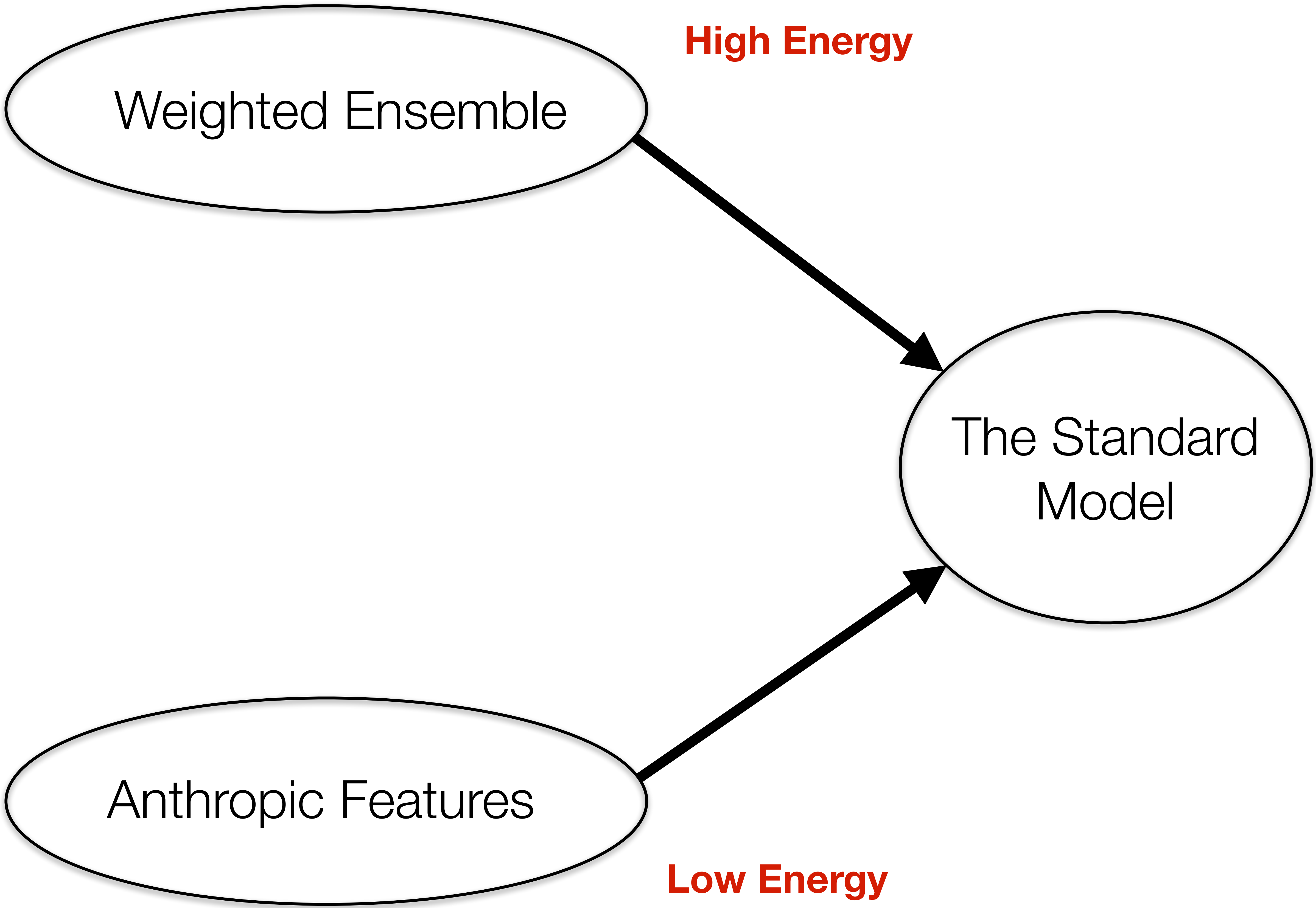
(concerns existence of observers)

vs.

Aesthetics

(concerns happiness of observers)


High Energy
Unique Theory
The Standard Model
Anthropic Features
Low Energy


Weighted Ensemble
High Energy
The Standard Model
Anthropic Features
Low Energy

Required anthropic features

- A large hierarchy
- At least one massless photon
  (to get atomic physics)
- A substantial variety of (semi)stable charged particles
  (playing a role analogous to nuclei and electrons, so that we get interesting atomic physics; just hydrogen and helium is too boring.)
- No massless charged particles

"A massless electron means that the Bohr radius of an atom—half a nanometer in the real world—would be infinite. In a world without compact atoms, valence chemical bonding would have no meaning. All matter would be insubstantial—and life as we know it would not exist! ***On top of all that, the vacuum would be unstable to the formation of a plasma of e^+e^- pairs."***

(C. Quigg, R. Shrock, Phys.Rev. D79 (2009) 096002)

The need for a large hierarchy

Maximal number of building blocks with mass m_p of an object that does not collapse into a black hole or fly apart

$$\left(\frac{m_{\text{Planck}}}{m_p}\right)^3$$

Brain with 10^{27} building blocks requires a hierarchy of 10^{-9}

Stars require an even bigger hierarchy:

Fred Adams, "Constraints on Alternate Universes: Stars and habitable planets with different fundamental constants", arXiv:1511.06958

"We find the limit $\alpha_G/\alpha < 10^{-34}$, which shows that habitable universes must have a large hierarchy between the strengths of the gravitational force and the electromagnetic force".

$$\alpha_G \equiv \frac{Gm_{\mathrm{p}}^2}{\hbar c}$$

The Hierarchy Problem

So far, approaches towards solving the hierarchy problem have failed for over two decades.

Perhaps this is because of two serious mistakes:

1. Ignoring anthropic arguments
   (“It is a deep mystery that this number is so small”)

2. Ignoring distributions
   (without this information exact statements are impossible)

The Technical Hierarchy Problem

Renormalization of scalar masses

$$\mu^2_{\text{phys}} = \mu^2_{\text{bare}} + \sum_i a_i \Lambda^2$$

Computable statistical cost of about 10^{-34} for the observed hierarchy. This is the "technical hierarchy problem".

Renormalization of fermion masses

$$\lambda_{\text{phys}} = \lambda_{\text{bare}} \left(\sum_i b_i \log(\Lambda/Q) \right)$$

Statistical cost determined by landscape distribution of λ_{bare}

The Technical Hierarchy Problem

Renormalization of scalar masses

$$\mu^2_{\rm phys} = \mu^2_{\rm bare} + \sum_i a_i \Lambda^2$$ **Bad**

Computable statistical cost of about 10^{-34} for the observed hierarchy. This is the "technical hierarchy problem".

Renormalization of fermion masses

$$\lambda_{\rm phys} = \lambda_{\rm bare} \left(\sum_i b_i \log(\Lambda/Q) \right)$$ **Good**

Statistical cost determined by landscape distribution of $\lambda_{\rm bare}$

The Technical Hierarchy Problem

Renormalization of scalar masses

$$\mu^2_{\rm phys} = \mu^2_{\rm bare} + \sum_i a_i \Lambda^2$$

Bad

Computable statistical cost of about 10^{-34} for the observed hierarchy. This is the "technical hierarchy problem".

Renormalization of fermion masses

$$\lambda_{\rm phys} = \lambda_{\rm bare} \left(\sum_i b_i \log(\Lambda/Q) \right)$$

Unknown

Statistical cost determined by landscape distribution of $\lambda_{\rm bare}$

The Single Higgs Hypothesis

If we accept the current status quo, apparently nature has chosen to pay the huge price of a single scalar that creates the hierarchy.

It remains to be shown that is cheaper than having fundamental Dirac particles with small masses, or than solutions to the technical hierarchy problem (susy, compositenes, ….) but we will assume that it is.

Then this price is going to be payed only once: **there should be at most one light scalar.**

Anomaly arguments

Geng and Marshak (1989)

A single SM family without a right-handed neutrino is the smallest non-trivial chiral anomaly-free representation of $SU(3)\times SU(2)\times U(1)$.

OK, but:

- There are **three** families.
- There probably are right-handed neutrinos.
- Why is the smallest representation preferred anyway?

See also:

Minahan, Ramond, Warner (1990), *Geng and Marshak (1990)*

Anomalies and Charge Quantization

An anomaly free set of chiral fermions with irrational charges
(which can even get their masses from the SM Higgs)

$$(3, 2, \frac{1}{6} - \frac{x}{3}) + (\bar{3}, 1, -\frac{2}{3} + \frac{x}{3}) + (\bar{3}, 1, \frac{1}{3} + \frac{x}{3})$$
$$+(1, 2, -\frac{1}{2} + x) + (1, 1, 1 - x) + (1, 1, -x)$$

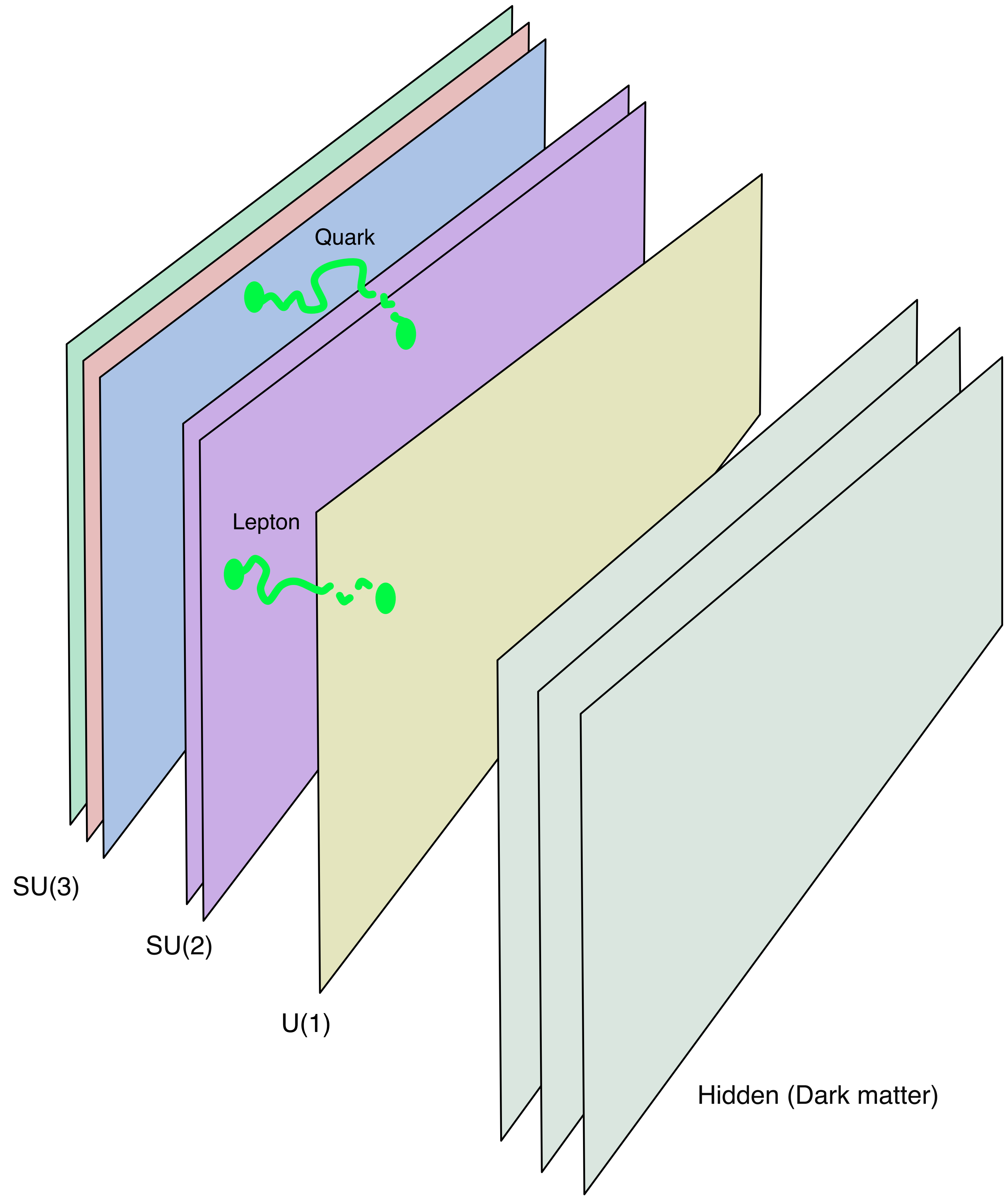
The Ensemble

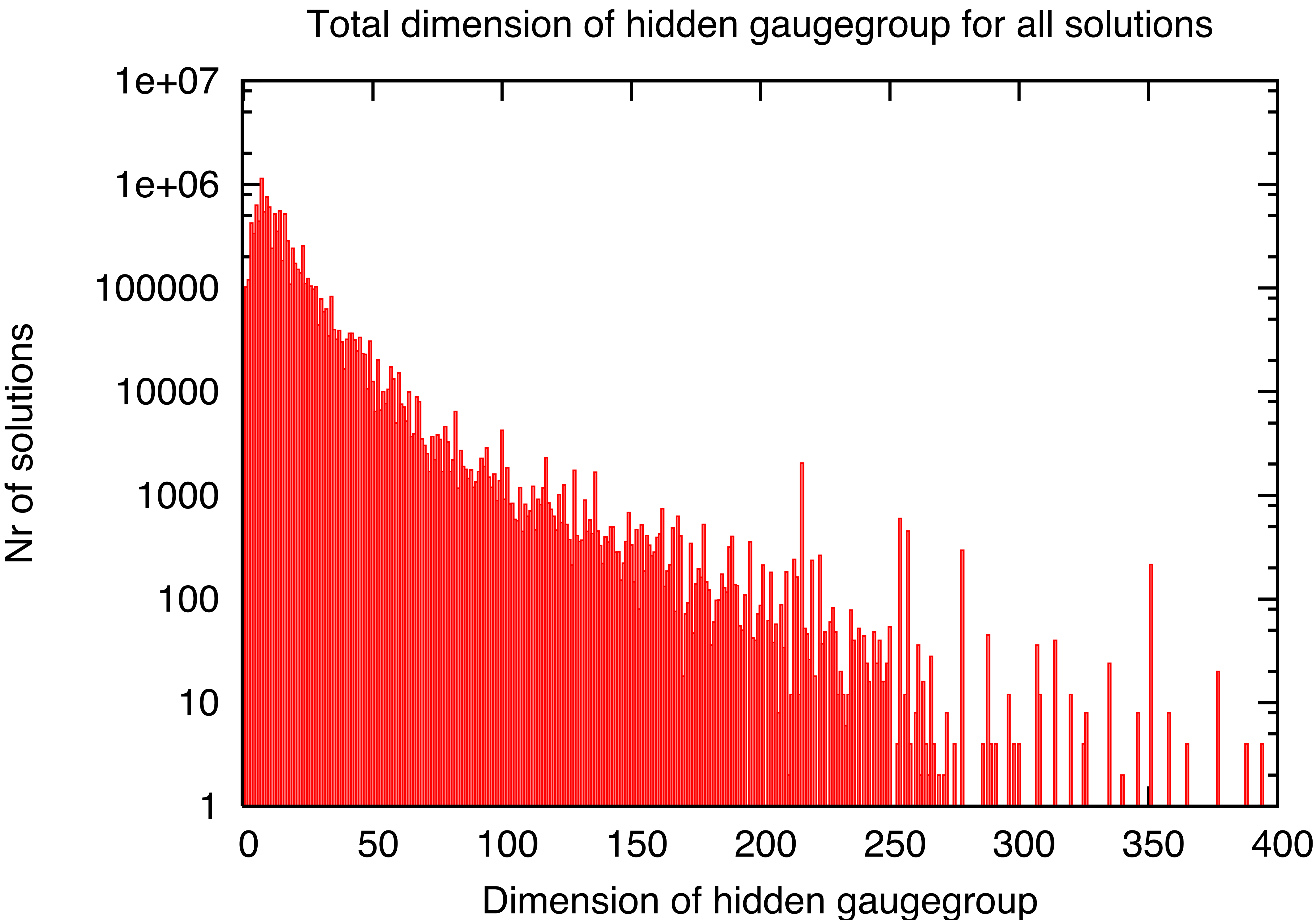
We would like to enumerate all QFT's with a gauge group and chiral matter. All non-chiral matter is assumed to be heavy, with the exception of at most one scalar field, the Higgs. We demand that after the Higgs gets a vev, and that when all possible dynamical symmetry breakings have been taken into account, at least one massless photon survives, and all charged particles are massive.

This condition is very restrictive, but still has an infinite number of solutions in QFT.

So at this point we invoke string theory. Its main rôle is to restrict the representations. It also provides a more fundamental rationale for anomaly cancellation.

Intersecting Brane Models





Dijkstra, Huiszoon, Schellekens (2004)

Type-II RCFT orientifolds

Intersecting Brane Models

We will assume that all matter and the Higgs boson are massless particles in intersecting brane models. Then the low-energy gauge groups is a product of $U(N)$, $O(N)$ and $Sp(N)$ factors.

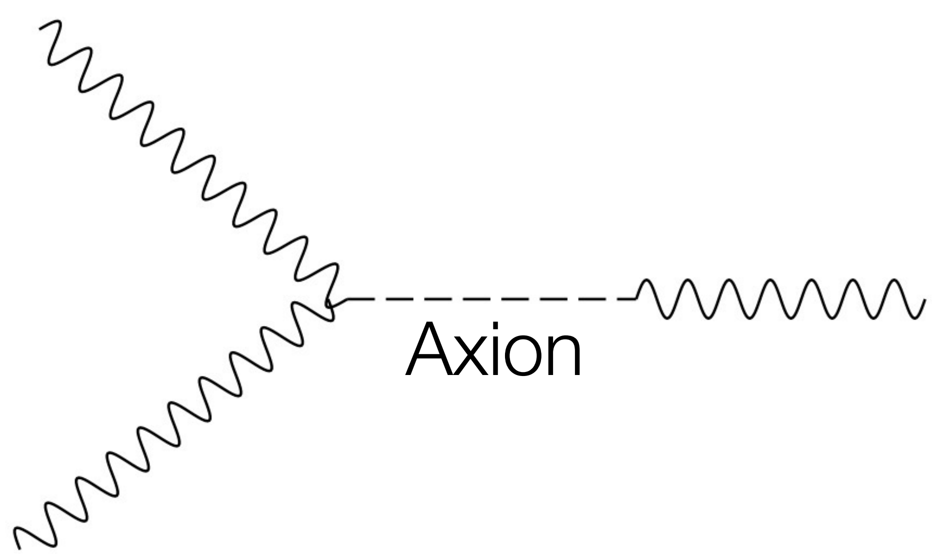
The low energy gauge group is assumed to come from S stacks of branes.
There can be additional branes that do not give rise to massless gauge bosons: $O(1)$ or $U(1)$ with a massive vector boson due to axion mixing.

All matter (fermions as well a the Higgs) are bi-fundamentals, symmetric or anti-symmetric tensors, adjoints or vectors (open strings with one end on a neutral brane)

We start with $S = 1$, and increase S until we find a solution.
($S = 1$ is easily ruled out, so the first case of interest is $S = 2$)

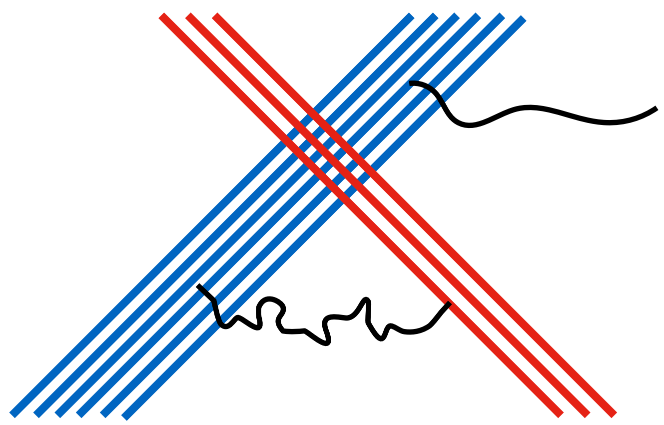
Intersecting Brane Models

- Intersections of branes in extra dimensions determine the massless spectrum.
- Brane multiplicities are subject to a constraint: tadpole cancellation (automatically implies absence of triangle anomalies in QFT).
- Massless photons may mix with axions and acquire a mass.



(Green-Schwarz mechanism)

Two stack models



$$SU(M) \times SU(N) \times U(1)$$

(assuming unitary branes)

$$Y = q_a Q_a + q_b Q_b$$

$q_a, \ q_b$ determined by axion couplings

Q	$(M, N, q_a + q_b)$
U	$(A, 1, 2q_a)$
D	$(\overline{M}, 1, -q_a)$
S	$(S, 1, 2q_a)$
X	$(M, \overline{N}, q_a - q_b)$
L	$(1, \overline{N}, -q_b)$
T	$(1, S, 2q_b)$
E	$(1, A, 2q_b)$

Tadpole Equations

$$
\begin{aligned}
(S+U)\tilde{q}_a &= C_1 \\
(T+E)\tilde{q}_b &= -C_2 \\
(D+8U)\tilde{q}_a &= (4+M)C_1 + NC_2 \\
L\tilde{q}_b + D\tilde{q}_a &= 0 \\
2E\tilde{q}_b + 2U\tilde{q}_a &= C_1 - C_2
\end{aligned}
$$

$$\tilde{q}_a \equiv Mq_a,\ \tilde{q}_b \equiv Nq_b$$

$$C_1 = -(Q-X)\tilde{q}_b$$

$$C_2 = (Q+X)\tilde{q}_a$$

Note that Q,U,D,L,E,S,T,X denote both the name and the multiplicity of a representation

($q_a = 0$ and/or $q_b = 0$ must be treated separately)

Abelian theories

Single $U(1)$: Higgs must break it, no electromagnetism left

$U(1) \times U(1)$: No solution to anomaly cancellation for two stacks

So in two-stack models we need at least one non-abelian factor in the high-energy theory.

Strong Interactions

It is useful to have a non-abelian factor in the low-energy theory as well, since the elementary particle charge spectrum is otherwise too poor. We need some additional interaction to bind these particles into bound states with larger charges (hadrons and nuclei in our universe).

For this to work there has to be an approximately conserved baryon number. This means that we need an $SU(M)$ factor with $M \geq 3$, and that this $SU(M)$ factor does not become part of a larger group at the "weak" scale.

Note that $SU(2)$ does not have baryon number, and the weak scale is near the constituent mass scale. We cannot allow baryon number to be broken at that scale.

But let's just call this an additional assumption.

Higgs Choice

This implies that at least one non-abelian factor is not broken by the Higgs. We take this factor to be $U(M)$.

Therefore we do not consider bi-fundamental Higgses breaking both $U(M)$ and $U(N)$. We assume that $U(N)$ is the broken gauge factor. Then the only Higgs choices are **L**,**T** and **E**.

We will assume that $U(M)$ it is strongly coupled in the IR-regime and stronger than $U(N)$.

$$SU(M)\times U(1)\ \ (i.e.\ N{=}1)$$

Higgs can only break $U(1)$, but then there is no electromagnetism.

Hence there will be a second non-abelian factor, broken by the Higgs.

$$M = 3,\ N = 2$$

Higgs = L

Decompose L, E, T: chiral charged leptons avoided only if

$$L = E,\ T = 0$$

Substitute in tadpole equation:

$$S\tilde{q}_a = \left(\frac{5 - N - M}{2M}\right) C_1$$

For $M = 3,\ N = 2$: $S = 0$

Therefore we get standard QCD without symmetric tensors.

$M = 3, \ N = 2$

Quark sector pairing

$$Q(3, q_a) + Q(3, q_a + 2q_b) + X(3, q_a) + X(3, q_a - 2q_b) - U(3, -2q_a) - D(3, q_a)$$

$Q + X - D = 0$

$Q = U$ **if and only if** $q_a + 2q_b = -2q_a$

or

$X = U$ **if and only if** $q_a - 2q_b = -2q_a$

In both cases we get an $SU(5)$ type charge relation, and hence standard charge quantization

$$M = 3,\ N = 2$$

Quark sector pairing

$$Q(3, q_a) + Q(3, q_a + 2q_b) + X(3, q_a) + X(3, q_a - 2q_b) - U(3, -2q_a) - D(3, q_a)$$

$$Q + X - D = 0$$

$Q = U$ **if and only if** $q_a + 2q_b = -2q_a$

or

$X = U$ **if and only if** $q_a - 2q_b = -2q_a$

In both cases we get an $SU(5)$ type charge relation, and hence standard charge quantization

$M = 3,\ N = 2$

Quark sector pairing

$$Q(3, q_a) + Q(3, q_a + 2q_b) + X(3, q_a) + X(3, q_a - 2q_b) - U(3, -2q_a) - D(3, q_a)$$

$$Q + X - D = 0$$

$Q = U$ **if and only if** $q_a + 2q_b = -2q_a$

or

$X = U$ **if and only if** $q_a - 2q_b = -2q_a$

In both cases we get an $SU(5)$ type charge relation, and hence standard charge quantization

$M = 3, \ N = 2$

Quark sector pairing

$$Q(3, q_a) + Q(3, q_a + 2q_b) + X(3, q_a) + X(3, q_a - 2q_b) - U(3, -2q_a) - D(3, q_a)$$

$$Q + X - D = 0$$

$Q = U$ **if and only if** $q_a + 2q_b = -2q_a$

or

$X = U$ **if and only if** $q_a - 2q_b = -2q_a$

In both cases we get an $SU(5)$ type charge relation, and hence standard charge quantization

$M = 3,\ N = 2$

Hence either $Q = 0$ ***or*** $X = 0$; the choice is irrelevant.

Take $X = 0$.

Then $D = Q = U,\ T = 0,\ L = E$

Remaining anomaly conditions: $L = Q$

Hence the only solution is a standard model family, occurring Q times.

The branes **a** and **b** are in principle unrelated, and can generally not be combined to a $U(5)$ stack. Hence no GUT proton decay.

This solution is just the well-known $S(U(3)\times U(2))$ model which produces the correct charge quantisation.

$$M = 3,\ N = 2$$

Higgs = T

The symmetric tensor can break $SU(2)\times U(1)$ in two ways, either to $U(1)$, in the same way as **L**, or to $SO(2)$.

Breaking to $U(1)$ (same subgroup as L)

No allowed Higgs couplings to give mass to the charged components of L, E and T, so we must require $E = L = T = 0$. Then there is no solution.

Breaking to $SO(2)$

Then $SO(2)$ must be electromagnetism. Y-charges forbid cubic T couplings, so $T = 0$ to avoid massless charged leptons. Quark charge pairing (to avoid chiral QED, broken by QCD) requires $Q = -X$. **If we also require $S = 0$, everything vanishes.**

(Note: stronger dynamical assumption: $S = 0$)

$M > 3$ **and/or** $N > 2$

- No solution for quark pairing for $M{>}3$
- Non-trivial solutions with quark and lepton pairing exist for $M{=}3,\ N{>}2$

  (This involves considering the most general $Q{+}\Lambda$, where Q is the external $U(1)$, and Λ a generator in the flavor group, left unbroken by dynamical symmetry breaking)

- All of them satisfy standard model charge quantization, even though $M{+}N \neq 5$
- But massless charged leptons can be avoided only for $N{=}2$

Conclusions (part 1)

- The Standard Model is the unique anthropic solution within the set of two-stack models.

- Family structure (and hence family repetition), charge quantization, the weak interactions and the Higgs choice are all derived.

- Standard Model charge quantization works the same way, for any value of N, even if $N+3 \neq 5$.

- The GUT extension offers **no advantages** (unless susy is found).

- From the two-brane ansatz, the single Higgs hypothesis, and the anthropic atomic physics requirements one can derive the Standard Model family structure **without any prior knowledge of quarks, leptons and their charges.**

Ensemble	Brane models (weighted)	QFT (not weighted)
Class	Two brane model (minimal choice)	Simple Lie Algebras
Gauge Group	SU(3) x SU(2) x U(1)	SU(5)
Particle representation	Output	(5)+(10*)
GUT scale Higgs	Not Needed	(24)
Breaking pattern	Not Needed	Choose SM, Not SU(4) x U(1)
SM Higgs	Output	(5) (doublet-triplet splitting problem)

Larger Landscapes

So far we were limited to two gauge stacks plus neutral branes.

But the single Higgs assumption allows an extension to the entire intersecting brane landscape.

The single Higgs always comes from one or two Higgs branes.

All other matter must get a mass from that Higgs.

This means that all other branes must intersect the Higgs brane(s).

This will certainly not determine the Standard Model uniquely, but perhaps it stands out as the simplest possibility.

Bi-fundamental Higgs

To investigate this we have to know how gauge groups break if Higgses from brane intersections get a vev.

These Higgses are bi-fundamentals or rank-2 tensors.

The gauge groups consist of of factors $U(N), O(N)$ or $USp(2N)$

A classic paper on this subject was written by Ling-Fong Li in 1974

PHYSICAL REVIEW D VOLUME 9, NUMBER 6 15 MARCH 1974

Group theory of the spontaneously broken gauge symmetries*

Ling-Fong Li
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 28 September 1973)

The patterns of symmetry breaking in the gauge theories are investigated systematically in the general rotation groups and unitary groups, with Higgs scalars in the various representations up to second-rank tensors. The occurrences of the fermion mass relations and pseudo-Goldstone bosons are also discussed in various cases.

But this only deals with $SU(M)\times SU(N)$ and $SO(M)\times SO(N)$

Branes produce $U(N), O(N)$ or $USp(2N)$ groups.

So we need to generalize Ling-Fong Li to mixed bi-fundamentals and symplectic groups.

$O(M) \times O(N)$

The invariant potential can be easily seen to be

$$V = -\tfrac{1}{2}\mu^2\phi_{i\alpha}\phi_{i\alpha} + \tfrac{1}{4}\lambda_1(\phi_{i\alpha}\phi_{i\alpha})^2 + \tfrac{1}{4}\lambda_2(\phi_{i\alpha}\phi_{i\beta})(\phi_{j\alpha}\phi_{j\beta}). \tag{4.4}$$

The minimum is then given by

$$\frac{\partial V}{\partial \phi_{i\alpha}} = -\mu^2\phi_{i\alpha} + \lambda_1(\phi_{j\beta}\phi_{j\beta})\phi_{i\alpha} + \lambda_2\phi_{i\beta}(\phi_{j\alpha}\phi_{j\beta}) = 0. \tag{4.5}$$

It is convenient to introduce the matrix X, defined by

$$X_{ij} = \sum_{\beta=1}^{M} \phi_{i\beta}\phi_{j\beta} = \phi\phi^T. \tag{4.6}$$

The matrix X defined this way is real and symmetric, and can be diagonalized by an orthogonal transformation. We can choose $X_{ij} = \delta_{ij}X_j$ to rewrite Eq. (4.5) as

$$\left(-\mu^2 + \lambda_1 \sum_{j=1}^{N} X_j + \lambda_2 X_i\right)\phi_{i\alpha} = 0. \tag{4.7}$$

Solution:

$$X_i = \frac{\mu^2}{\lambda_1 k + \lambda_2}, \quad i = 1, \dots, k$$

$$X_i = 0, \quad i = k+1, \dots, N$$

and

$$V = \frac{k\mu^4}{\lambda_1 k + \lambda_2}. \tag{4.8}$$

For $\lambda_2 < 0$, the potential is a monotonically increasing function of k. The minimum is at $k = 1$, and the X takes the form

$$X = b \begin{bmatrix} 1 & & & & & \\ & 0 & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix}, \quad b = \frac{\mu^2}{\lambda_1 + \lambda_2}. \tag{4.9}$$

For $\lambda_2 > 0$, the potential is a monotonically decreasing function of k. Hence the minimum is at the largest value of k allowed, which should be N. However, this would imply that X is a multiple of the $N \times N$ identity matrix

$$X = c\underline{1}_N = c \begin{bmatrix} 1 & & & & \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & 1 \end{bmatrix}. \tag{4.10}$$

But X is constructed from the $N \times M$ matrix ($N \geq M$) by

$$X = \phi\phi^T.$$

Equation (4.10) implies that if we consider each row as an M-component vector, all these N vectors are orthogonal to each other, which is impossible for $N > M$. Therefore the largest value of k allowed is M, not N, and the solutions for X and ϕ are

$$X = c^2 \begin{bmatrix} \underline{1}_M & & & & \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & 0_{NM} \end{bmatrix}, \quad c^2 = \frac{\mu^2}{N\lambda_1 + \lambda_2}$$

$O(M-1) \times O(M-1)$

$O(M) \times O(N-M)$

The general case

$$
\begin{aligned}
V = & - \mu^2 \phi_{i\alpha}\phi^*_{i\alpha} + \tfrac{1}{2}\lambda_1 \phi_{i\alpha}\phi^*_{i\alpha}\phi_{j\beta}\phi^*_{j\beta} + \tfrac{1}{2}\lambda_2 \phi_{i\alpha}\phi^*_{i\beta}\phi_{j\beta}\phi^*_{j\alpha} \\
& + \tfrac{1}{2}\epsilon_C \lambda_3 \phi_{i\alpha} C_{\alpha\beta}\phi_{j\beta}\phi^*_{j\gamma} C_{\gamma\delta}\phi^*_{i\delta} + \tfrac{1}{2}\epsilon_D \lambda_4 \phi_{i\alpha} D_{ij}\phi_{j\beta}\phi^*_{k\beta} D_{kl}\phi^*_{l\alpha} \\
& + \epsilon_D \tfrac{1}{4}\lambda_5 \phi_{j\alpha} C_{\alpha\beta}\phi_{k\beta} D_{kl}\phi_{l\gamma} C_{\gamma\delta}\phi_{i\delta} D_{ij} + \tfrac{1}{4}\lambda_5^* \phi^*_{j\alpha} C_{\alpha\beta}\phi^*_{k\beta} D_{kl}\phi^*_{l\gamma} C_{\gamma\delta}\phi^*_{i\delta} D_{ij}
\end{aligned}
$$

$$
V = -\mu^2 \phi_{i\alpha}\phi^*_{i\alpha} + \tfrac{1}{2}\lambda_1 V_1 + \tfrac{1}{2}\lambda_2 V_2 + \tfrac{1}{2}\lambda_3 V_3 + \tfrac{1}{2}\lambda_4 V_4 + \tfrac{1}{4}\lambda_5 V_5 + \tfrac{1}{4}\lambda_5^* V_5^*
$$

Groups	D	C	reality condition	coupling constants
$U(N)\times U(M)$	none	none	none	λ_1, λ_2
$O(N)\times O(M)$	δ	δ	$\phi = \phi^*$	λ_1, λ_2
$USp(2N)\times USp(2M)$	Ω	Ω	$\phi^* = \Omega^T\phi\Omega$	λ_1, λ_2
$O(N)\times U(M)$	δ	none	none	$\lambda_1, \lambda_2, \lambda_4$
$U(N)\times USp(2M)$	none	Ω	none	$\lambda_1, \lambda_2, \lambda_3$
$O(N)\times USp(2M)$	δ	Ω	none	$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$

$$\Omega^T = -\Omega; \quad \Omega^T\Omega = 1$$

Symmetric bi-fundamentals (identical group types)

Only two coupling constants. Equations of motion:

$$\mu^2\phi_{i\alpha} = \lambda_1\phi_{i\alpha}\left(\phi_{j\beta}\phi^*_{j\beta}\right) + \lambda_2\phi_{i\beta}\phi_{j\alpha}\phi^*_{j\beta}$$

$i = 1,\ldots,M \quad \alpha = 1,\ldots,N$

We can improve on Ling-Fong Li by using singular value decompositions:

$\phi = URV$; R diagonal, real, non-negative

Three cases:
$$\begin{cases} \phi \text{ complex}; \quad U, V \text{ unitary} \\ \phi \text{ real}; \quad U, V \text{ orthogonal} \\ \phi \text{ quaternionic}; \quad U, V \text{ symplectic} \end{cases}$$

Quaternionic: $\phi^* = \Omega_1\phi\Omega_2$

Symmetric bi-fundamentals

Singular value decompositions work even if the matrix is not square.

No need to introduce $\phi^\dagger\phi$ or $\phi\phi^T$

Substituting this into the equations of motion immediately gives the solution of Ling-Fong Li, and generalises it to $USp(2M)\times USp(2N)$

A-symmetric bi-fundamentals

More terms in the potential.
Need more than one quadratic combination

$$(\phi^\dagger\phi, \phi\phi^\dagger, \phi^T\phi, \phi\phi^T)$$

These cannot be simultaneously diagonalised.

Singular value decompositions also fail if left and right matrices are of different types.
Not enough parameters even to block-diagonalise.

Equations of motion

$$\begin{aligned}\mu^2\phi_{i\alpha} &= \lambda_1\phi_{i\alpha}\left(\phi_{j\beta}\phi^*_{j\beta}\right) + \lambda_2\phi_{i\beta}\phi_{j\alpha}\phi^*_{j\beta} + \epsilon_C\lambda_3\phi_{i\delta}C_{\delta\beta}\phi_{j\beta}\phi^*_{j\gamma}C_{\gamma\alpha}\\ &+ \epsilon_D\lambda_4\phi_{m\alpha}D_{mj}\phi_{j\beta}\phi^*_{k\beta}D_{ki} + \lambda^*_5C_{\alpha\beta}\phi^*_{j\beta}D_{jk}\phi^*_{k\gamma}C_{\gamma\delta}\phi^*_{m\delta}D_{mi}\end{aligned}$$

Three kinds of equations:

1. $\phi_{i\alpha} = 0$: four cubic equations (coefs of $\lambda_2, \ldots, \lambda_5$)
2. Difference equations: quadratic and quartic equations
3. Inhomogeneous equation

Assume all λ_i are unrelated

Inhomogeneous equation

Define $\phi_{i\alpha} = r\mu\chi_{i\alpha}$

Normalize by setting $\chi_{11} = 1$, $\chi_{1\alpha} = 0$, for $\alpha \geq 2$

Define "structure constants" (in suitable basis)

$$
\begin{aligned}
\rho_2 &= \chi_{j1}\chi^*_{j1} \\
\rho_3 &= \chi_{j2}\chi^*_{j2} \\
\rho_4 &= \chi_{j1}\chi_{j1} \\
\rho_5 &= \chi^*_{j2}\chi^*_{j2}
\end{aligned}
$$

Group: $G(M) \times H(N) = [U(M)$ or $O(M)] \times [U(N)$ or $S(N)]$

Then

$$r = \sqrt{\frac{1}{P\lambda_1 + Q}}$$

$$\begin{aligned} P &= \sum \chi_{i\alpha}\chi^*_{i\alpha} \\ Q &= \sum_{i=2}^{4} \lambda_i \rho_i + \lambda_5^* \rho_5 \end{aligned}$$

The structure constants are related to the potential

$$\begin{aligned} V &= -\mu^2 \phi_{i\alpha}\phi^*_{i\alpha} + \tfrac{1}{2}\mu^4 r^4 P\left(\lambda_1 P + \lambda_2\rho_2 + \lambda_3\rho_3 + \lambda_4\rho_4 + \lambda_5^*\rho_5\right) \\ &= -\mu^4 r^2 P + \tfrac{1}{2}\mu^4 r^4 P(\lambda_1 P + Q) \\ &= -\tfrac{1}{2}\mu^4 r^2 P \end{aligned}$$

Blocks

Solutions living in subspaces can be combined.
If they are disjoint, the homogeneous equations are automatically satisfied.

Solving the homogeneous equations requires that all blocks must have the same structure constants.

Hence all solutions are combinations of K identical basic blocks.

Blocks

$$\mathrm{A}_0 : \begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$\mathrm{B}_0 : \begin{pmatrix} 1 & 0 \\ i & 0 \end{pmatrix}$$

$$\mathrm{C}_x : \begin{pmatrix} 1 & 0 \\ 0 & x \end{pmatrix}$$

$$D : \begin{pmatrix} 1 & 0 \\ i & 0 \\ 0 & 1 \\ 0 & i \end{pmatrix}$$

$x = \sqrt{\frac{\lambda_5^*}{|\lambda_5|}}$ or $x = i\sqrt{\frac{\lambda_5^*}{|\lambda_5|}}$

Group	P	ρ_2	ρ_3	ρ_4	ρ_5	$\mathbb{X}$	p	q	Subgroup	Emb.
$U(N)\times U(M)$	1	1	–	–	–	A	1	1	$U(K)$	(0,0)
$O(N)\times O(M)$	1	1	–	–	–	A	1	1	$O(K)$	(0,0)
$USp(N)\times USp(M)$	2	2	–	–	–	C_1	2	2	$USp(2K)$	(0,0)
$O(N)\times U(M)$	1	1	–	1	–	A	1	1	$O(K)$	(0,1)
$O(N)\times U(M)$	2	2	–	0	–	B	2	1	$U(K)$	(3,0)
$U(N)\times USp(M)$	1	1	0	–	–	A_0	1	2	$U(K)$	(0,4)
$U(N)\times USp(M)$	2	1	1	–	–	C_1	2	2	$USp(2K)$	(2,0)
$O(N)\times USp(M)$	1	1	0	1	0	A_0	1	2	$O(K)$	(0,5)
$O(N)\times USp(M)$	2	2	0	0	0	B_0	2	2	$U(K)$	(3,4)
$O(N)\times USp(M)$	2	1	1	1	ω_5	C_y	2	2	$U(K)$	(3,4)
$O(N)\times USp(M)$	2	1	1	1	$-\omega_5$	C_{iy}	2	2	$U(K)$	(3,4)
$O(N)\times USp(M)$	4	2	2	0	0	D	4	2	$USp(2K)$	(6,0)

$$\omega_5 = \lambda_5/|\lambda_5|$$

$$y = \sqrt{\frac{\lambda_5^*}{|\lambda_5|}}$$

Full subgroup $G(N-K)\times H(M-K)\times$ Subgroup

Vacuum Energy

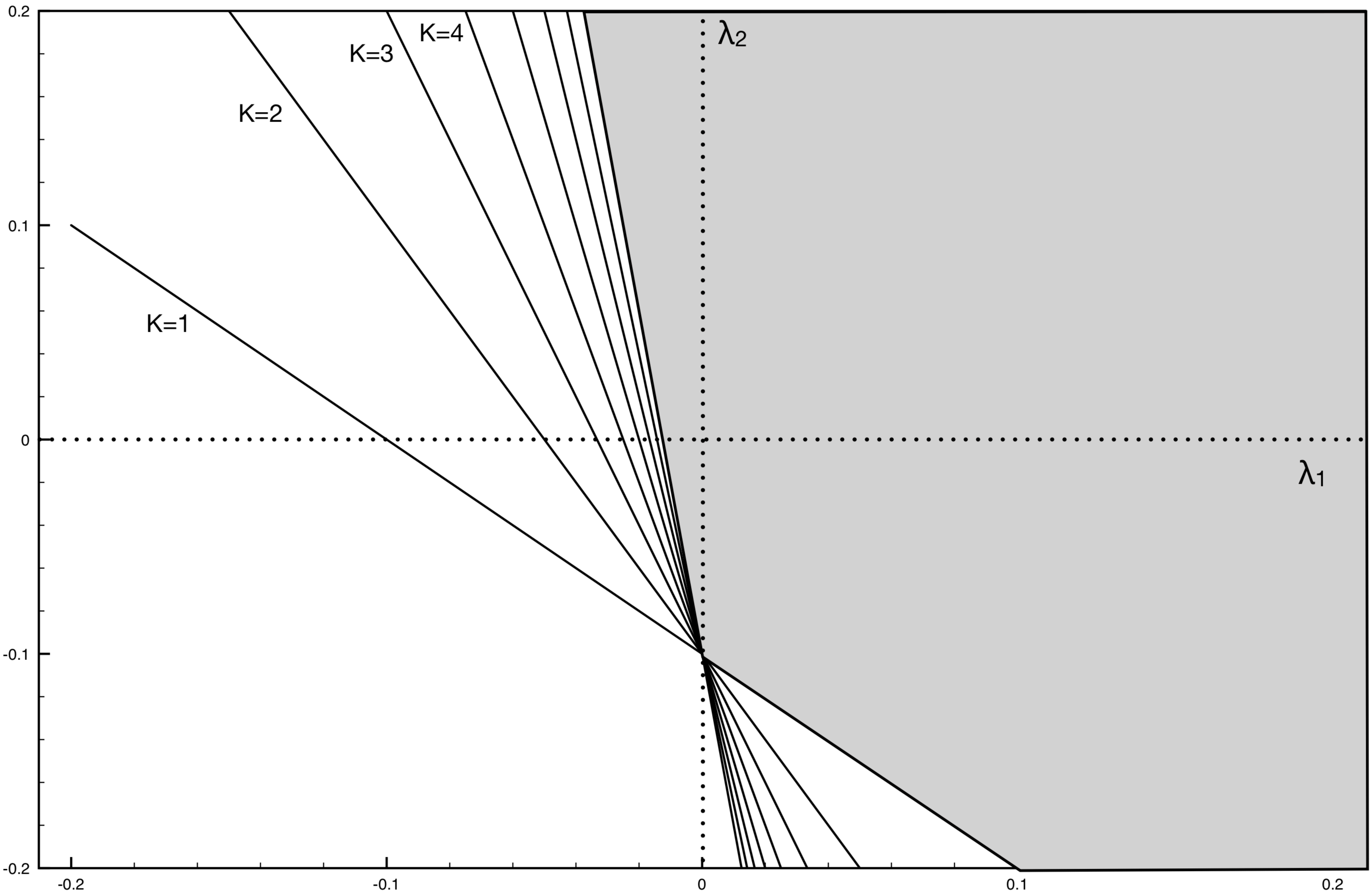
Energy of a solution with K blocks with parameters P and Q
($P = 1{,}2$ or 4, $Q = \lambda_2\rho_2 + \lambda_3\rho_3 + \lambda_4\rho_4 + \lambda_5^*\rho_5$)

$$E(K,P,Q) = -\frac{KP\mu^4}{2(KP\lambda_1 + Q)} = -\frac{\mu^4}{2(\lambda_1 + Q/KP)}$$

$Q < 0$: minimum for smallest K
$Q > 0$: minimum for largest K

$$r = \sqrt{\frac{1}{KP\lambda_1 + Q}}$$

Must be real



0.2
0.1
0
-0.1
-0.2
-0.2
-0.1
0
0.1
0.2
K=1
K=2
K=3
K=4
λ_2
λ_1

Rank-2 tensors

Group	K	Symmetric tensor	Anti-sym. tensor	Adjoint
$U(N)$	1	$U(N-1)\times O(1)$	$U(N-2)\times USp(2)$	$U(N-1)\times U(1)$
$U(2\ell)$	max	$O(2\ell)$	$USp(2\ell)$	$U(\ell)\times U(\ell)$
$U(2\ell+1)$	max	$O(2\ell+1)$	$USp(2\ell)\times U(1)$	$U(\ell)\times U(\ell+1)$
$O(N)$	1	$O(N-1)\times O(1)$	$O(N-2)\times U(1)$	-
$O(2\ell)$	max	$O(\ell)\times O(\ell)$	$U(\ell)$	-
$O(2\ell+1)$	max	$O(\ell)\times O(\ell+1)$	$U(\ell)\times O(1)$	-
$USp(2N)$	1	$USp(2N-2)\times U(1)$	$USp(2N-2)\times USp(2)$	-
$USp(4\ell)$	max	$U(2\ell)$	$USp(2\ell)\times USp(2\ell)$	-
$USp(4\ell+2)$	max	$U(2\ell+1)$	$USp(2\ell)\times USp(2\ell+2)$	-

First three items for general K

$$U(N) \;\to\; USp(2K)\times U(N-2K) \qquad \text{Anti-symmetric tensor}$$

Conclusions (Part 2)

- These are all single Higgs minima for all possible brane models
- All resulting subgroups are brane groups [no $SU(N)$ or $SO(N)$]
- Part 3 ?

Couplings

The $U(3)\times U(2)$ structure of this class of models implies one relation among the SM couplings, instead of the two of $SU(5)$

$$\frac{1}{\alpha_Y} = \frac{2}{3}\frac{1}{\alpha_s} + \frac{1}{\alpha_w}$$

see also:
Ibañez, Munos, Rigolin, 1998;
Blumenhagen, Kors, Lüst, Stieberger, 2007

Extrapolation this to higher energies we see that this is satisfied at 5.7×10^{13} GeV.

What happens at that scale and beyond is subject to speculation, but undoubtedly model-dependent.

New physics at that scale may be related to the QCD axion, the see-saw mechanism and Higgs stability.

Complete list of solutions

Nr.	M	N	q_a	q_b	Higgs	Q	U	D	S	X	L	E	T
1	1	2	2	-3	**L**	3	6	3	3	0	1	1	0
2	1	2	4	-1	**L**	2	1	1	0	0	2	3	1
3a	1	2	2	-1	**L**	3	4	1	3	-4	1	0	-1
3b	1	2	2	-1	**L**	2	2	1	1	-1	1	1	0
3c	1	2	2	-1	**L**	4	5	0	3	-4	0	1	-1
4	1	3	3	-2	**L**	2	3	2	1	0	1	1	0
5	1	3	3	-1	**E**	0	0	-2	-1	1	-2	1	0
6	1	4	4	-1	**L**	1	1	1	0	0	1	1	0
7	M	2	1	ρ	**T**	1	$-\rho$	$2M\rho$	$-\rho$	-1	$2M$	0	0
8	2	3	3	-2	**L**	1	1	1	0	0	1	1	0
9	3	2	2	-3	**L**	1	1	1	0	0	1	1	0

All chiral spectra without massless charged free leptons that can be obtained for all M and N with $q_a \neq 0$ and $q_b \neq 0$. Here $M = 1{,}2$ and ρ is a free integer parameter.

Complete list of solutions

Nr.	M	N	q_a	q_b	Higgs	Q	U	D	S	X	L	E	T
6	1	4	4	-1	**L**	1	1	1	0	0	1	1	0

This realizes the $SU(4)\times U(1)$ subgroup of SU(5).
The Higgs boson breaks this to $SU(3)\times U(1)$, QCD × QED.

But this implies $SU(5)$-type proton decay at the weak scale.

A family constitutes a single, complete $SU(4)$ Higgs multiplet.

Complete list of solutions

Nr.	M	N	q_a	q_b	Higgs	Q	U	D	S	X	L	E	T
8	2	3	3	-2	**L**	1	1	1	0	0	1	1	0

This is the same $SU(3)\times SU(2)\times U(1)$ subgroup of $SU(5)$ that gives rise to the Standard Model, but with a triplet Higgs instead of a doublet Higgs.

At low energies, there is a non-abelian $SO(4) \approx SU(2)\times SU(2)$ gauge group without conserved Baryon number.

The special case $q_a = 0$ (all M, N)

Anomaly cancellation:

$SU(M) \times SU(N) \times U(1)_Y$

$Q[(V, V, 1) + (V, \overline{V}, -1)] + \text{flavor-neutral } \mathbf{U}, \mathbf{D}, \mathbf{S} \text{ matter}$

For $M = 1{,}2$ this is vectorlike (hence massive)
For $M > 3$ there is no $U(1)$ in the flavor group that is non-chiral with respect to $SU(M)$, hence no electromagnetism.

Note: we treat Higgs and dynamical breaking on equal footing

The special case $q_b = 0$ (all M, N)

Anomaly cancellation:

$$SU(M) \times SU(N) \times U(1)_Y$$

$$Q[(V, V, 1) + (\overline{V}, V, -1)] + Y\text{-neutral } \mathbf{L},\ \mathbf{E},\ \mathbf{T} \text{ matter}$$

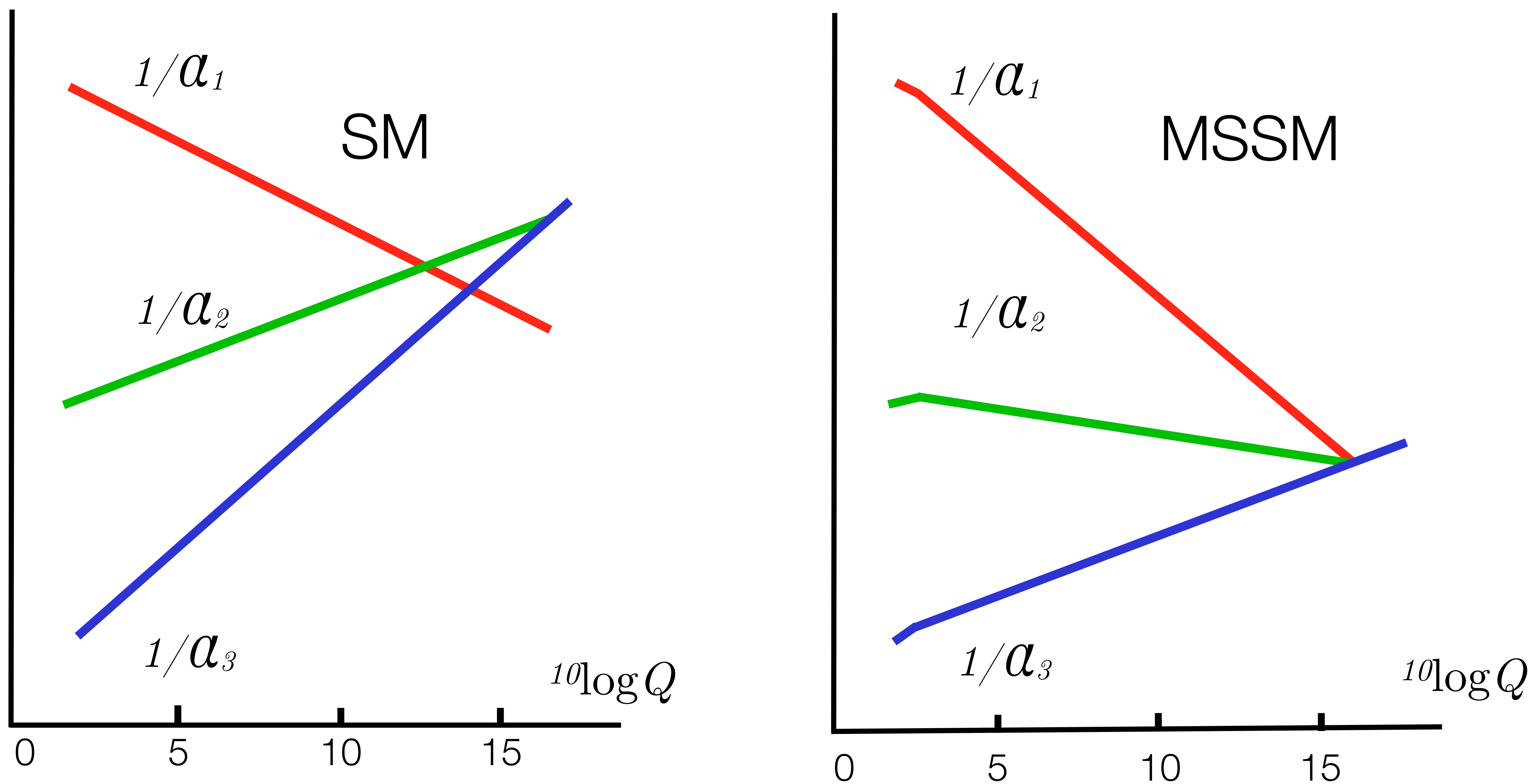
For $N = 1{,}2$ this is vector-like, and hence massive

For $N \geq 3$ the candidate Higgses do not break $U(1)_{\mathrm{Y}}$

Hence the Higgs just has to break $SU(N)$ to a real group, and this is indeed possible, for example Higgs = **T**, breaking $SU(N)$ to $SO(N)$

$$Q[(V, V, 1) + (\overline{V}, V, -1) + 2M(1, V, 0)]$$

No charged leptons; Baryon number is gauged, so baryogenesis would be problematic.



$1/\alpha_1$
SM
$1/\alpha_2$
$1/\alpha_3$
${}^{10}\log Q$
0
5
10
15
$1/\alpha_1$
MSSM
$1/\alpha_2$
$1/\alpha_3$
${}^{10}\log Q$
0
5
10
15

We will show that in a certain minimal string setting where GUT realizations are available, anthropic arguments work far better:

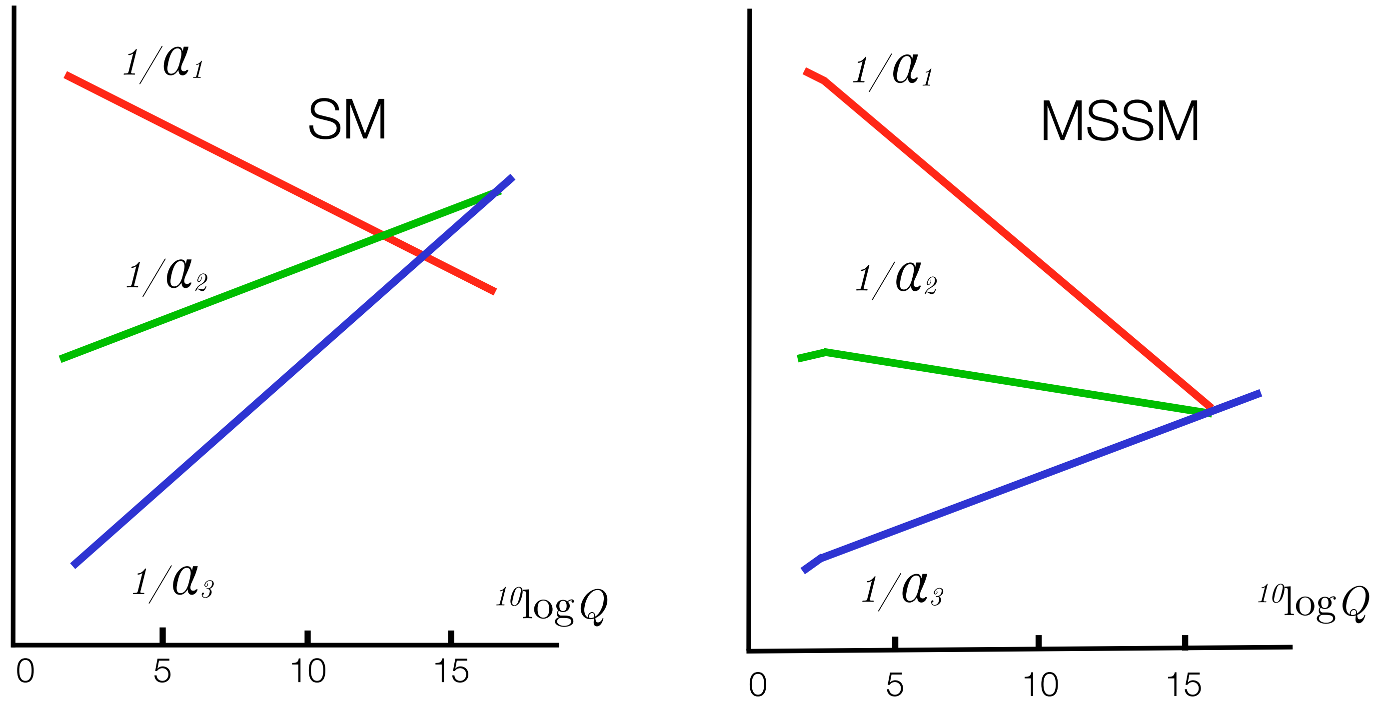
- Gauge group determined to be $SU(3) \times SU(2) \times U(1)$.
- Matter determined to be a number of standard families.
- Correct charge quantization without GUTs.
- Standard Model Higgs determined.

Assuming at least one unbroken non-abelian and at least one unbroken electromagnetic interaction

GUTs, Anomalies and Charge Quantization

If there is no low-energy supersymmetry, the three gauge coupling constants do not converge.

This removes one of the arguments in favor of GUTs.



But the arguments based on family structure and charge quantization remain valid.

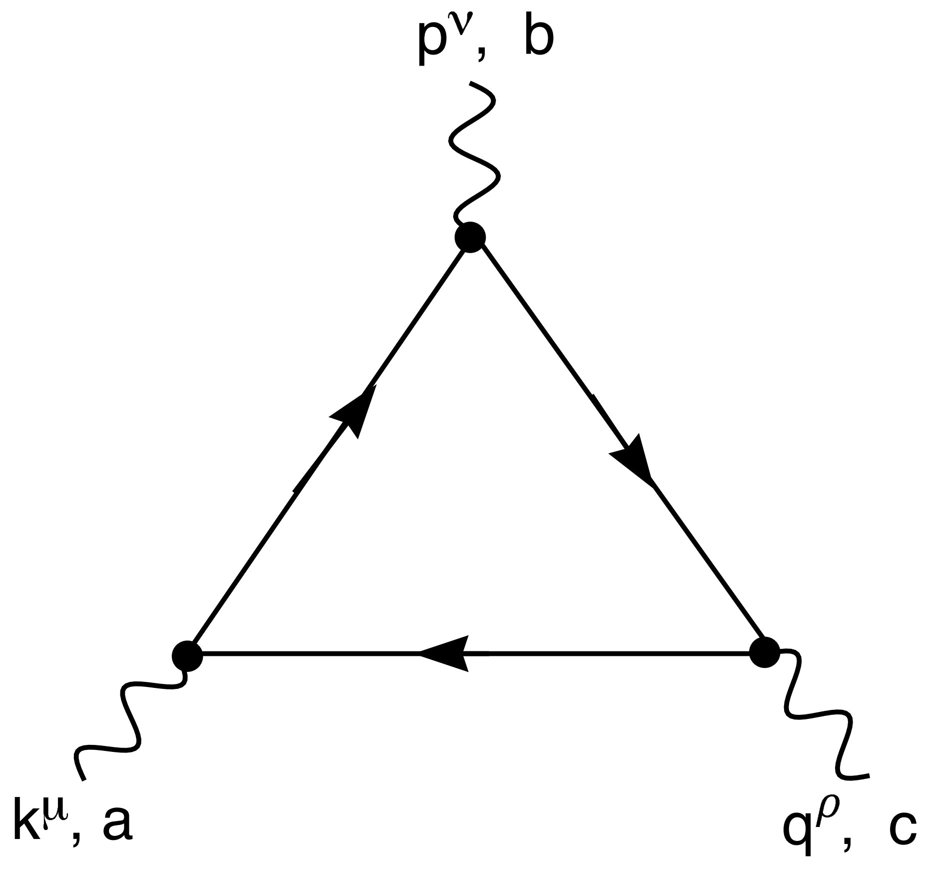
GUTs, Anomalies and Charge Quantization

The observed charged quantization is excellent evidence for BSM physics.

Imagine we end up with a consistent theory of quantum gravity that imposes no constraints on QFT. Then this would allow particles with arbitrary real charges. It is hard to accept that we just happen to live in a universe with quantized charges.

One often hears the arguments that anomaly cancellation imposes charge quantization.

Triangle anomalies



		SU(3)	SU(2)	$SU(3)^2$ x U(1)	$SU(2)^2$ x U(1)	$U(1)^3$	(Grav) x U(1)
Q	$(3,2,1/6)$	2	0	1/3	1/2	1/36	1
U*	$(3^*,1,-2/3)$	-1	0	-2/3	0	-8/9	-2
D*	$(3^*,1,1/3)$	-1	0	1/3	0	1/9	1
L	$(1,2,-1/2)$	0	0	0	-1/2	-1/4	-1
E*	$(1,1,1)$	0	0	0	0	1	1
	Sum	0	0	0	0	0	0

Anomaly arguments

Geng and Marshak (1989)
A single SM family (without right-handed neutrino) is the smallest non-trivial chiral anomaly-free representation of $SU(3)\times SU(2)\times U(1)$.

OK, but:

- There are **three** families.
- There probably are right-handed neutrinos.
- Why is the smallest representation preferred anyway?

See also:
Minahan, Ramond, Warner (1990), *Geng and Marshak (1990)*

GUTs, anomalies and Charge Quantization

Anomaly cancellation does *not* impose charge quantization:

One can add scalars or Dirac fermions of arbitrary real charge.

But even for chiral matter anomaly cancellation is not enough:
one could add an entire family with rescaled charges.

Such rescalings are not possible if one wishes to couple the
extra family to the SM Higgs.

GUTs, anomalies and Charge Quantization

One can try to impose one-family charge quantization on all three families by requiring that they all couple to the same Higgs.

But even that does not work:
One can have chiral fermions with irrational charges (in SM units) that get their mass from the SM Higgs

$$(3, 2, \frac{1}{6} - \frac{x}{3}) + (\bar{3}, 1, -\frac{2}{3} + \frac{x}{3}) + (\bar{3}, 1, \frac{1}{3} + \frac{x}{3})$$
$$+(1, 2, -\frac{1}{2} + x) + (1, 1, 1 - x) + (1, 1, -x)$$

Charge Quantization

We need some kind of BSM physics to explain charge quantization.

Our working hypothesis is there exists at least some BSM physics related to quantum gravity: a fundamental theory that imposes restrictions on the allowed QFT’s.

In other words, we are not going to end up with a consistent theory of quantum gravity that couples to ***any*** QFT.

The most promising, perhaps only candidate for such a theory is string theory.

String theory is likely to quantize the charges.
(although not necessarily in the right way)

If we already have string theory, do we also need GUTs?

The String Theory Landscape

String theory certainly does not predict the Standard Model uniquely.
As far as we know it leads to a huge ensemble ("landscape") of possibilities, realized in a multiverse. All of this is still in its infancy, but non-uniqueness of the QFT choice has been clear from the very beginning.

At this point, people tend to get nervous and start asking: but how do you ever falsify that statement? Those people should understand that the opposite point of view has the same problem. If you believe derive the Standard Model can be derived (the standard "Einstein" paradigm), you must have reasons to believe that it is unique.

But the only thing unique about it is that it is the only QFT we can observe, in principle.

Carl Sagan once said: "extraordinary claims requires extraordinary evidence".

But what is the most extraordinary claim, that there might (theoretically at least) exist other universes with other realizations of QFT, or that what we can see is all there is?

Either one of these claims can ultimately only be established by determining the fundamental theory and counting how many alternatives to the Standard Model it contains.

The String Theory Landscape

String theory certainly not predict the Standard Model uniquely. As far as we know it leads to a huge ensemble (“landscape”) of possibilities, realized in a multiverse.

So then how can we hope to derive the Standard Model?

We still have two clues, that are inevitable in a large landscape:

- Anthropic arguments
- Landscape distributions

The String Theory Landscape

The anthropic argument we will use is that the spectrum must be sufficiently complicated. In our universe this is achieved by quarks binding into protons and neutrons, which bind into nuclei, which together with electrons form atoms.

We cannot really derive this from $SU(3) \times SU(2) \times U(1)$, and hence we can certainly not expect to be able to derive this from any QFT that is more complicated.

But in some simpler theories the existence of a complicated set of bound states can be plausibly ruled out.

The String Theory Landscape

More complicated QFT's that *cannot* be anthropically ruled out certainly exist, for example

$$SU(5)\times SU(2)\times U(1)$$

With fifth-integer fractionally charged quarks.

So anthropic arguments alone will not do, given our current knowledge about strongly interacting gauge theories.

The String Theory Landscape

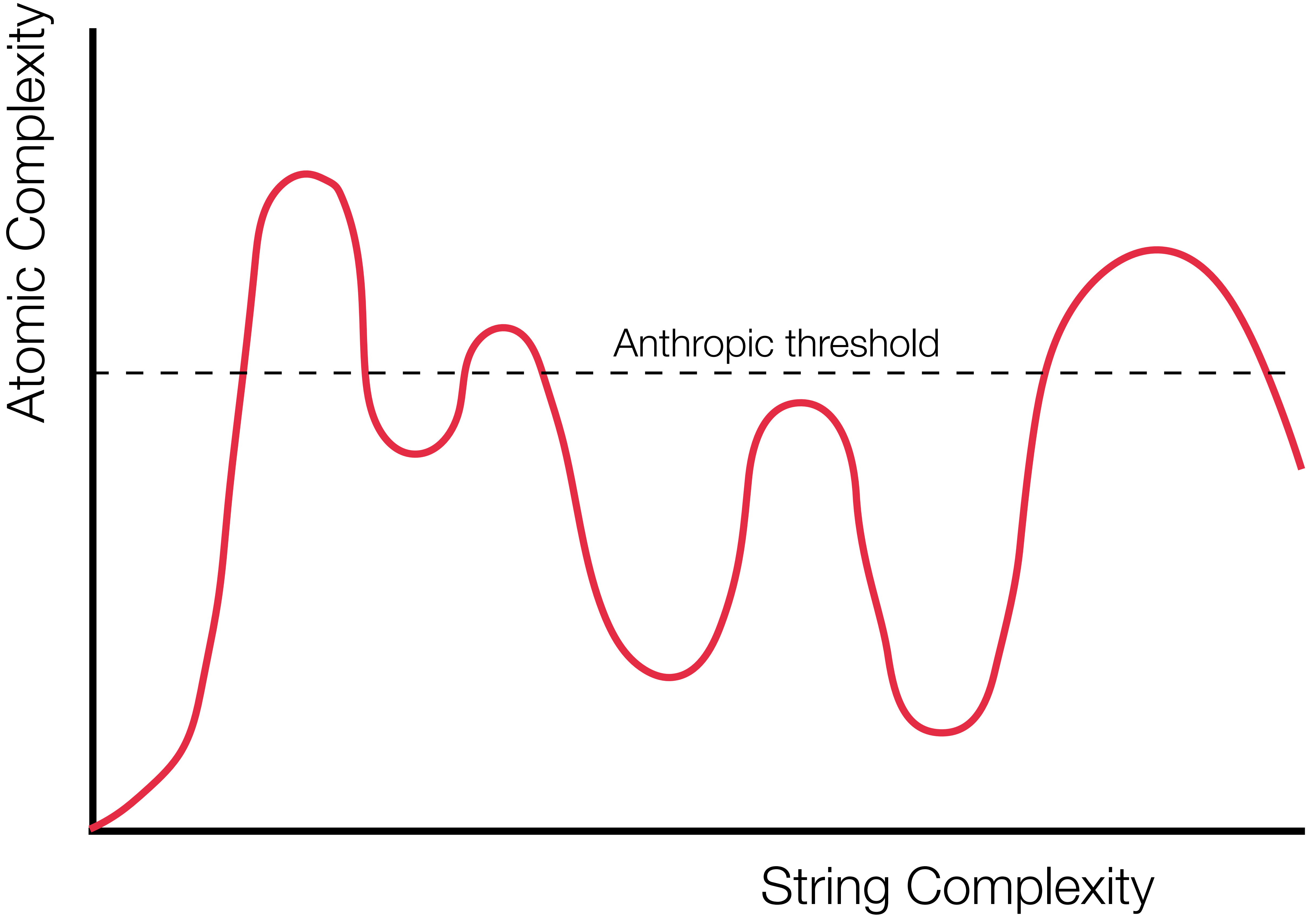
The String Theory Landscape

The hope is then that we can establish that the Standard Model is the simplest one with a complicated spectrum.

The String Theory Landscape

The hope is then that we can establish that the Standard Model is the simplest one with a complicated spectrum.

Then one may also hope that landscape statistics prefers simpler QFT’s over more complicated ones.



Atomic Complexity
Anthropic threshold
String Complexity

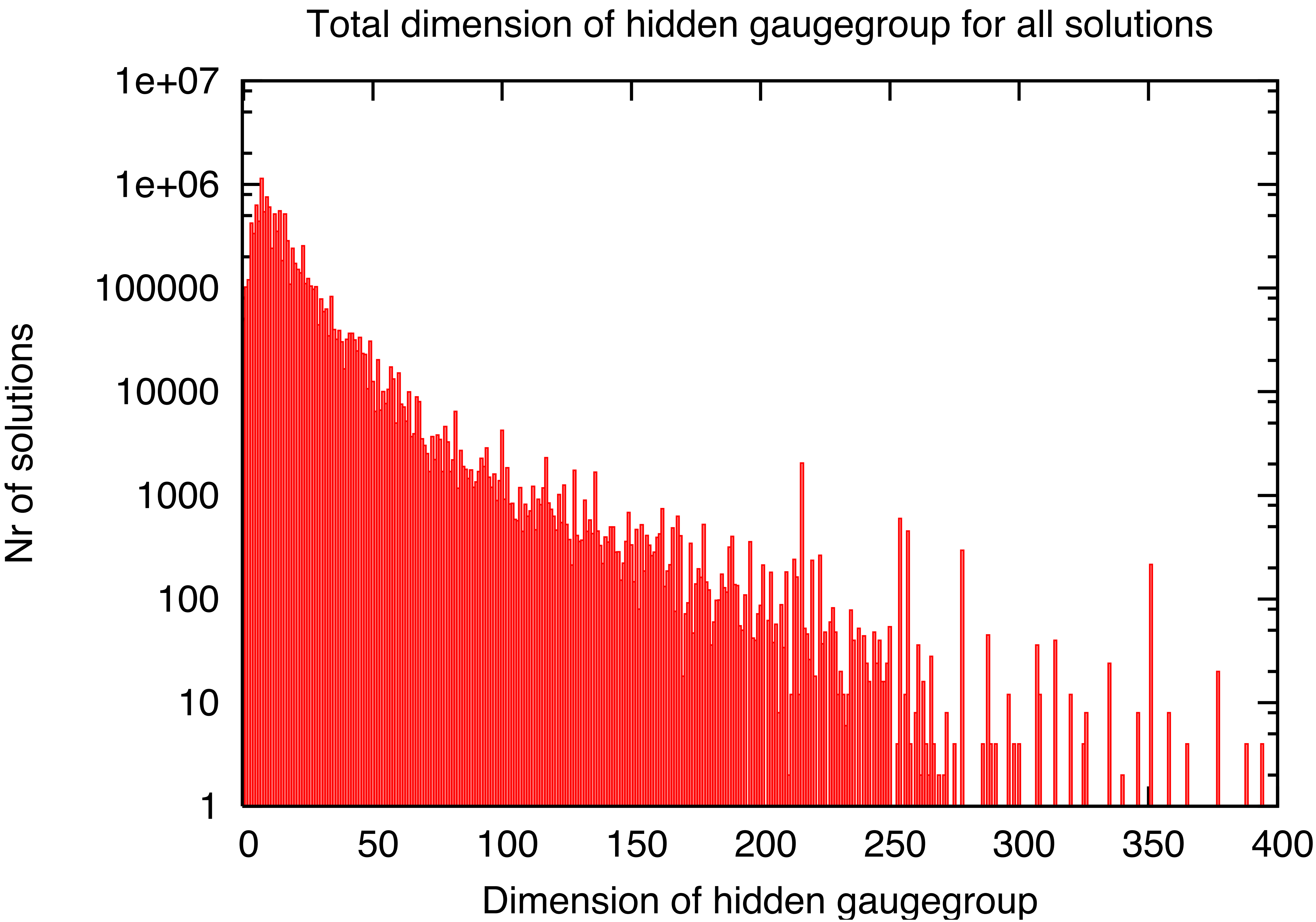
The String Theory Landscape

The hope is then that we can establish that the Standard Model is the simplest one with a complicated spectrum.

Then one may also hope that landscape statistics prefers simpler QFT’s over more complicated ones.

Here “simpler” means smaller gauge groups, smaller representations, fewer participating building blocks (e.g. membranes).

In string theory all these quantities are indeed fundamental limited, and hence their distribution will approach zero for large values.



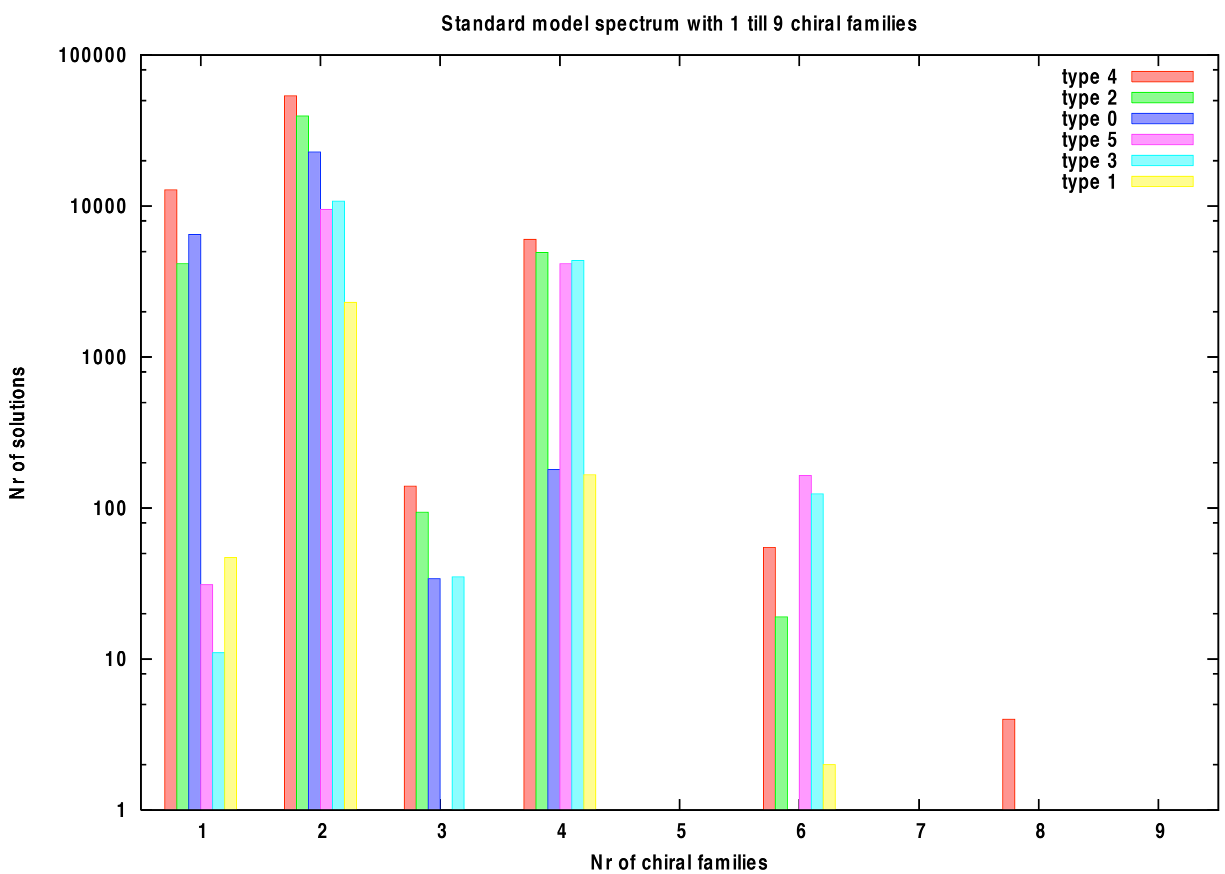
Dijkstra, Huiszoon, Schellekens (2004)

Type-II RCFT orientifolds

The String Theory Landscape

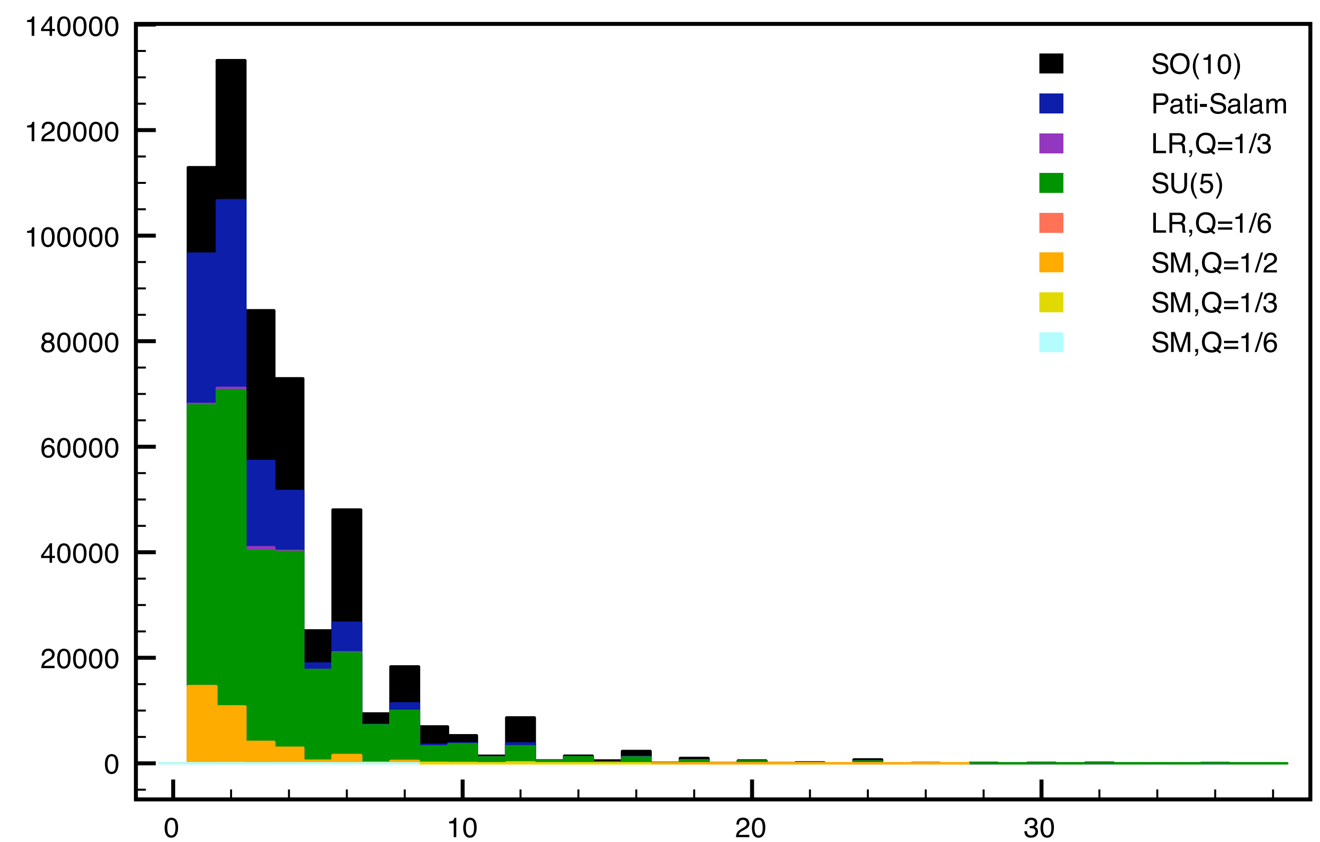
Unfortunately, the fact that we observe three families rather than one is counter evidence…

Type-II RCFT orientifolds



Dijkstra, Huiszoon, Schellekens (2004)

Heterotic Strings

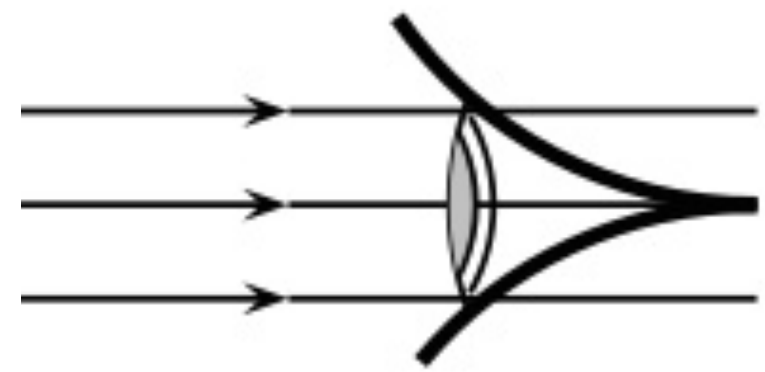


Gato-Rivera, Schellekens (2010)

Towards a derivation of the Standard Model

Main anthropic assumption:

To have observers we will need electromagnetism and a handful of particles with various charges.



We are not asking for a particular quantization, and we are *not* requiring particles of charge 6 (Carbon) to exist, but too simple sets will not do (e.g. charges $-1,1,2$: just Hydrogen and Helium)

So perhaps one could just “emulate” atomic physics with some fundamental particles with charges $-1,1,2,\ldots,N$ for sufficiently large N: fundamental “electrons” and “nuclei”.

Towards a derivation of the Standard Model

Pure QED with a set of charged particles has some problems:
No fusion-fueled stars, no stellar nucleosynthesis, baryogenesis difficult, ….

But we focus on another problem, namely that there has to be a hierarchy between the Planck scale and the masses of the building blocks of life.

Maximal number of building blocks with mass m_p of an object that does not collapse into a black hole

$$\left(\frac{m_{\text{Planck}}}{m_p}\right)^3$$

Brain with 10^{27} building blocks requires a hierarchy of 10^{-9}

Towards a derivation of the Standard Model

So to get a substantial number of light atoms, we have to solve a hierarchy problem for each of the constituents.

In the Standard Model this is solved by getting the particle masses from a single Higgs.

There may be landscape distribution arguments to justify this.

Is having N light fermions* statistically more costly than having a single light boson? (The N fermions can be either elementary nuclei or the two light quarks and the electron; then $N=3$)

(*) Our nuclei can be bosons and fermions, but that is not essential

The Hierarchy Problem

Renormalization of scalar masses

$$\mu^2_{\text{phys}} = \mu^2_{\text{bare}} + \sum_i a_i \Lambda^2$$

Computable statistical cost of about 10^{-34} for the observed hierarchy. This is the “hierarchy problem”.

Renormalization of fermion masses

$$\lambda_{\text{phys}} = \lambda_{\text{bare}} \left(\sum_i b_i \log(\Lambda/Q) \right)$$

Statistical cost determined by landscape distribution of λ_{bare}

The Hierarchy Problem

It is certainly possible that one fundamental scalar wins against N fermions for moderate N (even for $N \geq 3$).

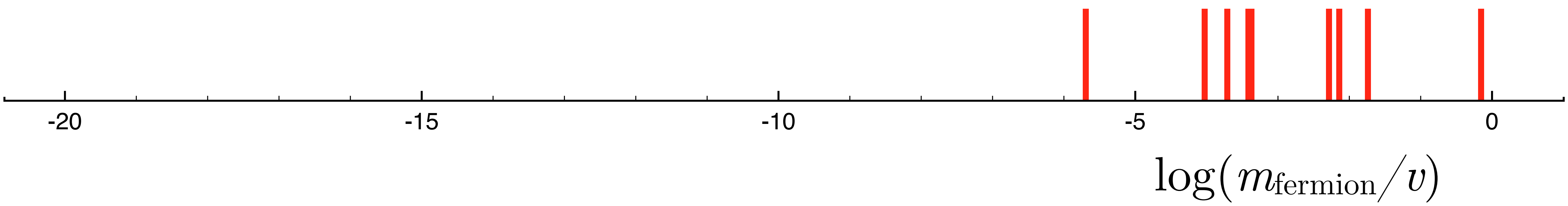
This depends on the landscape distribution of Yukawa couplings and Dirac masses of vector-like particle.

There is circumstantial experimental evidence that these distributions do not favor small values

cf. Harnik, Kribs, Perez, "A universe without weak interactions"

The Hierarchy Problem

- The charged quark and lepton Yukawa coupling distributions may be flat on a log scale*, but not over a large range.



- String theory has a large number of massless vector-like particles, but none of them have been seen, suggesting that they acquire masses, with a distribution that suppresses small masses.

(*) Donoghue, 1997

The Hierarchy Problem

One would also have to show that one fundamental scalar wins against dynamical Higgs mechanism or low energy supersymmetry.

Not enough is known theoretically to decide this, so we take experiment as our guiding principle.

Currently it seems we have a single Higgs + nothing.

This suggests that in a landscape the Higgs is not the *origin* but the *solution* of the Hierarchy problem: it could be the optimal way to create the anthropically required large hierarchy.

This would immediately imply that there is only a single Higgs.

No Higgs?

Statistically, no Higgs is better than one.
If there is a credible alternative to the SM with only dynamical symmetry breaking, that would be a serious competitor.

But generically these theories will have a number of problems.

Consider the SM without a Higgs. It is well-known that in that case the QCD chiral condensate will act like a composite Higgs and give mass to the quarks. The photon survives as a massless particle.

But the quark masses are not tuneable, and the leptons do not acquire a mass.

Massless charged leptons turn the entire universe into an opaque particle-antiparticle plasma.
(C. Quigg, R. Shrock, Phys.Rev. D79 (2009) 096002)

Lessons:

1. Dynamical Symmetry Breaking can play the role of the Higgs mechanism
2. Dynamical Symmetry Breaking should not make the photon massive
3. There should not be any massless charged leptons

String Theory Input

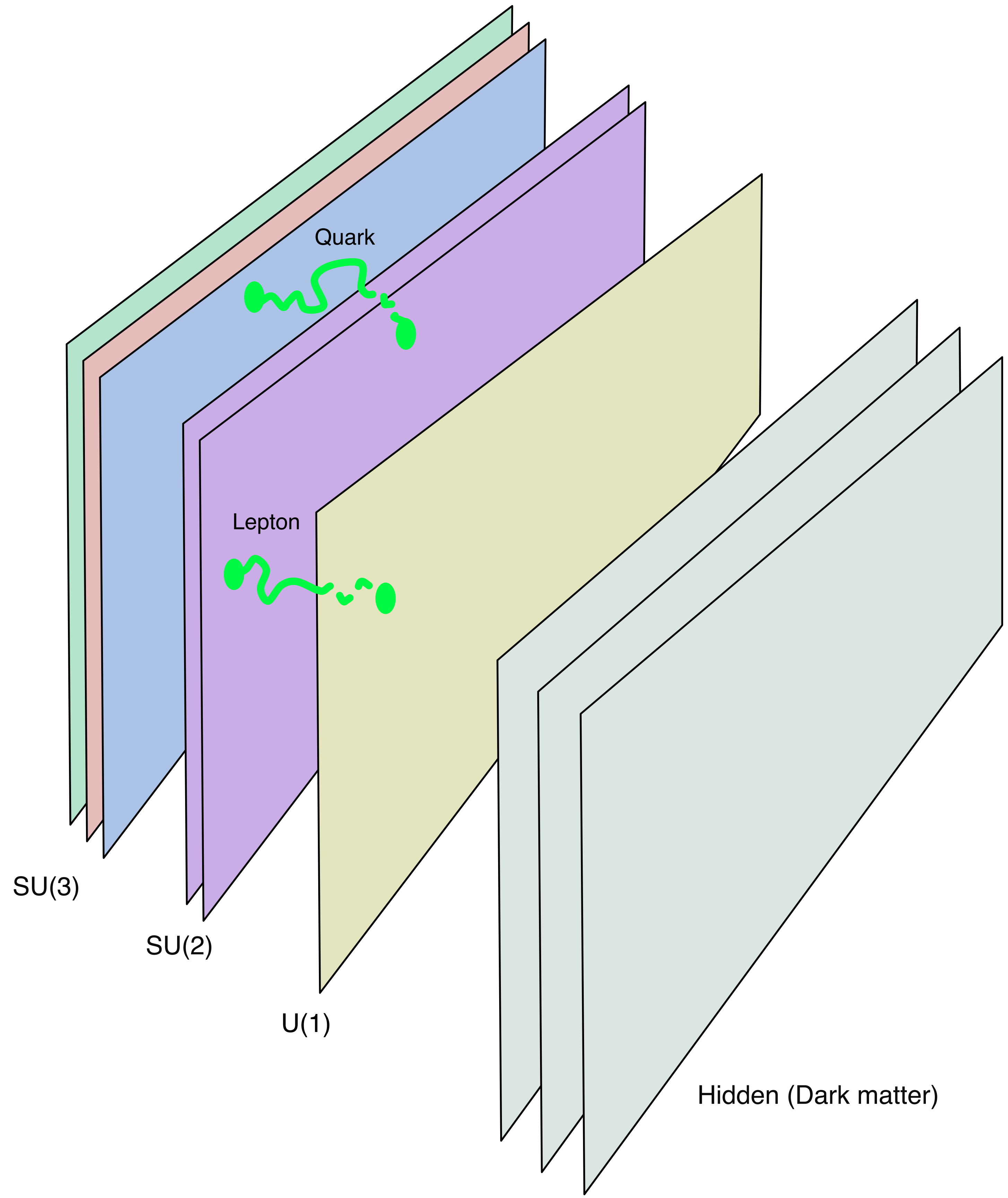
We would like to enumerate all QFT’s with a gauge group and chiral matter. All non-chiral matter is assumed to be heavy, with the exception of at most one scalar field, the Higgs. We demand that after the Higgs gets a vev, and that when all possible dynamical symmetry breakings have been taken into account, at least one massless photon survives, and all charged leptons* are massive.

This condition is very restrictive, but still has an infinite number of solutions in QFT.

So at this point we invoke string theory. Its main rôle is to restrict the representations. It also provides a more fundamental rationale for anomaly cancellation.

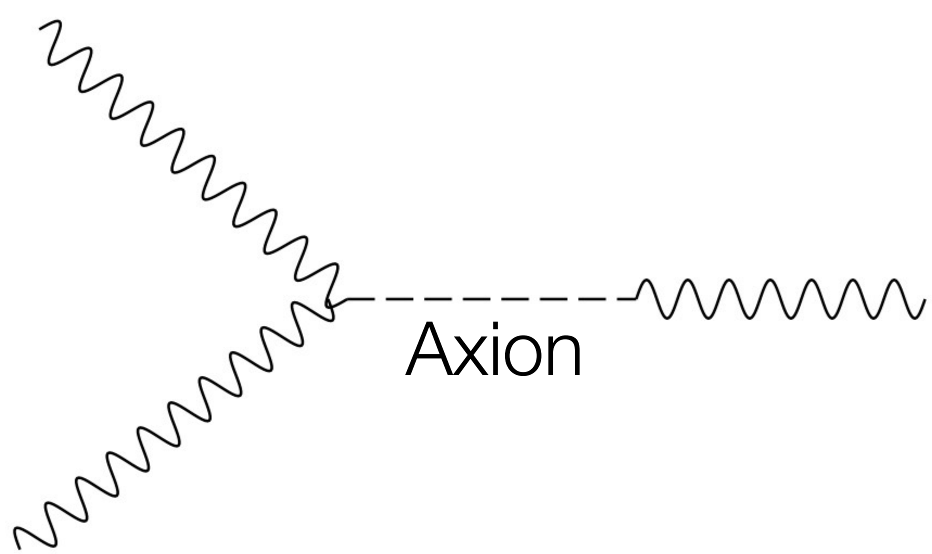
*lepton: a fermion not coupling to any non-abelian vector boson

Intersecting Brane Models



Intersection brane models

- Intersections of branes in extra dimensions determine the massless spectrum.
- Brane multiplicities are subject to a constraint: tadpole cancellation (automatically implies absence of triangle anomalies in QFT).
- Massless photons may mix with axions and acquire a mass.



(Green-Schwarz mechanism)

Intersecting Brane Models

We will assume that all matter and the Higgs bosons are massless particles in intersecting brane models. Then the low-energy gauge groups is a product of $U(N)$, $O(N)$ and $Sp(N)$ factors.

The low energy gauge group is assumed to come from S stacks of branes.
There can be additional branes that do not give rise to massless gauge bosons: $O(1)$ or $U(1)$ with a massive vector boson due to axion mixing.

All matter (fermions as well a the Higgs) are bi-fundamentals, symmetric or anti-symmetric tensors, adjoints or vectors (open strings with one end on a neutral brane)

We start with $S = 1$, and increase S until we find a solution.

Intersecting Brane Models: $S=1$

Chan-Paton group can be $U(N)$, $O(N)$ or $Sp(N)$, but only $U(N)$ can be chiral.

Matter can be symmetric or anti-symmetric tensors or vectors.
Chiral multiplicities $S,\ A,\ K$; charges $2q,\ 2q,\ q$.

Anomaly cancellation:

$$\begin{aligned}
KNq^3 + \tfrac{1}{2}N(N+1)S(2q)^3 + \tfrac{1}{2}N(N-1)A(2q)^3 &= 0 \\
KNq + \tfrac{1}{2}N(N+1)S(2q) + \tfrac{1}{2}N(N-1)A(2q) &= 0 \\
Kq + (N+2)S(2q) + (N-2)A(2q) &= 0
\end{aligned}$$

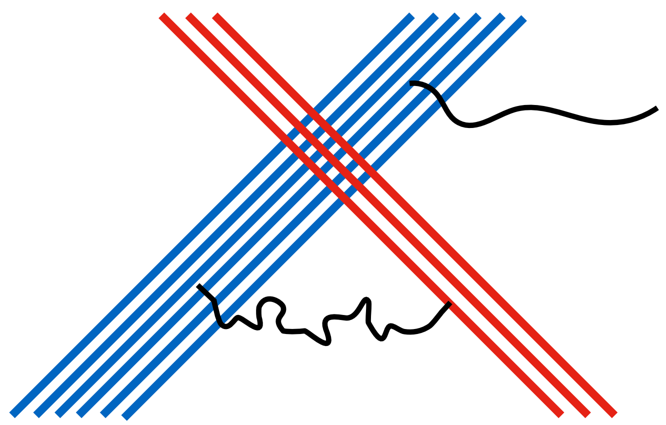
Solutions: $K=S=A=0$ or $q=0$. In the former case, there is no chiral spectrum, in the latter case no electromagnetism.

Higgs symmetry breaking could still produce a $U(1)$, but the choice of Higgses is limited to vectors and rank-2 (anti)-symmetric tensors or adjoints.

Adjoints never turn a chiral spectrum into a non-chiral one.

The others produces a $U(1)$ only for a symmetric tensor breaking $SU(2)$ to $SO(2)$. But $SU(2)$ is not chiral, so by assumption all matter then has Planck scale masses.

Two stack models



$$SU(M) \times SU(N) \times U(1)$$

(We have only considered unitary branes so far)

$$Y = q_a Q_a + q_b Q_b$$

$q_a, \ q_b$ determined by axion couplings

Q	$(M, N, q_a + q_b)$
U	$(A, 1, 2q_a)$
D	$(\overline{M}, 1, -q_a)$
S	$(S, 1, 2q_a)$
X	$(M, \overline{N}, q_a - q_b)$
L	$(1, \overline{N}, -q_b)$
T	$(1, S, 2q_b)$
E	$(1, A, 2q_b)$

Anomalies

$SU(M) \times SU(N) \times U(1)$

S W Y

There are six kinds of anomalies:

$$\left.\begin{array}{l}\texttt{SSS}\\ \texttt{WWW}\end{array}\right\}$$ From tadpole cancellation: also for $M,\ N < 3$

YYY

SSY

WWY

GGY Mixed gauge-gravity

At most one linear combination of the $U(1)$'s is anomaly-free

Anomalies

$$\begin{aligned}
(S+U)\tilde{q}_a &= C_1 \\
(T+E)\tilde{q}_b &= -C_2 \\
(D+8U)\tilde{q}_a &= (4+M)C_1 + NC_2 \\
L\tilde{q}_b + D\tilde{q}_a &= 0 \\
2E\tilde{q}_b + 2U\tilde{q}_a &= C_1 - C_2
\end{aligned}$$

$$\tilde{q}_a \equiv M q_a,\ \tilde{q}_b \equiv N q_b$$

$$C_1 = -(Q-X)\tilde{q}_b$$

$$C_2 = (Q+X)\tilde{q}_a$$

Only five independent ones. In most cases of interest,the stringy $SU(2)^3$ anomaly is not an independent constraint.

Cubic charge dependence can be linearized.

($q_a = 0$ and/or $q_b = 0$ must be treated separately)

Abelian theories

Single $U(1)$: Higgs must break it, no electromagnetism left

$U(1) \times U(1)$: No solution to anomaly cancellation for two stacks

So in two-stack models we need at least one non-abelian factor in the high-energy theory.

Strong Interactions

It is useful to have a non-abelian factor in the low-energy theory as well, since the elementary particle charge spectrum is otherwise too poor. We need some additional interaction to bind these particles into bound states with larger charges (hadrons and nuclei in our universe).

For this to work there has to be an approximately conserved baryon number. This means that we need an $SU(M)$ factor with $M \geq 3$, and that this $SU(M)$ factor does not become part of a larger group at the "weak" scale.

Note that $SU(2)$ does not have baryon number, and the weak scale is near the constituent mass scale. We cannot allow baryon number to be broken at that scale.

But let's just call this an additional assumption.

Higgs Choice

This implies that at least one non-abelian factor is not broken by the Higgs. We take this factor to be $U(M)$.

Therefore we do not consider bi-fundamental Higgses breaking both $U(M)$ and $U(N)$. We assume that $U(N)$ is the broken gauge factor. Then the only Higgs choices are **L**,**T** and **E**.

We will assume that $U(M)$ it is strongly coupled in the IR-regime and stronger than $U(N)$.

$SU(M) \times U(1)$ $(i.e.\ N{=}1)$

Higgs can only break $U(1)$, but then there is no electromagnetism.

Hence there will be a second non-abelian factor, broken by the Higgs.

$M = 3, \ N = 2$

Higgs = L

Decompose L, E, T: chiral charged leptons avoided only if

$$L = E, \ T = 0$$

Substitute in anomaly equation:

$$S\tilde{q}_a = \left(\frac{5 - N - M}{2M}\right) C_1$$

For $M = 3, \ N = 2$: $S = 0$

Therefore we get standard QCD without symmetric tensors.

$M = 3,\ N = 2$

Quark sector

$$Q(3, q_a) + Q(3, q_a + 2q_b) + X(3, q_a) + X(3, q_a - 2q_b) - U(3, -2q_a) - D(3, q_a)$$

$$Q + X - D = 0$$

$Q = U$ **if and only if** $q_a + 2q_b = -2q_a$

or

$X = U$ **if and only if** $q_a - 2q_b = -2q_a$

In both cases we get an $SU(5)$ type charge relation, and hence standard charge quantization

$$M = 3,\ N = 2$$

Quark sector

$$Q(3, q_a) + Q(3, q_a + 2q_b) + X(3, q_a) + X(3, q_a - 2q_b) - U(3, -2q_a) - D(3, q_a)$$

$$Q + X - D = 0$$

$Q = U$ **if and only if** $q_a + 2q_b = -2q_a$

or

$X = U$ **if and only if** $q_a - 2q_b = -2q_a$

In both cases we get an $SU(5)$ type charge relation, and hence standard charge quantization

$M = 3,\ N = 2$

Quark sector

$$Q(3, q_a) + Q(3, q_a + 2q_b) + X(3, q_a) + X(3, q_a - 2q_b) - U(3, -2q_a) - D(3, q_a)$$

$$Q + X - D = 0$$

$Q = U$ **if and only if** $q_a + 2q_b = -2q_a$

or

$X = U$ **if and only if** $q_a - 2q_b = -2q_a$

In both cases we get an $SU(5)$ type charge relation, and hence standard charge quantization

$M = 3,\ N = 2$

Quark sector

$$Q(3, q_a) + Q(3, q_a + 2q_b) + X(3, q_a) + X(3, q_a - 2q_b) - U(3, -2q_a) - D(3, q_a)$$

$$Q + X - D = 0$$

$Q = U$ **if and only if** $q_a + 2q_b = -2q_a$

or

$X = U$ **if and only if** $q_a - 2q_b = -2q_a$

In both cases we get an $SU(5)$ type charge relation, and hence standard charge quantization

$M = 3,\ N = 2$

Hence either $Q = 0$ ***or*** $X = 0$; the choice is irrelevant.

Take $X = 0$.

Then $D = Q = U,\ T = 0,\ L = E$

Remaining anomaly conditions: $L = Q$

Hence the only solution is a standard model family, occurring Q times.

The branes **a** and **b** are in principle unrelated, and can generally not be combined to a $U(5)$ stack

$M = 3,\ N = 2$

Higgs = T

The symmetric tensor can break $SU(2)\times U(1)$ in two ways, either to $U(1)$, in the same way as **L**, or to $SO(2)$.

Breaking to $U(1)$ (same subgroup as L)

No allowed Higgs couplings to give mass to the charged components of L, E and T, so we must require $E = L = T = 0$. Then there is no solution.

Breaking to $SO(2)$

Then $SO(2)$ must be electromagnetism. Y-charges forbid cubic T couplings, so $T = 0$ to avoid massless charged leptons. Quark charge pairing (to avoid chiral QED, broken by QCD) requires $Q = -X$. **If we also require $S = 0$, everything vanishes.**

Note: stronger dynamical assumption: $S = 0$

$M > 3$ and/or $N > 2$: lepton pairing

Lepton charge pairing: $- L + (N-1)E + (N+1)T = 0$

Combined with the five anomaly constraints this gives the following solution

$$\begin{aligned} U\tilde{q}_a &= \tfrac{3+M}{6} C_1 \\ S\tilde{q}_a &= \tfrac{3-M}{6} C_1 \\ D\tilde{q}_a &= N C_2 - \tfrac{M}{3} C_1 \\ L\tilde{q}_b &= -N C_2 + \tfrac{M}{3} C_1 \\ E\tilde{q}_b &= -\tfrac{1}{2} C_2 + \tfrac{M}{6} C_1 \\ T\tilde{q}_b &= -\tfrac{1}{2} C_2 - \tfrac{M}{6} C_1 \end{aligned}$$

$$C_1 = -(Q - X)\tilde{q}_b$$

$$C_2 = (Q + X)\tilde{q}_a$$

For $M = 3,\ S = 0$ automatically!

$M > 3$ and/or $N > 2$: quark pairing

$Q \neq -X$: Left-handed and righthanded quark representations have different dimensions. Then no subgroup of $SU(N)$ is non-chiral.
Hence dynamical symmetry breaking breaks $SU(N)$ completely.

But $SU(N) \times U(1)$ *does* contain a current that is non-chiral.
Note that now U and D participate, which are neutral under $SU(N)$, but carry a $U(1)$ charge. The surviving $U(1)$ symmetry must be a linear combination

$$Q_{\rm em} = \Lambda + Y,$$

where $\Lambda \in SU(N)$. There can be at most one such $U(1)$ factor.
This is the only symmetry that can survive DSB+Higgs breaking.

($Q = -X$: see paper)

$M > 3$ and/or $N > 2$

$\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_N)$ (surviving Higgs + any DSB)

Charges of Q: $q_a + q_b + \lambda_i$

Charges of X: $q_a - q_b - \lambda_i$

Charges of D: $-q_a$

Charges of U, S: $2q_a$

Lepton Charges: $q_b + \lambda_i;\ 2q_b + \lambda_i + \lambda_j$

Define $q_b + \lambda_i = \alpha q_a$

Quark charge pairing is possible only for $\alpha = 0,\ \pm 3$

All solutions satisfy Standard Model charge quantization!

$M > 3$ and/or $N > 2$

We can obtain a solution for any Q and X

$$\Lambda : n \times \{-q_b\} + n_+ \times \{-q_b + 3q_a\} + n_- \times \{-q_b - 3q_a\}$$

$$n_+ = \frac{Q}{R}$$

$$n_- = -\frac{X}{R}$$

$$N = n + n_+ + n_-$$

$$R = -(Q+X)\frac{\tilde{q}_a}{\tilde{q}_b} \in \mathbb{Z}$$

The trace of Λ must vanish

$$\mathrm{Tr}\ \Lambda = \tilde{q}_b \left(\frac{3}{M} - 1\right)$$

Hence $M = 3$!

$M > 3$ and/or $N > 2$

The spectrum can be computed

$$D = n(Q+X)$$

$$U = (N-n)(Q+X)$$

$$L = nR$$

$$E = \frac{1}{2}\left(N-n+1\right)R$$

$$T = -\frac{1}{2}\left(N-n-1\right)R$$

Absence of massless charged leptons only for $N=2$!

Conclusions

- The Standard Model is the only anthropic solution within the set of two-stack models.
- Family structure, charge quantization, the weak interactions and the Higgs choice are all derived.
- Standard Model charge quantization works the same way, for any value of N, even if $N+3 \neq 5$.
- The GUT extension offers no advantages, only problems (doublet-triplet splitting)
- Only if all couplings converge (requires susy), GUTs offer an advantage.
- The general class is like a GUT with its intestines removed, keeping only the good parts: **GUTs without guts**.

Couplings

The $U(3)\times U(2)$ structure of this class of models implies one relation among the SM couplings, instead of the two of $SU(5)$

$$\frac{1}{\alpha_Y} = \frac{2}{3}\frac{1}{\alpha_s} + \frac{1}{\alpha_w}$$

see also:
Ibañez, Munos, Rigolin, 1998;
Blumenhagen, Kors, Lüst, Stieberger, 2007

Extrapolation this to higher energies we see that this is satisfied at 5.7×10^{13} GeV (1.4×10^{16} GeV for susy).

Proton decay by SU(5) vector bosons would be far too large, but generically we do not have such bosons in the spectrum. There is no SU(5) in any limit.

But what happens at that scale?

If it is the string scale, one would still expect quantum-gravity related proton decay, which would be much too large.

But there are many ways out.

Complete list of solutions

Nr.	M	N	q_a	q_b	Higgs	Q	U	D	S	X	L	E	T
1	1	2	2	-3	**L**	3	6	3	3	0	1	1	0
2	1	2	4	-1	**L**	2	1	1	0	0	2	3	1
3a	1	2	2	-1	**L**	3	4	1	3	-4	1	0	-1
3b	1	2	2	-1	**L**	2	2	1	1	-1	1	1	0
3c	1	2	2	-1	**L**	4	5	0	3	-4	0	1	-1
4	1	3	3	-2	**L**	2	3	2	1	0	1	1	0
5	1	3	3	-1	**E**	0	0	-2	-1	1	-2	1	0
6	1	4	4	-1	**L**	1	1	1	0	0	1	1	0
7	M	2	1	ρ	**T**	1	$-\rho$	$2M\rho$	$-\rho$	-1	$2M$	0	0
8	2	3	3	-2	**L**	1	1	1	0	0	1	1	0
9	3	2	2	-3	**L**	1	1	1	0	0	1	1	0

All chiral spectra without massless charged free leptons that can be obtained for all M and N with $q_a \neq 0$ and $q_b \neq 0$. Here $M = 1{,}2$ and ρ is a free integer parameter.

Complete list of solutions

Nr.	M	N	q_a	q_b	Higgs	Q	U	D	S	X	L	E	T
6	1	4	4	-1	**L**	1	1	1	0	0	1	1	0

This realizes the $SU(4)\times U(1)$ subgroup of SU(5).

The Higgs boson breaks this to $SU(3)\times U(1)$, QCD × QED.

But this implies $SU(5)$-type proton decay at the weak scale.

A family constitutes a single, complete $SU(4)$ Higgs multiplet.

Complete list of solutions

Nr.	M	N	q_a	q_b	Higgs	Q	U	D	S	X	L	E	T
8	2	3	3	-2	**L**	1	1	1	0	0	1	1	0

This is the same $SU(3)\times SU(2)\times U(1)$ subgroup of $SU(5)$ that gives rise to the Standard Model, but with a triplet Higgs instead of a doublet Higgs.

At low energies, there is a non-abelian $SO(4) \approx SU(2)\times SU(2)$ gauge group without conserved Baryon number.

The special case $q_a = 0$ (all M,N)

Anomaly cancellation:

$SU(M) \times SU(N) \times U(1)_Y$

$Q[(V, V, 1) + (V, \overline{V}, -1)] + \text{flavor-neutral } \mathbf{U},\ \mathbf{D},\ \mathbf{S} \text{ matter}$

For $M = 1{,}2$ this is vectorlike (hence massive)
For $M > 3$ there is no $U(1)$ in the flavor group that is non-chiral with respect to $SU(M)$, hence no electromagnetism.

Note: we treat Higgs and dynamical breaking on equal footing

The special case $q_b = 0$ (all M,N)

Anomaly cancellation:

$$SU(M) \times SU(N) \times U(1)_Y$$

$$Q[(V,V,1) + (\overline{V},V,-1)] + Y\text{-neutral } \mathbf{L},\ \mathbf{E},\ \mathbf{T} \text{ matter}$$

For $N = 1{,}2$ this is vector-like, and hence massive

For $N \geq 3$ the candidate Higgses do not break $U(1)_{\mathrm{Y}}$

Hence the Higgs just has to break $SU(N)$ to a real group, and this is indeed possible, for example Higgs = **T**, breaking $SU(N)$ to $SO(N)$

$$Q[(V,V,1) + (\overline{V},V,-1) + 2M(1,V,0)]$$

No charged leptons; Baryon number is gauged, so baryogenesis would be problematic.