

# Real(Complex) Calculations in the Standard Model

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Higgs XS WG



Veltman80, 24 June 2011



## Real or Complex?

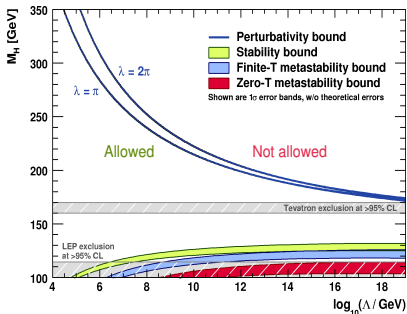


The real discovery is the one which enables me to stop doing philosophy when I want to.

# Theoretical and indirect exp. Higgs constr

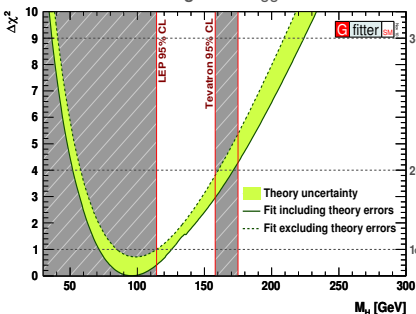
The Higgs boson should be light but not too light...

Perturbativity and (meta)stability bounds  
versus the SM cut-off scale  $\exists$



The SM Higgs must steer a narrow course between two disastrous situations if it is to survive up to the Planck scale  $M_p \sim 2 \cdot 10^{18}$  GeV

EW fit not including direct Higgs searches

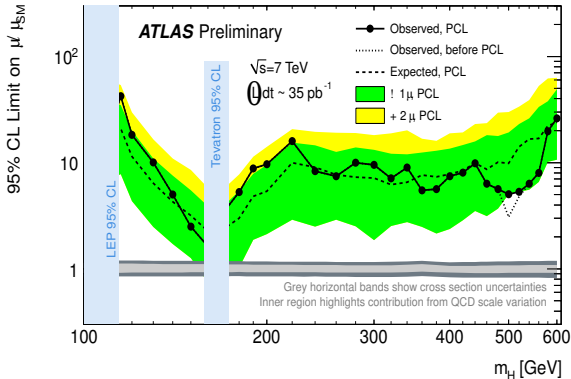


Central value  $\pm 1\sigma$ :  $M_H = 96^{+30}_{-25}$  GeV  
95% CL upper limit: 170 GeV

# SM Higgs Exclusion Potential

ATLAS combination of individual channels for 2010 data

Preliminary



Combination using maximum likelihood fit taking into account correlated nuisance parameters

Uncertainty on cross section included in SM expectation#

Not yet reached at Tevatron who exclude 158–173 GeV at 95% CL [arXiv:1103.3233]#

Power-constrained\* limits (PCL) computed from  $\text{CL}_{\text{s}+\text{b}}$  of test statistics using toy MC

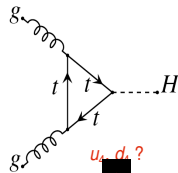
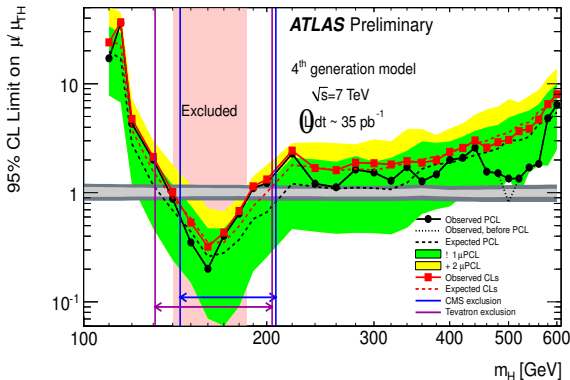
\*Fluctuations below  $-1\#$  wrt. median of expected limits are not allowed  
 Corresponding  $\text{CL}_{\text{s}}$  limits provide reduced exclusion (overcoverage)



# 4-Gen SM Higgs Exclusion Potential

Gluon fusion to Higgs via triangular heavy-quark loop sensitive to 4<sup>th</sup> generation

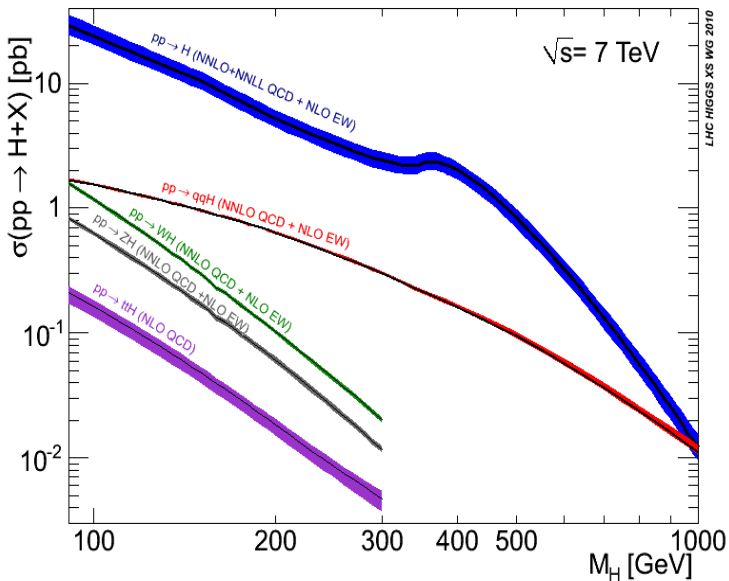
Preliminary



4<sup>th</sup> generation ?  
K-factor of  $\sim 3^2 = 9$   
using sequential model

95% CL exclusion of  $140 < M_H < 185$  GeV in SM4





UNITARITY AND CAUSALITY IN A RENORMALIZABLE  
FIELD THEORY WITH UNSTABLE PARTICLES

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**Synopsis**

The problems of unitarity, causality and renormalizability are treated in a field theory containing an unstable particle. Perturbation theory is suitably modified and leads to an implicit equation for complete propagators. The  $S$ -matrix constructed with these propagators and connecting stable particle states only is shown to be unitary, renormalizable and causal. It is also shown to give rise to interpolating Heisenberg fields which verify the original field equations.

1. *Introduction.* In recent years many authors have treated the problem of unstable particles in the framework of quantum field theory. Salam<sup>1)</sup> and Sachs<sup>2)</sup> gave suitable definitions of mass and particle in terms of its field theoretical propagator. In these definitions are in agreement with the situation.

In the present paper we study another aspect of unstable particles, namely the questions of unitarity and renormalization in perturbation theory. To be more precise we have a situation where an unstable scalar particle, say  $\phi$ , decays into two identical stable scalar particles, say  $\psi$  and  $\psi$ . In the perturbation theory for such a model one starts with fields  $A$  and  $\phi$ , obeying the Klein-Gordon equation. The interaction between  $A$  and  $\phi$  is other in some way specified by an interaction Lagrangian. The perturbation theory leads then, however, to a very undesirable situation: unstable  $A$ -particles appear at infinite times in the states. A realistic theory cannot have this feature. The unstable particle states from the in- and out- states. The problem of unitarity of the resulting truncated  $S$ -matrix can now be stated as follows: consider the Hilbert space of the in- and out- states. Is it then possible to construct by suitable modification of the theory an  $S$ -matrix which is unitary in this Hilbert space?

\*) Present address: CERN, Genève.



# Last few years

## Feynmanians versus Unitarians





# Contentious Matters

Melrose, Veltman et al.

$$\begin{aligned}
 I_N (4 \text{ dim}) &= \sum_{B \in I_N} C_B D_0(B) + \sum_{V \in I_N} C_T C_0(V) \\
 &+ \sum_{S \in I_N} C_S B_0(S) + \sum_{T \in I_N} C_T A_0(T)
 \end{aligned}$$

**Find the most efficient algorithm to compute coefficients**  
 ( $\oplus$  find the analogous two-loop basis).



# Highlights in Loops



- **NNLO QCD:** Catani, Grazzini, DeFlorian, Anastasiou, Petriello, . . .
- **Mostly QCD (Unitarians):** Bern, Dixon, Kosower, Kunszt, Zanderighi, K. Ellis, . . .
- **Mostly QCD (Numerical):** Ossola, Papadopolous, Pittau, . . .
- **Mostly EW (Feynmanians):** Denner, Dittmaier, Binoth . . .
- **Two-Loop EW:** Actis, Passarino, Uccirati, Sturm, . . .



# Feynmanians

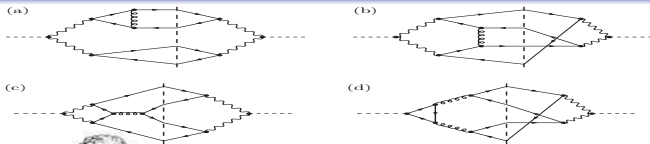


Figure 3: Categories of cut diagrams contributing to the QCD corrections.

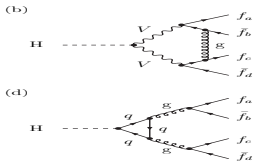
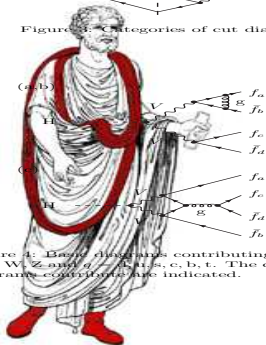


Figure 4: Basic diagrams contributing to the virtual QCD corrections for  $H \rightarrow 4f$  where  $V = W, Z$  and  $f = q, l, s, c, b, t$ . The categories of QCD corrections, (a)–(d), to which the diagrams contribute are indicated.



## Unitarions

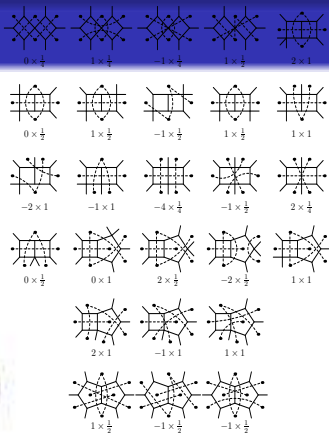


FIG. 75. The 26 different integrals which are allowed, by the hypothesis of dual conformal symmetry, to contribute to the amplitude  $M_6^{(2),D=4}$ . Beneath each diagram is the coefficient with which the corresponding integral, defined according to the rules reviewed in fig. 1, enters into our result for  $M_6^{(2),D=4}$ . An overall factor of  $1/16$  is suppressed and it is understood that one should sum over the 12 cyclic and reflection permutations of the external legs. In each coefficient, the second factor is a symmetry factor that accounts for overcounting in this sum.

# Anatomy of a Two–Loop EW Calculation

*with unstable particles*

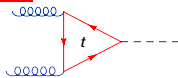


# Calculation & Techniques

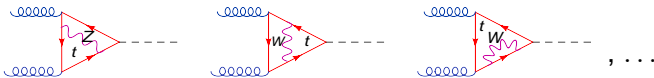
...some diagrams contributing to the EW 2-loop corrections

## ■ $gg \rightarrow H$ :

LO

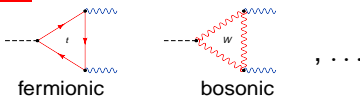


NLO



## ■ $H \rightarrow \gamma\gamma$ :

LO



fermionic

bosonic

NLO

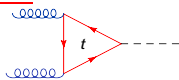


# Calculation & Techniques

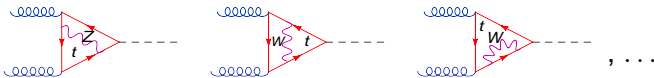
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## ■ $gg \rightarrow H$ :

LO

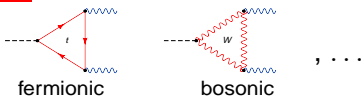


NLO



## ■ $H \rightarrow \gamma\gamma$ :

LO



NLO

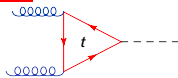


# Calculation & Techniques

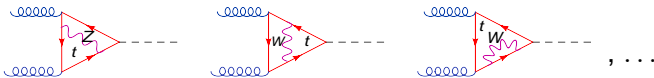
...some diagrams contributing to the EW 2-loop corrections

## ■ $gg \rightarrow H$ :

LO

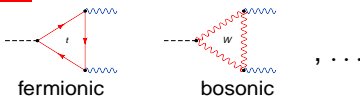


NLO



## ■ $H \rightarrow \gamma\gamma$ :

LO



fermionic

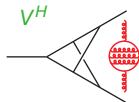
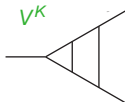
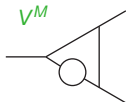
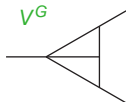
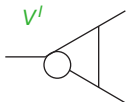
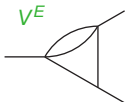
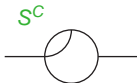
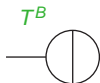
bosonic

NLO





# List-of-diagrams: all what is needed



# Extracting Collinear divergencies

## Theorem

*Coefficients of collinear logarithms are integrals of one-loop functions*

$$\begin{array}{c}
 \begin{array}{c}
 \text{wavy line } m \\
 \diagup \quad \diagdown \\
 m \quad m
 \end{array} \\
 \\
 \begin{array}{c}
 \begin{array}{c}
 p_2 \\
 \diagup \quad \diagdown \\
 M_5 \quad M_4 \\
 \diagdown \quad \diagup \\
 -P \quad m \\
 \diagdown \quad \diagup \\
 m \quad p_1
 \end{array} \\
 = \\
 \ln \frac{m^2}{s} \int_0^1 dy \\
 \begin{array}{c}
 p_2 \\
 \diagup \quad \diagdown \\
 M_5 \quad M_4 \\
 \diagdown \quad \diagup \\
 -P \quad \text{wavy line } (1-y)p_1 \\
 \diagdown \quad \diagup \\
 M_3 \quad \text{wavy line } yp_1
 \end{array} \\
 + \text{finite part}
 \end{array}
 \end{array}$$

## Theorem I

## General results I

Coll. behavior of arbitrary two-loop  $q$ -scalar, UV-finite diagrams

$$\begin{array}{c} p \\ \text{wavy line} \end{array} \rightarrow \begin{array}{c} q \\ \text{dashed line} \end{array} \rightarrow \begin{array}{c} m \\ \text{circle} \end{array} \rightarrow \begin{array}{c} q_a^{\mu_1} \dots q_a^{\mu_m} \\ \text{grey circle} \end{array} = \ln \frac{m^2}{s} \int_0^1 dz \begin{array}{c} zp \\ \text{wavy line} \end{array} \rightarrow \begin{array}{c} q_a^{\mu_1} \dots q_a^{\mu_m} \\ \text{grey circle} \end{array} \leftarrow \begin{array}{c} (1-z)p \\ \text{wavy line} \end{array} + \text{coll. fin.}$$



## Theorem II

## General results II

## Generalization to tensor integrals

$$\begin{aligned}
 & \text{Diagram: A shaded circle with mass } m \text{ and internal momenta } q^{\mu_1} \dots q^{\mu_m}. \text{ External momenta } p, q, \text{ and } q+p \text{ are shown.} \\
 & = \ln \frac{m^2}{s} \left[ 1 - \frac{\epsilon}{2} \Delta_{\mathcal{W}}(s) - \frac{\epsilon}{4} \ln \frac{m^2}{s} \right] \\
 & \times \int_0^1 dz (-z)^r (1-z)^p \text{Diagram: A shaded circle with mass } m \text{ and internal momenta } q^{\mu_1} \dots q^{\mu_m}. \text{ External momenta } zp \text{ and } (1-z)p \text{ are shown.} + \text{c. f.}
 \end{aligned}$$



# Extracting Ultraviolet divergencies

$$\begin{aligned}
 V^I &= \text{Diagram} = \frac{1}{\pi^4} \int \underbrace{\frac{d^n q_1 d^n q_2}{[1][2][3][4][5]}}_x, \\
 &= C_\epsilon \int_0^1 dx \int dS_3(y_1, y_2, y_3) [x(1-x)]^{-\epsilon/2} (1-y_1)^{\epsilon/2-1} V^{-1-\epsilon}
 \end{aligned}$$

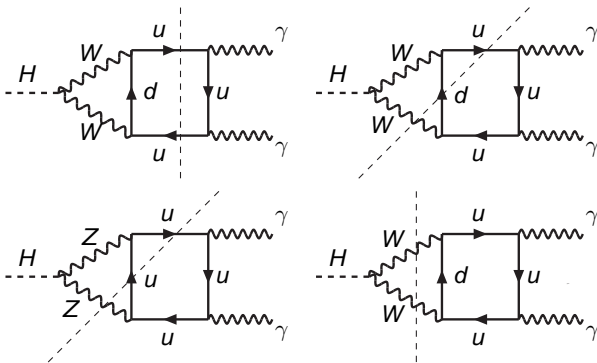
[1] =  $q_1^2 + m_1^2$   
 [2] =  $(q_1 - q_2)^2 + m_2^2$   
 [3] =  $q_2^2 + m_3^2$   
 [4] =  $(q_2 + p_1)^2 + m_4^2$   
 [5] =  $(q_2 + P)^2 + m_5^2$

The **single pole** can always be expressed in terms of **1L**.

$$V^I = \text{Diagram 1} \times \text{Diagram 2} + \text{finite part.}$$



# Around threshold



# $1/\beta$ -behavior

$$\begin{aligned}
 & \text{Diagram 1} = - \text{Diagram 2} \\
 & \times \text{Diagram 3} \\
 & + \left( \text{reg. part at } \beta = 0 \right)
 \end{aligned}$$

The diagram on the left is a triangle with mass  $m$  on all three sides. A dashed line labeled  $H$  enters from the left vertex. A bubble is attached to the bottom-left side of the triangle. The top and bottom horizontal edges of the triangle are decorated with wavy lines representing gluons.

The diagram on the right is a circle with mass  $m$  on its left and right sides, connected to a solid line. A minus sign is placed to the left of the circle.

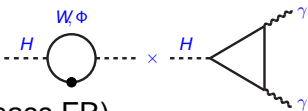
The diagram below the multiplication sign is a triangle with mass  $m$  on all three sides. A dashed line labeled  $H$  enters from the left vertex. A black dot is located on the bottom-left side of the triangle. The top and bottom horizontal edges of the triangle are decorated with wavy lines representing gluons.

The plus sign is followed by the text  $(\text{reg. part at } \beta = 0)$ .

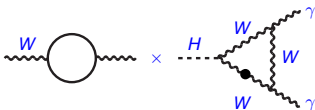


# Origin of $1/\beta$

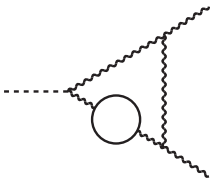
- (1-loop diagrams)  $\otimes$  (H wave-function FR)



- (1-loop diagrams)  $\otimes$  (W mass FR)

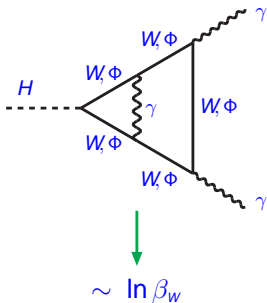


- Pure 2-loop diagrams

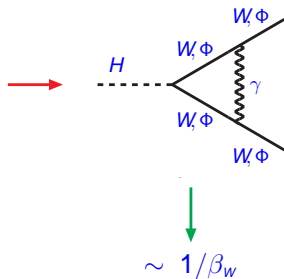




# Logarithmic singularities

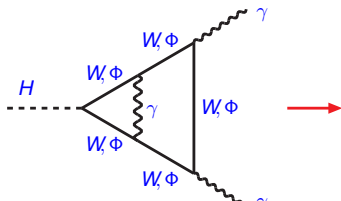


Remnant of  
Coulomb  
singularity

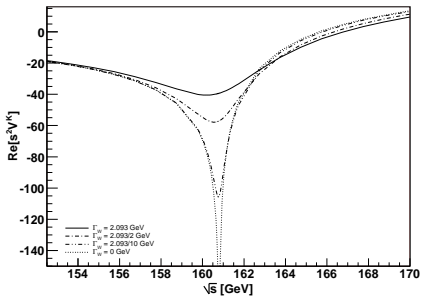


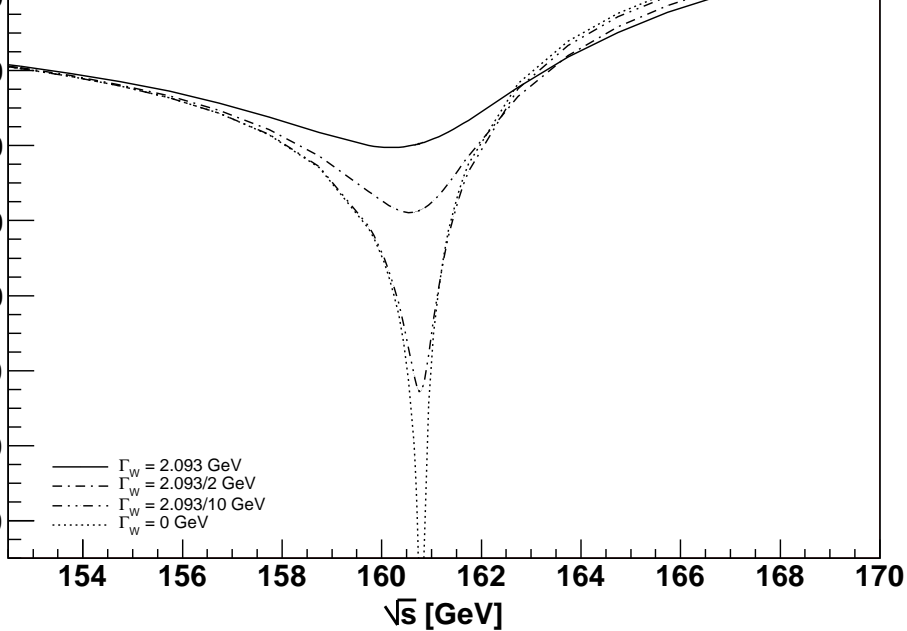
Complex for Coulomb

## Cure for logarithmic singularities



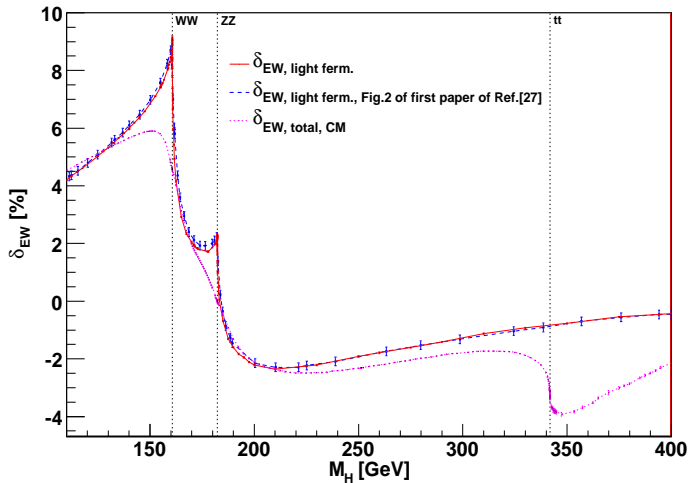
Complex W Mass

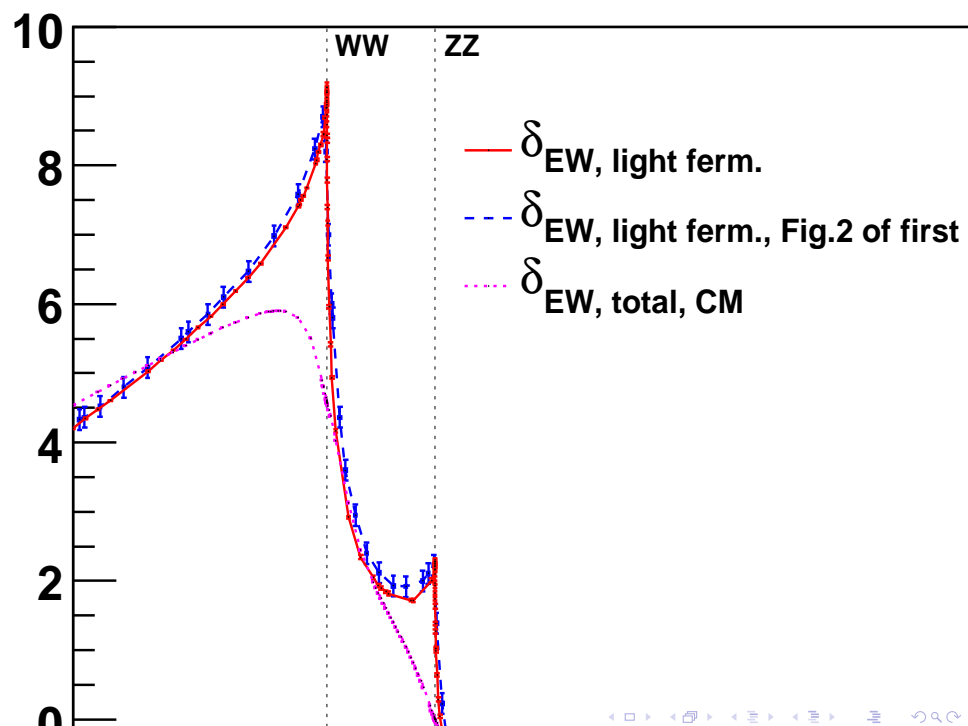




## Comparison I

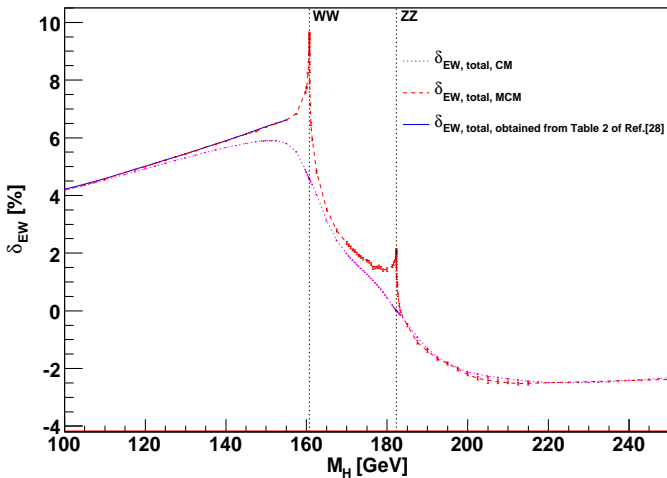
## Comparing





## Comparison II

## Comparing



# Dealing with experimenters today



## Dialogue Concerning the Two Chief World Systems: Salviati, Sagredo, Simplicio

**TH** How do you want to proceed? Full scenario?

**EX** No, we separate Higgs production and decay, and MCs implement an ad-hoc Breit-Wigner

**TH** Hope you are not going for high-mass!

**EX** Up to 600 GeV via  $ggF(+VBF)$  ( $H \rightarrow WW \rightarrow l\nu qq$ )

**TH** Then you got problems, the three bricks need a proper definition:

- ① The full  $\mathcal{S}$ -matrix element is  $S \oplus B$
- ②  $S$  is [ production  $\otimes$  propagation  $\otimes$  decay ]
- ③ each of them must be defined consistently

**EX** We are working with a mass spectrum peak, but what about the on-shell mass peak? Are there other definitions?

**TH** This I told you before



## Conclusion?

- What is the best way of dealing with experimenters? Well,

*that all true believers break their eggs at the convenient end.*

*Jonathan Swift's Travels into Several Remote Nations of the World*

- Exclusions are approaching

*Good tests kill flawed theories; we remain alive to guess again.*

*Karl Popper*

*El sueño de la razón produce monstruos*

*Francisco Goya*



## Moving towards modernity

### Which

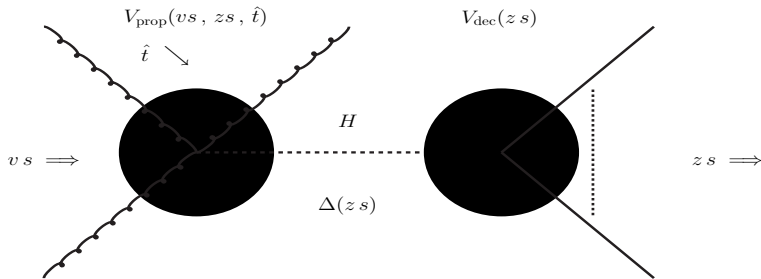
best language to simulate intuition?

- production of on-shell Higgs
- intermediate Breit–Wigner
- Higgs on-shell decay
- *production* of a Higgs at its complex pole
- Dyson resummed propagator
- Higgs *decay* at its complex pole

### Right column

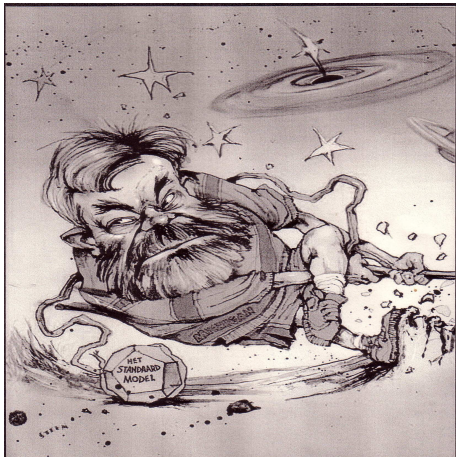
cannot yet produce fast answers, that's why the PO oblivion





$$\begin{aligned} &\Rightarrow \sigma_{gg \rightarrow H+X}(vs, \hat{t}, zs) \frac{(zs)^2}{|zs - s_H|^2} \frac{\Gamma_{H \rightarrow f}(zs)}{(zs)^{1/2}} + \text{NR} \\ &= \sigma_{gg \rightarrow H+X}(vs, \hat{t}, s_H) \frac{|s_H|^2}{|zs - s_H|^2} \frac{\Gamma_{H \rightarrow f}(s_H)}{|s_H|^{1/2}} + \text{NR}, \end{aligned}$$





*Age is an issue  
of mind  
over matter.  
If you don't  
mind, it  
doesn't  
matter.*

*Mark Twain*

