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THEORETICAL ASPECTS OF AZIMUTHAL AND SPIN ASYMMETRIES

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- Introduction
- Twist expansion in DIS
- Transverse momentum dependence in SIDIS
- QCD corrections for azimuthal asymmetries
- Summary and conclusions

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INTRODUCTION
'DEEP' STRUCTURE OF HADRONS

Objects in study of hadron structure (given QCD) include:

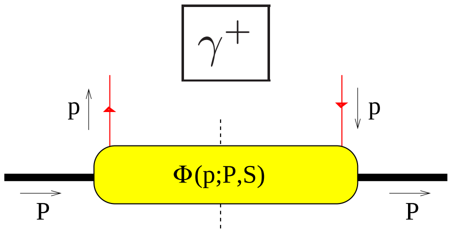
$\langle P+q\vert\psi^\dagger(\xi)\Gamma\psi(\xi)\vert P\rangle$	Elastic scattering $\Rightarrow$ form factor $G_\Gamma(q)$ $\Rightarrow$ spatial density/excitation spectrum
$\langle P\vert\psi^\dagger(0)\Gamma\psi(\xi)\vert P\rangle\big\vert_{\underbrace{\xi^+=\xi_T=0}_{LC}}$	DIS cross section $\Rightarrow$ DF $f_\Gamma(x)$ $\Rightarrow$ (light-cone) parton momentum distribution
$\langle P\vert\psi^\dagger(\xi)\Gamma\psi(\xi)\vert P\rangle$	$\Rightarrow$ charge: $Q_\Gamma = G_\Gamma(0) = \int dx\; f_\Gamma(x)$
$\langle P\vert\psi^\dagger(0)\Gamma D_T(\eta)\,\psi(\xi)\vert P\rangle\big\vert_{LC}$	DIS cross section at $1/Q$ $\Rightarrow$ (lightcone) higher twist correlation
$\langle P\vert\psi^\dagger(0)\Gamma\psi(\xi^-,\xi_T)\vert P\rangle\big\vert_{\underbrace{\xi^+=0}_{LF}}$	SIDIS or DY azimuthal asymmetry $\Rightarrow$ DF $f_\Gamma(x,p_T)$ $\Rightarrow$ p_T-dependent distribution
$\langle 0\vert\psi(\xi)\vert P_h\rangle\,\langle P_h\vert\psi^\dagger(0)\Gamma\vert 0\rangle\big\vert_{LF}$	SIDIS or e^+e^- azimuthal asymmetry $\Rightarrow$ FF $D_\Gamma(z,k_T)$ $\Rightarrow$ parton fragmentation (or decay) function
$\langle P+\Delta\vert\psi^\dagger(0)\Gamma\psi(\xi)\vert P\rangle\big\vert_{LC}$	DVCS cross section $\Rightarrow$ skewed parton distribution

(LEADING) QUARK DISTRIBUTION FUNCTIONS

Hadron structure enters hard processes via quark-quark correlator

$$\Phi_{ij}(x) = \int \frac{d\xi^-}{2\pi}\, e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}_j(0)\,\mathcal{U}(0,\xi)\,\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

Leading part



This part is using symmetry and hermiticity constraints parameterized with distribution functions depending on $x = p^+/P^+$, in DIS measured at $x_{_B} = Q^2/2P\cdot q$ and in DY at $x_{_A} = P_A\cdot q/P_A\cdot P_B$

Parameterization for a spin 1/2 hadron:

$$\Phi^{\text{twist}-2}(x) = \frac{1}{2}\Big\{ f_1(x)\,\not n_+ + S_{_L}\, g_1(x)\gamma_5\,\not n_+ + h_1(x)\,\not S_{_T}\,\not n_+\gamma_5 \Big\}$$

- The functions exist for every quark flavor: $f_1^q(x) = q(x)$, $g_1^q(x) = \Delta q(x)$, $h_1^q(x) = \delta q(x)$
- Leading correlator is of the nonlocal form $\psi_+^\dagger(0)\psi_+(\xi)$ and can be expanded (OPE) into local twist-2 operators of the form $\overline{\psi}\,\gamma^+\,(D^+)^n\psi = \psi_+^\dagger\,(D^+)^n\psi_+$
- Subleading terms are order (M/P^+)

INTERPRETING THE LEADING QUARK STRUCTURE

For a spin 1/2 hadron (e.g. nucleon) the quark distributions functions enter in a quark production matrix. Omitting the higher twist entries one obtains in quark$\otimes$nucleon spin space

$$M^{(\mathrm{prod})} \quad = \quad \begin{array}{cc} \begin{pmatrix} f_1 + g_1 & 0 & 0 & 2\,h_1 \\ 0 & f_1 - g_1 & 0 & 0 \\ 0 & 0 & f_1 - g_1 & 0 \\ 2\,h_1 & 0 & 0 & f_1 + g_1 \end{pmatrix} & \begin{array}{c} \mathrm{R}\rightarrow \\ \leftarrow\mathrm{R} \\ \mathrm{L}\rightarrow \\ \leftarrow\mathrm{L} \end{array} \\ \begin{array}{cccc} \mathrm{R}\rightarrow & \leftarrow\mathrm{R} & \mathrm{L}\rightarrow & \leftarrow\mathrm{L} \end{array} & \end{array}$$

- The three distribution functions are in principle independent!
- Positivity of production matrix gives $f_1(x) \geq 0$ and $|g_1(x)| \leq f_1(x)$ but also the more stringent (Soffer) bound

$$|h_1(x)| \leq \frac{1}{2}\left(f_1(x) + g_1(x)\right)$$

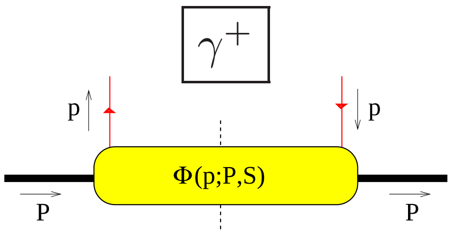
- The function h_1 is chirally odd and is not accessible in DIS

(SUBLEADING) QUARK DISTRIBUTION FUNCTIONS

Hadron structure enters hard processes via quark-quark correlator

$$\Phi_{ij}(x) = \int \frac{d\xi^-}{2\pi}\, e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}_j(0)\,\mathcal{U}(0,\xi)\,\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

Leading part



This part is using symmetry and hermiticity constraints parameterized with distribution functions depending on $x = p^+/P^+$, in DIS measured at $x_B = Q^2/2P\cdot q$ and in DY at $x_A = P_A\cdot q/P_A\cdot P_B$

Parameterization including subleading order for a spin 1/2 hadron:

$$\begin{aligned}\Phi(x) \;=\; & \frac{1}{2}\left\{ f_1(x)\,\not n_+ + S_L\, g_1(x)\gamma_5\,\not n_+ + h_1(x)\,\frac{[\not S_T,\not n_+]\gamma_5}{2}\right\} \\ & + \frac{M}{2P^+}\left\{ e(x) + g_T(x)\,\gamma_5\,\not S_T + S_L\, h_L(x)\,\frac{[\not n_+,\not n_-]\gamma_5}{2}\right\} \\ & + \frac{M}{2P^+}\left\{ f_T(x)\,\epsilon_T^{\rho\sigma} S_{T\rho}\gamma_\sigma - i\,S_L\, e_L(x)\,\gamma_5 + h(x)\,\frac{i\,[\not n_+,\not n_-]}{2}\right\}\end{aligned}$$

T-odd

[T-odd parts will be needed for fragmentation functions]

Subleading quark-quark correlators are via the QCD-eom connected to twist-3 operators of the type $\psi_+^\dagger\,(D^+)^m D_T^\alpha\,(D^+)^{n-m}\psi_+$, appearing in specific (nonlocal) quark-quark-gluon correlator

$$\Phi^\alpha_{D\,ij}(x,y) \sim F.T.\ \langle P,S|\overline{\psi}_j(0)\,\mathcal{U}(0,\eta)\,iD_T^\alpha(\eta)\,\mathcal{U}(\eta,\xi)\,\psi_i(\xi)|P,S\rangle\Big|_{LC},$$

Parameterization for a spin 1/2 hadron:

$$\begin{aligned}\Phi^\alpha_D(x,y) &= \frac{1}{2}\Big\{G_D(x,y)\,i\epsilon_T^{\alpha\beta}S_{T\beta}\,\not{n}_+ + \tilde{G}_D(x,y)\,S_T^\alpha\,\gamma_5\,\not{n}_+ \\ &\qquad + H_D(x,y)\,S_L\,\gamma_5\gamma_T^\alpha\,\not{n}_+ + E_D(x,y)\,\gamma_T^\alpha\,\not{n}_+\Big\},\end{aligned}$$

Equations of motion $[(i\,\not{D} - m)\psi = 0]$:

$$\int dy\,\Big[G_D(x,y) + \tilde{G}_D(x,y)\Big] = x\,g_T(x) - \frac{m}{M}\,h_1(x) + ix\,f_T(x)$$

$$2\int dy\ H_D(x,y) = x\,h_L(x) - \frac{m}{M}\,g_1(x) - ix\,e_L(x)$$

$$2\int dy\ E_D(x,y) = x\,e(x) + \frac{m}{M}\,f_1(x) + ix\,h(x)$$

WHERE DO THE FUNCTIONS SHOW UP

- Distribution functions can be measured in DIS or DY
- Time reversal invariance $\Rightarrow$ no T-odd distribution functions
- twist t of correlation functions $\Rightarrow$ behavior $(1/Q)^{t-2}$
- cross sections are chirally even $\Rightarrow$ in a specific DIS cross section only one chiral-even function will appear
(except for possible quark-mass effects)

$$\sigma_{OO} \Longrightarrow \sum_{q,\bar{q}} e_q^2\, f_1^q(x_{_B})$$

$$\sigma_{LL} \Longrightarrow \lambda_e\, S_{_L} \sum_{q,\bar{q}} e_q^2\, g_1^q(x_{_B})$$

$$\langle \cos\phi_S^\ell\ \sigma_{LT}\rangle \Longrightarrow \lambda_e\, |S_{_T}|\, \frac{M}{Q} \sum_{q,\bar{q}} e_q^2\, g_T^q(x_{_B})$$

SEMI-INCLUSIVE LEPTOPRODUCTION

- Flavor tagging
- Polarimetry
- AZIMUTHAL ASYMMETRIES

TRANSVERSE MOMENTS

- We need SIDIS, DY or 2PI e^+e^-(two hadrons!) to probe transverse momentum balance
- In SIDIS e.g.

$$q_T \;=\; \underbrace{q + x_{_B}\, P}_{\uparrow\; p_{\rm quark}} - \underbrace{\frac{P_h}{z_h}}_{\uparrow\; p'_{\rm quark}} \;=\; \overbrace{\frac{P_{h\perp}}{z_h}}^{\rm EXP\;\downarrow}$$

- SIDIS (q_T-integrated): Sensitive to p_T-integrated correlations.

$$\int d^2q_T \int d^2p_T\, d^2p'_T\;\delta^2(p_T+q_T-p'_T)\ldots =$$
$$= \left[\int d^2p_T\ldots\right]\left[\int d^2p'_T\ldots\right]$$

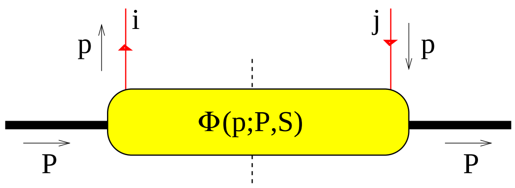
- SIDIS (q_T-weighted): Sensitive to p_T-weighted correlations

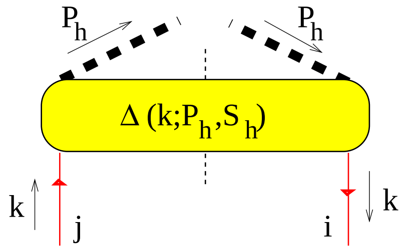
$$\int d^2q_T\; q_T^\alpha \int d^2p_T\, d^2p'_T\;\delta^2(p_T+q_T-p'_T)\ldots =$$
$$= \left[\int d^2p_T\ldots\right]\underbrace{\left[\int d^2p'_T\; p'^{\alpha}_T\ldots\right]}_{\downarrow} - \underbrace{\left[\int d^2p_T\; p^{\alpha}_T\ldots\right]}_{\downarrow}\left[\int d^2p'_T\ldots\right]$$

transverse moments

- Experimentally: Weighting with $q_T^\ell \Rightarrow \int d^2P_{h\perp}\; |P_{h\perp}|\cos(\phi_h^\ell)\ldots$

SOFT PART IN SIDIS





Hard inclusive (polarized) deep inelastic leptoproduction involves the soft part Φ integrated over all momenta except $p^+ \equiv x\,P^+$

$$\Phi_{ij}(x) = \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}_j(0)\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

Hard semi-inclusive (polarized) deep inelastic lepton-hadron scattering involves soft part Φ integrated over p^- with $p^+ \equiv x\,P^+$ and p_T

$$\Phi_{ij}(x,p_T) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\; e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}_j(0)\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=0}$$

Fragmentation into a hadron involves soft part integrated over k^- leaving $P_h^- = z k^-$ and $P_{h\perp} = -z k_T$

$$\Delta_{ij}(z,k_T) = \int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3}\; e^{ik\cdot\xi}\,\langle 0|\psi_i(\xi)|P_h,X\rangle\langle P_h,X|\overline{\psi}_j(0)|0\rangle\Big|_{\xi^-=0}$$

QUARK SPIN STRUCTURE

For azimuthal asymmetries in semi-inclusive leptoproduction or Drell-Yan scattering one needs transverse momentum dependence

Dirac structure for a polarized (spin 1/2) hadron:

$$\Phi_0(x, p_T) = \left\{ f_1(x, p_T^2) + i\, h_1^\perp(x, p_T^2)\, \frac{\not{p}_T}{M} \right\} \not{n}_+$$

$$\Phi_L(x, p_T) = \left\{ S_L\, g_{1L}(x, p_T^2)\, \gamma_5 + S_L\, h_{1L}^\perp(x, p_T^2) \gamma_5\, \frac{\not{p}_T}{M} \right\} \not{n}_+$$

$$\Phi_T(x, p_T) = \left\{ f_{1T}^\perp(x, p_T^2)\, \frac{\epsilon_{T\,\rho\sigma} p_T^\rho S_T^\sigma}{M} + g_{1T}(x, p_T^2)\, \frac{\boldsymbol{p}_T \cdot \boldsymbol{S}_T}{M}\, \gamma_5 \right.$$
$$\left. + h_{1T}(x, p_T^2)\, \gamma_5\, \not{S}_T + h_{1T}^\perp(x, p_T^2)\, \frac{\boldsymbol{p}_T \cdot \boldsymbol{S}_T}{M}\, \frac{\gamma_5\, \not{p}_T}{M} \right\} \not{n}_+$$

- The functions $h_1^\perp$ and $f_{1T}^\perp$ are T-odd
- Operator structure $\psi_+^\dagger(0)\psi_+(\xi)$ allows a density interpretation.
- For fragmentation: $f \rightarrow D$, $g \rightarrow G$, $h \rightarrow H$ or hat-notation

FULL QUARK & HADRON SPIN STRUCTURE

Quark p_T-dependent distributions

$$
\begin{array}{c}
\begin{array}{cccc}
\quad \text{R} \rightarrow \quad & \quad \leftarrow \text{R} \quad & \quad \text{L} \rightarrow \quad & \quad \leftarrow \text{L} \quad
\end{array} \\
\begin{pmatrix}
f_1 + g_{1L} & \frac{|p_T|}{M}\, e^{i\phi}\, g_{1T} & \frac{|p_T|}{M}\, e^{-i\phi}\, h_{1L}^{\perp} & 2\, h_1 \\
\frac{|p_T|}{M}\, e^{-i\phi}\, g_{1T} & f_1 - g_{1L} & \frac{|p_T|^2}{M^2}\, e^{-2i\phi}\, h_{1T}^{\perp} & -\frac{|p_T|}{M}\, e^{-i\phi}\, h_{1L}^{\perp} \\
\frac{|p_T|}{M}\, e^{i\phi}\, h_{1L}^{\perp} & \frac{|p_T|^2}{M^2}\, e^{2i\phi}\, h_{1T}^{\perp} & f_1 - g_{1L} & -\frac{|p_T|}{M}\, e^{i\phi}\, g_{1T} \\
2\, h_1 & -\frac{|p_T|}{M}\, e^{i\phi}\, h_{1L}^{\perp} & -\frac{|p_T|}{M}\, e^{-i\phi}\, g_{1T} & f_1 + g_{1L}
\end{pmatrix}
\end{array}
$$

- Matrix representation in Dirac space allows derivation of bounds

  Bacchetta, Boglione, Henneman and M, PRL 58 (2000) 712

- Straightforward extensions for gluons

  J. Rodrigues and PM, PRD (2001), hep-ph/0009343

  . . . and spin 1 hadrons

  A. Bacchetta and PM, PRD 62 (2000) 114004, hep-ph/0007120

- Several of these functions are chiral-odd!

FULL FRAGMENTATION SPIN STRUCTURE

Quark fragmentation functions accessible via azimuthal asymmetries

$$
\begin{array}{cccc}
R \rightarrow & \leftarrow R & L \rightarrow & \leftarrow L \\
\left(\begin{array}{cccc}
D_1 + G_{1L} & \frac{|k_T|}{M}\, e^{i\phi}\, G_{1T} & \frac{|k_T|}{M}\, e^{-i\phi}\, H_{1L}^{\perp} & 2\, H_1 \\
\frac{|k_T|}{M}\, e^{-i\phi}\, G_{1T}^{*} & D_1 - G_{1L} & \frac{|k_T|^2}{M^2}\, e^{-2i\phi}\, H_{1T}^{\perp} & -\frac{|k_T|}{M}\, e^{-i\phi}\, H_{1L}^{\perp} \\
\frac{|k_T|}{M}\, e^{i\phi}\, H_{1L}^{\perp *} & \frac{|k_T|^2}{M^2}\, e^{2i\phi}\, H_{1T}^{\perp} & D_1 - G_{1L} & -\frac{|k_T|}{M}\, e^{i\phi}\, G_{1T} \\
2\, H_1 & -\frac{|k_T|}{M}\, e^{i\phi}\, H_{1L}^{\perp *} & -\frac{|k_T|}{M}\, e^{-i\phi}\, G_{1T}^{*} & D_1 + G_{1L}
\end{array}\right)
\end{array}
$$

- The fragmentation functions depend on the ratio of (light-cone) momenta $z = P_h^-/k^-$ and on the transverse momentum of the quark wrt hadron (k_T)
- T-odd functions appear as imaginary parts of off-diagonal entries for fragmentation functions: $G_{1T}+i\,D_{1T}^{\perp}$ and $H_{1L}^{\perp}+i\,H_1^{\perp}$
- T-odd functions appear in single (or triple) spin asymmetries!
- For ρ-production one needs extension to spin 1

A. Bacchetta and P.M., hep-ph/0104176

FRAGMENTATION FUNCTIONS FOR SPIN 0

Quark fragmentation functions into unpolarized or spin 0 hadrons as entries in the quark decay matrix in Dirac space

$$M^{(\mathrm{dec})} = \begin{array}{cc} \begin{pmatrix} D_1 & i\,\frac{|k_T|\,e^{-i\phi}}{M_h}\,H_1^{\perp} \\[2em] -i\,\frac{|k_T|\,e^{+i\phi}}{M_h}\,H_1^{\perp} & D_1 \end{pmatrix} & \begin{array}{c} \mathrm{R} \\[2em] \mathrm{L} \end{array} \\ \begin{array}{cc} \mathrm{R} & \qquad\qquad \mathrm{L} \end{array} & \end{array}$$

- Note for fragmentation into spin 0 hadrons (e.g pions) there are two (!) fragmentation functions, D_1 and $H_1^{\perp}$

J. Collins, NP B396 (1993) 161

- In quark decay time reversal invariance cannot be used $\longrightarrow$ T-odd fragmentation functions, in this case $H_1^{\perp}$
- The function $H_1^{\perp}$ is also chiral-odd!
  It can be used to probe chiral-odd distribution functions

INTEGRATED AND WEIGHTED DISTRIBUTIONS

From the p_T-dependent distributions we recover the p_T-integrated results and obtain p_T-weighted distributions

$$\begin{aligned}
\Phi(x) &= \int dp^-\, d^2\boldsymbol{p}_T\; \Phi(p,P,S) \\
&= \int \frac{d\xi^-}{2\pi}\, e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}(0)\,\mathcal{U}\,\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0} \\
\Rightarrow\quad \Phi_\partial^\alpha(x) &= \int dp^-\, d^2\boldsymbol{p}_T\; p_T^\alpha\, \Phi(p,P,S) \\
&= \int \frac{d\xi^-}{2\pi}\, e^{ip\cdot\xi}\, \langle P,S|\, i\partial^\alpha \left(\overline{\psi}(0)\,\mathcal{U}\,\psi(\xi)\right)|P,S\rangle\Big|_{\xi^+=\xi_T=0}
\end{aligned}$$

$$\Phi(x) = \frac{1}{2}\left\{ f_1\, \not{n}_+ + S_L\, g_1 \gamma_5\, \not{n}_+ + h_1\, \frac{[\,\not{S}_T,\, \not{n}_+]\gamma_5}{2} \right\}$$

$$\begin{aligned}
\frac{1}{M}\,\Phi_\partial^\alpha(x) = \frac{1}{2}\Bigg\{ & g_{1T}^{(1)}\, S_T^\alpha\, \gamma_5\, \not{n}_+ + S_L\, h_{1L}^{\perp(1)}\, \frac{[\not{n}_+,\gamma^\alpha]\gamma_5}{2} \\
& + f_{1T}^{\perp(1)}\, \epsilon_T^{\alpha\beta} S_{T\beta}\, \not{n}_+ + h_1^{\perp(1)}\, \frac{i\,[\not{n}_+,\gamma^\alpha]}{2} \Bigg\}
\end{aligned}$$

T-odd

Transverse moments: $g_{1T}^{(1)} = g_{1T}^{(1)}(x) \equiv \int d^2\boldsymbol{p}_T\, \frac{\boldsymbol{p}_T^2}{2M^2}\, g_{1T}(x,\boldsymbol{p}_T)$

WHERE DO THE FUNCTIONS SHOW UP

- twist t of correlation functions
  $\Rightarrow$ behavior $(1/Q)^{t-2}$
- k_T-dependent functions
  $\Rightarrow$ azimuthal dependence $(\phi_\ell,\ \phi_h,\ \phi_S)$
- cross sections are chirally even

$$\sigma_{000} \Longrightarrow f_1 \otimes D_1$$

$$\langle \frac{Q_T}{M_h} \sin\phi_h^\ell \ \sigma_{L00} \rangle \Longrightarrow \lambda_e \frac{M}{Q} \left(e - \frac{m}{Mx} f_1 \right) \otimes H_1^\perp$$

- $\#$ of spin vectors is even in case of a T-even combination

$$\sigma_{LL0} \Longrightarrow \lambda_e\, S_L\, g_1 \otimes D_1$$

$$\langle \cos(\phi_S^\ell + \phi_{S_h}^\ell)\ \sigma_{0TT} \rangle \Longrightarrow |S_T|\,|S_{hT}|\,h_1 \otimes H_1$$

$$\langle \frac{Q_T}{M} \cos(\phi_S^\ell - \phi_h^\ell)\ \sigma_{LT0} \rangle \Longrightarrow \lambda_e\,|\boldsymbol{S}_T|\,g_{1T}^{(1)} \otimes D_1$$

- $\#$ of spin vectors is odd in case of a T-odd combination (single spin asymmetries)

$$\langle \frac{Q_T}{M_h} \sin(\phi_h^\ell + \sin\phi_S^\ell)\ \sigma_{0T0} \rangle \Longrightarrow |S_T|\,h_1 \otimes H_1^{\perp(1)}$$

QCD DYNAMICS

RELATIONS IMPOSED BY LORENTZ INVARIANCE

Recall the various $\langle\overline{\psi}(0)\psi(\xi)\rangle$ correlators

$$
\begin{aligned}
\Phi(x) &= \frac{1}{2}\left\{f_1\,\not{n}_+ + S_L\,g_1\gamma_5\,\not{n}_+ + h_1\,\frac{[\not{S}_T,\not{n}_+]\gamma_5}{2}\right\}\\
&\quad + \frac{M}{2P^+}\left\{e + g_T\,\gamma_5\,\not{S}_T + S_L\,h_L\,\frac{[\not{n}_+,\not{n}_-]\gamma_5}{2}\right\}\\
&\quad + \frac{M}{2P^+}\left\{f_T\,\epsilon_T^{\rho\sigma}\,S_{T\rho}\gamma_\sigma - i\,S_L\,e_L\,\gamma_5 + h\,\frac{i\,[\not{n}_+,\not{n}_-]}{2}\right\}
\end{aligned}
$$

T-odd

$$
\begin{aligned}
\frac{1}{M}\,\Phi_\partial^\alpha(x) &= \frac{1}{2}\left\{g_{1T}^{(1)}\,S_T^\alpha\,\gamma_5\,\not{n}_+ + S_L\,h_{1L}^{\perp(1)}\,\frac{[\not{n}_+,\gamma^\alpha]\gamma_5}{2}\right.\\
&\quad\left. + f_{1T}^{\perp(1)}\,\epsilon_T^{\alpha\beta}S_{T\beta}\,\not{n}_+ + h_1^{\perp(1)}\,\frac{i\,[\not{n}_+,\gamma^\alpha]}{2}\right\}
\end{aligned}
$$

T-odd

Covariant form for $\langle\overline{\psi}(0)\,\psi(\xi)\rangle \Rightarrow$ Lorentz invariance relations

$$
\underbrace{g_T - g_1}_{g_2} = \frac{d}{dx}\,g_{1T}^{(1)} \qquad\qquad \underbrace{h_L - h_1}_{\frac{1}{2}\,h_2} = -\,\frac{d}{dx}\,h_{1L}^{\perp(1)}
$$

T-odd

$$
f_T = -\,\frac{d}{dx}\,f_{1T}^{\perp(1)} \qquad\qquad h = -\,\frac{d}{dx}\,h_1^{\perp(1)}
$$

A.P. Bukhvostov, E.A. Kuraev, L.N. Lipatov; Tangerman & M

LIGHTCONE CORRELATORS

$$\Phi(x) = \int dp^-\, d^2\boldsymbol{p}_T\ \Phi(p,P,S)$$

$$= \int \frac{d\xi^-}{2\pi}\ e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}(0)\,\mathcal{U}\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

$$\Rightarrow \quad \Phi_\partial^\alpha(x) = \int dp^-\, d^2\boldsymbol{p}_T\ p_T^\alpha\,\Phi(p,P,S)$$

$$= \int \frac{d\xi^-}{2\pi}\ e^{ip\cdot\xi}\,\langle P,S|i\partial^\alpha\left(\overline{\psi}(0)\,\mathcal{U}\psi(\xi)\right)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

$$\Phi_D^\alpha(x,y) = \int dp^-\, d^2\boldsymbol{p}_T\, dp_1^-\, d^2p_{1T}\,\Phi_D(p,p_1,P,S)$$

$$= \int \frac{d\xi^-}{2\pi}\,\frac{d\eta^-}{2\pi}\ e^{i\,p_1\cdot(\xi-\eta)+ip\cdot\eta}$$

$$\times\ \langle P,S|\overline{\psi}(0)\,\mathcal{U}\,iD^\alpha(\eta)\,\mathcal{U}\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=\eta^+=\eta_T=0}$$

$$\Phi_D^\alpha(x) = \int dp^-\, d^2\boldsymbol{p}_T\, d^4p_1\,\Phi_D(p,p_1,P,S)$$

$$= \int \frac{d\xi^-}{2\pi}\ e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}(0)\,\mathcal{U}\,iD^\alpha\,\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

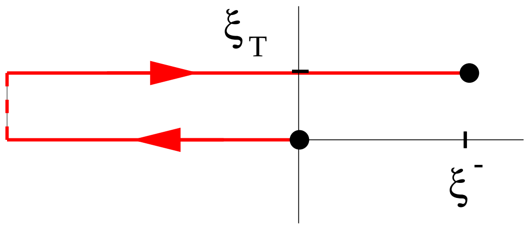
Similarly $\Phi_A^\alpha(x,y)$, $\Phi_F^{+\alpha}(x,y)$, etc.

Ellis, Furmanski, Petronzio; Jaffe, Soldate
Belitsky, Braun, . . .

Color gauge invariance and twist analysis

Including longitudinal gluons and transverse momenta we get:

$$\Phi(x,\boldsymbol{p}_T) = \int \frac{d\xi^-\,d^2\boldsymbol{\xi}_T}{2\,(2\pi)^3}\; e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}(0)\,\mathcal{U}(0,-\infty)\,\mathcal{U}(-\infty,\xi^-)\,\psi(\xi)|P,S\rangle\Big|_{\xi^+=0}$$



$$\mathcal{U}(x,y) = \mathcal{P}\,e^{-ig\int_x^y d\eta^-\,A^+(\eta)}\Big|_{x_T=y_T=\eta_T}$$

Integrated results:

$$\Phi(x) = \int \frac{d\xi^-}{4\pi}\; e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}(0)\,\mathcal{U}\,\psi(\xi)|P,S\rangle\Big|_{LC}$$

$$\begin{aligned}\Phi_\partial^\alpha(x) &= \int \frac{d\xi^-}{4\pi}\; e^{ip\cdot\xi}\Big\{\langle P,S|\overline{\psi}(0)\,\mathcal{U}\,iD_T^\alpha\psi(\xi)|P,S\rangle\Big|_{LC} \\ &\qquad -\langle P,S|\overline{\psi}(0)\,\mathcal{U}\int_{-\infty}^{\xi^-} d\eta^-\,\mathcal{U}\,g\,F^{+\alpha}(\eta)\,U\,\psi(\xi)|P,S\rangle\Big|_{LC}\Big\} \\ &= \Phi_D^\alpha(x) - \Phi_A^\alpha(x)\end{aligned}$$

In $A^+ = 0$ gauge: $F^{+\alpha} = \partial^+ A_T^\alpha \Longrightarrow A_T^\alpha(\xi) = \int_{-\infty}^{\xi^-} d\eta^-\,F^{+\alpha}(\eta)$

EQUATIONS OF MOTION

$(i\,\not{D} - m)\psi = 0 \qquad \Longrightarrow$

$$
\begin{aligned}
\frac{1}{M}\,\Phi_D^\alpha(x) \;=\; \frac{1}{2}\Bigg\{ & \left(x g_T - \frac{m}{M}\,h_1\right) S_T^\alpha\,\gamma_5\,\not{n}_+ \\
& + S_L\left(x h_L - \frac{m}{M}\,g_1\right)\frac{[\not{n}_+,\gamma^\alpha]\gamma_5}{2} \\
& - x f_T\;\epsilon_T^{\alpha\beta} S_{T\beta}\,\not{n}_+ \\
& - x h\;\frac{i\,[\not{n}_+,\gamma^\alpha]}{4}\Bigg\}
\end{aligned}
$$

T-odd

T-odd

Interaction dependent pieces: $\Phi_A \equiv \Phi_D - \Phi_\partial$

$$
\begin{aligned}
\frac{1}{M}\,\Phi_A^\alpha(x) \;=\; \frac{1}{2}\Bigg\{ & \underbrace{\left(x g_T - g_{1T}^{(1)} - \frac{m}{M}\,h_1\right)}_{x\tilde{g}_T} S_T^\alpha\,\gamma_5\,\not{n}_+ \\
& + S_L\left(x h_L - h_{1L}^{\perp(1)} - \frac{m}{M}\,g_1\right)\frac{[\not{n}_+,\gamma^\alpha]\gamma_5}{2} \\
& - \left(x f_T + f_{1T}^{\perp(1)}\right)\epsilon_T^{\alpha\beta} S_{T\beta}\,\not{n}_+ \\
& - \left(x h + 2h_1^{\perp(1)}\right)\frac{i\,[\not{n}_+,\gamma^\alpha]}{4}\Bigg\}
\end{aligned}
$$

T-odd

T-odd

ISOLATING TWIST-3 AND TRANSVERSE MOMENTS

$$g_T(x) = \int_x^1 dy\, \frac{g_1(y)}{y} + \left[\tilde{g}_T(x) - \int_x^1 dy\, \frac{\tilde{g}_T(y)}{y}\right]$$

$$\frac{g_{1T}^{(1)}(x)}{x} = \int_x^1 dy\, \frac{g_1(y)}{y} - \int_x^1 dy\, \frac{\tilde{g}_T(y)}{y}$$

$$h_L(x) = 2x \int_x^1 dy\, \frac{h_1(y)}{y^2} + \left[\tilde{h}_L(x) - 2x \int_x^1 dy\, \frac{\tilde{h}_L(y)}{y^2}\right]$$

$$\frac{h_{1L}^{\perp(1)}(x)}{x^2} = -\int_x^1 dy\, \frac{h_1(y)}{y^2} + \int_x^1 dy\, \frac{\tilde{h}_L(y)}{y^2}$$

$$f_T(x) = \left[\tilde{f}_T(x) - \int_x^1 dy\, \frac{\tilde{f}_T(y)}{y}\right]$$

$$\frac{f_{1T}^{\perp(1)}(x)}{x} = \int_x^1 dy\, \frac{\tilde{f}_T(y)}{y}$$

$$h(x) = \left[\tilde{h}(x) - 2x \int_x^1 dy\, \frac{\tilde{h}(y)}{y^2}\right]$$

$$\frac{h_1^{\perp(1)}(x)}{x^2} = \int_x^1 dy\, \frac{\tilde{h}(y)}{y^2}$$

$$\frac{G_T(z)}{z} = -\int_z^1 dy\, \frac{G_1(y)}{y^2} + \left[\frac{\tilde{G}_T(z)}{z} + \int_z^1 dy\, \frac{\tilde{G}_T(y)}{y^2}\right]$$

$$G_{1T}^{(1)}(z) = -\int_z^1 dy\, \frac{G_1(y)}{y^2} + \int_z^1 dy\, \frac{\tilde{G}_T(y)}{y^2}$$

$$H_L(z) = -2\int_z^1 dy\, \frac{H_1(y)}{y} + \left[\tilde{H}_L(z) + 2\int_z^1 dy\, \frac{\tilde{H}_L(y)}{y}\right]$$

$$z\, H_{1L}^{\perp(1)}(z) = \int_z^1 dy\, \frac{H_1(y)}{y} - \int_z^1 dy\, \frac{\tilde{H}_L(y)}{y}$$

$$\frac{D_T(z)}{z} = \left[\frac{\tilde{D}_T(z)}{z} + \int_z^1 dy\, \frac{\tilde{D}_T(y)}{y^2}\right]$$

$$D_{1T}^{\perp(1)}(z) = -\int_z^1 dy\, \frac{\tilde{D}_T(y)}{y^2}$$

$$H(z) = \left[\tilde{H}(z) + 2\int_z^1 dy\, \frac{\tilde{H}(y)}{y}\right]$$

$$z\, H_1^{\perp(1)}(z) = -\int_z^1 dy\, \frac{\tilde{H}(y)}{y}$$

EVOLUTION OF DISTRIBUTION FUNCTIONS

$$\frac{d}{d\tau}\, f(x,\tau) = \frac{\alpha_s(\tau)}{2\pi} \int_x^1 \frac{dy}{y}\, P^{[f]}\left(\frac{x}{y}\right) f(y,\tau),$$

$$\frac{d}{d\tau}\, [f]_n(\tau) = \frac{\alpha_s(\tau)}{2\pi}\, A_n^{[f]}\, [f]_n(\tau)$$

Twist-2 functions

$$P^{[f_1]}(\beta) = C_F \left[\frac{3}{2}\,\delta(1-\beta) + \frac{1+\beta^2}{(1-\beta)_+}\right]$$

$$P^{[g_1]}(\beta) = C_F \left[\frac{3}{2}\,\delta(1-\beta) + \frac{1+\beta^2}{(1-\beta)_+}\right]$$

$$P^{[h_1]}(\beta) = C_F \left[\frac{3}{2}\,\delta(1-\beta) + \frac{2\beta}{(1-\beta)_+}\right]$$

Interaction-dependent functions (for large N_c)

$$P^{[\tilde{g}_T]}(\beta) = P^{[\tilde{f}_T]}(\beta) = \frac{N_c}{2}\left[\frac{1}{2}\,\delta(1-\beta) + \frac{1+\beta}{(1-\beta)_+}\right]$$

$$P^{[\tilde{h}_L]}(\beta) = P^{[\tilde{h}]}(\beta) = \frac{N_c}{2}\left[\frac{1}{2}\,\delta(1-\beta) + \frac{3\beta-1}{(1-\beta)_+}\right]$$

$$P^{[\tilde{e}]}(\beta) = P^{[\tilde{e}_L]}(\beta) = \frac{N_c}{2}\left[\frac{1}{2}\,\delta(1-\beta) + \frac{3-\beta}{(1-\beta)_+}\right]$$

Braun, Balitsky, Koike, Tanaka, Ji, Osborne, ...

Large N_c

From the twist-2 and interaction-dependent functions one obtains the evolution of the transverse moments

Azimuthal asymmetries (large N_c, omitting quark mass-terms)

$$\frac{d}{d\tau}\, g_{1T}^{(1)}(x,\tau) = \frac{\alpha_s(\tau)}{4\pi}\, N_c \int_x^1 dy \left\{ \left[\frac{1}{2}\,\delta(y-x) + \frac{x^2+xy}{y^2(y-x)_+} \right] g_{1T}^{(1)}(y,\tau) + \frac{x^2}{y^2}\, g_1(y,\tau) \right\}$$

$$\frac{d}{d\tau}\, h_{1L}^{\perp(1)}(x,\tau) = \frac{\alpha_s(\tau)}{4\pi}\, N_c \int_x^1 dy \left\{ \left[\frac{1}{2}\,\delta(y-x) + \frac{3x^2-xy}{(y-x)_+} \right] h_{1L}^{\perp(1)}(y,\tau) - \frac{x}{y}\, h_1(y,\tau) \right\}$$

Evolution of transverse moments is also driven by the twist-2 polarized quark distribution functions g_1 and h_1.

EVOLUTION OF FRAGMENTATION FUNCTIONS

We note a generalized reciprocity valid for twist-2 *and* interaction-dependent functions:

$$P^{[f]}(\beta) = \frac{\mathcal{N}(\beta)}{(1-\beta)_+}$$

$$P^{[zD]}(\beta) = \frac{\beta^2\,\mathcal{N}(1/\beta)}{(1-\beta)_+}$$

$$A_n^{[f]} = \mathcal{A}(n) - 2\sum_{j=1}^{n}\frac{1}{j}$$

$$A_{n+1}^{[D]} = \mathcal{A}\left(-(n+1)\right) - 2\sum_{j=1}^{n}\frac{1}{j}.$$

$$P^{[f_1]}(\beta) = C_F\left[\frac{1+\beta^2}{(1-\beta)_+}\right]$$

$$P^{[g_1]}(\beta) = C_F\left[\frac{1+\beta^2}{(1-\beta)_+}\right]$$

$$P^{[h_1]}(\beta) = C_F\left[\frac{2\beta}{(1-\beta)_+}\right]$$

$$P^{[zD_1]}(\beta) = C_F\left[\frac{1+\beta^2}{(1-\beta)_+}\right]$$

$$P^{[zG_1]}(\beta) = C_F\left[\frac{1+\beta^2}{(1-\beta)_+}\right]$$

$$P^{[zH_1]}(\beta) = C_F\left[\frac{2\beta}{(1-\beta)_+}\right]$$

$$P^{[\tilde{g}_T]}(\beta) = \frac{N_c}{2}\left[\frac{1+\beta}{(1-\beta)_+}\right]$$

$$P^{[\tilde{h}_L]}(\beta) = \frac{N_c}{2}\left[\frac{3\beta-1}{(1-\beta)_+}\right]$$

$$P^{[\tilde{e}]}(\beta) = \frac{N_c}{2}\left[\frac{3-\beta}{(1-\beta)_+}\right]$$

$$P^{[z\tilde{G}_T]}(\beta) = \frac{N_c}{2}\left[\frac{1+\beta}{(1-\beta)_+}\right]$$

$$P^{[z\tilde{H}_L]}(\beta) = \frac{N_c}{2}\left[\frac{3\beta-1}{(1-\beta)_+}\right]$$

$$P^{[z\tilde{E}]}(\beta) = \frac{N_c}{2}\left[\frac{3-\beta}{(1-\beta)_+}\right]$$

Large N_c

Like for the distribution functions one has evolution equations for the transverse moments with driving from G_1 and H_1:

$$\begin{aligned}
\frac{d}{d\tau}\, zG_{1T}^{(1)}(z,\tau) &= \frac{\alpha_s(\tau)}{4\pi}\, N_c \int_z^1 dy \left\{ \left[\frac{1}{2}\,\delta(y-z) + \frac{y+z}{y(y-z)_+}\right] yG_{1T}^{(1)}(y,\tau) \right. \\
&\qquad \left. - \frac{y^2+z^2}{2y^2 z}\, G_1(y,\tau) \right\} \\
\frac{d}{d\tau}\, zH_{1L}^{\perp(1)}(z,\tau) &= \frac{\alpha_s(\tau)}{4\pi}\, N_c \int_z^1 dy \left\{ \left[\frac{1}{2}\,\delta(y-z) + \frac{3y-z}{y(y-z)_+}\right] yH_{1L}^{\perp(1)}(y,\tau) \right. \\
&\qquad \left. + \frac{1+\ln(z/y)}{y}\, H_1(y,\tau) \right\}
\end{aligned}$$

Evolution equation for Collins function obtained as imaginary part of the equation for $H_{1L}^{\perp}$

$$\boxed{\frac{d}{d\tau}\, zH_1^{\perp(1)}(z,\tau) = \frac{\alpha_s}{4\pi}\, N_c \int_z^1 dy \left[\frac{1}{2}\,\delta(y-z) + \frac{3y-z}{y(y-z)_+}\right] yH_1^{\perp(1)}(y,\tau)}$$

- No driving term in the large N_c evolution
- No mixing with gluons

SUMMARY

- p_T-dependent distribution and fragmentation functions are needed to understand azimuthal asymmetries
- Operator structure of lowest transverse moments of the p_T-dependent functions involves (known) twist-2 and twist-3 operators
- This also enables us to find their evolution, which in the limit $N_c \to \infty$ can be simply written down

A. Henneman, D. Boer and PM, hep-ph/0104nnn