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AZIMUTHAL (AND SPIN) ASYMMETRIES IN SEMIINCLUSIVE LEPTOPRODUCTION

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- Inclusive and semiinclusive leptoproduction
- Parton transverse momentum
- Operator structure within QCD
- QCD corrections for azimuthal asymmetries
- Summary and conclusions

in collaboration with
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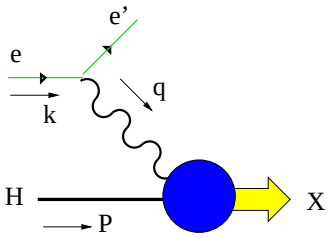
$\ell H \longrightarrow \ell' X$

Scale : $-q^2 \equiv Q^2$ (Hard : $Q^2 \to \infty$)



$$x_{_B} = \frac{Q^2}{2P\cdot q}$$
$$y = \frac{P\cdot q}{P\cdot k}$$

Hadronic tensor

$$2M\, W^{(\ell H)}_{\mu\nu}(q;PS) = \frac{1}{(2\pi)}\int \frac{d^3P_X}{(2\pi)^3 2P^0_X}(2\pi)^4\delta^4(q+P-P_X) \times \langle PS|J_\mu(0)|P_X\rangle\langle P_X|J_\nu(0)|PS\rangle$$

Inclusive scattering (unpolarized lepton and hadron, γ-exchange)[1]

$$2MW_S^{\mu\nu}(q,P) = \underbrace{\left(-g^{\mu\nu} + \hat{q}^\mu\hat{q}^\nu - \hat{t}^\mu\hat{t}^\nu\right)}_{-g_\perp^{\mu\nu}} F_1 + \hat{t}^\mu\hat{t}^\nu \underbrace{\left(\frac{F_2}{2x_{_B}} - F_1\right)}_{F_L}$$

Cross section

$$\frac{d\sigma_{OO}}{dx_{_B}dy} = \frac{4\pi\,\alpha^2\,x_{_B}s}{Q^4}\left\{\left(1-y+\frac{1}{2}\,y^2\right)F_T + (1-y)\,F_L\right\}$$

Parton model

$$\begin{aligned} F_T(x_{_B},Q) &= \frac{1}{2}\sum_{q,\bar{q}} e_q^2\, f_1^q(x_{_B}) \\ F_L(x_{_B},Q) &= \mathcal{O}(\alpha_s) \end{aligned}$$

Importance

Obtaining PDF's, $q(x) = f_1^q(x)$, and (via evolution) $g(x)$, which encode basic information on the (soft) structure of the nucleon, and which can be analysed within QCD framework (OPE, ...)

[1]

$$\begin{aligned} \hat{q}^\mu &= \frac{q^\mu}{Q} \\ \hat{t}^\mu &= \frac{\tilde{P}^\mu}{\sqrt{\tilde{P}^2}} = \frac{1}{\sqrt{\tilde{P}^2}}\left(P^\mu - \frac{P\cdot q}{q^2}\,q^\mu\right) \end{aligned}$$

Via known leptonic tensor

$$L_{\mu\nu}^{(\ell H)}(k,k',\lambda_e) = 2k_\mu k'_\nu + 2k_\nu k'_\mu - Q^2 g_{\mu\nu} + 2i\,\lambda_e\,\epsilon_{\mu\nu\rho\sigma}q^\rho k^\sigma$$

Expand lepton momentum in $\hat{q}$, $\hat{t}$ and perpendicular vector $\hat{\ell}$

$$k^\mu = \frac{Q}{2}\,\hat{q}^\mu + \frac{(2-y)\,Q}{2y}\,\hat{t}^\mu + \frac{Q\sqrt{1-y}}{y}\,\hat{\ell}^\mu$$

Lepton probe:

$$\begin{aligned} L_{(\ell H)}^{\mu\nu} = \frac{Q^2}{y^2}\Bigg[& -2\left(1-y+\frac{1}{2}\,y^2\right)g_\perp^{\mu\nu} \\ & +4\,(1-y)\,\hat{t}^\mu\hat{t}^\nu \\ & +4\,(1-y)\left(\hat{\ell}^\mu\hat{\ell}^\nu + \frac{1}{2}\,g_\perp^{\mu\nu}\right) \\ & +2\,(2-y)\sqrt{1-y}\;\hat{t}^{\{\mu}\hat{\ell}^{\nu\}} \\ & -i\,\lambda_e\,y(2-y)\,\epsilon_\perp^{\mu\nu} \\ & -2i\,\lambda_e\,y\sqrt{1-y}\;\hat{t}^{\,[\mu}\epsilon_\perp^{\nu]\rho}\hat{\ell}_\rho\Bigg] \end{aligned}$$

Inclusive scattering off a polarized target with $S = S_L \frac{P}{M} + S_\perp + \ldots$ using polarized leptons (γ-exchange)

$$2MW_A^{\mu\nu}(q,P,S) = \underbrace{-i\,S_L\,\frac{\epsilon^{\mu\nu\rho\sigma}P_\rho q_\sigma}{P\cdot q}}_{-i\,S_L\,\epsilon_\perp^{\mu\nu}}\,g_1 + i\,\frac{2Mx_B}{Q}\,\hat{t}^{\,[\mu}\epsilon_\perp^{\nu]\rho}S_{\perp\rho}\,g_T$$

Cross sections

$$\frac{d\sigma_{LL}}{dx_B\,dy} = \frac{4\pi\,\alpha^2}{Q^2}\,\lambda_e\,S_L\,\left(1-\frac{y}{2}\right)g_1$$

$$\frac{d\sigma_{LT}}{dx_B\,dy} = -\frac{4\pi\,\alpha^2}{Q^2}\,\lambda_e\,|S_\perp|\,\cos\,\phi_S^\ell\,\frac{2Mx_B}{Q}\sqrt{1-y}\;g_T$$

Parton model

$$g_1(x_B,Q) = \frac{1}{2}\sum_{q,\bar{q}} e_q^2\,g_1^q(x_B)$$

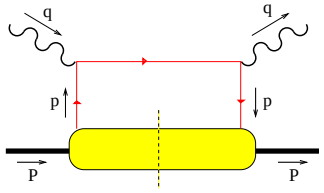
$$g_T(x_B,Q) = (g_1+g_2)(x_B,Q) = \frac{1}{2}\sum_{q,\bar{q}} e_q^2\,g_T^q(x_B)$$

Importance

Polarized PDF's $\Delta q(x) = g_1^q(x)$ and (via evolution) $\Delta g(x)$, again basic information on quark and gluon structure. Via g_2 access to quark-quark-gluon matrix elements (higher twist operators)

Large scale Q leads in a natural way to the use of lightlike vectors: $n_+^2 = n_-^2 = 0$ and $n_+\cdot n_- = 1$

$$\left.\begin{array}{l} q^2 = -Q^2 \\ P^2 = M^2 \\ 2\,P\cdot q = \frac{Q^2}{x_B} \end{array}\right\} \longleftrightarrow \left\{\begin{array}{l} P = \frac{x_B M^2}{Q\sqrt{2}}\,n_- + \frac{Q}{x_B\sqrt{2}}\,n_+ \\ \\ q = \frac{Q}{\sqrt{2}}\,n_- - \frac{Q}{\sqrt{2}}\,n_+ \end{array}\right.$$



part	'components' $-$	$+$	
HARD	$\sim Q$	$\sim Q$	
$H \to q$	$\sim 1/Q$	$\sim Q$	$\to \int dp^-\,d^2p_T\,\ldots$

Note: the lightlike vectors may be rescaled into dimensionful vectors $\hat{p} = (Q/x_B\sqrt{2})n_+$ and $\hat{n} = (x_B\sqrt{2}/Q)n_-$

$$P = \hat{p} + \frac{M^2}{2}\,\hat{n}$$

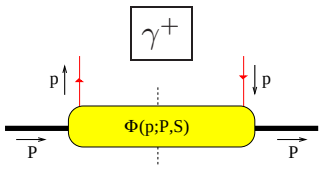
$$q = -x_B\,\hat{p} + \frac{Q^2}{2x_B}\,\hat{n}$$

(LEADING) QUARK DISTRIBUTION FUNCTIONS

Hadron structure enters hard processes via quark-quark correlator

$$\Phi_{ij}(x) = \int \frac{d\xi^-}{2\pi}\, e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}_j(0)\,\mathcal{U}(0,\xi)\,\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

Leading part



This part is using symmetry and hermiticity constraints parameterized with distribution functions depending on $x = p^+/P^+$, in DIS measured at $x_{_B} = Q^2/2P\cdot q$ and in DY at $x_{_A} = P_A\cdot q/P_A\cdot P_B$

Parameterization for a spin 1/2 hadron:

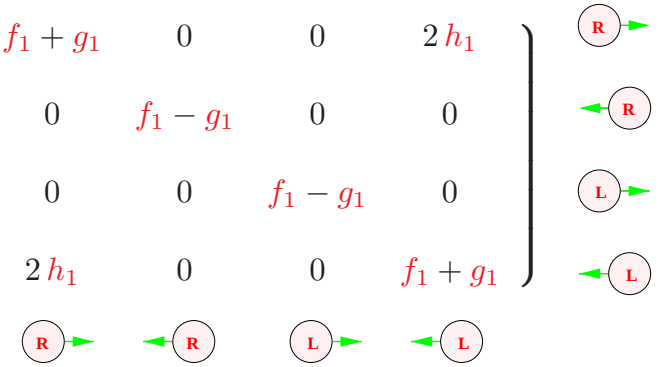
$$\Phi^{\text{twist}-2}(x) = \frac{1}{2}\Big\{ f_1(x)\,\not{n}_+ + S_{_L}\, g_1(x)\gamma_5\,\not{n}_+ + h_1(x)\; \$_{_T}\,\not{n}_+\gamma_5 \Big\}$$

- The functions exist for every quark flavor: $f_1^q(x) = q(x)$, $g_1^q(x) = \Delta q(x)$, $h_1^q(x) = \delta q(x)$
- Leading correlator is of the nonlocal form $\psi_+^\dagger(0)\psi_+(\xi)$ and can be expanded (OPE) into local twist-2 operators of the form $\overline{\psi}\,\gamma^+\,(D^+)^n\psi = \psi_+^\dagger\,(D^+)^n\psi_+$
- Part $\sim (M/P^+)$, with $g_T^q(x)$, is needed at order $1/Q$ in the g_2 structure function ($\psi_+^\dagger\,(D^+)^m D_{_T}^\alpha\,(D^+)^{n-m}\psi_+$ operators)

INTERPRETING THE LEADING QUARK STRUCTURE

For a spin 1/2 hadron (e.g. nucleon) the quark distribution functions enter in a quark production matrix. Omitting the higher twist entries one obtains in quark⊗nucleon spin space

$$M^{(\text{prod})} = \begin{pmatrix} f_1+g_1 & 0 & 0 & 2\,h_1 \\ 0 & f_1-g_1 & 0 & 0 \\ 0 & 0 & f_1-g_1 & 0 \\ 2\,h_1 & 0 & 0 & f_1+g_1 \end{pmatrix}$$



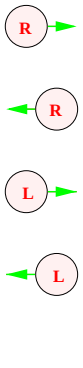
- The three distribution functions are in principle independent!
- Positivity of production matrix gives $f_1(x) \geq 0$ and $|g_1(x)| \leq f_1(x)$ but also the more stringent (Soffer) bound

$$|h_1(x)| \leq \frac{1}{2}\,(f_1(x) + g_1(x))$$

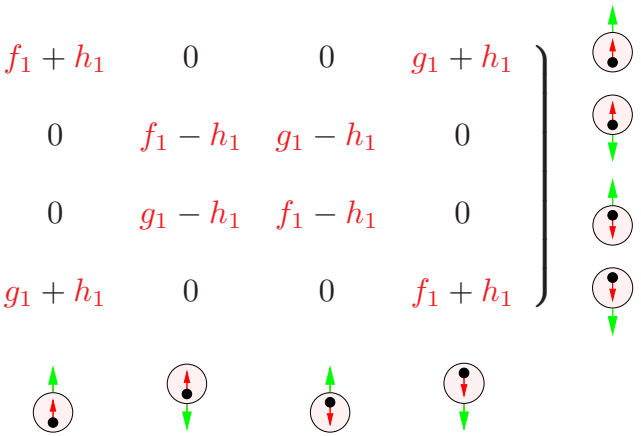
- The function h_1 is chirally odd and is not accessible in DIS

Effects of changing basis:

$$M^{(\mathrm{prod})} = \begin{pmatrix} f_1+g_1 & 0 & 0 & 2\,h_1 \\ 0 & f_1-g_1 & 0 & 0 \\ 0 & 0 & f_1-g_1 & 0 \\ 2\,h_1 & 0 & 0 & f_1+g_1 \end{pmatrix}$$



$$M^{(\mathrm{prod})} = \begin{pmatrix} f_1+h_1 & 0 & 0 & g_1+h_1 \\ 0 & f_1-h_1 & g_1-h_1 & 0 \\ 0 & g_1-h_1 & f_1-h_1 & 0 \\ g_1+h_1 & 0 & 0 & f_1+h_1 \end{pmatrix}$$

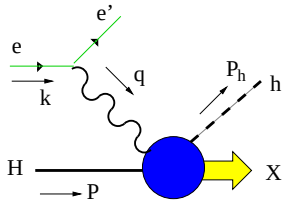


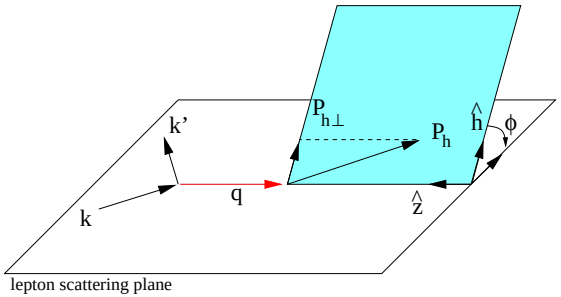
Objects in study of hadron structure (given QCD) include:

$\langle P+q\|\psi^\dagger(\xi)\Gamma\psi(\xi)\|P\rangle$	Elastic scattering $\Rightarrow$ form factor $G_\Gamma(q)$ $\Rightarrow$ spatial density/excitation spectrum
$\langle P\|\psi^\dagger(0)\Gamma\psi(\xi)\|P\rangle\big\|_{\underbrace{\xi^+=\xi_T=0}_{LC}}$	DIS cross section $\Rightarrow$ DF $f_\Gamma(x)$ $\Rightarrow$ (light-cone) parton momentum distribution
$\langle P\|\psi^\dagger(\xi)\Gamma\psi(\xi)\|P\rangle$	$\Rightarrow$ charge: $Q_\Gamma = G_\Gamma(0) = \int dx\ f_\Gamma(x)$
$\langle P\|\psi^\dagger(0)\Gamma D_T(\eta)\,\psi(\xi)\|P\rangle\big\|_{LC}$	DIS cross section at $1/Q$ $\Rightarrow$ (lightcone) higher twist correlation
$\langle P\|\psi^\dagger(0)\Gamma\psi(\xi^-,\xi_T)\|P\rangle\big\|_{\underbrace{\xi^+=0}_{LF}}$	SIDIS or DY azimuthal asymmetry $\Rightarrow$ DF $f_\Gamma(x,p_T)$ $\Rightarrow$ p_T-dependent distribution
$\langle 0\|\psi(\xi)\|P_h\rangle\,\langle P_h\|\psi^\dagger(0)\Gamma\|0\rangle\big\|_{LF}$	SIDIS or e^+e^- azimuthal asymmetry $\Rightarrow$ FF $D_\Gamma(z,k_T)$ $\Rightarrow$ parton fragmentation (or decay) function
$\langle P+\Delta\|\psi^\dagger(0)\Gamma\psi(\xi)\|P\rangle\big\|_{LC}$	DVCS cross section $\Rightarrow$ skewed parton distribution

$\ell H \longrightarrow \ell' h X$

$-q^2 \equiv Q^2 \to \infty$





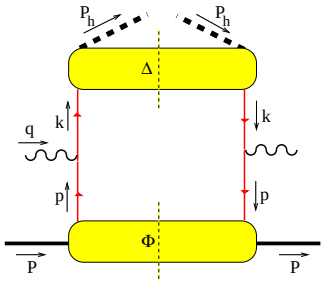
$$x_B = \frac{Q^2}{2P\cdot q}$$
$$y = \frac{P\cdot q}{P\cdot k}$$
$$z_h = \frac{P\cdot P_h}{P\cdot q}$$
$$Q_T = \frac{|P_{h\perp}|}{z_h}$$

Hadronic tensor

$$2M\mathcal{W}^{(\ell H)}_{\mu\nu}(q;PS;P_hS_h)$$
$$= \frac{1}{(2\pi)^4}\int \frac{d^3P_X}{(2\pi)^3 2P_X^0}(2\pi)^4\delta^4(q+P-P_X-P_h)$$
$$\times\langle PS|J_\mu(0)|P_X;P_hS_h\rangle\langle P_X;P_hS_h|J_\nu(0)|PS\rangle$$

Large scale Q leads in a natural way to the use of lightlike vectors: $n_+^2 = n_-^2 = 0$ and $n_+\cdot n_- = 1$

$$\left.\begin{array}{l} q^2 = -Q^2 \\ P^2 = M^2 \\ P_h^2 = M_h^2 \\ 2\,P\cdot q = \frac{Q^2}{x_B} \\ 2\,P_h\cdot q = -z_h\,Q^2 \end{array}\right\} \longleftrightarrow \left\{\begin{array}{l} P_h = \frac{z_h Q}{\sqrt{2}}\,n_- + \frac{M_h^2}{z_h Q\sqrt{2}}\,n_+ \\ q = \frac{Q}{\sqrt{2}}\,n_- - \frac{Q}{\sqrt{2}}\,n_+ + q_T \\ P = \frac{x_B M^2}{Q\sqrt{2}}\,n_- + \frac{Q}{x_B\sqrt{2}}\,n_+ \end{array}\right.$$



	$-$	$+$	
$q \to h$	$\sim Q$	$\sim 1/Q$	$\to \int dk^+ \ldots$
HARD	$\sim Q$	$\sim Q$	
$H \to q$	$\sim 1/Q$	$\sim Q$	$\to \int dp^- \ldots$

Three external momenta (P, P_h, q) → transverse direction relevant

Non-collinearity: $q_T = q + x_B\,P - \dfrac{P_h}{z_h} = -\dfrac{P_{h\perp}}{z_h}$

Semi-inclusive scattering (unpolarized lepton and hadron) [2]

$$2M\mathcal{W}_S^{\mu\nu}(q,P,P_h) = -g_\perp^{\mu\nu}\,\mathcal{H}_T + \hat{t}^\mu\hat{t}^\nu\,\mathcal{H}_L + \hat{t}^{\{\mu}\hat{h}^{\nu\}}\,\mathcal{H}_{LT} + \left(2\,\hat{h}^\mu\hat{h}^\nu + g_\perp^{\mu\nu}\right)\mathcal{H}_{TT}$$

Cross section

$$\frac{d\sigma_O}{dx_{_B}\,dy\,dz_h d^2q_{_T}} = \frac{4\pi\,\alpha^2\,s}{Q^4}\,x_{_B}\,z_h\Big\{\left(1-y+\frac{1}{2}\,y^2\right)\mathcal{H}_T + (1-y)\,\mathcal{H}_L - (2-y)\sqrt{1-y}\,\cos\phi_h^\ell\;\mathcal{H}_{LT} + (1-y)\,\cos 2\phi_h^\ell\;\mathcal{H}_{TT}\Big\}$$

[2]

$$\hat{q}^\mu = \frac{q^\mu}{Q}, \qquad \hat{t}^\mu = \frac{q^\mu + 2x_{_B}\,P^\mu}{Q},$$
$$g_\perp^{\mu\nu} = g^{\mu\nu} - \hat{q}^\mu\hat{q}^\nu + \hat{t}^\mu\hat{t}^\nu$$
$$q_T^\mu = q^\mu + x_{_B}\,P^\mu - \frac{P_h^\mu}{z_h} = -\frac{P_{h\perp}^\mu}{z_h} \equiv -Q_T\,\hat{h}^\mu$$

Semi-inclusive scattering (polarized lepton, but unpolarized hadron)

$$2M\mathcal{W}_A^{\mu\nu}(q,P,P_h) = -i\hat{t}^{[\mu}\hat{h}^{\nu]}\,\mathcal{H}'_{LT}$$

Cross section

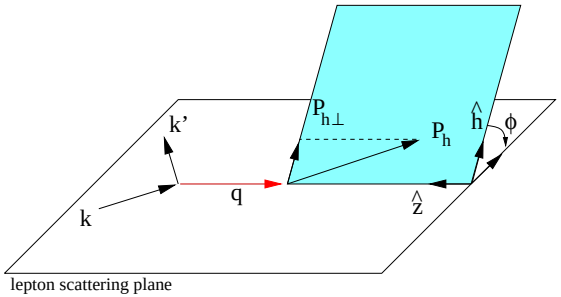
$$\frac{d\sigma_L}{dx_{_B}\,dy\,dz_h d^2q_{_T}} = \lambda_e\,\frac{4\pi\,\alpha^2}{Q^2}\,z_h\,\sqrt{1-y}\,\sin\phi_h^\ell\;\mathcal{H}'_{LT}$$

For a polarized target or using polarimetry with

$$S = S_L\,\frac{P}{M} + S_T + \ldots$$

many more structure functions appear

$\Rightarrow$ look at parton results to select relevant observables



$\ell H \longrightarrow \ell' h X$

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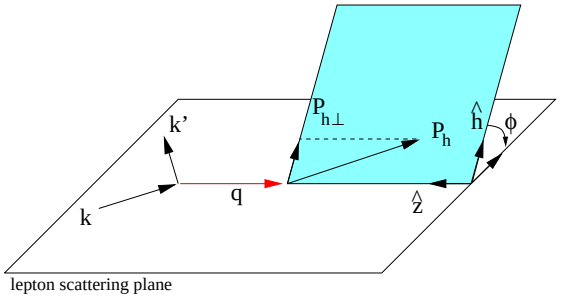
$$y = \frac{P\cdot q}{P\cdot k}$$
$$z_h = \frac{P\cdot P_h}{P\cdot q}$$
$$Q_T = \frac{|P_{h\perp}|}{z_h}$$

$|p_T|$-averaged cross sections (leading in $1/Q$ and $\alpha_s(Q^2)$)

$$\frac{d\sigma_{OO}}{dx_B\,dy\,dz_h} = \frac{2\pi\alpha^2\,s}{Q^4}\,x_B\,\Big(1+(1-y)^2\Big)\,\sum_{q,\bar q} e_q^2\,f_1^q(x_B)\,D_1^q(z_h)$$

$$\frac{d\sigma_{LL}}{dx_B\,dy\,dz_h} = \frac{2\pi\alpha^2\,s}{Q^4}\,x_B\,y(2-y)\,\lambda_e\,S_L\,\sum_{q,\bar q} e_q^2\,g_1^q(x_B)\,D_1^q(z_h)$$



$\ell H \longrightarrow \ell' h X$

$$x_B = \frac{Q^2}{2P\cdot q}$$
$$y = \frac{P\cdot q}{P\cdot k}$$
$$z_h = \frac{P\cdot P_h}{P\cdot q}$$
$$Q_T = \frac{|P_{h\perp}|}{z_h}$$

Azimuthally weighted cross sections

$$\langle W\rangle_{\underbrace{P_e P_H P_h}_{\uparrow}} \equiv \int d\phi^\ell\,d^2\boldsymbol{q}_T\,\underbrace{W}_{\uparrow}\,\frac{d\sigma_{P_eP_H}}{dx_B\,dy\,dz_h\,d\phi^\ell\,d^2\boldsymbol{q}_T}$$

(O,L,T) $\qquad$ $W(Q_T,\phi_h^\ell,\phi_S^\ell,\phi_{S_h}^\ell)$

e.g.

$$\langle 1\rangle_{OOO} = \frac{2\pi\alpha^2\,s}{Q^4}\,x_B\,\Big(1+(1-y)^2\Big)\,\sum_{q,\bar q} e_q^2\,f_1^q(x_B)\,D_1^q(z_h)$$

SEMI-INCLUSIVE LEPTOPRODUCTION

- inclusive DIS $\longleftrightarrow$ $q(x) = f_1^q(x)$ and $g(x)$ via evolution
- Polarization $\longleftrightarrow$ $\Delta q(x) = g_1^q(x) = q_R(x) - q_L(x)$, $\Delta G(x)$

- SIDIS $\longleftrightarrow$ tagging to get f_1^q and g_1^q for $q = u,\ d,\ s,\ c$
- Polarimetry $\longleftrightarrow$ measure $\delta q(x) = h_1^q(x) = q_\uparrow(x) - q_\downarrow(x)$
- Azimuthal dependence $\longleftrightarrow$ study intrinsic quark $\boldsymbol{k}_T$
- Single spin asymmetries $\longleftrightarrow$ T-odd fragmentation functions

TRANSVERSE MOMENTS

- We need SIDIS, DY or 2PI e^+e^-(two hadrons!) to probe transverse momentum balance
- In SIDIS e.g.

$$\underset{\text{EXP}}{\underset{\downarrow}{q_T}} = q + x_B\,P - \frac{P_h}{z_h} = \underset{\downarrow}{\frac{P_{h\perp}}{z_h}}$$

$$\uparrow \quad \uparrow$$

$$p_{\text{quark}} \quad p'_{\text{quark}}$$

- SIDIS (q_T-integrated): Sensitive to p_T-integrated correlations.

$$\int d^2q_T \int d^2p_T\, d^2p'_T\ \delta^2(p_T + q_T - p'_T)\ldots = \left[\int d^2p_T \ldots\right]\left[\int d^2p'_T \ldots\right]$$

- SIDIS (q_T-weighted): Sensitive to p_T-weighted correlations

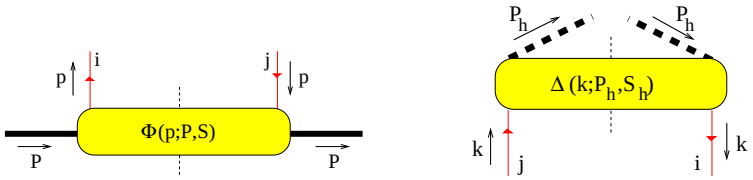
$$\int d^2q_T\ q_T^\alpha \int d^2p_T\, d^2p'_T\ \delta^2(p_T + q_T - p'_T)\ldots = \left[\int d^2p_T \ldots\right]\left[\int d^2p'_T\ p_T'^\alpha \ldots\right] - \left[\int d^2p_T\ p_T^\alpha \ldots\right]\left[\int d^2p'_T \ldots\right]$$

$$\downarrow \qquad \downarrow$$

transverse moments

- Experimentally: Weighting with $q_T^\ell \Rightarrow \int d^2P_{h\perp}\ |P_{h\perp}|\cos(\phi_h^\ell)\ldots$

SOFT PART IN SIDIS



Hard inclusive (polarized) deep inelastic leptoproduction involves the soft part Φ integrated over all momenta except $p^+ \equiv x\,P^+$

$$\Phi_{ij}(x) = \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}_j(0)\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

Hard semi-inclusive (polarized) deep inelastic lepton-hadron scattering involves soft part Φ integrated over p^- with $p^+ \equiv x\,P^+$ and p_T

$$\Phi_{ij}(x,p_T) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\; e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}_j(0)\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=0}$$

Fragmentation into a hadron involves soft part integrated over k^- leaving $P_h^- = zk^-$ and $P_{h\perp} = -zk_T$

$$\Delta_{ij}(z,k_T) = \int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3}\; e^{ik\cdot\xi}\,\langle 0|\psi_i(\xi)|P_h,X\rangle\langle P_h,X|\overline{\psi}_j(0)|0\rangle\Big|_{\xi^-=0}$$

QUARK SPIN STRUCTURE

For azimuthal asymmetries in semi-inclusive leptoproduction or Drell-Yan scattering one needs transverse momentum dependence

Dirac structure for a polarized (spin 1/2) hadron:

$$\Phi_0(x,p_T) = \left\{ f_1(x,p_T^2) + i\,h_1^\perp(x,p_T^2)\,\frac{\not{p}_T}{M}\right\}\not{n}_+$$

$$\Phi_L(x,p_T) = \left\{ S_L\,g_{1L}(x,p_T^2)\,\gamma_5 + S_L\,h_{1L}^\perp(x,p_T^2)\gamma_5\,\frac{\not{p}_T}{M}\right\}\not{n}_+$$

$$\Phi_T(x,p_T) = \left\{ f_{1T}^\perp(x,p_T^2)\,\frac{\epsilon_{T\,\rho\sigma}p_T^\rho S_T^\sigma}{M} + g_{1T}(x,p_T^2)\,\frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M}\,\gamma_5 \right.$$
$$\left. + h_{1T}(x,p_T^2)\,\gamma_5\,\not{S}_T + h_{1T}^\perp(x,p_T^2)\,\frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M}\,\frac{\gamma_5\,\not{p}_T}{M}\right\}\not{n}_+$$

- For fragmentation functions time reversal symmetry does not give constraints
- The functions $h_1^\perp$ and $f_{1T}^\perp$ are T-odd
- Operator structure $\psi_+^\dagger(0)\psi_+(\xi)$ allows a density interpretation.
- For fragmentation: $f \to D$, $g \to G$, $h \to H$ or hat-notation

FULL QUARK & HADRON SPIN STRUCTURE

Quark p_T-dependent distributions

$$\begin{pmatrix} f_1 + g_{1L} & \frac{|p_T|}{M} e^{i\phi} g_{1T} & \frac{|p_T|}{M} e^{-i\phi} h_{1L}^{\perp} & 2\, h_1 \\ \frac{|p_T|}{M} e^{-i\phi} g_{1T} & f_1 - g_{1L} & \frac{|p_T|^2}{M^2} e^{-2i\phi} h_{1T}^{\perp} & -\frac{|p_T|}{M} e^{-i\phi} h_{1L}^{\perp} \\ \frac{|p_T|}{M} e^{i\phi} h_{1L}^{\perp} & \frac{|p_T|^2}{M^2} e^{2i\phi} h_{1T}^{\perp} & f_1 - g_{1L} & -\frac{|p_T|}{M} e^{i\phi} g_{1T} \\ 2\, h_1 & -\frac{|p_T|}{M} e^{i\phi} h_{1L}^{\perp} & -\frac{|p_T|}{M} e^{-i\phi} g_{1T} & f_1 + g_{1L} \end{pmatrix}$$

- Three new functions with straightforward interpretation: g_{1T} corresponds to probability of finding longitudinally polarized quarks in a transversely polarized target; $h_{1L}^{\perp}$ to that of transversely polarized quarks in a longitudinally polarized target
- Naming convention: quark polarization/chirality: f, g, or h; target polarization in subscript L or T
- Matrix representation in Dirac space allows derivation of bounds

  Bacchetta, Boglione, Henneman and M, PRL 58 (2000) 712
- h-functions are chiral-odd!

FULL FRAGMENTATION SPIN STRUCTURE

Quark fragmentation functions accessible via azimuthal asymmetries

$$\begin{pmatrix} D_1 + G_{1L} & \frac{|k_T|}{M} e^{i\phi} G_{1T} & \frac{|k_T|}{M} e^{-i\phi} H_{1L}^{\perp} & 2\, H_1 \\ \frac{|k_T|}{M} e^{-i\phi} G_{1T}^{*} & D_1 - G_{1L} & \frac{|k_T|^2}{M^2} e^{-2i\phi} H_{1T}^{\perp} & -\frac{|k_T|}{M} e^{-i\phi} H_{1L}^{\perp} \\ \frac{|k_T|}{M} e^{i\phi} H_{1L}^{\perp *} & \frac{|k_T|^2}{M^2} e^{2i\phi} H_{1T}^{\perp} & D_1 - G_{1L} & -\frac{|k_T|}{M} e^{i\phi} G_{1T} \\ 2\, H_1 & -\frac{|k_T|}{M} e^{i\phi} H_{1L}^{\perp *} & -\frac{|k_T|}{M} e^{-i\phi} G_{1T}^{*} & D_1 + G_{1L} \end{pmatrix}$$

- The fragmentation functions depend on the ratio of (light-cone) momenta $z = P_h^- / k^-$ and on the transverse momentum of the quark wrt hadron (k_T)
- T-odd functions appear as imaginary parts of off-diagonal entries for fragmentation functions: $G_{1T} + i\, D_{1T}^{\perp}$ and $H_{1L}^{\perp} + i\, H_1^{\perp}$
- T-odd functions appear in single (or triple) spin asymmetries!
- For ρ-production one needs extension to spin 1

  A. Bacchetta and P.M., hep-ph/0104176

FRAGMENTATION FUNCTIONS FOR SPIN 0

Quark fragmentation functions into unpolarized or spin 0 hadrons as entries in the quark decay matrix in Dirac space

$$M^{(\mathrm{dec})} = \begin{pmatrix} D_1 & i\frac{|k_T|\,e^{-i\phi}}{M_h}\,H_1^\perp \\ -i\frac{|k_T|\,e^{+i\phi}}{M_h}\,H_1^\perp & D_1 \end{pmatrix} \begin{matrix} \mathrm{R} \\ \mathrm{L} \end{matrix}$$

$$\begin{matrix} \mathrm{R} & \mathrm{L} \end{matrix}$$

- Note for fragmentation into spin 0 hadrons (e.g pions) there are two (!) fragmentation functions, D_1 and $H_1^\perp$

  J. Collins, NP B396 (1993) 161

- In quark decay time reversal invariance cannot be used $\longrightarrow$ T-odd fragmentation functions, in this case $H_1^\perp$

- The function $H_1^\perp$ is also chiral-odd!
  It can be used to probe chiral-odd distribution functions

INTEGRATED AND WEIGHTED DISTRIBUTIONS

From the p_T-dependent distributions we recover the p_T-integrated results and obtain p_T-weighted distributions

$$\begin{aligned} \Phi(x) &= \int dp^-\, d^2\boldsymbol{p}_T\; \Phi(p,P,S) \\ &= \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}(0)\,\mathcal{U}\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0} \\ \Rightarrow\quad \Phi_\partial^\alpha(x) &= \int dp^-\, d^2\boldsymbol{p}_T\; p_T^\alpha\,\Phi(p,P,S) \\ &= \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\,\langle P,S|\,i\partial^\alpha\left(\overline{\psi}(0)\,\mathcal{U}\psi(\xi)\right)|P,S\rangle\Big|_{\xi^+=\xi_T=0} \end{aligned}$$

$$\begin{aligned} \Phi(x) &= \frac{1}{2}\left\{ f_1\,\not{n}_+ + S_L\,g_1\gamma_5\,\not{n}_+ + h_1\,\frac{[\,\not{S}_T,\,\not{n}_+]\gamma_5}{2}\right\} \\ \frac{1}{M}\,\Phi_\partial^\alpha(x) &= \frac{1}{2}\left\{ g_{1T}^{(1)}\,S_T^\alpha\,\gamma_5\,\not{n}_+ + S_L\,h_{1L}^{\perp(1)}\,\frac{[\,\not{n}_+,\gamma^\alpha]\gamma_5}{2}\right. \\ &\qquad \left. + f_{1T}^{\perp(1)}\,\epsilon_T^{\alpha\beta}S_{T\beta}\,\not{n}_+ + h_1^{\perp(1)}\,\frac{i\,[\,\not{n}_+,\gamma^\alpha]}{2}\right\} \end{aligned}$$

T-odd

Transverse moments: $g_{1T}^{(1)} = g_{1T}^{(1)}(x) \equiv \int d^2\boldsymbol{p}_T\,\frac{\boldsymbol{p}_T^2}{2M^2}\,g_{1T}(x,\boldsymbol{p}_T)$

WHERE DO THE FUNCTIONS SHOW UP

- twist t of correlation functions
  $\Rightarrow$ behavior $(1/Q)^{t-2}$
- k_T-dependent functions
  $\Rightarrow$ azimuthal dependence (ϕ_ℓ, ϕ_h, ϕ_S)
- cross sections are chirally even

$$\sigma_{000} \Longrightarrow f_1 \otimes D_1$$

$$\langle \frac{Q_T}{M_h} \sin\phi_h^\ell \; \sigma_{L00} \rangle \Longrightarrow \lambda_e \frac{M}{Q} \left(e - \frac{m}{Mx} f_1 \right) \otimes H_1^\perp$$

- # of spin vectors is even in case of a T-even combination

$$\sigma_{LL0} \Longrightarrow \lambda_e S_L \, g_1 \otimes D_1$$

$$\langle \cos(\phi_S^\ell + \phi_{S_h}^\ell) \; \sigma_{0TT} \rangle \Longrightarrow |S_T| \, |S_{hT}| \, h_1 \otimes H_1$$

$$\langle \frac{Q_T}{M} \cos(\phi_S^\ell - \phi_h^\ell) \; \sigma_{LT0} \rangle \Longrightarrow \lambda_e \, |\boldsymbol{S}_T| \, g_{1T}^{(1)} \otimes D_1$$

- # of spin vectors is odd in case of a T-odd combination (single spin asymmetries)

$$\langle \frac{Q_T}{M_h} \sin(\phi_h^\ell + \sin\phi_S^\ell) \; \sigma_{0T0} \rangle \Longrightarrow |S_T| \, h_1 \otimes H_1^{\perp(1)}$$

RELATIONS IMPOSED BY LORENTZ INVARIANCE

Recall the various $\langle \overline{\psi}(0) \psi(\xi) \rangle$ correlators

$$\Phi(x) = \frac{1}{2} \left\{ f_1 \not{n}_+ + S_L \, g_1 \gamma_5 \not{n}_+ + h_1 \frac{[\not{S}_T, \not{n}_+]\gamma_5}{2} \right\}$$
$$+ \frac{M}{2P^+} \left\{ e + g_T \, \gamma_5 \not{S}_T + S_L \, h_L \frac{[\not{n}_+, \not{n}_-]\gamma_5}{2} \right\}$$
$$\text{T-odd} \quad + \frac{M}{2P^+} \left\{ f_T \, \epsilon_T^{\rho\sigma} S_{T\rho} \gamma_\sigma - i \, S_L \, e_L \, \gamma_5 + h \frac{i[\not{n}_+, \not{n}_-]}{2} \right\}$$

$$\frac{1}{M} \Phi_\partial^\alpha(x) = \frac{1}{2} \left\{ g_{1T}^{(1)} \, S_T^\alpha \, \gamma_5 \not{n}_+ + S_L \, h_{1L}^{\perp(1)} \frac{[\not{n}_+, \gamma^\alpha]\gamma_5}{2} \right.$$
$$\text{T-odd} \quad \left. + f_{1T}^{\perp(1)} \, \epsilon_T^{\alpha\beta} S_{T\beta} \not{n}_+ + h_1^{\perp(1)} \frac{i[\not{n}_+, \gamma^\alpha]}{2} \right\}$$

Covariant form for $\langle \overline{\psi}(0) \, \psi(\xi) \rangle \Rightarrow$ Lorentz invariance relations

$$\underbrace{g_T - g_1}_{g_2} = \frac{d}{dx} g_{1T}^{(1)} \qquad \underbrace{h_L - h_1}_{\frac{1}{2} h_2} = -\frac{d}{dx} h_{1L}^{\perp(1)}$$

$$\text{T-odd} \quad f_T = -\frac{d}{dx} f_{1T}^{\perp(1)} \qquad h = -\frac{d}{dx} h_1^{\perp(1)}$$

A.P. Bukhvostov, E.A. Kuraev, L.N. Lipatov; Tangerman & M

$$\Phi(x) = \int dp^-\, d^2\boldsymbol{p}_T\; \Phi(p,P,S)$$

$$= \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}(0)\,\mathcal{U}\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

$$\Rightarrow\quad \Phi^\alpha_\partial(x) = \int dp^-\, d^2\boldsymbol{p}_T\; p_T^\alpha\, \Phi(p,P,S)$$

$$= \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\, \langle P,S|i\partial^\alpha\left(\overline{\psi}(0)\,\mathcal{U}\psi(\xi)\right)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

$$\Phi^\alpha_D(x,y) = \int dp^-\, d^2\boldsymbol{p}_T\, dp_1^-\, d^2p_{1T}\; \Phi_D(p,p_1,P,S)$$

$$= \int \frac{d\xi^-}{2\pi}\,\frac{d\eta^-}{2\pi}\; e^{i\,p_1\cdot(\xi-\eta)+ip\cdot\eta}$$
$$\times\, \langle P,S|\overline{\psi}(0)\,\mathcal{U}\,iD^\alpha(\eta)\,\mathcal{U}\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=\eta^+=\eta_T=0}$$

$$\Phi^\alpha_D(x) = \int dp^-\, d^2\boldsymbol{p}_T\, d^4p_1\; \Phi_D(p,p_1,P,S)$$

$$= \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}(0)\,\mathcal{U}\,iD^\alpha\,\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

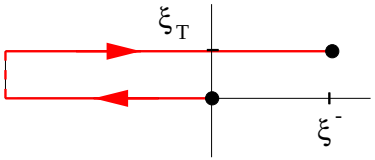
Similarly $\Phi^\alpha_A(x,y)$, $\Phi^{+\alpha}_F(x,y)$, etc.

Ellis, Furmanski, Petronzio
Jaffe, Soldate
Belitsky, Braun, ...

Color gauge invariance and twist analysis

Including longitudinal gluons and transverse momenta we get:

$$\Phi(x,\boldsymbol{p}_T) = \int \frac{d\xi^-d^2\boldsymbol{\xi}_T}{2\,(2\pi)^3}\; e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}(0)\,\mathcal{U}(0,-\infty)$$
$$\mathcal{U}(-\infty,\xi^-)\psi(\xi)|P,S\rangle\big|_{\xi^+=0}$$



$$\mathcal{U}(x,y) = \mathcal{P}\, e^{-ig\int_x^y d\eta^-\, A^+(\eta)}\Big|_{x_T=y_T=\eta_T}$$

Integrated results:

$$\Phi(x) = \int \frac{d\xi^-}{4\pi}\; e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}(0)\,\mathcal{U}\psi(\xi)|P,S\rangle\Big|_{LC}$$

$$\Phi^\alpha_\partial(x) = \int \frac{d\xi^-}{4\pi}\; e^{ip\cdot\xi}\Big\{\langle P,S|\overline{\psi}(0)\,\mathcal{U}\,iD^\alpha_T\psi(\xi)|P,S\rangle\Big|_{LC}$$
$$-\langle P,S|\overline{\psi}(0)\,\mathcal{U}\int_{-\infty}^{\xi^-} d\eta^-\,\mathcal{U}\,g\,F^{+\alpha}(\eta)\,U\,\psi(\xi)|P,S\rangle\Big|_{LC}\Big\}$$
$$= \Phi^\alpha_D(x) - \Phi^\alpha_A(x)$$

In $A^+=0$ gauge: $F^{+\alpha} = \partial^+ A^\alpha_T \Longrightarrow A^\alpha_T(\xi) = \int_{-\infty}^{\xi^-} d\eta^-\, F^{+\alpha}(\eta)$

EQUATIONS OF MOTION

$(i\not{D} - m)\psi = 0 \qquad \Longrightarrow$

$$\frac{1}{M}\,\Phi_D^\alpha(x) = \frac{1}{2}\left\{\left(x g_T - \frac{m}{M}\,h_1\right) S_T^\alpha\,\gamma_5\,\not{n}_+ + S_L\left(x h_L - \frac{m}{M}\,g_1\right)\frac{[\not{n}_+,\gamma^\alpha]\gamma_5}{2} - x f_T\;\epsilon_T^{\alpha\beta} S_{T\beta}\,\not{n}_+ - x h\,\frac{i\,[\not{n}_+,\gamma^\alpha]}{4}\right\}$$

T-odd

T-odd

Interaction dependent pieces: $\Phi_A \equiv \Phi_D - \Phi_\partial$

$$\frac{1}{M}\,\Phi_A^\alpha(x) = \frac{1}{2}\left\{\underbrace{\left(x g_T - g_{1T}^{(1)} - \frac{m}{M}\,h_1\right)}_{x\tilde{g}_T} S_T^\alpha\,\gamma_5\,\not{n}_+ + S_L\left(x h_L - h_{1L}^{\perp(1)} - \frac{m}{M}\,g_1\right)\frac{[\not{n}_+,\gamma^\alpha]\gamma_5}{2} - \left(x f_T + f_{1T}^{\perp(1)}\right)\epsilon_T^{\alpha\beta} S_{T\beta}\,\not{n}_+ - \left(x h + 2h_1^{\perp(1)}\right)\frac{i\,[\not{n}_+,\gamma^\alpha]}{4}\right\}$$

T-odd

T-odd

ISOLATING TWIST-3 AND TRANSVERSE MOMENTS

$$g_T(x) = \int_x^1 dy\;\frac{g_1(y)}{y} + \left[\tilde{g}_T(x) - \int_x^1 dy\;\frac{\tilde{g}_T(y)}{y}\right]$$

$$\frac{g_{1T}^{(1)}(x)}{x} = \int_x^1 dy\;\frac{g_1(y)}{y} - \int_x^1 dy\;\frac{\tilde{g}_T(y)}{y}$$

$$h_L(x) = 2x\int_x^1 dy\;\frac{h_1(y)}{y^2} + \left[\tilde{h}_L(x) - 2x\int_x^1 dy\;\frac{\tilde{h}_L(y)}{y^2}\right]$$

$$\frac{h_{1L}^{\perp(1)}(x)}{x^2} = -\int_x^1 dy\;\frac{h_1(y)}{y^2} + \int_x^1 dy\;\frac{\tilde{h}_L(y)}{y^2}$$

$$f_T(x) = \left[\tilde{f}_T(x) - \int_x^1 dy\;\frac{\tilde{f}_T(y)}{y}\right]$$

$$\frac{f_{1T}^{\perp(1)}(x)}{x} = \int_x^1 dy\;\frac{\tilde{f}_T(y)}{y}$$

$$h(x) = \left[\tilde{h}(x) - 2x\int_x^1 dy\;\frac{\tilde{h}(y)}{y^2}\right]$$

$$\frac{h_1^{\perp(1)}(x)}{x^2} = \int_x^1 dy\;\frac{\tilde{h}(y)}{y^2}$$

Moments: $[f]_n \equiv \int dx\; x^{n-1}\, f(x)$

$$[g_T]_n = \frac{1}{n}\,[g_1]_n + \frac{n-1}{n}\,[\tilde{g}_T]_n + \frac{m}{M}\,\frac{n-1}{n}\,[h_1]_{n-1}$$
$$[g_{1T}^{(1)}]_n = \frac{1}{n+1}\left([g_1]_{n+1} - [\tilde{g}_T]_{n+1} - \frac{m}{M}\,[h_1]_n\right)$$

$$[h_L]_n = \frac{2}{n+1}\,[h_1]_n + \frac{n-1}{n+1}\,[\tilde{h}_L]_n + \frac{m}{M}\,\frac{n-1}{n+1}\,[g_1]_{n-1}$$
$$[h_{1L}^{\perp(1)}]_n = -\frac{1}{n+2}\left([h_1]_{n+1} - [\tilde{h}_L]_{n+1} - \frac{m}{M}\,[g_1]_n\right)$$

$$[f_T]_n = \frac{n-1}{n}\,[\tilde{f}_T]_n$$
$$[f_{1T}^{\perp(1)}]_n = \frac{1}{n+1}[\tilde{f}_T]_{n+1}$$

$$[h]_n = \frac{n-1}{n+1}\,[\tilde{h}]_n$$
$$[h_1^{\perp(1)}]_n = \frac{1}{n+2}\,[\tilde{h}]_{n+1}$$

$$\frac{G_T(z)}{z} = -\int_z^1 dy\;\frac{G_1(y)}{y^2} + \left[\frac{\tilde{G}_T(z)}{z} + \int_z^1 dy\;\frac{\tilde{G}_T(y)}{y^2}\right]$$
$$G_{1T}^{(1)}(z) = -\int_z^1 dy\;\frac{G_1(y)}{y^2} + \int_z^1 dy\;\frac{\tilde{G}_T(y)}{y^2}$$

$$H_L(z) = -2\int_z^1 dy\;\frac{H_1(y)}{y} + \left[\tilde{H}_L(z) + 2\int_z^1 dy\;\frac{\tilde{H}_L(y)}{y}\right]$$
$$z\,H_{1L}^{\perp(1)}(z) = \int_z^1 dy\;\frac{H_1(y)}{y} - \int_z^1 dy\;\frac{\tilde{H}_L(y)}{y}$$

$$\frac{D_T(z)}{z} = \left[\frac{\tilde{D}_T(z)}{z} + \int_z^1 dy\;\frac{\tilde{D}_T(y)}{y^2}\right]$$
$$D_{1T}^{\perp(1)}(z) = -\int_z^1 dy\;\frac{\tilde{D}_T(y)}{y^2}$$

$$H(z) = \left[\tilde{H}(z) + 2\int_z^1 dy\;\frac{\tilde{H}(y)}{y}\right]$$
$$z\,H_1^{\perp(1)}(z) = -\int_z^1 dy\;\frac{\tilde{H}(y)}{y}$$

Moments: $[D]_n \equiv \int dz\; z^{n-1}\, f(z)$

$$[G_T]_n = -\frac{1}{n+1}\,[G_1]_n + \frac{n+2}{n+1}\,[\tilde{G}_T]_n + \frac{m}{M_h}\,\frac{n+2}{n+1}\,[H_1]_{n+1}$$
$$[G_{1T}^{(1)}]_{n+1} = -\frac{1}{n+1}\left([G_1]_n - [\tilde{G}_T]_n - \frac{m}{M_h}\,[H_1]_{n+1}\right)$$

$$[H_L]_n = -\frac{2}{n}\,[H_1]_n + \frac{n+2}{n}\,[\tilde{H}_L]_n + \frac{m}{M_h}\,\frac{n+2}{n}\,[G_1]_{n+1}$$
$$[H_{1L}^{\perp(1)}]_{n+1} = \frac{1}{n}\left([H_1]_n - [\tilde{H}_L]_n - \frac{m}{M_h}\,[G_1]_{n+1}\right)$$

$$[D_T]_n = \frac{n+2}{n+1}\,[\tilde{D}_T]_n$$
$$[D_{1T}^{\perp(1)}]_{n+1} = -\frac{1}{n+1}\,[\tilde{D}_T]_n$$

$$[H]_n = \frac{n+2}{n}\,[\tilde{H}]_n$$
$$[H_1^{\perp(1)}]_{n+1} = -\frac{1}{n}\,[\tilde{H}]_n$$

$$\frac{d}{d\tau}\,f(x,\tau) = \frac{\alpha_s(\tau)}{2\pi}\int_x^1 \frac{dy}{y}\,P^{[f]}\left(\frac{x}{y}\right) f(y,\tau),$$
$$\frac{d}{d\tau}\,[f]_n(\tau) = \frac{\alpha_s(\tau)}{2\pi}\,A_n^{[f]}\,[f]_n(\tau)$$

Twist-2 functions

$$P^{[f_1]}(\beta) = C_F\left[\frac{3}{2}\,\delta(1-\beta) + \frac{1+\beta^2}{(1-\beta)_+}\right]$$
$$P^{[g_1]}(\beta) = C_F\left[\frac{3}{2}\,\delta(1-\beta) + \frac{1+\beta^2}{(1-\beta)_+}\right]$$
$$P^{[h_1]}(\beta) = C_F\left[\frac{3}{2}\,\delta(1-\beta) + \frac{2\beta}{(1-\beta)_+}\right]$$

Interaction-dependent functions (for large N_c)

$$P^{[\tilde{g}_T]}(\beta) = P^{[\tilde{f}_T]}(\beta) = \frac{N_c}{2}\left[\frac{1}{2}\,\delta(1-\beta) + \frac{1+\beta}{(1-\beta)_+}\right]$$
$$P^{[\tilde{h}_L]}(\beta) = P^{[\tilde{h}]}(\beta) = \frac{N_c}{2}\left[\frac{1}{2}\,\delta(1-\beta) + \frac{3\beta-1}{(1-\beta)_+}\right]$$
$$P^{[\tilde{e}]}(\beta) = P^{[\tilde{e}_L]}(\beta) = \frac{N_c}{2}\left[\frac{1}{2}\,\delta(1-\beta) + \frac{3-\beta}{(1-\beta)_+}\right]$$

Braun, Balitsky, Koike, Tanaka, Ji, Osborne, ...

EVOLUTION OF AZIMUTHAL ASYMMETRIES

Large N_c

From the twist-2 and interaction-dependent functions one obtains the evolution of the transverse moments

Azimuthal asymmetries (large N_c, omitting quark mass-terms)

$$\frac{d}{d\tau}\, g_{1T}^{(1)}(x,\tau) = \frac{\alpha_s(\tau)}{4\pi}\, N_c \int_x^1 dy \left\{ \left[\frac{1}{2}\,\delta(y-x) + \frac{x^2+xy}{y^2(y-x)_+} \right] g_{1T}^{(1)}(y,\tau) + \frac{x^2}{y^2}\, g_1(y,\tau) \right\}$$

$$\frac{d}{d\tau}\, h_{1L}^{\perp(1)}(x,\tau) = \frac{\alpha_s(\tau)}{4\pi}\, N_c \int_x^1 dy \left\{ \left[\frac{1}{2}\,\delta(y-x) + \frac{3x^2-xy}{(y-x)_+} \right] h_{1L}^{\perp(1)}(y,\tau) - \frac{x}{y}\, h_1(y,\tau) \right\}$$

Evolution of transverse moments is also driven by the twist-2 polarized quark distribution functions g_1 and h_1.

EVOLUTION OF FRAGMENTATION FUNCTIONS

We note a generalized reciprocity valid for twist-2 *and* interaction-dependent functions:

$$P^{[f]}(\beta) = \frac{\mathcal{N}(\beta)}{(1-\beta)_+} \qquad A_n^{[f]} = \mathcal{A}(n) - 2\sum_{j=1}^{n}\frac{1}{j}$$

$$P^{[zD]}(\beta) = \frac{\beta^2\,\mathcal{N}(1/\beta)}{(1-\beta)_+} \qquad A_{n+1}^{[D]} = \mathcal{A}\left(-(n+1)\right) - 2\sum_{j=1}^{n}\frac{1}{j}.$$

$$P^{[f_1]}(\beta) = C_F\left[\frac{1+\beta^2}{(1-\beta)_+}\right] \qquad P^{[zD_1]}(\beta) = C_F\left[\frac{1+\beta^2}{(1-\beta)_+}\right]$$

$$P^{[g_1]}(\beta) = C_F\left[\frac{1+\beta^2}{(1-\beta)_+}\right] \qquad P^{[zG_1]}(\beta) = C_F\left[\frac{1+\beta^2}{(1-\beta)_+}\right]$$

$$P^{[h_1]}(\beta) = C_F\left[\frac{2\beta}{(1-\beta)_+}\right] \qquad P^{[zH_1]}(\beta) = C_F\left[\frac{2\beta}{(1-\beta)_+}\right]$$

$$P^{[\tilde{g}_T]}(\beta) = \frac{N_c}{2}\left[\frac{1+\beta}{(1-\beta)_+}\right] \qquad P^{[z\tilde{G}_T]}(\beta) = \frac{N_c}{2}\left[\frac{1+\beta}{(1-\beta)_+}\right]$$

$$P^{[\tilde{h}_L]}(\beta) = \frac{N_c}{2}\left[\frac{3\beta-1}{(1-\beta)_+}\right] \qquad P^{[z\tilde{H}_L]}(\beta) = \frac{N_c}{2}\left[\frac{3\beta-1}{(1-\beta)_+}\right]$$

$$P^{[\tilde{e}]}(\beta) = \frac{N_c}{2}\left[\frac{3-\beta}{(1-\beta)_+}\right] \qquad P^{[z\tilde{E}]}(\beta) = \frac{N_c}{2}\left[\frac{3-\beta}{(1-\beta)_+}\right]$$

$$\frac{d}{d\tau}\,D(z,\tau) = \frac{\alpha_s(\tau)}{2\pi}\int_z^1 \frac{dy}{y}\,P^{[f]}\left(\frac{z}{y}\right) f(y,\tau),$$

$$\frac{d}{d\tau}\,[D]_n(\tau) = \frac{\alpha_s(\tau)}{2\pi}\,A_n^{[D]}\,[D]_n(\tau)$$

Twist-2 functions

$$P^{[zD_1]}(\beta) = C_F\left[\frac{3}{2}\,\delta(1-\beta) + \frac{1+\beta^2}{(1-\beta)_+}\right]$$

$$P^{[zG_1]}(\beta) = C_F\left[\frac{3}{2}\,\delta(1-\beta) + \frac{1+\beta^2}{(1-\beta)_+}\right]$$

$$P^{[zH_1]}(\beta) = C_F\left[\frac{3}{2}\,\delta(1-\beta) + \frac{2\beta}{(1-\beta)_+}\right]$$

Interaction-dependent functions (for large N_c)

$$P^{[z\tilde{G}_T]}(\beta) = P^{[z\tilde{D}_T]}(\beta) = \frac{N_c}{2}\left[\frac{1}{2}\,\delta(1-\beta) + \frac{1+\beta}{(1-\beta)_+}\right]$$

$$P^{[z\tilde{H}_L]}(\beta) = P^{[z\tilde{H}]}(\beta) = \frac{N_c}{2}\left[\frac{1}{2}\,\delta(1-\beta) + \frac{3\beta-1}{(1-\beta)_+}\right]$$

$$P^{[z\tilde{E}]}(\beta) = P^{[z\tilde{E}_L]}(\beta) = \frac{N_c}{2}\left[\frac{1}{2}\,\delta(1-\beta) + \frac{3-\beta}{(1-\beta)_+}\right]$$

Large N_c

Like for the distribution functions one has evolution equations for the transverse moments with driving from G_1 and H_1:

$$\frac{d}{d\tau}\,zG_{1T}^{(1)}(z,\tau) = \frac{\alpha_s(\tau)}{4\pi}\,N_c\int_z^1 dy\left\{\left[\frac{1}{2}\,\delta(y-z) + \frac{y+z}{y(y-z)_+}\right] yG_{1T}^{(1)}(y,\tau) - \frac{y^2+z^2}{2y^2z}\,G_1(y,\tau)\right\}$$

$$\frac{d}{d\tau}\,zH_{1L}^{\perp(1)}(z,\tau) = \frac{\alpha_s(\tau)}{4\pi}\,N_c\int_z^1 dy\left\{\left[\frac{1}{2}\,\delta(y-z) + \frac{3y-z}{y(y-z)_+}\right] yH_{1L}^{\perp(1)}(y,\tau) + \frac{1+\ln(z/y)}{y}\,H_1(y,\tau)\right\}$$

Evolution equation for Collins function obtained as imaginary part of the equation for $H_{1L}^{\perp}$

$$\frac{d}{d\tau}\,zH_1^{\perp(1)}(z,\tau) = \frac{\alpha_s}{4\pi}\,N_c\int_z^1 dy\left[\frac{1}{2}\,\delta(y-z) + \frac{3y-z}{y(y-z)_+}\right] yH_1^{\perp(1)}(y,\tau)$$

- No driving term in the large N_c evolution
- No mixing with gluons

ANOMALOUS DIMENSIONS

Large Q^2 behavior of the moments $\propto [\alpha_s(Q^2)]^{d_n}$

Twist-2: $d_n = -6\, a_n\, C_F/(11\, N_c - 2\, N_f)$
For large N_c: $d_n = -6\, a_n/11$

function	C	a_0	a_1	a_2	a_3	validity
f_1	+		0	-7/6	-25/12	
g_1	−		0	-7/6	-25/12	
h_1	+	+3/2	-1/2	-3/2	-13/6	
$\tilde{g}_T$	−		-1/2	-2	-17/6	large N_c
$\tilde{h}_L$	+		-5/2	-3	-7/2	large N_c
$\tilde{e}$	−		+3/2	-1	-13/6	large N_c
$g_{1T}^{(1)}$	+	-1/2	-2	-17/6	-41/12	large N_c
$h_{1L}^{\perp(1)}$	−	-5/2	-3	-7/2	-47/12	large N_c
zD_1	+		0	-7/6	-25/12	
zG_1	−		0	-7/6	-25/12	
zH_1	+	+3/2	-1/2	-3/2	-13/6	
$z\tilde{G}_T$ and $z\tilde{D}_T$	−	-1/2	-2	-17/6	-41/12	large N_c
$z\tilde{H}_L$ and $z\tilde{H}$	+	+3/2	-1	-13/6	-35/12	large N_c
$z\tilde{E}$	−	-5/2	-3	-7/2	-47/12	large N_c
$zG_{1T}^{(1)}$ and $zD_{1T}^{\perp(1)}$	+		-1/2	-2	-17/6	large N_c
$zH_{1L}^{\perp(1)}$ and $zH_1^{\perp(1)}$	−		+3/2	-1	-13/6	large N_c

Note: $\int dz\ zH_1^{\perp(1)}(z) = 0$ (Schaefer & Teryaev)

$\int dz\ e(z) = \infty$ (Jaffe & Ji)

SUMMARY

- p_T-dependent distribution and fragmentation functions ⇔ azimuthal asymmetries
- Complementary to study of twist-3

$$\frac{M}{Q} \text{ contribution } \Rightarrow \text{ measurement of } Q_T \ll Q$$

- Access to $h_1^q(x)$ (transversity) in the nucleon

For results including weak interactions, see
D. Boer, R. Jakob and PM, NP B564 (2000) 471;
hep-ph/9907504

- Operator structure of lowest transverse moments of the p_T-dependent functions involves (known) twist-2 and twist-3 operators
- This also enables us to find their evolution, which in the limit $N_c \to \infty$ can be simply written down

A. Henneman, D. Boer and PM, hep-ph/0104271