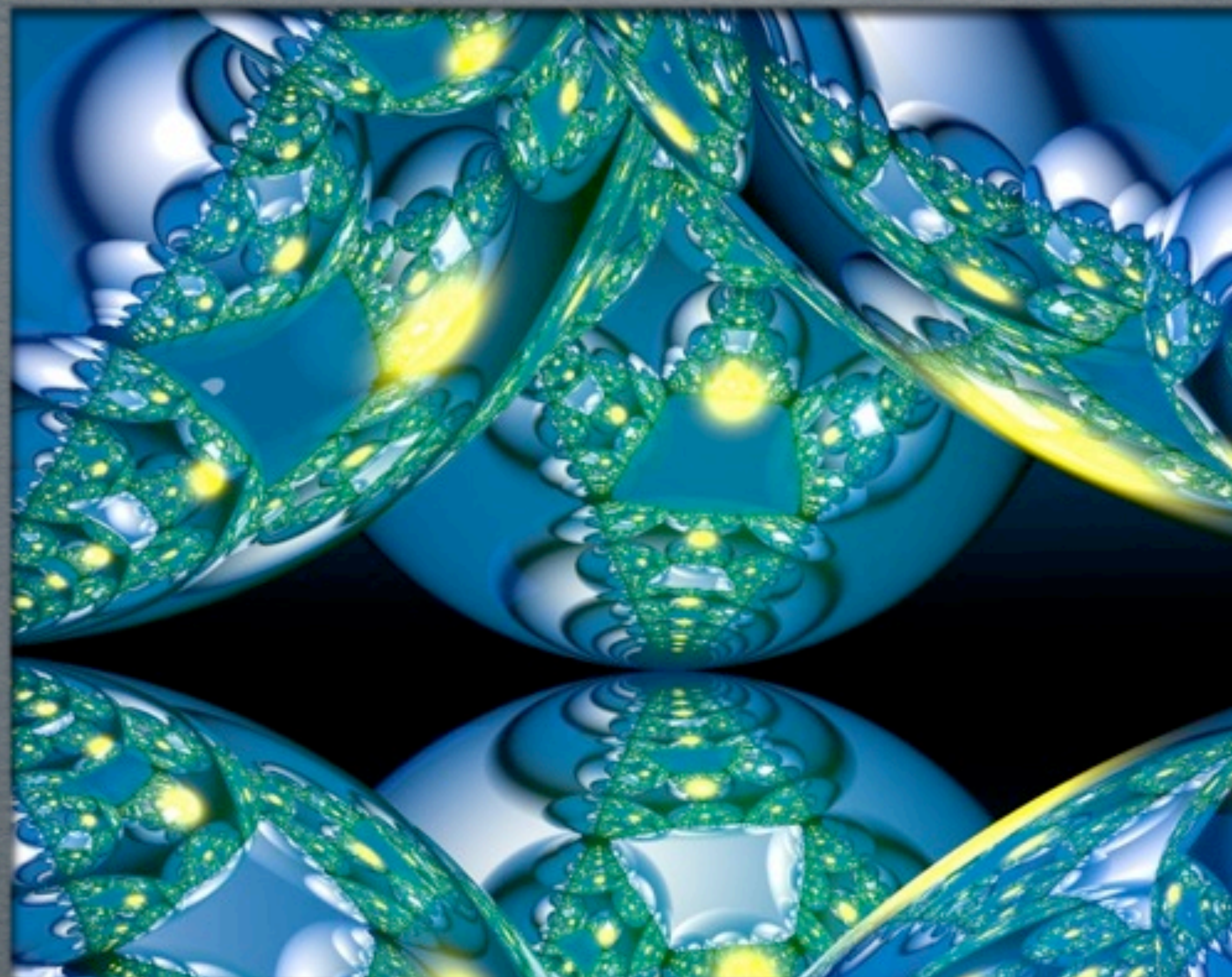
Academic Lectures

Beyond the Standard Model



Supersymmetry

Supersymmetry

A symmetry between fermions and bosons

De-Motivations

* No mass degeneracies among SM-particles with different spin:
  Not an exact symmetry.
* No SM-particles are each others partners:
  Doubling of the spectrum.
* Even that is not enough;
  Two Higgses are needed.
* Nucleon stability is not automatic.
* Huge number of parameters.

Motivations

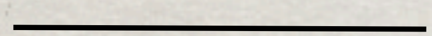
- Nice
- Finiteness
- String Theory
- Hierarchy problem
- Dark Matter
- Coupling constant convergence

Motivations

	Confidence level
Nice	~1%
Finiteness	~.001%
String Theory	~10%
Hierarchy problem	~10%
Dark Matter	~15%
Coupling constant convergence	~20%

The Hierarchy Problem

Loop correction to scalar masses



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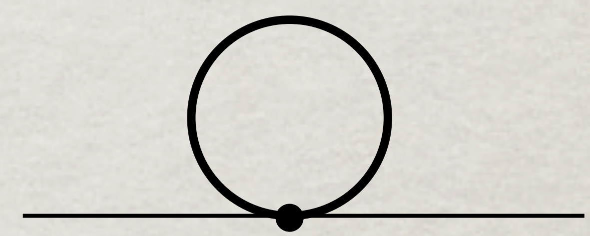


$$\int d^4k \frac{1}{k^2 - m^2} \approx g\Lambda^2$$

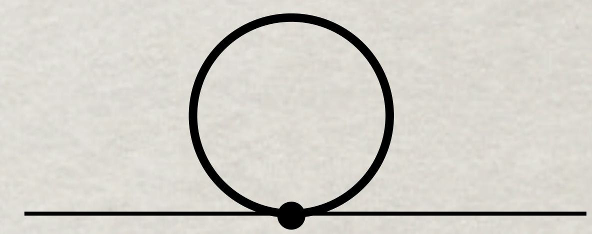
$$m^2_{\text{phys}} = m^2_{\text{bare}} + g\Lambda^2 << \Lambda^2$$

Fine tuning

Fermions vs. Scalars

Fermion  + 

$$\delta_m \propto g^2 m \log(\Lambda/m)$$

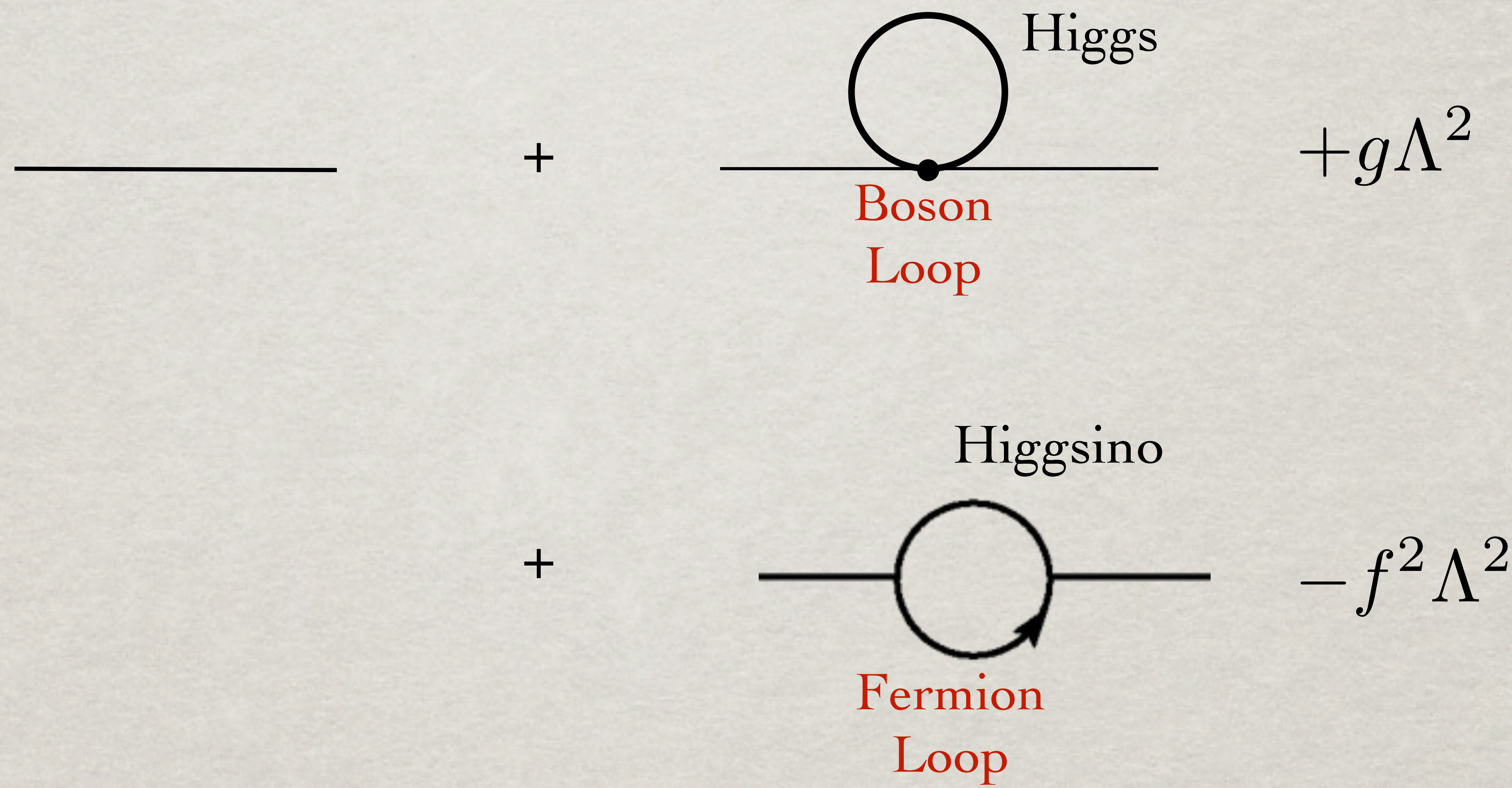
Scalar + 

$$\delta_m = g\Lambda^2$$

* Correction is quadratic instead of logarithmic
* Correction is not proportional to mass

Small fermion mass gets small corrections: no fine-tuning.

How does Susy solve this?



Fermion-Boson cancellation (if couplings match)

Technical naturalness

- Dirac Naturalness:
  Parameters should be of order 1 in natural units.

- ‘t Hooft naturalness (Technical Naturalness):
  A parameter is naturally small if setting it to zero enhances the symmetry of the theory.

Naturalness

Some examples

Natural: $\frac{m_{\rm top}}{M_Z}$

Unnatural, but technically natural: $\frac{m_e}{m_\tau}$

$$i\bar{\psi}_L\gamma^\mu D_\mu\psi_L + i\bar{\psi}_R\gamma^\mu D_\mu\psi_R + m\bar{\psi}_L\psi_R + m\bar{\psi}_R\psi_L$$

for $m = 0$ we can rotate ψ_L and ψ_R
by *separate* phases (chiral symmetry)

Unnatural (by any definition) $\frac{m_Z}{m_{\rm Planck}}$
(In SM + gravity)

Susy & The Hierarchy problem

- A priori, supersymmetry *only* solves the *technical* naturalness problem.
- It does not explain why M_{weak} is much smaller than M_{planck}.
  (cf. QCD and "dimensional transmutation")
- In fact, susy has a Higgs mass parameter that is unnatural (but technically natural): μ
- In supersymmetric theories additional mechanisms exist that do explain this ratio (require large M_{top}).
- Cosmological constant hierarchy problem much worse, and not solved by Susy

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Does this justify (more than) doubling the particle spectrum?

The Wess-Zumino model

$$\mathcal{L} = \mathcal{L}_{\text{boson}} + \mathcal{L}_{\text{fermion}}$$

$$\mathcal{L}_{\text{boson}} = \eta^{\mu\nu}\partial_\mu\phi^\dagger\partial_\nu\phi \qquad \mathcal{L}_{\text{fermion}} = i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi$$

$$\sigma^\mu = (1, \vec{\tau}) \qquad \bar{\sigma}^\mu = (1, -\vec{\tau})$$

$$\gamma^\mu = \begin{pmatrix} 0 & -i\sigma^\mu \\ -i\bar{\sigma}^\mu & 0 \end{pmatrix} \qquad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Free complex boson + left-handed (Weyl)fermion

Susy transformation on the scalar

(ψ is a 2-component spinor)

$$\delta_\varepsilon \phi = \sqrt{2}\varepsilon\psi \equiv \sqrt{2}\varepsilon^\alpha \psi_\alpha$$

ϵ is a constant spinor

$$\delta_\varepsilon \mathcal{L}_{\text{scalar}} = \sqrt{2}\left(\varepsilon \partial^\mu \psi \partial_\mu \phi^\dagger + \bar{\varepsilon} \partial^\mu \bar{\psi} \partial_\mu \phi\right)$$

Now we look for a transformation of the fermion Lagrangian to cancel this; let us try

$$\delta_\varepsilon \psi_\alpha = i\lambda(\sigma^\mu \bar{\varepsilon})_\alpha \partial_\mu \phi = i\lambda \sigma^\mu_{\alpha\dot{\beta}} \bar{\varepsilon}^{\dot{\beta}} \partial_\mu \phi$$

$$\delta_\varepsilon \mathcal{L}_{\text{fermion}} \quad = \lambda(\varepsilon\sigma^\mu\partial_\mu\phi^\dagger\bar{\sigma}^\nu\partial_\nu\psi - \bar{\psi}\bar{\sigma}^\mu\sigma^\nu\bar{\varepsilon}\partial_\mu\partial_\nu\phi)$$
$$= \lambda(-\varepsilon\sigma^\mu\bar{\sigma}^\nu\psi\partial_\mu\partial_\nu\phi^\dagger - \bar{\psi}\bar{\sigma}^\mu\sigma^\nu\bar{\varepsilon}\partial_\mu\partial_\nu\phi)$$

Because of the symmetric appearance of the derivatives we may replace $[\sigma^\mu\bar{\sigma}^\nu]_\alpha^\beta$ by

$$\frac{1}{2}[\sigma^\mu\bar{\sigma}^\nu + \sigma^\nu\bar{\sigma}^\mu]_\alpha^\beta = \eta^{\mu\nu}\delta_\alpha^\beta$$

Cancels bosonic variation if

$$\lambda = -\sqrt{2}$$

SUSY Transformations

$$\delta_\varepsilon \phi = \sqrt{2}\varepsilon\psi \equiv \sqrt{2}\varepsilon^\alpha \psi_\alpha$$

$$\delta_\varepsilon \psi_\alpha = -i\sqrt{2}\sigma^0(\sigma^\mu \bar{\varepsilon})_\alpha \partial_\mu \phi$$

Define an operator that generates this transformation on all fields

$$\left(\varepsilon Q + \bar{Q}\bar{\varepsilon}\right) X = \delta_\varepsilon X$$

$$X = \phi \text{ or } \psi$$

Susy Commutator

$$\left[\varepsilon_1 Q + \bar{Q}\bar{\varepsilon}_1, \varepsilon_2 Q + \bar{Q}\bar{\varepsilon}_2\right] = -2i(\varepsilon_2\sigma^\mu\bar{\epsilon}_1 - \varepsilon_1\sigma^\mu\bar{\epsilon}_2)\partial_\mu$$

Or, equivalently

$$\left\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\right\} = 2i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu$$

Susy Algebra

$$[Q_\alpha, P_\mu] = 0$$

$$\{Q_\alpha, Q_\beta\} = 0$$

$$\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0$$

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu$$

Non-trivial extension of the Poincaré Algebra.

Vacuum Energy

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu$$

Implies:

$$H = P^0 = \frac{1}{4}(\bar{Q}_1 Q_1 + Q_1 \bar{Q}_1 + \bar{Q}_2 Q_2 + Q_2 \bar{Q}_2)$$

Therefore

$$\langle\Psi| H |\Psi\rangle = \frac{1}{4}(|Q_1 |\Psi\rangle|^2 + |\bar{Q}_1 |\Psi\rangle|^2) + |Q_2 |\Psi\rangle|^2 + |\bar{Q}_2 |\Psi\rangle|^2)$$

If the vacuum is supersymmetric

$$Q_\alpha |0\rangle = \bar{Q}_{\dot{\alpha}} |0\rangle = 0 \longrightarrow \langle 0| H |0\rangle = 0$$

Vacuum energy is ±∞ in non-supersymmetric QFT

Supermultiplets

- **Chiral Multiplet(*)**
  complex scalar + left-handed fermion

- **Vector Multiplet**
  Vector + Majorana fermion

- **Graviton Multiplet**
  Graviton + Gravitino

(*)CPT-Conjugate:
complex scalar + right-handed fermion [not needed]

(Extended supersymmetry)

$$Q^i_\alpha; \quad i = 1, \ldots N$$

Generates larger multiplets.

- N=1: previous slide.
- N=2: smallest multiplet has two complex scalars,
  plus a left and a right-handed spinor.
  Non-chiral!
- N=4: smallest multiplet contains a vector, six complex
  scalars plus four left- and right-handed spinors.
  Finite!
- N=8: smallest multiplet contains graviton.
- N>8: smallest multiplet contains spin 5/2

Supermultiplets (N=1)

- **Chiral Multiplet(*)**
  complex scalar + left-handed fermion

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  Vector + Majorana fermion

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Physical state counting

Type	Example	# d.o.f
Real Scalar	π^0	1
Complex Scalar	π^+, π^-	2
Dirac fermion	$e_L^-, e_R^-, e_L^+, e_R^+$	4
Weyl fermion	e_L^-, e_R^+ (mass = 0)	2
Majorana fermion	ν_R, ν_L^c(charge = 0)	2
Vector boson	photon	2
Gravitino		2
Graviton		2

The SSM

SM particle	SSM partner	Multiplet
e_L^- (l_L^-)	selectron1 (slepton1)	Chiral
e_L^+ (l_L^+)	selectron2 (slepton2)	Chiral
q_L	squark1	Chiral
q_R	squark2	Chiral
photon	photino (Majorana fermion)	Vector
gluon	gluino (Eight Majorana fermions)	Vector
W^+, W^-, Z	Wino$^{\pm}$, Zino (Three Majorana fermions)	Vector
Higgs	???	Chiral

The Susy Higgs

$$\phi : \quad (1, 2, \frac{1}{2}) \rightarrow \text{ Weyl fermion}(1, 2, \frac{1}{2})_L$$

$\downarrow$ CPT

$$\phi^* : \quad (1, 2, -\frac{1}{2}) \rightarrow \text{ Weyl fermion}(1, 2, -\frac{1}{2})_L$$

The Susy Higgs

$$\phi : \quad (1, 2, \frac{1}{2}) \rightarrow \text{ Weyl fermion}(1, 2, \frac{1}{2})_L$$

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Two distinct options for supermultiplet of SM Higgs

The Susy Higgs

$$\phi : \quad (1, 2, \frac{1}{2}) \rightarrow \text{ Weyl fermion}(1, 2, \frac{1}{2})_L \qquad H_2$$

CPT ↓ $\qquad$ ~~CPT~~ ↓

$$\phi^* : \quad (1, 2, -\frac{1}{2}) \rightarrow \text{ Weyl fermion}(1, 2, -\frac{1}{2})_L \qquad H_1$$

Two distinct options for supermultiplet of SM Higgs

Both are needed to cancel anomalies

The MSSM (1)

MSSM spectrum:

- Quarks + sQuarks
- Leptons+sLeptons
- Gauge bosons + gauginos
- H_1, H_2 + Higgsinos
- **+ NOTHING**

Interactions

SSM action:

$$\int d^4x(d^2\theta\mathcal{L}_F + \text{c.c}) + \int d^4x d^4\theta\mathcal{L}_D$$

"F-terms"

"D-terms"

Origin of:

Most interactions;
Gauge kinetic terms

Scalar and fermion kinetic terms and their gauge couplings

Easy

Hard

Standard Model Lagrangian

$$-\frac{1}{4}\sum_{I=1}^{12} F^{I}_{\mu\nu}F^{\mu\nu,I} \quad + \quad i\sum_{\ell=1}^{15}\bar{\psi}_\ell \gamma^\mu D_\mu \psi_\ell$$

$$+ \quad (D_\mu\phi)^\dagger(D^\mu\phi) - \mu^2\phi^\dagger\phi - \tfrac{1}{4}\lambda(\phi^\dagger\phi)^2 \quad +$$

$$g^{\alpha\beta}_{\mathcal{U}}\bar{\psi}^{\mathcal{Q},\alpha}_L\,[\mathbf{C}\phi^*]\psi^{\mathcal{U},\beta}_R + g^{\alpha\beta}_{\mathcal{D}}\bar{\psi}^{\mathcal{Q},\alpha}_L\phi\psi^{\mathcal{D},\beta}_R + g^{\alpha\beta}_{\mathcal{E}}\bar{\psi}^{\mathcal{L},\alpha}_L\phi\psi^{\mathcal{E},\beta}_R + \text{c.c.}$$

(+ neutrino contributions)

$(+F_{\mu\nu}\tilde{F}^{\mu\nu}$ terms)

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(+ neutrino contributions)

($+F_{\mu\nu}\tilde{F}^{\mu\nu}$ terms)

Superspace

Extend space-time by four anti-commuting variables

$$\theta_\alpha, \alpha = 1,2 \qquad \bar{\theta}_\alpha, \alpha = 1,2$$ (Spinor index)

$$\{\theta_\alpha, \theta_\beta\} = 0$$

$$\{\bar{\theta}_\alpha, \theta_\beta\} = 0$$

$$\{\bar{\theta}_\alpha, \bar{\theta}_\beta\} = 0$$

$$\theta^2 \equiv \theta_1 \theta_2$$

$$\bar{\theta}^2 \equiv \bar{\theta}_1 \bar{\theta}_2$$

$$\theta_1^2 = \theta_2^2 = \bar{\theta}_1^2 = \bar{\theta}_2^2 = 0$$

Superspace integrals

$$\begin{cases} \int d^4x d^2\theta \\ \int d^4x d^2\theta d^2\bar{\theta} \equiv \int d^4x d^4\theta \end{cases}$$

Chiral Superfields

For each supermultiplet (ϕ, ψ_L) define a *Chiral Superfield*

$$\phi_L(x,\theta) = \varphi(x) + \sqrt{2}\theta\psi_L(x) + \theta^2 F(x)$$

$$F(x): \text{ auxiliary field}$$

Conjugate:

$$\phi_L^\dagger(x,\bar{\theta}) = \varphi^*(x) + \sqrt{2}\bar{\psi}_L(x)\bar{\theta} + \bar{\theta}^2 F^*(x)$$

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Chiral Superfield

$$\phi_L(x,\theta) = \varphi(x) + \sqrt{2}\theta\psi_L(x) + \theta^2 F(x)$$ + Nothing

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Vector Superfields

To describe vector bosons we need an additional kind of superfield

Vector Superfield

$$V(x,\theta,\bar{\theta}) = -\theta\rho_\mu\bar{\theta}V^\mu + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D$$

V^μ is a vector boson

λ is a Majorana fermion

D is the auxiliary field

Satisfies $V = V^\dagger$

Interactions

SSM action:

$$\int d^4x(d^2\theta\mathcal{L}_F + \text{c.c}) + \int d^4x d^4\theta \mathcal{L}_D$$

Supersymmetric if and only if

$\mathcal{L}_F$ is only a function of θ (and NOT of $\bar{\theta}$)

$\rightarrow$ $\mathcal{L}_F$ is a chiral superfield

$\mathcal{L}_D = (\mathcal{L}_D)^\dagger$

$\rightarrow$ $\mathcal{L}_D$ is a vector superfield

} **Built with Fundamental Superfields**

The superpotential

F-terms $\int d^4x(d^2\theta\mathcal{L}_F + \text{c.c})$

$d^2\theta \equiv$ "Expand in θ and keep only the quadratic terms"

$\mathcal{L}_F = \text{gauge kinetic terms} + W(\phi)$

(Superpotential)

Superpotential:

$W(\phi) =$ Any polynomial in the superfields

(but NOT their conjugates)

Contains most of the information about couplings

Rules:

- **Superpotential contains all allowed terms**
- **Renormalizability: at most order 3 in superfield**

D-terms

$$\int d^4x d^4\theta \mathcal{L}_D$$

$$\mathcal{L}_D = (\mathcal{L}_D)^\dagger$$

$d^4\theta \equiv$ "expand to order $\theta^2\bar{\theta}^2$ and take its coefficient"

Example

$$\mathcal{L}_D = \phi^\dagger e^{2gV} \phi$$

Yields

$$-|D_\mu\varphi|^2 - i\psi\sigma^\mu D_\mu\bar{\psi} + 2ig[\varphi^*\lambda\psi - \varphi\bar{\lambda}\bar{\psi}] + FF^* + g\varphi^* D\varphi$$

Example

(Wess-Zumino model + interactions)

A single chiral superfield ϕ, with superpotential

$$W(\phi) = \frac{1}{2}m\phi^2 + \frac{1}{3}\lambda\phi^3$$

$$\phi(x,\theta) = \varphi(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x)$$

Coefficient of θ^2:

$$m(\varphi F - \tfrac{1}{2}\psi^2) + \lambda(F\varphi^2 - \varphi\psi^2)$$

Kinetic terms (from D-terms)

$$-\partial_\mu\varphi\partial^\mu\varphi + i\psi\sigma^\mu\partial_\mu\bar{\psi} + FF^*$$

Elimination of auxiliary fields

Complete action:

$$-\partial_\mu\varphi\partial^\mu\varphi + i\psi\rho^\mu\partial_\mu\bar{\psi} + FF^* + [m(\varphi F - \tfrac{1}{2}\psi^2) + \lambda(F\varphi^2 - \varphi\psi^2) + \text{c.c}]$$

Equation of motion for F:

$$F = -m\varphi^* - \lambda^*(\varphi^*)^2$$

Substitute back into action:

$$\mathcal{L} = -\partial_\mu\varphi\partial^\mu\varphi + i\psi\rho^\mu\partial_\mu\bar{\psi} - \tfrac{1}{2}m(\psi^2 + \bar{\psi}^2) - \lambda\varphi\psi^2 - \lambda^*\varphi^*\bar{\psi}^2 - |m\varphi + \lambda\varphi^2|^2$$

Elimination of auxiliary fields

Complete action:

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Substitute back into action:

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Note: Zero electric charge (Majorana mass allowed)

Remarks

$$\mathcal{L} = -\partial_\mu\varphi\partial^\mu\varphi + i\psi\rho^\mu\partial_\mu\bar{\psi} - \frac{1}{2}m(\psi^2 + \bar{\psi}^2) - \lambda\varphi\psi^2 - \lambda^*\varphi^*\bar{\psi}^2 - |m\varphi + \lambda\varphi^2|^2$$

- Scalar and fermion have equal mass
- Trilinear terms can be read off directly from the superpotential ([scalar][fermion]^2)
- Quartic terms derived from cubic and quadratic terms in the superpotential. (no additional parameters).

The MSSM interactions: The good

Yukawa's

$$\mathcal{L}_Y = \quad g_{\mathcal{U}}^{\alpha\beta}\bar{\psi}_L^{\mathcal{Q},\alpha}\,[\mathbf{C}\phi^*]\psi_R^{\mathcal{U},\beta} \quad +g_{\mathcal{N}}^{\alpha\beta}\bar{\psi}_L^{\mathcal{L},\alpha}\,[\mathbf{C}\phi^*]\psi_R^{\mathcal{N},\beta} + \\ g_{\mathcal{D}}^{\alpha\beta}\bar{\psi}_L^{\mathcal{Q},\alpha}\phi\psi_R^{\mathcal{D},\beta} \quad +g_{\mathcal{E}}^{\alpha\beta}\bar{\psi}_L^{\mathcal{L},\alpha}\phi\psi_R^{\mathcal{E},\beta} + \mathrm{c.c.}$$

The MSSM interactions: The good

Yukawa's

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$(3,2,\frac{1}{6})$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\mathcal{Q}$
$(3^*,1,-\frac{2}{3})$	u_L^c	$\bar{\mathcal{U}}$
$(3^*,1,\frac{1}{3})$	d_L^c	$\bar{\mathcal{D}}$
$(1,2,-\frac{1}{2})$	$\begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$	$\mathcal{L}$
$(1,1,1)$	e_L^+	$\bar{\mathcal{E}}$
$(1,1,0)$	ν_L^c	$\bar{\mathcal{N}}$

MSSM interactions: The good

Yukawa's

$$\mathcal{L}_Y = \quad g_{\mathcal{U}}^{\alpha\beta}\bar{\psi}_L^{\mathcal{Q},\alpha}\,[\mathbf{C}\phi^*]\psi_R^{\mathcal{U},\beta} \quad + g_{\mathcal{N}}^{\alpha\beta}\bar{\psi}_L^{\mathcal{L},\alpha}\,[\mathbf{C}\phi^*]\psi_R^{\mathcal{N},\beta} + $$
$$g_{\mathcal{D}}^{\alpha\beta}\bar{\psi}_L^{\mathcal{Q},\alpha}\phi\psi_R^{\mathcal{D},\beta} \quad + g_{\mathcal{E}}^{\alpha\beta}\bar{\psi}_L^{\mathcal{L},\alpha}\phi\psi_R^{\mathcal{E},\beta} + \text{c.c.}$$

$\alpha, \beta :$ family labels

$\psi^{\mathcal{Q}} :$ fermion field

$\mathcal{Q} :$ corresponding superfield

MSSM interactions: The good

Yukawa's

$$\mathcal{L}_Y = \quad g_{\mathcal{U}}^{\alpha\beta}\bar{\psi}_L^{\mathcal{Q},\alpha}\,[\mathbf{C}\phi^*]\psi_R^{\mathcal{U},\beta} \quad +g_{\mathcal{N}}^{\alpha\beta}\bar{\psi}_L^{\mathcal{L},\alpha}\,[\mathbf{C}\phi^*]\psi_R^{\mathcal{N},\beta} + \\ g_{\mathcal{D}}^{\alpha\beta}\bar{\psi}_L^{\mathcal{Q},\alpha}\phi\psi_R^{\mathcal{D},\beta} \quad +g_{\mathcal{E}}^{\alpha\beta}\bar{\psi}_L^{\mathcal{L},\alpha}\phi\psi_R^{\mathcal{E},\beta} + \text{c.c.}$$

Note: the Higgs field ϕ is needed with and without conjugate

Hence both H_1 and H_2 are needed to get all required Yukawa's

$$g_{\mathcal{D}}\mathcal{Q}H_1\bar{\mathcal{D}} + g_{\mathcal{E}}\mathcal{L}H_1\bar{\mathcal{E}}$$

$$g_{\mathcal{U}}\mathcal{Q}H_2\bar{\mathcal{U}} + g_{\mathcal{N}}\mathcal{L}H_2\bar{\mathcal{N}}$$

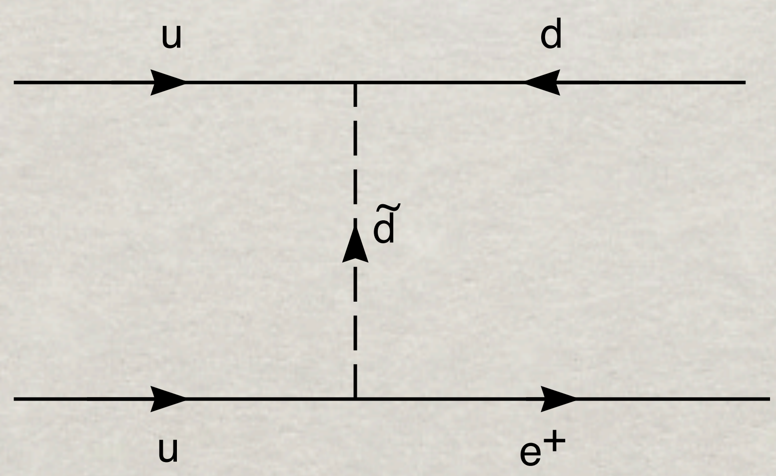
MSSM interactions: The bad

Some undesirable terms are also allowed:

$$\mathcal{QL}\bar{\mathcal{D}};\ \mathcal{LL}\bar{\mathcal{E}};\ \bar{\mathcal{U}}\bar{\mathcal{U}}\bar{\mathcal{D}};\ \mathcal{L}H_2$$

Violate Baryon number and/or Lepton number

Not allowed in SM because of odd number of fermions



Disastrous unless very small, or sparticles very heavy

MSSM interactions: The bad

How to prevent this?
Note that

$$\mathcal{Q}\mathcal{L}\bar{\mathcal{D}};\ \mathcal{L}\mathcal{L}\bar{\mathcal{E}};\ \bar{\mathcal{U}}\bar{\mathcal{U}}\bar{\mathcal{D}};\ \mathcal{L}H_2$$

Violate B-L.

Hence we may postulate B-L as an exact symmetry of nature
(not possible for B and L separately!)

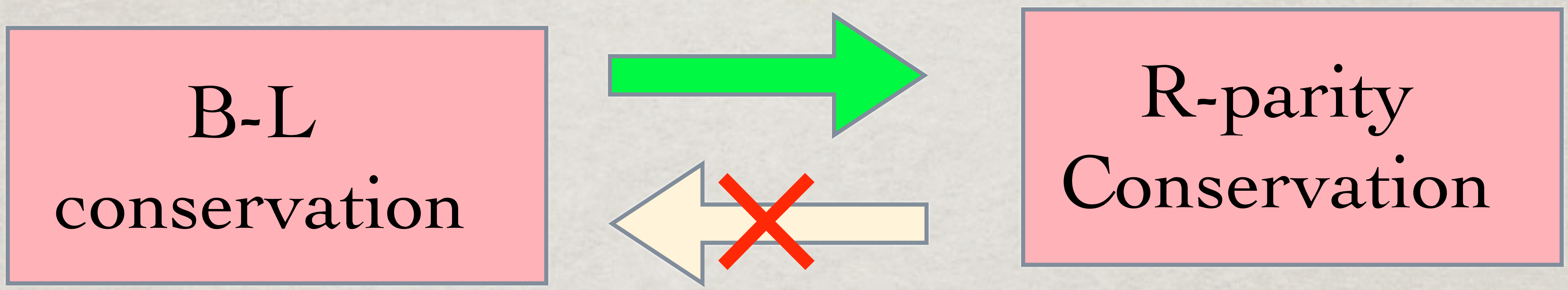
But this would forbid Majorana neutrino masses!

A less restrictive constraint is R-parity

$$R_p = (-1)^{3(B-L)+2S}$$

R-parity

$R_p = (-1)^{3(B-L)+2S}$ S: Spin



SM particles: R-parity +
Superpartners: R-parity -

Two important consequences

- **Superpartners are pair-produced**
- **Lightest superpartner is stable (LSP)**

MSSM interactions: The ugly

Another undesirable term

$$\mu H_1 H_2$$

This gives an equal mass to Higgses and Higgsinos

$$\mu^2(|h_1|^2 + |h_2|^2)$$

Problems:

- Natural size: M_{Planck} or M_{GUT}
  (but technically natural)
- Positive definite: No "Mexican hat"
  (but susy still unbroken)

"The μ problem"

Supersymmetry Breaking

At low energy, susy is broken.
At high energy it can be:

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  Then it must be symmetry of gravity as well:
  Supergravity.
  This symmetry is not a symmetry of the vacuum:
  Spontaneous breaking.

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  Accidental low-energy symmetry:
  Explicit breaking

Spontaneously broken supergravity

Vacuum not invariant:

$$Q_\alpha|0> \neq \ 0$$

This would lead to a massless
Goldstone particle in the spectrum;
Because susy is fermionic this particle is fermion:
The Goldstino.

Supergravity implies that supersymmetry is a local
symmetry. The gauge boson is a spin-3/2 particle:
The Gravitino.

Symmetry breaking now leads to a Higgs-like mechanism:
The Gravitino eats the Goldstino and become massive

Soft susy breaking

Parametrization of broken supersymmetry (independent of how it is broken).

Soft supersymmetry breaking term:
term in the action that breaks susy, but not its good properties at high energies:
"non-renormalization theorems".

In particular, these terms respect the absence of quadratic divergencies for scalar masses:
The hierarchy problem is solved in the technical sense.

Allowed soft breaking terms

Allowed:

$$m_{ij}\varphi_i\varphi_j^* \ ; \quad \alpha_{ij}\varphi_i\varphi_j + \text{c.c} \ ; \quad \beta_{ijk}\varphi_i\varphi_j\varphi_k + \text{c.c} \ ; \quad \mu(\lambda\lambda + \bar{\lambda}\bar{\lambda})$$

λ can be any gaugino in the theory

φ_i can be any scalar in the theory

All superpartners plus the Higgs can get a mass after susy-breaking, but before SU(3) × SU(2) × U(1) breaking.

Not allowed:

Fourth order scalar terms.

MSSM (2)
SOFT BREAKING PARAMETERS

$$\mathcal{L}_{\rm soft} = \quad - \sum_i m_i^2 |\varphi_i|^2$$
$$- \frac{1}{2} \sum_a M_a \bar{\lambda}_a \lambda_a$$
$$+[m_{12}^2 h_1 h_2 + c.c]$$
$$+[g_{\mathcal{U}} A_{\mathcal{U}} \varphi_{\mathcal{Q}} \varphi_{\bar{\mathcal{U}}} h_2 + g_{\mathcal{N}} A_{\mathcal{N}} \varphi_{\mathcal{L}} \varphi_{\bar{\mathcal{N}}} h_2 + c.c]$$
$$+[g_{\mathcal{D}} A_{\mathcal{D}} \varphi_{\mathcal{Q}} \varphi_{\bar{\mathcal{D}}} h_1 + g_{\mathcal{L}} A_{\mathcal{L}} \varphi_{\mathcal{L}} \varphi_{\bar{l}} h_1 + \mathrm{c.c}]$$

MSSM (2)
SOFT BREAKING PARAMETERS

$$\mathcal{L}_{\rm soft} = -\sum_i m_i^2 |\varphi_i|^2$$ (5 × 9) + 2

$$-\frac{1}{2}\sum_a M_a \bar{\lambda}_a \lambda_a$$ 3

$$+[m_{12}^2 h_1 h_2 + c.c]$$ 1

$$+[g_{\mathcal{U}} A_{\mathcal{U}} \varphi_{\mathcal{Q}} \varphi_{\bar{\mathcal{U}}} h_2 + g_{\mathcal{N}} A_{\mathcal{N}} \varphi_{\mathcal{L}} \varphi_{\bar{\mathcal{N}}} h_2 + c.c]$$

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Lots of additional parameters(*) 105 (+19 SM)

(*) Ignoring neutrino masses

MSSM (2)
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105

(+19 SM)

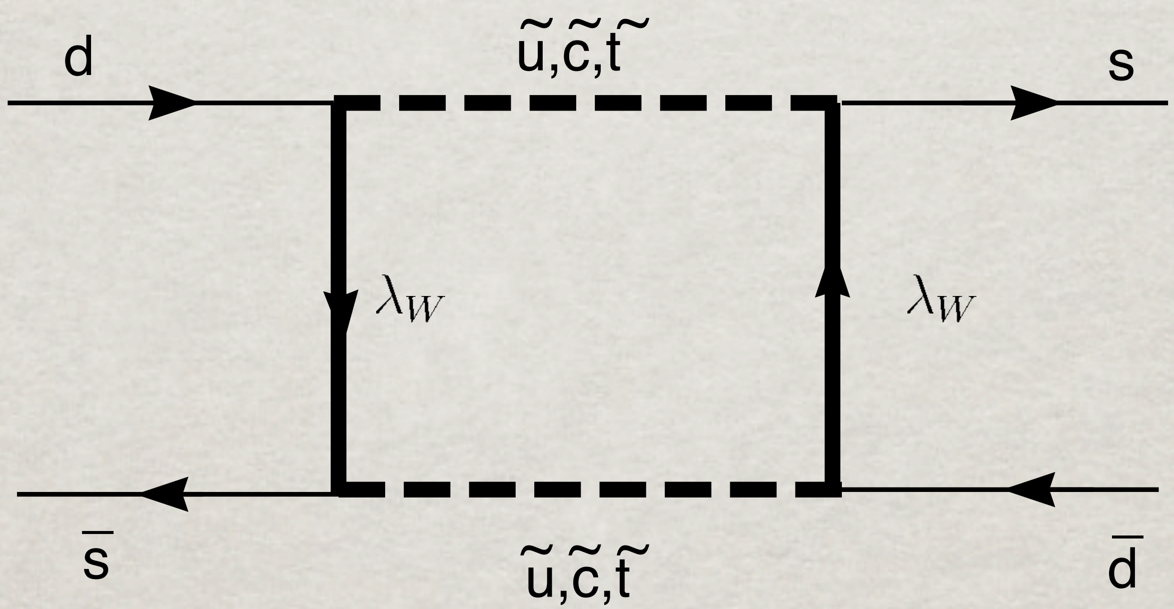
Lots of additional parameters(*)

But: just a parametrization of an unknown breaking mechanism

(*) Ignoring neutrino masses

Constraints

Just an example: the K_L-K_S mass difference



$$\frac{1}{M^2}\left(\frac{\Delta\tilde{m}^2_{\mathcal{U}}}{\tilde{m}^2_{\mathcal{U}}}\right) < 10^{-7}\ \mathrm{GeV}^{-2}$$

ADDITIONAL ASSUMPTIONS

To reduce the parameter space often some model-inspired assumptions are made for the parameter values at some high (GUT?) scale.

$$(m_i)^2 = (m_0)^2\mathbf{1} \quad \text{[Universal scalar mass]}$$

$$M_a = m_{\frac{1}{2}} \quad \text{[Universal gaugino mass]}$$

$$A_x = Am_0\mathbf{1} \quad \text{[Universal three-point coupling]}$$

Then there are just 5 additional parameters:

$$\mu, m_{1/2}, m_0^2, m_{12}^2 \text{ and } A$$

The Higgs System

The complete Higgs potential is

$$V(h_1, h_2) = \mu_1^2|h_1|^2 + \mu_2^2|h_2|^2 - (m_{12}^2 h_1 h_2 + \text{c.c})$$
$$+\frac{1}{8}(g_1^2 + g_2^2)(|h_1|^2 - |h_2|^2)^2 + \frac{1}{2}g_2^2|h_1^\dagger h_2|^2$$

h_i : Scalar in Higgs superfield H_i

$$\mu_i^2 = |\mu|^2 + m_{h_i}^2$$

g_i : Gauge coupling

$m_{h_i}^2$ can be negative

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F-terms

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D-terms

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Soft breaking

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$m_{h_i}^2$ can be negative

Higgs alignment

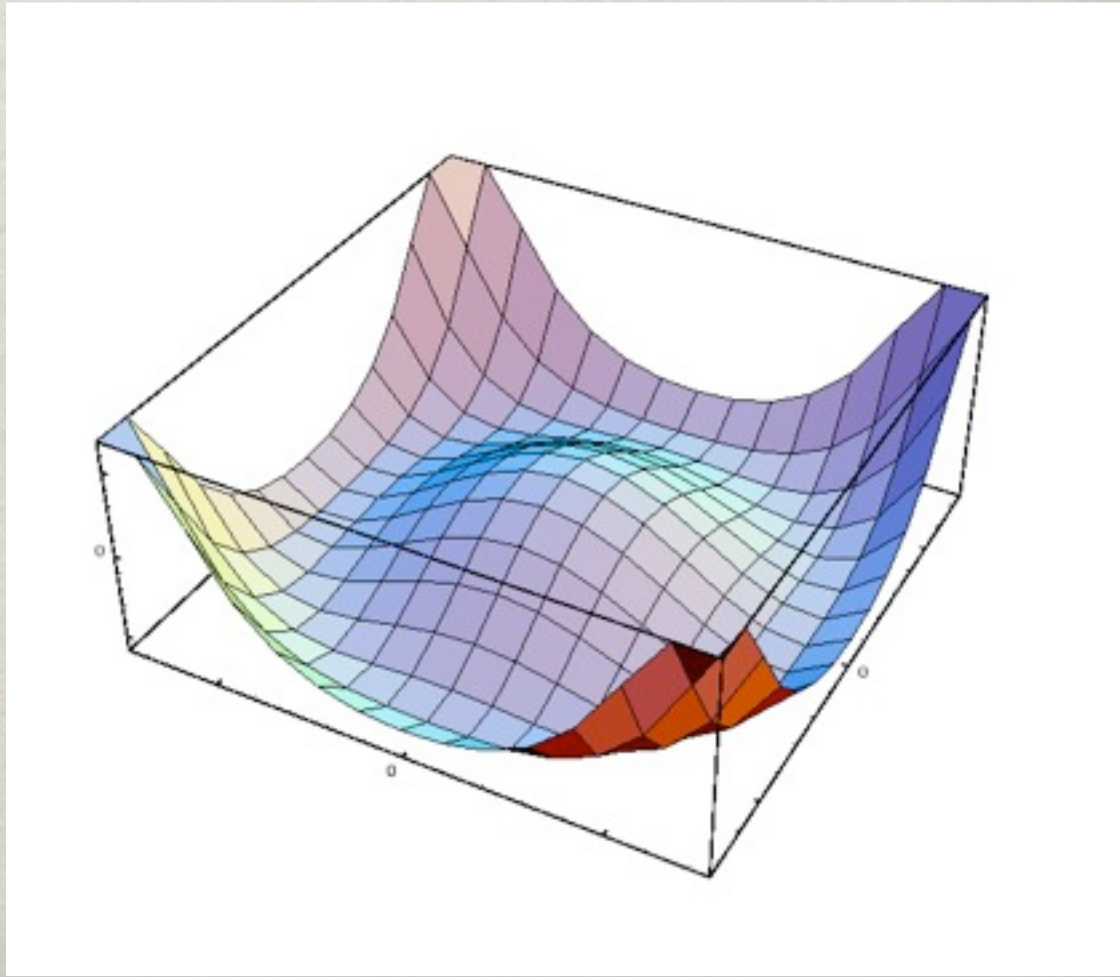
$$H_1 : (1, 2, -\frac{1}{2}) \qquad \langle h_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$$

$$T_3 + Y = 0$$

$$H_2 : (1, 2, \frac{1}{2}) \qquad \langle h_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

h_1 direction is irrelevant

h_2 direction with respect to h_1 *is* relevant:
If not exactly aligned, the photon is massive!



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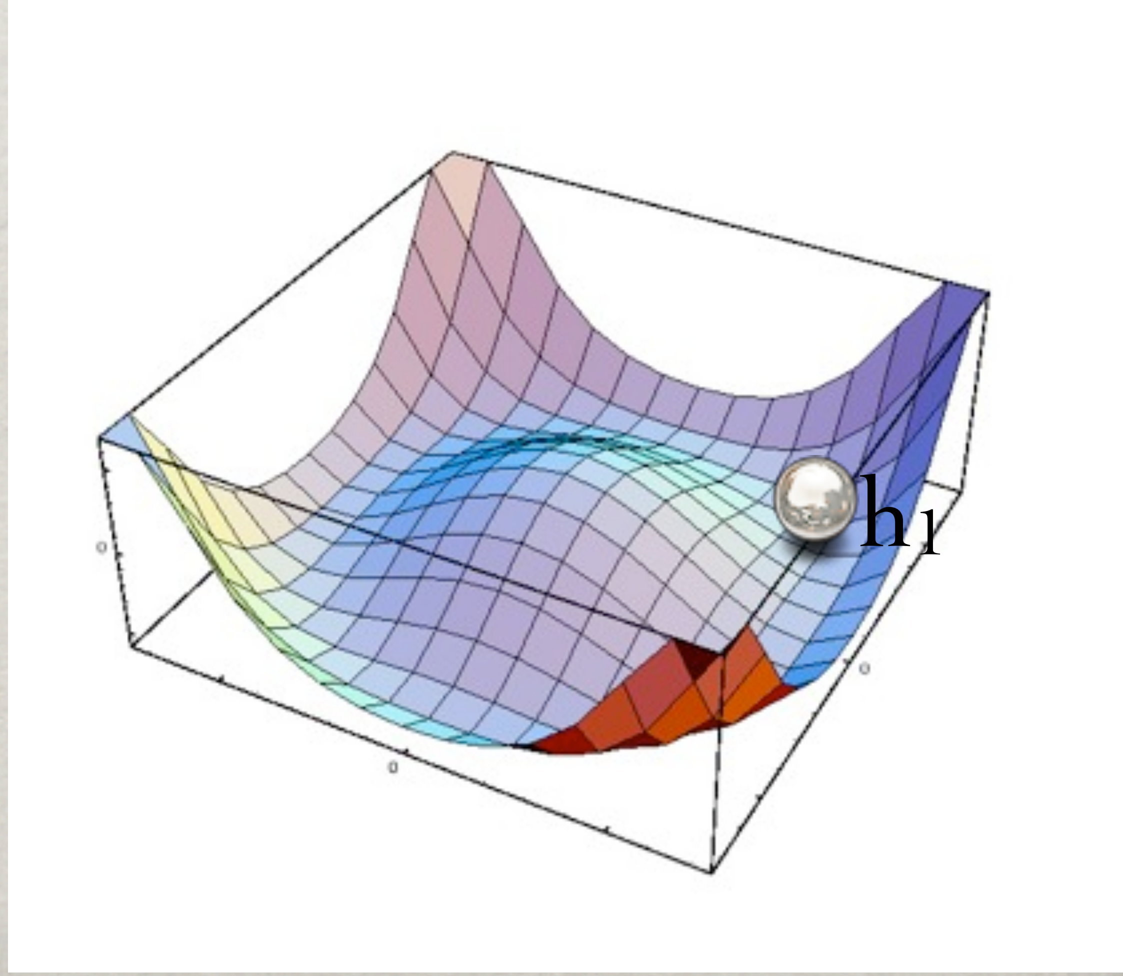

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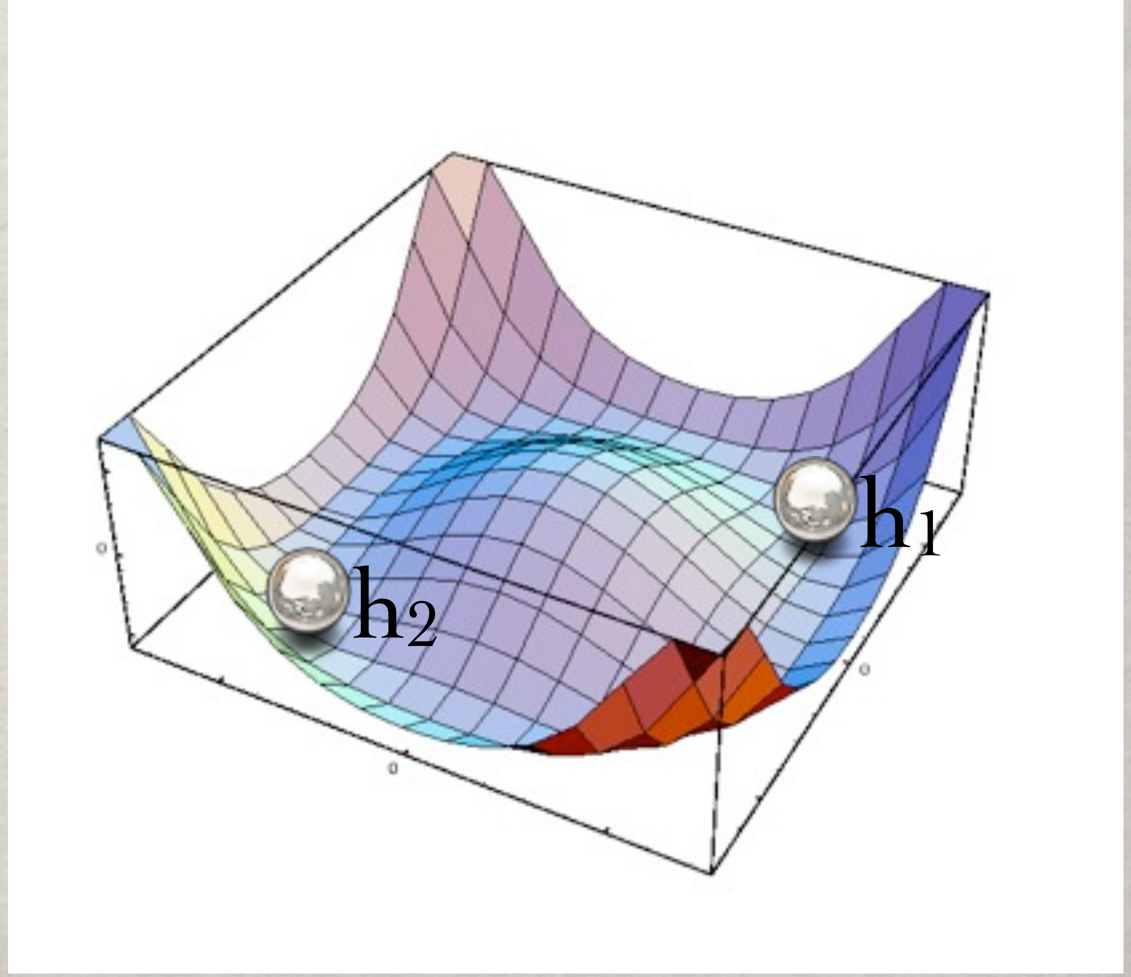


Higgs alignment

Parametrize the h_2 direction

$$\langle h_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 \ e^{i\alpha} \sin\gamma \\ v_2 \ e^{i\eta} \cos\gamma \end{pmatrix}$$

Then:

$$\begin{aligned} V(v_1, v_2, \alpha, \eta, \gamma) = \quad & \mu_1^2 v_1^2 + \mu_2^2 v_2^2 - 2m_{12}^2 v_1 v_2 \cos\eta \cos\gamma \\ & + \tfrac{1}{8}(g_1^2 + g_2^2)(v_1^2 - v_2^2)^2 + \tfrac{1}{2} g_2^2 g_2^2 v_1^2 v_2^2 \sin^2\gamma \end{aligned}$$

Minimum: $\sin\gamma = 0$

(not true for general two-Higgs potential)

Radiative Breaking

$$V(h_1, h_2) = \mu_1^2|h_1|^2 + \mu_2^2|h_2|^2 - (m_{12}^2 h_1 h_2 + \text{c.c})$$
$$+\tfrac{1}{8}(g_1^2 + g_2^2)(|h_1|^2 - |h_2|^2)^2 + \tfrac{1}{2}g_2^2|h_1^\dagger h_2|^2$$

Flat direction in quartic potential: $h_2 = e^{i\alpha} C h_1^\dagger$

$(hCh \equiv h_i \epsilon_{ij} h_j = 0)$

Quadratic terms: $(\mu_1^2 + \mu_2^2 - 2m_{12}^2 \cos\alpha)|h_1|^2$

Positivity condition: $\mu_1^2 + \mu_2^2 \geq 2|m_{12}^2|$

Negative determinant: $|m_{12}^2|^2 > \mu_1^2 \mu_2^2$

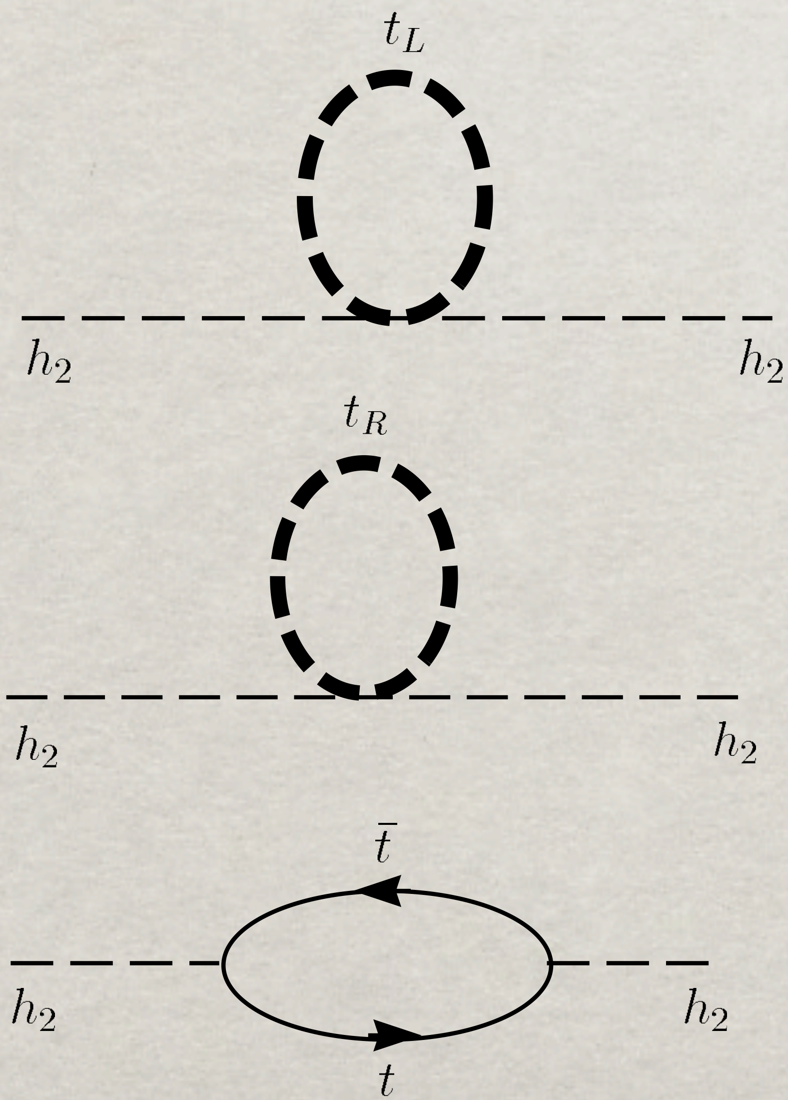
Incompatible if $\mu_1 = \mu_2$ (universal scalar masses)

Radiative Breaking

At some large scale:

$$m_{h_1} = m_{h_2} = m_0 \quad \rightarrow \quad \mu_1 = \mu_2$$

Then both masses start running separately



h_2 couples to t
h_1 couples to b

Physical Higgses

h_1,h_2: Eight real d.o.f.
Three are "eaten" by W, Z
Hence five massive scalars left (vs. just one in SM).

Electric charges per Higgs: $2 \times 0, +1, -1$

Charged Higges: two are eaten, two survive: H^+, H^-

Neutral Higges: Common phase is eaten;
Scales and relative phase survive: h_0, H_0, A_0

Physical Higgses

$$V(h_1, h_2) = \mu_1^2|h_1|^2 + \mu_2^2|h_2|^2 - (m_{12}^2 h_1 h_2 + \mathrm{c.c})$$
$$+\tfrac{1}{8}(g_1^2 + g_2^2)(|h_1|^2 - |h_2|^2)^2 + \tfrac{1}{2}g_2^2|h_1^\dagger h_2|^2$$

Invariant under $h_i \to h_i^\dagger$

Provided m_{12}^2 is real (can be chosen real w.l.o.g.)

This symmetry can be extended to an approximate CP symmetry of the full Lagrangian.

Neutral mass eigenstates are approximate CP eigenstates

Physical Higgses

$$V(h_1, h_2) = \mu_1^2 |h_1|^2 + \mu_2^2 |h_2|^2 - (m_{12}^2 h_1 h_2 + \text{c.c})$$
$$+\tfrac{1}{8}(g_1^2 + g_2^2)(|h_1|^2 - |h_2|^2)^2 + \tfrac{1}{2} g_2^2 |h_1^\dagger h_2|^2$$

Relative phase: Odd under CP, massless if $m_{12}^2 = 0$

$$m_{A^0}^2 = \frac{m_{12}^2}{\cos\beta \sin\beta}$$

$$\tan\beta \equiv \frac{v_2}{v_1}$$

Charged Higgs masses $M_{\rm w}^2 + m_{A^0}^2$

Physical Higgses

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Physical Higgses

Neutral, CP even

$$m^2_{H^0,h^0} = \tfrac{1}{2}\left(m^2_{A^0} + M^2_{\rm z} \pm \sqrt{(m^2_{A^0} + M^2_{\rm z})^2 - 4m^2_{A^0}M^2_{\rm z}\cos^2 2\beta}\right)$$

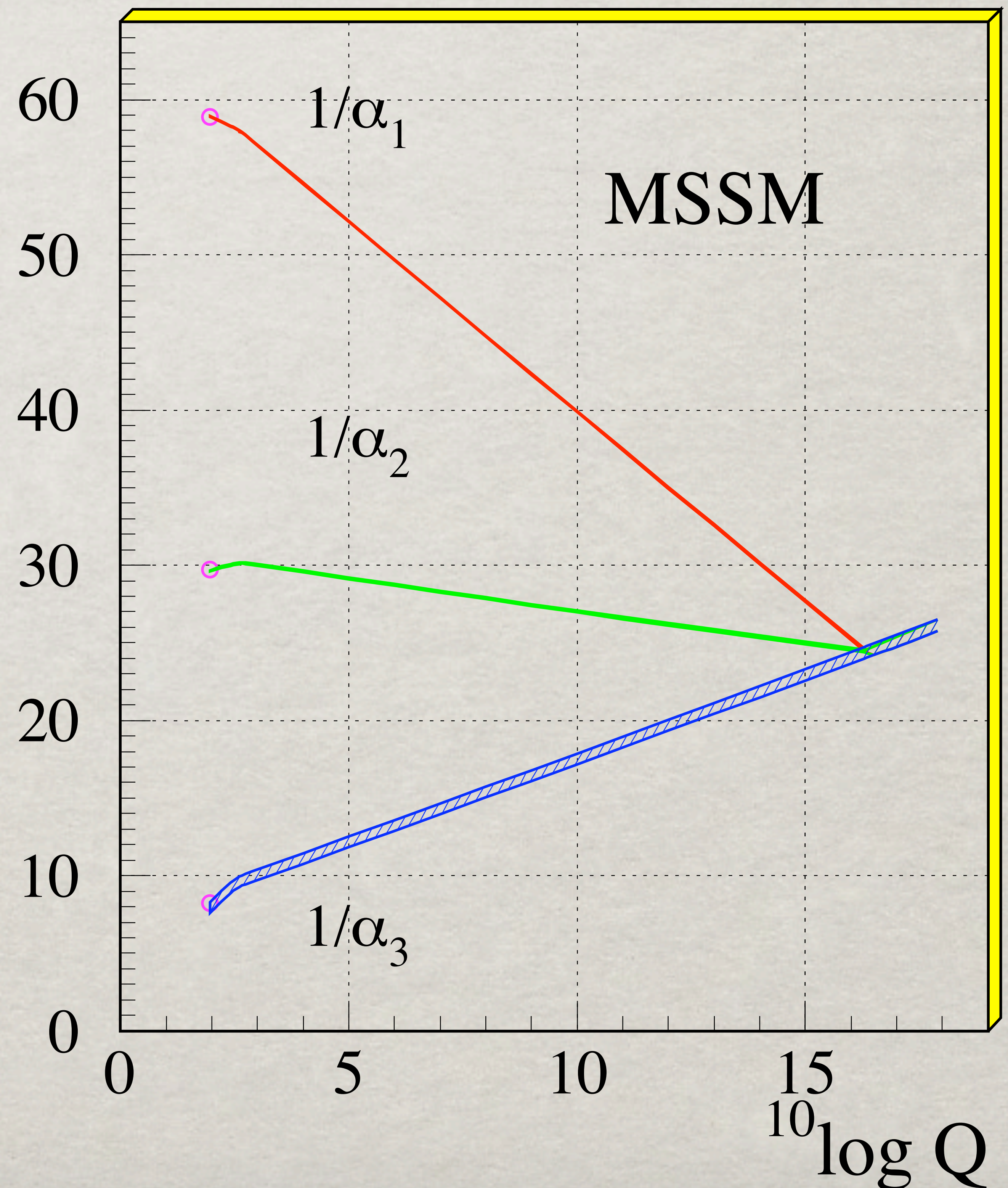
Lightest one has mass below M_Z

Loop corrections (due to top quark loops)

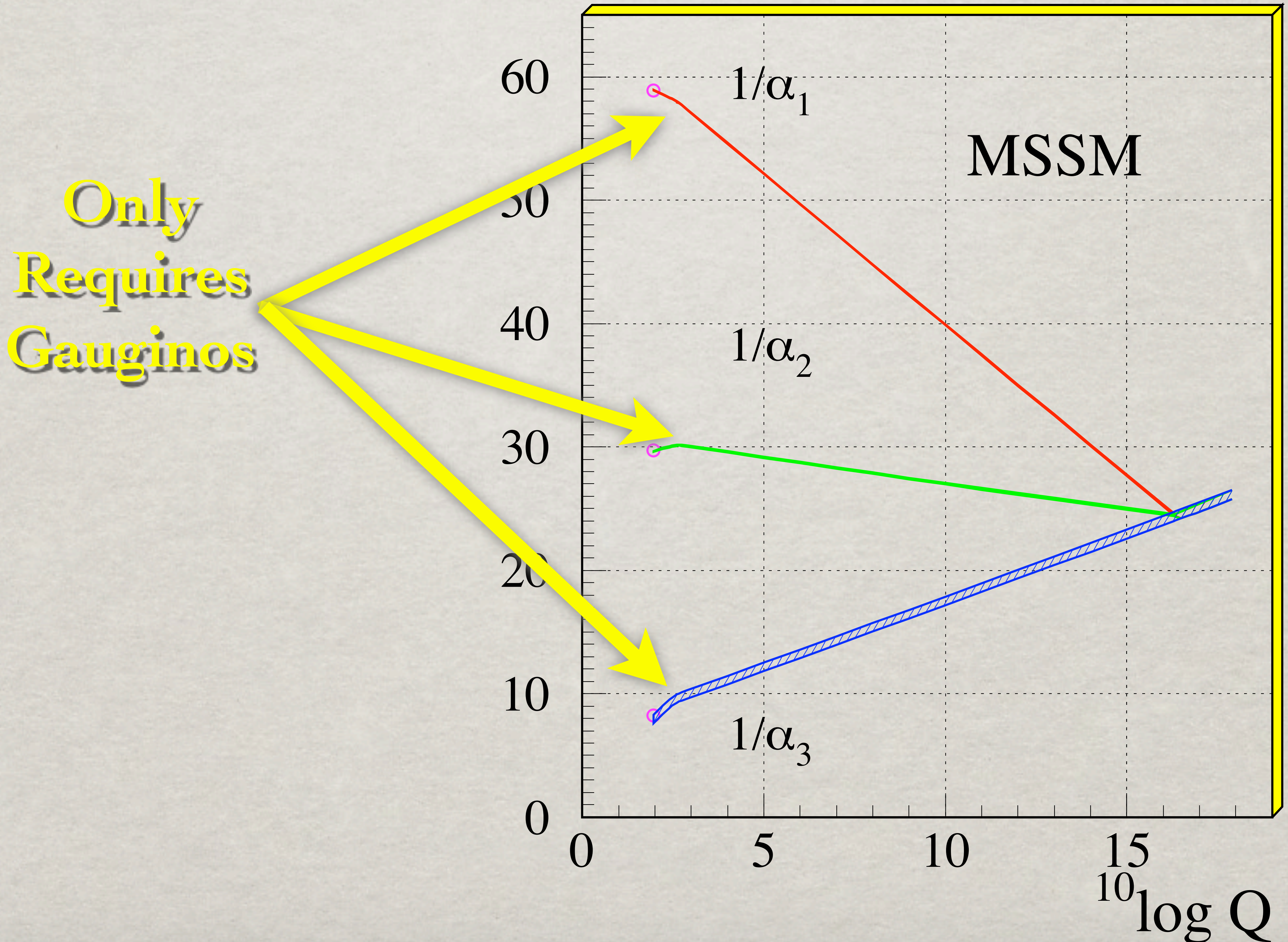
$$\Delta M^2 = \frac{3}{8\pi^2}\frac{g_2^2 m_t^4}{M^2_{\rm w}\sin^2\beta}\log(1+\frac{m_0^2}{m_t^2})$$

$$M_{h_0} < 135 \text{ GeV}$$

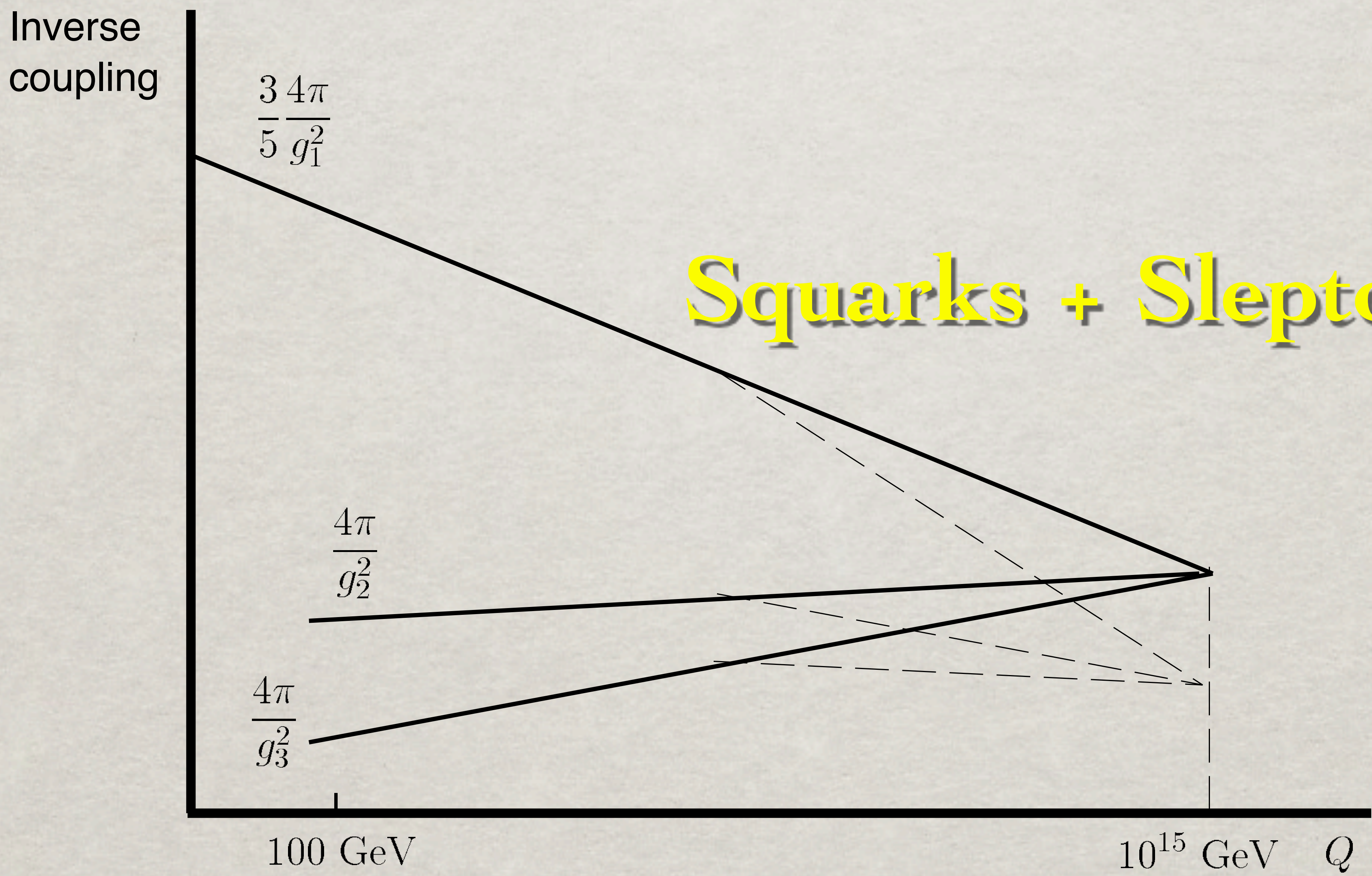
Coupling Constant Unification



Coupling Constant Unification



Split Supersymmetry



Split Supersymmetry

- Hierarchy Problem
- Dark Matter
- Coupling Constant Convergence

Split Supersymmetry

Hierarchy Problem	☑ Gauginos, sFermions
Dark Matter	☑ Gauginos
Coupling Constant Convergence	☑ Gauginos

Split Supersymmetry

- ~~Hierarchy Problem~~ — ☑ Gauginos, ~~sFermions~~
- Dark Matter — ☑ Gauginos
- Coupling Constant Convergence — ☑ Gauginos

(Arkani-Hamed, Dimopoulos, Giudice, Romanino 2004)

Susy at Hadron Colliders

Boselab

March 25, 1983

Experimental Consequences of Supersymmetry

10:00 C. Zachos - Introduction to Supersymmetry P.1

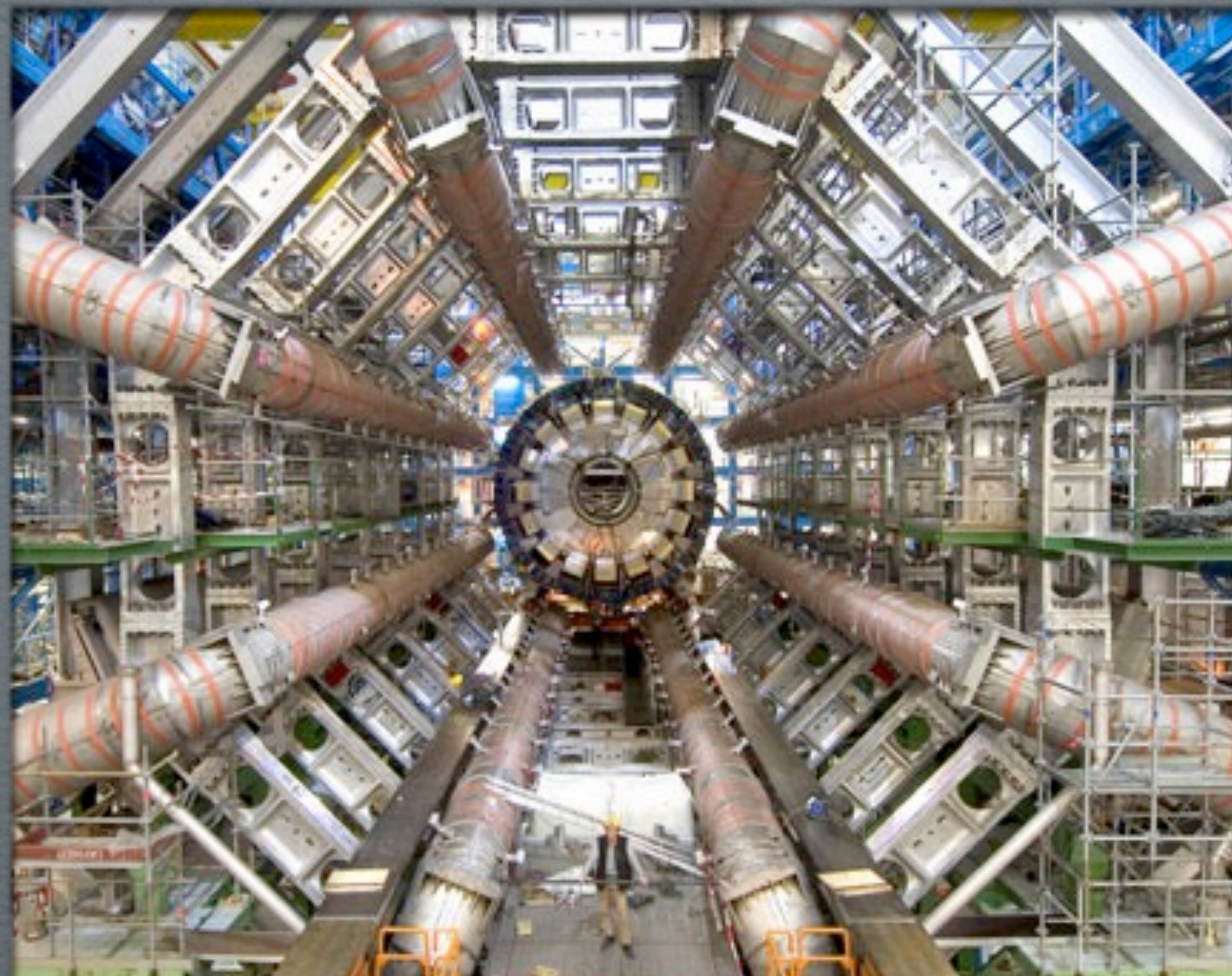
11:00 T. Taylor - Proton Decay P.12

1:30 S. Dawson - Limits on Superparticles P.24

2:30 R. Huerta - Electron-Positron Collisions P.53

3:15 Coffee Break

4:00 A. Schellekens - Hadron-Hadron Colliders P.63



Conclusion