Academic Lectures

Beyond the Standard Model

Guiding principles

- Consistency — Quantum Gravity
- Experiment — Dark Matter
  (Baryogenesis, inflation...
- Esthetics — Choices:
  Groups & Representations
- Naturalness — Cosmological constant,
  Gauge Hierarchy,
  up, down quark masses,
  electron mass, neutrino masses,
  θ_{QCD}, ...

Ideas

- Grand Unification
- Technicolor
- Composite models
- (Low energy) supersymmetry
- Peccei-Quinn mechanism
- See-Saw mechanism
- Large extra dimensions
- Little Higgs models
- String Theory
- ......

Ideas

- Grand Unification ✓
- Technicolor
- Composite models
- (Low energy) supersymmetry ✓
- Peccei-Quinn mechanism
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- Large extra dimensions
- Little Higgs models
- String Theory
- ……

Grand Unified Theories (GUTs)

Based on two (possibly accidental) facts:

- Group theoretical structure of SM gauge groups and representations
- Apparent convergence of SM couplings

Unity Of All Elementary Particle Forces.
H. Georgi, S.L. Glashow (Harvard U.) . 1974.
Published in **Phys.Rev.Lett.32:438-441,1974**.

TOPCITE = 2000+

Cited 2842 times

Hierarchy Of Interactions In Unified Gauge Theories.
H. Georgi, Helen R. Quinn, Steven Weinberg (Harvard U.) . Print-74-1122 Rev. (HARVARD), PRINT-74-1122 (HARVARD), (Received Aug 1974). 12pp.
Published in **Phys.Rev.Lett.33:451-454,1974**. (Also in *Mohapatra, R. N. (ed.), Lai, C. H. (ed.): Gauge Theories Of Fundamental Interactions*, 428-431, and in *Froggatt, C.D., Nielsen, H.B.: Origin of symmetries* 334-337)

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Abstracts

Strong, electromagnetic, and weak forces are conjectured to arise from a single fundamental interaction based on the gauge group SU(5)

Georgi-Glashow

We present a general formalism for calculating the renormalization effects which make strong interactions strong in simple gauge theories of strong, electromagnetic, and weak interactions. In an SU(5) model the superheavy gauge bosons arising in the spontaneous breakdown to observed interactions have mass perhaps as large as 10^{17} GeV, almost the Planck mass. Mixing-angle predictions are substantially modified.

Georgi-Quinn-Weinberg

The Standard Model

Input:

The Standard Model

Input:

- Quantum Field Theory

The Standard Model

Input:

- Quantum Field Theory
- Choice of Gauge Group

The Standard Model

Input:

- Quantum Field Theory
- Choice of Gauge Group
- Choice of spins and representations

The Standard Model

Input:

- Quantum Field Theory
- Choice of Gauge Group
- Choice of spins and representations
- Absence of interactions with dimension > 4

The Standard Model

Input:

- Quantum Field Theory
- Choice of Gauge Group
  $SU(3) \times SU(2) \times U(1)$
- Choice of spins and representations
- Absence of interactions with dimension > 4

The Standard Model

Input:

- Quantum Field Theory
- Choice of Gauge Group

  $SU(3) \times SU(2) \times U(1)$
- Choice of spins and representations

  3 families + Higgs + right-handed neutrinos
- Absence of interactions with dimension > 4

Dimensions

- Boson: 1
- Fermion: 3/2
- Derivative: 1

Allowed: $\partial_\mu \phi \partial^\mu \phi$ $\bar{\psi}\gamma^\mu \partial_\mu \psi$ $F_{\mu\nu}F^{\mu\nu}$

$\bar{\psi}\gamma^\mu A_\mu \psi$ $\bar{\psi}\psi$ ϕ^2, ϕ^3, ϕ^4 $\phi\bar{\psi}\psi$

Not Allowed: $(\bar{\psi}\psi)^2$ ϕ^5 $\bar{\psi}\gamma^\mu\gamma^\nu F_{\mu\nu}\psi$

Disallowed interactions have a coupling constant of dimension $(\text{mass})^{-n}$

Can be consistently omitted.

Gauge Theories

Lagrangian: $$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu,a} + i\bar{\psi}\gamma^\mu D_\mu\psi$$

Covariant derivative:

$$D_\mu = \partial_\mu - igT^aA^a_\mu$$

Normalization:

$$\mathrm{Tr}\ T^aT^b = \frac{1}{2}\delta^{ab}$$

Standard Model Lagrangian

$$-\frac{1}{4}\sum_{I=1}^{12} F^I_{\mu\nu}F^{\mu\nu,I} \quad + \quad i\sum_{\ell=1}^{15}\bar{\psi}_\ell\gamma^\mu D_\mu\psi_\ell$$

$$+ \quad (D_\mu\phi)^\dagger(D^\mu\phi) - \mu^2\phi^\dagger\phi - \tfrac{1}{4}\lambda(\phi^\dagger\phi)^2 \quad +$$

$$g^{\alpha\beta}_{\mathcal{U}}\bar{\psi}^{\mathcal{Q},\alpha}_L\,[\mathbf{C}\phi^*]\psi^{\mathcal{U},\beta}_R + g^{\alpha\beta}_{\mathcal{D}}\bar{\psi}^{\mathcal{Q},\alpha}_L\phi\psi^{\mathcal{D},\beta}_R + g^{\alpha\beta}_{\mathcal{E}}\bar{\psi}^{\mathcal{L},\alpha}_L\phi\psi^{\mathcal{E},\beta}_R + \mathrm{c.c.}$$

(+ neutrino contributions)

(+ $F_{\mu\nu}\tilde{F}^{\mu\nu}$ terms)

Standard Model Lagrangian

$$-\frac{1}{4}\sum_{I=1}^{12} F^{I}_{\mu\nu}F^{\mu\nu,I} \quad + \quad i\sum_{\ell=1}^{15}\bar{\psi}_\ell\gamma^\mu D_\mu\psi_\ell$$

3(3) 0(0)

$$+ \quad (D_\mu\phi)^\dagger(D^\mu\phi) - \mu^2\phi^\dagger\phi - \tfrac{1}{4}\lambda(\phi^\dagger\phi)^2 \quad +$$

2(2)

$$g^{\alpha\beta}_{\mathcal{U}}\bar{\psi}^{\mathcal{Q},\alpha}_L\,[\mathbf{C}\phi^*]\psi^{\mathcal{U},\beta}_R + g^{\alpha\beta}_{\mathcal{D}}\bar{\psi}^{\mathcal{Q},\alpha}_L\phi\psi^{\mathcal{D},\beta}_R + g^{\alpha\beta}_{\mathcal{E}}\bar{\psi}^{\mathcal{L},\alpha}_L\phi\psi^{\mathcal{E},\beta}_R + \text{c.c.}$$

54(13)

(+ neutrino contributions)

Parameters:
62(19)

(+ $F_{\mu\nu}\tilde{F}^{\mu\nu}$ terms) 3(1)

Running Parameters

Running Parameters

Running Parameters

The parameters cannot be computed (within SM).
They must be measured.

But the results of such a measurement are scale dependent.

This scale dependence is calculable from loop corrections.

Running Parameters

Define some reference process to measure a parameter, for example the QCD coupling.

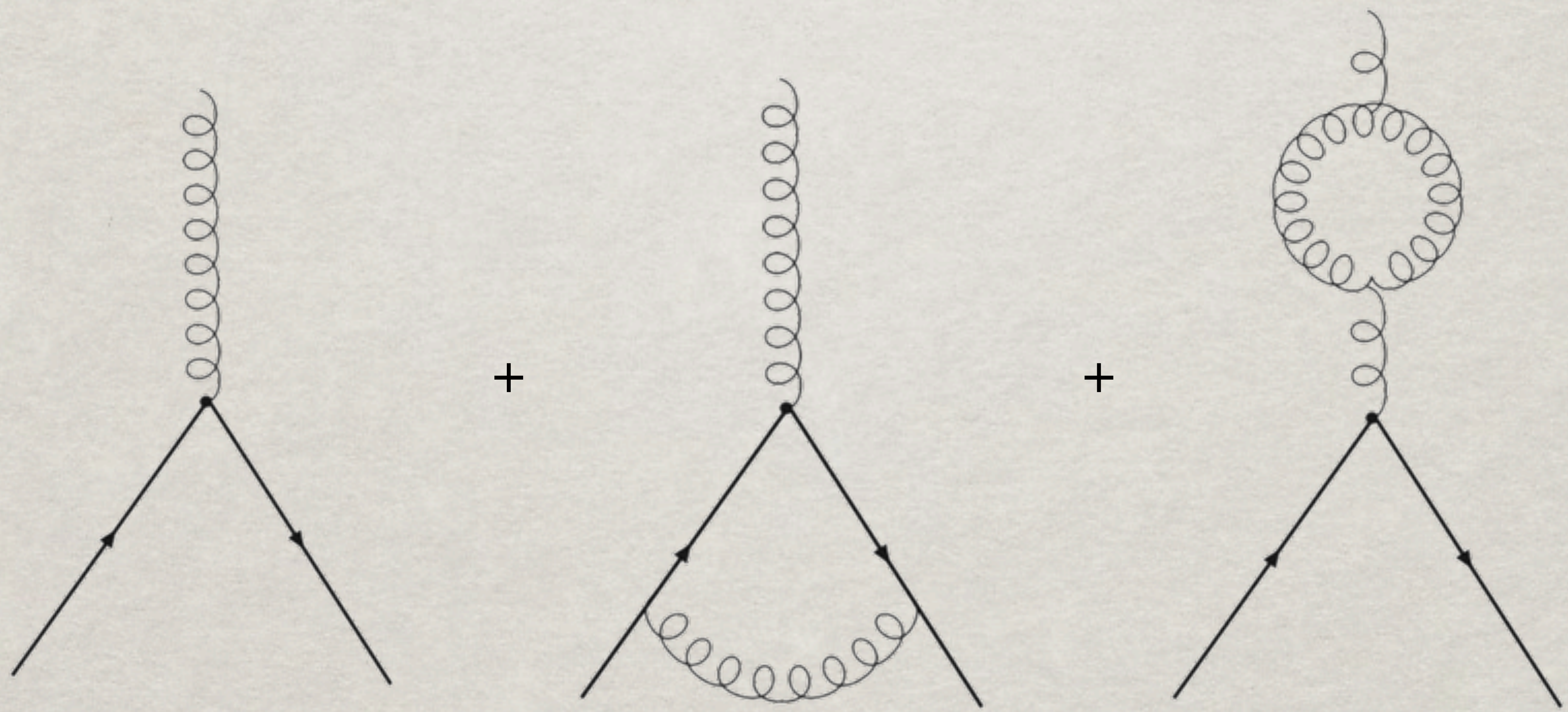
We cannot directly compare the experimental measurement to a single diagram.

There is an infinity of diagrams contributing to any process, but luckily higher orders in the coupling constant are suppressed.

Suppose the reference process is gluon-quark scattering.

Running Parameters

Some contributions to this process are:

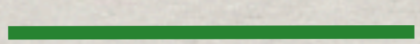


Scalar field theory

To avoid inessential complications due to spins consider a scalar field theory

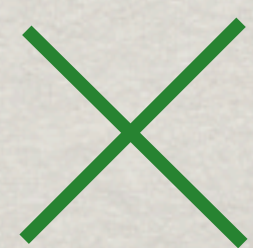
$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{24}g\phi^4$$

Feynman rules



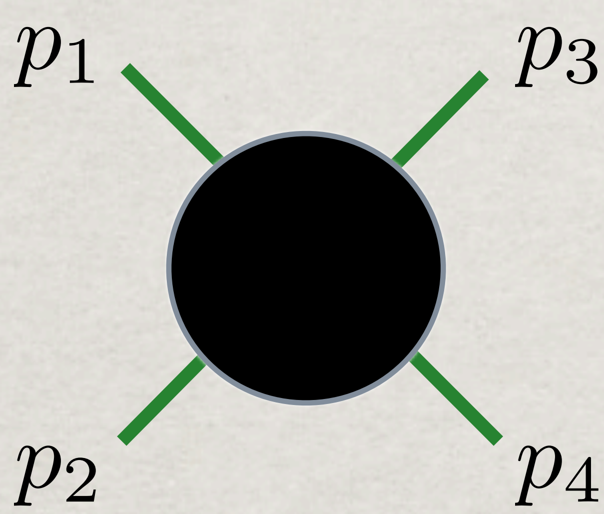
k

$$\frac{-i}{k^2 - m^2}$$



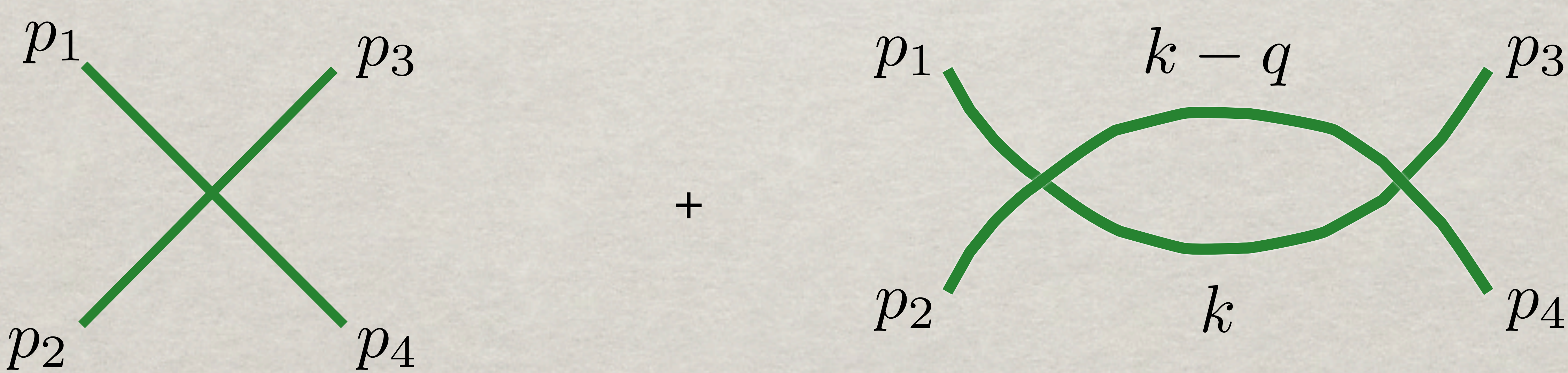
$$-ig$$

Reference process for measuring g



$$q = p_1 + p_2 = p_3 + p_4$$

Lowest order contributions:



Loop diagram:

$$\frac{1}{2}(-ig)^2 \int \frac{d^4k}{(2\pi)^4} \left(\frac{-i}{k^2 - m^2}\right) \left(\frac{-i}{(k-q)^2 - m^2}\right)$$

Feynman's trick:

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{(xA + (1-x)B)^2}$$

Change of variables:

$$l^\mu = k^\mu - xq^\mu$$

$$\frac{1}{2}\frac{g^2}{(2\pi)^4}\int_0^1 dx \int dl_0 \int d^3l \frac{1}{(l_0^2 - \vec{l}^2 + x(1-x)q^2 - m^2)^2}$$

Wick rotation: $l_0 = il_4$

Polar coordinates in 4D Euclidean space:

$$d^4l = l^3 dl d\Omega_3 \qquad \int d\Omega_3 = 2\pi^2$$

Result:

$$i\frac{g^2}{16\pi^2}\int_0^1 dx \int_0^\infty dl \frac{l^3}{(l^2 + x(1-x)Q^2 + m^2)^2}$$

$Q^2 = -q^2$

Momentum integral:

$$\int dl \frac{l^3}{(l^2+a)^2} = \frac{1}{2}\log(l^2+a) + \frac{1}{2}\ \frac{a}{l^2+a}$$

Diverges for large momenta:
Introduce a “cut-off” parameter Λ

$$i\frac{g^2}{32\pi^2}\int_0^1 dx \log\left[\left(\frac{\Lambda^2 + x(1-x)Q^2 + m^2}{x(1-x)Q^2+m^2}\right) - 1\right]$$

Note: x-integral is well-defined

Now consider the limit $m^2 << Q^2 << \Lambda^2$

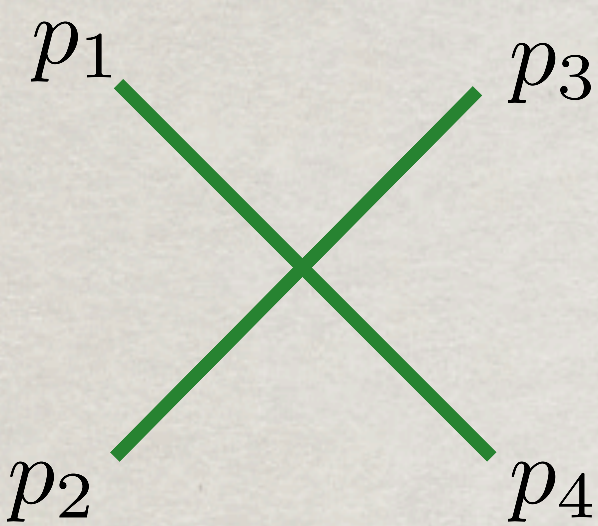
$$i\frac{g^2}{32\pi^2}\int_0^1 dx\left[\log(\frac{\Lambda^2}{Q^2}) - \log[x(1-x)+\frac{m^2}{Q^2}] - 1\right]$$

$$\approx \frac{ig^2}{32\pi^2}\log\left(\frac{\Lambda^2}{Q^2}\right)$$

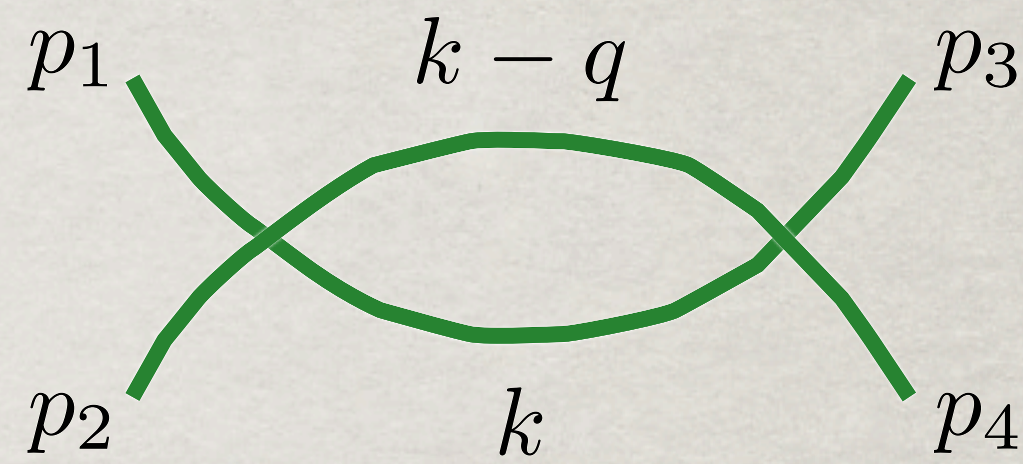
Note: if $Q^2 << m^2 << \Lambda^2$ we get

$$\frac{ig^2}{32\pi^2}\log\left(\frac{\Lambda^2}{m^2}\right)$$

and the Q^2 drops out ("decoupling")



+



=

$$-i\left[g-\frac{g^2}{32\pi^2}\log\left(\frac{\Lambda^2}{Q^2}\right)\right]$$

Note: increases with Q

Running Couplings

Suppose the cross-section for the reference process is*

$$V(Q) = g_{\text{bare}} - g_{\text{bare}}^n b_0 \log(\frac{\Lambda}{Q})$$

Now define the physical coupling constant as

$$g_{\text{phys}} \equiv V(\mu)$$

(Reference scale μ)

Inverting this relation (ignoring higher orders)

$$g_{\text{bare}} = g_{\text{phys}}(\mu) - g_{\text{phys}}^n b_0 \log(\frac{\mu}{\Lambda})$$

(*) n=2 for scalars, n=3 for gauge theories

Running Couplings

This substitution should remove all dependence on Λ in all processes

$$g_{\text{bare}} = g_{\text{phys}}(\mu) - g^{n}_{\text{phys}} b_0 \log(\frac{\mu}{\Lambda})$$

This implies the existence of powers of logs in higher orders.
These "leading logs" can be summed to all orders.

For the reference process itself we get

$$V(Q) = g_{\text{phys}}(\mu) - g^{n}_{\text{phys}}(\mu) b_0 \log(\frac{\mu}{Q}) + \text{higher order}$$

Running Couplings

$$V(Q) = g_{\text{phys}}(\mu) - g^n_{\text{phys}}(\mu) b_0 \log(\frac{\mu}{Q}) + \text{higher order}$$

The higher orders must be such that V(Q) is independent of the reference scale μ.

Hence

$$\mu \frac{d}{d\mu} V(Q) = 0$$

Or

$$\mu \frac{d}{d\mu} g_{\text{phys}}(\mu) - b_0 g^n_{\text{phys}}(\mu) = 0$$

The β-function

$$\mu \frac{d}{d\mu} g_{\mathrm{phys}}(\mu) - b_0 g_{\mathrm{phys}}^n(\mu) = 0$$

The second term is the first term in an expansion.
In general we get

$$\mu \frac{d}{d\mu} g_{\mathrm{phys}}(\mu) = \beta(g_{\mathrm{phys}}(\mu))$$

With

$$\beta(g) = b_0 g^n + b_1 g^{2n-1} + b_2 g^{3n-1} \dots$$

The renormalization group equation

Consider now any other physical quantity G.
We distinguish the explicit dependence on μ trough $\log\ (\mu/Q)$ from the dependence through g_{phys} using partial derivatives

$$0 = \mu \frac{d}{d\mu} G(Q) = \left[\mu \frac{\partial}{\partial \mu} + \mu \frac{d g_{\mathrm{phys}}(\mu)}{d\mu} \frac{\partial}{\partial g_{\mathrm{phys}}} \right] G(g_{\mathrm{phys}}, \mu, Q)$$

or

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial_g} \right] G(g, \mu, Q) = 0$$

Solving the RGE

Solution:

$$G(g,\mu,Q) = G(\bar{g}(\log(Q/\mu),Q,Q)$$

With

$$\frac{d}{dt}\bar{g}(t) = \beta(\bar{g}(t))$$

With the boundary condition

$$\bar{g}(0) = g$$

i.e. $\bar{g}(Q\!=\!\mu) = g_{\mathrm{phys}}$

Solving the RGE

Solution:

$$G(g,\mu,Q) = G(\bar{g}(\log(Q/\mu),Q,Q)$$

Kills all logs!

With

$$\frac{d}{dt}\bar{g}(t) = \beta(\bar{g}(t))$$

With the boundary condition

$$\bar{g}(0) = g$$

i.e. $\bar{g}(Q{=}\mu) = g_{\mathrm{phys}}$

One loop running

$$\frac{d}{dt}\bar{g}(t) = \beta(\bar{g}(t)) \; = b_0 g^n$$

Solution:

$$\bar{g}^{n-1}(t) = \frac{g^{n-1}}{(1-(n-1)b_0 t g^{n-1})}$$

$$t = \; \log\,(Q/\mu)$$

Asymptotic Behavior

$$\bar{g}^{n-1}(t) = \frac{g^{n-1}}{(1-(n-1)b_0 t g^{n-1})}$$

$b_0 > 0$ Running coupling increases until it reaches a singularity (Landau Pole).

$b_0 < 0$ Running coupling decreases; Asymptotic freedom.

Standard Model gauge couplings

$$\bar{g}^{n-1}(t) = \frac{g^{n-1}}{(1-(n-1)b_0 t g^{n-1})}$$

$$n = 3; \quad \frac{1}{g^2} \text{ is a linear function of } t$$

$$b_0 = \frac{1}{96\pi^2}\left(2I_2(R_f) + \frac{1}{2}I_2(R_s) - 11I_2(A)\right)$$

$$\mathrm{Tr}_R T^a T^b = \tfrac{1}{2} I_2(R)\delta^{ab}$$

$$\frac{1}{g^2}\begin{cases}\text{Decreases with t for QED, Y}\\ \text{Increases with t for QCD and Weak interactions}\end{cases}$$

Standard Model gauge couplings

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Nobel 2004

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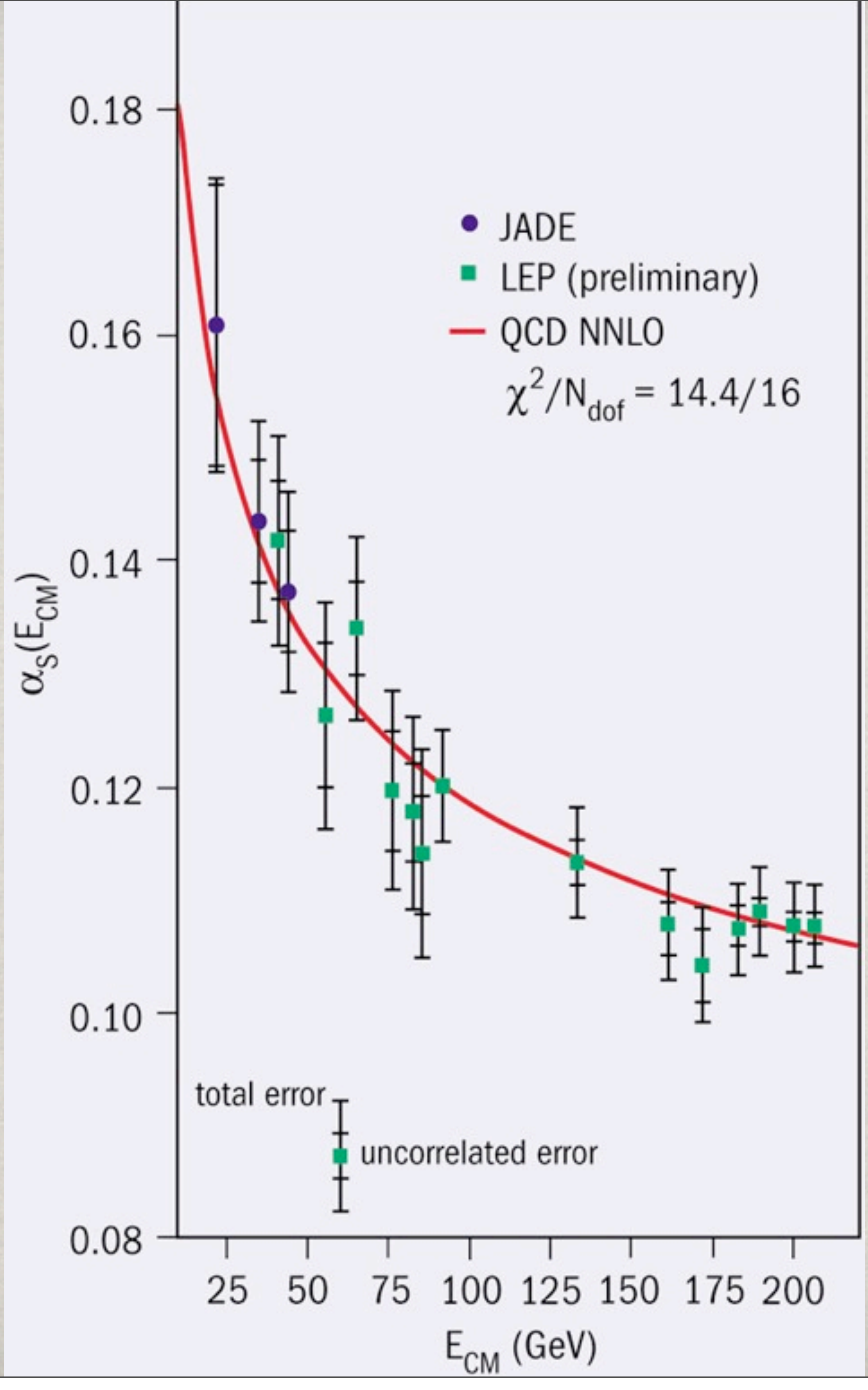
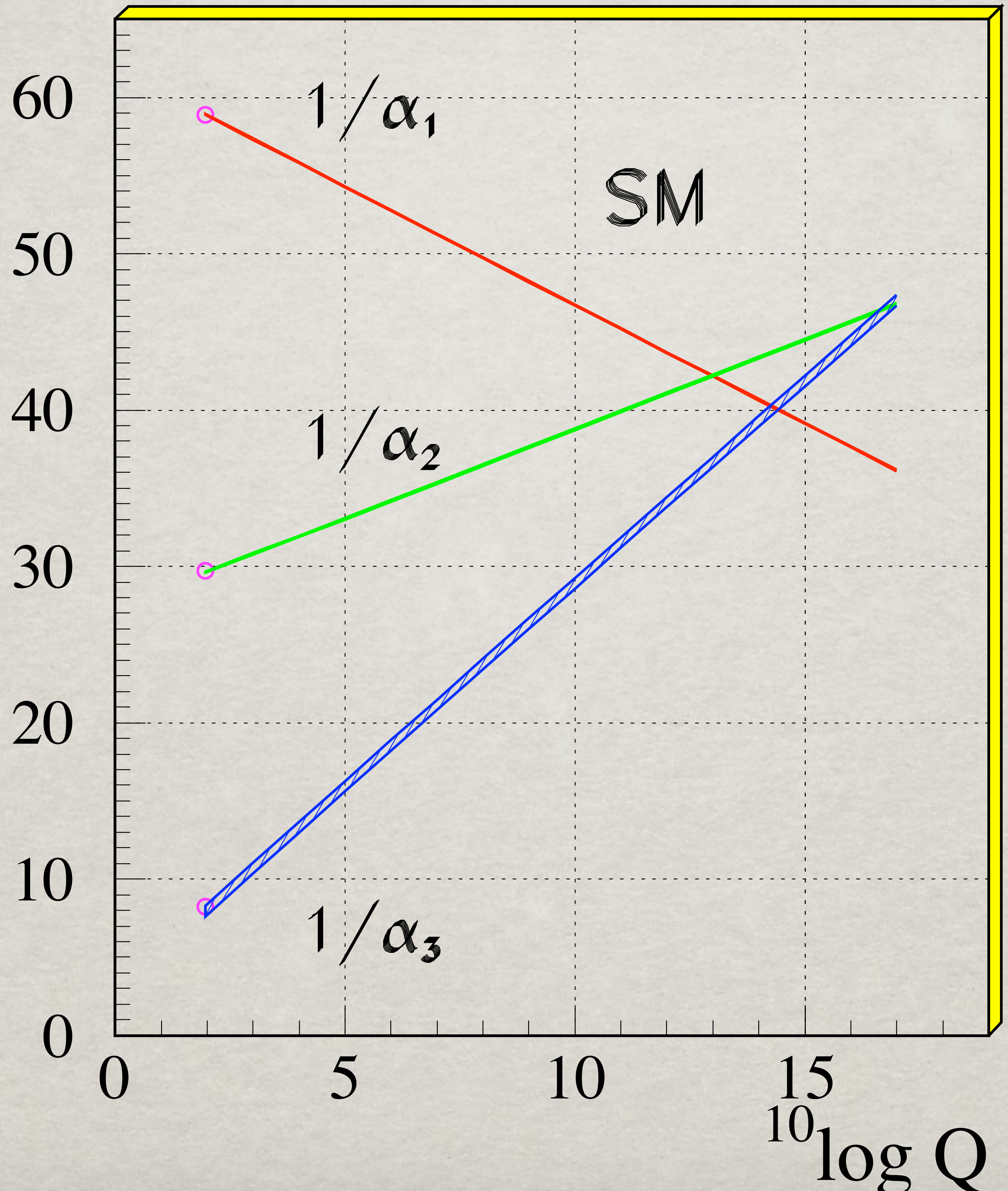
$$\frac{1}{g^2}\begin{cases}\text{Decreases with t for QED, Y}\\ \text{Increases with t for QCD and Weak interactions}\end{cases}$$



JADE
LEP (preliminary)
QCD NNLO
$\chi^2/N_{dof} = 14.4/16$
0.18
0.16
0.14
0.12
0.10
0.08
$\alpha_S(E_{CM})$
total error
uncorrelated error
25 50 75 100 125 150 175 200
E_{CM} (GeV)



(1991)

Remarks

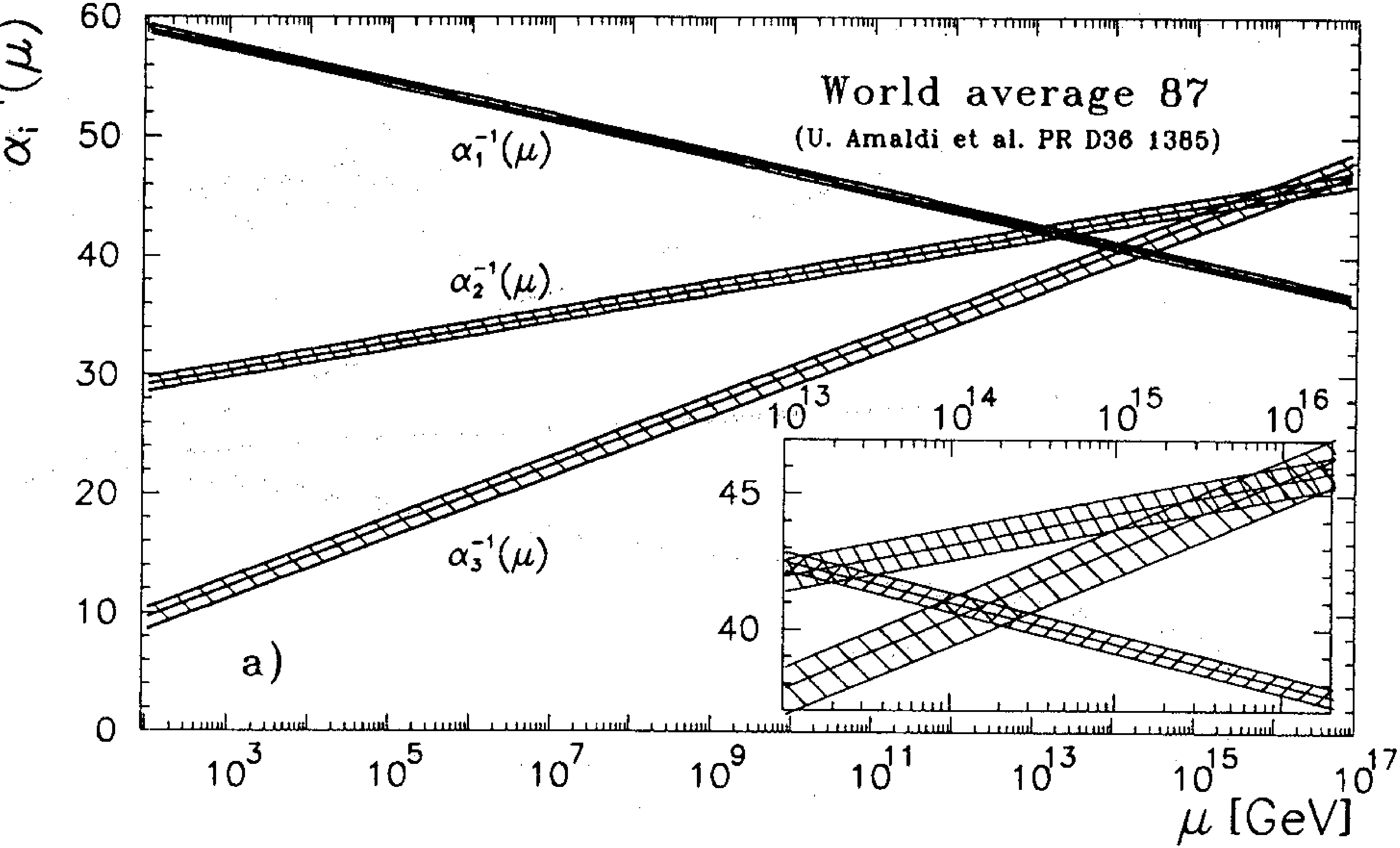
- The top line represents $U(1)_Y$, not QED ($Q=T_3+Y$)
- Contributing matter: 3 families + Higgs
- Some choice of normalization*
- One-loop running only

$$(*) \quad i\bar{\psi}\gamma^\mu(\partial_\mu - eYA_\mu)\psi; \quad [Y,Y]=0$$

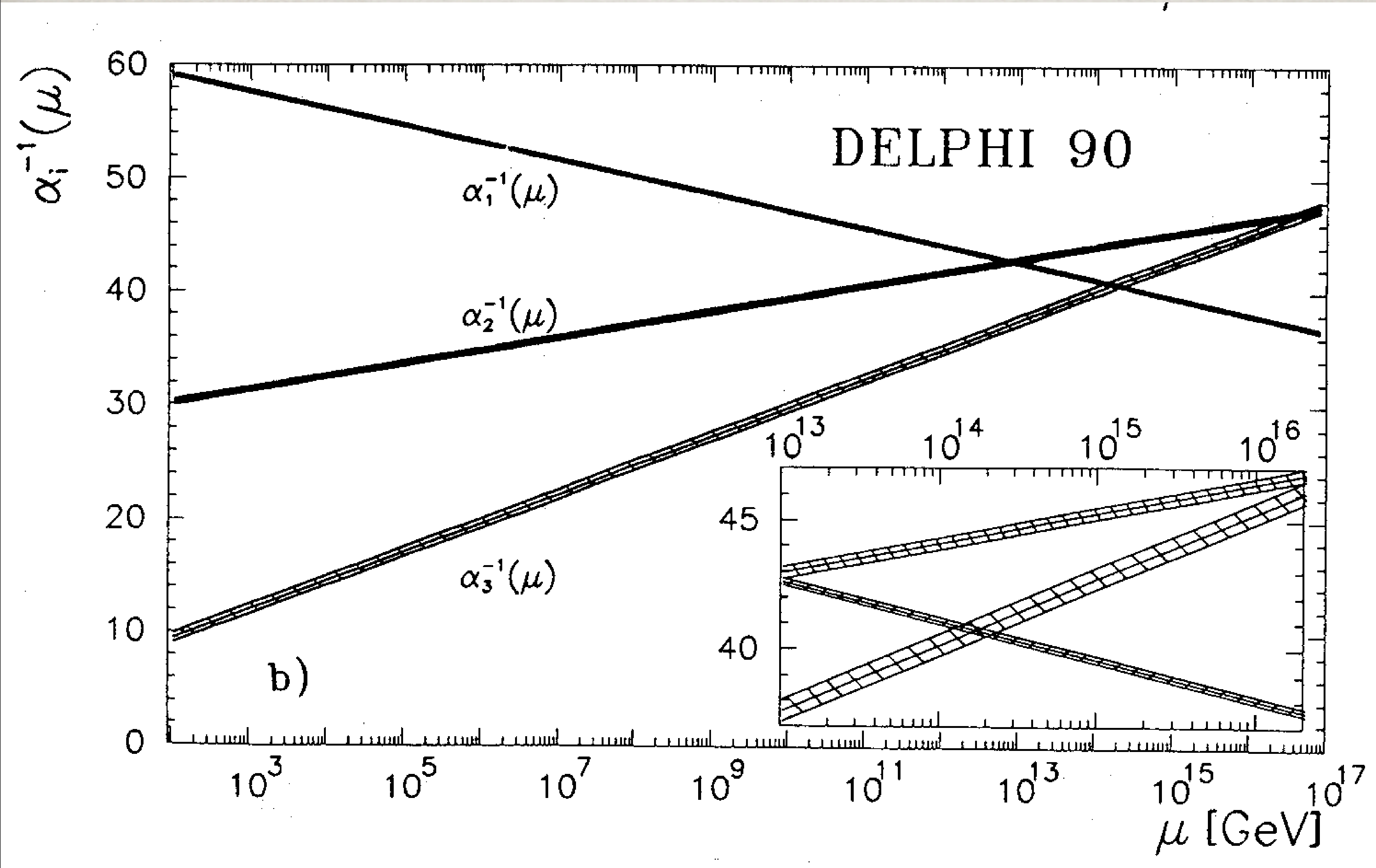
No canonical normalization

$$i\bar{\psi}\gamma^\mu(\partial_\mu - gT^aA^a_\mu)\psi; \quad [T^a,T^b]=if^{abc}T^c$$

Amaldi, de Boer and Furstenau (1991)



Amaldi, de Boer and Furstenau (1991)



Standard Model Group Structure

Some group theory

$$U \approx 1 - i\theta^a T^a$$

U: Group element (Unitary)
T: Lie algebra generator (Hermitean)

Generators T^a appear in gauge couplings.
They satisfy the relation

$$[T^a, T^b] = i f^{abc} T^c$$

Conjugate representation

$$\begin{aligned} U &\rightarrow U^* \\ T^a &\rightarrow -(T^a)^* \end{aligned}$$

Lie Groups

Group	Number of Generators
SU(N)	N^2-1
SO(N)	½N(N-1)
Sp(N)	½N(N+1)
E_6	78
E_7	133
E_8	248
G_2	14
F_4	52

Left-handed representation

The Standard Model is written in terms of left- righthanded quarks and leptons.

But instead of the electron field one could use the positron field.

This is just a change of variables:

$$\psi = C^{-1}(\gamma^0)^T(\psi^c)^* = C^\dagger(\bar{\psi}^c)^T$$
$$\bar{\psi} = -(\psi^c)^T C \ ,$$

C is a unitary matrix that satisfies

$$\gamma_\mu^T = -C\gamma_\mu C^{-1}$$

Some representations

Vector: $SU(N), SO(N), Sp(N)$: dimension N

Adjoint: $(T^a)_{bc} = -i f^{abc}$

dimension = number of generators

Spinor:

$SO(2n)$: dimension 2^{n-1}

$SO(2n+1)$: dimension 2^n

Transformation of chiral spinors

$$\gamma_\mu^T = -C\gamma_\mu C^{-1}$$

The matrix

$$\gamma_5 \equiv \gamma^5 = i\gamma_0\gamma_1\gamma_2\gamma_3$$

transforms as follows

$$C\gamma_5 C^{-1} = (\gamma_5)^T$$

The chiral projection operators are

$$P_L = \tfrac{1}{2}(1+\gamma_5); \quad P_R = \tfrac{1}{2}(1-\gamma_5)$$

therefore:

$$\psi_R = P_R\psi = P_R C^{-1}(\gamma^0)^T(\psi^c)^* = C^{-1}(P_R)^T(\gamma^0)^T(\psi^c)^*$$
$$= C^{-1}(\gamma^0)^T(P_L)^T(\psi^c)^* = C^{-1}(\gamma^0)^T(P_L)^*(\psi^c)^* = C^{-1}(\gamma^0)^T(\psi_L^c)^*$$

Transformation of gauge couplings

A right-handed quark/lepton gauge coupling

$$i\bar{\psi}_R\gamma^\mu D_\mu\psi_R = -i(\psi_L^c)^T C\gamma^\mu D_\mu C^\dagger(\bar{\psi}_L^c)^T$$
$$= -i(\psi_L^c)^T C\gamma^\mu D_\mu C^{-1}(\bar{\psi}_L^c)^T$$
$$= i(\psi_L^c)^T(\gamma^\mu)^T D_\mu(\bar{\psi}_L^c)^T$$

with $\quad D_\mu = \partial_\mu - igT^a A_\mu^a$

Is transformed to:

$$i(\bar{\psi}^c)_L\gamma^\mu(\partial_\mu + ig(T^a)^* A_\mu^a)\psi_L^c$$

Conjugate representations
(opposite charge)

$$U = 1 - i\theta^a T^a$$
$$U^* = 1 + i\theta^a (T^a)^*$$

Standard model in left-handed representation

left and right-handed fields

$$SU(3) \times SU(2) \times U(1)_Y \to SU(3) \times U(1)_{\text{em}}$$

$$Q = T_3 + Y$$

$$(3, 2, \tfrac{1}{6})_L \to (3, \tfrac{2}{3})_L + (3, -\tfrac{1}{3})_L$$

$$(3, 1, \tfrac{2}{3})_R \to (3, \tfrac{2}{3})_R$$

$$(3, 1, -\tfrac{1}{3})_R \to (3, -\tfrac{1}{3})_R$$

$$(1, 2, -\tfrac{1}{2})_L \to (1, -1)_L + (1, 0)_L$$

$$(1, 1, -1)_R \to (1, -1)_R$$

$$(1, 1, 0)_R \to (1, 0)_R$$

Left-handed only

$$SU(3) \times SU(2) \times U(1)_Y$$

$$(3, 2, \tfrac{1}{6}) \quad \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$(3^*, 1, -\tfrac{2}{3}) \quad u_L^c$$

$$(3^*, 1, \tfrac{1}{3}) \quad d_L^c$$

$$(1, 2, -\tfrac{1}{2}) \quad \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$$

$$(1, 1, 1) \quad e_L^+$$

$$(1, 1, 0) \quad \nu_L^c$$

Advantage: allows additional internal symmetries

SU(5)

Unitary 5 x 5 matrices with determinant 1.

Standard model embedding:

$$U = \begin{pmatrix} U_3 & 0 \\ 0 & U_2 \end{pmatrix}$$

$$U_3 = e^{i\phi}\hat{U}_3 \qquad \text{QCD}$$

$$U_2 = e^{i\chi}\hat{U}_2 \qquad \text{Weak}$$

$$3\phi + 2\chi = 0 \bmod 2\pi$$

One phase left free: $U(1)_Y$

$$\mathrm{diag}(e^{-\frac{1}{3}i\phi}, e^{-\frac{1}{3}i\phi}, e^{-\frac{1}{3}i\phi}, e^{\frac{1}{2}i\phi}, e^{\frac{1}{2}i\phi})$$

Gauge coupling

Lagrangian:

$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu,a} + i\bar{\psi}_L\gamma^\mu D_\mu\psi_L \qquad (a = 1, \ldots, 24)$$

Covariant derivative:

$$D_\mu = \partial_\mu - igT^a A^a_\mu$$

Normalization: (Vector Representation) $\quad \mathrm{Tr}\; T^a T^b = \frac{1}{2}\delta^{ab}$

$$\text{24 gauge bosons} \begin{cases} \text{8 gluons} \\ + W^+ + W^- + Z \\ + \text{photon} \\ + \text{12 additional ones (X, Y)} \end{cases}$$

Content of the (5)

$$\mathbf{5} \to (3, 1, -\frac{1}{3}x) + (1, 2, \frac{1}{2}x) \quad x = ???$$

Normalization:

$$\mathrm{Tr}\ T^a T^b = \frac{1}{2}\delta^{ab}$$

Implies standard normalization for QCD, Weak, but also:

$$T_Y = \sqrt{3/5}\ \mathrm{diag}\ (-\tfrac{1}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}, \tfrac{1}{2}, \tfrac{1}{2})$$

Coupling relation

$$i\bar{\psi}_L(\partial_\mu - ig_5 T_Y A^Y_\mu)\psi_L$$

=

$$i\bar{\psi}_L(\partial_\mu - i\sqrt{3/5}\; g_5 Y A^Y_\mu)\psi_L$$

=

$$i\bar{\psi}_L(\partial_\mu - ig_1 Y A^Y_\mu)\psi_L$$

Therefore

$$g_1 = \sqrt{3/5} g_5$$

$$g_2 = g_3 = g_5$$

$$\sin^2\theta_W = \frac{g_1^2}{g_1^2 + g_2^2} = \frac{3}{8}$$

Content of the (5)

$$(3, 1, -\tfrac{1}{3}) + (1, 2, \tfrac{1}{2})$$

Not a left-handed SM particle,
so use the (5*) instead:

$$(3^*, 1, \tfrac{1}{3}) + (1, 2, -\tfrac{1}{2})$$

Content of the (5)

$$(3, 1, -\tfrac{1}{3}) + (1, 2, \tfrac{1}{2})$$

Not a left-handed SM particle, so use the (5*) instead:

$$(3^*, 1, \tfrac{1}{3}) + (1, 2, -\tfrac{1}{2})$$

$(3, 2, \frac{1}{6})$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$
$(3^*, 1, -\frac{2}{3})$	u_L^c
$(3^*, 1, \frac{1}{3})$	d_L^c
$(1, 2, -\frac{1}{2})$	$\begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$
$(1, 1, 1)$	e_L^+
$(1, 1, 0)$	ν_L^c

Content of the (5)

$$(3, 1, -\tfrac{1}{3}) + (1, 2, \tfrac{1}{2})$$

Not a left-handed SM particle, so use the (5*) instead:

$$(3^*, 1, \tfrac{1}{3}) + (1, 2, -\tfrac{1}{2})$$

$(3, 2, \frac{1}{6})$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$
$(3^*, 1, -\frac{2}{3})$	u_L^c
$\boxed{(3^*, 1, \frac{1}{3})}$	d_L^c
$\boxed{(1, 2, -\frac{1}{2})}$	$\begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$
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$\boxed{(3^*, 1, \frac{1}{3})}$	d_L^c
$\boxed{(1, 2, -\frac{1}{2})}$	$\begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$
$(1, 1, 1)$	e_L^+
$(1, 1, 0)$	ν_L^c

$$\Sigma = (d_1^c, d_2^c, d_3^c, e^-, \nu)$$

Tensor representations

Vector $\phi_i \to U_{ij}\phi_j$

Tensor $\phi^i\hat{\phi}^j \to U_{ik}U_{jl}\phi^l\hat{\phi}^k$

$$\phi^i\hat{\phi}^j = \underbrace{\frac{1}{2}(\phi^i\hat{\phi}^j - \phi^j\hat{\phi}^i)}_{\text{Anti-symmetric}} + \underbrace{\frac{1}{2}(\phi^i\hat{\phi}^j + \phi^j\hat{\phi}^i)}_{\text{Symmetric}}$$

$\frac{1}{2}N(N-1)$ components

$\frac{1}{2}N(N+1)$ components

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Anti-symmetric: $\frac{1}{2}N(N-1)$ components

Symmetric: $\frac{1}{2}N(N+1)$ components

SU(5) 10 15

Contents of the (10)

$$\mathbf{10} = (3^*, 1, -\tfrac{2}{3}) + (1, 1, 1) + (3, 2, \tfrac{1}{6})$$

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$\boxed{(3^*, 1, \frac{1}{3})}$	d^c_L
$\boxed{(1, 2, -\frac{1}{2})}$	$\begin{pmatrix} \nu_L \\ e^-_L \end{pmatrix}$
$(1, 1, 1)$	e^+_L
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Contents of the (10)

$$\mathbf{10} = (3^*, 1, -\tfrac{2}{3}) + (1,1,1) + (3, 2, \tfrac{1}{6})$$

$$\Delta = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix}$$

$(3, 2, \frac{1}{6})$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$
$(3^*, 1, -\frac{2}{3})$	u_L^c
$(3^*, 1, \frac{1}{3})$	d_L^c
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SU(5) Matter

$$(5^*) + (10)$$

SU(5) Matter

$$3 \times [\quad (5^*) + (10) \quad]$$

SU(5) Matter

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+ N right-handed neutrinos

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+ N right-handed neutrinos

+ 2 Higgses, Ψ and H

SU(5) Matter

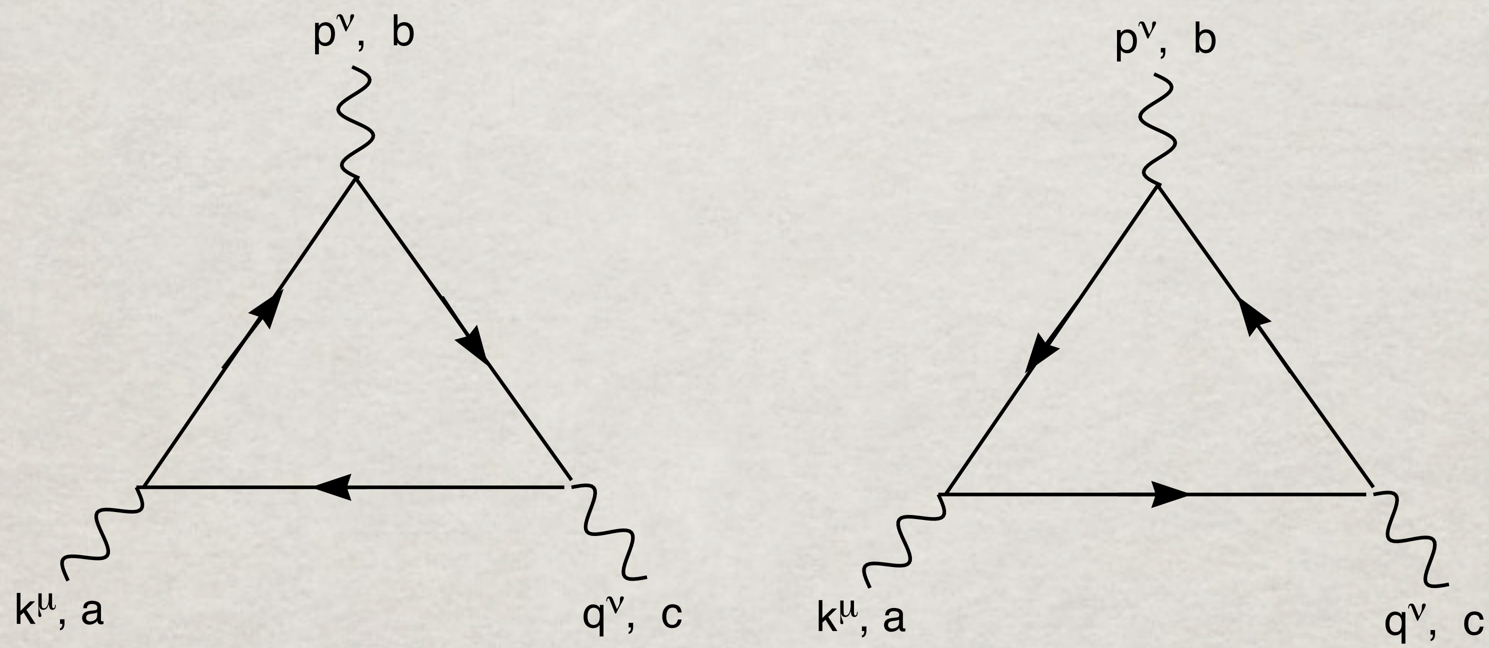
$$3 \times [\quad (5^*) + (10) \quad]$$

+ N right-handed neutrinos

+ 2 Higgses, Ψ and H

SU(5) “explains” structure of a standard model family?

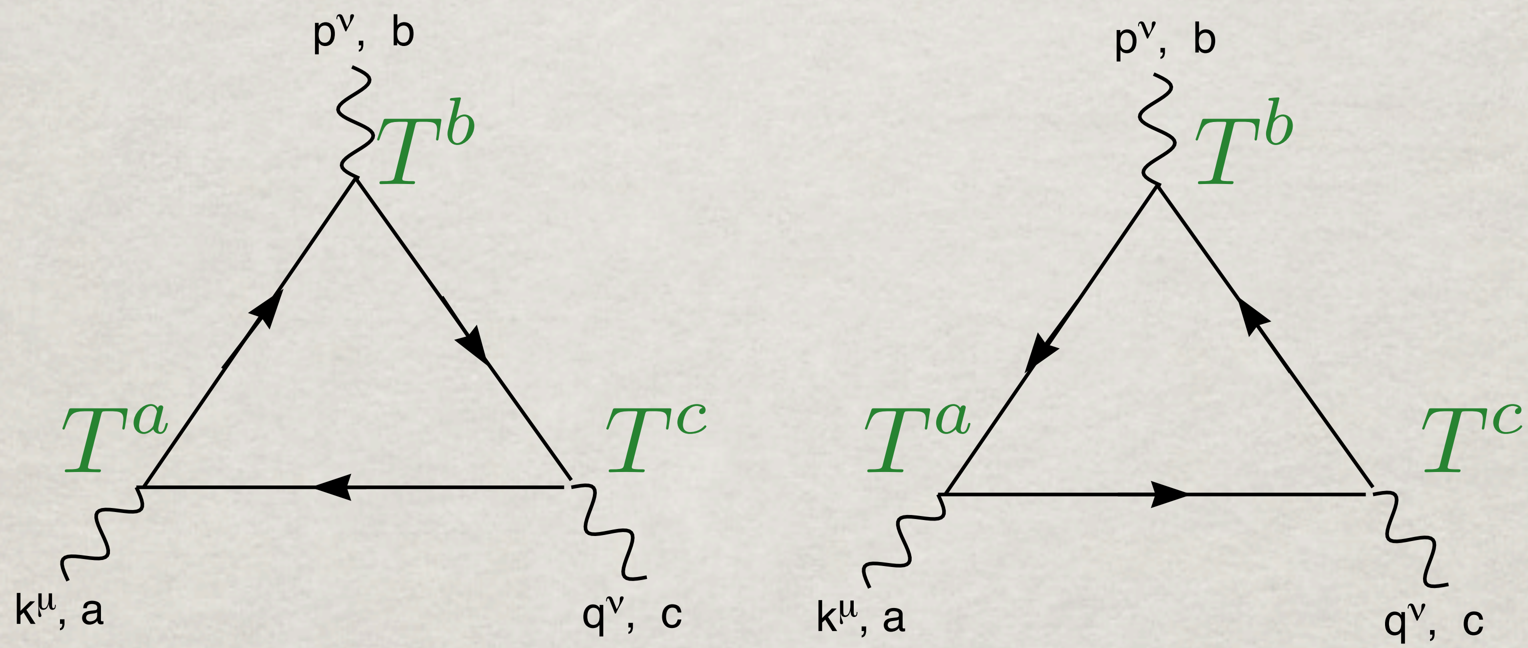
Anomalies



Break gauge invariance and unitarity if the loops contain a γ_5

Must cancel!

Anomalies



Break gauge invariance and unitarity if the loops contain a γ_5

Must cancel!

$$\mathrm{Tr}\ T^a\{T^b, T^c\} = 0$$

Standard model Anomalies

Standard Model

- $SU(3)^3$
- $SU(2)^3$ *(trivial)*
- $U(1)^3$
- $SU(3)^2 \times U(1)$
- $SU(2)^2 \times U(1)$
- $(\text{Gravity})^2 \times U(1)$

SU(5)

- $SU(5)^3$

Standard Model Anomalies

$$(3, 2, q_1) + (3^*, 1, q_2) + (3^*, 1, q_3) + (1, 2, q_4) + (1, 1, q_5)$$

Charges are constrained by

$$6q_1^3 + 3q_2^3 + 3q_3^3 + 2q_4^3 + q_5^3 = 0$$

$$2q_1 + q_2 + q_3 = 0$$

$$3q_1 + q_4 = 0$$

$$6q_1 + 3q_2 + 3q_3 + 2q_4 + q_5 = 0$$

Fix all q's up to normalization

SU(5) anomalies

Representation	Anomaly	SU(N)
(5*) (conjugate vector)	-1	-1
(10) (anti-symmetric tensor)	1	N-4

Charge Quantization

In the SM, the relation $Q_{electron}$=-Q_{proton} is exact
(because of anomaly cancellation)

But it is possible to add non-chiral particles with any (even irrational) charges.

In SU(5) theories Q_{em} is a non-abelian generator with fixed normalization.

All SM representations satisfy the rule

$$t/3 + s/2 + Y = 0 \text{ mod } 1$$

This implies that all unconfined charges are integer

Furthermore the theory contains magnetic monopoles

('t Hooft; Polyakov)

Symmetry Breaking

$$SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$$

Requires a Higgs boson Ψ

This is a scalar in some SU(5) representation.
After symmetry breaking there must be a ground state that is invariant under SU(3) × SU(2) × U(1).

Hence the SM decomposition of the Higgs must contain a singlet.

SU(5) Higgs Choice

Simplest choice: The adjoint (same representation as the gauge bosons):

$$\phi^i(\phi^j)^* - \frac{1}{N}\delta^{ij}\sum_k \phi^k(\phi^k)^*$$

$$\mathbf{24} \rightarrow \underbrace{(8,1,0) + (1,3,0) + (1,1,0)}_{\text{Massive}} + \underbrace{(3,2,-\tfrac{5}{6}) + (3^*,2,\tfrac{5}{6})}_{\text{“Eaten”}}$$

This gives a mass to the 12 non-SM gauge-bosons

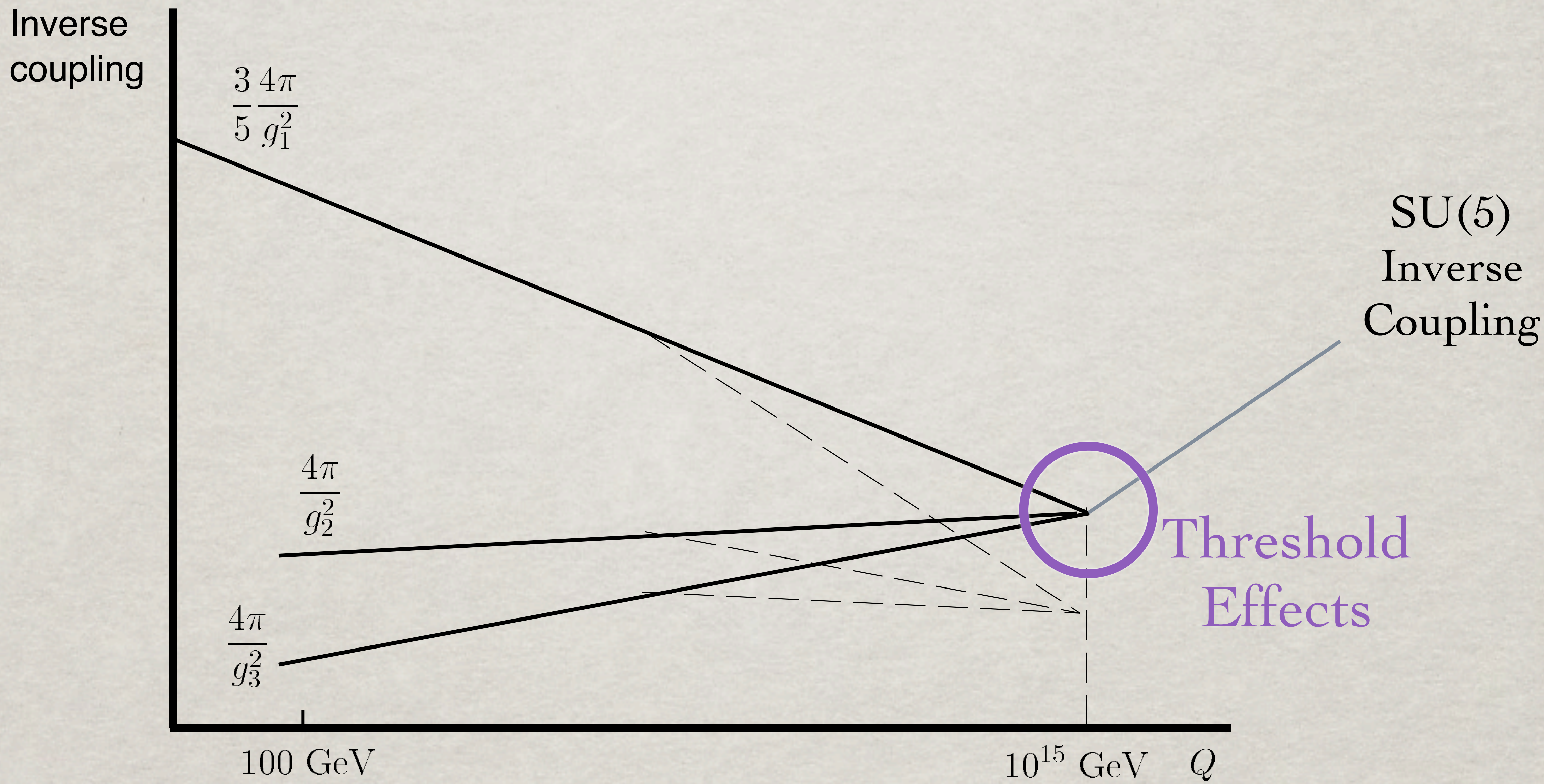
Coupling Unification

Below the SU(5) Higgs scale:

SM gauge bosons massless
X, Y massive: they "decouple".

From here on the three SM couplings
go their own way.

Inverse coupling
$\frac{3}{5}\frac{4\pi}{g_1^2}$
SU(5) Inverse Coupling
$\frac{4\pi}{g_2^2}$
$\frac{4\pi}{g_3^2}$
100 GeV
10^{15} GeV
Q



Inverse coupling
$\frac{3}{5}\frac{4\pi}{g_1^2}$
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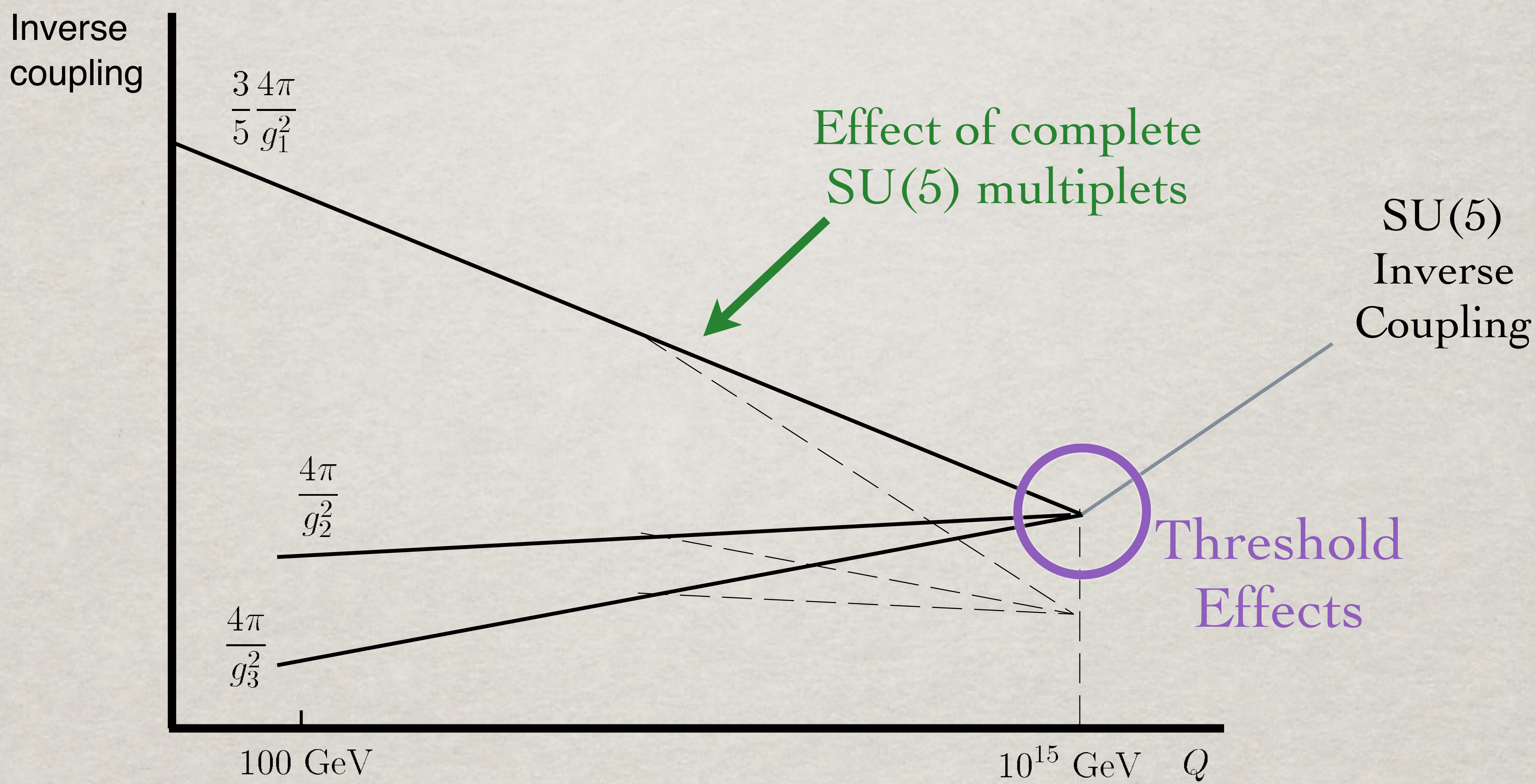
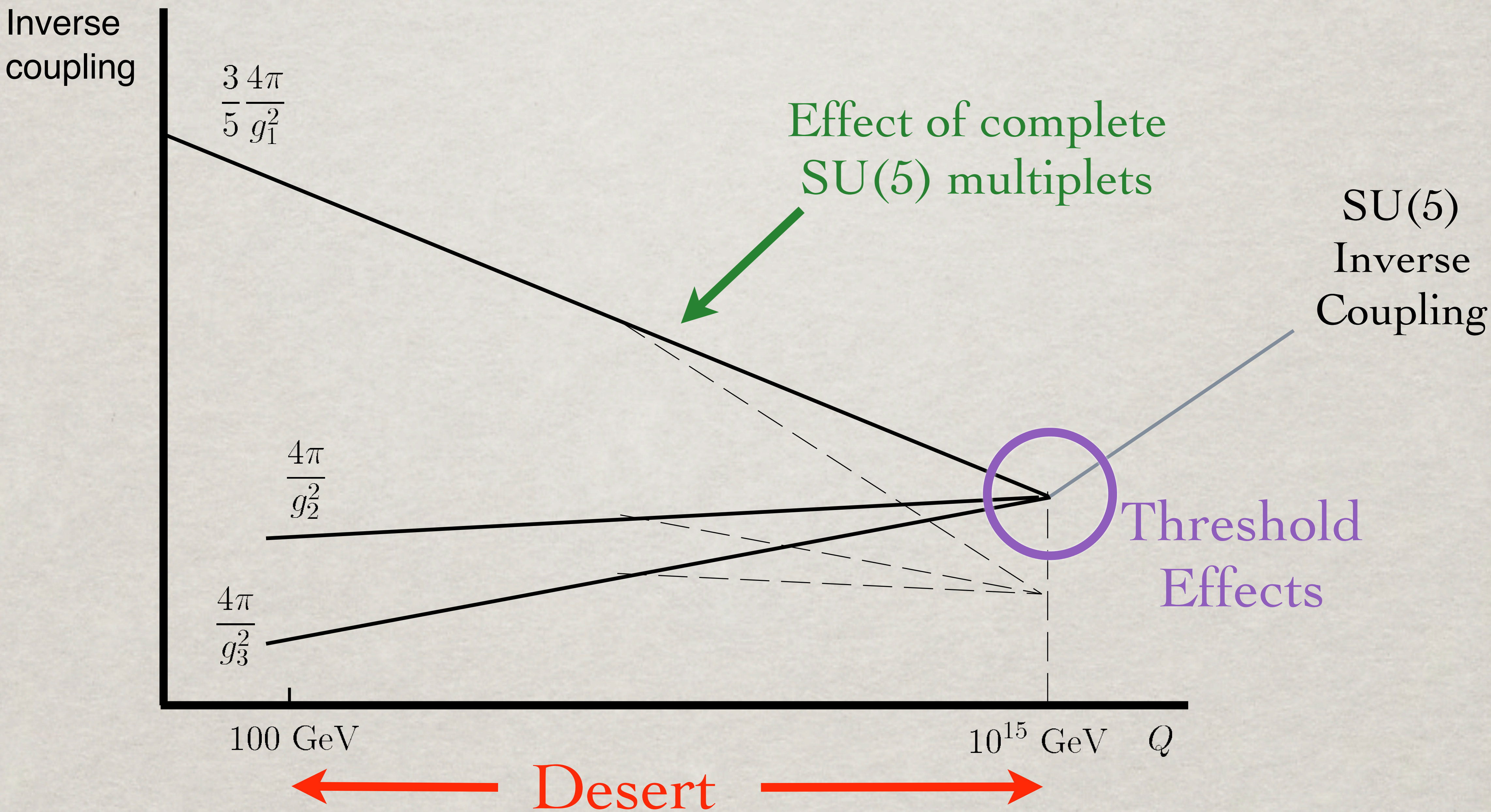
100 GeV
10^{15} GeV
Q
SU(5) Inverse Coupling
Threshold Effects


Inverse coupling
$\frac{3}{5}\frac{4\pi}{g_1^2}$
Effect of complete SU(5) multiplets
SU(5) Inverse Coupling
$\frac{4\pi}{g_2^2}$
Threshold Effects
$\frac{4\pi}{g_3^2}$
100 GeV
10^{15} GeV
Q


Inverse
coupling
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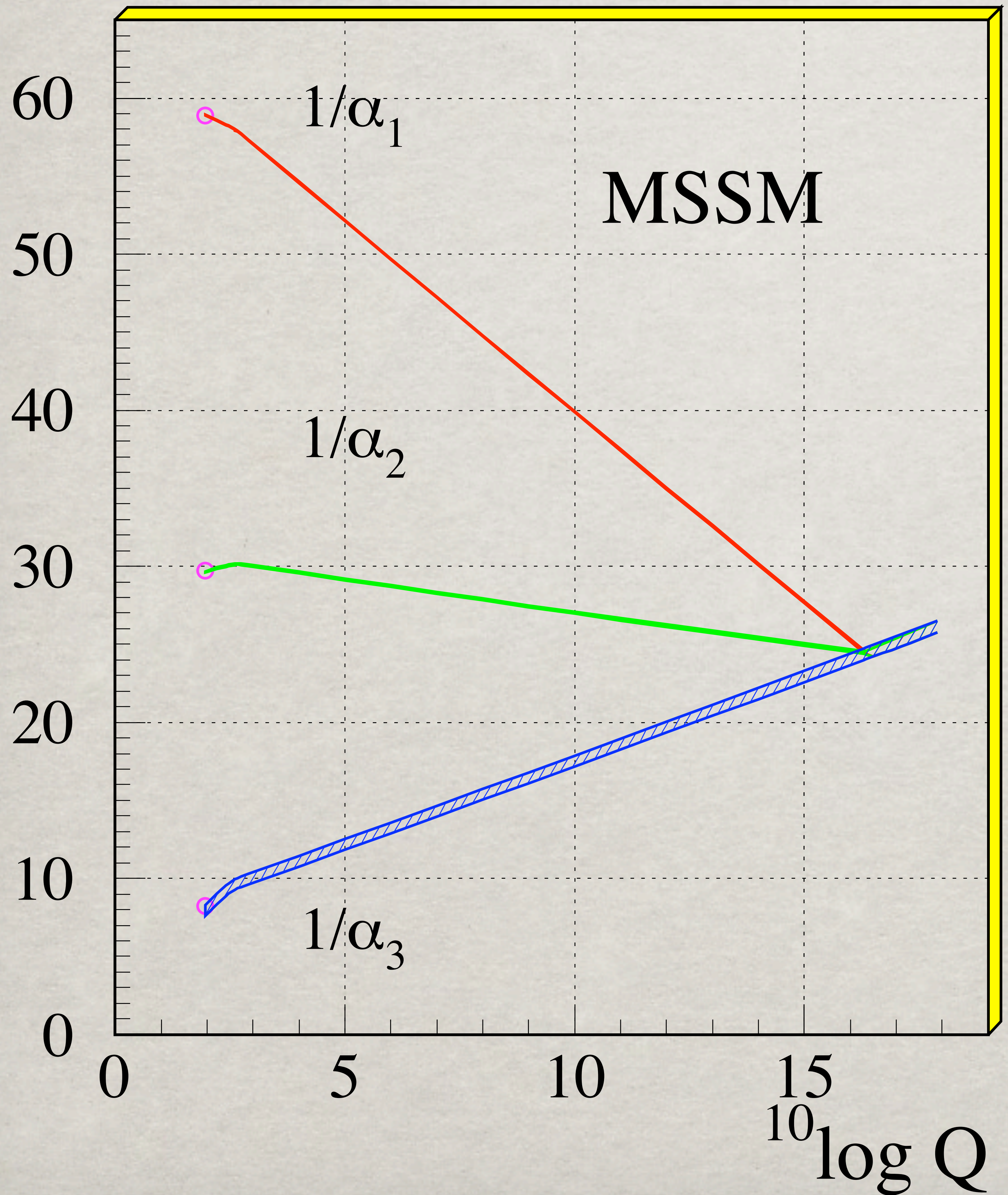
Effect of complete
SU(5) multiplets
SU(5)
Inverse
Coupling
$\frac{4\pi}{g_2^2}$
Threshold
Effects
$\frac{4\pi}{g_3^2}$
100 GeV
10^{15} GeV
Q
Desert

Coupling Unification

- Two parameters: SU(5)-couping and scale
- Three observables: SM couplings
- U(1) normalization 3/5 is essential
- Additional unbroken SU(5) matter does not affect unification.
- Seemed to work in 1980, but not anymore.

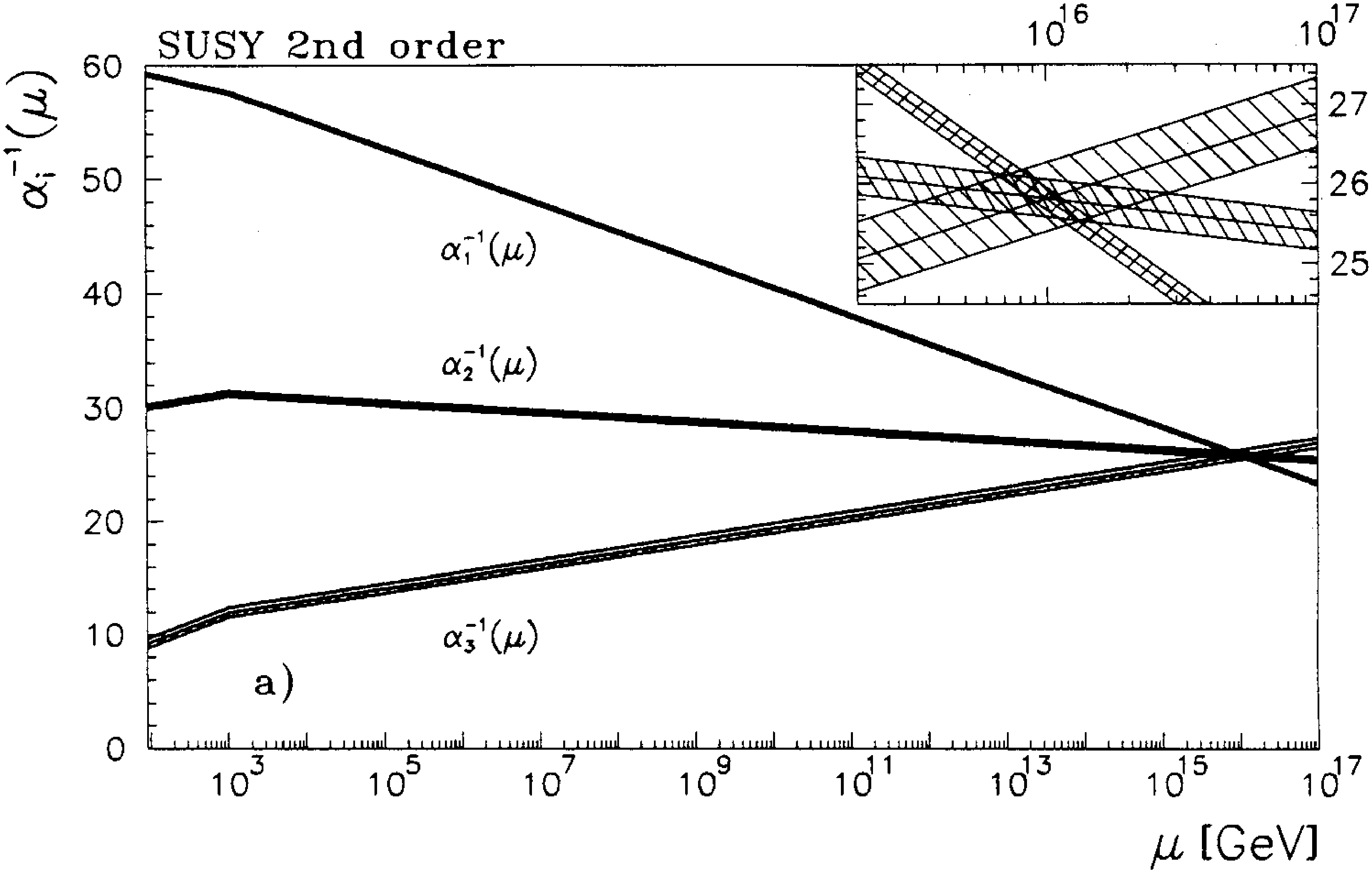
Amaldi, de Boer and Furstenau (1991)

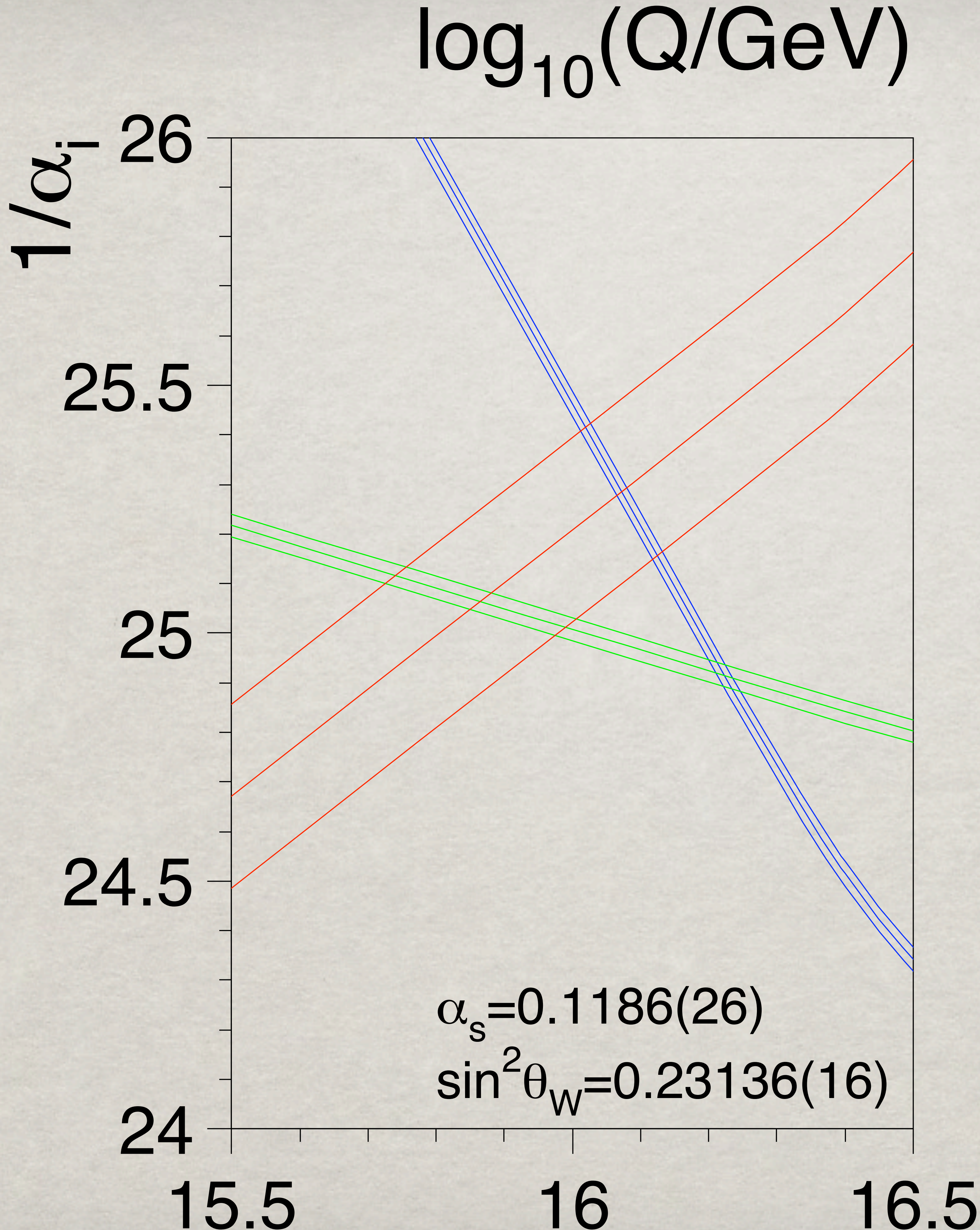


$$\alpha_1 = (5/3)\alpha^{\overline{\mathrm{MS}}} / \cos^2\theta_W^{\overline{\mathrm{MS}}}$$
$$\alpha_2 = \alpha^{\overline{\mathrm{MS}}} / \sin\theta_W^{\overline{\mathrm{MS}}},$$
$$\alpha_3 = \alpha_s^{\overline{\mathrm{MS}}},$$

Note:
One extra parameter!

Amaldi, de Boer and Furstenau (1991)





De Boer and Sander, 2003

Proton Decay

The heavy X and Y bosons mix quarks and leptons.
They couple as follows

$$T^a A^a_\mu = X^1_{\mu,i} T^1(i,4) + X^2_{\mu,i} T^2(i,4) + Y^1_{\mu,i} T^1(i,5) + Y^2_{\mu,i} T^2(i,5)$$

With:

$$T^1(i,j)_{kl} = \tfrac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

$$T^2(i,j)_{kl} = \tfrac{1}{2} i(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk})$$

Coupling to the quarks and leptons:

(5*) $$\bar{\Sigma}\gamma^\mu[-(T^a)^*]A^a_\mu \Sigma$$

(10) $$\bar{\Delta}_{\alpha\beta}\gamma^\mu[T^a_{\alpha\gamma}\delta_{\beta\delta} + T^a_{\beta\delta}\delta_{\gamma\alpha}]A^a_\mu \Delta_{\gamma\delta}$$

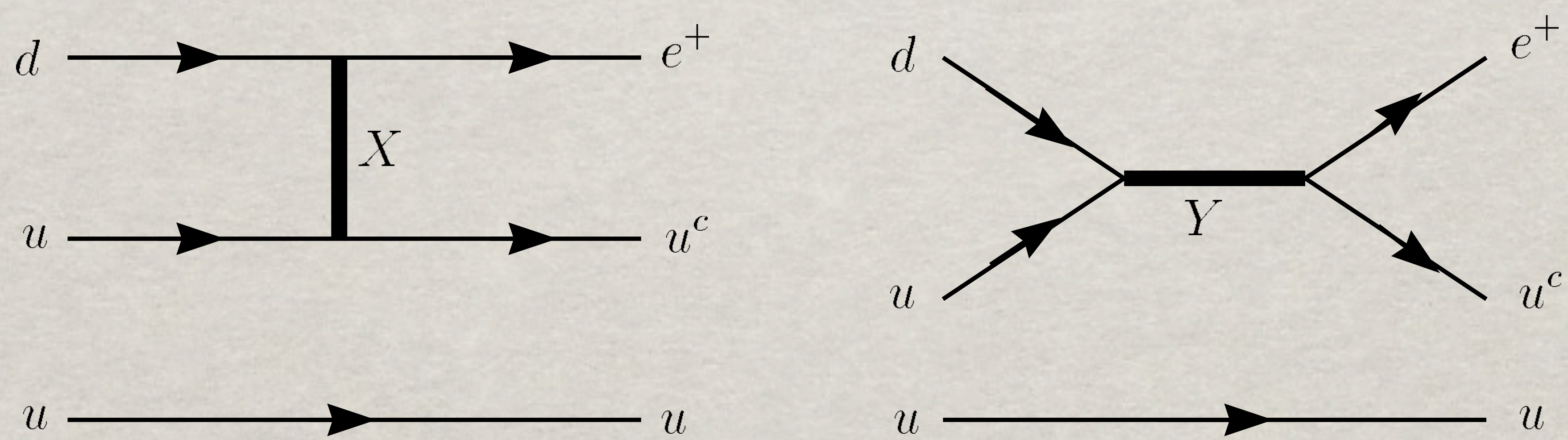
Interaction vertices (one family):

$$\mathcal{L}_X = \frac{g_5}{\sqrt{2}} X_\mu^- [\bar{e}^- \gamma_\mu d^c + \bar{d}\gamma_\mu e^+ - \bar{u}^c \gamma_\mu u] + \text{c.c}$$

$$\mathcal{L}_Y = \frac{g_5}{\sqrt{2}} Y_\mu^- [\ \overline{\nu}\gamma_\mu d^c - \bar{u}\gamma_\mu e^+ - \bar{u}^c \gamma_\mu d] + \text{c.c}$$

B, L broken, B-L preserved

Some example of proton decay processes $\quad p \to e^+ \pi$



Family mixing

We have no idea if the up and down quarks are related to the electron, muon or tau.

They can be expected to mix.
So we get something like:

$$\mathcal{L}_X = \frac{g_5}{\sqrt{2}} X^-_\mu [\bar{\mathcal{E}}^-_\alpha [U^\dagger_\mathcal{E} V_\mathcal{D}]_{\alpha\beta} \gamma_\mu \mathcal{D}^c_\beta + \bar{\mathcal{D}}_\alpha [U^\dagger_\mathcal{D} V_\mathcal{E}]_{\alpha\beta} \gamma_\mu \mathcal{E}^+_\beta + \bar{\mathcal{U}}^c_\alpha [V^\dagger_\mathcal{U} U_\mathcal{U}]_{\alpha\beta} \gamma_\mu \mathcal{U}_\beta$$

$$\mathcal{L}_Y = \frac{g_5}{\sqrt{2}} Y^-_\mu [-\bar{\mathcal{N}}_\alpha [U^\dagger_\mathcal{N} V_\mathcal{D}]_{\alpha\beta} \gamma_\mu \mathcal{E}^+_\beta + \bar{\mathcal{U}}_\alpha [U^\dagger_\mathcal{U} V_\mathcal{E}]_{\alpha\beta} \gamma_\mu \mathcal{E}^+_\beta + \bar{\mathcal{U}}^c_\alpha [V^\dagger_\mathcal{U} U_\mathcal{D}]_{\alpha\beta} \gamma_\mu \mathcal{D}_\beta$$

But we cannot rotate proton decay away $(p \to \tau^+ \pi)$: there are too many other channels.

Proton Decay Width

$$\Gamma = \left(\frac{g_5}{M_{\rm X}}\right)^4 C|\psi(0)|^2(E_{qq})^2$$

- $\psi(r)$: quark-quark wavefunction
- E_{qq}: quark-quark energy
- C: numerical constant

MSSM estimate 10^{36} years
(GQW: 6 x 10^{31} years)

Current limit $10^{32\pm1}$ years

Fermion Masses

At least two Higgses are required namely for

- Breaking SU(5) to SM (Ψ)
- Breaking SM to QCD x QED (H)

Obviously Ψ cannot give mass to the SM fermions.

H is the SM Higgs, but must now be an SU(5) multiplet.

Note that the SM Higgs transforms as a lepton doublet: (1,2,-1/2). Hence there is an obvious choice:

$$H=(5)$$

(and there are less obvious choices...)

Fermion Masses

down quark, charged lepton masses:

$$g_1^{\alpha\beta}\psi_i^\alpha(\mathbf{5}^*)C\psi_{kl}^\beta(\mathbf{10})H_m^*\delta_{ik}\delta_{lm} + \text{c.c}$$

up quark masses:

$$g_2^{\alpha\beta}\psi_{ij}^\alpha(\mathbf{10})C\psi_{kl}^\beta(\mathbf{10})H_m\epsilon_{ijklm} + \text{c.c}\ .$$

neutrino masses:

$$g_{\text{neutrino}}^{\alpha\beta}\psi_i^\alpha(\mathbf{5}^*)C\psi^\beta(\mathbf{1})H_m\delta_{im} + \text{c.c}$$

Fermion Masses

- Only three couplings, vs. four in SM

- Hence one relation

$$M = M_{\mathcal{D}} = M_{\mathcal{E}}^{\dagger} = \frac{v}{\sqrt{2}} g_1$$

- This implies $\left\{ \begin{array}{l} m_d = m_e \\ m_s = m_\mu \\ m_b = m_\tau \end{array} \right.$

Fermion Masses

- These relations are subject to “running”.
- Therefore not as bad as they look.
  $(m_b \approx 6\ \mathrm{GeV})$
- But still wrong when comparing ratios.

$$\frac{m_\mu}{m_e} \neq \frac{m_s}{m_d}$$

- Not valid in string GUTs.

Higgs Problems

- In the SM, the Higgs breaks uniquely to QCD x QED.
  But the SU(5) Higgs Ψ can break SU(5) to
  SU(3) x SU(2) x U(1) or SU(4) x U(1).
  [Vacuum selection problem].

- The (5) of SU(5) contains not only the SM Higgs, but also a color triplet, which mediates proton decay (and must therefore be heavy).
  [Doublet-triplet splitting problem].

- If the color component of H gets a vev, QCD is broken instead of $SU(2)_W$
  [Alignment problem].

- The two Higges have totally different mass-scales
  [Hierarchy problem].

Higgs Problems

Higgs vev of Ψ can be diagonalized, and can be either

$$\langle\Phi\rangle = \text{ diag } (v, v, v, v, -4v) \qquad \text{SU(4) x U(1)}$$

$$\langle\Phi\rangle = \text{ diag } (v, v, v, -\tfrac{3}{2}v, -\tfrac{3}{2}v) \qquad \text{SU(3) x SU(2) x U(1)}$$

(Ignoring H)

Most general Higgs potential (dimension <= 4)

$$V(\Phi, H) = -(\mu_5)^2 H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 - \tfrac{1}{2}\mu^2 \text{ Tr } \Phi^2 + \tfrac{1}{4}a(\text{ Tr } \Phi^2)^2 + \tfrac{1}{2}b \text{ Tr } \Phi^4$$
$$+ \alpha H^\dagger H \text{ Tr } \Phi^2 + \beta H^\dagger \Phi^2 H \ .$$

Higgs Problems

For suitable parameter values, a possible minimum is

$$\langle \Phi \rangle = \text{ diag } (v, v, v, (-\tfrac{3}{2} - \tfrac{1}{2}\epsilon)v, (-\tfrac{3}{2} + \tfrac{1}{2}\epsilon)v); \qquad \langle H \rangle = \frac{1}{\sqrt{2}}(0, 0, 0, 0, v_0)^T$$

$$v_0 << v$$

Induced H mass

$$-\mu_5^2 + \tfrac{15}{2}\alpha v^2 + \tfrac{9}{2}\beta v^2$$

Should be $<< v^2$

Fine-tuning!

SO(10)

SU(5) can be enlarged further

$$SO(10) \supset SU(5) \times U(1) \supset SU(3) \times SU(2) \times U(1)$$

One family fits nicely in a spinor representation of SO(10)

$$\mathbf{16} \to \mathbf{5}^* + \mathbf{10} + \mathbf{1}$$

Automatically anomaly free; no “manual” cancellations required.

And we get three right-handed neutrinos for free!

Several paths to SM

$$
\begin{aligned}
SO(10) &\to SU(5) \times U(1) \\
&\to SU(5) \\
&\to SU(3) \times SU(2) \times U(1)
\end{aligned}
$$

$$
\begin{aligned}
SO(10) &\to SU(4) \times SU(2) \times SU(2) \\
&\to SU(3) \times SU(2) \times SU(2) \times U(1)_1 \\
&\to SU(3) \times SU(2) \times U(1)_2 \times U(1)_1 \\
&\to SU(3) \times SU(2) \times U(1)\ .
\end{aligned}
$$

Several Higgses required

Several paths to SM

B-L

$$
\begin{aligned}
SO(10) &\to SU(5) \times U(1) \\
&\to SU(5) \\
&\to SU(3) \times SU(2) \times U(1)
\end{aligned}
$$

$$
\begin{aligned}
SO(10) &\to SU(4) \times SU(2) \times SU(2) \\
&\to SU(3) \times SU(2) \times SU(2) \times U(1)_1 \\
&\to SU(3) \times SU(2) \times U(1)_2 \times U(1)_1 \\
&\to SU(3) \times SU(2) \times U(1) \ .
\end{aligned}
$$

Several Higgses required

Several paths to SM

B-L

$$SO(10) \to SU(5) \times U(1)$$
$$\to SU(5)$$
$$\to SU(3) \times SU(2) \times U(1)$$

$$SO(10) \to SU(4) \times SU(2) \times SU(2)$$

Pati-Salam model

$$\to SU(3) \times SU(2) \times SU(2) \times U(1)_1$$
$$\to SU(3) \times SU(2) \times U(1)_2 \times U(1)_1$$
$$\to SU(3) \times SU(2) \times U(1) \ .$$

Several Higgses required

Several paths to SM

B-L

$$
\begin{aligned}
SO(10) &\to SU(5) \times U(1) \\
&\to SU(5) \\
&\to SU(3) \times SU(2) \times U(1)
\end{aligned}
$$

$$
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&\to SU(3) \times SU(2) \times SU(2) \times U(1)_1 \\
&\to SU(3) \times SU(2) \times U(1)_2 \times U(1)_1 \\
&\to SU(3) \times SU(2) \times U(1)\ .
\end{aligned}
$$

Pati-Salam model

LR-Symmetric model

Several Higgses required

Further extensions

$$E_6 \supset SO(10) \times U(1)$$

$$27 \to 16 + 10 + 1$$

Noteworthy because E_6 appears naturally as the gauge group of $E_8 \times E_8$ Heterotic Strings compactified on Calabi-Yau manifolds.

(But GUT scale too large...)

Conclusions

- The GUT idea is still alive, although not in its minimal form.
- Apparent coupling constant convergence does seem to hint at something.
- The family structure comes out very nicely (especially in SO(10))
- Hard to believe that this is all accidental.
- But clearly we have not been able to get the details right yet.