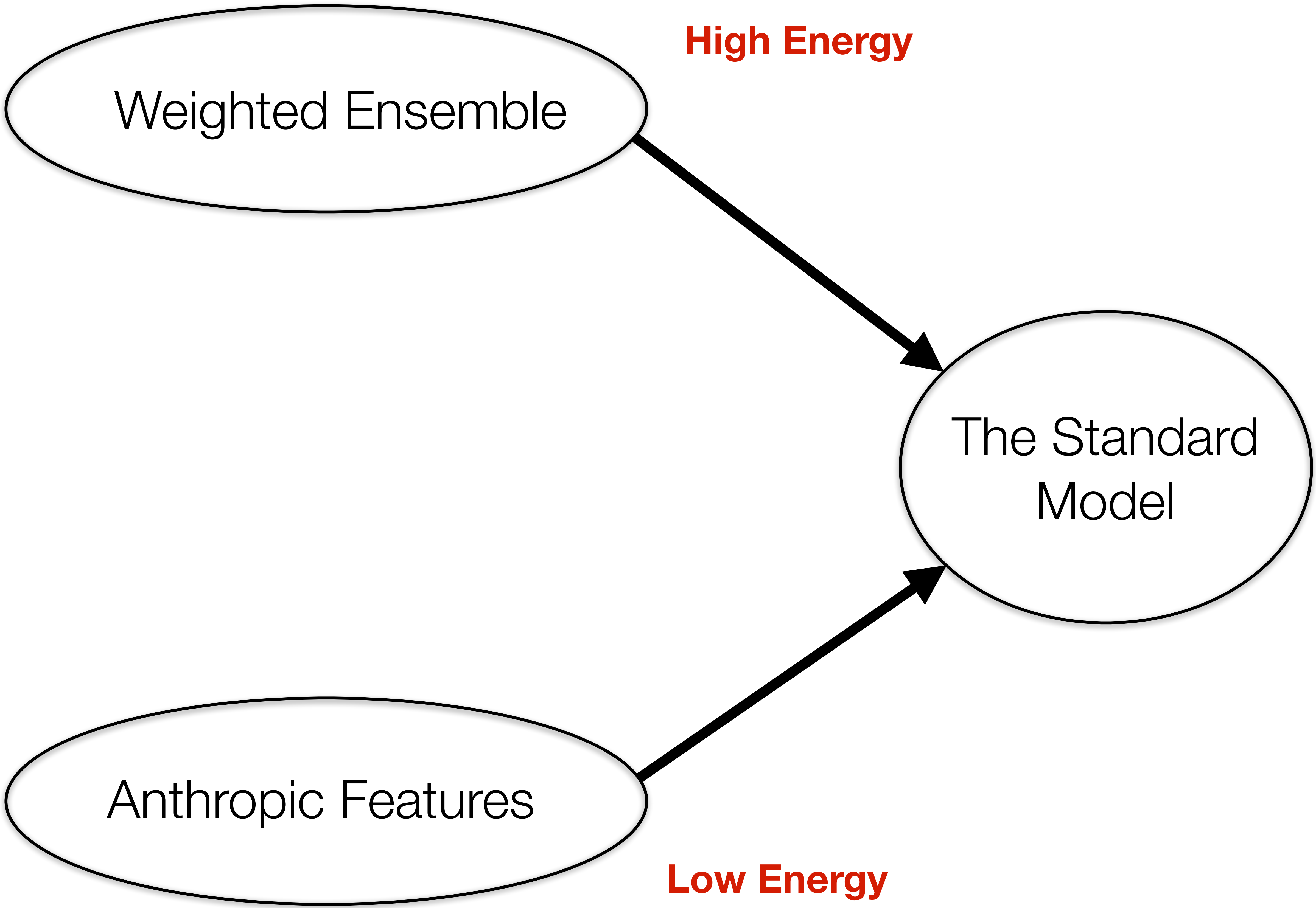
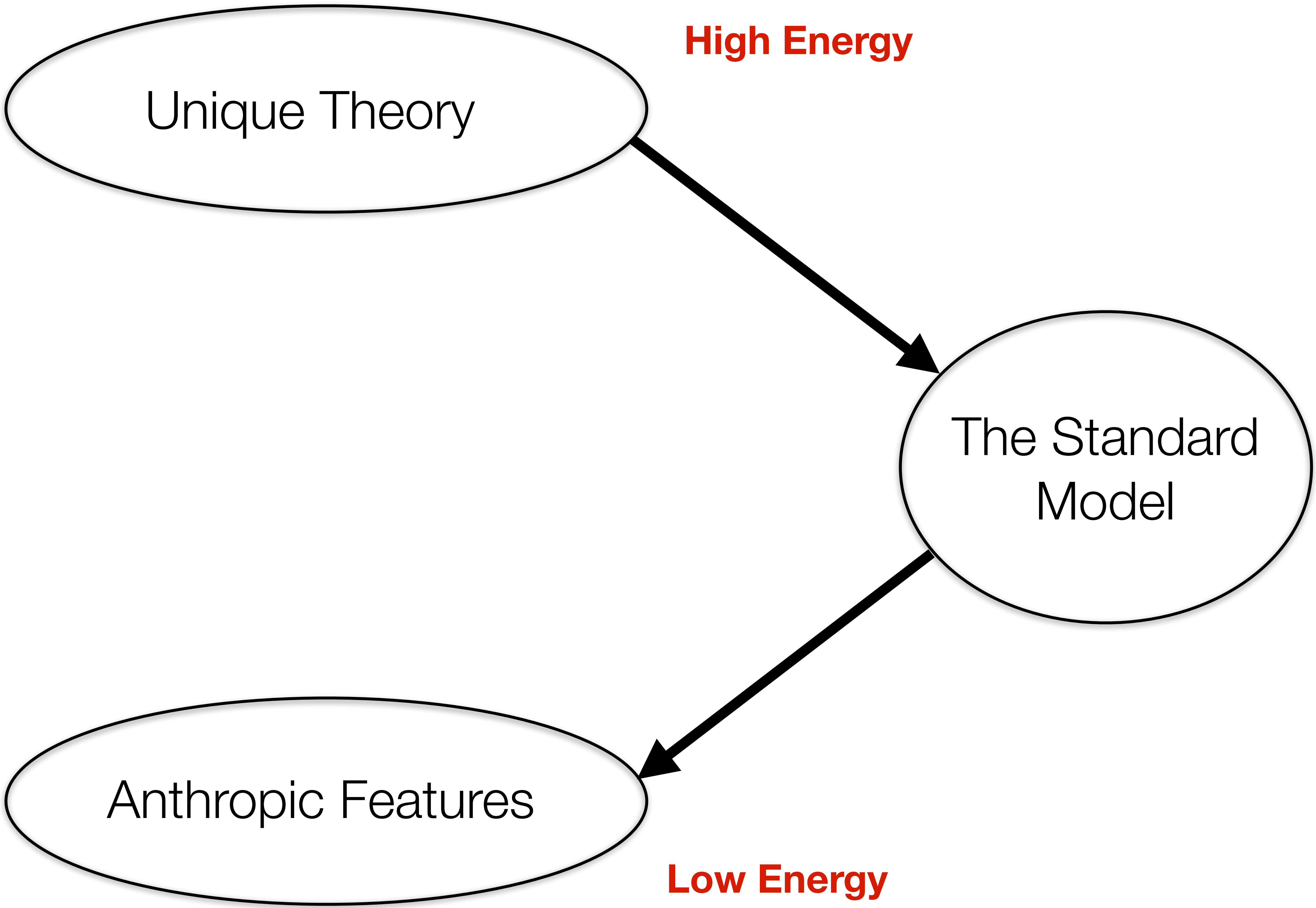
A derivation of the Standard Model

Based on Nucl.Phys. B883 (2014) 529-580 with B. Gato Rivera


Weighted Ensemble
High Energy
The Standard Model
Anthropic Features
Low Energy


High Energy
Unique Theory
The Standard Model
Anthropic Features
Low Energy

Anthropics

(concerns existence of observers)

vs.

Aesthetics

(concerns happiness of observers)

Required anthropic features

- A large hierarchy
- At least one massless photon
  (to get atomic physics)
- A substantial variety of (semi)stable charged particles
  (playing a role analogous to nuclei and electrons, so that we get interesting atomic physics; just hydrogen and helium is too boring.)
- No massless charged particles

"A massless electron means that the Bohr radius of an atom—half a nanometer in the real world—would be infinite. In a world without compact atoms, valence chemical bonding would have no meaning. All matter would be insubstantial—and life as we know it would not exist! ***On top of all that, the vacuum would be unstable to the formation of a plasma of e^+e^- pairs."***

(C. Quigg, R. Shrock, Phys.Rev. D79 (2009) 096002)

The need for a large hierarchy

Maximal number of building blocks with mass m_p of an object that does not collapse into a black hole or fly apart

$$\left(\frac{m_{\text{Planck}}}{m_p}\right)^3$$

Brain with 10^{27} building blocks requires a hierarchy of 10^{-9}

Stars require an even bigger hierarchy:

Fred Adams, "Constraints on Alternate Universes: Stars and habitable planets with different fundamental constants", arXiv:1511.06958

"We find the limit $\alpha_G/\alpha < 10^{-34}$, which shows that habitable universes must have a large hierarchy between the strengths of the gravitational force and the electromagnetic force".

$$\alpha_G \equiv \frac{G m_{\text{p}}^2}{\hbar c}$$

The Hierarchy Problem

So far, approaches towards solving the hierarchy problem have failed for over two decades.

Perhaps this is because of two serious mistakes:

1. Ignoring anthropic arguments
   (“It is a deep mystery that this number is so small”)

2. Ignoring distributions
   (without this information exact statements are impossible)

The Technical Hierarchy Problem

Renormalization of scalar masses

$$\mu^2_{\text{phys}} = \mu^2_{\text{bare}} + \sum_i a_i \Lambda^2$$

Bad

Computable statistical cost of about 10^{-34} for the observed hierarchy. This is the "technical hierarchy problem".

Renormalization of fermion masses

$$\lambda_{\text{phys}} = \lambda_{\text{bare}} \left(\sum_i b_i \log(\Lambda/Q) \right)$$

Good **Unknown**

Statistical cost determined by landscape distribution of λ_{bare}

The Single Higgs Hypothesis

If we accept the current status quo, apparently nature has chosen to pay the huge price of a single scalar that creates the hierarchy.

It remains to be shown that is cheaper than having fundamental Dirac particles with small masses, or than solutions to the technical hierarchy problem (susy, compositenes, ….) but we will assume that it is.

Then this price is going to be payed only once: **there should be at most one light scalar.**

No 750 GeV scalar!

Anomaly arguments

Geng and Marshak (1989)

A single SM family without a right-handed neutrino is the smallest non-trivial chiral anomaly-free representation of $SU(3)\times SU(2)\times U(1)$.

OK, but:

- There are **three** families.
- There probably are right-handed neutrinos.
- Why is the smallest representation preferred anyway?

See also:

Minahan, Ramond, Warner (1990), *Geng and Marshak (1990)*

Anomalies and Charge Quantization

An anomaly free set of chiral fermions with irrational charges
(which can even get their masses from the SM Higgs)

$$(3,2,\frac{1}{6}-\frac{x}{3})+(\bar{3},1,-\frac{2}{3}+\frac{x}{3})+(\bar{3},1,\frac{1}{3}+\frac{x}{3})$$
$$+(1,2,-\frac{1}{2}+x)+(1,1,1-x)+(1,1,-x)$$

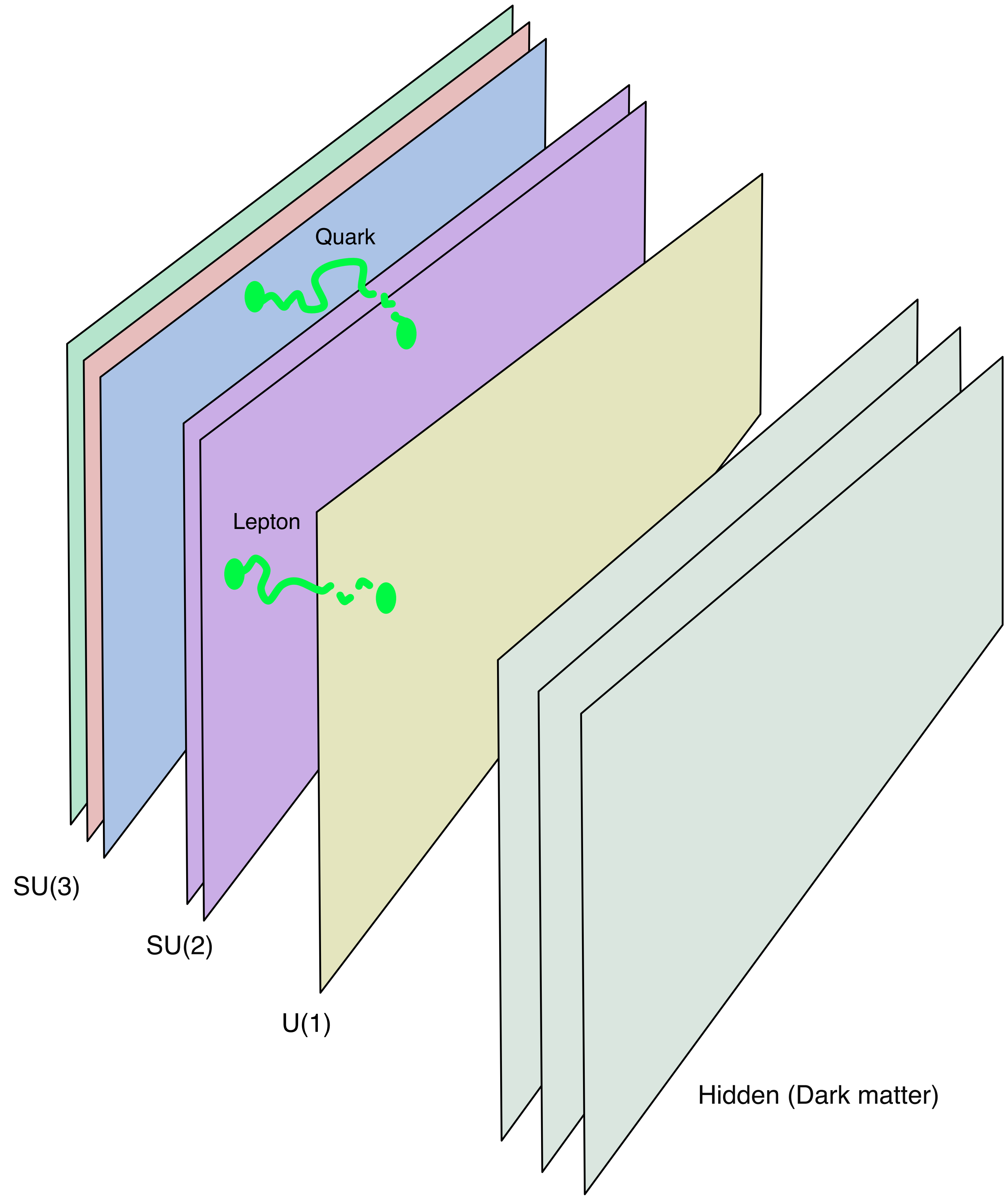
The Ensemble

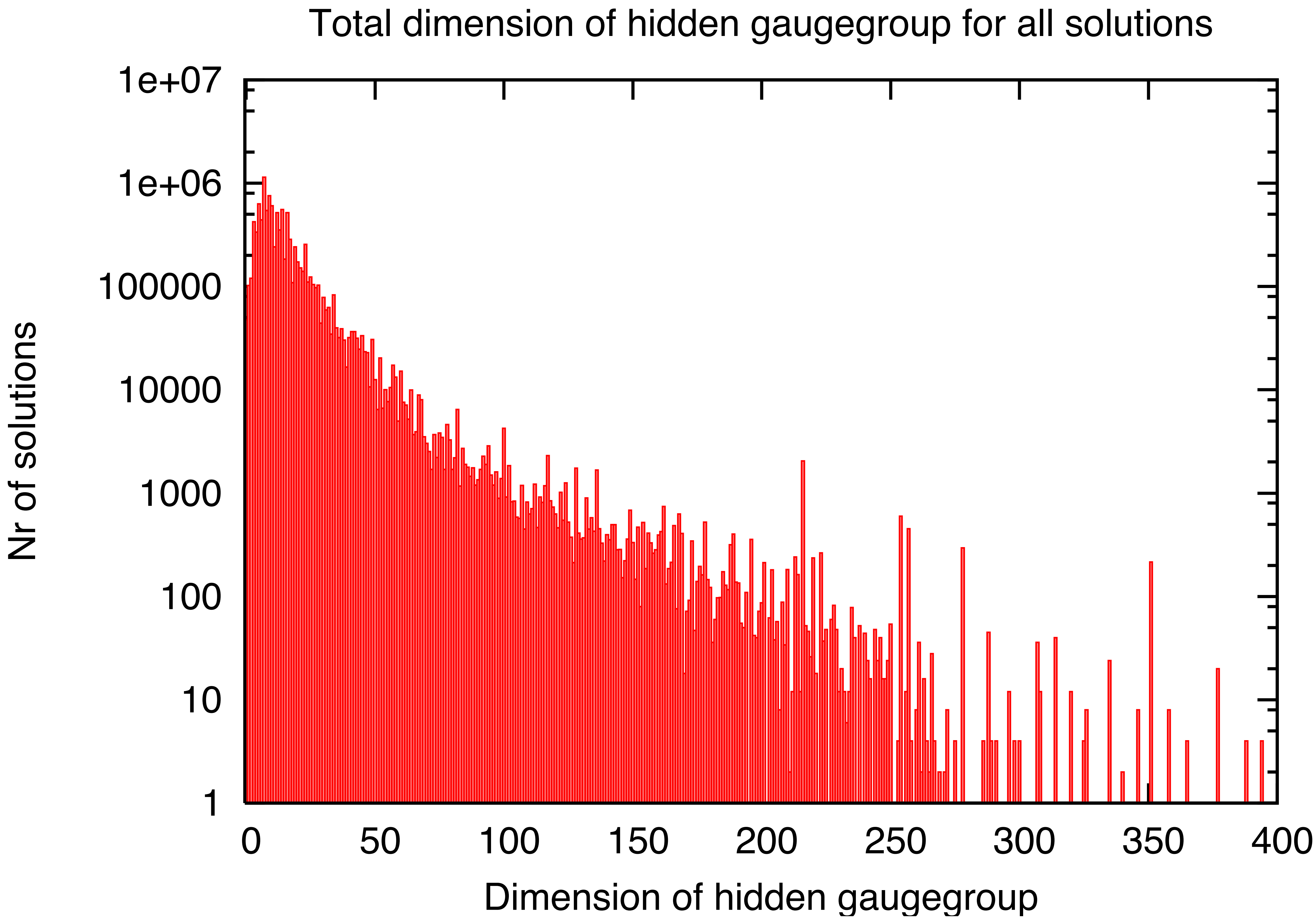
We would like to enumerate all QFT’s with a gauge group and chiral matter. All non-chiral matter is assumed to be heavy, with the exception of at most one scalar field, the Higgs. We demand that after the Higgs gets a vev, and that when all possible dynamical symmetry breakings have been taken into account, at least one massless photon survives, and all charged particles are massive.

This condition is very restrictive, but still has an infinite number of solutions in QFT.

So at this point we invoke string theory. Its main rôle is to restrict the representations. It also provides a more fundamental rationale for anomaly cancellation.

Intersecting Brane Models





Dijkstra, Huiszoon, Schellekens (2004)

Type-II RCFT orientifolds

Intersecting Brane Models

We will assume that all matter and the Higgs boson are massless particles in intersecting brane models. Then the low-energy gauge groups is a product of $U(N)$, $O(N)$ and $Sp(N)$ factors.

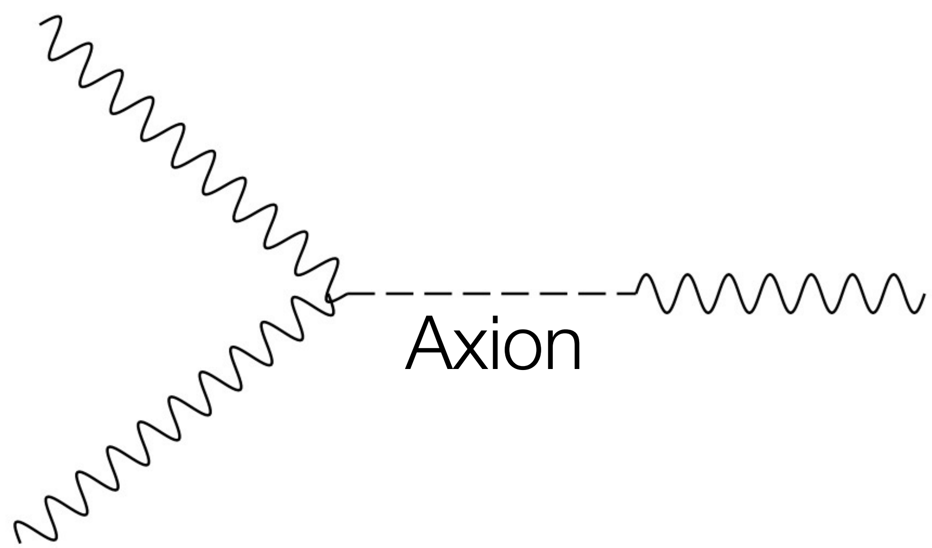
The low energy gauge group is assumed to come from S stacks of branes.
There can be additional branes that do not give rise to massless gauge bosons: $O(1)$ or $U(1)$ with a massive vector boson due to axion mixing.

All matter (fermions as well a the Higgs) are bi-fundamentals, symmetric or anti-symmetric tensors, adjoints or vectors (open strings with one end on a neutral brane)

We start with $S = 1$, and increase S until we find a solution.
($S = 1$ is easily ruled out, so the first case of interest is $S = 2$)

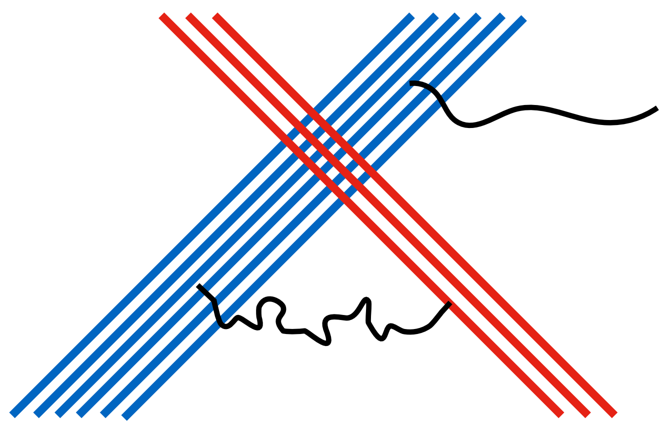
Intersecting Brane Models

- Intersections of branes in extra dimensions determine the massless spectrum.
- Brane multiplicities are subject to a constraint: tadpole cancellation (automatically implies absence of triangle anomalies in QFT).
- Massless photons may mix with axions and acquire a mass.



(Green-Schwarz mechanism)

Two stack models



$$SU(M) \times SU(N) \times U(1)$$

(assuming unitary branes)

$$Y = q_a Q_a + q_b Q_b$$

$q_a, \; q_b$ determined by axion couplings

Q	$(M, N, q_a + q_b)$
U	$(A, 1, 2q_a)$
D	$(\overline{M}, 1, -q_a)$
S	$(S, 1, 2q_a)$
X	$(M, \overline{N}, q_a - q_b)$
L	$(1, \overline{N}, -q_b)$
T	$(1, S, 2q_b)$
E	$(1, A, 2q_b)$

Tadpole Equations

$$\begin{aligned}
(S+U)\tilde{q}_a &= C_1 \\
(T+E)\tilde{q}_b &= -C_2 \\
(D+8U)\tilde{q}_a &= (4+M)C_1 + NC_2 \\
L\tilde{q}_b + D\tilde{q}_a &= 0 \\
2E\tilde{q}_b + 2U\tilde{q}_a &= C_1 - C_2
\end{aligned}$$

$$\tilde{q}_a \equiv Mq_a,\ \tilde{q}_b \equiv Nq_b$$

$$C_1 = -(Q-X)\tilde{q}_b$$

$$C_2 = (Q+X)\tilde{q}_a$$

Note that Q,U,D,L,E,S,T,X denote both the name and the multiplicity of a representation

($q_a = 0$ and/or $q_b = 0$ must be treated separately)

Abelian theories

Single $U(1)$: Higgs must break it, no electromagnetism left

$U(1)\times U(1)$: No solution to anomaly cancellation for two stacks

So in two-stack models we need at least one non-abelian factor in the high-energy theory.

Strong Interactions

It is useful to have a non-abelian factor in the low-energy theory as well, since the elementary particle charge spectrum is otherwise too poor. We need some additional interaction to bind these particles into bound states with larger charges (hadrons and nuclei in our universe).

For this to work there has to be an approximately conserved baryon number.
This means that we need an $SU(M)$ factor with $M \geq 3$, and that this $SU(M)$ factor does not become part of a larger group at the "weak" scale.

Note that $SU(2)$ does not have baryon number, and the weak scale is near the constituent mass scale. We cannot allow baryon number to be broken at that scale.

But let's just call this an additional assumption.

Higgs Choice

This implies that at least one non-abelian factor is not broken by the Higgs. We take this factor to be $U(M)$.

Therefore we do not consider bi-fundamental Higgses breaking both $U(M)$ and $U(N)$. We assume that $U(N)$ is the broken gauge factor. Then the only Higgs choices are **L**,**T** and **E**.

We will assume that $U(M)$ it is strongly coupled in the IR-regime and stronger than $U(N)$.

$$SU(M)\times U(1)\ \ (i.e.\ N{=}1)$$

Higgs can only break $U(1)$, but then there is no electromagnetism.

Hence there will be a second non-abelian factor, broken by the Higgs.

$$M = 3,\ N = 2$$

Higgs = L

Decompose L, E, T: chiral charged leptons avoided only if

$$L = E,\ T = 0$$

Substitute in tadpole equation:

$$S\tilde{q}_a = \left(\frac{5 - N - M}{2M}\right) C_1$$

For $M = 3,\ N = 2$: $S = 0$

Therefore we get standard QCD without symmetric tensors.

$$M = 3, \; N = 2$$

Quark sector pairing

$$Q(3, q_a) + Q(3, q_a + 2q_b) + X(3, q_a) + X(3, q_a - 2q_b) - U(3, -2q_a) - D(3, q_a)$$

$$Q + X - D = 0$$

$Q = U$ **if and only if** $q_a + 2q_b = -2q_a$

or

$X = U$ **if and only if** $q_a - 2q_b = -2q_a$

In both cases we get an $SU(5)$ type charge relation, and hence standard charge quantization

$M = 3,\ N = 2$

Hence either $Q = 0$ ***or*** $X = 0$; the choice is irrelevant.

Take $X = 0$.

Then $D = Q = U,\ T = 0,\ L = E$

Remaining anomaly conditions: $L = Q$

Hence the only solution is a standard model family, occurring Q times.

The branes **a** and **b** are in principle unrelated, and can generally not be combined to a $U(5)$ stack. Hence no GUT proton decay.

This solution is just the well-known $S(U(3)\times U(2))$ model which produces the correct charge quantisation.

$$M = 3,\ N = 2$$

Higgs = T

The symmetric tensor can break $SU(2)\times U(1)$ in two ways, either to $U(1)$, in the same way as **L**, or to $SO(2)$.

Breaking to $U(1)$ (same subgroup as **L**)

No allowed Higgs couplings to give mass to the charged components of L, E and T, so we must require $E = L = T = 0$. Then there is no solution.

Breaking to $SO(2)$

Then $SO(2)$ must be electromagnetism. Y-charges forbid cubic T couplings, so $T = 0$ to avoid massless charged leptons. Quark charge pairing (to avoid chiral QED, broken by QCD) requires $Q = -X$. **If we also require $S = 0$, everything vanishes.**

(Note: stronger dynamical assumption: $S = 0$)

$M > 3$ and/or $N > 2$

- No solution for quark pairing for $M>3$
- Non-trivial solutions with quark and lepton pairing exist for $M=3,\ N>2$
  (This involves considering the most general $Q+\Lambda$, where Q is the external $U(1)$, and Λ a generator in the flavor group, left unbroken by dynamical symmetry breaking)
- All of them satisfy standard model charge quantization, even though $M+N \neq 5$
- But massless charged leptons can be avoided only for $N=2$

Conclusions

- The Standard Model is the unique anthropic solution within the set of two-stack models.

- Family structure (and hence family repetition), charge quantization, the weak interactions and the Higgs choice are all derived.

- Standard Model charge quantization works the same way, for any value of N, even if $N+3 \neq 5$.

- The GUT extension offers **no advantages** (unless susy is found).

- From the two-brane ansatz, the single Higgs hypothesis, and the anthropic atomic physics requirements one can derive the Standard Model family structure **without any prior knowledge of quarks, leptons and their charges.**

Ensemble	Brane models (weighted)	QFT (not weighted)
Class	Two brane model (minimal choice)	Simple Lie Algebras
Gauge Group	SU(3) x SU(2) x U(1)	SU(5)
Particle representation	Output	(5)+(10*)
GUT scale Higgs	Not Needed	(24)
Breaking pattern	Not Needed	Choose SM, Not SU(4) x U(1)
SM Higgs	Output	(5) (doublet-triplet splitting problem)

Couplings

The $U(3)\times U(2)$ structure of this class of models implies one relation among the SM couplings, instead of the two of $SU(5)$

$$\frac{1}{\alpha_Y} = \frac{2}{3}\frac{1}{\alpha_s} + \frac{1}{\alpha_w}$$

see also:
Ibañez, Munos, Rigolin, 1998;
Blumenhagen, Kors, Lüst, Stieberger, 2007

Extrapolation this to higher energies we see that this is satisfied at 5.7×10^{13} GeV.

What happens at that scale and beyond is subject to speculation, but undoubtedly model-dependent.

New physics at that scale may be related to the QCD axion, the see-saw mechanism and Higgs stability.


AIA

Complete list of solutions

Nr.	M	N	q_a	q_b	Higgs	Q	U	D	S	X	L	E	T
1	1	2	2	-3	**L**	3	6	3	3	0	1	1	0
2	1	2	4	-1	**L**	2	1	1	0	0	2	3	1
3a	1	2	2	-1	**L**	3	4	1	3	-4	1	0	-1
3b	1	2	2	-1	**L**	2	2	1	1	-1	1	1	0
3c	1	2	2	-1	**L**	4	5	0	3	-4	0	1	-1
4	1	3	3	-2	**L**	2	3	2	1	0	1	1	0
5	1	3	3	-1	**E**	0	0	-2	-1	1	-2	1	0
6	1	4	4	-1	**L**	1	1	1	0	0	1	1	0
7	M	2	1	ρ	**T**	1	$-\rho$	$2M\rho$	$-\rho$	-1	$2M$	0	0
8	2	3	3	-2	**L**	1	1	1	0	0	1	1	0
9	3	2	2	-3	**L**	1	1	1	0	0	1	1	0

All chiral spectra without massless charged free leptons that can be obtained for all M and N with $q_a \neq 0$ and $q_b \neq 0$. Here $M = 1{,}2$ and ρ is a free integer parameter.

Complete list of solutions

Nr.	M	N	q_a	q_b	Higgs	Q	U	D	S	X	L	E	T
6	1	4	4	-1	**L**	1	1	1	0	0	1	1	0

This realizes the $SU(4)\times U(1)$ subgroup of SU(5).

The Higgs boson breaks this to $SU(3)\times U(1)$, QCD × QED.

But this implies $SU(5)$-type proton decay at the weak scale.

A family constitutes a single, complete $SU(4)$ Higgs multiplet.

Complete list of solutions

Nr.	M	N	q_a	q_b	Higgs	Q	U	D	S	X	L	E	T
8	2	3	3	-2	**L**	1	1	1	0	0	1	1	0

This is the same $SU(3)\times SU(2)\times U(1)$ subgroup of $SU(5)$ that gives rise to the Standard Model, but with a triplet Higgs instead of a doublet Higgs.

At low energies, there is a non-abelian $SO(4) \approx SU(2)\times SU(2)$ gauge group without conserved Baryon number.

The special case $q_a = 0$ (all M,N)

Anomaly cancellation:

$SU(M) \times SU(N) \times U(1)_Y$

$Q[(V, V, 1) + (V, \overline{V}, -1)] + \text{flavor-neutral } \mathbf{U}, \mathbf{D}, \mathbf{S} \text{ matter}$

For $M = 1{,}2$ this is vectorlike (hence massive)
For $M > 3$ there is no $U(1)$ in the flavor group that is non-chiral with respect to $SU(M)$, hence no electromagnetism.

Note: we treat Higgs and dynamical breaking on equal footing

The special case $q_b = 0$ (all M, N)

Anomaly cancellation:

$SU(M) \times SU(N) \times U(1)_Y$

$Q[(V, V, 1) + (\overline{V}, V, -1)] + Y\text{-neutral } \mathbf{L}, \mathbf{E}, \mathbf{T} \text{ matter}$

For $N = 1{,}2$ this is vector-like, and hence massive

For $N \geq 3$ the candidate Higgses do not break $U(1)_{\mathrm{Y}}$

Hence the Higgs just has to break $SU(N)$ to a real group, and this is indeed possible, for example Higgs = **T**, breaking $SU(N)$ to $SO(N)$

$Q[(V, V, 1) + (\overline{V}, V, -1) + 2M(1, V, 0)]$

No charged leptons; Baryon number is gauged, so baryogenesis would be problematic.