

# RCFT STRING MODEL BUILDING

BERT SCHELLEKENS



Ηράκλειον

13-10-2009

# THE STANDARD MODEL IN STRING THEORY

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But we still have to establish that this is true, and that gets a little harder now.

# THE STANDARD MODEL IN STRING THEORY

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How can we do that?

- Find it! (not going to happen)
- Try to find some fairly generic prediction  
(extra  $U(1)$ 's, large extra dimensions, susy features, F-theory features....)
- Decide if what we already know looks like it came from the string theory landscape (standard model structure, hierarchies...)

# THE STANDARD MODEL IN STRING THEORY

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## String theory and the discrete standard model structure

### ● What string theory gets right:

- Small representations
- Anomaly cancellation

### ● What string theory gets wrong:

- Gauge group too large  
(Extra  $U(1)$ 's, in particular B-L, extra non-abelian factors)
- Wrong number of families
- Too many singlets (incl. moduli)
- Mirror fermions
- Fractional Charges (for color singlets)

# THE STANDARD MODEL IN STRING THEORY

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For all these failures one can find some excuses:

- Artefacts of some method.
- Artefacts of Susy.
- Non-perturbative effects.
- Massive in generic string theory.
- Invisible (“hidden sector”).
- Fractional charges may be confined.
- Anthropic arguments.
- .....

But we still know only a few small corners of the landscape



# THE STANDARD MODEL IN STRING THEORY

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The best-controlled string realizations are:

- Heterotic Strings  
(SM particles realized as closed strings)
- Orientifolds  
(SM particles realized as open strings)

# METHODS

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- Free fields (bosons, fermions, orbifolds)
  - + Easy
  - Limited scope
- Interacting (rational) CFT's
  - Hard, but usable at least for spectra.
  - Larger scope than free CFT's
- Geometric (Calabi-Yau, ...)
  - Hard
  - + Most complete

# RCFT: HETEROTIC VS ORIENTIFOLD

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During the last five years, orientifolds were scanned systematically for Standard model spectra

*Dijkstra, Huiszoon, Schellekens [200.000 out of  $10^{19}$ ]*

*Gmeiner, Blumenhagen, Honecker, Lust, T. Weigand [0 out of  $10^9$ ]*

*Anastasopoulos, Dijkstra, Kiritsis, Schellekens [1900]*

*Kiritsis, Lennek, Schellekens [0]*

*Gmeiner, Honecker [100.000 out of  $10^{23}$ ]*

No comparable results exist for heterotic strings. All we have are Hodge number scans<sup>1</sup>, and fermionic construction scans<sup>2</sup> not focused on the Standard Model

(1) *Lutken, Ross (1988)*

*Schellekens, Yankielowicz (1989)*

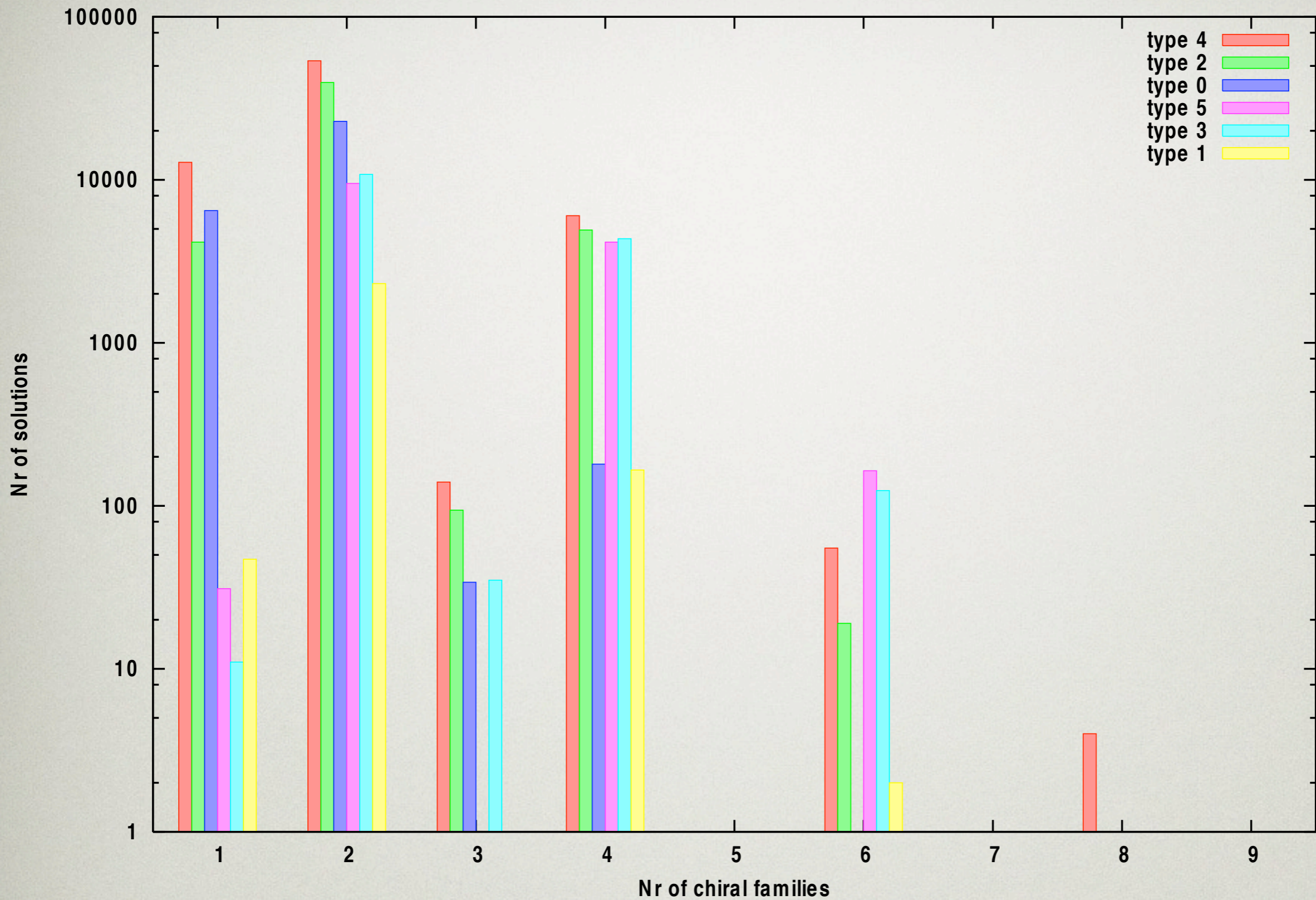
*Fuchs, Klemm, Scheich, Schmidt (1989)*

*Kreuzer, Skarke (1992)*

*Donagi, Faraggi (2004), Donagi, Wendland (2008)*

*Kiritsis, Lennek, Schellekens (2008)*

(2) *Dienes (2006)*



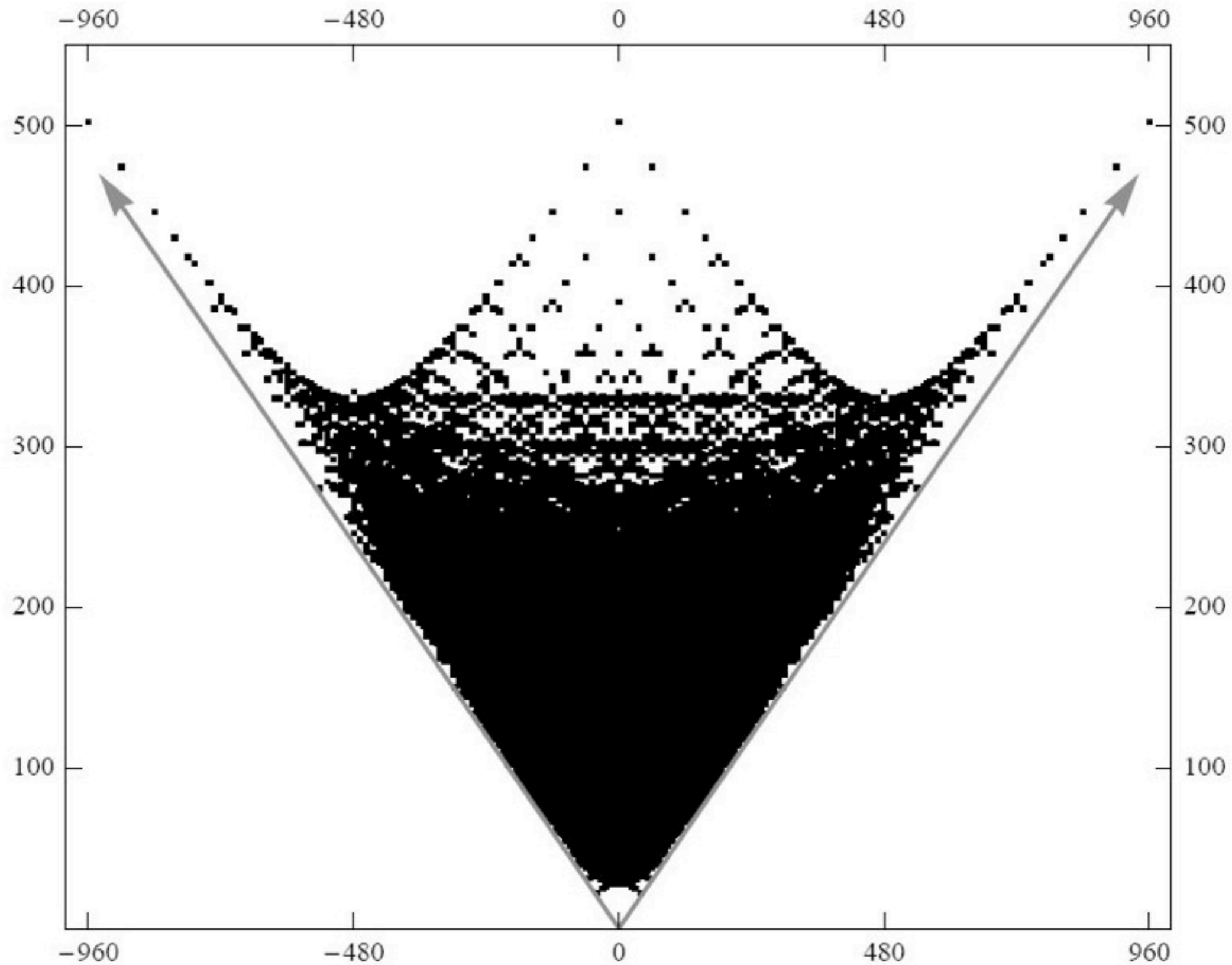


Figure 1: A plot of the Hodge numbers of the Kreuzer–Skarke list.  $\chi = 2(h^{11} - h^{21})$  is plotted horizontally and  $h^{11} + h^{21}$  is plotted vertically. The oblique axes bound the region  $h^{11} \geq 0, h^{21} \geq 0$ .

# GEPNER MODELS

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Tensor product of an NSR model in 4 space time dimensions with a number of N=2 minimal CFT's with total central charge 9.

## *Heterotic:*

Partition function  $\sum_{i,j} \chi_i(\tau) M_{ij} \chi_j(\bar{\tau})$

Map the NSR model to  $SO(10) \times E_8$  in the bosonic sector.  
M not necessarily symmetric; Standard model in  $SO(10)$

## *Orientifold:*

Partition function with symmetric matrix M (type-II)  
Mod out world-sheet orientation

Add boundary states, Standard Model from intersecting branes.

Gauge group:  $U(3) \times Sp(2) \times U(1) \times U(1)$

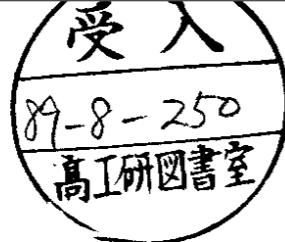
Q U D L E	7 x	(V , V , 0 , 0 )	chirality 3
	3 x	(V , 0 , V , 0 )	chirality -3
	3 x	(V , 0 , V* , 0 )	chirality -3
	9 x	(0 , V , 0 , V )	chirality 3
	5 x	(0 , 0 , V , V )	chirality -3
	3 x	(0 , 0 , V , V* )	chirality 3
H	6 x	(V , 0 , 0 , V )	
	10 x	(0 , V , V , 0 )	
	2 x	(Ad , 0 , 0 , 0 )	
	2 x	(A , 0 , 0 , 0 )	
	6 x	(S , 0 , 0 , 0 )	
	14 x	(0 , A , 0 , 0 )	
	10 x	(0 , S , 0 , 0 )	
	9 x	(0 , 0 , Ad , 0 )	
	6 x	(0 , 0 , A , 0 )	
	14 x	(0 , 0 , S , 0 )	
	3 x	(0 , 0 , 0 , Ad)	
	4 x	(0 , 0 , 0 , A )	
6 x	(0 , 0 , 0 , S )		

No hidden sector

B-L Massive (axion mixing)

Gauge group:

Exactly  $SU(3) \times SU(2) \times U(1)$



NEW MODULAR INVARIANTS FOR  $N=2$  TENSOR PRODUCTS  
AND FOUR-DIMENSIONAL STRINGS

A. N. Schellekens

and

S. Yankielowicz<sup>\* †</sup>

*CERN, 1211 Geneva 23, Switzerland*

ABSTRACT

The construction of modular invariant partition functions of tensor products of  $N = 2$  superconformal field theories is clarified and extended by means of a recently proposed method using simple currents, *i.e.* primary fields with simple fusion rules. Apart from providing a conceptually much simpler way of understanding space-time and world-sheet supersymmetry projections in modular invariant string theories, this makes a large class of modular invariant partition functions accessible for investigation. We demonstrate this by constructing thousands of (2,2), (1,2) and (0,2) string theories in four dimensions, including more than 40 new three generation models.

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<sup>\*</sup> Work supported in part by the US-Israel Binational Science Foundation, and the Israel Academy of Science.

<sup>†</sup> Permanent address: School of Physics and Astronomy, Raymond and Beverley Sackler Faculty of Exact Sciences, Tel-Aviv University, Israel.



# HETEROTIC STRINGS CONSIDERED:

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Right	Left
NSR	$SO(10) \times E_8$
N=2 minimal $k_1$	N=2 minimal $k_1$
N=2 minimal $k_2$	N=2 minimal $k_2$
...	...
N=2 minimal $k_{n-1}$	N=2 minimal $k_{n-1}$
N=2 minimal $k_n$	N=2 minimal $k_n$

# HETEROTIC STRINGS CONSIDERED:

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Right	Left
NSR	$SO(10) \times E_8$
N=2 minimal $k_1$	N=2 minimal $k_1$
N=2 minimal $k_2$	N=2 minimal $k_2$
...	...
N=2 minimal $k_{n-1}$	N=2 minimal $k_{n-1}$
N=2 minimal $k_n$	N=2 minimal $k_n$

$$\sum_i \frac{3k_i}{k_i + 2} = 9$$



# HETEROTIC STRINGS CONSIDERED:

---

Modular invariance: bosonic string map(\*)

Right	Left
NSR	$SO(10) \times E_8$
N=2 minimal $k_1$	N=2 minimal $k_1$
N=2 minimal $k_2$	N=2 minimal $k_2$
...	...
N=2 minimal $k_{n-1}$	N=2 minimal $k_{n-1}$
N=2 minimal $k_n$	N=2 minimal $k_n$

(\*) *Lerche, Lüst, Schellekens, 1986*

# HETEROTIC STRINGS CONSIDERED:

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World sheet susy: “alignment currents”




Right	Left
NSR	$SO(10) \times E_8$
N=2 minimal $k_1$	N=2 minimal $k_1$
N=2 minimal $k_2$	N=2 minimal $k_2$
...	...
N=2 minimal $k_{n-1}$	N=2 minimal $k_{n-1}$
N=2 minimal $k_n$	N=2 minimal $k_n$

# HETEROTIC STRINGS CONSIDERED:

---

Space-time susy: chiral algebra extension



Right	Left
NSR	$SO(10) \times E_8$
N=2 minimal $k_1$	N=2 minimal $k_1$
N=2 minimal $k_2$	N=2 minimal $k_2$
...	...
N=2 minimal $k_{n-1}$	N=2 minimal $k_{n-1}$
N=2 minimal $k_n$	N=2 minimal $k_n$

Start with the diagonal invariant, and modify it with simple currents **without requiring** worldsheet or space-time supersymmetry in the left (bosonic) sector, but impose them on the fermionic sector.

Simple currents: discrete symmetries of CFT's allow us to write down huge numbers of partition function matrices  $M_{ij}$ . In general, these matrices are asymmetric.

This gives (2,2), (2,1) and (2,0) heterotic strings with chiral fermions in (16)'s of  $SO(10)$  or (27)'s of  $E_6$ .

## Result

A huge “phone-book” of tables of (2,2) and (2,1) spectra.

## Number of families:

Quantized in certain units  $\Delta$  for each of the 168 combinations of Gepner models.

The following values occur for the 120, 96, 72, 60, 48, 40, 36, 32, 24, 12, 8, 6, 4 and 0.

There is one known way to get multiples of 3:  
Use (1,16,16,16) with exceptional invariants in all three factors with  $k=16$  (Gepner, unpublished).

This allowed us to get 3-family (2,2), (2,1) and (2,0) models with gauge groups  $SO(10)$  or  $E_6$  (44 distinct ones)

## 6. Outlook and conclusions

Clearly the method we have advocated in this paper greatly extends the list of four-dimensional string theories accessible to exploration. However, this is by no means all one can do. Up to now we have always kept an unbroken  $SO(10) \times E_8$  Kac-Moody algebra on the left. However, just as one can break the left-moving “space-time” and world-sheet supersymmetries, one can break this KM-algebra as well. To do so, one simply starts with characters of some conformal sub-algebra of  $SO(10) \times E_8$ . Of course one wants to get the full  $SO(10) \times E_8$  algebra on the *right*, in order to be able to map this sector to a fermionic one. But this can always be achieved by putting some projection matrices in front of the right-moving characters to add the missing  $SO(10) \times E_8$  roots.

This opens the way to constructing string theories whose gauge group is something a bit closer to the standard model than  $SO(10)$ , perhaps even  $SU(3) \times SU(2) \times U(1)^n$  (where  $n$  is almost inevitably larger than 1). There is no reason why one could not get 3 generations in such a model, and in fact there could well be many more models than those listed in table III, since the center of the conformal field theory one starts with is even larger. We hope to come back to this in the future.



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Twenty years later, we have reached the future...  
(work in progress with Beatriz Gato-Rivera)

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Twenty years later, we have reached the future...

(work in progress with Beatriz Gato-Rivera)

Meanwhile this idea was used by Blumenhagen en Wiskirchen (1996)

See also Kreuzer (2009)

# BREAKING SO(10)

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Consider  $SU(3) \times SU(2) \times U(1)_{30} \times U(1)_{20} \subset SO(10)$

We extend this to  $SO(10)$ , but only in the fermionic sector, then map it to NSR.

This should give chiral families of  $SU(3) \times SU(2) \times U(1)$ .

Indeed, it does, but there was a major disappointment:

**All these spectra contain fractionally charged particles.**

This was easily seen to be a very general result.

*(A.N. Schellekens Phys. Lett. B237, 363, 1990).*

# GUTS VERSUS STRINGS

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In heterotic strings, unification ( $SO(10)$  or  $E_6$ ) seems “natural” (bosonic string map, spin-connection embedded in  $E_8$ )

But one beautiful feature of  $SU(5)$  GUTs, an explanation for the observed charge quantization, is lost when one breaks the GUT group in CFT.

*This may be avoided:*

- Massive or non-chiral fractional charges
- Additional confinement groups
- Higher level affine Lie-algebras
- Non-GUT  $U(1)$  normalization
- Other string theories (orientifolds, F-theory ....)

But only in the first case the nice heterotic realization of GUTs would remain more or less intact.

This was too hard to analyse in 1989.

# Modular Invariant Partition Function:

$$\sum_{ij} \chi_i(\bar{\tau}) M_{il}^{\text{proj}} M_{lk}(J_1, \dots, J_n) \chi_k(\tau)$$

Worldsheet susy  
Space-time susy  
SO(10) projection

$N \times N$  matrix  
for  $n$  simple currents

For  $K$  minimal models:

$$N = 3 \times 2 \times 60 \times 20 \times \prod_i^K N_i$$

(3,3,3,3,3)

368.640.000.000

## Potentially a huge landscape:

For  $K$  currents of order  $p$  (prime)

*(B. Gato-Rivera, A.N. Schellekens, Comm. Math. Phys. 145, 85 (1992))*

$$N_{\text{MIPF}} = \prod_{l=0}^{K-1} (1 + p^l)$$

The seven  $\mathbf{Z}_5$  factors in  $SU(3) \times SU(2) \times U_{30} \times U_{20} \times (\mathbf{k}=3)^5$  contribute a factor

1.202.088.011.709.312

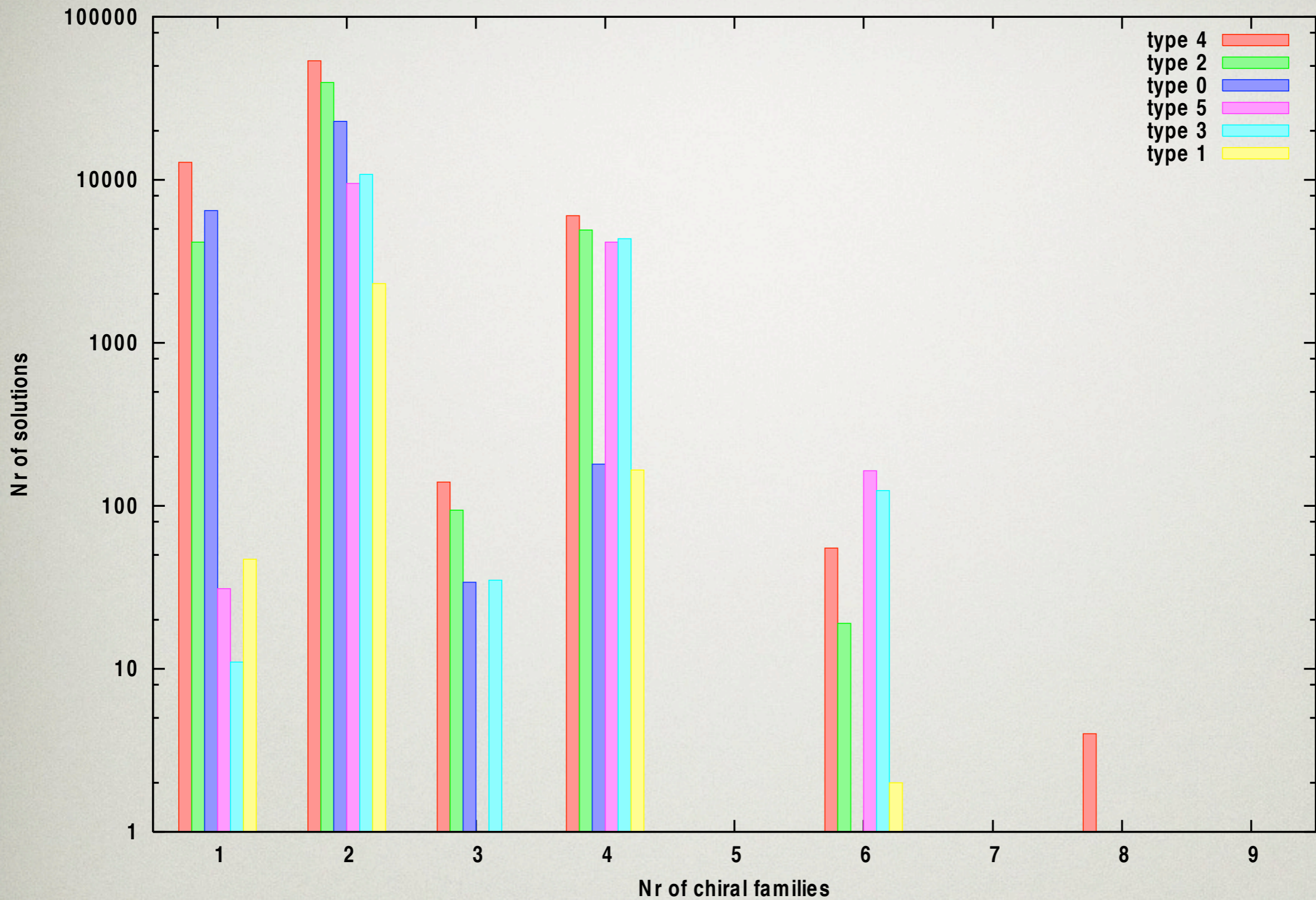
This is reduced by at most  $5! \times 2^8$  (permutations, outer automorphisms),  
and enhanced by a factor 8 for  $(\mathbf{Z}_3)^2$  and an unknown, huge factor for  $(\mathbf{Z}_2)^2 \times (\mathbf{Z}_4)^6$

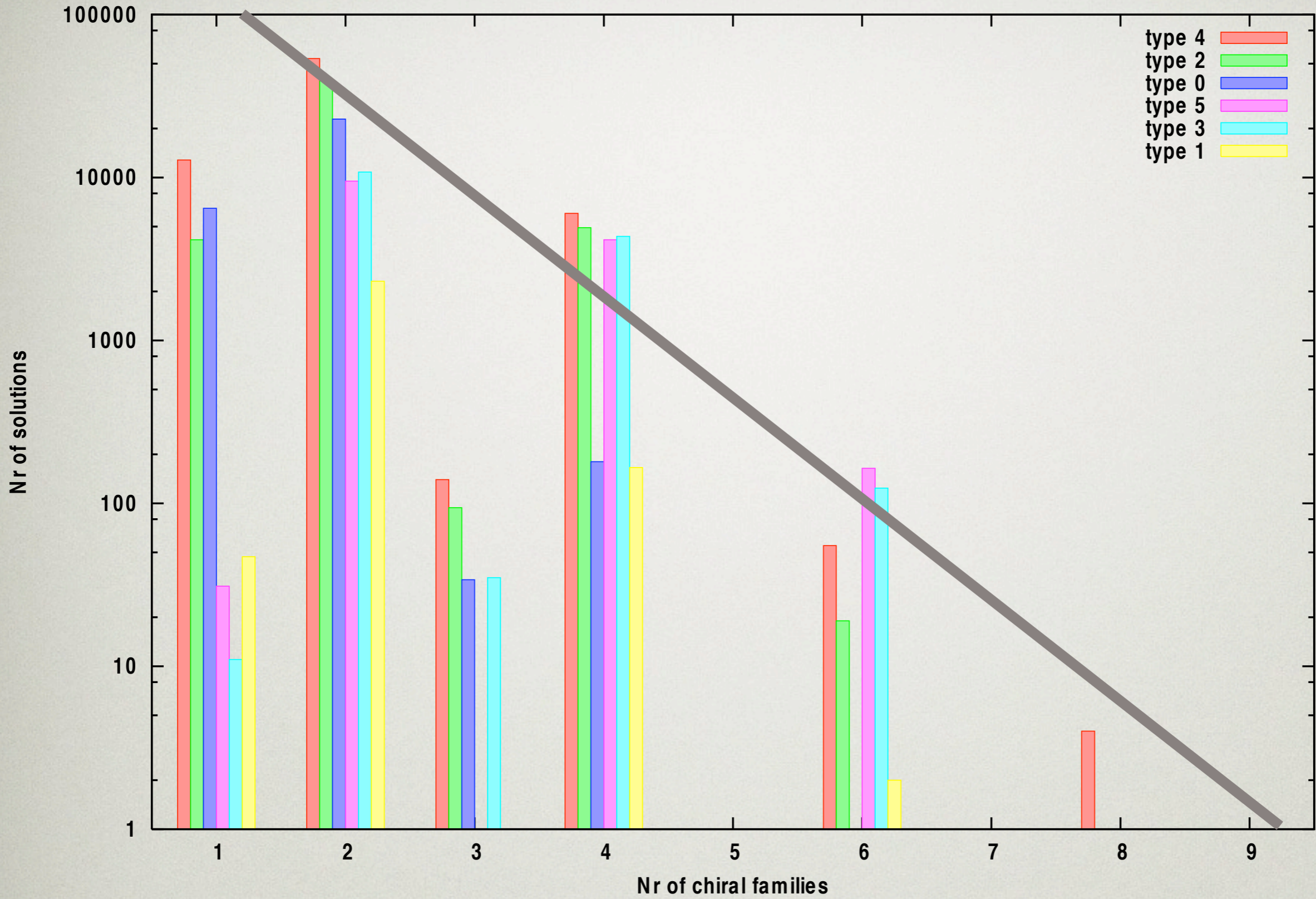
Some questions that remained unanswered in 1989:

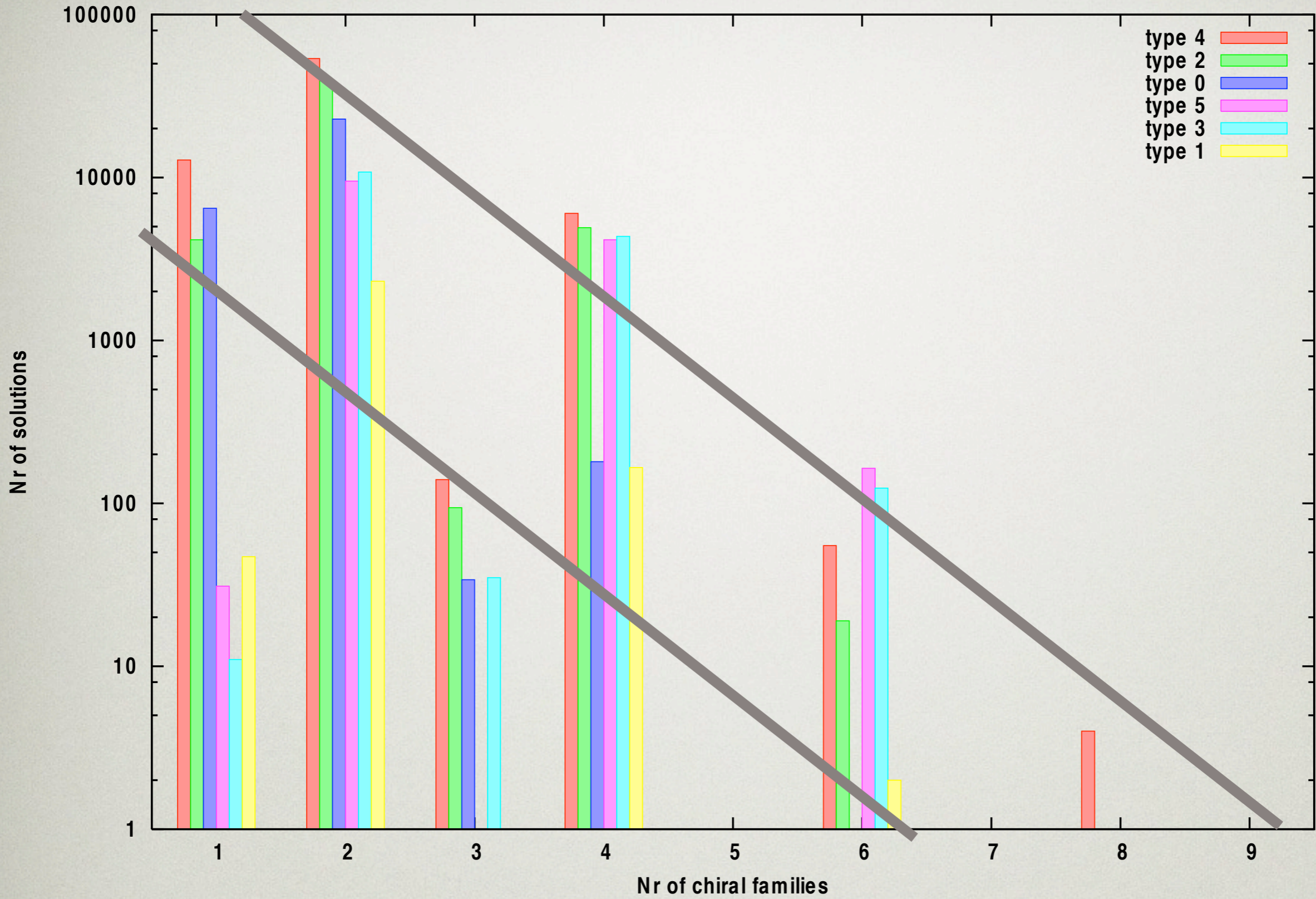
- How is  $\Delta$  affected by breaking  $SO(10)$  and world-sheet supersymmetry?
- Are the fractionally charge particles chiral?
- What do distributions of families look like?
- Can we get three families of  $SU(3) \times SU(2) \times U(1)$ ?

# THE NUMBER OF FAMILIES

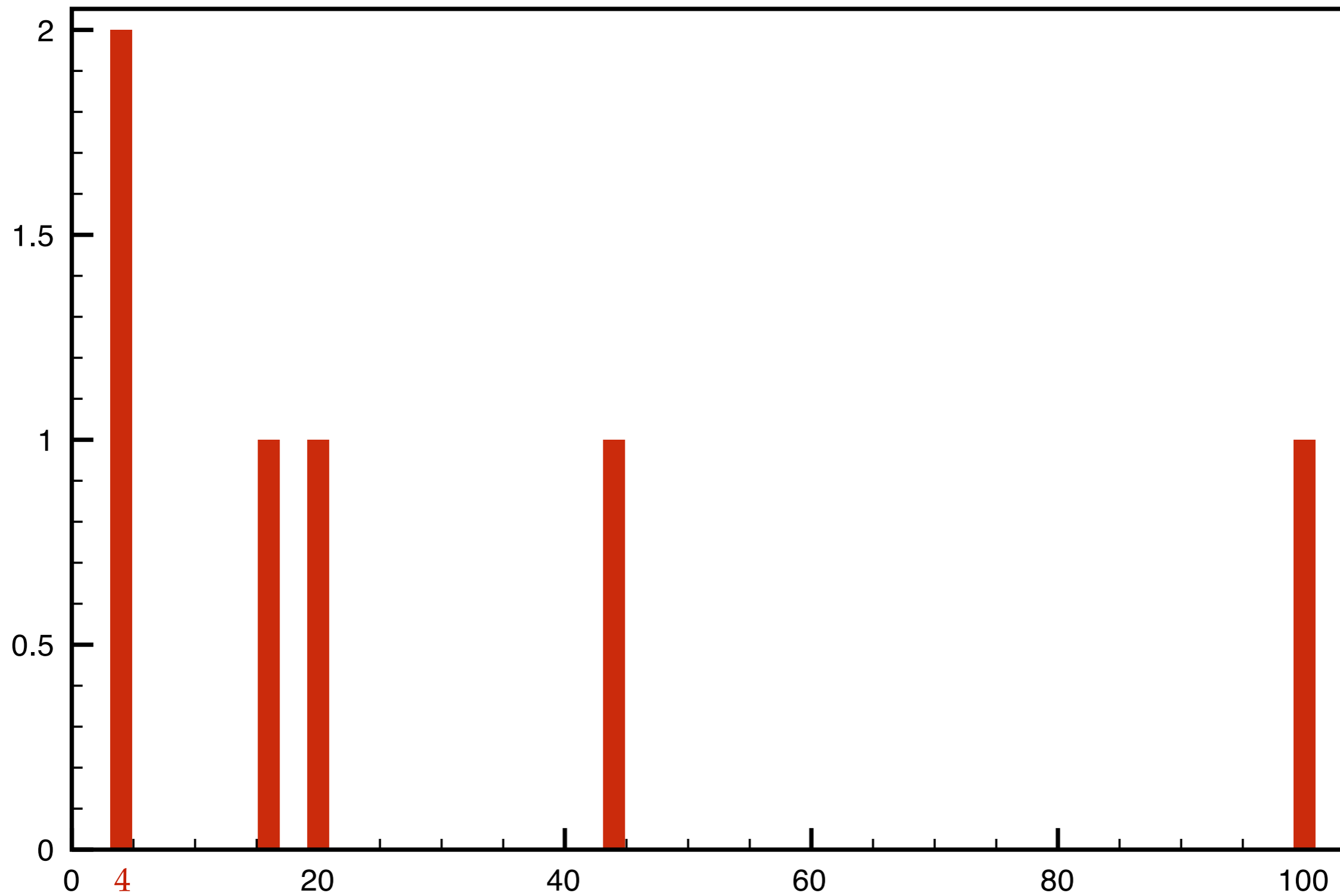




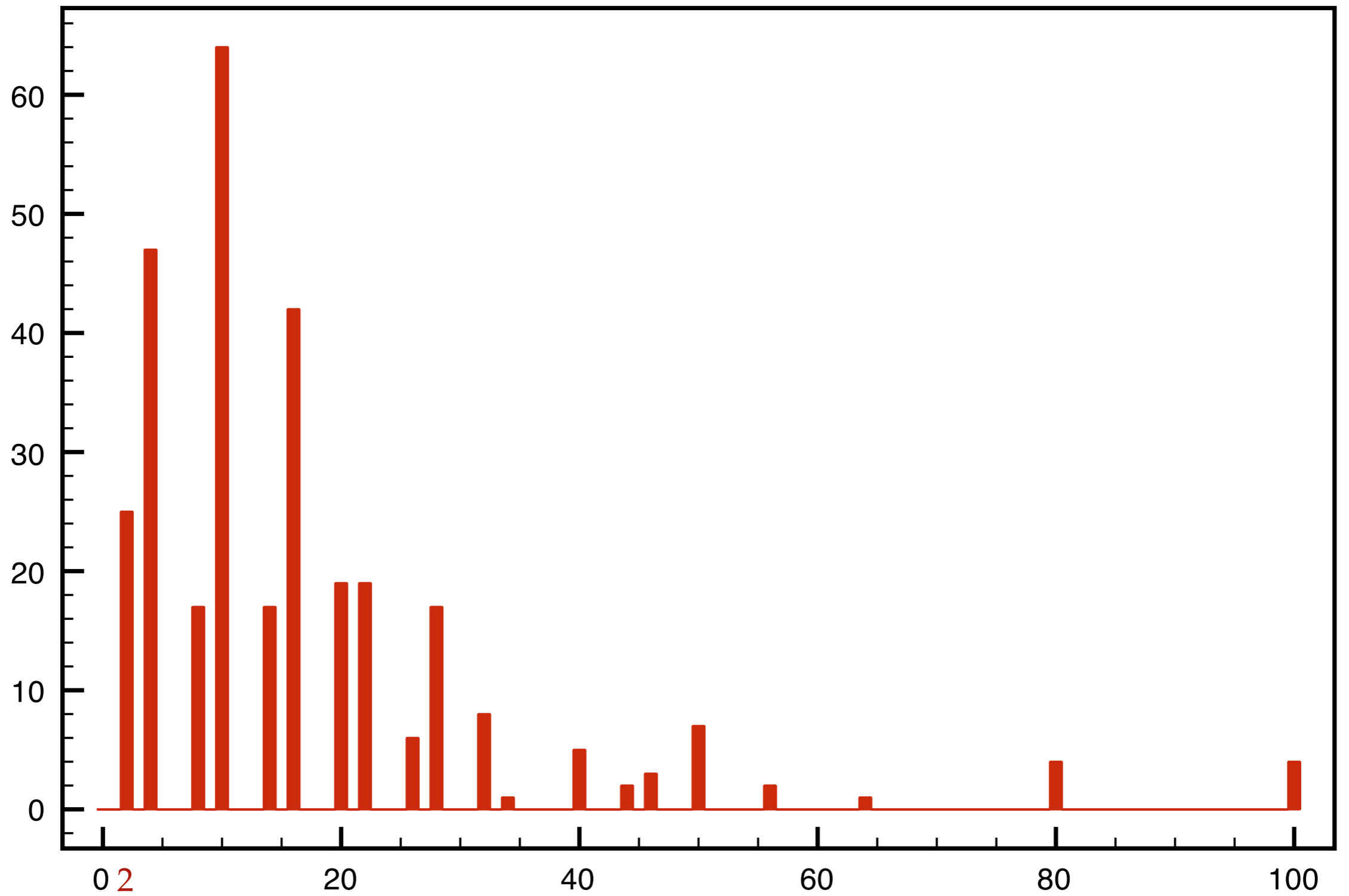




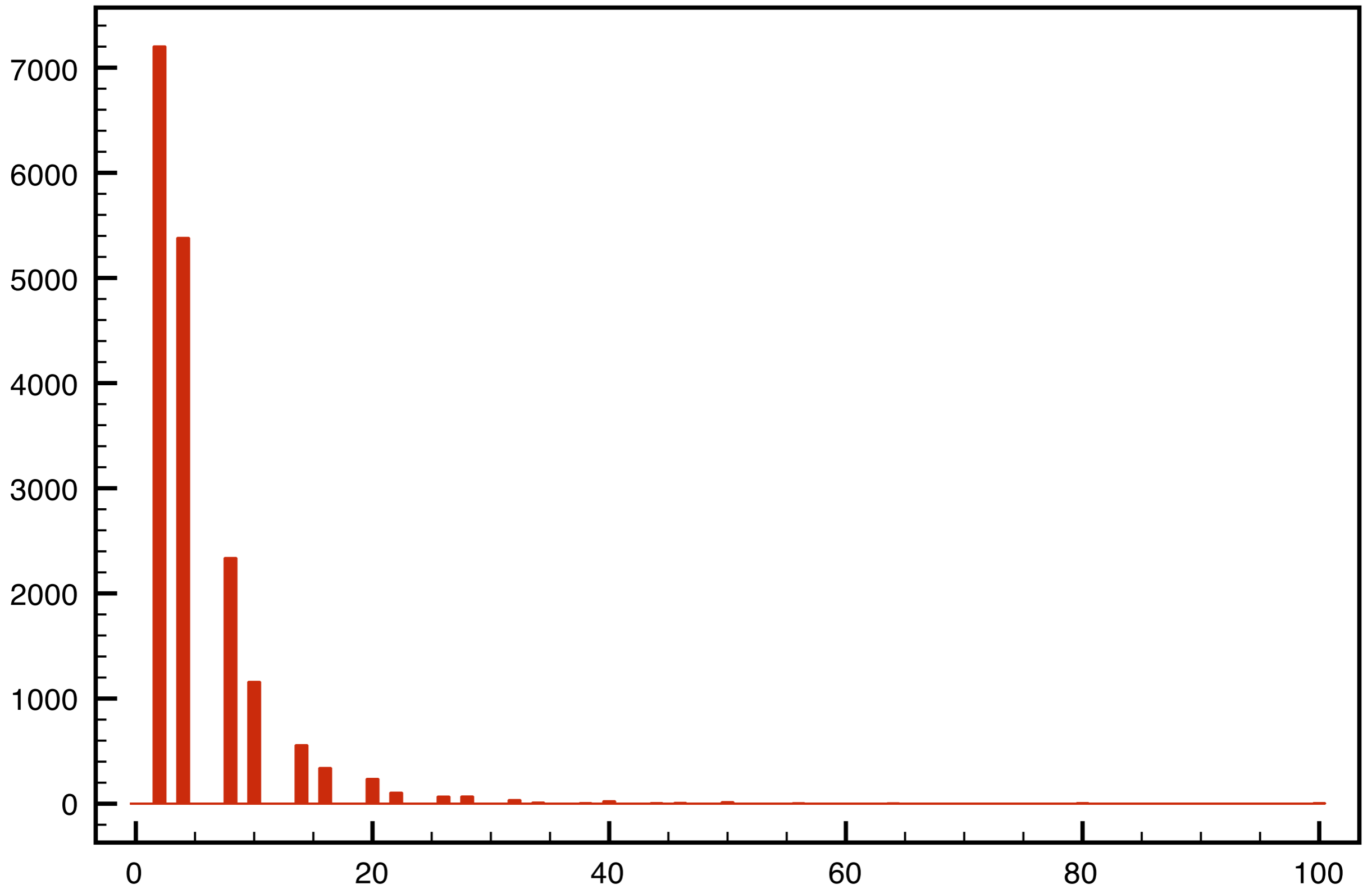
# Tensor product (3,3,3,3,3)



(2,2) models: gauge group  $E_6$



(2,0) models: various gauge groups; using one simple current



(2,0) models: various gauge groups; using two simple currents

# Non-exceptional Gepner Models

$\Delta$  is reduced in many cases.

In essentially all cases,  $\Delta$  is a multiple of six.

In a few cases, it is a multiple of 12 or 0.

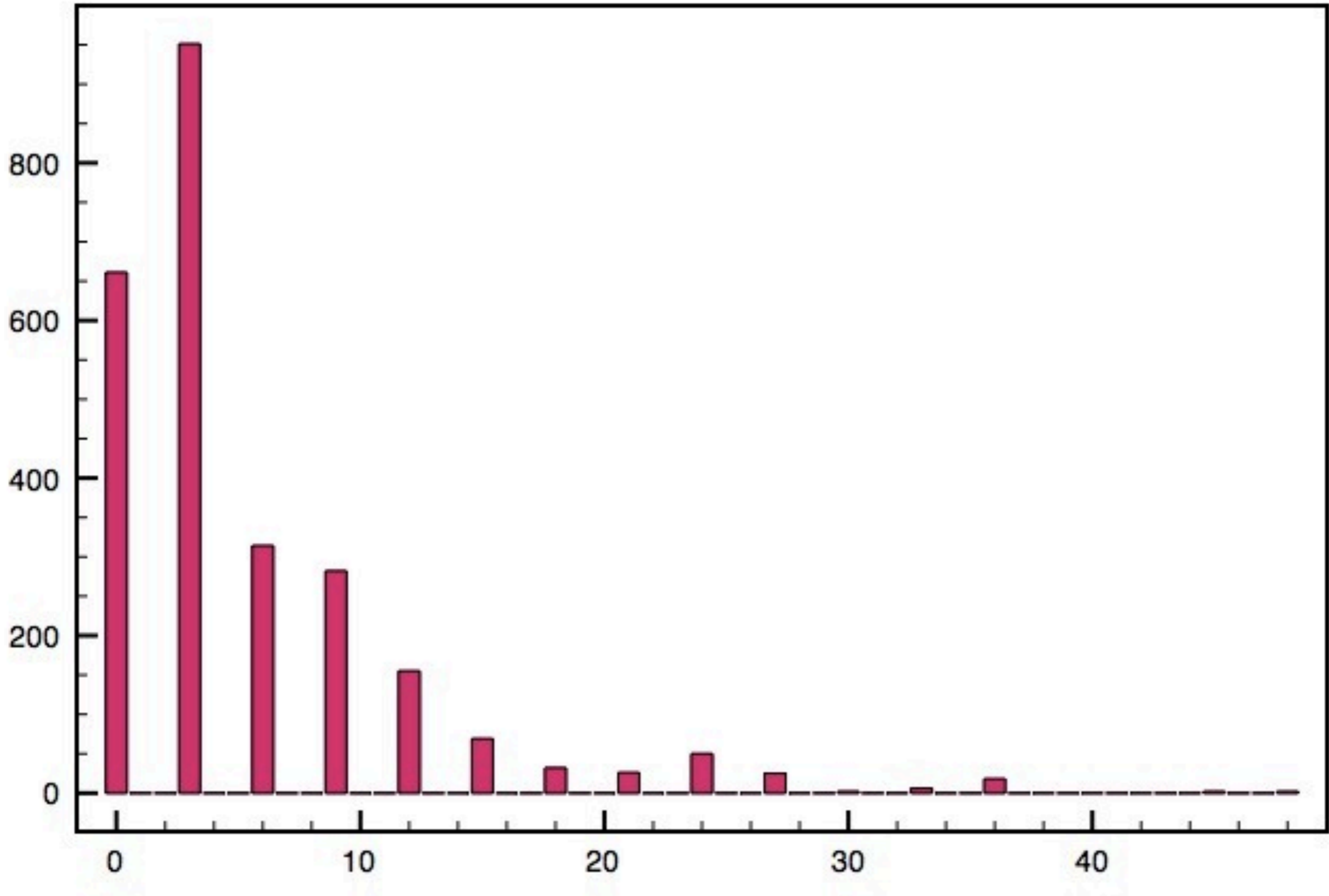
In five cases, it is a multiple of 2 but NOT of 3.

No three-family models, even if we break  $SO(10)$

# THREE FAMILY MODELS



$(1, 16_E, 16_E, 16_E)$



$$SU(3) \times SU(2) \times SU(2) \times U(1)$$

Representation	Particles	Multiplicity
$(3,2,1,\frac{1}{6})$	Q	3
$(3^*,1,2,-\frac{1}{6})$	$U^* + D^*$	$4+1^*$
$(1,2,1,-\frac{1}{2})$	L	$5+2^*$
$(1,1,2,\frac{1}{2})$	$E^* + N^*$	$5+2^*$
$(3^*,1,1,\frac{1}{3})$	$D^*$	$5+5^*$
$(1,2,2,0)$	$H_1 + H_2$	9
$(1,1,0,0)$	singlets	80
$(1,1,1,\frac{1}{3})$	<div style="border: 2px solid black; border-radius: 15px; padding: 20px; width: fit-content; margin: auto;"> <p style="font-size: 2em; margin: 0;">Charge</p> <p style="font-size: 3em; margin: 0;">1/3</p> </div>	$41+41^*$
$(1,1,2,-\frac{1}{6})$		$20+20^*$
$(1,2,1,-\frac{1}{6})$		$19+19^*$
$(3,1,1,0)$		$17+17^*$
$(3,1,1,\frac{1}{3})$		$8+8^*$
$(3,2,1,-\frac{1}{6})$		$3+3^*$
$(3^*,1,2,\frac{1}{6})$		$3+3^*$
$(1,2,2,\frac{1}{3})$		$2+2^*$
$(1,1,1,-\frac{2}{3})$		$2+2^*$

# FRACTIONAL CHARGES

(1,4,4,4,4)

Minimal charge	Chiral	Non-chiral
$\frac{1}{6}$	1048538	16614
$\frac{1}{3}$	709334	65809
$\frac{1}{2}$	12037	228183
1	0	219493

23% non-chiral

(6,6,6,6)

Minimal charge	Chiral	Non-chiral
$\frac{1}{6}$	0	0
$\frac{1}{3}$	0	0
$\frac{1}{2}$	41240	1076404
1	0	973604

98.5% non-chiral

(Always at least a Pati-Salam extension)

$(3,3,3,3,3)$

Minimal charge	Chiral	Non-chiral
$\frac{1}{6}$	0	0
$\frac{1}{3}$	0	0
$\frac{1}{2}$	853368	401795(*)
1	0	2409517

76% non-chiral

(\*) includes cases with just  $SU(3) \times SU(2) \times U(1) \times U(1)^6$

(5,5,5,12)

Minimal charge	Chiral	Non-chiral
$\frac{1}{6}$	0	0
$\frac{1}{3}$	0	0
$\frac{1}{2}$	0	262987
1	0	755413

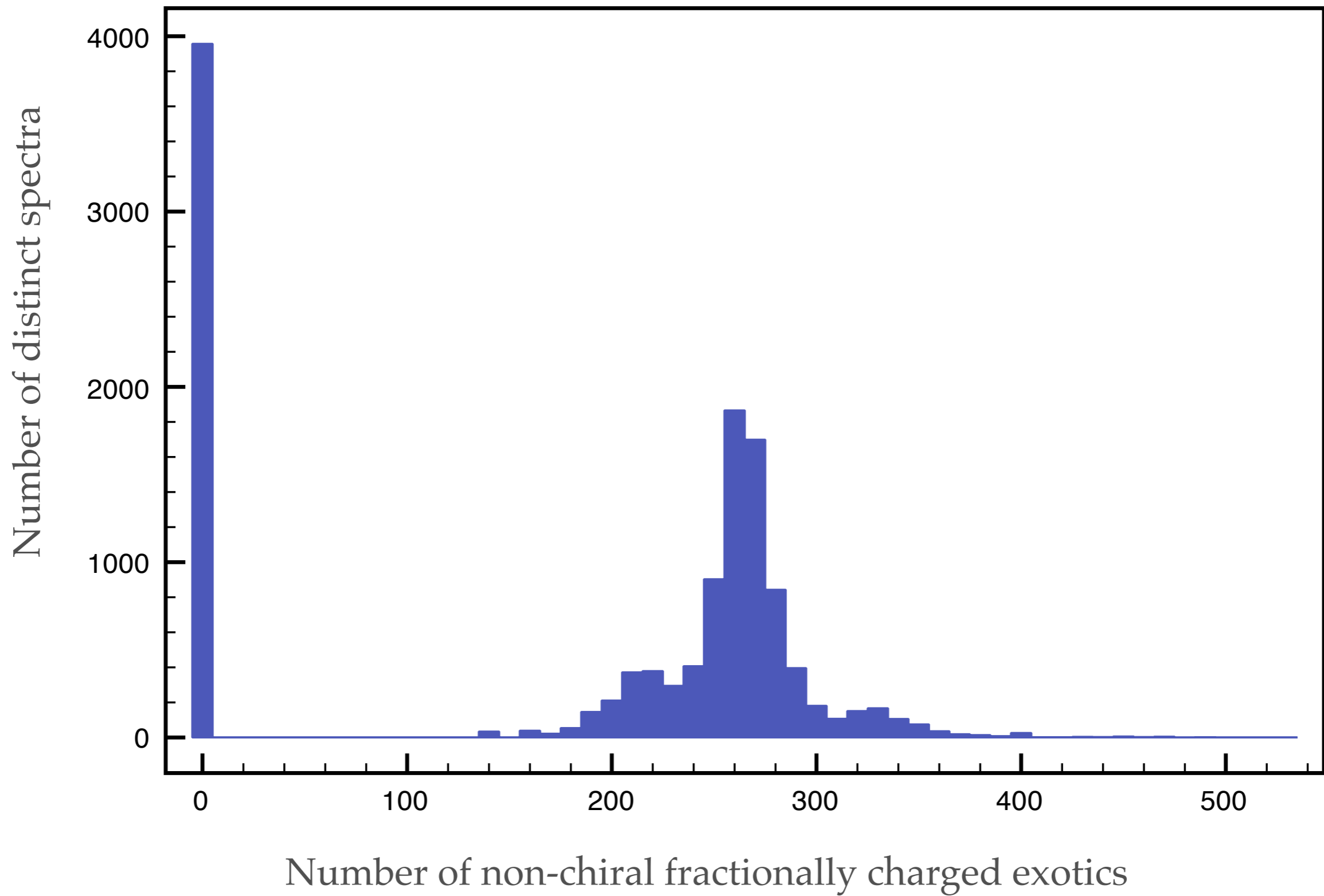
100% non-chiral

It seems to be easy to get only *non-chiral* fractional charges.

Any chance of getting only *massive* fractional charges?



$(3,3,3,3,3)$



# HETEROTIC WEIGHT LIFTING

B. Gato-Rivera and A.N Schellekens

[arXiv:0910.1526](https://arxiv.org/abs/0910.1526)

# HOW TO BUILD (2,0) HETEROTIC STRINGS

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Two approaches are used:

- Build a (2,2) theory and map the fermionic sector to a bosonic one using the bosonic string map.  
*Disadvantage: misses a lot of the (2,0) landscape.*
- Build a (0,0) theory and impose susy on the fermionic sector.  
Only known way: free fermions or bosons.  
*Disadvantage: misses a lot of the interacting CFT landscape.*

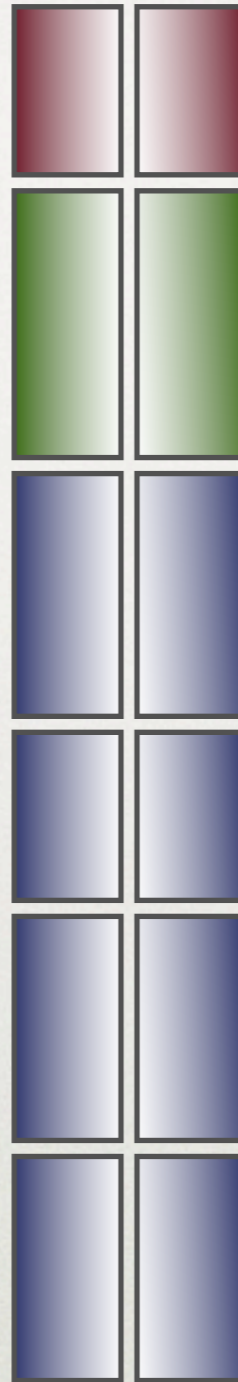
We would like to go beyond that

A standard Gepner model uses the first method:

Fermionic

Bosonic

NSR



SO(10)

$E_8$

$N=2, k_1$

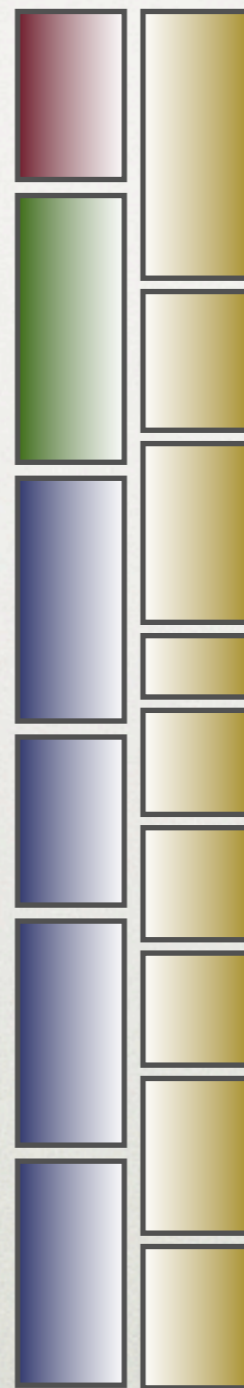
$N=2, k_2$


$N=2, k_3$

$N=2, k_4$

What we would like is:

# General (2,0) model in RCFT



  
N=0  
building  
block

Modular invariance makes this very hard

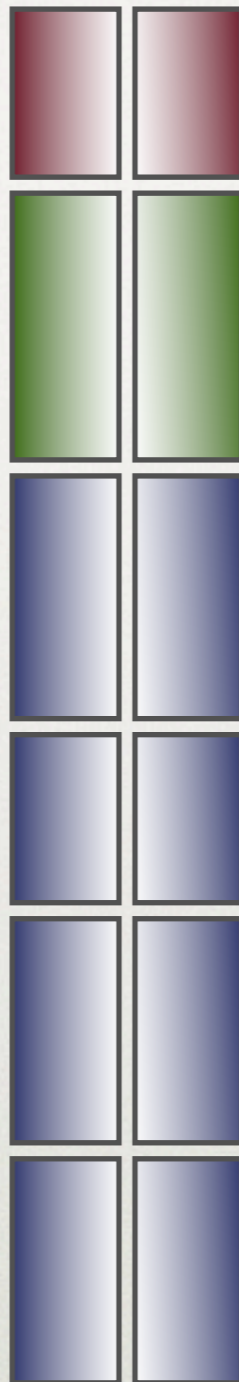
$$P(\tau, \bar{\tau}) = \sum_{ij} \chi_i(\tau) M_{ij} \xi_j(\bar{\tau})$$

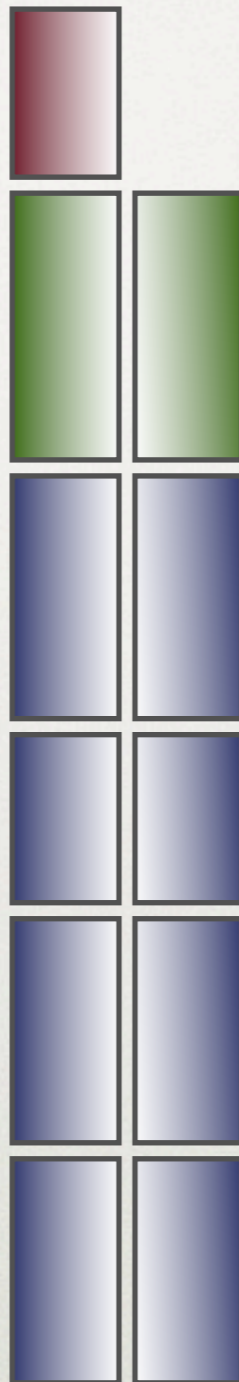
$$P\left(-\frac{1}{\tau}, -\frac{1}{\bar{\tau}}\right) = P(\tau, \bar{\tau})$$

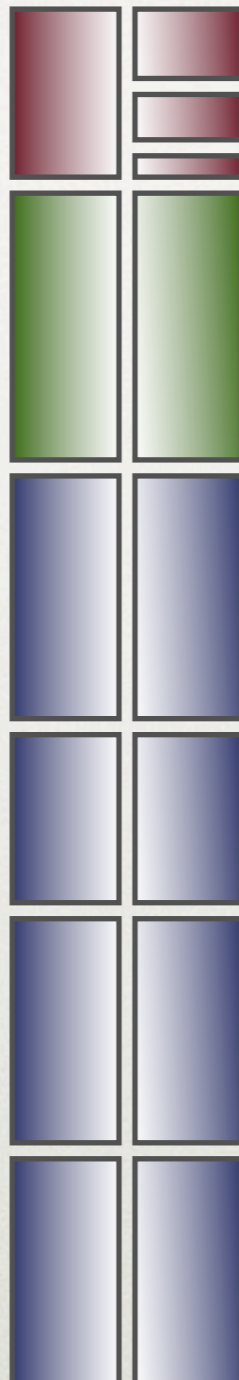
Has a canonical solution,  $M_{ij} = \delta_{ij}$ , if the left and the right CFT are identical, so that  $\chi = \xi$



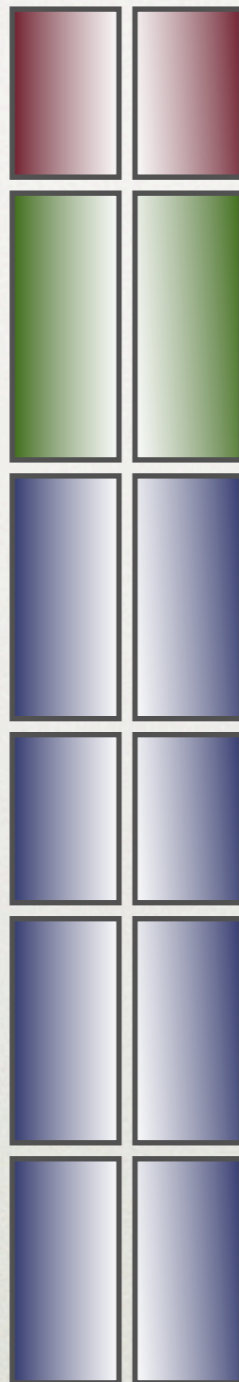
What we did so far is:

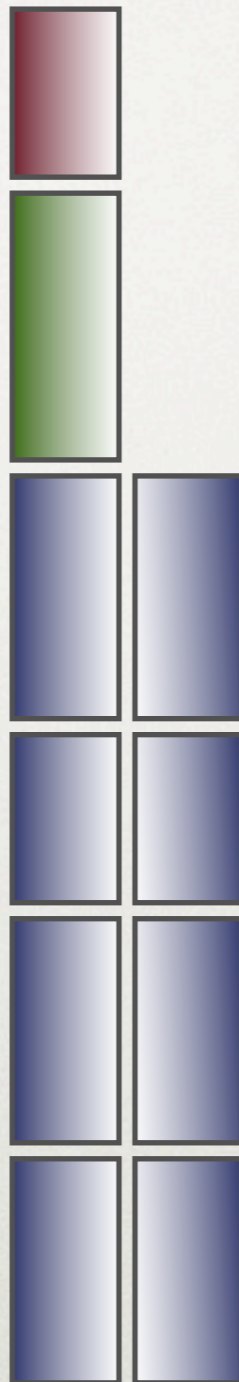


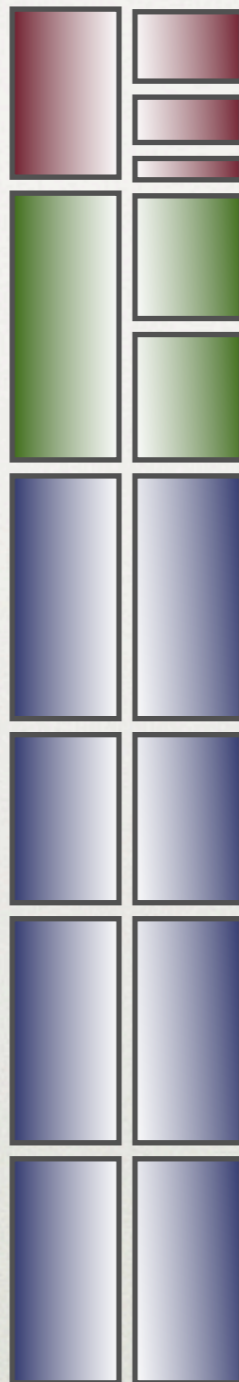




One may also try:





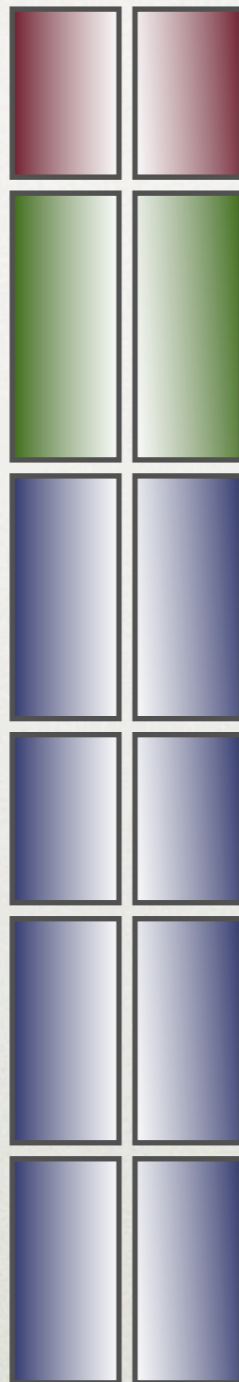


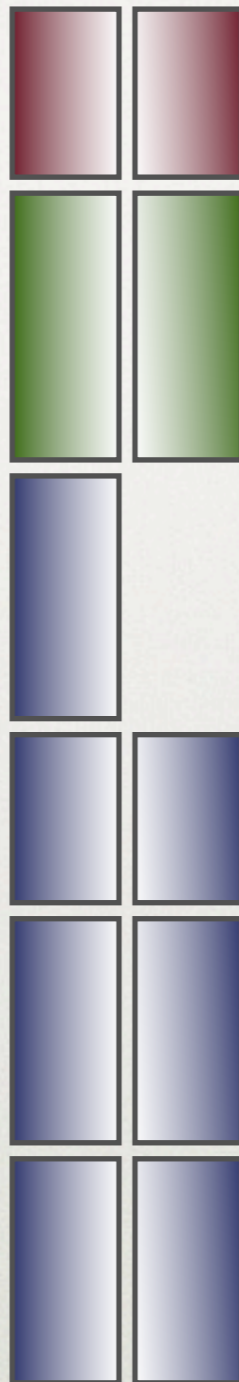


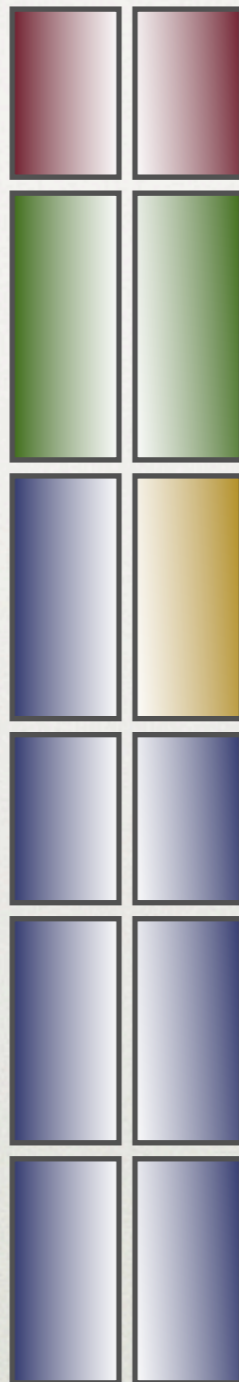
This is straightforward, but has not been  
tried except in a few simple cases

*(Blumenhagen & Wisskirchen, 1996)*

What we would really like is something like this:



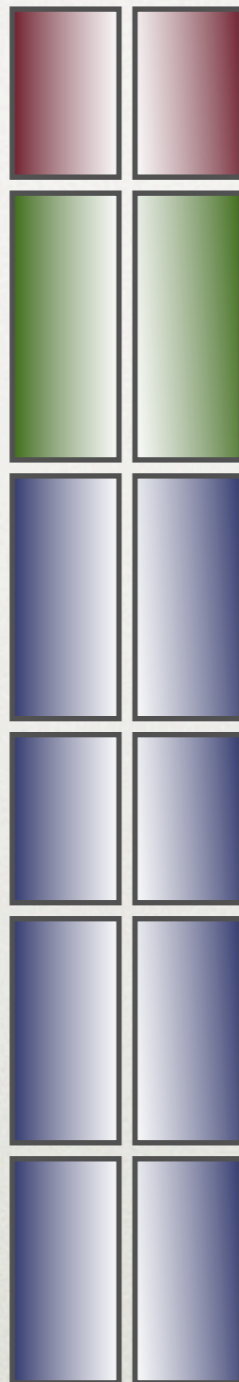


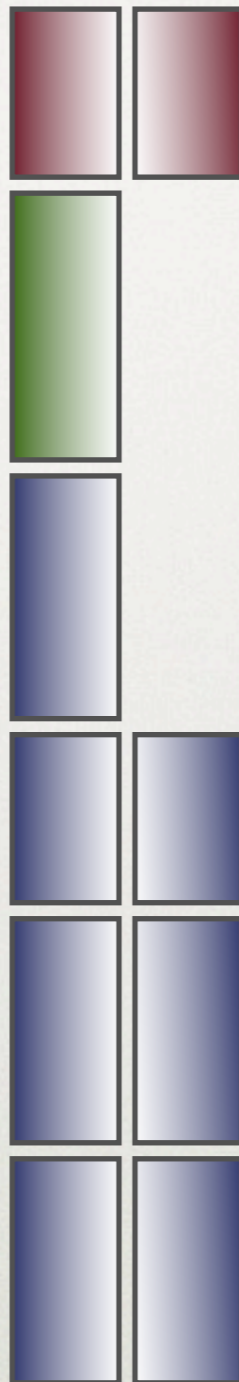


... but we have to find a  $N=0$  cft with the same  $S$ ,  $T$ , and central charge as some  $N=2$  model, without being identical to it.

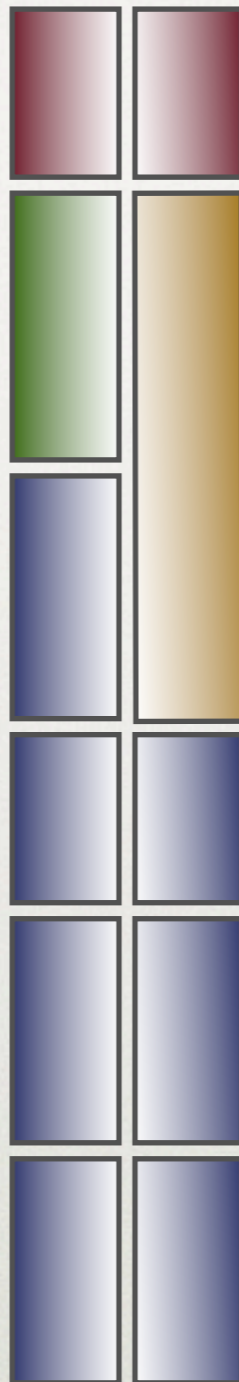
This looks difficult.

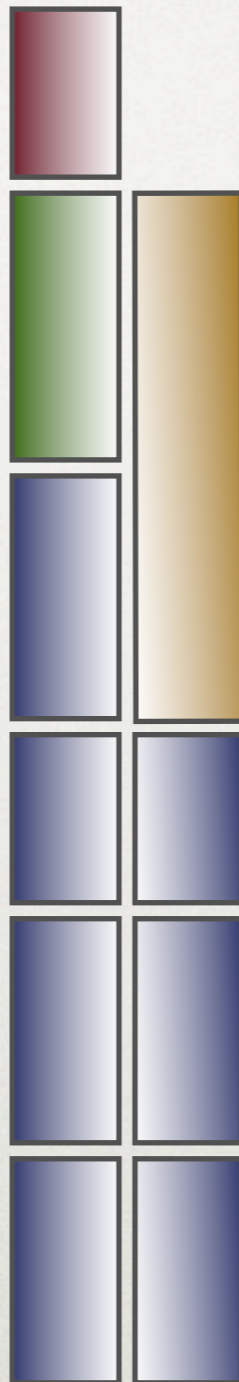
But there is something else we could try:

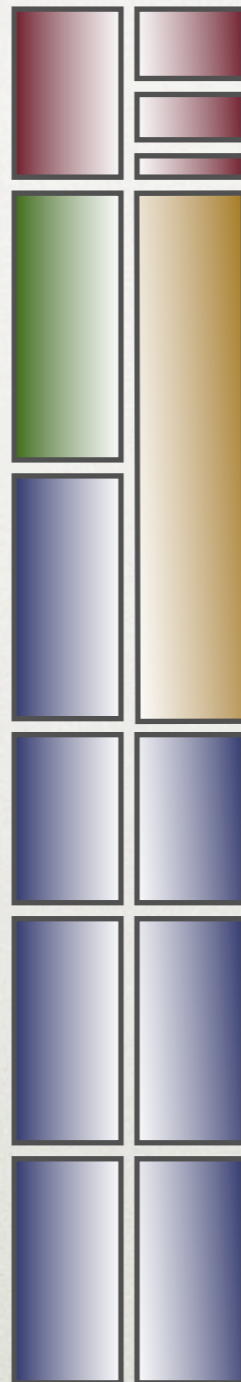












So our goal is to find, for some minimal  $N=2$  model with central charge  $c$ , a replacement that has central charge  $c+8$ , and exactly the same  $S$  and  $T$  matrices.

Hence it must have the same number of primaries, and the same spectrum, up to integers.

Minimal N=2 model at level k:

$$c = \frac{3k}{k+2}$$

Coset description:

$$\frac{SU(2)_k \times SO(2)}{U_{k+2}}$$

Plus “field identification”

*(Gepner; Schellekens and Yankielowicz, 1989)*

Field identification is a formal simple current extension of the coset CFT by a current of spin 0. This relates multiple vacua.

This “extends” the chiral algebra so that the identity representation is doubled, and roughly half the states (that do not satisfy the G/H selection rules) are removed.

The coset CFT may be thought of as tensor product

$$SU(2)_k \times SO(2) \times U(1)_{k+2}^c$$

Where  $U(1)^c$  is the “complement”: an auxiliary representation of the modular group with complex conjugate S and T matrices, and  $c = -1 + 8N$

Now we remove the field identification extension, and consider

$$SU(2)_{k+2} \times SO(2) \times \frac{E_8}{U_{k+2}}$$

In other word, we embed the  $U(1)$  in  $E_8$  instead of  $SU(2) \times SO(2)$ .

Next we identify a CFT  $X_7$  which can be combined with  $U_{k+2}$  to  $E_8$ , so that

$$E_8 = [U_{k+2} \times X_7]_{\text{ext}}$$

Then we can write the CFT as

$$SU(2)_{k+2} \times SO(2) \times X_7$$

And finally we re-establish the equivalent of the field identification, as a standard, higher spin extension

The result is guaranteed, by construction, to have the same S and T matrices as the original minimal model.

But the spectrum is different

Standard coset field  $h_i^G - h_j^H \quad (j \in i)$

Replacement  $h_i^G + h_j^{H^c}$

$$h_j^{H^c} = -h_j^H \pmod{1}$$

All weight of H and  $H^c$  are positive

Therefore standard weights are lifted:

$$h_i^G + h_j^{H^c} > h_i^G - h_j^H$$

(but equal mod 1)



The simplest class of examples: find a  $U(1)$  in  $E_8$  through subgroup embeddings:

For example the Standard Model  $U(1), Y$

$$\begin{aligned}SU(3) \times SU(2) \times U(1)_{30} \times U(1)_{20} &\subset SO(10) \\SO(10) \times SO(6) &\subset E_8\end{aligned}$$

This implies

$$\frac{E_8}{U_{30}} = A_{2,1} A_{1,1} A_{4,1}$$

And hence

$$(N = 2, k = 13) \sim A_{1,13} U_4 A_{2,1} A_{1,1} A_{4,1}$$

Extended by the current  $(J, v, 0, J, 0)$

The minimal  $N=2$ ,  $k=13$  model has 420 primaries.  
We have compared the  $S$  and  $T$  matrices explicitly,  
and they are identical.

But many states in the spectrum are shifted:

136 massless ( $h \leq 1$ ) are lifted(\*)  
81 massive ones become massless  
37 are massless before and after  
166 are massive before and after

(\*) Including all Ramond ground states

# COMPUTING THE SPECTRUM

---

Amazingly easy: start with the **full** spectrum of a standard Gepner model. For example, all states associated with a massless space-time spinor in the fermionic sector

$$\sum_j M_{ij}(\dim_1, h_1, \dots, \dim_n, h_n)_j$$

To compute the consequences of “lifting” factor  $k$ , just replace  $\dim_k$  and  $h_k$  by the corresponding values in the lift CFT

# CHIRAL SPECTRA?

---

All R ground states are lifted.

Hence no extension  $SO(10) \rightarrow E_6$

But also all chiral families are removed.

The diagonal MIPF yields, for (4,4,8,13)

Before lifting:

$$75(27) + 3(\overline{27}) + 450(1) \text{ of } E_6$$

After lifting:

$$20 \times (10) + 1088(1) \text{ of } SO(10)$$

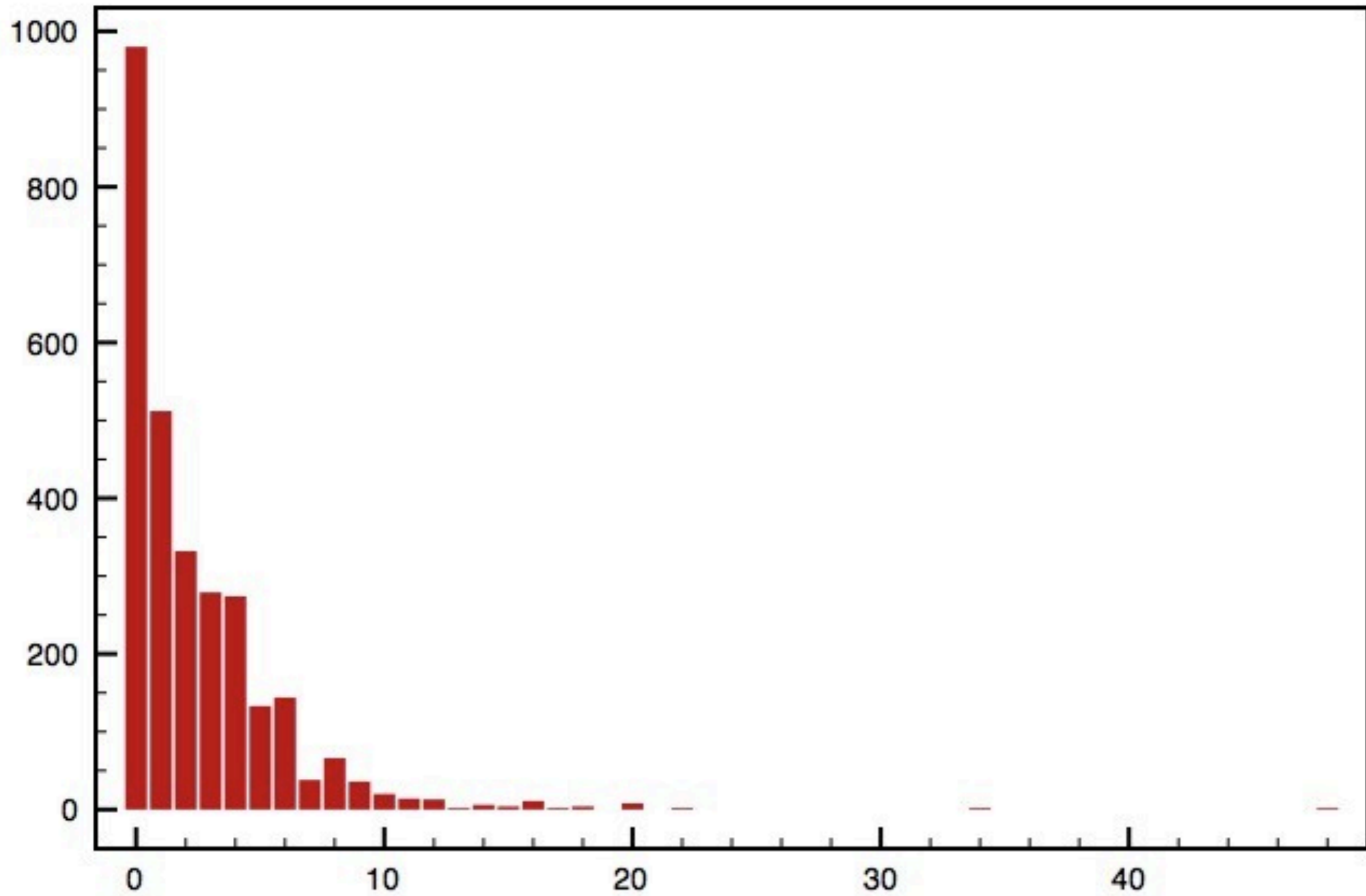
# CHIRAL SPECTRA?

---

But now we can break all non-essential symmetries in the bosonic sector. In particular world-sheet susy.

So we do not need Ramond to get massless fermions!

And this is what came out:



$(3, 3, \widehat{13}, 28)$

For the first time in Heterotic Gepner models,  
family quantization in units of 1!

Many cases with three families.

~ Exponential fall-off with the number of families

Not as steep as in orientifolds.

Three families relatively much more common.

# CHECKS

---

- Extensive computations of unlifted spectra, which are in full agreement with the 1989 results

- Anomalies!

*Schellekens, Warner (1987):*

Modular invariance  $\rightarrow \propto (\text{Tr}F^2 - \text{Tr}R^2)$

These spectra are not obtained from any known compactification, so this is the only way to see that the anomalies must factorize exactly like this

$\text{Tr} F^2$  gets contributions from all gauge group factors.

Hence no anomaly as long as  $E_8$  is not broken.

Here it is broken, and sometimes there are anomalies, which factorize as expected

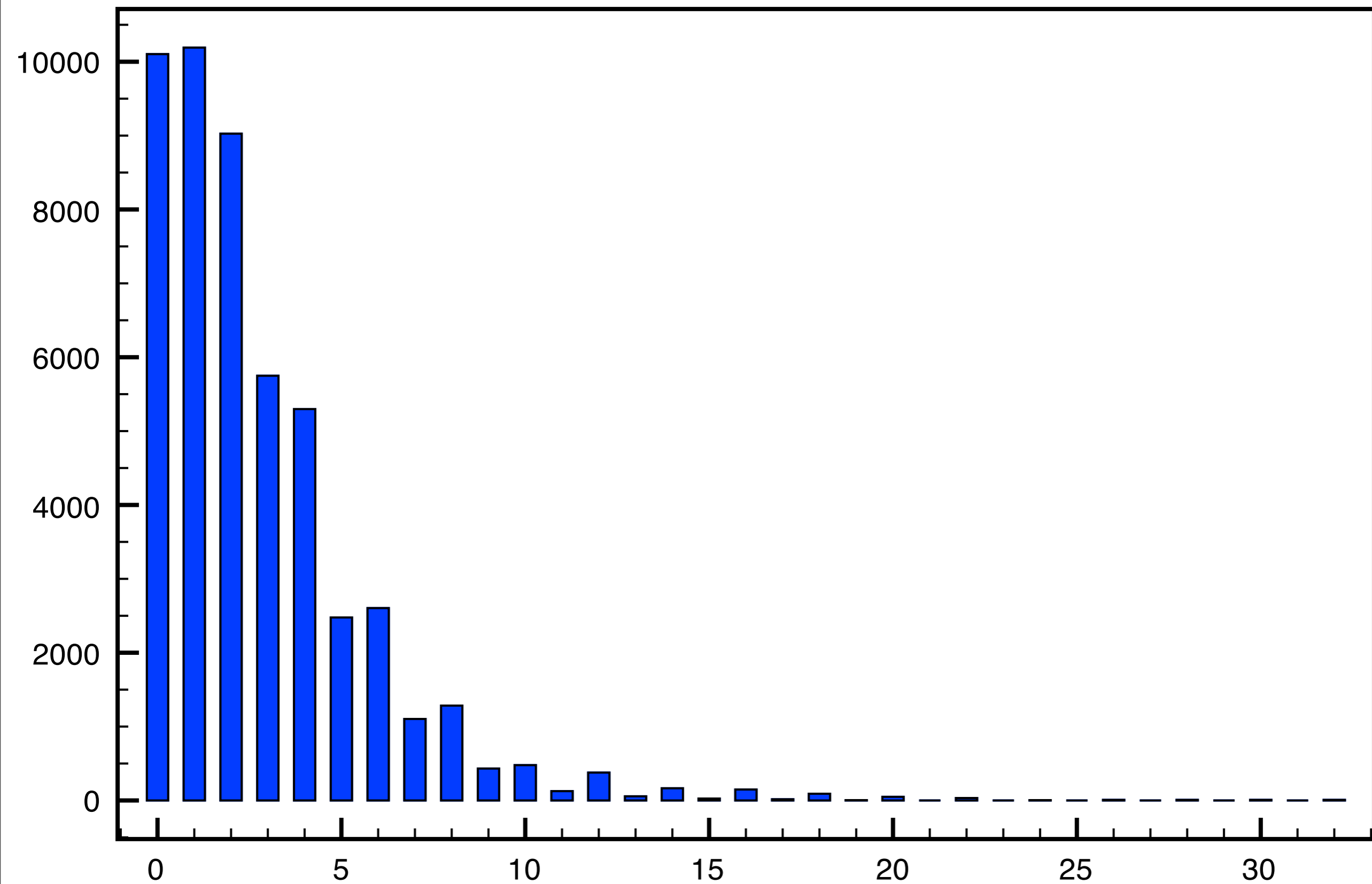


# OTHER LIFTS

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- Using some simple computations we found 30 more.
- There will be many more. We simply stopped looking for them.
- For several values of  $k$  there is more than one.
- We do not even have a proof that their number is finite.
- There are also double lifts. Perhaps many. Perhaps also triple and quadruple lifts.
- Single lifts give rise to about 450 lifted Gepner models. We have only examined about 10% of these.
- About half of them yield 3-family models. Anywhere from a handful to a few thousand.

$k$	Lift	Lifted	Lowered	Unchanged
1	$E_6 \times A_1$	4	1	4
2	$A_7$	7	1	12
3	$[D_6 \times U_{10}]_{\text{ext}}$	10	3	22
4	$D_5 \times A_2$	21	4	23
5	$A_6 \times A_1$	32	8	29
5	$[E_6 \times U_{42}]_{\text{ext}}$	24	11	37
6	$[A_6 \times U_{112}]_{\text{ext}}$	33	15	39
8	$A_4 \times A_3$	65	29	37
9	$[A_6 \times U_{154}]_{\text{ext}}$	76	41	39
11	$[E_6 \times U_{78}]_{\text{ext}}$	104	61	39
11	$[D_6 \times U_{26}]_{\text{ext}}$	98	60	45
12	$A_6 \times U_4$	125	66	39
13	$A_4 \times A_2 \times A_1$	136	81	37
14	$[A_4 \times A_2 \times U_{480}]_{\text{ext}}$	147	105	47
14	$[A_6 \times U_{224}]_{\text{ext}}$	153	95	41
17	$[E_6 \times U_{114}]_{\text{ext}}$	202	105	37
17	$[A_4 \times A_2 \times U_{570}]_{\text{ext}}$	198	133	41
19	$E_6 \times U_{14}$	228	119	42
20	$[A_6 \times U_{308}]_{\text{ext}}$	243	143	42
23	$[D_6 \times U_{50}]_{\text{ext}}$	300	161	41
26	$A_6 \times U_8$	349	199	39
30	$[A_6 \times U_{448}]_{\text{ext}}$	417	235	46
41	$[E_6 \times U_{258}]_{\text{ext}}$	610	297	44
41	$[A_6 \times U_{602}]_{\text{ext}}$	606	325	48
42	$[A_6 \times U_{616}]_{\text{ext}}$	627	337	46
44	$[A_6 \times U_{644}]_{\text{ext}}$	673	361	42
44	$[A_4 \times A_2 \times U_{1380}]_{\text{ext}}$	659	465	56
47	$[E_6 \times U_{294}]_{\text{ext}}$	728	367	46
54	$A_6 \times U_{16}$	857	455	51
58	$A_4 \times A_2 \times U_8$	923	611	56
86	$[A_6 \times U_{1232}]_{\text{ext}}$	1501	741	52
89	$[E_6 \times U_{546}]_{\text{ext}}$	1556	705	49
238	$A_4 \times A_2 \times U_{32}$	4959	2729	73
1,1	$A_2 \times A_1 \times A_2 \times A_1$	16	1	14



$(3, \hat{8}, 8, 8)$

# SOME FEATURES

---

- Many different gauge groups, from just  $SU(3) \times SU(2) \times U(1)$  to  $SO(10)$ .
- (\*)Additional non-abelian gauge group from the lift CFT.
- Distribution of number of mirrors from a few tens to zero.
- At least one example of a Pati-Salam like model with no mirrors at all.
- Another example with  $3 \times (16) + (10)$  of  $SO(10)$  (exactly the minimal  $SO(10)$  susy-GUT).
- Examples with just(\*)  $SU(3) \times SU(2) \times U(1)$  and B-L broken by anomalies.
- Large spread of number of singlets and fractionally charged particles, but not including 0.

Looks more and more like orientifold results...

# APPROACHING THE SM

---

An example from  $(3, \hat{8}, 8, 8)$

Gauge group:

$$SU(3) \times SU(2) \times U(1) \times [SU(2)_8 \times SO(2) \times SU(4) \times SU(5)]$$

(anomalous "B-L")

Spectrum:

$$3 \times (Q + U^c + D^c + L + E^c) + 3 \times (D + D^c) + 3 \times (H_1 + H_2)$$

+ 250 singlets

+ 172 fractionally charged particles



## Fractional charges:

Non-chiral.

Only half-integer (no sixth or third).

Confined by  $SU(2)_8$

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**Singlets:** (of  $SU(3) \times SU(2) \times U(1)$ )

Only 38 are B-L singlets. Only three are absolute singlets.

Many are in nontrivial  $SU(4)$  and  $SU(5)$  reps.



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Many are in nontrivial  $SU(4)$  and  $SU(5)$  reps.

## The bad news:

$U^c$ ,  $D^c$  and  $E^c$  are in the triplet representation of  $SU(2)_8$ ; Higgs candidates are singlets.

# QUESTIONS

---

- What, if any, is the geometric interpretation of these models?
- Are they related to other constructions, and how?
- Is there a related Landau-Ginzburg description?
- What are their strong coupling duals?
- Is there an exact mirror symmetry?
- Is it possible to classify all the lifts?
- Are there any generic bad features that rule out this entire class phenomenologically?
- What can be said in general about charge quantization and confinement?
- Is there a simple rule for family number quantization?
- How close can we get to the MSSM spectrum?
- Without supersymmetry, how close can we get to the SM spectrum?

# CONCLUSIONS

---

- Asymmetric Gepner models provide a huge and largely unexplored part of the landscape.
- Family distributions peak at small values.
- Three families still hard to get.
- Fractional charges occur, but are reasonably often non-chiral.
- Many other possibilities exist.

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