TRANSVERSE SPIN
an introduction

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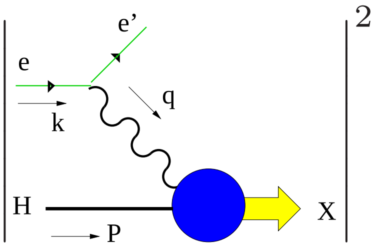
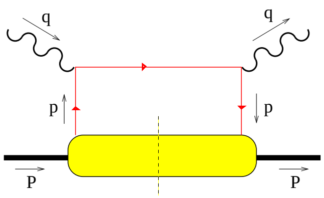
- Introduction
- Transverse spin distribution
  Properties, chirality, bounds
- Parton transverse momenta
  SIDIS, azimuthal asymmetries
- Recent developments
- Summary

- Focus is on transverse spin in deep inelastic leptoproduction
- The hard scale Q in deep inelastic scattering allows factorization of the cross section in hard (partonic) cross section and soft (hadronic) part(s) $\Longleftrightarrow$ forward/diagonal matrix elements

DIS:

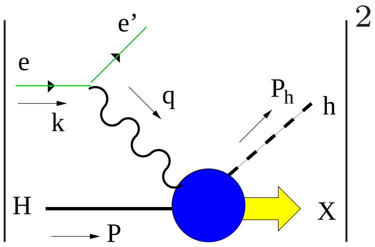


$\propto$

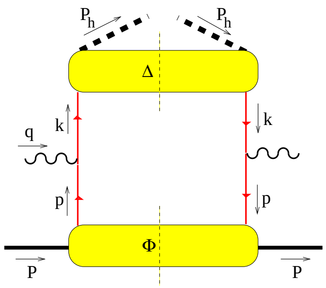


$+\,\mathcal{O}\left(\frac{1}{Q}\right)$

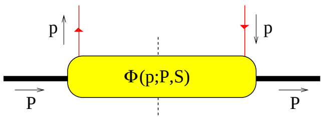
SIDIS:

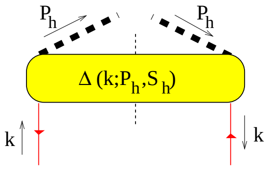


$\propto$



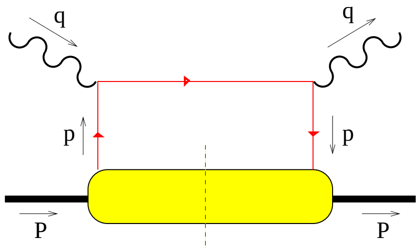
$+\,\mathcal{O}\left(\frac{1}{Q}\right)$

- Parametrization  $\Longrightarrow$ distribution functions

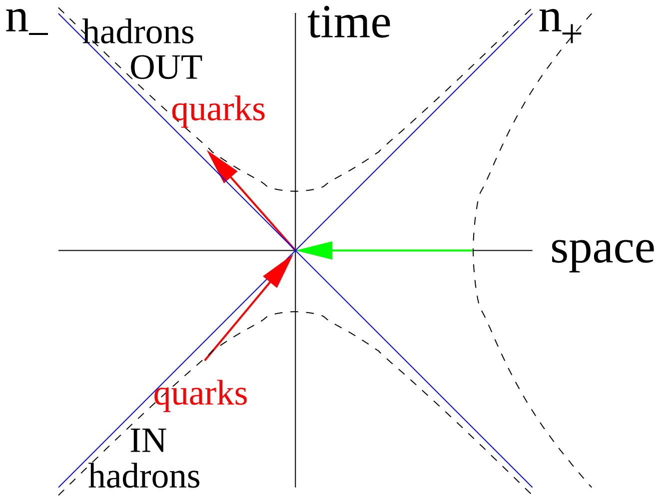
- Parametrization  $\Longrightarrow$ fragmentation functions

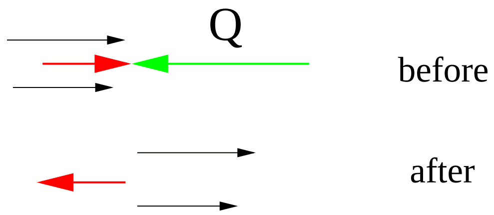
Large scale Q leads in a natural way to the use of lightlike vectors: $n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$

$$\left.\begin{array}{l} q^2 = -Q^2 \\ P^2 = M^2 \\ 2\,P\cdot q = \dfrac{Q^2}{x_B} \end{array}\right\} \longleftrightarrow \left\{\begin{array}{l} P = \dfrac{x_B\,M^2}{Q\sqrt{2}}\,n_- + \dfrac{Q}{x_B\sqrt{2}}\,n_+ \\[2ex] q = \dfrac{Q}{\sqrt{2}}\,n_- - \dfrac{Q}{\sqrt{2}}\,n_+ \end{array}\right.$$



part	'components' $-$	$+$	
HARD	$\sim Q$	$\sim Q$	
$H \rightarrow q$	$\sim 1/Q$	$\sim Q$	$\rightarrow \int dp^- d^2p_T \ldots$





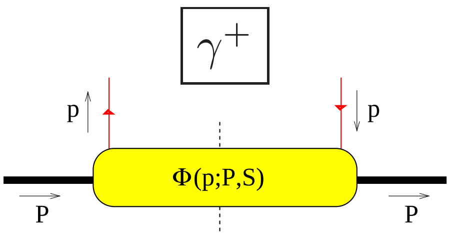
space−time development
of deep−inelastic scattering

(LEADING) QUARK DISTRIBUTION FUNCTIONS

Hadron structure enters hard processes via quark-quark correlator

$$\Phi_{ij}(x) = \int \frac{d\xi^-}{2\pi}\, e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}_j(0)\,\mathcal{U}(0,\xi)\,\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

Leading part



This part is using symmetry and hermiticity constraints parameterized with distribution functions depending on $x = p^+/P^+$

in DIS measured at $x_B = Q^2/2P\cdot q$
in DY measured at $x_A = P_A\cdot q/P_A\cdot P_B$

Parameterization of leading part for an unpolarized hadron:

$$\Phi^{\text{twist}-2}(x) = \frac{1}{2}\, f_1(x)\, \not{n}_+$$

- This function exist for every quark flavor: $f_1^q(x) = q(x)$
- Leading correlator $\Phi(x)\,\gamma^+$ has operator structure $\psi_+^\dagger(0)\psi_+(\xi)$ and represents a density
- $(\Phi\gamma^+)_{ij}^T$ is a semi-definite quark production matrix
- The nonlocal operator can be expanded (OPE) into local twist-2 operators of the form $\overline{\psi}\,\gamma^+\,(D^+)^n\psi = \psi_+^\dagger\,(D^+)^n\psi_+$

Dirac structure in an unpolarized or spin 0 hadron target:

$$\Phi(x)\gamma^+ = f_1(x)\,\underbrace{\frac{1}{2}\,\gamma^-\gamma^+}_{\mathcal{P}_+}$$

Explicit production matrix $(\Phi\gamma^+)^T$ in chiral representation

$$M_{ij}^{(\mathrm{prod})} \quad = \quad \begin{array}{cc} \begin{pmatrix} f_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & f_1 \end{pmatrix} & \begin{array}{c} \mathrm{R} \\ \\ \\ \mathrm{L} \end{array} \\ \begin{array}{cccc} \mathrm{R} & \quad & \quad & \mathrm{L} \end{array} & \end{array}$$

The projector $\mathcal{P}_+$ leaves only two (good) quark states

These can be choosen to be chiral eigenstates:

$$\psi_{R/L} \equiv \frac{1}{2}(1 \pm \gamma_5)\psi : \qquad |\mathrm{R}\rangle \quad \text{and} \quad |\mathrm{L}\rangle$$

or they can be choosen to be transverse spin eigenstates:

$$\psi_{\uparrow/\downarrow} \equiv \frac{1}{2}(1 \pm \gamma^\alpha\gamma_5)\psi : \qquad |\uparrow\rangle \quad \text{and} \quad |\downarrow\rangle$$

(Note: $[\mathcal{P}_{R/L}, \mathcal{P}_+] = [\mathcal{P}_{\uparrow/\downarrow}, \mathcal{P}_+] = 0$)

- To get arbitrary forward matrix element one needs all (also off-diagonal) spin matrix elements.
- Use density matrix to characterize polarized target

$$\rho = \sum_\alpha |\alpha\rangle p_\alpha \langle\alpha| \qquad \text{with} \qquad \sum_\alpha p_\alpha = 1$$

- ρ is hermitean, $\rho^2 = \rho$ (projector) for a pure state
- For spin 1/2, $\rho = \rho(S)$ characterized by a spin vector, in rest frame given by $\boldsymbol{S} \equiv (\boldsymbol{S}_T, S_L)$

$$\rho(S) = \frac{1}{2}(1 + \boldsymbol{\sigma}\cdot\boldsymbol{S}) = \frac{1}{2}\left(\begin{array}{cc} 1 + S_L & S_T^1 - i\,S_T^2 \\ S_T^1 + i\,S_T^2 & 1 - S_L \end{array}\right)$$

- For a pure state $|\boldsymbol{S}| = 1$
- In any frame $S^\mu(P)$ with $P\cdot S = 0$ and $-1 \le S^2 \le 0$

Extension ...

- For spin 1, $\rho = \rho(S,T)$ characterized by a vector S and a symmetric traceless second rank tensor T with $P.S = P_\mu T^{\mu\nu}$ $= T^{\mu\nu}P_\nu = 0$

TARGET SPIN (SPIN 1/2)

Momentum: $$P^\mu \equiv P^+\, n_+^\mu + \frac{M^2}{2P^+}\, n_-^\mu$$

Spin vector: $$S^\mu \equiv S_{L}\,\frac{P^+}{M}\, n_+^\mu - S_{L}\,\frac{M}{2P^+}\, n_-^\mu + S_{T}^\mu$$

Incorporate spin via S-dependent production matrix $M = (\Phi\gamma^+)^T$:

$$M(S) = \text{Tr}\left[\underbrace{\overbrace{\rho(\overbrace{S}^{\text{spin vector}})}^{\text{density matrix}}\tilde{M}}_{\uparrow}\right] = \sum_\alpha p_\alpha\, \tilde{M}_{\alpha\alpha}$$

Matrix in spin space

Explicitly:

$$M(S) = M_0 + S_{L}\, M_{L} + S_{T}^1 M_{T}^1 + S_{T}^2 M_{T}^2$$

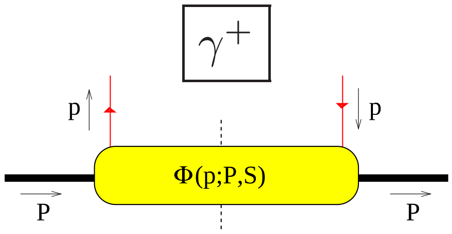
$$\updownarrow$$

$$\tilde{M}_{mm'} = \begin{pmatrix} M_0 + M_{L} & M_{T}^1 - i\, M_{T}^2 \\ M_{T}^1 + i\, M_{T}^2 & M_0 - M_{L} \end{pmatrix}$$

Back to the quark-quark correlator

$$\Phi_{ij}(x) = \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}_j(0)\,\mathcal{U}(0,\xi)\,\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

Leading part



This part is using symmetry and hermiticity constraints parameterized with distribution functions depending on $x = p^+/P^+$

Parameterization of leading part for a spin 1/2 hadron:

$$\Phi^{\text{twist}-2}(x) = \frac{1}{2}\left\{f_1(x) + S_L\, g_1(x)\gamma_5 + h_1(x)\,\gamma_5\,\not{S}_T\right\}\not{n}_+$$

The functions exist for every flavor: $g_1^q(x) = \Delta q(x)$, $h_1^q(x) = \delta q(x)$

Explicit chiral representation for $M_{ij}^{(\text{prod})}(S) = (\Phi\gamma^+)_{ij}^T$ again leaves only two (good) states in Dirac space

$$M_{ij}(S) \quad = \quad \begin{array}{cc} \left(\begin{array}{cccc} f_1 + S_L\, g_1 & 0 & 0 & (S_T^1 + i\,S_T^2)\,h_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (S_T^1 - i\,S_T^2)\,h_1 & 0 & 0 & f_1 - S_L\, g_1 \end{array}\right) & \begin{array}{c} \mathrm{R} \\ \\ \\ \mathrm{L} \end{array} \\ \begin{array}{cccc} \mathrm{R} & \quad & \quad & \mathrm{L} \end{array} & \end{array}$$

THE FULL COLLINEAR QUARK STRUCTURE

For a spin 1/2 hadron (e.g. nucleon) the quark production matrix in quark$\otimes$nucleon spin space can be given by

$$M^{(\mathrm{prod})} \quad = \quad \begin{array}{cc} \begin{pmatrix} f_1 + g_1 & 0 & 0 & 2\,h_1 \\ 0 & f_1 - g_1 & 0 & 0 \\ 0 & 0 & f_1 - g_1 & 0 \\ 2\,h_1 & 0 & 0 & f_1 + g_1 \end{pmatrix} & \begin{array}{c} \mathrm{R}\rightarrow \\ \leftarrow\mathrm{R} \\ \mathrm{L}\rightarrow \\ \leftarrow\mathrm{L} \end{array} \\ \begin{array}{cccc} \mathrm{R}\rightarrow & \leftarrow\mathrm{R} & \mathrm{L}\rightarrow & \leftarrow\mathrm{L} \end{array} & \end{array}$$

- The three distribution functions are in principle independent!
- Positivity of production matrix gives $f_1(x) \geq 0$ and $|g_1(x)| \leq f_1(x)$ but also the more stringent (Soffer) bound

$$|h_1(x)| \leq \frac{1}{2}\left(f_1(x) + g_1(x)\right)$$

- The function h_1 is chirally odd (off-diagonal when using quark chirality eigenstates)

Effects of changing basis:

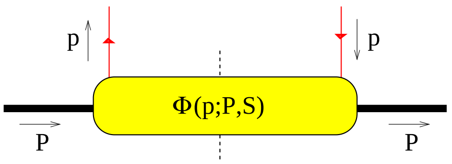
$$M^{(\mathrm{prod})} = \begin{array}{cc} \begin{pmatrix} f_1+g_1 & 0 & 0 & 2\,h_1 \\ 0 & f_1-g_1 & 0 & 0 \\ 0 & 0 & f_1-g_1 & 0 \\ 2\,h_1 & 0 & 0 & f_1+g_1 \end{pmatrix} & \begin{array}{c} \mathrm{R}\rightarrow \\ \leftarrow\mathrm{R} \\ \mathrm{L}\rightarrow \\ \leftarrow\mathrm{L} \end{array} \\ \begin{array}{cccc} \mathrm{R}\rightarrow & \leftarrow\mathrm{R} & \mathrm{L}\rightarrow & \leftarrow\mathrm{L} \end{array} & \end{array}$$

$$M^{(\mathrm{prod})} = \begin{pmatrix} f_1+h_1 & 0 & 0 & g_1+h_1 \\ 0 & f_1-h_1 & g_1-h_1 & 0 \\ 0 & g_1-h_1 & f_1-h_1 & 0 \\ g_1+h_1 & 0 & 0 & f_1+h_1 \end{pmatrix}$$

Back to the quark-quark correlator

$$\Phi_{ij}(x) = \int \frac{d\xi^-}{2\pi}\, e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}_j(0)\,\mathcal{U}(0,\xi)\,\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

Subleading $(1/P^+)$ part



This part is using symmetry and hermiticity constraints also parameterized with (distribution) functions depending on $x = p^+/P^+$

$[x \iff x_B]$
$[M/P^+ \iff M/Q]$

Parameterization including subleading order for a spin 1/2 hadron:

$$\Phi(x) = \frac{1}{2}\left\{ f_1(x)\,\not n_+ + S_L\, g_1(x)\gamma_5\,\not n_+ + h_1(x)\,\frac{[\,\not S_T,\,\not n_+]\gamma_5}{2}\right\}$$

$$+ \frac{M}{2P^+}\left\{ e(x) + g_T(x)\,\gamma_5\,\not S_T + S_L\, h_L(x)\,\frac{[\not n_+,\,\not n_-]\gamma_5}{2}\right\}$$

T-odd

$$+ \frac{M}{2P^+}\left\{ f_T(x)\,\epsilon_T^{\rho\sigma} S_{T\rho}\gamma_\sigma - i\,S_L\,e_L(x)\,\gamma_5 + h(x)\,\frac{i\,[\not n_+,\,\not n_-]}{2}\right\}$$

[T-odd parts will be needed for fragmentation functions]

WHERE DO THE FUNCTIONS SHOW UP

- Distribution functions can be measured in DIS or DY
- Time reversal invariance $\Rightarrow$ no T-odd distribution functions
- twist t of correlation functions $\Rightarrow$ behavior $(1/Q)^{t-2}$
- cross sections are chirally even $\Rightarrow$ in a specific DIS cross section only one chiral-even function will appear because the hard part (except for quark-mass effects) is chirality-conserving.

$$\sigma_{OO} \Longrightarrow \sum_{q,\bar{q}} e_q^2\, f_1^q(x_{_B})$$

$$\sigma_{LL} \Longrightarrow \lambda_e\, S_{_L} \sum_{q,\bar{q}} e_q^2\, g_1^q(x_{_B})$$

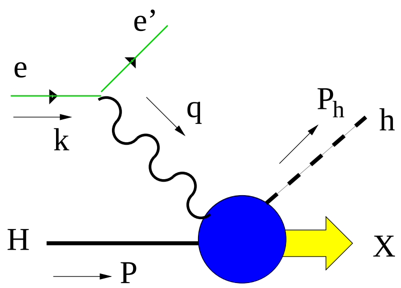
$$\langle \cos\phi_S^\ell \;\times\; \sigma_{LT}\rangle \Longrightarrow \lambda_e\, |S_{_T}|\, \frac{M}{Q} \sum_{q,\bar{q}} e_q^2\, g_T^q(x_{_B})$$

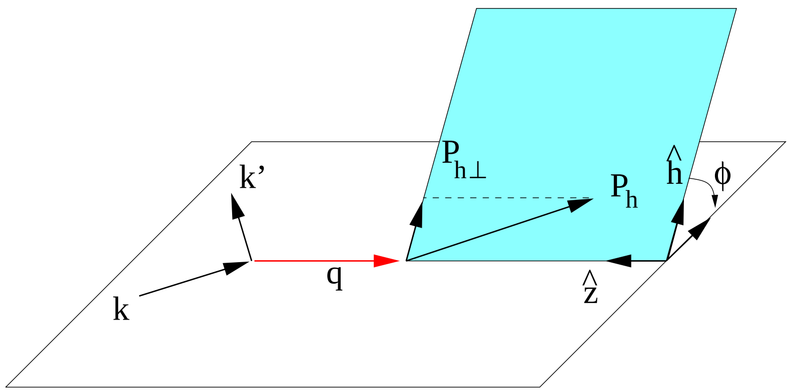
- To access chiral-odd functions (in particular h_1) one needs to combine two soft chiral-odd parts, e.g. SIDIS, DY, or 2-particle inclusive e^+e^-

1-PARTICLE INCLUSIVE LEPTOPRODUCTION

$$\ell H \longrightarrow \ell' h X$$

$$-q^2 \equiv Q^2 \to \infty$$





$$x_B = \frac{Q^2}{2P\cdot q}$$

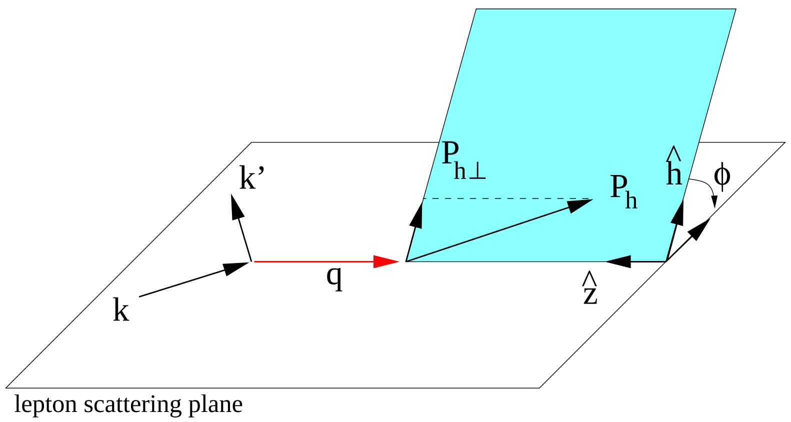
$$y = \frac{P\cdot q}{P\cdot k}$$

$$z_h = \frac{P\cdot P_h}{P\cdot q}$$

$$Q_T = \frac{|P_{h\perp}|}{z_h}$$

Hadronic tensor

$$\begin{aligned} &2M\mathcal{W}^{(\ell H)}_{\mu\nu}(q; PS; P_h S_h) \\ &\quad = \frac{1}{(2\pi)^4}\int \frac{d^3P_X}{(2\pi)^3 2P_X^0}(2\pi)^4\delta^4(q + P - P_X - P_h) \\ &\qquad \times \langle PS|J_\mu(0)|P_X; P_h S_h\rangle\langle P_X; P_h S_h|J_\nu(0)|PS\rangle \end{aligned}$$



$$\ell H \longrightarrow \ell' h X$$

$$x_B = \frac{Q^2}{2P\cdot q}$$

$$y = \frac{P\cdot q}{P\cdot k}$$

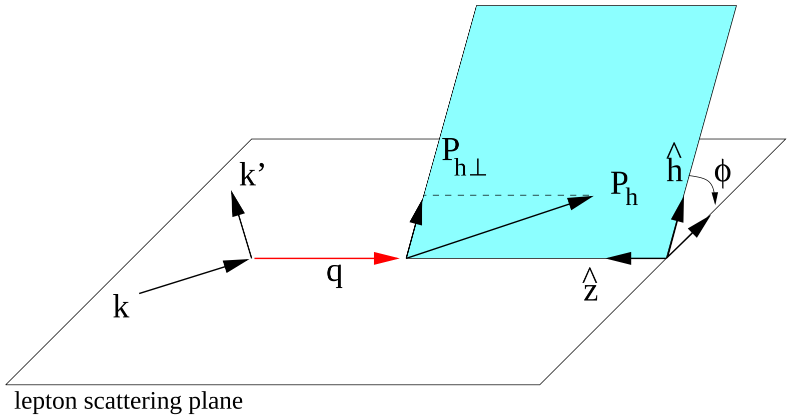
$$z_h = \frac{P\cdot P_h}{P\cdot q}$$

$$Q_T = \frac{|P_{h\perp}|}{z_h}$$

$|p_T|$-averaged cross sections (leading in $1/Q$ and $\alpha_s(Q^2)$)

$$\frac{d\sigma_{OO}}{dx_B\,dy\,dz_h} = \frac{2\pi\alpha^2\,s}{Q^4}\,x_B\,\left(1+(1-y)^2\right)\,\sum_{a,\bar{a}} e_a^2 f_1^a(x_B)\,D_1^a(z_h)$$

$$\frac{d\sigma_{LL}}{dx_B\,dy\,dz_h} = \frac{2\pi\alpha^2\,s}{Q^4}\,x_B\,y(2-y)\,\lambda_e\,S_L\,\sum_{a,\bar{a}} e_a^2 g_1^a(x_B)\,D_1^a(z_h)$$



$$\ell H \longrightarrow \ell' h X$$

$$x_B = \frac{Q^2}{2P\cdot q}$$

$$y = \frac{P\cdot q}{P\cdot k}$$

$$z_h = \frac{P\cdot P_h}{P\cdot q}$$

$$Q_T = \frac{|P_{h\perp}|}{z_h}$$

Azimuthally weighted cross sections

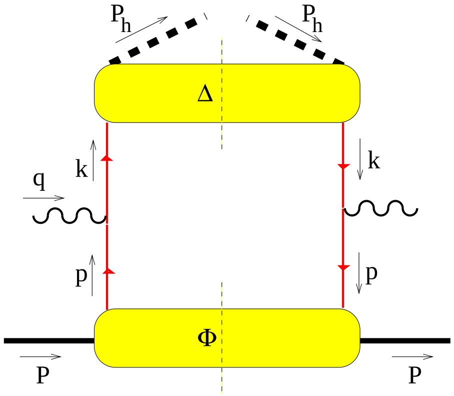
$$\langle W\rangle_{\underbrace{P_e P_H P_h}_{\uparrow\ (O,L,T)}} \equiv \int d\phi^\ell\, d^2\boldsymbol{q}_T\, \underbrace{W}_{\uparrow\ W(Q_T,\phi_h^\ell,\phi_S^\ell,\phi_{S_h}^\ell)}\, \frac{d\sigma_{P_e P_H}}{dx_B\, dy\, dz_h\, d\phi^\ell\, d^2\boldsymbol{q}_T}$$

e.g.

$$\langle 1\rangle_{OOO} = \frac{2\pi\alpha^2\, s}{Q^4}\, x_B\, \left(1+(1-y)^2\right) \sum_{a,\bar{a}} e_a^2 f_1^a(x_B)\, D_1^a(z_h)$$

Large scale Q leads in a natural way to the use of lightlike vectors: $n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$

$$\left.\begin{array}{l} q^2 = -Q^2 \\ P^2 = M^2 \\ P_h^2 = M_h^2 \\ 2\,P\cdot q = \frac{Q^2}{x_B} \\ 2\,P_h \cdot q = -z_h\,Q^2 \end{array}\right\} \longleftrightarrow \left\{\begin{array}{l} P_h = \frac{z_h\,Q}{\sqrt{2}}\,n_- + \frac{M_h^2}{z_h\,Q\sqrt{2}}\,n_+ \\ \\ q = \frac{Q}{\sqrt{2}}\,n_- - \frac{Q}{\sqrt{2}}\,n_+ + q_T \\ \\ P = \frac{x_B\,M^2}{Q\sqrt{2}}\,n_- + \frac{Q}{x_B\sqrt{2}}\,n_+ \end{array}\right.$$



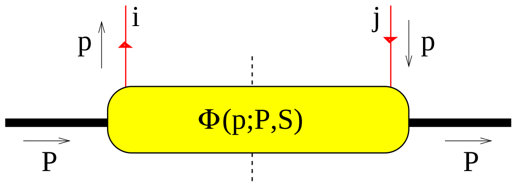
	$-$	$+$	
$q \to h$	$\sim Q$	$\sim 1/Q$	$\to \int dk^+ \ldots$
HARD	$\sim Q$	$\sim Q$	
$H \to q$	$\sim 1/Q$	$\sim Q$	$\to \int dp^- \ldots$

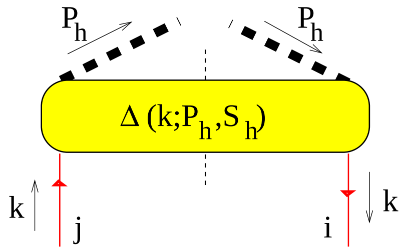
Three external momenta $(P,\, P_h,\, q) \to$ transverse direction relevant

$$q_T = q + x_B\,P - \frac{P_h}{z_h} = -\frac{P_{h\perp}}{z_h}$$

No integrations $\int d^2p_T \ldots$ and $\int d^2k_T \ldots$ in soft parts!

SOFT PART IN SIDIS





Hard inclusive (polarized) deep inelastic leptoproduction involves the soft part Φ integrated over all momenta except $p^+ \equiv x\,P^+$

$$\Phi_{ij}(x) = \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}_j(0)\psi_i(\xi)|P,S\rangle\bigg|_{\xi^+=\xi_T=0}$$

Hard semi-inclusive (polarized) deep inelastic lepton-hadron scattering involves soft part Φ integrated over p^- fixing $p^+ \equiv x\,P^+$ and p_T

$$\Phi_{ij}(x,p_T) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\; e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}_j(0)\psi_i(\xi)|P,S\rangle\bigg|_{\xi^+=0}$$

Fragmentation into a hadron involves soft part integrated over k^- leaving $P_h^- = z k^-$ and $P_{h\perp} = -z k_T$

$$\Delta_{ij}(z,k_T) = \int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3}\; e^{ik\cdot\xi}\,\langle 0|\psi_i(\xi)|P_h,X\rangle\langle P_h,X|\overline{\psi}_j(0)|0\rangle\bigg|_{\xi^-=0}$$

QUARK SPIN STRUCTURE

For azimuthal asymmetries in semi-inclusive leptoproduction or Drell-Yan scattering one needs transverse momentum dependence

Dirac structure for a polarized (up to spin 1/2) hadron:

$$\Phi_0(x,p_T) = \left\{ f_1(x,p_T^2) + i\, h_1^\perp(x,p_T^2)\,\frac{\not p_T}{M} \right\} \not n_+$$

$$\Phi_L(x,p_T) = \left\{ S_L\, g_{1L}(x,p_T^2)\,\gamma_5 + S_L\, h_{1L}^\perp(x,p_T^2)\gamma_5\,\frac{\not p_T}{M} \right\} \not n_+$$

$$\Phi_T(x,p_T) = \left\{ f_{1T}^\perp(x,p_T^2)\,\frac{\epsilon_{T\,\rho\sigma}p_T^\rho S_T^\sigma}{M} + g_{1T}(x,p_T^2)\,\frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M}\,\gamma_5 + h_{1T}(x,p_T^2)\,\gamma_5\,\not S_T + h_{1T}^\perp(x,p_T^2)\,\frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M}\,\frac{\gamma_5\,\not p_T}{M} \right\} \not n_+$$

$$\Phi_{LL}(x,p_T) = \ldots$$

- Operator structure is still $\psi_+^\dagger(0)\psi_+(\xi)$, hence the functions have a straightforward density interpretation.
- The functions $h_1^\perp$ and $f_{1T}^\perp$ are T-odd, vanishing for distributions, but allowed in fragmentation ($f \to D$, $g \to G$, $h \to H$) or hat-notation

FULL QUARK & HADRON SPIN STRUCTURE

Quark p_T-dependent distributions

$$
\begin{array}{c}
\begin{array}{cccc}
\text{R}\rightarrow & \leftarrow\text{R} & \text{L}\rightarrow & \leftarrow\text{L}
\end{array}
\\
\begin{pmatrix}
f_1 + g_{1L} & \frac{|p_T|}{M}\, e^{i\phi}\, g_{1T} & \frac{|p_T|}{M}\, e^{-i\phi}\, h_{1L}^{\perp} & 2\, h_1 \\
\frac{|p_T|}{M}\, e^{-i\phi}\, g_{1T} & f_1 - g_{1L} & \frac{|p_T|^2}{M^2}\, e^{-2i\phi}\, h_{1T}^{\perp} & -\frac{|p_T|}{M}\, e^{-i\phi}\, h_{1L}^{\perp} \\
\frac{|p_T|}{M}\, e^{i\phi}\, h_{1L}^{\perp} & \frac{|p_T|^2}{M^2}\, e^{2i\phi}\, h_{1T}^{\perp} & f_1 - g_{1L} & -\frac{|p_T|}{M}\, e^{i\phi}\, g_{1T} \\
2\, h_1 & -\frac{|p_T|}{M}\, e^{i\phi}\, h_{1L}^{\perp} & -\frac{|p_T|}{M}\, e^{-i\phi}\, g_{1T} & f_1 + g_{1L}
\end{pmatrix}
\end{array}
$$

- Matrix representation in Dirac space allows derivation of bounds
- Several of these functions are chiral-odd!
- In appropriately weighted cross sections one can access particular transverse moments

$$
g_{1T}^{(n)}(x) = \int d^2p_T\, \underbrace{\left(\frac{\boldsymbol{p}_T^2}{2M^2}\right)^n g_{1T}(x, p_T^2)}_{g_{1T}^{(n)}(x, p_T^2)}
$$

BOUNDS ON k_T DEPENDENT FUNCTIONS

$$\begin{array}{cccc}
\mathrm{R}\rightarrow & \leftarrow\mathrm{R} & \mathrm{L}\rightarrow & \leftarrow\mathrm{L} \\
\left(\begin{array}{cccc}
f_1 + g_{1L} & \frac{|p_T|}{M}\, e^{i\phi}\, g_{1T} & \frac{|p_T|}{M}\, e^{-i\phi}\, h_{1L}^\perp & 2\, h_1 \\
\frac{|p_T|}{M}\, e^{-i\phi}\, g_{1T} & f_1 - g_{1L} & \frac{|p_T|^2}{M^2}\, e^{-2i\phi}\, h_{1T}^\perp & -\frac{|p_T|}{M}\, e^{-i\phi}\, h_{1L}^\perp \\
\frac{|p_T|}{M}\, e^{i\phi}\, h_{1L}^\perp & \frac{|p_T|^2}{M^2}\, e^{2i\phi}\, h_{1T}^\perp & f_1 - g_{1L} & -\frac{|p_T|}{M}\, e^{i\phi}\, g_{1T} \\
2\, h_1 & -\frac{|p_T|}{M}\, e^{i\phi}\, h_{1L}^\perp & -\frac{|p_T|}{M}\, e^{-i\phi}\, g_{1T} & f_1 + g_{1L}
\end{array}\right) & & &
\end{array}$$

Bounds from 1-dimensional subspaces:

$$f_1 \geq 0$$
$$|g_{1L}| \leq f_1$$

Bounds from 2-dimensional subspaces:

$$|h_1| \leq \frac{1}{2}\,(f_1 + g_{1L}) \leq f_1$$
$$|h_{1T}^{\perp(1)}| \leq \frac{1}{2}\,(f_1 - g_{1L}) \leq f_1$$
$$|g_{1T}^{(1)}| \leq \frac{|\boldsymbol{p}_T|}{2M}\sqrt{(f_1 + g_{1L})\,(f_1 - g_{1L})} \leq \frac{|\boldsymbol{p}_T|}{2M}\, f_1$$
$$|h_{1L}^{\perp(1)}| \leq \frac{|\boldsymbol{p}_T|}{2M}\sqrt{(f_1 + g_{1L})\,(f_1 - g_{1L})} \leq \frac{|\boldsymbol{p}_T|}{2M}\, f_1$$

Note: including T-odd functions: $g_{1T} \longrightarrow g_{1T} + i\, f_{1T}^\perp$ and $h_{1L}^\perp \longrightarrow h_{1L}^\perp + i\, h_1^\perp$

FRAGMENTATION FUNCTIONS FOR SPIN 0

(Leading) quark fragmentation functions into unpolarized or spin 0 hadrons enter as elements in the quark decay matrix $M = \Delta\gamma^-$ in Dirac space

$$M^{(\mathrm{dec})} = \begin{array}{cc} \begin{pmatrix} D_1 & 0 & 0 & i\frac{|k_T|\,e^{-i\phi}}{M_h}\,H_1^\perp \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -i\frac{|k_T|\,e^{+i\phi}}{M_h}\,H_1^\perp & 0 & 0 & D_1 \end{pmatrix} & \begin{array}{c} \mathrm{R} \\ \\ \\ \mathrm{L} \end{array} \\ \begin{array}{cccc} \mathrm{R} & \quad & \quad & \mathrm{L} \end{array} & \end{array}$$

- Note for fragmentation into spin 0 hadrons (e.g pions) there are two (!) fragmentation functions, D_1 and $H_1^\perp$

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- In quark decay time reversal invariance cannot be used $\longrightarrow$ T-odd fragmentation functions, in this case $H_1^\perp$
- The function $H_1^\perp$ is also chiral-odd!
  It can be used to probe chiral-odd distribution functions

INTEGRATED AND WEIGHTED DISTRIBUTIONS

From the p_T-dependent distributions we recover the p_T-integrated results and obtain p_T-weighted distributions

$$
\begin{aligned}
\Phi(x) &= \int dp^-\, d^2\boldsymbol{p}_T\; \Phi(p,P,S) \\
&= \int \frac{d\xi^-}{2\pi}\, e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}(0)\,\mathcal{U}\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0} \\
\Rightarrow\quad \Phi_\partial^\alpha(x) &= \int dp^-\, d^2\boldsymbol{p}_T\; p_T^\alpha\, \Phi(p,P,S) \\
&= \int \frac{d\xi^-}{2\pi}\, e^{ip\cdot\xi}\, \langle P,S|\, i\partial^\alpha \left(\overline{\psi}(0)\,\mathcal{U}\psi(\xi)\right)|P,S\rangle\Big|_{\xi^+=\xi_T=0}
\end{aligned}
$$

$$
\begin{aligned}
\Phi(x) &= \frac{1}{2}\left\{ f_1 + S_L\, g_1\gamma_5 + h_1\,\gamma_5\,\not{S}_T \right\} \not{n}_+ \\
\frac{1}{M}\,\Phi_\partial^\alpha(x) &= \frac{1}{2}\left\{ g_{1T}^{(1)}\, S_T^\alpha\,\gamma_5 + S_L\, h_{1L}^{\perp(1)}\,\gamma^\alpha\gamma_5 \right. \\
&\qquad \left. + f_{1T}^{\perp(1)}\,\epsilon_T^{\alpha\beta} S_{T\beta} - i\, h_1^{\perp(1)}\,\gamma^\alpha \right\} \not{n}_+
\end{aligned}
$$

T-odd

Transverse moments: $g_{1T}^{(1)} = g_{1T}^{(1)}(x) \equiv \int d^2\boldsymbol{p}_T\, \frac{\boldsymbol{p}_T^2}{2M^2}\, g_{1T}(x,\boldsymbol{p}_T)$

WHERE DO THE FUNCTIONS SHOW UP

- twist t of correlation functions
  $\Rightarrow$ behavior $(1/Q)^{t-2}$
- k_T-dependent functions
  $\Rightarrow$ azimuthal dependence (ϕ_ℓ, ϕ_h, ϕ_S)
- cross sections are chirally even

$$\langle 1 \rangle_{000} \Longrightarrow f_1 \otimes D_1$$

$$\langle \frac{Q_T}{M_h} \sin\phi_h^\ell \rangle_{L00} \Longrightarrow \lambda_e \frac{M}{Q} \left(e - \frac{m}{Mx} f_1 \right) \otimes H_1^\perp$$

- # of spin vectors is even in case of a T-even combination

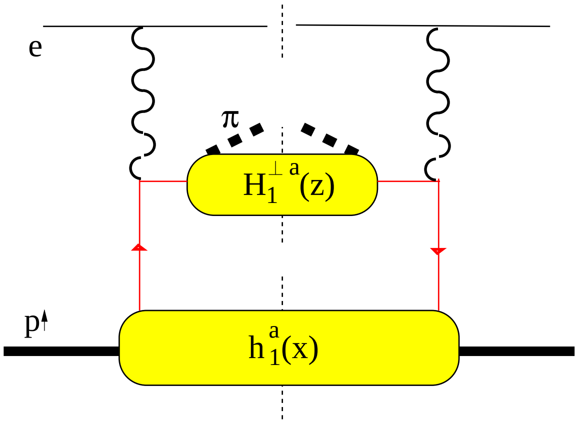
$$\langle 1 \rangle_{LL0} \Longrightarrow \lambda_e S_L\, g_1 \otimes D_1$$

$$\langle \cos(\phi_S^\ell + \phi_{S_h}^\ell) \rangle_{0TT} \Longrightarrow |S_T|\,|S_{hT}|\, h_1 \otimes H_1$$

$$\langle \frac{Q_T}{M} \cos(\phi_S^\ell - \phi_h^\ell) \rangle_{LT0} \Longrightarrow \lambda_e\, |\boldsymbol{S}_T|\, g_{1T}^{(1)} \otimes D_1$$

- # of spin vectors is odd in case of a T-odd combination (single spin asymmetries)

$$\langle \frac{Q_T}{M_h} \sin(\phi_h^\ell + \sin\phi_S^\ell) \rangle_{0T0} \Longrightarrow |S_T|\, h_1 \otimes H_1^{\perp(1)}$$



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$$\left\langle \frac{Q_T}{M_\pi} \sin(\phi_h^\ell + \phi_S^\ell) \right\rangle_{OTO} = \frac{2\pi\alpha^2\, s}{Q^4} \, |\boldsymbol{S}_T| \, 2(1-y) \sum_{a,\bar{a}} e_a^2 \, x_B \, h_1^a(x_B) H_1^{\perp(1)a}(z_h)$$

How large can $H_1^{\perp(1)}(z)$ become?

$$|H_1^{\perp(1)}(z, -z\boldsymbol{k}_T)| \quad = \quad |\frac{\boldsymbol{k}_T^2}{2M_\pi^2} H_1^\perp(z, -z\boldsymbol{k}_T)| \leq \frac{|\boldsymbol{k}_T|}{2M_\pi} D_1(z, -z\boldsymbol{k}_T)$$

With assumption

$$D_1(z, -z\boldsymbol{k}_T) \quad = \quad D_1(z) \frac{R_\pi^2(z)}{\pi\, z^2} \, e^{-\boldsymbol{k}_T^2 R_\pi^2}$$

one finds

$$|H_1^{\perp(1)}(z)| \leq \underbrace{\frac{\sqrt{\pi}}{4M_\pi R_\pi(z)}}_{\mathcal{O}(1)} D_1(z)$$

RECENT DEVELOPMENTS AND REMARKS

- Transverse momentum dependence needed for full forward quark spin structure

  Bacchetta, Boglione, Henneman, M, PRL 85 (2000) 712

- All results apply to CURRENT fragmentation!

  M, EPIC2000 workshop, hep-ph/0010199

- At order α_s and in hadronic single-spin asymmetries one also needs p_T-dependent gluon distribution and fragmentation functions

  Rodrigues & M, PRD 63 (2001) 094021

- Spin 1 distribution and fragmentation functions
  T-odd and chiral-odd fragmentation function without k_T

  Ji, PRD 49 (1994) 114
  Bacchetta & M, PL B 518 (2001) 85

- Evolution of h_1 is known

  Baldracchini et al., FP 29/30 (1981) 505
  Artru, Mekhfi, ZPC 45 (1990) 669

RECENT DEVELOPMENTS AND REMARKS

- Evolution of subleading functions (g_T, e and h_L) known
  Particularly simple in large N_c limit

  Ali, Braun, Hiller, PL B266 (1991) 117
  Balitsky, Braun, Koike, Tanaka, PRL 77 (1996) 3078
  Ji, Osborne, EPJ C9 (1999) 487

- Operator structure of transverse momentum dependent functions involves known twist-2 and twist-3 operators
  - Complementarity with higher twist in DIS
  - Evolution can be written down

  Henneman, Boer & M, NPB 620 (2002) 331

- Particular transverse moments, e.g. $H_1^{\perp(1)}$, are not affected by Sudakov factors

  Boer, PRD 62 (2000) 094029

- To model functions in soft part one needs models such as bag models, spectator models

  Jakob, Mulders, Rodrigues, NPA 626 (1997) 937
  Bacchetta, Kundu, Metz & M, PLB 506 (2001) 155 and hep-ph/0201091

SUMMARY AND CONCLUSIONS

Inclusive leptoproduction

- Quark DF $f_1^q(x)$ and $\bar{f}_1^q(x)$ for $q = u$, d, s, c
- Chirality distributions $g_1^q(x)$ and $\bar{g}_1^q(x)$
- Sum rules ($\longrightarrow$ moments of DF) and evolution (coupling to gluons)
- Gluon distributions, $G(x)$ and $\Delta G(x)$
- Higher twist distributions ($\longrightarrow$ quark-gluon correlations)

Semi-inclusive leptoproduction

1. **Flavor sensitivity** . . .
2. **Full collinear spin structure** . . .
3. **Full forward spin structure (with k_T)** . . .

Hadroproduction

1. **Full forward spin structure (with k_T) accessible** . . .
2. **Need for specific situations** . . .

SUMMARY AND CONCLUSIONS

Semi-inclusive leptoproduction

1. **Flavor sensitivity**
   - Production via (favored) FF: $u \longrightarrow \pi^+$, $\bar{u} \longrightarrow \pi^-$, ...

$$\sum_q e_q^2\, f_1^q(x) \Rightarrow \sum_q e_q^2\, D_1^{q\rightarrow h}(z)\, f_1^q(x)$$

   - Gluon DF via charm production
2. **Full collinear spin structure**
   - Access to chiral-odd DF h_1^q (need transverse pol.)
   - No mixing with gluons under evolution for chiral-odd DF
3. **Full forward spin structure**
   - p_T-dependent DF $\Leftrightarrow$ QCD dynamics (higher twist)
   - Obtained from azimuthal dependence, in most cases requiring also polarization
   - T-odd FF (for spin 0 and 1/2 necessarily p_T-dependent) appear in single-spin asymmetries
   - The FF $H_1^\perp$ (chiral-odd and T-odd) can be used to access h_1^q via single-spin asymmetries

SUMMARY AND CONCLUSIONS

Hadroproduction

1. **Full spin structure (with k_T)**
   - Access to chiral-odd DF h_1^q (need transverse pol.)
   - ... and many more functions obtained by looking at azimuthal dependences, in most cases requiring also polarization
2. **Need for specific situations**
   - lepton pair production (Drell-Yan)
   - Single-spin asymmetries pointing to production via T-odd fragmentation functions