Radiative Corrections in Quantum Field Theory

Martinus Veltman 80th birthday

J. Iliopoulos

Amsterdam, June 24, 2011

Happy Birthday, Tini

Radiative Corrections

in Quantum Field Theory

Radiative Corrections in Quantum Field Theory

- Because of Tini’s contributions

Radiative Corrections

in Quantum Field Theory

- Because of Tini's contributions

- Because they are at the basis of all successes of Q.F.T.

Veltman, M. Physica 29
1963 186–207

UNITARITY AND CAUSALITY IN A RENORMALIZABLE FIELD THEORY WITH UNSTABLE PARTICLES

M. VELTMAN *)

Instituut voor theoretische fysica der Rijksuniversiteit, Utrecht, Nederland

Synopsis

The problems of unitarity, causality and renormalizability are treated in a field theory containing an unstable particle. Perturbation theory is suitably modified and leads to an implicit equation for complete propagators. The S-matrix constructed with these propagators and connecting stable particle states only is shown to be unitary, renormalizable and causal. It is also shown to give rise to interpolating Heisenberg fields which verify the original field equations.

1. *Introduction*. In recent years many authors discussed unstable particles in the framework of quantum field theory. In particular, Matthews and Salam[1]) gave suitable definitions of mass and lifetime of an unstable particle in terms of its field theoretical propagator. As shown by Jacob and Sachs[2]) these definitions are in agreement with the experimental situation.

In the present paper we study another aspect of a field theory with unstable particles, namely the questions of unitarity, causality and renormalization in perturbation theory. To be more explicit suppose that we have a situation where an unstable scalar particle, say A-particle, can decay into two identical stable scalar particles, say φ-particles. In setting up perturbation theory for such a model one starts by introducing the "bare" fields A and φ, obeying the Klein-Gordon equation and coupled to each other in some way specified by an interaction Lagrangian. Ordinary perturbation theory leads then, however, to a very undesirable feature, namely the unstable A-particles appear at infinite times in incoming and outgoing states. A realistic theory cannot have this feature, but if one just removes the unstable particle states from the in- and out-states one is faced with the problem of unitarity of the resulting truncated S-matrix. The problem can now be stated as follows: consider the Hilbert space of stable particle states. Is it then possible to construct by suitable modification of perturbation theory an S-matrix which is unitary in this Hilbert space. The answer is

*) Present address: CERN, Genève.

thesis of Ref. 5). Note, however, that (5) is not quite the same as the SU(6) result[9] which in our notation reads

$$(f_\rho^2/4\pi) = (G_{\pi NN}^2/4\pi)(g_V/g_A)^2(m_M^2/m_B^2)$$
$$= (9/25)(G_{\pi NN}^2/4\pi)(m_M^2/m_B^2), \qquad (6)$$

where m_M and m_B, respectively, stand for the mean masses of the meson 35-plet and the baryon 56-plet of SU(6).

It is a pleasure to thank Professor Peter G. O. Freund for stimulating conversations which led to this investigation.

*This work supported in part by the U. S. Atomic Energy Commission.
†Alfred P. Sloan Foundation Fellow.

[1]M. Gell-Mann, Phys. Rev. 125, 1067 (1962).
[2]M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960); Y. Nambu, Phys. Rev. Letters 4, 380 (1960); J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo Cimento 17, 757 (1960).
[3]Y. Tomozawa, to be published.
[4]S. Weinberg, to be published.
[5]J. J. Sakurai, Ann. Phys. (N.Y.) 11, 1 (1960).
[6]See also M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961). These authors define a constant γ_ρ, which is equivalent to our $(\frac{1}{2}f_\rho)$.
[7]Equation (5) gives a ρ width of 160 MeV. Possible off-the-mass-shell corrections are discussed in footnote 10 of Ref. 8. It may also be mentioned that if we use $G_{\pi NN}^2/4\pi$, which gives the correct pion lifetime when inserted in the Goldberger-Treiman relation, then the predicted ρ width is changed to 120 MeV in good agreement with observation.
[8]K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters 16, 255 (1966). See also Riazuddin and Fayyazuddin, Phys. Rev. 147, 1071 (1966).
[9]F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters 13, 299 (1964).

DIVERGENCE CONDITIONS AND SUM RULES*

M. Veltman†
Brookhaven National Laboratory, Upton, New York
(Received 29 July 1966)

Recently several sum rules have been derived employing current commutation[1] rules and divergence conditions for those currents. As is well known, the application of commutation rules involves the manipulation of the so-called Schwinger terms,[2] and where some of these calculations avoid such complications, others may be criticized in this respect. An alternative derivation of these sum rules, based on assumptions other than current commutation relations may, therefore, be of help in understanding the mechanism involved.

Consider the vector current of hadrons that is coupled to leptons and photons. Neglecting higher-order electromagnetic (em) and weak interactions one customarily assumes, following Feynman and Gell-Mann,[3]

$$\partial_\mu \vec{J}_\mu^V = 0. \qquad (1)$$

As is well known, em interactions break this law for the charged components of $\vec{J}_\mu$. Similarly the weak interactions break (1) for the neutral component because they carry off a nonzero charge. (Remember that $\vec{J}_\mu$ is the hadron current only.) We will try to find the first-order em and weak effects on (1).

According to the principle of minimality, we find the em effect on (1) by substituting $\partial_\mu - ieA_\mu$ for ∂_μ applying to a (negative) charged field. Thus, neglecting here the case that $\vec{J}_\mu^V$ itself contains derivatives, we find

$$\partial_\mu \vec{J}_\mu^V = +ie\vec{A}_\mu \times \vec{J}_\mu^V,$$
$$(\vec{A}_\mu \times \vec{J}_\mu)^i = i\epsilon_{ijk} A_\mu^j J_\mu^k, \qquad (2)$$

where $\vec{A}_\mu$ is an isotopic vector whose first two components are zero. Equation (2) is already sufficient to derive the Cabibbo-Radicati[4] sum rule.

In accordance with the observations made above, we generalize (2) to include also first-order weak interaction effects:

$$\partial_\mu \vec{J}_\mu^V = ie\vec{A}_\mu \times \vec{J}_\mu^V + ig\vec{l}_\mu \times \vec{J}_\mu^V. \qquad (3)$$

Here l_μ represents the lepton current.[5] Equation (3) is valid if no axial currents are present. The generalization to include axial currents also requires some care. Let us intro-

8.C.1
7.A.1

Nuclear Physics B7 (1968) 637-650. North-Holland Publ. Comp., Amsterdam

PERTURBATION THEORY OF MASSIVE YANG-MILLS FIELDS

M. VELTMAN
Institute for Theoretical Physics, University of Utrecht, The Netherlands
and
Laboratoire de Physique Théorique et Hautes Energies, Orsay *

Received 10 September 1968

Abstract: Perturbation theory of massive Yang-Mills fields is investigated with the help of the Bell-Treiman transformation. Primitive diagrams containing one closed loop are shown to be convergent if there are more than four external vector boson lines. The investigation presented does not exclude the possibility that the theory is renormalizable.

1. INTRODUCTION

The structure of weak interactions discovered so far suggests very strongly the existence of charged, and possibly also neutral vector mesons. First of all, there is the current × current form of leptonic and semi-leptonic weak interactions, and possibly also the non-leptonic weak interactions; secondly there is the structure of the hadron currents, very similar to the structure of the electromagnetic currents in the sense that they may be thought of as to be constructed with the help of some gauge principle. More precisely, Gell-Mann [1] suggested current commutation rules for vector and axial-vector currents that can be understood as a simple extension of the commutation rules known for electromagnetic currents; as is well known these commutation rules have led to a large number of sucesses, in particular the so-called low-energy theorems.

These very same low-energy theorems have been derived also by means of some gauge principle [2]. One is led then to divergence equations for the currents of weak interactions of the form

$$\partial_\mu \boldsymbol{J}_\mu = -g\, \boldsymbol{W}_\mu \times \boldsymbol{J}_\mu \ , \tag{1}$$

a very natural extension of the equation

$$\partial_\mu \boldsymbol{J}_\mu = -e \boldsymbol{A}_\mu \times \boldsymbol{J}_\mu \ , \tag{2}$$

* Laboratoire associé au C.N.R.S.
Postal address: Laboratoire de Physique Théorique et Hautes Energies, Bâtiment 211, Faculté des Sciences, 91-Orsay, France.

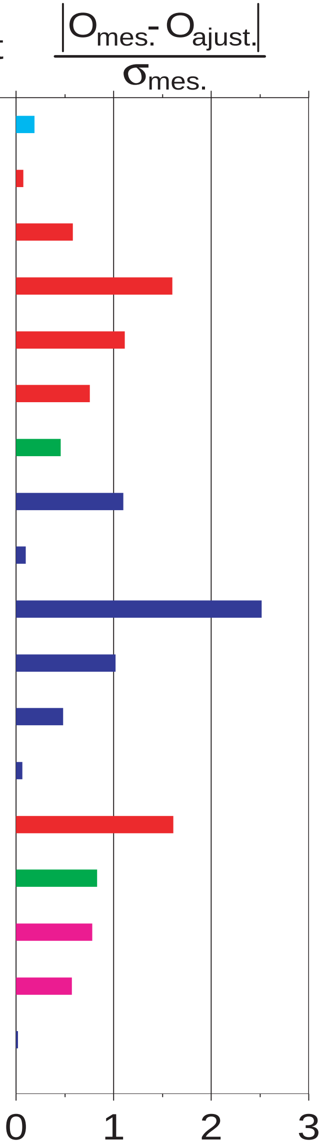
These papers, gave us to-day

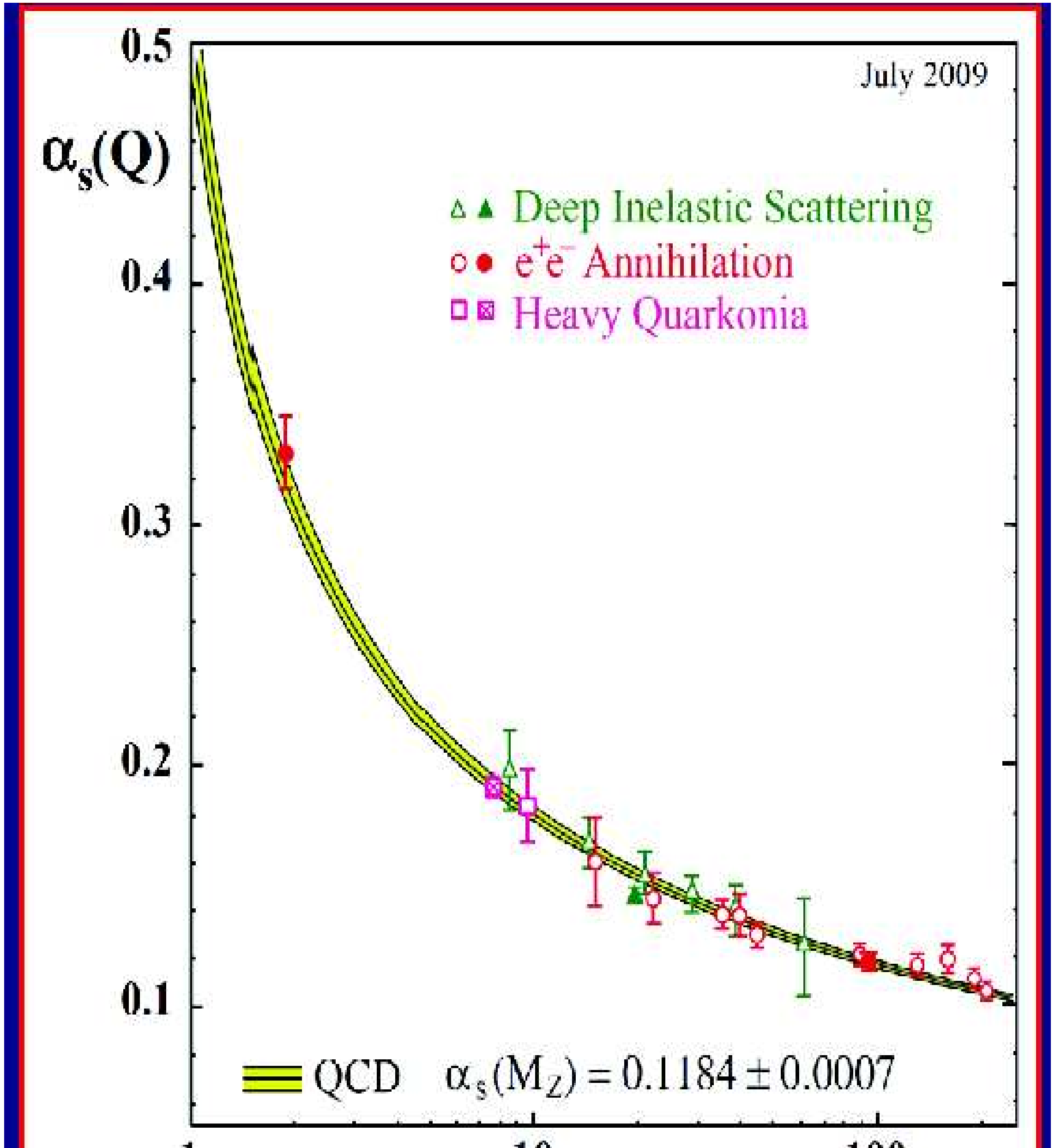
THE STANDARD MODEL

THE STANDARD MODEL

HAS BEEN ENORMOUSLY SUCCESSFUL

Observable	Mesure	Ajustement
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02761 ± 0.00036	0.02768
m_Z [GeV]	91.1875 ± 0.0021	91.1873
Γ_Z [GeV]	2.4952 ± 0.0023	2.4965
σ^0_{had} [nb]	41.540 ± 0.037	41.481
R_l	20.767 ± 0.025	20.739
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01642
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21638 ± 0.00066	0.21566
R_c	0.1720 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0997 ± 0.0016	0.1037
$A^{0,c}_{fb}$	0.0706 ± 0.0035	0.0742
A_b	0.925 ± 0.020	0.935
A_c	0.670 ± 0.026	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.398
Γ_W [GeV]	2.133 ± 0.069	2.094
m_t [GeV]	178.0 ± 4.3	178.1





0.5
July 2009
$\alpha_s(Q)$
Deep Inelastic Scattering
e^+e^- Annihilation
Heavy Quarkonia
0.4
0.3
0.2
0.1
QCD $\alpha_s(M_Z) = 0.1184 \pm 0.0007$

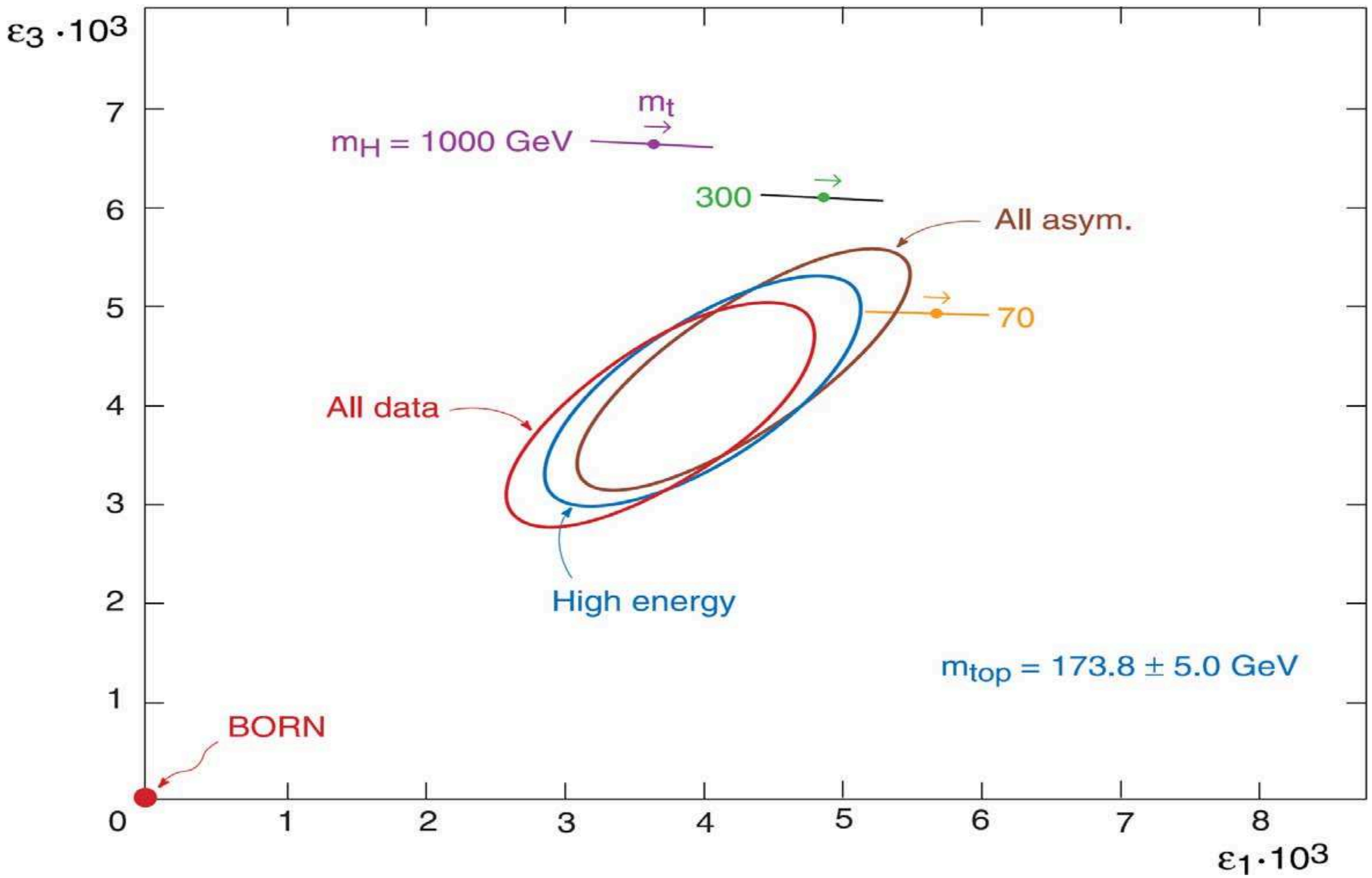
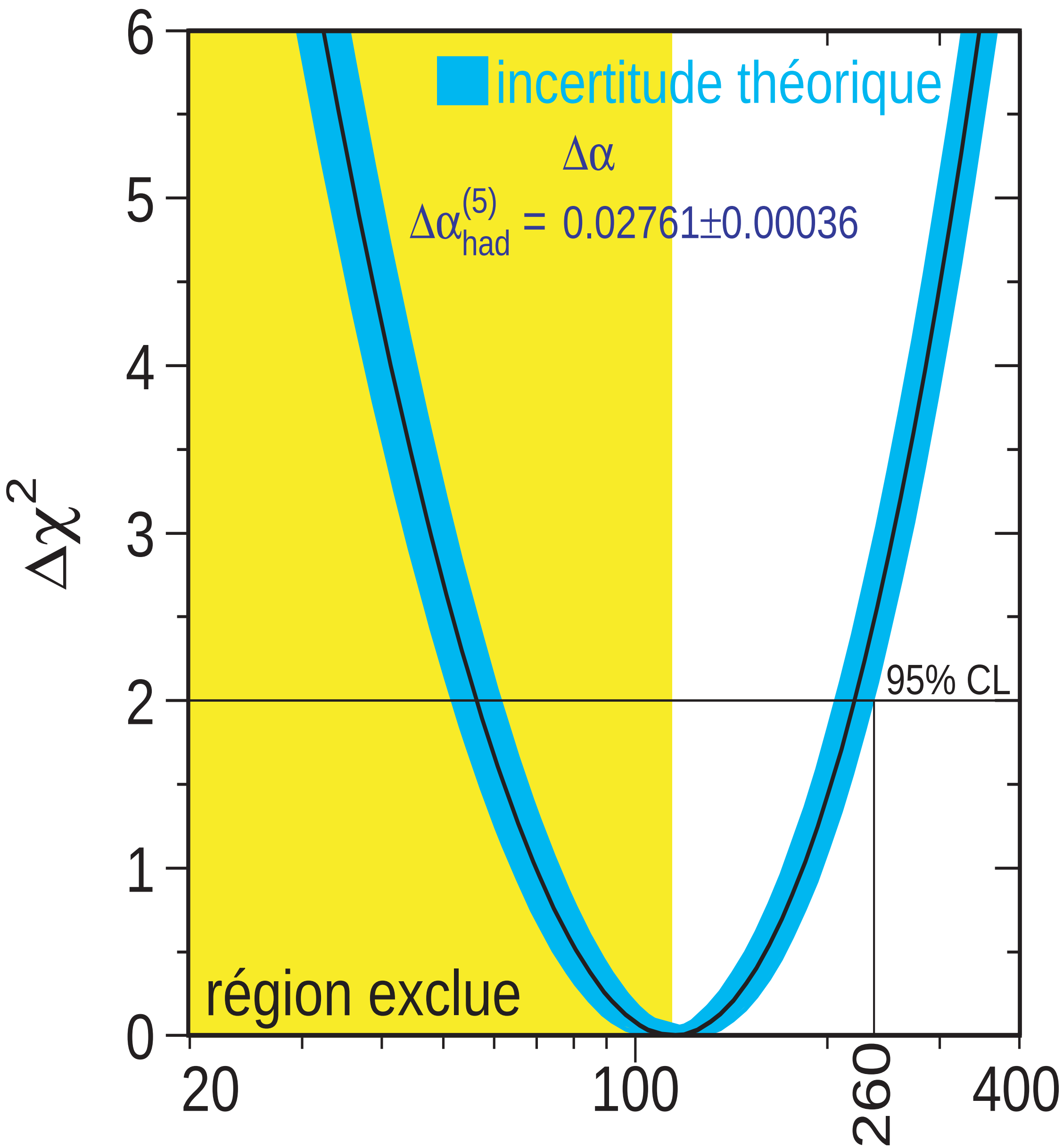


Figure 6: Data vs theory in the ϵ_3-ϵ_1 plane (notations as in fig.5)

$$\epsilon_1 = \frac{3G_F m_t^2}{8\sqrt{2}\pi^2} - \frac{3G_F m_W^2}{4\sqrt{2}\pi^2} \tan^2\theta_W \ln\frac{m_H}{m_Z} + ... \quad (1)$$

$$\epsilon_3 = \frac{G_F m_W^2}{12\sqrt{2}\pi^2} \ln\frac{m_H}{m_Z} - \frac{G_F m_W^2}{6\sqrt{2}\pi^2} \ln\frac{m_t}{m_Z} + ... \quad (2)$$



incertitude théorique
Δα
Δα(5)had = 0.02761±0.00036
95% CL
région exclue
Δχ2
0
1
2
3
4
5
6
20
100
260
400

What we have learnt

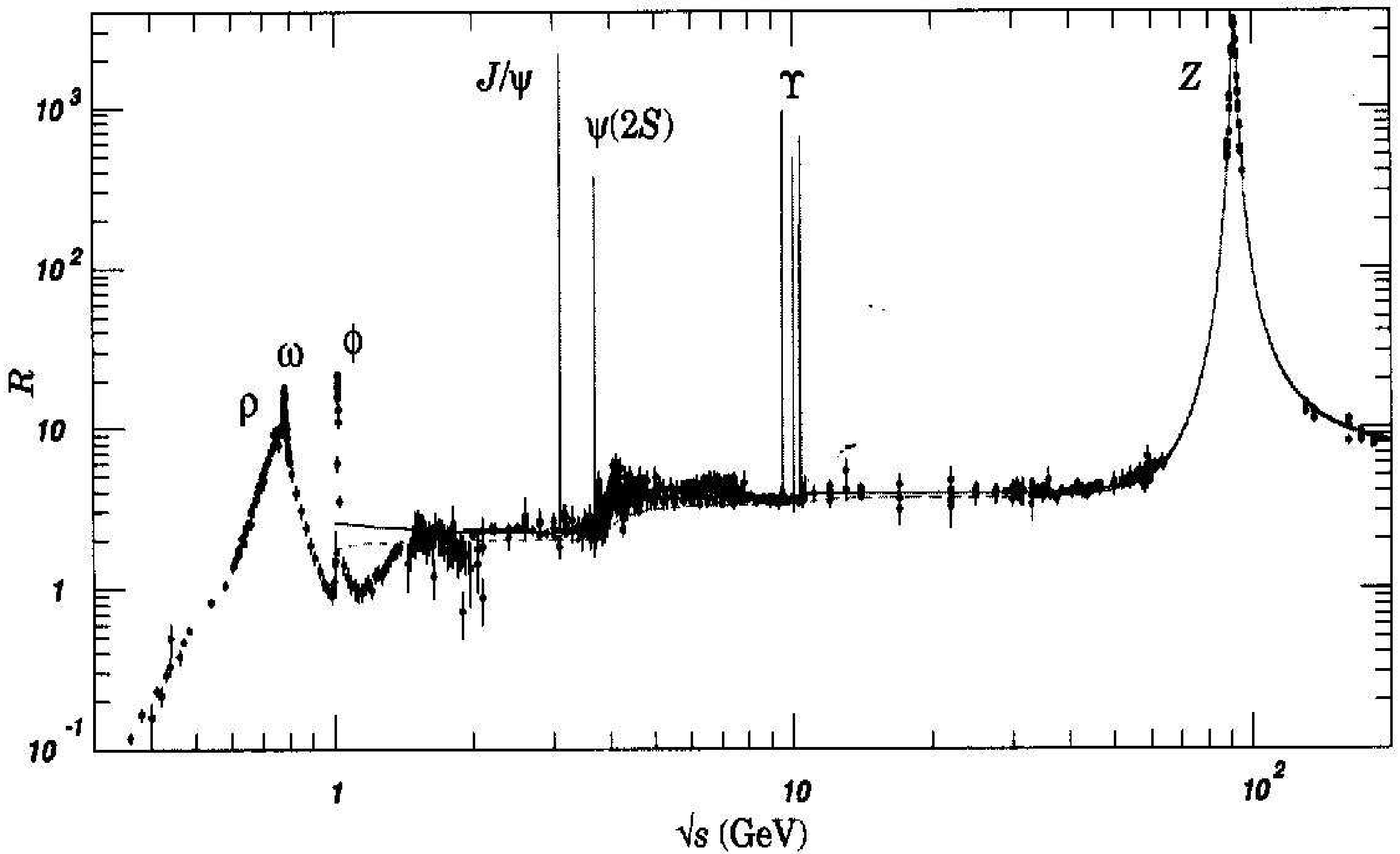
What we have learnt

Perturbation theory is remarkably reliable

What we have learnt

Perturbation theory is remarkably reliable

Outside the region of strong interactions


J/ψ
ψ(2S)
Υ
Z
ω
φ
ρ
R
10³
10²
10
1
10⁻¹
1
10
10²
√s (GeV)

-We know of no reason for this remarkable validity

Dyson's argument:

$A_n \sim \alpha^n (2n-1)!!$

Perturbation theory breaks down when $A_n \sim A_{n+1}$

$2n + 1 \sim \alpha^{-1}$

For QED $n >> 1$; For QCD ???

I want to exploit this experimental fact and argue that the available precision tests of the Standard Model allow us to claim with reasonable confidence that new physics will be unravelled at the LHC.
The argument assumes the validity of perturbation theory and it will fail if the latter fails. But, as we just saw, perturbation theory breaks down only when strong interactions become important. But new strong interactions imply new physics.

Precision measurements at one energy scale

allow us to guess New Physics at the next scale

EXAMPLES:
1) Yukawa's prediction of the π meson.

The Physics was accurate, the details were not

EXAMPLES:
1) Yukawa's prediction of the π meson.

The Physics was accurate, the details were not

2) Tini's work of the 60's gave us:
-The neutral currents
-The W bosons
-.......

In the same way New Physics may be predicted for LHC

Major task of Tevatron and LHC

Study the Higgs sector of the theory.

$m_H^2 \sim \frac{\lambda}{g^2} m_W^2$

$0 < \lambda < 1$

Major task of Tevatron and LHC

Study the Higgs sector of the theory.

Limits on the Standard Model Higgs mass:

1) $m_H \geq 114$ GeV (Exp.)

2) $m_H \; \leq \; \sim 200$ GeV (From global fit)

3) $m_H \leq \mathcal{O}$(1TeV) (Validity of perturbation)

4) $m_H \geq \mathcal{O}$(130GeV) (Vacuum stability)

$\mathbf{m_H^2} \sim \lambda$

$$\frac{d\lambda}{dt} = \frac{3}{4\pi^2}[\lambda^2 + 3\lambda h_t^2 - 9h_t^4 + ...]$$

Validity of perturbation

The Landau pole does not occur up to Λ

Validity of perturbation

The Landau pole does not occur up to Λ

$$\Lambda \sim 1TeV \rightarrow m_H \leq 0.8TeV$$

Validity of perturbation

The Landau pole does not occur up to Λ

$$\Lambda \sim 1TeV \rightarrow m_H \leq 0.8TeV$$

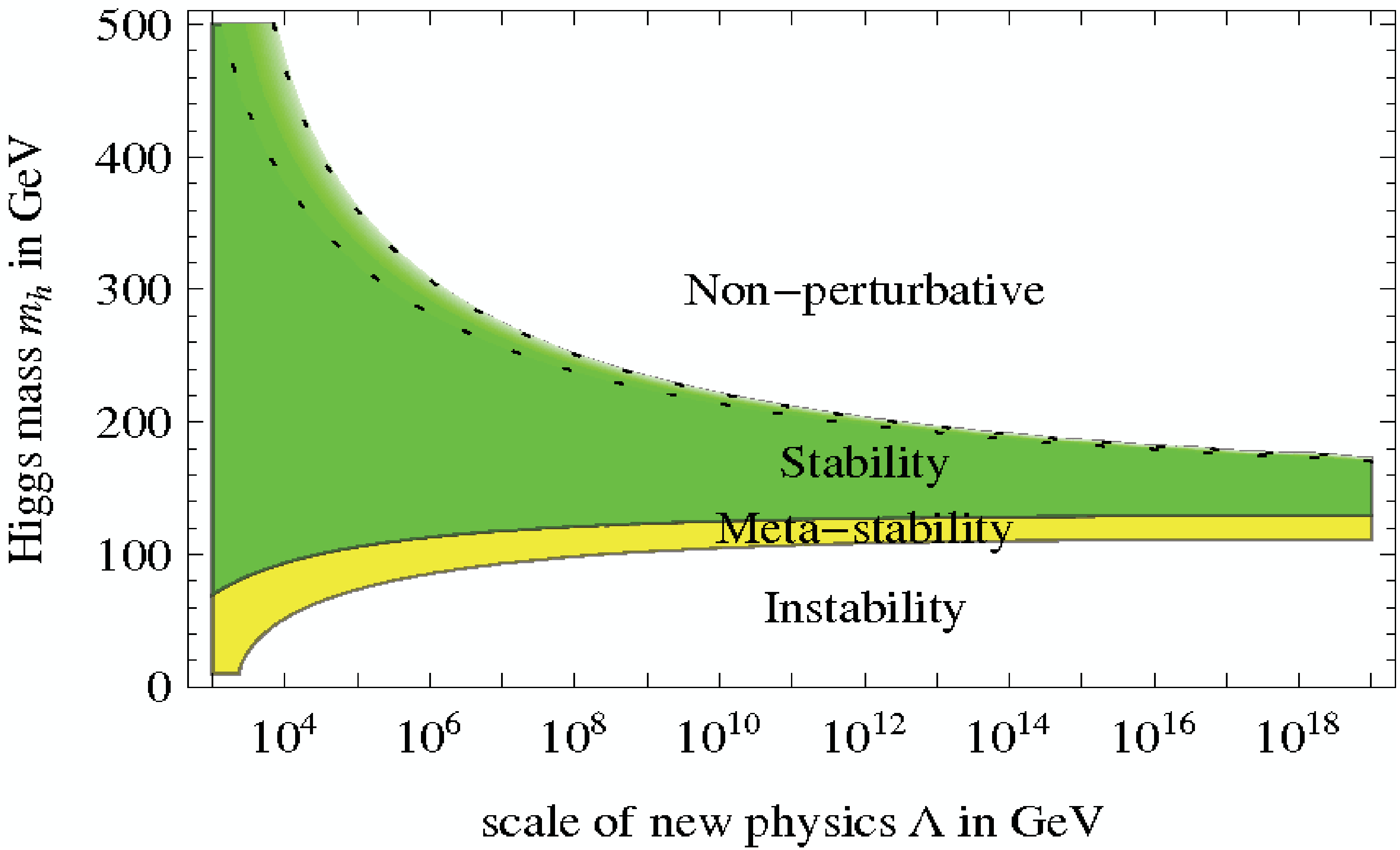
$$\Lambda \sim 10^{16}GeV \rightarrow m_H \leq 180GeV$$

Vacuum stability

$$\lambda > 0$$

for $\Lambda \sim 10^{16} GeV$

$$m_H \geq 130 GeV$$



500
400
300
200
100
0
Higgs mass m_h in GeV
Non−perturbative
Stability
Meta−stability
Instability
10^4
10^6
10^8
10^{10}
10^{12}
10^{14}
10^{16}
10^{18}
scale of new physics Λ in GeV

Can we predict the value of the Higgs mass in the Standard Model?
(No New Physics assumed!)

$$m_Z/m_H = C \tag{3}$$

$$C = \frac{m_Z}{m_H} = \frac{\sqrt{g_1^2 + g_2^2}}{\sqrt{8\lambda}} \tag{4}$$

$$
\begin{aligned}
16\pi^2\beta_{g_1} &= g_1^3\frac{1}{10} \\
16\pi^2\beta_{g_2} &= -g_2^3\frac{43}{6} \\
16\pi^2\beta_{\lambda} &= 12\lambda^2 - \frac{9}{5}g_1^2\lambda - 9g_2^2\lambda + \frac{27}{100}g_1^4 + \frac{9}{10}g_1^2g_2^2 + \frac{9}{4}g_2^4
\end{aligned}
\tag{5}
$$

$$\beta_z = \beta_{\eta_1} + \beta_{\eta_2} =$$

$$= \frac{-\lambda w}{16\pi^2 \rho z} \left[\left(\frac{27}{100}\rho^2 + \frac{9}{10}\rho + \frac{9}{4} \right) z^2 - \left(2\rho^2 + \frac{54}{5}\rho - \frac{16}{3} \right) z \right.$$

$$\left. + 12(\rho + 1)^2 \right]$$

$$\eta_1 = \frac{g_1^2}{\lambda} \;\; ; \;\; \eta_2 = \frac{g_2^2}{\lambda} \;\; ; \;\; z = \eta_1 + \eta_2 \;\; ; \;\; \rho = \frac{\eta_1}{\eta_2} \;\; ; \;\; w = \eta_1 \eta_2$$

$\Rightarrow \beta_z$ **has no zeroes for** $z > 0$

Possible (Predictable) LHC Results

Possible (Predictable) LHC Results

1) A Light Higgs is found

Possible (Predictable) LHC Results

1) A Light Higgs is found

The Standard Model is **complete**

Possible (Predictable) LHC Results

1) A Light Higgs is found

The Standard Model is **complete**

No new Strong Interactions $\Rightarrow$

Perturbation theory is reliable$\Rightarrow$

Possible (Predictable) LHC Results

1) A Light Higgs is found

The Standard Model is **complete**

No new Strong Interactions $\Rightarrow$

Perturbation theory is reliable $\Rightarrow$

$m_H^2 \sim \alpha M^2$

Hierarchy, or “Fine tuning”

Possible (Predictable) LHC Results

2) A Light Higgs is NOT found

Possible (Predictable) LHC Results

2) A Light Higgs is NOT found

Perturbation theory breaks down $\Rightarrow$

New Strong Interactions

Possible (Predictable) LHC Results

THE ABSENCE OF A LIGHT HIGGS

IMPLIES NEW PHYSICS

BUT A LIGHT HIGGS IS UNSTABLE

WITHOUT NEW PHYSICS

CONCLUSIONS

THE TIME FOR SPECULATIONS WILL BE SOON OVER!

L.H.C. IS WORKING

CONCLUSIONS

THE TIME FOR SPECULATIONS WILL BE SOON OVER!

L.H.C. IS WORKING

THE COMPUTATION OF THE RADIATIVE CORRECTIONS

IS THE MOST URGENT THEORETICAL TASK

CONCLUSIONS

THE TIME FOR SPECULATIONS WILL BE SOON OVER!

L.H.C. IS WORKING

THE COMPUTATION OF THE RADIATIVE CORRECTIONS

IS THE MOST URGENT THEORETICAL TASK

CONCLUSIONS

TINI CANNOT RETIRE

NEVER BEFORE AN EXPERIMENTAL FACILITY WAS LOADED WITH SO GREAT EXPECTATIONS