Frascati, 9 June 1999
http://www.nat.vu.nl/~mulders (Talks)

Perspectives in Polarized Leptoproduction

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Free University, Amsterdam

- Probing hadronic matter in leptoproduction
- Quark and gluon correlations in hadrons
- Theory:
  Intrinsic transverse momentum of partons
- Experiment:
  Higher twist
  Azimuthal asymmetries

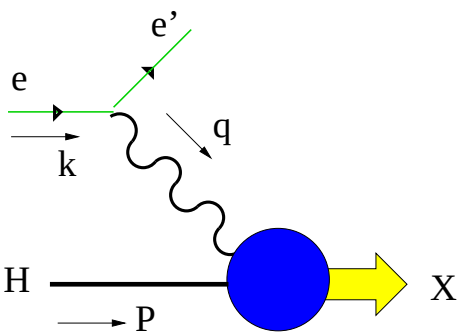
[1]collaborators: D. Boer, J. Rodrigues, A. Henneman, R. Jakob, M. Boglione, A. Bacchetta, O. Teryaev

INCLUSIVE LEPTOPRODUCTION

(INCLUSIVE) LEPTOPRODUCTION

$\ell H \longrightarrow \ell' X$

Scale : $-q^2 \equiv Q^2 \quad (\text{Hard} : Q^2 \to \infty)$



$$x_B = \frac{Q^2}{2P \cdot q}$$

$$y = \frac{P \cdot q}{P \cdot k}$$

Hadronic tensor

$$2M\, W_{\mu\nu}^{(\ell H)}(q; PS)$$
$$= \frac{1}{(2\pi)} \int \frac{d^3 P_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta^4(q + P - P_X)$$
$$\times \langle PS|J_\mu(0)|P_X\rangle\langle P_X|J_\nu(0)|PS\rangle$$

Inclusive scattering (unpolarized lepton and hadron, γ-exchange)[2]

$$2MW_S^{\mu\nu}(q,P) = \underbrace{\left(-g^{\mu\nu} + \hat{q}^\mu\hat{q}^\nu - \hat{t}^\mu\hat{t}^\nu\right)}_{-g_\perp^{\mu\nu}} F_1 + \hat{t}^\mu\hat{t}^\nu \underbrace{\left(\frac{F_2}{2x_{_B}} - F_1\right)}_{F_L}$$

Cross section

$$\frac{d\sigma_{OO}}{dx_{_B}\,dy} = \frac{4\pi\,\alpha^2\,x_{_B}\,s}{Q^4}\left\{\left(1 - y + \frac{1}{2}\,y^2\right)F_T + (1-y)\,F_L\right\}$$

Parton model

$$F_T(x_{_B},Q) \overset{DIS}{=} \frac{1}{2}\sum_a e_a^2\, f_1^a(x_{_B})$$

$$F_L(x_{_B},Q) \overset{DIS}{=} 0$$

2

$$\hat{q}^\mu = \frac{q^\mu}{Q}$$

$$\hat{t}^\mu = \frac{\tilde{P}^\mu}{\sqrt{\tilde{P}^2}} = \frac{1}{\sqrt{\tilde{P}^2}}\left(P^\mu - \frac{P\cdot q}{q^2}\,q^\mu\right)$$

Inclusive scattering (polarized leptons and hadrons, γ-exchange)

$$2MW_A^{\mu\nu}(q,P,S) = \underbrace{-i\,\lambda\,\frac{\epsilon^{\mu\nu\rho\sigma}P_\rho q_\sigma}{P\cdot q}}_{-i\,\lambda\,\epsilon_\perp^{\mu\nu}}\,g_1 + i\,\frac{2Mx_{_B}}{Q}\,\hat{t}^{\,[\mu}\epsilon_\perp^{\nu]\rho}S_{\perp\rho}\,g_T$$

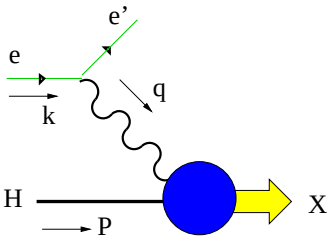
Cross section

$$\frac{d\sigma_{LL}}{dx_{_B}dy} = \lambda_e\,\frac{4\pi\,\alpha^2}{Q^2}\left\{\lambda\left(1-\frac{y}{2}\right)g_1 - |S_\perp|\,\cos\phi_S^\ell\,\frac{2Mx_{_B}}{Q}\sqrt{1-y}\;g_T\right\}$$

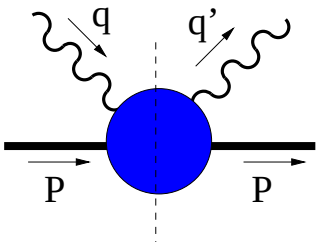
Parton model

$$g_1(x_{_B},Q) = \frac{1}{2}\sum_a e_a^2\,g_1^a(x_{_B})$$

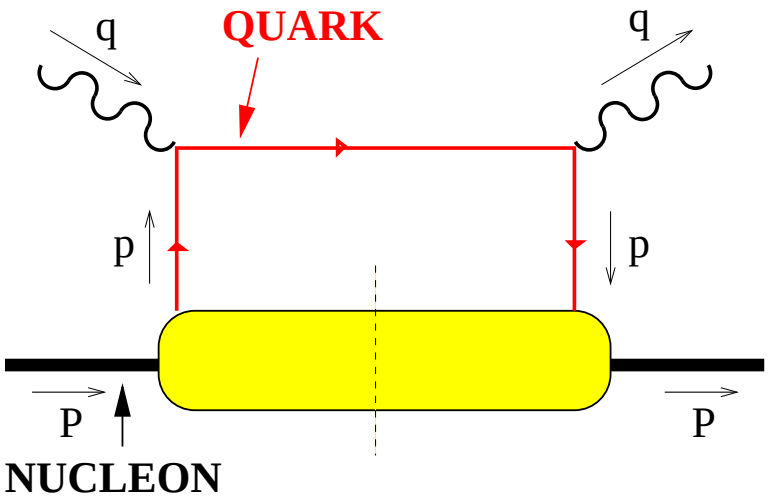
$$g_T(x_{_B},Q) = (g_1+g_2)(x_{_B},Q) = \sum_a e_a^2\,g_T^a(x_{_B})$$



$$\sigma \propto \mathcal{I}m\,T_{\mu\nu} =$$



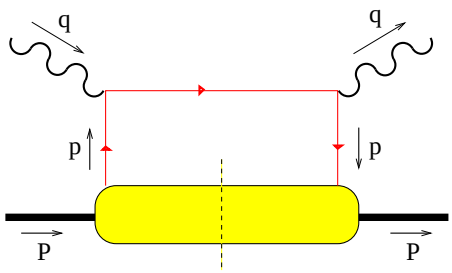
Parton picture (at large Q^2)



+ higher twist stuff $\sim (1/Q)^{t-2}$
+ perturbative corrections $\sim \alpha_s^n(Q^2)\,\ln^m Q^2$
+ ...

Large scale Q leads in a natural way to the use of lightlike vectors: $n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$

$$\left.\begin{array}{l} q^2 = -Q^2 \\ P^2 = M^2 \\ 2\,P\cdot q = \dfrac{Q^2}{x_{_B}} \end{array}\right\} \longleftrightarrow \left\{\begin{array}{l} P = \dfrac{x_{_B} M^2}{Q\sqrt{2}}\, n_- + \dfrac{Q}{x_{_B}\sqrt{2}}\, n_+ \\[2ex] q = \dfrac{Q}{\sqrt{2}}\, n_- - \dfrac{Q}{\sqrt{2}}\, n_+ \end{array}\right.$$



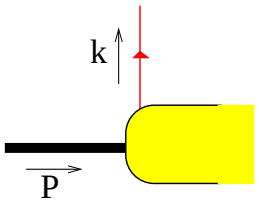
part	'components' $-$	$+$	
HARD	$\sim Q$	$\sim Q$	
$H \to q$	$\sim 1/Q$	$\sim Q$	$\to \int dp^- \ldots$

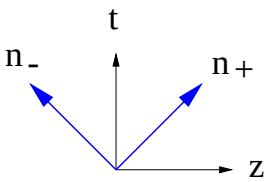
Note: the lightlike vectors may be rescaled into dimensionful vectors $\hat{p} = (Q/x_{_B}\sqrt{2})n_+$ and $\hat{n} = (x_{_B}\sqrt{2}/Q)n_-$

$$P = \hat{p} + \frac{M^2}{2}\,\hat{n}$$

$$q = -x_{_B}\,\hat{p} + \frac{Q^2}{2x_{_B}}\,\hat{n}$$

In a hard scattering process hadrons are about lightlike!





$$n_+^2 = n_-^2 = 0$$
$$n_+ \cdot n_- = 1$$

A hard scale (e.g. exchanged photon) provides naturally the lightlike directions such that P^+ can be considered large

$$P^\mu \equiv P^+ \, n_+^\mu + \frac{M^2}{2P^+} \, n_-^\mu$$

collinear ↑ ↓ suppressed

$$k^\mu \equiv x P^+ \, n_+^\mu + k_T^\mu + \mathcal{O}\left(\frac{1}{P^+}\right)$$

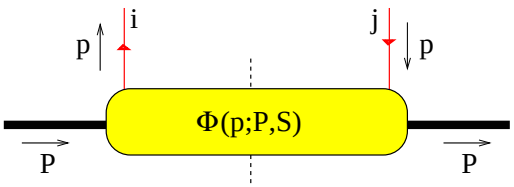
↑ beyond collinearity

Similarly for spin:

$$S^\mu \equiv \lambda \frac{P^+}{M} \, n_+^\mu + S_T^\mu + \mathcal{O}\left(\frac{1}{P^+}\right)$$

SOFT PART IN DIS

Hard inclusive (polarized) deep inelastic lepton-hadron scattering allows isolation of the soft part Φ integrated over all momenta except $p^+ \equiv x\,P^+$



$$\begin{aligned}\Phi(x) &= \int dp^-\, d^2\boldsymbol{p}_T\; \Phi(p,P,S)\Big|_{p^+ = x\,P^+} \\ &= \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}(0)\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}\end{aligned}$$

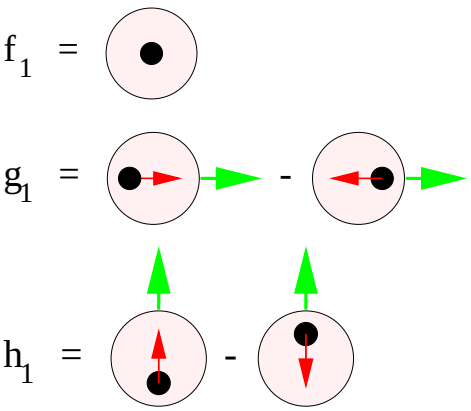
It is parametrized in terms of distribution functions

$$\Phi(x) = \frac{1}{2}\left\{ f_1(x)\;\not{n}_+ + \lambda\, g_1(x)\,\gamma_5\,\not{n}_+ + h_1(x)\,\frac{[\not{S}_T,\not{n}_+]\gamma_5}{2}\right\} + \mathcal{O}\left(\frac{M}{P^+}\right)$$

INTERPRETATION OF SOFT PARTS

The (leading) distribution functions f_1, g_1 and h_1 have a natural interpretation as densities in momentum space

$$\begin{aligned} f_1 &\sim \langle \overline{\psi}(0)\,\gamma^+\psi(\xi)\rangle \sim \langle \psi_+^\dagger(0)\,\psi_+(\xi)\rangle \\ &\sim \langle b^\dagger(p^+)\,b(p^+) - d(p^+)\,d^\dagger(p^+)\rangle \end{aligned}$$



chirality distribution

transverse spin distribution
chirally odd!
(off-diagonal in chiral basis)

For each of the distributions there are many aspects to be discussed (they are the subject under discussion at workshops like DISyy, SPINyy and NUCLEONyy, etc.)

- Interpretation as (lightcone) momentum distributions
- Flavor dependence
  $f_1^u(x) = u(x), g_1^u(x) = \Delta u(x), \ldots$
- positivity, e.g. $f_1^a(x) \geq 0$
  bounds, e.g. $|g_1^a(x)| \leq f_1^a(x)$
- Symmetry relations and antiquark distributions
  $\bar{f}_1(x) = -f_1(-x),\ \bar{g}_1(x) = g_1(-x)$
- Sum rules

$$\int_{-1}^{1} dx\ f_1^a(x) = \int_0^1 dx\ (f_1^a(x) - \bar{f}_1^a(x)) = n_a^{\text{val}}$$

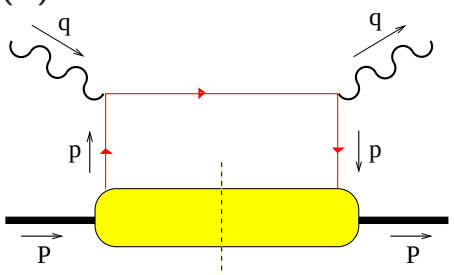
$$\int_{-1}^{1} dx\ (g_1^u(x) - g_1^d(x)) = g_A \quad \text{(Bjorken sumrule)}$$

THE FULL CALCULATION OF $W_{\mu\nu}$

Field theoretical approach

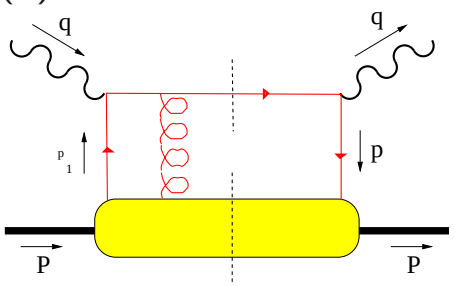
- Introduce soft parts that allow inclusion of hadrons in the field theoretical framework of QCD
- If $|q^2| \to \infty$ only a limited number of diagrams survives, each of them with characteristic behavior

(1)



Parton model result

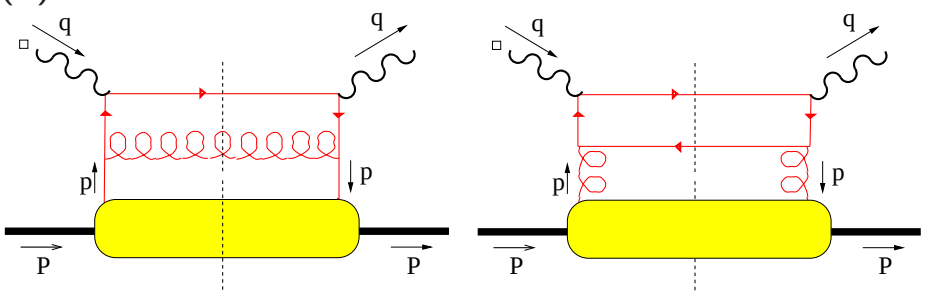
(2)



- 'Physical' A_T gluons
  [e.m. gauge invariance at $\mathcal{O}(1/Q)$]
- Longitudinal A^+ gluons
  [create link, color gauge invariance]

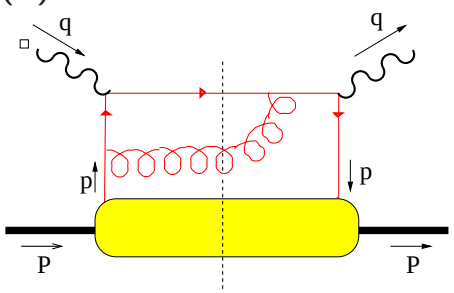
THE FULL CALCULATION OF $W_{\mu\nu}$

(3)



$\alpha_s \log(Q^2/\mu^2)$ corrections [DGLAP equations]

(5)

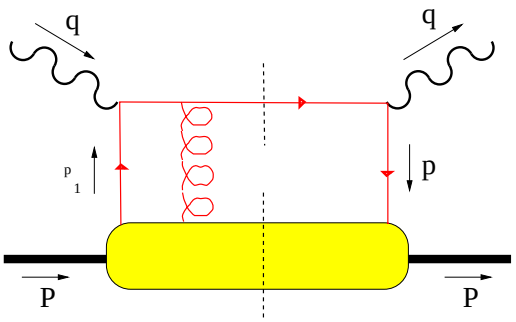


(Process dependent) α_s corrections

$$F_L = \frac{F_2}{x} - 2F_1 \sim \alpha_s(Q^2) \times \ldots$$

LONGITUDINAL GLUONS

Soper, Collins, . . . , Radyushkin, . . .



$\Longrightarrow$ In DIS one also probes quark-quark-gluon correlations

$$\overline{\psi}(0) A^{\alpha}(\eta)\, \psi(\xi)$$

- At leading order, $(1/Q)^0$ one needs to take along:

$$\Phi_A^+ \;\propto\; \langle P,S|\overline{\psi}(0) \left[g \int_0^{\xi^-} d\eta^-\, A^+(\eta) \right] \psi(\xi)|P,S\rangle$$

  etc.

- This leads to appearance of color gauge invariant form

$$\Phi(x) = \int \frac{d\xi^-}{4\pi}\, e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}(0)\, \mathcal{P} e^{-ig\int d\eta^\mu A_\mu(\eta)} \psi(\xi)|P,S\rangle \Big|_{LC}$$

- Practical hint: work in lightcone gauge ($A^+ = 0$): No link and no gluon diagrams at leading order!

EXTENSION TO $1/Q$

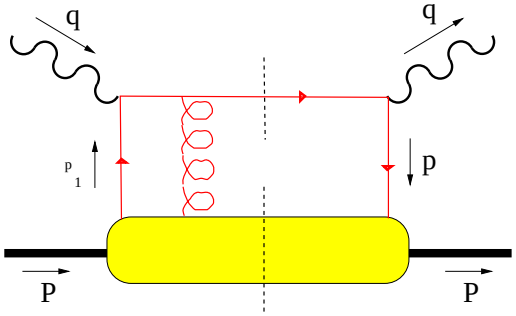
Jaffe, Ji, ...

Extension to order $1/Q$ requires:

- Inclusion of M/P^+ 'handbag'contributions in $\Phi(x)$:

$$\Phi(x) = \frac{1}{2}\left\{ f_1 \, \not{n}_+ + \lambda \, g_1 \gamma_5 \, \not{n}_+ + h_1 \, \frac{[\not{s}_T, \not{n}_+]\gamma_5}{2} \right\} + \frac{M}{2P^+}\left\{ e + g_T \, \gamma_5 \, \not{s}_T + \lambda \, h_L \, \frac{[\not{n}_+, \not{n}_-]\gamma_5}{2} \right\}$$

- Inclusion of diagrams with (transverse) gluons in soft part:



$\overline{\psi}(0) A_T^{\alpha}(\eta) \, \psi(\xi)$

- Use of equations of motion relates gluonic matrix elements to above quark-quark correlation function
- Partonic graph (although no partonic interpretation!) + gluon graph ensure electromagnetic gauge invariance at $\mathcal{O}(1/Q)^1$

Twist-3 functions in polarized DIS

Reaction: $\vec{\ell} + H^{\uparrow} \to \ell + X$

Cross sections:

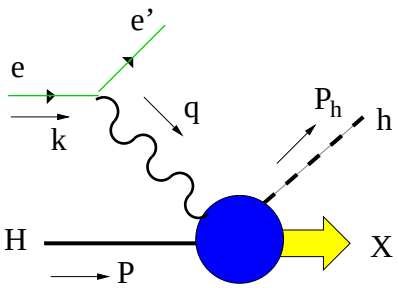
$$\frac{d\sigma_{LT}}{dx_{_B}\, dy\, d\phi^{\ell}} = -\frac{4\,\alpha^2\, s}{Q^4}\,\lambda_e\,|\boldsymbol{S}_{_T}|\,\cos(\phi_S^{\ell})\,y\sqrt{1-y}\,\frac{M x_{_B}^2}{Q}\underbrace{\sum_{a,\bar{a}} e_a^2\, g_T^a(x_{_B})}_{g_T(x_{_B},Q^2)}$$

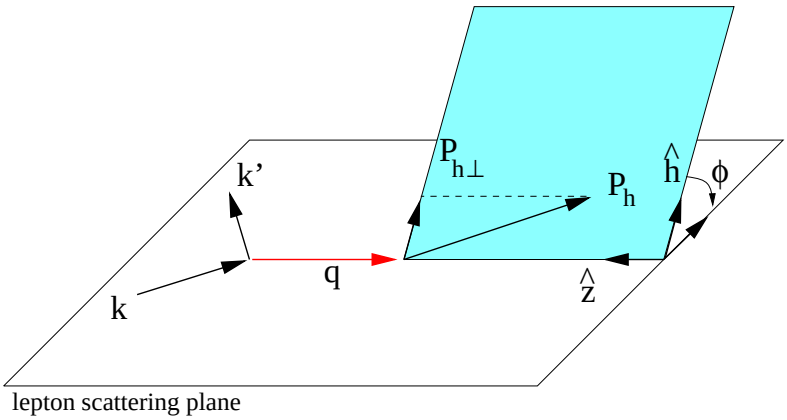
- The compensation of the transverse index of the target spin requires a transverse index in the matrix element, which effectively means a transverse gluon index, $\langle\overline{\psi}A_T^{\alpha}\psi\rangle$, leading to suppression factor M/Q (higher twist).
- In 1-particle inclusive leptoproduction S_T^{α} can be correlated with the transverse momentum of the outgoing hadron
  $\Rightarrow$ azimuthal asymmetries at leading order in $1/Q$

SEMI-INCLUSIVE LEPTOPRODUCTION

$$\ell H \longrightarrow \ell' h X$$

$$-q^2 \equiv Q^2 \to \infty$$





lepton scattering plane

$$x_B = \frac{Q^2}{2P \cdot q}$$

$$y = \frac{P \cdot q}{P \cdot k}$$

$$z_h = \frac{P \cdot P_h}{P \cdot q}$$

$$Q_T = \frac{|P_{h\perp}|}{z_h}$$

Hadronic tensor

$$2M\mathcal{W}_{\mu\nu}^{(\ell H)}(q; PS; P_h S_h)$$
$$= \frac{1}{(2\pi)^4} \int \frac{d^3 P_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta^4(q + P - P_X - P_h)$$
$$\times \langle PS | J_\mu(0) | P_X; P_h S_h \rangle \langle P_X; P_h S_h | J_\nu(0) | PS \rangle$$

Semi-inclusive scattering (unpolarized lepton and hadron) [3]

$$2M\mathcal{W}_S^{\mu\nu}(q,P,P_h) = -g_\perp^{\mu\nu}\,\mathcal{H}_T + \hat{t}^\mu\hat{t}^\nu\,\mathcal{H}_L + \hat{t}^{\{\mu}\hat{h}^{\nu\}}\,\mathcal{H}_{LT} + \left(2\,\hat{h}^\mu\hat{h}^\nu + g_\perp^{\mu\nu}\right)\mathcal{H}_{TT}$$

Cross section

$$\frac{d\sigma_O}{dx_{_B}\,dy\,dz_h d^2q_T} = \frac{4\pi\,\alpha^2\,s}{Q^4}\,x_{_B}\,z_h\left\{\left(1-y+\frac{1}{2}\,y^2\right)\mathcal{H}_T + (1-y)\,\mathcal{H}_L - (2-y)\sqrt{1-y}\,\cos\phi_h^\ell\,\mathcal{H}_{LT} + (1-y)\,\cos 2\phi_h^\ell\,\mathcal{H}_{TT}\right\}$$

[3]

$$\hat{q}^\mu = \frac{q^\mu}{Q}, \qquad \hat{t}^\mu = \frac{q^\mu + 2x_{_B}\,P^\mu}{Q},$$
$$g_\perp^{\mu\nu} = g^{\mu\nu} - \hat{q}^\mu\hat{q}^\nu + \hat{t}^\mu\hat{t}^\nu$$
$$q_T^\mu = q^\mu + x_{_B}\,P^\mu - \frac{P_h^\mu}{z_h} = -\frac{P_{h\perp}^\mu}{z_h} \equiv -Q_T\,\hat{h}^\mu$$

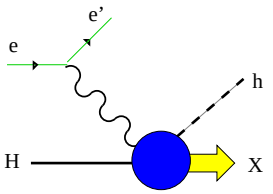
Semi-inclusive scattering (polarized lepton, but unpolarized hadron)

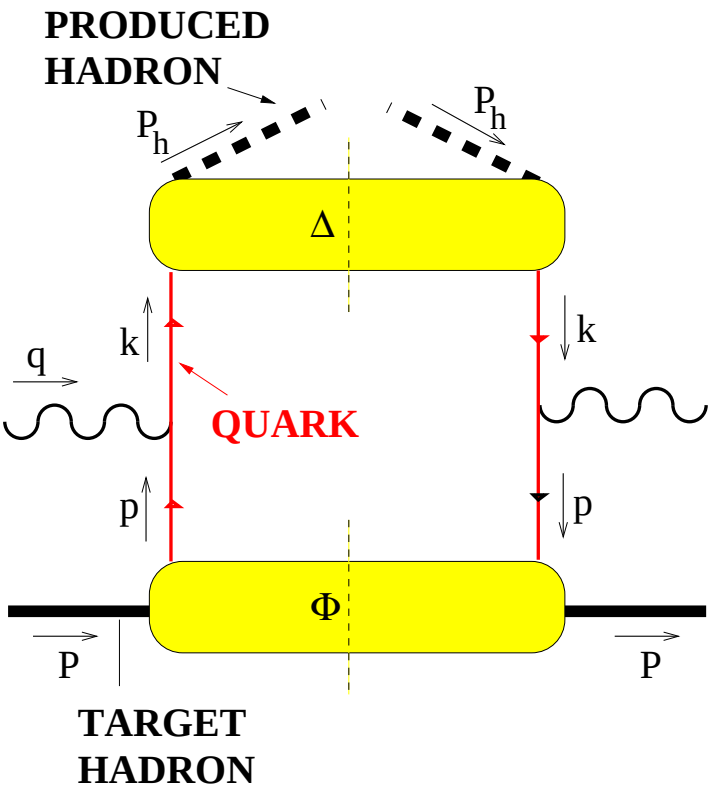
$$2M\mathcal{W}_A^{\mu\nu}(q,P,P_h) = -i\hat{t}^{\,[\mu}\hat{h}^{\nu]}\,\mathcal{H}'_{LT}$$

Cross section

$$\frac{d\sigma_L}{dx_{_B}dy\,dz_h d^2q_{_T}} = \lambda_e\,\frac{4\pi\,\alpha^2}{Q^2}\,z_h\,\sqrt{1-y}\,\sin\phi_h^\ell\;\mathcal{H}'_{LT}$$

Parton picture (at large Q^2)



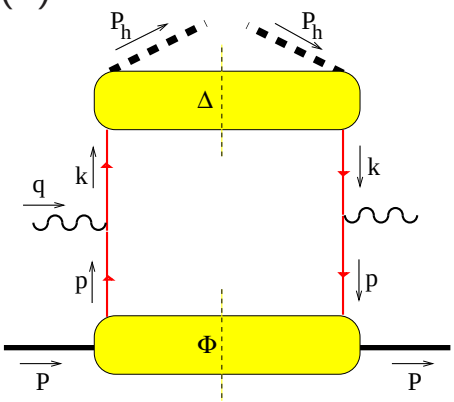


+ target fragmentation
+ higher twist stuff $\sim (1/Q)^{t-2}$
+ perturbative corrections $\sim \alpha_s^n(Q^2)\ \ln^m Q^2$
+ ...

Field theoretical approach

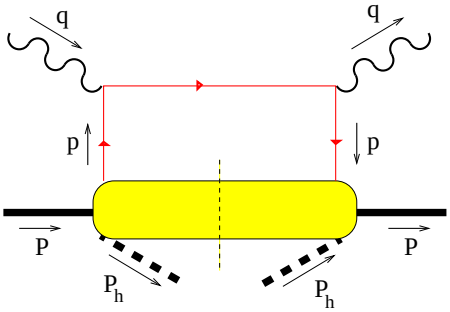
- Introduce soft parts that allow inclusion of hadrons in the field theoretical framework of QCD
- If $|q^2| \to \infty$ only a limited number of diagrams survives, each of them with characteristic behavior

(1)



Parton model result
[current fragmentation]

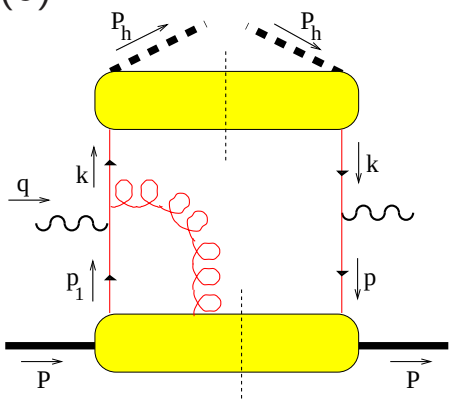
(2)



$P \cdot P_h \sim M^2$
[target fragmentation,
fracture functions]

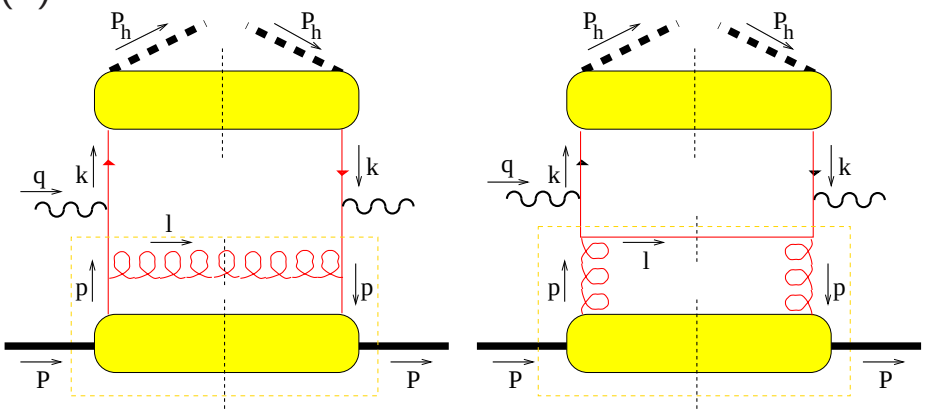
THE CALCULATION OF $\mathcal{W}_{\mu\nu}$

(3)



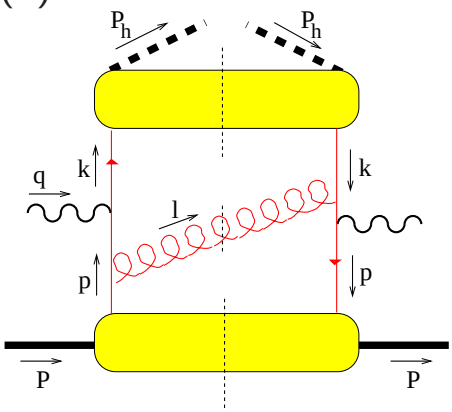
- 'Physical' A_T gluons
  [e.m. gauge invariance at $\mathcal{O}(1/Q)$]
- Longitudinal A^+ gluons
  [create link, color gauge invariance]

(4)



$\alpha_s \log(Q^2/\mu^2)$ corrections
[DGLAP equations]

(5)

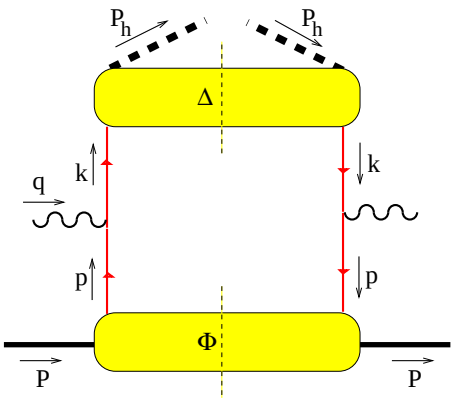


(Process dependent) α_s corrections
Compare inclusive DIS:

$$F_L = \frac{F_2}{x} - 2F_1 \sim \alpha_s(Q^2) \times \ldots$$

Large scale Q leads in a natural way to the use of lightlike vectors: $n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$

$$\left.\begin{aligned} q^2 &= -Q^2 \\ P^2 &= M^2 \\ P_h^2 &= M_h^2 \\ 2\,P\cdot q &= \frac{Q^2}{x_{_B}} \\ 2\,P_h\cdot q &= -z_h\,Q^2 \end{aligned}\right\} \longleftrightarrow \left\{\begin{aligned} P_h &= \frac{z_h\,Q}{\sqrt{2}}\,n_- + \frac{M_h^2}{z_h\,Q\sqrt{2}}\,n_+ \\ q &= \frac{Q}{\sqrt{2}}\,n_- - \frac{Q}{\sqrt{2}}\,n_+ + q_T \\ P &= \frac{x_{_B}\,M^2}{Q\sqrt{2}}\,n_- + \frac{Q}{x_{_B}\sqrt{2}}\,n_+ \end{aligned}\right.$$

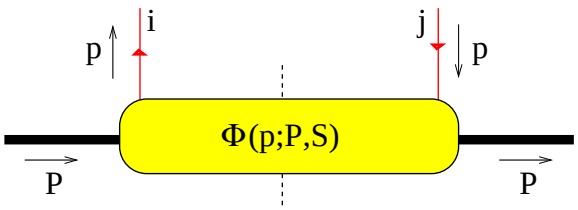


	$-$	$+$	
$q \to h$	$\sim Q$	$\sim 1/Q$	$\to \int dk^+ \ldots$
HARD	$\sim Q$	$\sim Q$	
$H \to q$	$\sim 1/Q$	$\sim Q$	$\to \int dp^- \ldots$

Three external momenta (P_A, P_B, q)

$\to$ transverse direction relevant

ANALYSIS OF SOFT PARTS: DISTRIBUTIONS



Hard 1-particle inclusive scattering allows isolation of $\Phi(x, p_T)$

$$\Phi(x, \boldsymbol{p}_T) = \int dp^- \, \Phi(p; P, S)\Big|_{p^+ = xP^+,\, p_T}$$

$$= \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3} \, e^{ip\cdot\xi} \, \langle P, S|\overline{\psi}(0)\psi(\xi)|P, S\rangle\Big|_{\xi^+=0}$$

$$\Phi(x, \boldsymbol{p}_T) = \frac{1}{2}\left\{ f_1 \, \not{n}_+ + \lambda \, g_{1L}\gamma_5 \, \not{n}_+ + g_{1T} \, \frac{(\boldsymbol{p}_T \cdot \boldsymbol{S}_T)}{M} \, \gamma_5 \, \not{n}_+ \right.$$
$$+ h_{1T} \, \frac{[\not{S}_T, \, \not{n}_+]\gamma_5}{2} + \lambda \, h_{1L}^\perp \, \frac{[\not{p}_T, \, \not{n}_+]\gamma_5}{2}$$
$$\left. + h_{1T}^\perp \, \frac{(\boldsymbol{p}_T \cdot \boldsymbol{S}_T)}{M} \, \frac{[\not{p}_T, \, \not{n}_+]\gamma_5}{2} \right\}$$

T-odd

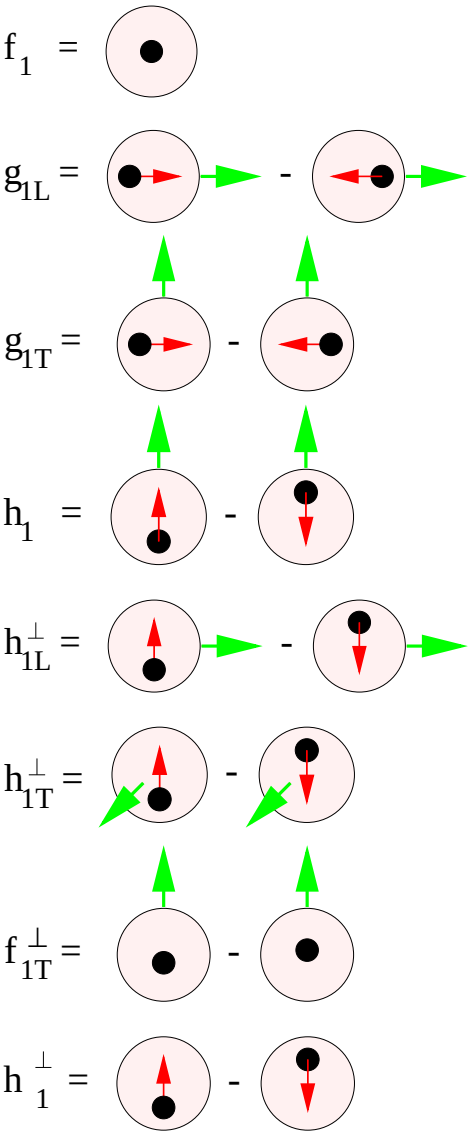
$$+ \frac{1}{2}\left\{ f_{1T}^\perp \, \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu p_T^\rho S_T^\sigma}{M} + h_1^\perp(x, p_T^2) \, \frac{i\,[\not{p}_T, \, \not{n}_+]}{2M} \right\}$$

$f_1 = f_1(x, \boldsymbol{p}_T^2)$, etc.

Ralston & Soper, Tangerman & M

INTERPRETATION OF SOFT PARTS

All these functions can be interpreted as momentum densities:

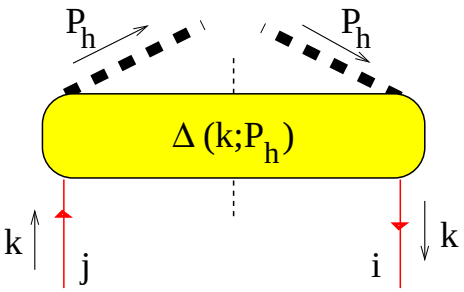


Longitudinally polarized quarks in a transversely polarized target

Transversely polarized quarks in a longitudinally polarized target

Unpolarized quarks in a transversely polarized target (T-odd)

Transversely polarized quarks in an unpolarized target (T-odd)



$$P_h = P_h^- \, n_- + \frac{M_h^2}{2P_h^-} \, n_+$$

$$k = \frac{1}{z} \, P_h^- \, n_- + k_T + \mathcal{O}\left(\frac{1}{P_h^-}\right)$$

Hard processes need the quantity (lightfront correlation)

$$\Delta(z, \boldsymbol{k}_T) = \int dk^- \; \Delta(k, P_h, S_h)\Big|_{k^- = P_h^-/z,\, k_T}$$

$$= \int \frac{d\xi^+ d^2\boldsymbol{\xi}_T}{(2\pi)^3} \, e^{ik\cdot\xi} \, \langle 0|\psi(\xi) a_h^\dagger a_h \overline{\psi}(0)|0\rangle\Big|_{\xi^- = 0}$$

Using parity and hermiticity one finds for an unpolarized hadron

$$\Delta(z, k_T) = z D_1(z, k_T') \, \not{n}_- + z H_1^\perp(z, k_T') \, \frac{i[\not{k}_T, \not{n}_-]}{2M_h} + \mathcal{O}\left(\frac{M_h}{P_h^-}\right)$$

- $k_T' = -z k_T$ is transverse momentum of hadron wrt quark.
- $H_1^\perp$ is T-odd (natural for fragmentation functions)

FROM THE LIGHTFRONT TO THE LIGHTCONE

$\boldsymbol{p}_T$-averaging: relevant for inclusive ℓH or $\boldsymbol{q}_T$-integrated DY

$$\Phi(x) = \int dp^- \, d^2\boldsymbol{p}_T \; \Phi(p,P,S)$$

$$= \int \frac{d\xi^-}{2\pi} \; e^{ip\cdot\xi} \, \langle P,S|\overline{\psi}(0)\mathcal{U}\psi(\xi)|P,S\rangle \Big|_{\xi^+=\xi_T=0}$$

$$\Phi(x) = \frac{1}{2}\left\{ f_1 \, \not{n}_+ + \lambda \, g_1 \gamma_5 \, \not{n}_+ + h_1 \, \frac{[\not{S}_T, \not{n}_+]\gamma_5}{2} \right\}$$

$$f_1 = f_1(x) = \int d^2\boldsymbol{p}_T \; f_1(x,\boldsymbol{p}_T)$$

$\boldsymbol{p}_T$-weighting: relevant for asymmetries in ℓH or DY

$$\Phi^\alpha_\partial(x) = \int dp^- \, d^2\boldsymbol{p}_T \; p_T^\alpha \, \Phi(p,P,S)$$

$$= \int \frac{d\xi^-}{2\pi} \; e^{ip\cdot\xi} \, \langle P,S|\overline{\psi}(0)\mathcal{U}\, i\partial^\alpha \, \mathcal{U}\psi(\xi)|P,S\rangle \Big|_{\xi^+=\xi_T=0}$$

$$\frac{1}{M}\,\Phi^\alpha_\partial(x) = \frac{1}{2}\left\{ g_{1T}^{(1)} \, S_T^\alpha \, \gamma_5 \, \not{n}_+ + \lambda \, h_{1L}^{\perp(1)} \, \frac{[\not{n}_+, \gamma^\alpha]\gamma_5}{2} \right.$$

T-odd

$$\left. + \, f_{1T}^{\perp(1)} \, \epsilon_T^{\alpha\beta} S_{T\beta} \, \not{n}_+ + h_1^{\perp(1)} \, \frac{i\,[\not{n}_+, \gamma^\alpha]}{2} \right\}$$

$$g_{1T}^{(1)} = g_{1T}^{(1)}(x) \equiv \int d^2\boldsymbol{p}_T \, \frac{\boldsymbol{p}_T^2}{2M^2} \, g_{1T}(x,\boldsymbol{p}_T)$$

EXTENSION TO SUBLEADING ORDER $(1/Q)$

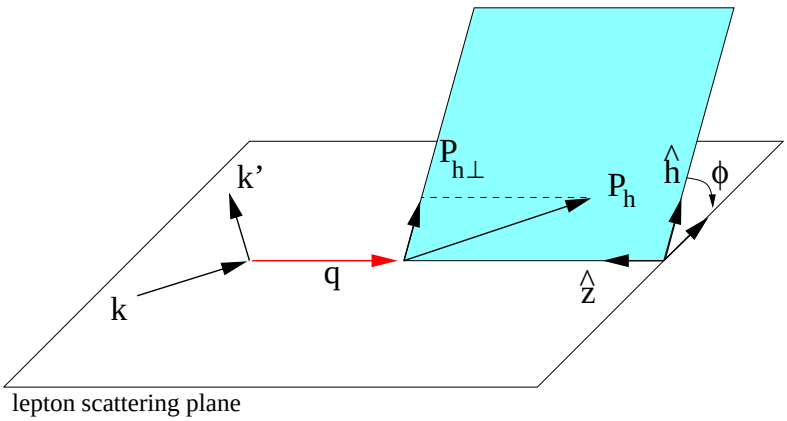
Including subleading order for the $\boldsymbol{p}_T$-averaged functions

$$\Phi(x) = \frac{1}{2}\left\{ f_1 \not{n}_+ + \lambda\, g_1 \gamma_5 \not{n}_+ + h_1 \frac{[\not{\$}_T, \not{n}_+]\gamma_5}{2} \right\}$$
$$+ \frac{M}{2P^+}\left\{ e + g_T\, \gamma_5 \not{\$}_T + \lambda\, h_L \frac{[\not{n}_+, \not{n}_-]\gamma_5}{2} \right\}$$
$$\text{T-odd} \qquad + \frac{M}{2P^+}\left\{ f_T\, \epsilon_T^{\rho\sigma} S_{T\rho}\gamma_\sigma - i\,\lambda\, e_L\, \gamma_5 + h \frac{i\,[\not{n}_+, \not{n}_-]}{2} \right\}$$

- $M/P^+ \propto M/Q \longrightarrow$ subleading
- The (twist-3) functions do not have a density interpretation
  $e = (m/Mx)\, f_1 + \tilde{e}$, $g_T = g_{1T}^{(1)}/x + (m/Mx)\, h_1 + \tilde{g}_T$,
  $h_L = -2\, h_{1L}^{\perp(1)}/x + (m/Mx)\, g_1 + \tilde{h}_L$
- From the general amplitude expansion for Φ (covariance), constrained by Hermiticity, P, and T one obtains:

$$\underbrace{g_T - g_1}_{g_2} = \frac{d}{dx} g_{1T}^{(1)} \qquad \underbrace{h_L - h_1}_{\frac{1}{2} h_2} = -\frac{d}{dx} h_{1L}^{\perp(1)}$$

$$f_T = -\frac{d}{dx} f_{1T}^{\perp(1)} \qquad h = -\frac{d}{dx} h_1^{\perp(1)}$$



$$\ell H \longrightarrow \ell' h X$$

$$x_{_B} = \frac{Q^2}{2P \cdot q}$$

$$y = \frac{P \cdot q}{P \cdot k}$$

$$z_h = \frac{P \cdot P_h}{P \cdot q}$$

$$Q_{_T} = \frac{|P_{h\perp}|}{z_h}$$

Cross sections (leading in $1/Q$)

$$\frac{d\sigma_{OO}}{dx_{_B}\, dy\, dz_h} = \frac{2\pi\alpha^2\, s}{Q^4}\, x_{_B} \left(1 + (1-y)^2\right) \sum_{a,\bar{a}} e_a^2 f_1^a(x_{_B})\, D_1^a(z_h)$$

$$\frac{d\sigma_{LL}}{dx_{_B}\, dy\, dz_h} = \frac{2\pi\alpha^2\, s}{Q^4}\, x_{_B}\, y(2-y)\, \lambda_e\, \lambda \sum_{a,\bar{a}} e_a^2 g_1^a(x_{_B})\, D_1^a(z_h)$$

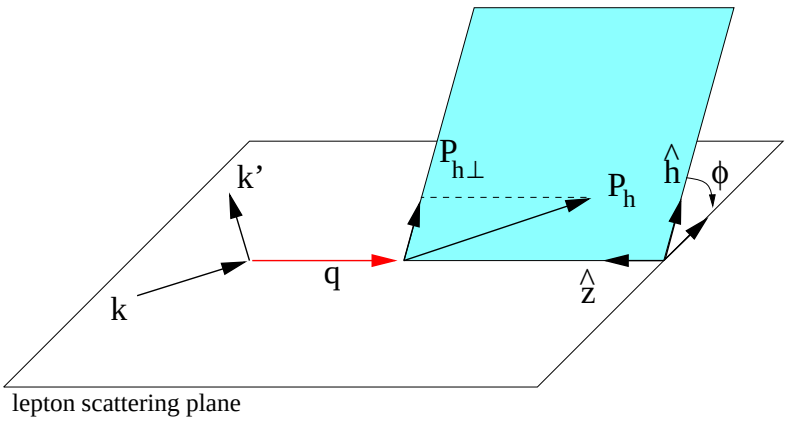
Where to find a specific correlation function?

- twist t of correlation functions
  $\Rightarrow$ behavior $(1/Q)^{t-2}$

- cross sections are chirally even
  Examples: $\propto f_1 \otimes D_1$
  $\propto h_1 \otimes H_1$
  $\propto e \otimes H_1^\perp$
  $\propto m\, f_1 \otimes H_1^\perp$

- # of spin vectors is even in case of a T-even combination
  Examples: $\propto \lambda_e\, \lambda\, g_1 \otimes D_1$
  $\propto \lambda_e\, |\boldsymbol{S}_T|\, g_{1T}^{(1)} \otimes D_1$
  $\propto |\boldsymbol{S}_T|\, |\boldsymbol{S}_{hT}|\, h_1 \otimes H_1$
  $\propto h_1^\perp \otimes H_1^\perp$

- # of spin vectors is odd in case of a T-odd combination (single spin asymmetries)
  Examples: $\propto \lambda_e\, e \otimes H_1^\perp$
  $\propto |\boldsymbol{S}_T|\, h_1 \otimes H_1^\perp$
  $\propto |\boldsymbol{S}_T|\, f_{1T}^\perp \otimes D_1$



$$\ell H \longrightarrow \ell' h X$$

$$x_B = \frac{Q^2}{2P \cdot q}$$

$$y = \frac{P \cdot q}{P \cdot k}$$

$$z_h = \frac{P \cdot P_h}{P \cdot q}$$

$$Q_T = \frac{|P_{h\perp}|}{z_h}$$

Azimuthally weighted cross sections

$$\langle W \rangle_{\underbrace{P_e P_H P_h}_{\uparrow}} \equiv \int d\phi^\ell \, d^2\boldsymbol{q}_T \, \underbrace{W}_{\uparrow} \, \frac{d\sigma_{P_e P_H}}{dx_B \, dy \, dz_h \, d\phi^\ell \, d^2\boldsymbol{q}_T}$$

(O, L, T) $\qquad$ $W(Q_T, \phi_h^\ell, \phi_S^\ell, \phi_{S_h}^\ell)$

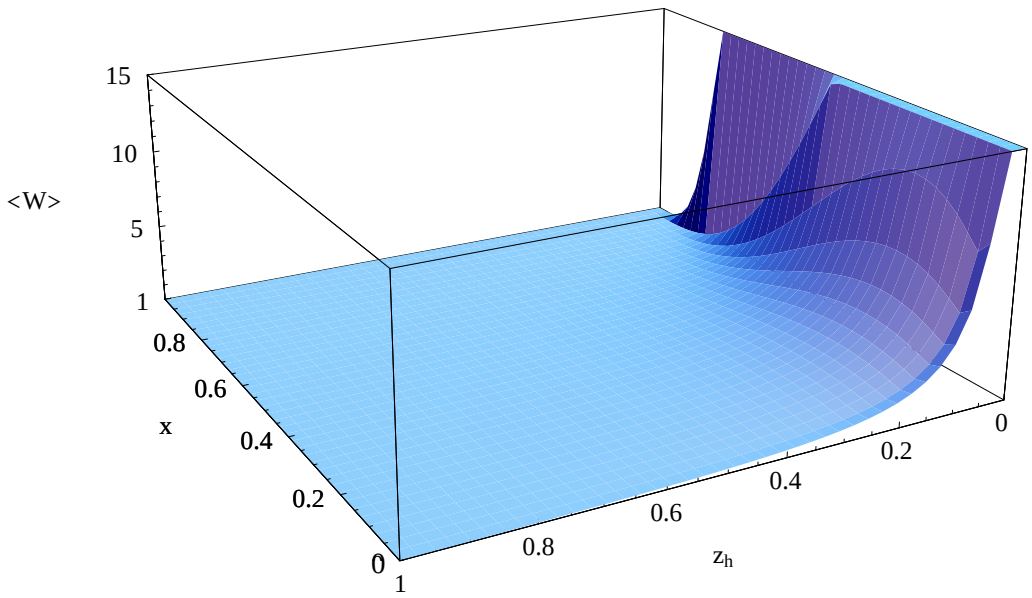
e.g.

$$\langle 1 \rangle_{OOO} = \frac{2\pi\alpha^2 \, s}{Q^4} \, x_B \left(1 + (1-y)^2\right) \sum_{a,\bar{a}} e_a^2 f_1^a(x_B) \, D_1^a(z_h)$$

π^+ production on a proton: $ep \to e\pi^+ X$

$$\langle 1 \rangle_{OOO} \quad \propto \sum_{a,\bar{a}} e_a^2 f_1^a(x_B)\, D_1^a(z_h)$$

For valence quarks one finds



AZIMUTHAL ASYMMETRIES FOR POLARIZED TARGETS (LEADING TWIST)

Azimuthal asymmetries for polarized targets (leading twist)

$$\left\langle \frac{Q_T}{M} \cos(\phi_h^\ell - \phi_S^\ell) \right\rangle_{LTO}$$
$$= \frac{4\pi\alpha^2\, s}{Q^4}\, \lambda_e\, |\boldsymbol{S}_T|\, y \left(1 - \frac{y}{2}\right) \sum_{a,\bar{a}} e_a^2\, x_B\, g_{1T}^{(1)a}(x_B) D_1^a(z_h)$$

$$\left\langle \frac{Q_T^2}{4MM_h} \sin(2\phi_h^\ell) \right\rangle_{OLO}$$
$$= -\frac{4\pi\alpha^2\, s}{Q^4}\, \lambda\, (1-y) \sum_{a,\bar{a}} e_a^2\, x_B\, h_{1L}^{\perp(1)a}(x_B) H_1^{\perp(1)a}(z_h)$$

$$\left\langle \frac{Q_T}{M_h} \sin(\phi_h^\ell + \phi_S^\ell) \right\rangle_{OTO}$$
$$= \frac{4\pi\alpha^2\, s}{Q^4}\, |\boldsymbol{S}_T|\, (1-y) \sum_{a,\bar{a}} e_a^2\, x_B\, h_1^a(x_B) H_1^{\perp(1)a}(z_h)$$

Note: there exists also an (OTO) $\sin(3\phi_h^\ell - \phi_S^\ell)$ asymmetry $\sim h_{1T}^{\perp(2)} H_1^{\perp(1)}$

J.C. Collins, NPB 396 (1993) 161
A. Kotzinian, NPB 441 (1995) 234
R.D. Tangerman and PJM, PLB 352 (1995) 129

ESTIMATE FOR $g_{1T}^{(1)}$

Leading functions: g_1, $g_{1T}^{(1)}$

Subleading function

$$\left.\begin{array}{rcl} g_T &=& \dfrac{g_{1T}^{(1)}}{x} + \tilde{g}_T \\ &=& g_1 + \dfrac{d}{dx}\, g_{1T}^{(1)} \end{array}\right\} \Rightarrow g_T = \int_x^1 dy\, \frac{g_1(y)}{y} + \underbrace{\left(\tilde{g}_T - \int_x^1 dy\, \frac{\tilde{g}_T(y)}{y} \right)}_{\bar{g}_T}$$

Approximation 1: $g_{1T}^{(1)}(x) \approx 0$

$$\begin{aligned} g_T(x) &= g_1(x) \\ \bar{g}_T(x) &= g_1(x) - \int_x^1 dy\, \frac{g_1(y)}{y} \end{aligned}$$

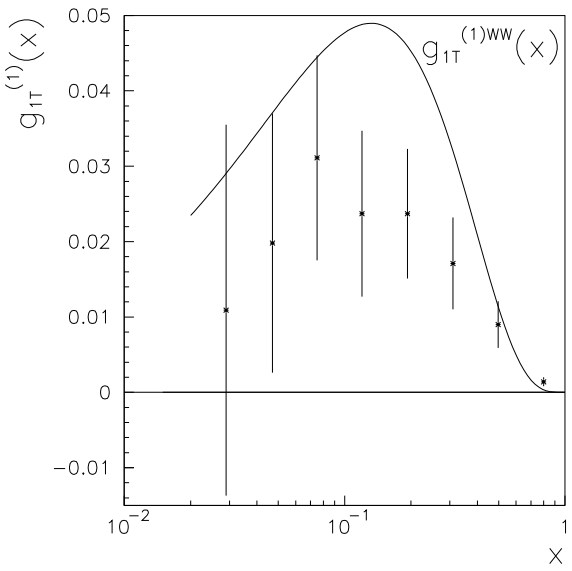
Approximation 2: $\bar{g}_T(x) \approx 0$

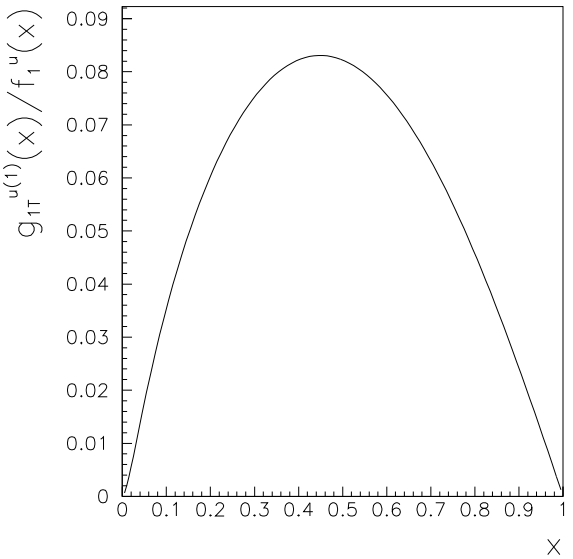
$$\begin{aligned} g_T(x) &= \int_x^1 dy\, \frac{g_1(y)}{y} \\ g_{1T}^{(1)}(x) &= x \int_x^1 dy\, \frac{g_1(y)}{y} = x\, g_T \end{aligned}$$

ESTIMATE OF THE LT ASYMMETRY IN SIDIS

Estimate of $g_{1T}^{(1)}$ A. Kotzinian & PJM, PRD 54 (1996) 1229

- datapoints: SLAC g_2-data: $g_{1T}^{(1)}(x) = -\int_x^1 dy\, g_2(y)$
- line: above relation with $g_2(x) = g_2^{WW}(x)$





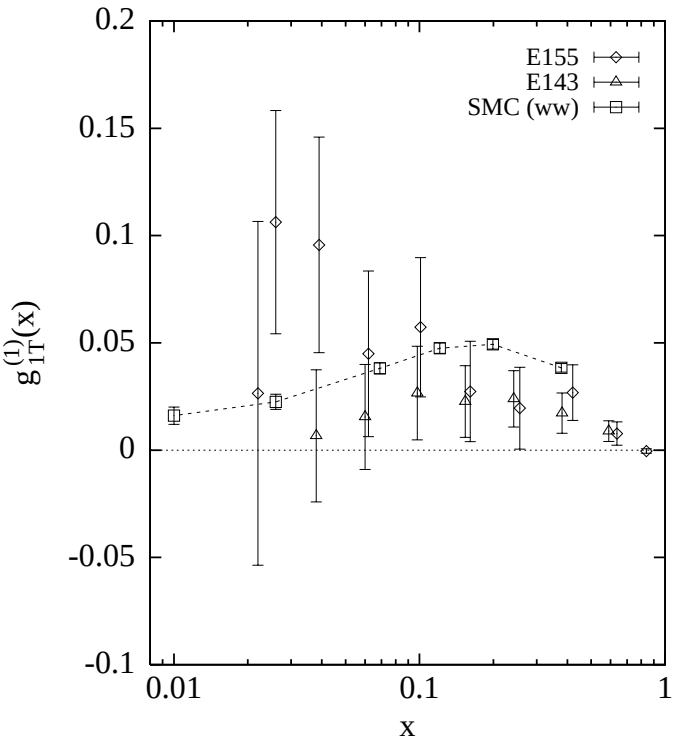
Resulting asymmetry in $\vec{e}\vec{p} \to e\pi^+ X$:

$$A_{LT} \propto \frac{g_{1T}^{(1)u}(x)}{f_1^u(x)}$$

Updated estimate of $g_{1T}^{(1)}$

M. Boglione & PJM

- datapoints: SLAC g_2-data: $g_{1T}^{(1)}(x) = -\int_x^1 dy\, g_2(y)$ (including E155, preliminary)
- line: above relation with $g_2(x) = g_2^{WW}(x)$



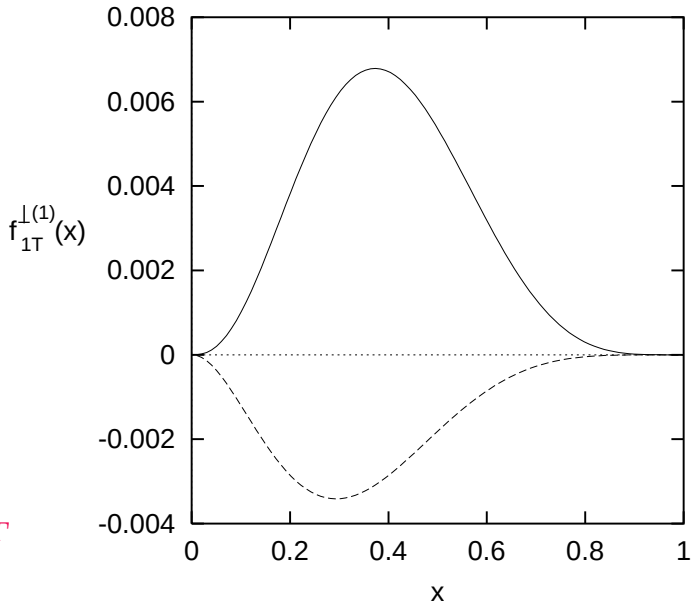
ESTIMATE OF $H_1^\perp$

Single spin asymmetry A_N **in** $pp^\uparrow \to \pi\, X$

Explanation 1: Scattering via an unpolarized quark

$$\sigma \;\propto\; \bar{f}_1 \otimes f_{1T}^{\perp} \otimes D_1$$

M. Anselmino, M. Boglione,
F. Murgia, PLB 362 (1995) 164

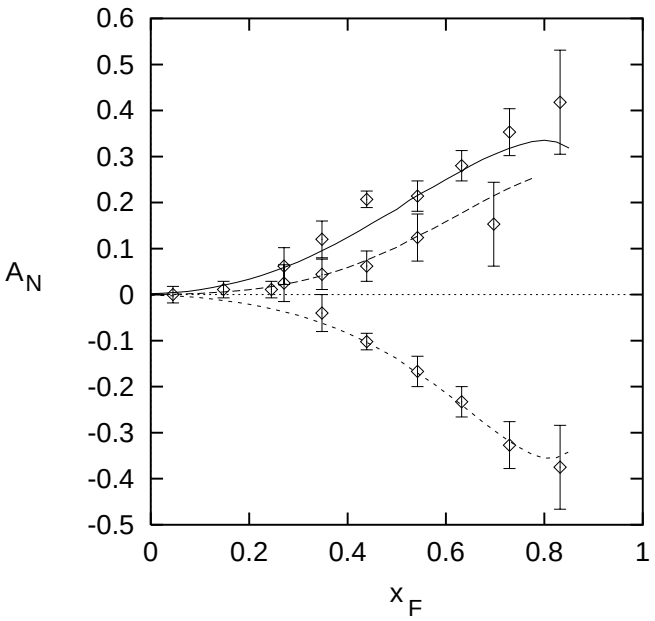


Sivers effect: $f_{1T}^{\perp}$

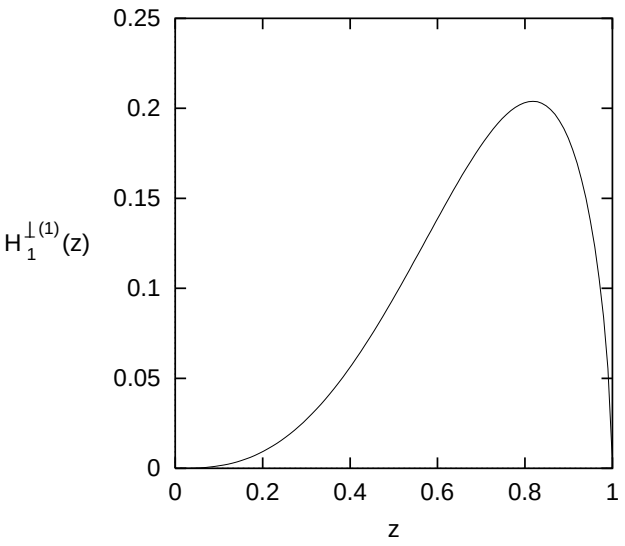
Single spin asymmetry A_N **in** $pp^\uparrow \to \pi X$

Explanation 2: Scattering via a transversely polarized quark

$$\sigma \;\propto\; \bar{f}_1 \otimes h_1 \otimes H_1^\perp$$



M. Anselmino, M Boglione,
F. Murgia, hep-ph/9901442



Collins effect: $H_1^\perp$

Single spin asymmetries (leading twist)

$$\left\langle \frac{Q_T}{M} \sin(\phi_h^\ell - \phi_S^\ell) \right\rangle_{OTO}$$
$$= \frac{2\pi\alpha^2\, s}{Q^4} |\boldsymbol{S}_T| \left(1 - y + \frac{1}{2}\, y^2\right) \sum_{a,\bar{a}} e_a^2\, x_B\, f_{1T}^{\perp(1)a}(x_B) D_1^a(z_h)$$

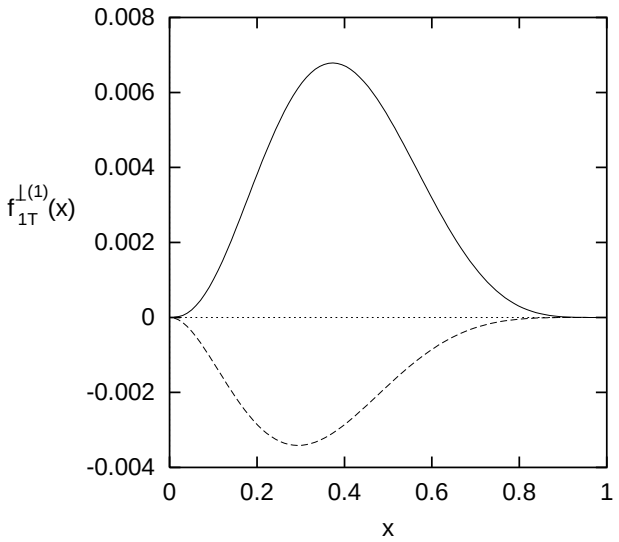
$$\left\langle \frac{Q_T}{M_h} \sin(\phi_h^\ell + \phi_S^\ell) \right\rangle_{OTO}$$
$$= \frac{2\pi\alpha^2\, s}{Q^4} |\boldsymbol{S}_T|\, 2(1-y) \sum_{a,\bar{a}} e_a^2\, x_B\, h_1^a(x_B) H_1^{\perp(1)a}(z_h)$$

D. Boer and PJM, PRD 57 (1998) 5780

Use $ep^\uparrow \to e\pi^+ X$ to distinguish different explanations of the asymmetry in $pp^\uparrow \to \pi^+ X$

M. Boglione and PJM, hep-ph/9903354

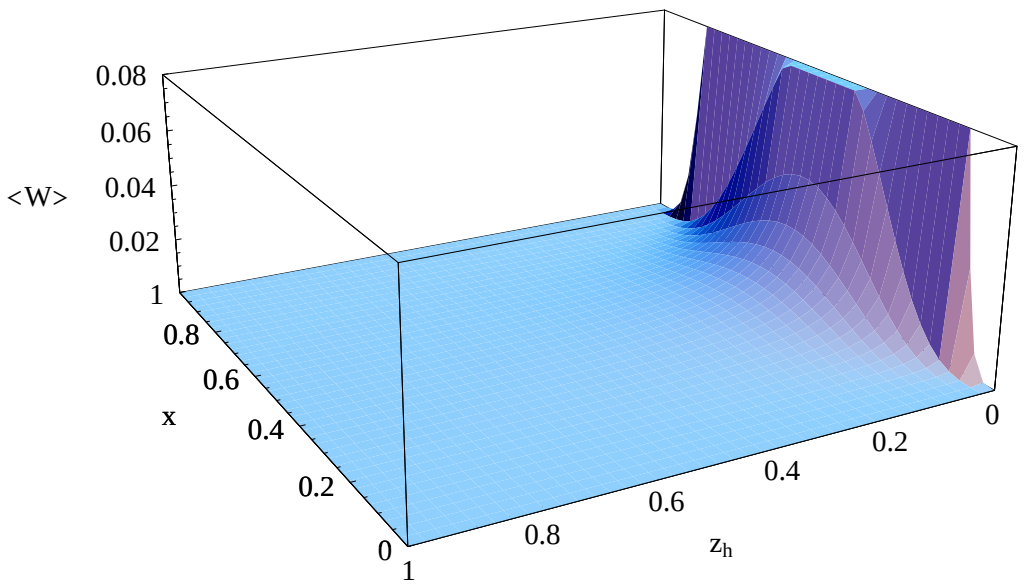
Use $f_{1T}^{\perp}$ from spin asymmetry A_N in $pp^{\uparrow} \to \pi\, X$



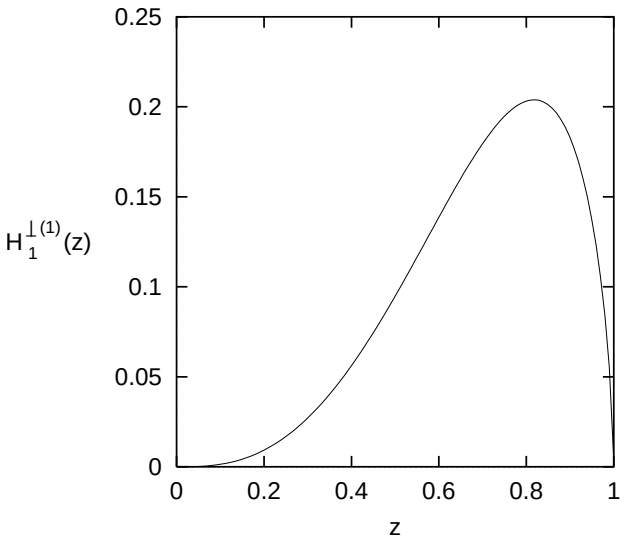
calculate for $ep^{\uparrow} \to \pi^{+}\, X$

$$\left\langle \frac{Q_T}{M}\, \sin(\phi_h^{\ell} - \phi_S^{\ell}) \right\rangle_{OTO}$$

$$\propto \sum_{a,\bar{a}} e_a^2\, x_B\, f_{1T}^{\perp(1)a}(x_B) D_1^a(z_h)$$



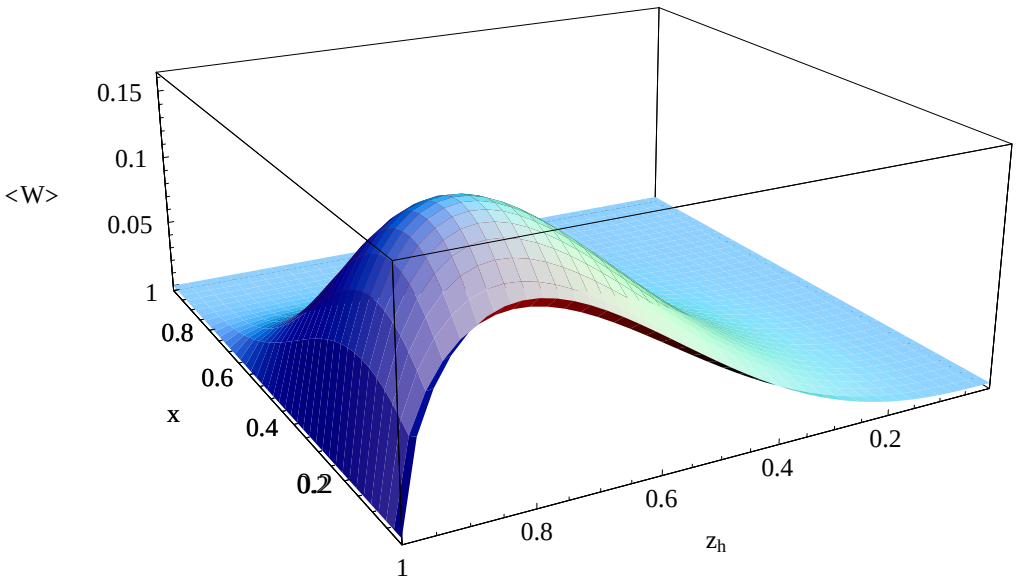
Use $H_1^\perp$ from spin asymmetry A_N in $pp^\uparrow \to \pi\, X$



calculate for $ep^\uparrow \to \pi^+ X$

$$\left\langle \frac{Q_T}{M_h} \sin(\phi_h^\ell + \phi_S^\ell) \right\rangle_{OTO}$$

$$\propto \sum_{a,\bar{a}} e_a^2\, x_B\, h_1^a(x_B) H_1^{\perp(1)a}(z_h)$$



SINGLE SPIN ASYMMETRIES FOR LONGITUDINALLY POLARIZED TARGET

Single spin asymmetries for longitudinally polarized target

Leading:

$$\left\langle \frac{Q_T^2}{4MM_h}\,\sin(2\phi_h^\ell)\right\rangle_{OLO} = -\frac{4\pi\alpha^2\,s}{Q^4}\,\lambda\,(1-y)\sum_{a,\bar a} e_a^2\,x_{_B}\,h_{1L}^{\perp(1)a}(x_{_B})H_1^{\perp(1)a}(z_h)$$

Subleading:

$$\left\langle \frac{Q_T}{M}\,\sin(\phi_h^\ell)\right\rangle_{OLO} = \frac{4\pi\alpha^2\,s}{Q^4}\,\lambda\,(2-y)\sqrt{1-y}\,\frac{2M_h}{Q} \times\sum_{a,\bar a} e_a^2\left\{x_{_B}h_{1L}^{\perp(1)a}(x_{_B})\,\frac{\tilde H^a(z_h)}{z_h} + x_{_B}\left(2\,h_{1L}^{\perp(1)a} - x_{_B}\,\tilde h_L^a(x_{_B})\right)\,H_1^{\perp(1)a}(z_h)\right\}$$

Note: $\Rightarrow \tilde H^a(z) = H^a(z) + 2z\,H_1^{\perp(1)a}(z) = z\,\frac{d}{dz}\left(z\,H_1^{\perp(1)a}\right)$

$\Rightarrow x\,h_L^a(x) - \frac{m}{M}\,g_{1L}^a(x) = -2\,h_{1L}^{\perp(1)a} + x\,\tilde h_L^a(x)$

$= x\,h_1^a - x\,\frac{d}{dx}\,h_{1L}^{\perp(1)a} - \frac{m}{M}\,g_{1L}^a(x)$

$\Rightarrow$ HERMES $\mathcal{A}_{OL}$ results (DIS99): $h_{1L}^{\perp(1)}(x) \approx 0$ (Efremov)

Azimuthal asymmetries for unpolarized targets (higher twist)

$$\left\langle \frac{Q_T}{M}\,\cos(\phi_h^\ell) \right\rangle_{OOO}$$
$$= -\frac{4\pi\alpha^2\,s}{Q^4}\,(2-y)\sqrt{1-y}\sum_{a,\bar a} e_a^2 \left\{ \frac{2M}{Q}\,x_{_B}^2\, f^{\perp(1)a}(x_{_B})\, D_1^a(z_h) + \frac{2M_h}{Q}\,x_{_B}\, f_1^a(x_{_B})\,\frac{\tilde D^{\perp(1)a}(z_h)}{z_h} \right\}$$

Note: $\tilde D^{\perp a}(z) = D^{\perp a}(z) - z D_1^a(z)$

J. Levelt and PJM, PRD 49 (1994) 96

$$\left\langle \frac{Q_T}{M}\,\sin(\phi_h^\ell) \right\rangle_{LOO}$$
$$= \frac{4\pi\alpha^2\,s}{Q^4}\,\lambda_e\, y\sqrt{1-y}\,\frac{2M}{Q}\sum_{a,\bar a} e_a^2\, x_{_B}^2\,\tilde e^a(x_{_B})\, H_1^{\perp(1)a}(z_h)$$

Note: $\tilde e^a(x) = e^a(x) - \frac{m_a}{M}\,\frac{f_1^a(x)}{x}$

J. Levelt and PJM, PLB 338 (1994) 357

ESTIMATE OF $h_{1L}^{\perp(1)}$ and h_L

Leading functions: h_1, $h_{1L}^{(1)}$, $h_{1T}^{(2)}$

Subleading function

$$\left.\begin{aligned} h_L &= -2\,\frac{h_{1L}^{\perp(1)}}{x} + \tilde{h}_L \\ &= h_1 - \frac{d}{dx}\,h_{1L}^{\perp(1)} \end{aligned}\right\} \Rightarrow \quad \begin{aligned} h_L &= 2x\int_x^1 dy\,\frac{h_1(y)}{y^2} \\ &\quad + \underbrace{\left(\tilde{h}_L - 2x\int_x^1 dy\,\frac{\tilde{h}_L(y)}{y^2}\right)}_{\bar{h}_L} \end{aligned}$$

Approximation 1: $h_{1L}^{\perp(1)}(x) \approx 0$

$$\begin{aligned} h_L(x) &= h_1(x) \\ \bar{h}_L(x) &= h_1(x) - 2x\int_x^1 dy\,\frac{h_1(y)}{y^2} \end{aligned}$$

Approximation 2: $\bar{h}_L(x) \approx 0$

$$\begin{aligned} h_L(x) &= 2x\int_x^1 dy\,\frac{h_1(y)}{y^2} \\ h_{1L}^{\perp(1)}(x) &= -x^2\int_x^1 dy\,\frac{h_1(y)}{y^2} = -\frac{1}{2}\,x\,h_L \end{aligned}$$

Leading functions: $H_1^{\perp(1)}$

Subleading function

$$\begin{aligned} H(z) &= -2z\, H_1^{\perp(1)}(z) + \tilde{H}(z) \\ &= z^3 \frac{d}{dz}\left(\frac{H_1^{\perp(1)}}{z}\right) \end{aligned}$$

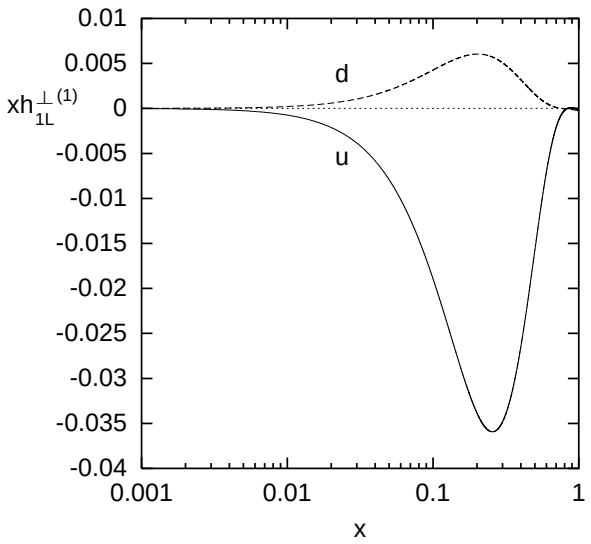
Approximation: $H_1^{\perp(1)}(z) \approx 0$

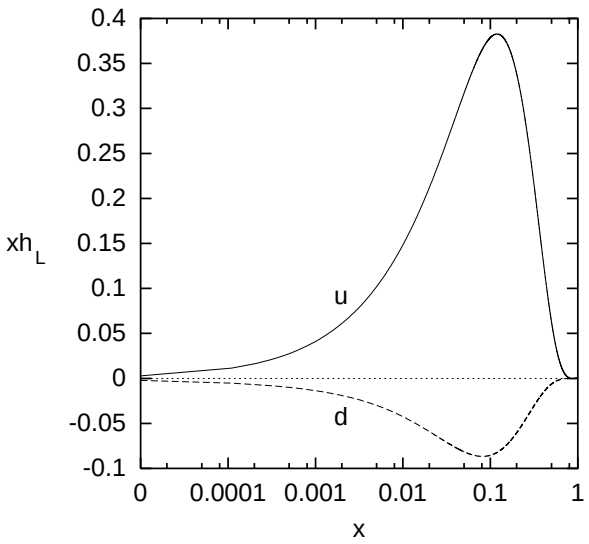
$$\begin{aligned} \frac{H(z)}{z} &= z^2 \frac{d}{dz}\left(\frac{H_1^{\perp(1)}}{z}\right) \approx 0 \\ \frac{\tilde{H}(z)}{z} &= \frac{d}{dz}\left(z\, H_1^{\perp(1)}\right) \approx 0 \end{aligned}$$

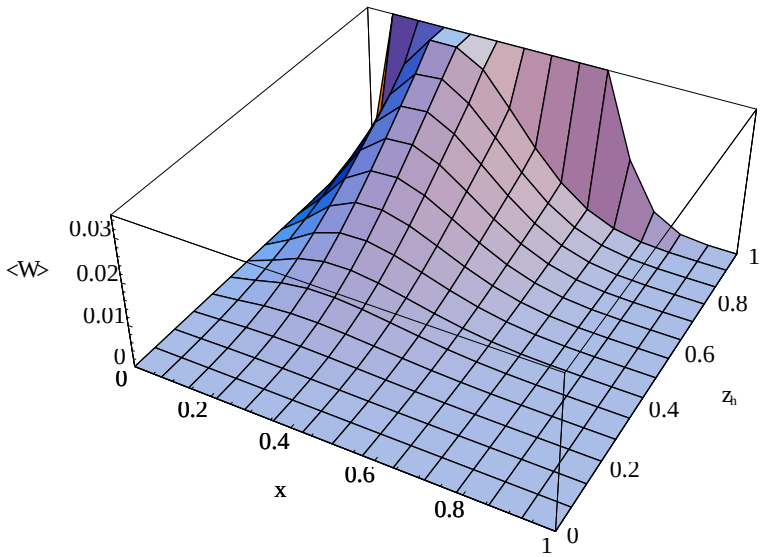
Note: The approximation $\tilde{H}(z) \approx 0$ leads to inconsistencies

M. Boglione and M. (PRELIMINARY)

Use $H_1^\perp$ from spin asymmetry A_N in $pp^\uparrow \to \pi\, X$ and h-distributions in approximation $\bar{h}_L = 0$







Calculate for $e\vec{p} \to \pi^+ X$

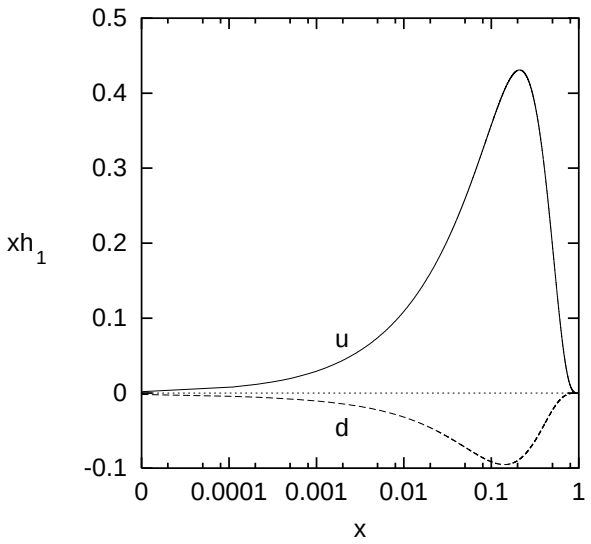
$$\left\langle \frac{Q_T}{M}\, \sin(\phi_h^\ell) \right\rangle_{OLO}$$

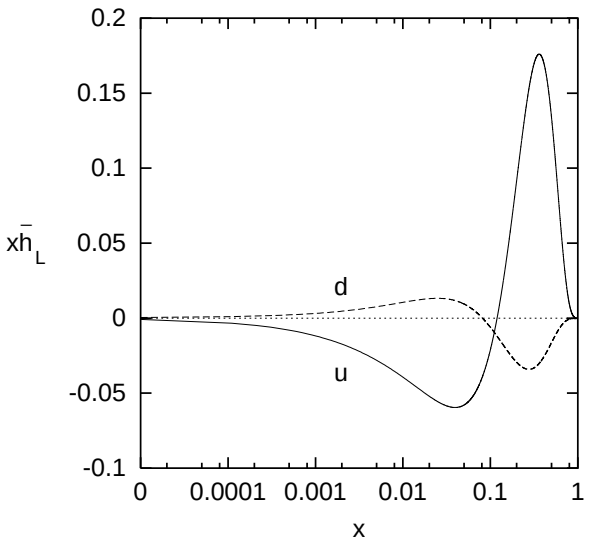
$$\propto -\sum_{a,\bar{a}} e_a^2 \left[x_B\, h_{1L}^{\perp(1)a}(x_B)\, \frac{\tilde{H}^a(z_h)}{z_h} \right.$$

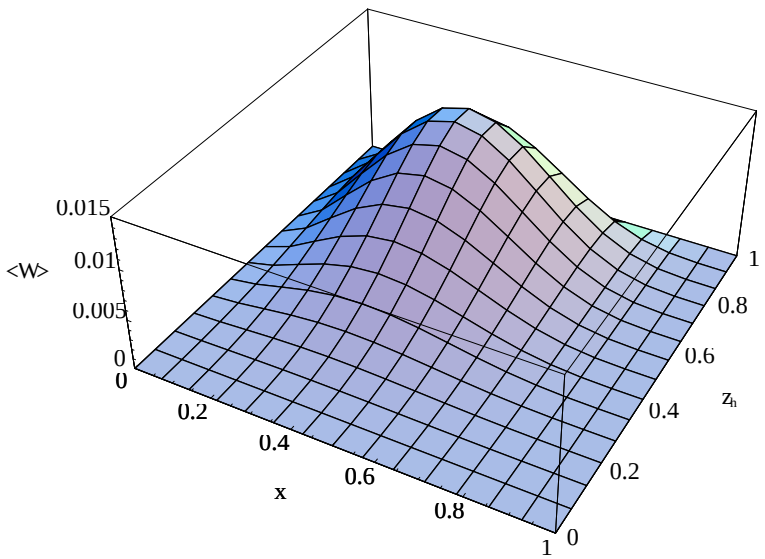
$$\left. - x_B^2\, h_L^a(x_B)\, H_1^{\perp(1)a}(z_h) \right]$$

M. Boglione and M. (PRELIMINARY)

Use $H_1^\perp$ from spin asymmetry A_N in $pp^\uparrow \to \pi\, X$ and h-distributions in approximation $\bar{h}_{1L}^{\perp(1)} = 0$







Calculate for $e\vec{p} \to \pi^+ X$

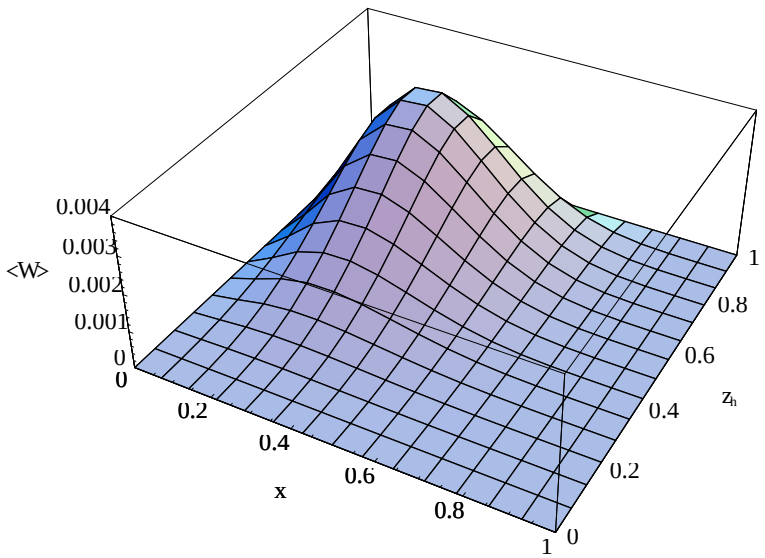
$$\left\langle \frac{Q_T}{M}\, \sin(\phi_h^\ell) \right\rangle_{OLO}$$

$$\propto -\sum_{a,\bar{a}} e_a^2 \left[x_B\, h_{1L}^{\perp(1)a}(x_B)\, \frac{\tilde{H}^a(z_h)}{z_h} \right.$$

$$\left. - x_B^2\, h_L^a(x_B)\, H_1^{\perp(1)a}(z_h) \right]$$

M. Boglione and M. (PRELIMINARY)

Results in the same approximations for the $\sin(2\phi_\ell^h)$ asymmetry:



Calculate for $e\vec{p} \to \pi^+ X$

Approximation: $\bar{h}_L = 0$

$$\left\langle \frac{Q_T}{M} \sin(2\phi_h^\ell) \right\rangle_{OLO}$$

$$\propto \sum_{a,\bar{a}} e_a^2 \, x_B \, h_{1L}^{\perp(1)a}(x_B) \, \tilde{H}_1^{\perp(1)a}(z_h)$$

Thus: be careful with conclusions!

DOUBLE T-ODD EFFECTS

Double T-odd effects

D. Sivers, PRD 41 (1990) 83 and PRD 43 (1991) 261
J. Collins, NPB 396 (1993) 161

Azimuthal asymmetry for unpolarized leptoproduction (leading twist)

$$\left\langle \frac{Q_T^2}{M M_h} \cos(2\phi_h^\ell) \right\rangle_{OOO} = \frac{16\pi\alpha^2\, s}{Q^4}\,(1-y) \sum_{a,\bar{a}} e_a^2\, x_{_B}\, h_1^{\perp(1)a}(x_{_B})\, H_1^{\perp(1)a}(z_h)$$

D. Boer and PJM, PRD 57 (1998) 5780

Similar azimuthal asymmetry in $e^+e^- \to \pi\pi\, X$

D. Boer, R. Jakob, PJM, NPB 504 (1997) 345
A. Efremov, O. Smirnova, L. Tkatchev, hep-ph/9812522

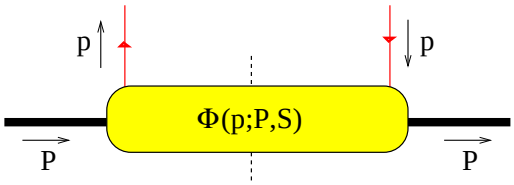
Conclusions and outlook

- Semi-inclusive processes provide opportunity to access effects due to quark transverse momentum. In particular the T-odd functions enable spin physics without spin
- Relations with twist-3 distributions (g_T and h_L)
- Systematic investigation of soft parts (distribution and fragmentation) formalizes many effects such as the Sivers effect, the Collins effect, handedness, initial and final state interactions, effects of gluonic poles
- Need for more detailed study of factorization properties (in progress, Henneman, Bacchetta, Leader)
- Need for quantitative estimates (using models)
  Scopetta, Diakonov, ...
  Jakob, M. and Rodrigues, NPA 626 (1997) 937

APPENDICES

D. Soper; J.C. Collins; R.L. Jaffe; ...

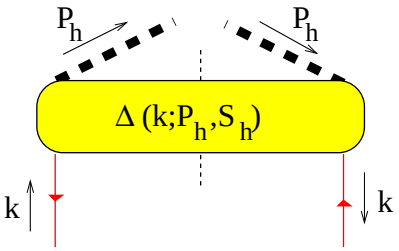
(1) distribution part



$$p^2 \sim p \cdot P \sim P^2 = M^2 \ll Q^2$$

$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4} \int d^4x \; e^{ip\cdot x} \; \langle P,S|\overline{\psi}_j(0)\psi_i(x)|P,S\rangle$$

(2) fragmentation part

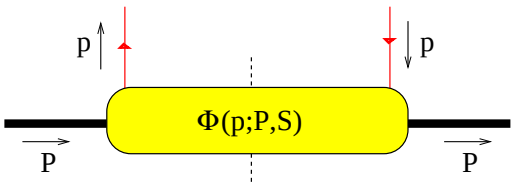


$$k^2 \sim k \cdot P_h \sim P_h^2 = M_h^2 \ll Q^2$$

$$\Delta_{ij}(k,P_h,S_h) = \sum_X \frac{1}{(2\pi)^4} \int d^4x \; e^{ik\cdot x}$$
$$\times \langle 0|\psi_i(x)|P_h,S_h;X\rangle\langle P_h,S_h;X|\overline{\psi}_j(0)|0\rangle$$

D. Soper; J.C. Collins; R.L. Jaffe; ...

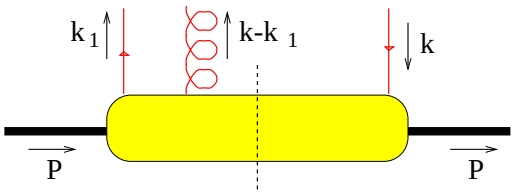
(1) Quark-quark correlator



$$p^2 \sim p \cdot P \sim P^2 = M^2 \ll Q^2$$

$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4} \int d^4x \; e^{ip\cdot x} \; \langle P,S|\overline{\psi}_j(0)\psi_i(x)|P,S\rangle$$

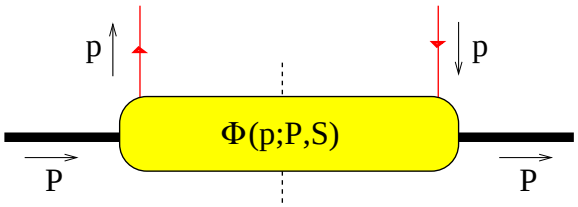
(2) Quark-quark-gluon correlator



$$k^2 \sim k \cdot P \sim k_1^2 \sim k_1 \cdot P \sim k \cdot k_1 \sim P^2 = M^2 \ll Q^2$$

$$\Phi^{\alpha}_{A\,ij}(k,k_1;P,S) = \frac{1}{(2\pi)^4} \int d^4x\, d^4y \; e^{i\,k_1\cdot(x-y)+ik\cdot y} \times \langle P,S|\overline{\psi}_j(0)\, gA^{\alpha}_T(y)\psi_i(x)|P,S\rangle$$

LORENTZ STRUCTURE OF SOFT PARTS



Hermiticity:

$$\Phi^\dagger(k,P,S) = \gamma_0\,\Phi(k,P,S)\,\gamma_0$$

Parity:

$$\Phi(k,P,S) = \gamma_0\,\Phi(\bar{k},\bar{P},-\bar{S})\,\gamma_0$$

$$[\bar{k} = (k^0,-\boldsymbol{k})]$$

$$\begin{aligned}\Phi(k,P,S) \;=\; & M\,A_1 + A_2\;\not{P} + A_3\,\not{k} + i\,A_4\,\frac{[\not{P},\not{k}]}{2M} \\ & + i\,A_5\,(k\cdot S)\gamma_5 + M\,A_6\;\not{S}\gamma_5 + A_7\,\frac{k\cdot S}{M}\;\not{P}\gamma_5 \\ & + A_8\,\frac{k\cdot S}{M}\,\not{k}\gamma_5 + A_9\,\frac{[\not{P},\not{S}]\gamma_5}{2} + A_{10}\,\frac{[\not{k},\not{S}]\gamma_5}{2} \\ & + A_{11}\,\frac{k\cdot S}{M}\,\frac{[\not{P},\not{k}]\gamma_5}{2M} + A_{12}\,\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho S^\sigma}{M}\end{aligned}$$

with real $A_i = A_i(k^2, k\cdot P)$

Time reversal invariance

$$\Phi^*(k,P,S) = \gamma_5 C\,\Phi(\bar{k},\bar{P},\bar{S})\,C^\dagger\gamma_5 \quad \Rightarrow \quad A_4 = A_5 = A_{12} = 0$$

SUMMARY OF PROFILE FUNCTIONS

DISTRIBUTIONS (T-even)					
$\Phi^{[\Gamma]}(x, \boldsymbol{k}_T)$		Dirac structure γ	$\gamma\,\gamma_5$	$[\gamma,\gamma]\gamma_5$	$1/\gamma_5$
twist 2	U	f_1			
	L		g_{1L}	$h_{1L}^\perp$	
	T		g_{1T}	h_1 $h_{1T}^\perp$	
twist 3	U	$f^\perp$			e
	L		$g_L^\perp$	h_L	
	T		g_T $g_T^\perp$	h_T $h_T^\perp$	

(9 independent)

DISTRIBUTIONS (T-odd)					
$\Phi^{[\Gamma]}(x, \boldsymbol{k}_T)$		Dirac structure γ	$\gamma\,\gamma_5$	$[\gamma,\gamma]\gamma_5$	$1/\gamma_5$
twist 2	U			$h_1^\perp$	
	L				
	T	$f_{1T}^\perp$			
twist 3	U			h	
	L	$f_L^\perp$			e_L
	T	f_T			e_T

(3 independent)

INTERACTION DEPENDENT PIECES

$$\Phi_D^\alpha(x) = \frac{M}{2}\left\{\left(x g_T - \frac{m}{M} h_1\right) S_T^\alpha \gamma_5 \not{n}_+ + \lambda\left(x h_L - \frac{m}{M} g_1\right)\frac{[\not{n}_+, \gamma^\alpha]\gamma_5}{2}\right\}$$

$$\Phi_\partial^\alpha(x) = \frac{M}{2}\left\{g_{1T}^{(1)} S_T^\alpha \gamma_5 \not{n}_+ + \lambda\, h_{1L}^{\perp(1)}\frac{[\not{n}_+, \gamma^\alpha]\gamma_5}{2}\right\}$$

$$\Phi_A^\alpha(x) = \frac{M}{2}\left\{x\tilde{g}_T S_T^\alpha \gamma_5 \not{n}_+ + \lambda\, x\tilde{h}_L\frac{[\not{n}_+, \gamma^\alpha]\gamma_5}{2}\right\}$$

$$x\tilde{g}_T = x g_T - g_{1T}^{(1)} - \frac{m}{M} h_1$$

$$x\tilde{h}_L = x h_L - h_{1L}^{\perp(1)} - \frac{m}{M} g_1$$

NOTE:

In parton model interaction-dependent parts vanish: $\Phi_A^\alpha = 0$

Combined with Lorentz invariance

$$\Rightarrow \quad g_T(x) = \int_x^1 dy\,\frac{g_1(y)}{y} + \frac{m}{M}\left[\frac{h_1(x)}{x} - \int_x^1 dy\,\frac{h_1(y)}{y^2}\right]$$

S. Wandzura, F. Wilczek

INTERACTION DEPENDENT PIECES (T-ODD)

$$\Phi_D^\alpha(x)\Big|_{T-odd} = \frac{M}{2}\left\{ x f_T\, \epsilon^\alpha{}_{\mu\nu\rho}\gamma^\mu n_-^\nu S_T^\rho + x\, h\, \frac{i[\gamma^\alpha, \not{n}_+]}{4} \right\}$$

$$\Phi_\partial^\alpha(x)\Big|_{T-odd} = \frac{M}{2}\left\{ -f_{1T}^{\perp(1)}\, \epsilon^\alpha{}_{\mu\nu\rho}\gamma^\mu n_-^\nu S_T^\rho - h_1^{\perp(1)}\, \frac{i[\gamma^\alpha, \not{n}_+]}{2} \right\}$$

$$\Phi_A^\alpha(x)\Big|_{T-odd} = \frac{M}{2}\left\{ x \tilde{f}_T\, \epsilon^\alpha{}_{\mu\nu\rho}\gamma^\mu n_-^\nu S_T^\rho + x\, \tilde{h}\, \frac{i[\gamma^\alpha, \not{n}_+]}{4} \right\}$$

$$x\tilde{f}_T = x f_T + f_{1T}^{(1)}$$
$$x\tilde{h} = xh + 2h_1^{\perp(1)}$$

In parton model interaction-dependent parts vanish: $\Phi_A^\alpha = 0$
Combined with Lorentz invariance

$$\Rightarrow \quad f_T(x) = h(x) = 0$$