APPENDICES

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- Topic 1
- Topic 2
- Topic 3

Leading functions: g_1, $g_{1T}^{(1)}$

Subleading function

$$\left.\begin{aligned} g_T &= \frac{g_{1T}^{(1)}}{x} + \tilde{g}_T \\ &= g_1 + \frac{d}{dx}\, g_{1T}^{(1)} \end{aligned}\right\} \Rightarrow g_T = \int_x^1 dy\, \frac{g_1(y)}{y} + \underbrace{\left(\tilde{g}_T - \int_x^1 dy\, \frac{\tilde{g}_T(y)}{y}\right)}_{\bar{g}_T}$$

Approximation 1: $g_{1T}^{(1)}(x) \approx 0$

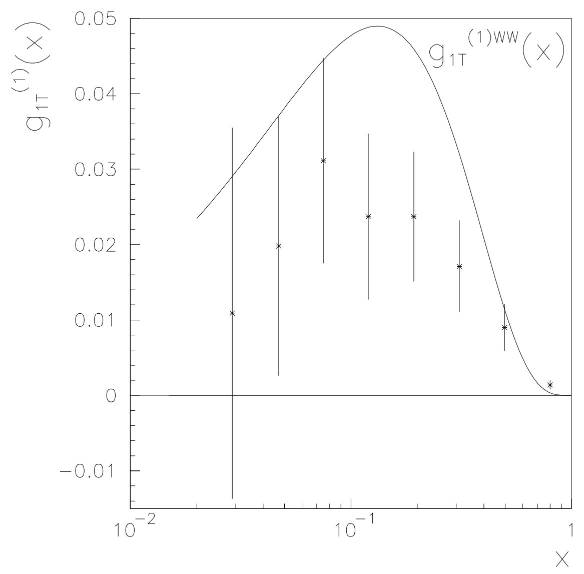
$$\begin{aligned} g_T(x) &= g_1(x) \\ \bar{g}_T(x) &= g_1(x) - \int_x^1 dy\, \frac{g_1(y)}{y} \end{aligned}$$

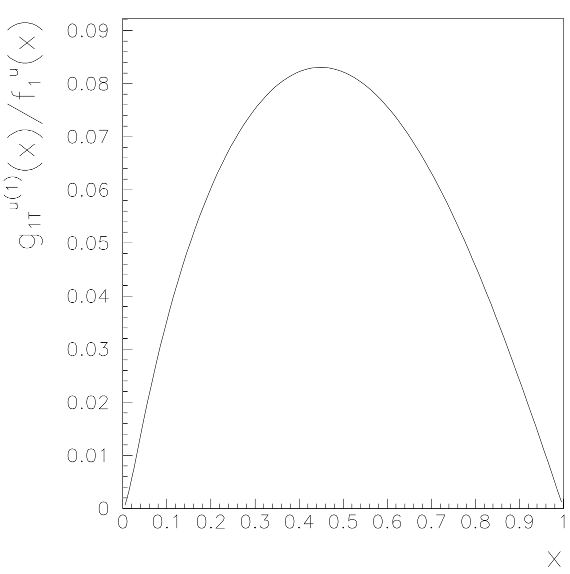
Approximation 2: $\bar{g}_T(x) \approx 0$

$$\begin{aligned} g_T(x) &= \int_x^1 dy\, \frac{g_1(y)}{y} \\ g_{1T}^{(1)}(x) &= x \int_x^1 dy\, \frac{g_1(y)}{y} = x\, g_T \end{aligned}$$

Estimate of $g_{1T}^{(1)}$ A. Kotzinian & PJM, PRD 54 (1996) 1229

- datapoints: SLAC g_2-data: $g_{1T}^{(1)}(x) = -\int_x^1 dy\, g_2(y)$
- line: above relation with $g_2(x) = g_2^{WW}(x)$





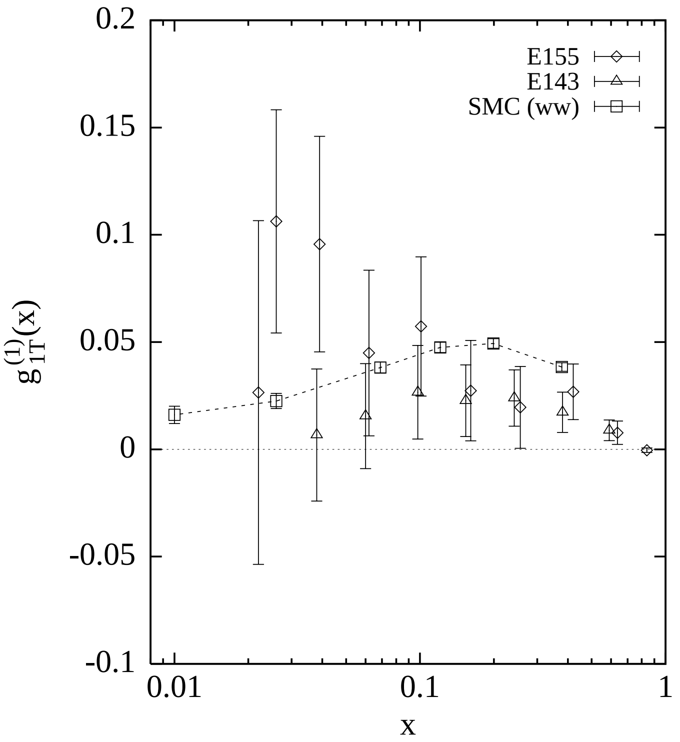
Resulting asymmetry in $\vec{e}\vec{p} \to e\pi^+ X$:

$$A_{LT} \propto \frac{g_{1T}^{(1)u}(x)}{f_1^u(x)}$$

Updated stimate of $g_{1T}^{(1)}$

M. Boglione & PJM

- datapoints: SLAC g_2-data: $g_{1T}^{(1)}(x) = -\int_x^1 dy\, g_2(y)$ (including E155, preliminary)
- line: above relation with $g_2(x) = g_2^{WW}(x)$



Leading functions: h_1, $h_{1L}^{(1)}$, $h_{1T}^{(2)}$

Subleading function

$$\left.\begin{aligned} h_L &= -2\,\frac{h_{1L}^{\perp(1)}}{x} + \tilde{h}_L \\ &= h_1 - \frac{d}{dx}\,h_{1L}^{\perp(1)} \end{aligned}\right\} \Rightarrow \quad \begin{aligned} h_L &= 2x\int_x^1 dy\,\frac{h_1(y)}{y^2} \\ &\quad + \underbrace{\left(\tilde{h}_L - 2x\int_x^1 dy\,\frac{\tilde{h}_L(y)}{y^2}\right)}_{\bar{h}_L} \end{aligned}$$

Approximation 1: $h_{1L}^{\perp(1)}(x) \approx 0$

$$\begin{aligned} h_L(x) &= h_1(x) \\ \bar{h}_L(x) &= h_1(x) - 2x\int_x^1 dy\,\frac{h_1(y)}{y^2} \end{aligned}$$

Approximation 2: $\bar{h}_L(x) \approx 0$

$$\begin{aligned} h_L(x) &= 2x\int_x^1 dy\,\frac{h_1(y)}{y^2} \\ h_{1L}^{\perp(1)}(x) &= -x^2\int_x^1 dy\,\frac{h_1(y)}{y^2} = -\frac{1}{2}\,x\,h_L \end{aligned}$$

Leading functions: $H_1^{\perp(1)}$

Subleading function

$$\begin{aligned} H(z) &= -2z\, H_1^{\perp(1)}(z) + \tilde{H}(z) \\ &= z^3\, \frac{d}{dz}\left(\frac{H_1^{\perp(1)}}{z}\right) \end{aligned}$$

Approximation: $H_1^{\perp(1)}(z) \approx 0$

$$\begin{aligned} \frac{H(z)}{z} &= z^2\, \frac{d}{dz}\left(\frac{H_1^{\perp(1)}}{z}\right) \approx 0 \\ \frac{\tilde{H}(z)}{z} &= \frac{d}{dz}\left(z\, H_1^{\perp(1)}\right) \approx 0 \end{aligned}$$

Note: The approximation $\tilde{H}(z) \approx 0$ leads to inconsistencies

BOUNDS ON k_T DEPENDENT FUNCTIONS

$$
\begin{pmatrix}
f_1 + g_{1L} & \frac{|p_T|}{M}\, e^{i\phi}\, g_{1T} & \frac{|p_T|}{M}\, e^{-i\phi}\, h_{1L}^\perp & 2\, h_1 \\
\frac{|p_T|}{M}\, e^{-i\phi}\, g_{1T} & f_1 - g_{1L} & \frac{|p_T|^2}{M^2}\, e^{-2i\phi}\, h_{1T}^\perp & -\frac{|p_T|}{M}\, e^{-i\phi}\, h_{1L}^\perp \\
\frac{|p_T|}{M}\, e^{i\phi}\, h_{1L}^\perp & \frac{|p_T|^2}{M^2}\, e^{2i\phi}\, h_{1T}^\perp & f_1 - g_{1L} & -\frac{|p_T|}{M}\, e^{i\phi}\, g_{1T} \\
2\, h_1 & -\frac{|p_T|}{M}\, e^{i\phi}\, h_{1L}^\perp & -\frac{|p_T|}{M}\, e^{-i\phi}\, g_{1T} & f_1 + g_{1L}
\end{pmatrix}
$$

Bounds from 1-dimensional subspaces:

$$f_1 \geq 0$$
$$|g_{1L}| \leq f_1$$

Bounds from 2-dimensional subspaces:

$$|h_1| \leq \frac{1}{2}\,(f_1 + g_{1L}) \leq f_1$$
$$|h_{1T}^{\perp(1)}| \leq \frac{1}{2}\,(f_1 - g_{1L}) \leq f_1$$
$$|g_{1T}^{(1)}| \leq \frac{|\boldsymbol{p}_T|}{2M}\sqrt{(f_1 + g_{1L})\,(f_1 - g_{1L})} \leq \frac{|\boldsymbol{p}_T|}{2M}\, f_1$$
$$|h_{1L}^{\perp(1)}| \leq \frac{|\boldsymbol{p}_T|}{2M}\sqrt{(f_1 + g_{1L})\,(f_1 - g_{1L})} \leq \frac{|\boldsymbol{p}_T|}{2M}\, f_1$$

Note: including T-odd functions: $g_{1T} \rightarrow g_{1T} + i\, f_{1T}^\perp$ and $h_{1L}^\perp \rightarrow h_{1L}^\perp + i\, h_1^\perp$

SHARPENING THE BOUNDS

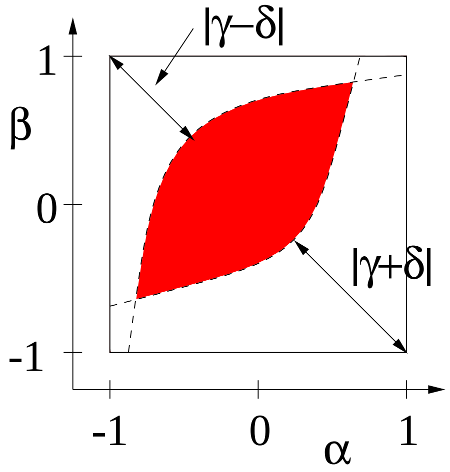
Use $f_1 = A + B$ and $g_1 = A - B$ and bounds to define

$$\begin{aligned}
h_1 &= \alpha\, A \\
h_{1T}^{\perp(1)} &= \beta\, B \\
g_{1T}^{(1)} + i\, f_{1T}^{\perp(1)} &= \gamma\, \frac{|p_T|}{M}\sqrt{AB} \\
h_{1L}^{\perp(1)} + i\, h_1^{\perp(1)} &= \delta\, \frac{|p_T|}{M}\sqrt{AB}
\end{aligned}$$

Positivity of eigenvalues:

$$\begin{aligned}
&A + B \geq 0 \\
&|\alpha\, A - \beta\, B| \leq A + B \qquad \text{i.e. } |h_{1T}| \leq f_1 \\
&|\gamma + \delta|^2 \leq (1-\alpha)(1+\beta) \\
&|\gamma - \delta|^2 \leq (1+\alpha)(1-\beta)
\end{aligned}$$

Red is allowed region for α and β



(Leading) soft part

$$
\begin{aligned}
(\Phi\gamma^+)_{ij,m'm}(x) \quad &\sim \quad \int \frac{d\xi^-}{2\pi}\, e^{ip\cdot\xi}\, \langle P, m'|\psi^\dagger_{+j}(0)\psi_{+i}(\xi)|P,m\rangle\Big|_{\xi^+=\xi_T=0} \\
&= \quad \sum_n \langle P_n|\psi_{+j}(0)|P,m'\rangle^* \langle P_n|\psi_{+i}(0)|P,m\rangle \\
&\qquad\qquad\qquad\qquad \times\ \delta\left(P_n^+ - (1-x)P^+\right)
\end{aligned}
$$

where

$$
\psi_+ \equiv \underbrace{\frac{1}{2}\gamma^-\gamma^+}_{\mathcal{P}_+}\,\psi \qquad \text{(good component)}
$$

They are in essence momentum space densities or (for fragmentation) decay probabilities

In particular, if $im = jm'$:

$$
(\Phi\gamma^+)_{ii,mm}(x) \geq 0
$$

This is the basis for positivity constraints

(LEADING) QUARK SPIN STRUCTURE

Dirac structure in an unpolarized or spin 0 hadron target:

$$\Phi(x)\gamma^+ = f_1(x)\mathcal{P}_+$$

Explicitly: Take chiral (Weyl) representation for $M_{ij}^{(\mathrm{prod})} = (\Phi\gamma^+)_{ij}^T$
This leaves only two basis states (right/left-handed partons)

$$M_{ij}^{(\mathrm{prod})} = \begin{array}{cc} \begin{pmatrix} f_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & f_1 \end{pmatrix} & \begin{array}{c} \mathrm{R} \\ \\ \\ \mathrm{L} \end{array} \\ \begin{array}{cccc} \mathrm{R} & \quad & \quad & \mathrm{L} \end{array} & \end{array}$$

(LEADING) QUARK SPIN STRUCTURE

Dirac structure in an unpolarized or spin 0 hadron target:

$$\Phi(x)\gamma^{+} = f_1(x)\mathcal{P}_{+}$$

Explicitly: Take chiral (Weyl) representation for $M_{ij}^{(\mathrm{prod})} = (\Phi\gamma^{+})^{T}_{ij}$
This leaves only two basis states (right/left-handed partons)

$$M_{ij}^{(\mathrm{prod})} \quad = \quad \begin{array}{cl} \begin{pmatrix} f_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & f_1 \end{pmatrix} & \begin{array}{l} \mathrm{R} \\ \\ \\ \mathrm{L} \end{array} \\ \begin{array}{cccc} \mathrm{R} & \quad & \quad & \mathrm{L} \end{array} & \end{array}$$

This is generally true for 'leading' structure'. E.g. Dirac structure for fragmentation into spin 0 hadron (includes T-odd structure):

$$\Delta(z,\boldsymbol{k}_T)\,\gamma^{-} = \left\{ D_1(z,\boldsymbol{P}_{h\perp}) + i\,H_1^{\perp}(z,\boldsymbol{P}_{h\perp})\,\frac{\not{k}_T}{M_h} \right\} \mathcal{P}_{-}$$

Explicitly: Take chiral (Weyl) representation for $M_{ij}^{(\mathrm{dec})} = (\Phi\gamma^{-})^{T}_{ij}$

$$M_{ij}^{(\mathrm{dec})} \quad = \quad \begin{array}{cl} \begin{pmatrix} D_1 & 0 & 0 & i\frac{|k_T|}{M_h}\,H_1^{\perp} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -i\frac{|k_T|}{M_h}\,H_1^{\perp} & 0 & 0 & D_1 \end{pmatrix} & \begin{array}{l} \mathrm{R} \\ \\ \\ \mathrm{L} \end{array} \\ \begin{array}{cccc} \mathrm{R} & \quad & \quad & \mathrm{L} \end{array} & \end{array}$$

TARGET SPIN DEPENDENCE (SPIN 1/2)

Spin vector: $$S^\mu \equiv S_L \frac{P^+}{M}\, n_+^\mu + S_T^\mu + \mathcal{O}\left(\frac{1}{P^+}\right)$$

In rest frame $S = (0, \boldsymbol{S}_T, S_L)$ and $S_L^2 + \boldsymbol{S}_T^2 \le 1$

Spin vector specifies a particular (in general mixed) initial state via density matrix $\rho(S) = \frac{1}{2}(1 + \boldsymbol{S}\cdot\boldsymbol{\sigma})$

$$\underset{\uparrow\atop\text{spin vector}}{M(S)} = \text{Tr}\left[\underset{\uparrow\atop\text{density matrix}}{\rho(S)}\,\underset{\downarrow\atop\text{Matrix in spin space}}{\tilde{M}}\right]$$

$$M(S) = M_0 + S_L\, M_L + S_T^1 M_T^1 + S_T^2 M_T^2$$

$$\updownarrow$$

$$\tilde{M}_{mm'} = \begin{pmatrix} M_0 + M_L & M_T^1 - i\, M_T^2 \\ M_T^1 + i\, M_T^2 & M_0 - M_L \end{pmatrix}$$

QUARK SPIN STRUCTURE

Dirac structure in a polarized (spin 1/2) hadron:

$$\Phi(x)\gamma^+ = \left\{ f_1(x) + S_L\, g_1(x)\,\gamma_5 + h_1(x)\,\gamma_5\ \not{S}_T \right\} \mathcal{P}_+$$

Explicitly: Take chiral (Weyl) representation for $M_{ij}^{(\mathrm{prod})}(S) = (\Phi\gamma^+)_{ij}^T$

$$M_{ij}(S) \quad = \quad \begin{array}{cc} \left(\begin{array}{cccc} f_1 + S_L\, g_1 & 0 & 0 & (S_T^1 + i\, S_T^2)\, h_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (S_T^1 - i\, S_T^2)\, h_1 & 0 & 0 & f_1 - S_L\, g_1 \end{array} \right) & \begin{array}{c} \mathrm{R} \\ \\ \\ \mathrm{L} \end{array} \\ \begin{array}{cccc} \mathrm{R} & \qquad & \qquad & \mathrm{L} \end{array} & \end{array}$$

As before, only two relevant basis states (right/left-handed partons)

QUARK & TARGET SPIN STRUCTURE

Combine into explicit spin structure on basis $+R$, $-R$, $+L$ and $-L$

$$
\tilde{M} \quad = \quad \begin{array}{cc}
\begin{pmatrix}
f_1 + g_1 & 0 & 0 & 2\,h_1 \\
0 & f_1 - g_1 & 0 & 0 \\
0 & 0 & f_1 - g_1 & 0 \\
2\,h_1 & 0 & 0 & f_1 + g_1
\end{pmatrix} & \begin{array}{c} R\rightarrow \\ \leftarrow R \\ L\rightarrow \\ \leftarrow L \end{array} \\
\begin{array}{cccc} R\rightarrow & \leftarrow R & L\rightarrow & \leftarrow L \end{array} &
\end{array}
$$

Any matrix element $\langle \alpha | \tilde{M} | \alpha \rangle \geq 0$

Diagonal elements $\Rightarrow$ trivial bounds

$$
\begin{gathered}
f_1(x) \geq 0 \\
|g_1(x)| \leq f_1(x)
\end{gathered}
$$

Eigenvalues $\longrightarrow$ less trivial (Soffer) bound

$$
|h_1(x)| \leq \frac{1}{2}\left(f_1(x) + g_1(x)\right)
$$

These are valid for quarks and antiquarks of every flavor

For semi-inclusive leptoproduction or Drell-Yan scattering one needs to consider transverse momentum dependence

Dirac structure for a polarized (spin 1/2) hadron:

$$\Phi_0(x,\boldsymbol{p}_T)\,\gamma^+ =$$
$$\left\{ f_1 x,\boldsymbol{p}_T) + i\,h_1^\perp(x,\boldsymbol{p}_T)\,\frac{\not{p}_T}{M}\right\}\mathcal{P}_+$$

$$\Phi_L(x,\boldsymbol{p}_T)\,\gamma^+ =$$
$$\left\{ S_L\,g_{1L}(x,\boldsymbol{p}_T)\,\gamma_5 + S_L\,h_{1L}^\perp(x,\boldsymbol{p}_T)\gamma_5\,\frac{\not{p}_T}{M}\right\}\mathcal{P}_+$$

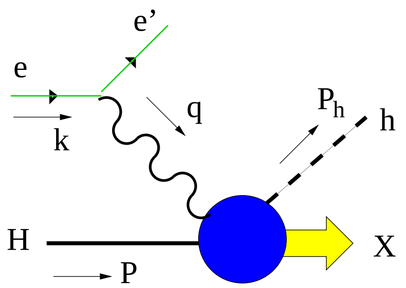
$$\Phi_T(x,\boldsymbol{p}_T)\,\gamma^+ =$$
$$\left\{ f_{1T}^\perp(x,\boldsymbol{p}_T)\,\frac{\epsilon_{T\,\rho\sigma}p_T^\rho S_T^\sigma}{M} + g_{1T}(x,\boldsymbol{p}_T)\,\frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M}\,\gamma_5 \right.$$
$$\left. + h_{1T}(x,\boldsymbol{p}_T)\,\gamma_5\,\not{S}_T + h_{1T}^\perp(x,\boldsymbol{p}_T)\,\frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M}\,\frac{\gamma_5\,\not{p}_T}{M}\right\}\mathcal{P}_+$$

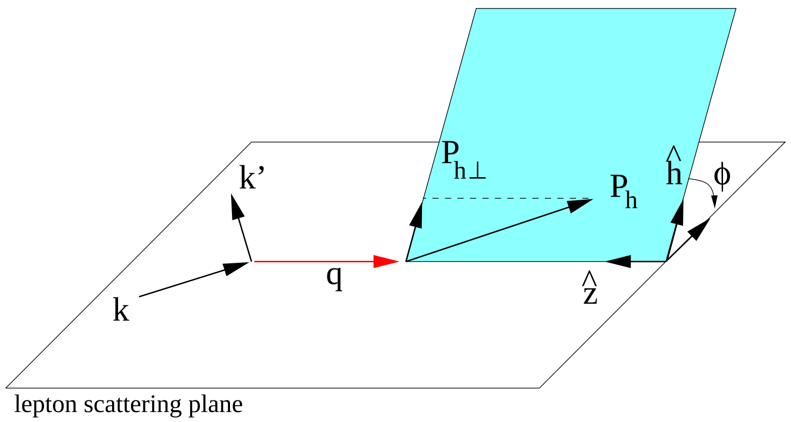
- The functions $h_1^\perp$ and $f_{1T}^\perp$ are T-odd
- For fragmentation: $f \longrightarrow D$, $g \longrightarrow G$, $h \longrightarrow H$ or hat-notation

1-PARTICLE INCLUSIVE LEPTOPRODUCTION

$$\ell H \longrightarrow \ell' h X$$

$$-q^2 \equiv Q^2 \rightarrow \infty$$





$$x_B = \frac{Q^2}{2P \cdot q}$$

$$y = \frac{P \cdot q}{P \cdot k}$$

$$z_h = \frac{P \cdot P_h}{P \cdot q}$$

$$Q_T = \frac{|P_{h\perp}|}{z_h}$$

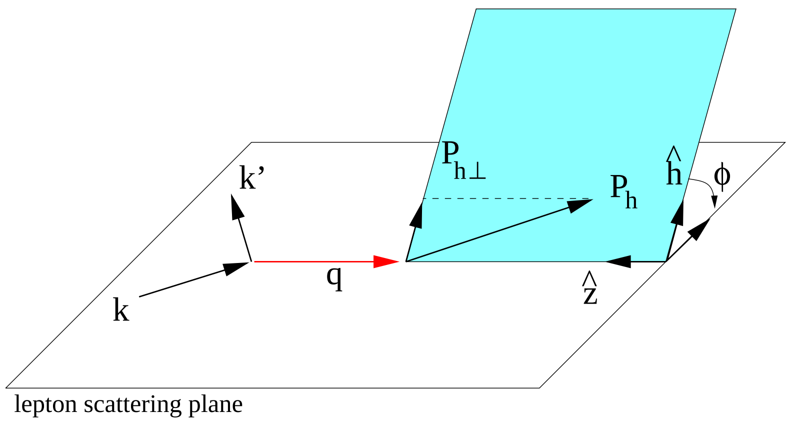
Hadronic tensor

$$2M\mathcal{W}^{(\ell H)}_{\mu\nu}(q; PS; P_h S_h)$$

$$= \frac{1}{(2\pi)^4} \int \frac{d^3 P_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta^4(q + P - P_X - P_h)$$

$$\times \langle PS | J_\mu(0) | P_X; P_h S_h \rangle \langle P_X; P_h S_h | J_\nu(0) | PS \rangle$$

CROSS SECTIONS (ℓH)



$$\ell H \longrightarrow \ell' h X$$

$$x_B = \frac{Q^2}{2P \cdot q}$$

$$y = \frac{P \cdot q}{P \cdot k}$$

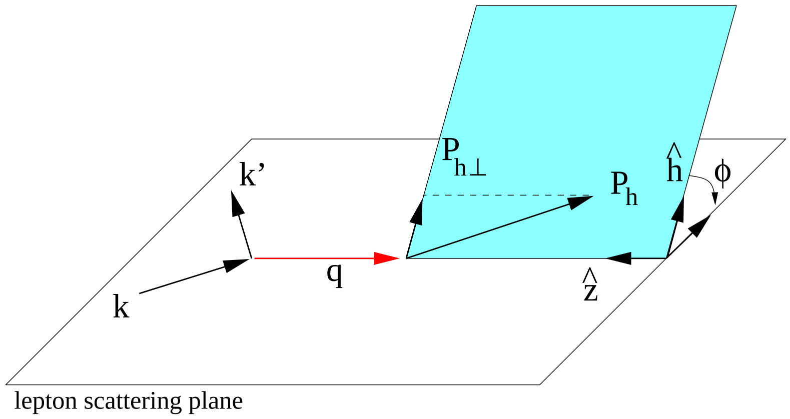
$$z_h = \frac{P \cdot P_h}{P \cdot q}$$

$$Q_T = \frac{|P_{h\perp}|}{z_h}$$

$|p_T|$-averaged cross sections (leading in $1/Q$ and $\alpha_s(Q^2)$)

$$\frac{d\sigma_{OO}}{dx_B\, dy\, dz_h} = \frac{2\pi\alpha^2\, s}{Q^4}\, x_B \left(1 + (1-y)^2\right) \sum_{a,\bar{a}} e_a^2 f_1^a(x_B)\, D_1^a(z_h)$$

$$\frac{d\sigma_{LL}}{dx_B\, dy\, dz_h} = \frac{2\pi\alpha^2\, s}{Q^4}\, x_B\, y(2-y)\, \lambda_e\, \lambda \sum_{a,\bar{a}} e_a^2 g_1^a(x_B)\, D_1^a(z_h)$$



$$\ell H \longrightarrow \ell' h X$$

$$x_B = \frac{Q^2}{2P\cdot q}$$
$$y = \frac{P\cdot q}{P\cdot k}$$
$$z_h = \frac{P\cdot P_h}{P\cdot q}$$
$$Q_T = \frac{|P_{h\perp}|}{z_h}$$

Azimuthally weighted cross sections

$$\langle W\rangle_{\underbrace{P_e P_H P_h}_{\substack{\uparrow \\ (O,L,T)}}} \equiv \int d\phi^\ell\, d^2\boldsymbol{q}_T \underbrace{W}_{\substack{\uparrow \\ W(Q_T,\phi_h^\ell,\phi_S^\ell,\phi_{S_h}^\ell)}} \frac{d\sigma_{P_e P_H}}{dx_B\, dy\, dz_h\, d\phi^\ell\, d^2\boldsymbol{q}_T}$$

e.g.

$$\langle 1\rangle_{OOO} = \frac{2\pi\alpha^2\, s}{Q^4}\, x_B \left(1+(1-y)^2\right) \sum_{a,\bar{a}} e_a^2 f_1^a(x_B)\, D_1^a(z_h)$$

Leading azimuthal asymmetries for polarized targets

$$\left\langle \frac{Q_T}{M}\cos(\phi_h^\ell - \phi_S^\ell)\right\rangle_{LTO} = \frac{4\pi\alpha^2\, s}{Q^4}\,\lambda_e\,|\boldsymbol{S}_T|\,y\left(1-\frac{y}{2}\right)\sum_{a,\bar{a}} e_a^2\,x_B\,g_{1T}^{(1)a}(x_B)D_1^a(z_h)$$

$$\left\langle \frac{Q_T^2}{4MM_h}\sin(2\phi_h^\ell)\right\rangle_{OLO} = -\frac{4\pi\alpha^2\, s}{Q^4}\,\lambda\,(1-y)\sum_{a,\bar{a}} e_a^2\,x_B\,h_{1L}^{\perp(1)a}(x_B)H_1^{\perp(1)a}(z_h)$$

$$\left\langle \frac{Q_T}{M_h}\sin(\phi_h^\ell + \phi_S^\ell)\right\rangle_{OTO} = \frac{4\pi\alpha^2\, s}{Q^4}\,|\boldsymbol{S}_T|\,(1-y)\sum_{a,\bar{a}} e_a^2\,x_B\,h_1^a(x_B)H_1^{\perp(1)a}(z_h)$$

Note: also (OTO) $\sin(3\phi_h^\ell - \phi_S^\ell)$ asymmetry $\sim h_{1T}^{\perp(2)}H_1^{\perp(1)}$

J.C. Collins, NPB 396 (1993) 161
A. Kotzinian, NPB 441 (1995) 234
R.D. Tangerman and PJM, PLB 352 (1995) 129

Subleading azimuthal asymmetries for unpolarized targets

$$\left\langle \frac{Q_T}{M}\cos(\phi_h^\ell)\right\rangle_{OOO} = -\frac{4\pi\alpha^2\,s}{Q^4}\,(2-y)\sqrt{1-y}\sum_{a,\bar{a}} e_a^2 \left\{ \frac{2M}{Q}\,x_B^2\,f^{\perp(1)a}(x_B)\,D_1^a(z_h) + \frac{2M_h}{Q}\,x_B\,f_1^a(x_B)\,\frac{\tilde{D}^{\perp(1)a}(z_h)}{z_h}\right\}$$

Note: $\tilde{D}^{\perp a}(z) = D^{\perp a}(z) - z D_1^a(z)$

J. Levelt and PJM, PRD 49 (1994) 96

$$\left\langle \frac{Q_T}{M}\sin(\phi_h^\ell)\right\rangle_{LOO} = \frac{4\pi\alpha^2\,s}{Q^4}\,\lambda_e\,y\sqrt{1-y}\,\frac{2M}{Q}\sum_{a,\bar{a}} e_a^2\,x_B^2\,\tilde{e}^a(x_B)\,H_1^{\perp(1)a}(z_h)$$

Note: $\tilde{e}^a(x) = e^a(x) - \frac{m_a}{M}\,\frac{f_1^a(x)}{x}$

J. Levelt and PJM, PLB 338 (1994) 357

Subleading $1/Q$ contributions in OLO single spin asymmetries

Leading:

$$\left\langle \frac{Q_T^2}{4MM_h}\,\sin(2\phi_h^\ell)\right\rangle_{OLO} = -\frac{4\pi\alpha^2\,s}{Q^4}\,\lambda\,(1-y)\sum_{a,\bar a} e_a^2\,x_{_B}\,h_{1L}^{\perp(1)a}(x_{_B})H_1^{\perp(1)a}(z_h)$$

Subleading:

$$\left\langle \frac{Q_T}{M}\,\sin(\phi_h^\ell)\right\rangle_{OLO} = \frac{4\pi\alpha^2\,s}{Q^4}\,\lambda\,(2-y)\sqrt{1-y}\,\frac{2M_h}{Q} \times \sum_{a,\bar a} e_a^2 \Bigg\{ x_{_B}h_{1L}^{\perp(1)a}(x_{_B})\,\frac{\tilde H^a(z_h)}{z_h} + x_{_B}\left(2\,h_{1L}^{\perp(1)a} - x_{_B}\,\tilde h_L^a(x_{_B})\right)\,H_1^{\perp(1)a}(z_h)\Bigg\}$$

Note: $\Rightarrow$ $\tilde H^a(z) = H^a(z) + 2z\,H_1^{\perp(1)a}(z) = z\,\frac{d}{dz}\left(z\,H_1^{\perp(1)a}\right)$

$\Rightarrow$ $x\,h_L^a(x) - \frac{m}{M}\,g_{1L}^a(x) = -2\,h_{1L}^{\perp(1)a} + x\,\tilde h_L^a(x) = x\,h_1^a - x\,\frac{d}{dx}\,h_{1L}^{\perp(1)a} - \frac{m}{M}\,g_{1L}^a(x)$

$\Rightarrow$ HERMES $\mathcal{A}_{OL}$ results (DIS99): $h_{1L}^{\perp(1)}(x) \approx 0$ (Efremov)

T-odd effects in initial and/or final state

D. Sivers, PRD 41 (1990) 83 and PRD 43 (1991) 261
J. Collins, NPB 396 (1993) 161

Leading single spin asymmetries in leptoproduction

$$\left\langle \frac{Q_T}{M_h} \sin(\phi_h^\ell + \phi_S^\ell) \right\rangle_{OTO} = \frac{2\pi\alpha^2\, s}{Q^4} \, |\boldsymbol{S}_T| \, 2(1-y) \sum_{a,\bar{a}} e_a^2 \, x_B \, h_1^a(x_B) H_1^{\perp(1)a}(z_h)$$

$$\left\langle \frac{Q_T}{M} \sin(\phi_h^\ell - \phi_S^\ell) \right\rangle_{OTO} = \frac{2\pi\alpha^2\, s}{Q^4} \, |\boldsymbol{S}_T| \left(1 - y + \frac{1}{2}\, y^2\right) \sum_{a,\bar{a}} e_a^2 \, x_B \, f_{1T}^{\perp(1)a}(x_B) D_1^a(z_h)$$

D. Boer and PJM, PRD 57 (1998) 5780

Both initial and final state effects have been employed to explain the observed single spin asymmetry in $pp^\uparrow \rightarrow \pi^+ X$

M. Boglione and PJM, PRD 60 (1999) 054007

Azimuthal asymmetry for unpolarized leptoproduction
A leading double T-odd effects (?)[1]

$$\left\langle \frac{Q_T^2}{M M_h} \cos(2\phi_h^\ell) \right\rangle_{OOO} = \frac{16\pi\alpha^2 s}{Q^4} (1-y) \sum_{a,\bar{a}} e_a^2 \, x_B \, h_1^{\perp(1)a}(x_B) \, H_1^{\perp(1)a}(z_h)$$

D. Boer and PJM, PRD 57 (1998) 5780

and higher order in α_s[2]

$$\int dz_h \; z_h \, \langle \cos(2\phi_h^\ell) \rangle_{OOO} = \frac{\pi\alpha^2 s}{Q^4} (1-y) \, F_L(x_B, Q^2)$$

T. Gehrmann, hep-ph/0003156

and higher order in $1/Q^2$...

[1] A similar azimuthal asymmetry as in $e^+e^- \rightarrow \pi\pi X$
(D. Boer, R. Jakob, PJM, NPB 504 (1997) 345
A. Efremov, O. Smirnova, L. Tkatchev, hep-ph/9812522)
[2] Using momentum sumrule for fragmentation functions

DISTRIBUTIONS (T-even)						
		Dirac structure				
$\Phi^{[\Gamma]}(x, \boldsymbol{k}_T)$		γ	$\gamma\,\gamma_5$	$[\gamma,\gamma]\gamma_5$		$1/\gamma_5$
twist 2	U	f_1				
	L		g_{1L}		$h_{1L}^{\perp}$	
	T		g_{1T}	h_1	$h_{1T}^{\perp}$	
twist 3	U	$f^{\perp}$				e
	L		$g_L^{\perp}$		h_L	
	T		$g_T \quad g_T^{\perp}$	h_T	$h_T^{\perp}$	

(9 independent)

FRAGMENTATION (T-even & T-odd)						
		Dirac structure				
$\Delta^{[\Gamma]}(z, -z\boldsymbol{k}_T)$		γ	$\gamma\,\gamma_5$	$[\gamma,\gamma]\gamma_5$		$1/\gamma_5$
twist 2	U	D_1			$H_1^{\perp}$	
	L		G_{1L}		$H_{1L}^{\perp}$	
	T	$D_{1T}^{\perp}$	G_{1T}	H_1	$H_{1T}^{\perp}$	
twist 3	U	$D^{\perp}$			H	E
	L	$D_L^{\perp}$	$G_L^{\perp}$		H_L	E_L
	T	D_T	$G_T \quad G_T^{\perp}$	H_T	$H_T^{\perp}$	E_T

(12 independent)

SUMMARY OF PROFILE FUNCTIONS

FRAGMENTATION (T-even & T-odd)					
$\Delta^{[\Gamma]}(z,-z\boldsymbol{k}_T)$		Dirac structure γ	$\gamma\,\gamma_5$	$[\gamma,\gamma]\gamma_5$	$1/\gamma_5$
twist 2	U	D_1		$H_1^\perp$	
	L		G_{1L}	$H_{1L}^\perp$	
	T	$D_{1T}^\perp$	G_{1T}	H_1 $H_{1T}^\perp$	
twist 3	U	$D^\perp$		H	E
	L	$D_L^\perp$	$G_L^\perp$	H_L	E_L
	T	D_T	G_T $G_T^\perp$	H_T $H_T^\perp$	E_T

$\boldsymbol{k}_T$-integrated results

FRAGMENTATION (T-even & T-odd)					
$\Delta^{[\Gamma]}(z)$		Dirac structure γ	$\gamma\,\gamma_5$	$[\gamma,\gamma]\gamma_5$	$1/\gamma_5$
twist 2	U	D_1			
	L		G_1		
	T			H_1	
twist 3	U			H	E
	L			H_L	E_L
	T	D_T	G_T		

SUMMARY OF PROFILE FUNCTIONS

DISTRIBUTION (T-even & T-odd)						
			Dirac structure			
$\Phi^{[\Gamma]}(x, \boldsymbol{k}_T)$			γ	$\gamma\,\gamma_5$	$[\gamma,\gamma]\gamma_5$	$1/\gamma_5$
twist 2	U		f_1		$h_1^\perp$	
	L			g_{1L}	$h_{1L}^\perp$	
	T		$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$	
twist 3	U		$f^\perp$		h	e
	L		$f_L^\perp$	$g_L^\perp$	h_L	e_L
	T		f_T	g_T $g_T^\perp$	h_T $h_T^\perp$	e_T

$\boldsymbol{k}_T$-integrated results

DISTRIBUTION (T-even & T-odd)						
			Dirac structure			
$\Phi^{[\Gamma]}(x)$			γ	$\gamma\,\gamma_5$	$[\gamma,\gamma]\gamma_5$	$1/\gamma_5$
twist 2	U		f_1			
	L			g_1		
	T				h_1	
twist 3	U				h	e
	L				h_L	e_L
	T		f_T	g_T		

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TRIVIAL ROTATIONS OF SPIN

Spin along z-axis:

$$\tilde{M}_{mm'} = \begin{pmatrix} M_0 + M_L & M_T^1 - i\,M_T^2 \\ M_T^1 + i\,M_T^2 & M_0 - M_L \end{pmatrix}$$

Spin along x-axis:

$$\tilde{M}_{mm'} = \begin{pmatrix} M_0 + M_T^1 & M_L + i\,M_T^2 \\ M_L - i\,M_T^2 & M_0 - M_T^1 \end{pmatrix}$$

Spin along y-axis:

$$\tilde{M}_{mm'} = \begin{pmatrix} M_0 + M_T^2 & -M_T^1 - i\,M_L \\ -M_T^1 + i\,M_L & M_0 - M_T^2 \end{pmatrix}$$

Effects of changing basis:

$$
M^{(\mathrm{prod})} = \begin{array}{cc}
\left(\begin{array}{cccc}
f_1+g_1 & 0 & 0 & 2\,h_1 \\
0 & f_1-g_1 & 0 & 0 \\
0 & 0 & f_1-g_1 & 0 \\
2\,h_1 & 0 & 0 & f_1+g_1
\end{array}\right) &
\begin{array}{c}
\mathrm{R}\rightarrow \\
\leftarrow\mathrm{R} \\
\mathrm{L}\rightarrow \\
\leftarrow\mathrm{L}
\end{array} \\
\begin{array}{cccc}
\mathrm{R}\rightarrow & \leftarrow\mathrm{R} & \mathrm{L}\rightarrow & \leftarrow\mathrm{L}
\end{array} &
\end{array}
$$

$$
M^{(\mathrm{prod})} = \begin{array}{cc}
\left(\begin{array}{cccc}
f_1 & g_1 & h_1 & -h_1 \\
g_1 & f_1 & h_1 & -h_1 \\
h_1 & h_1 & f_1 & -g_1 \\
-h_1 & -h_1 & -g_1 & f_1
\end{array}\right) &
\begin{array}{c}
\mathrm{R}\uparrow \\
\mathrm{R}\downarrow \\
\mathrm{L}\uparrow \\
\mathrm{L}\downarrow
\end{array} \\
\begin{array}{cccc}
\mathrm{R}\uparrow & \mathrm{R}\downarrow & \mathrm{L}\uparrow & \mathrm{L}\downarrow
\end{array} &
\end{array}
$$