Gravitational Waves

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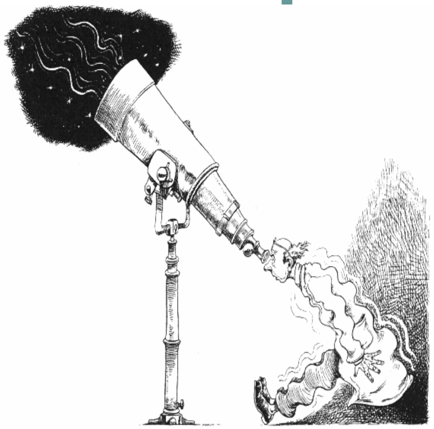
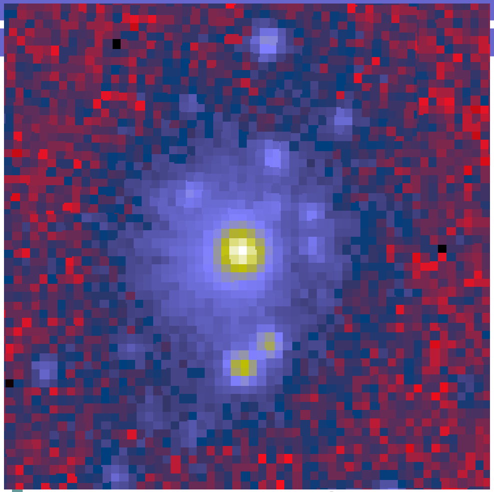
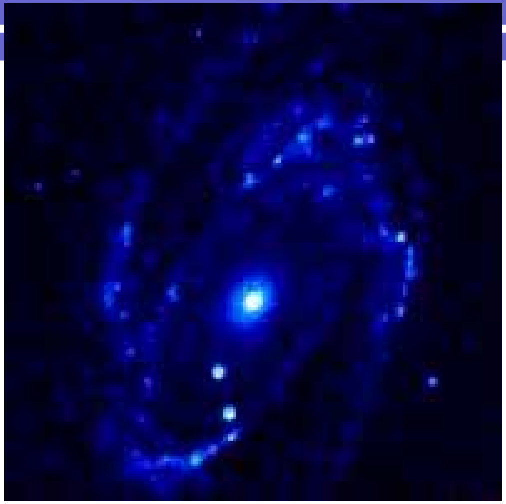
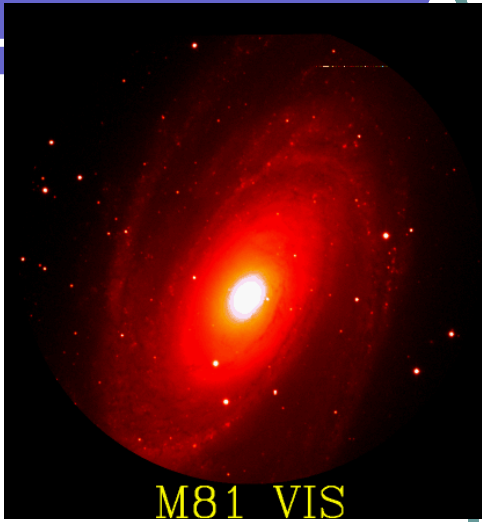


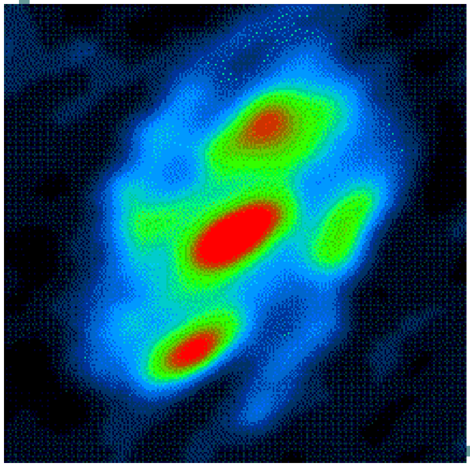
M81 galaxy

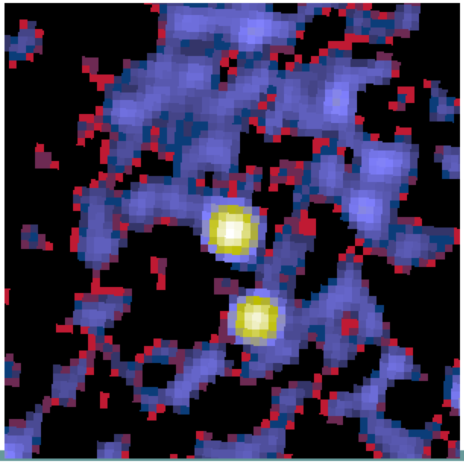

X-ray: 10 nm

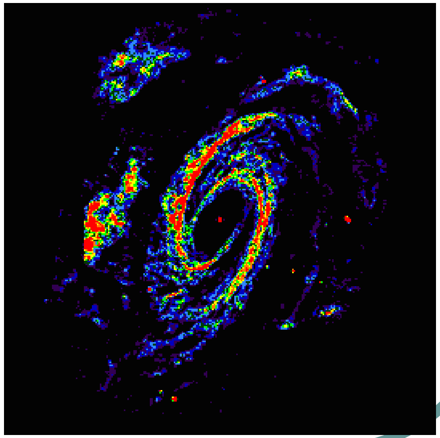

UV: 200 nm



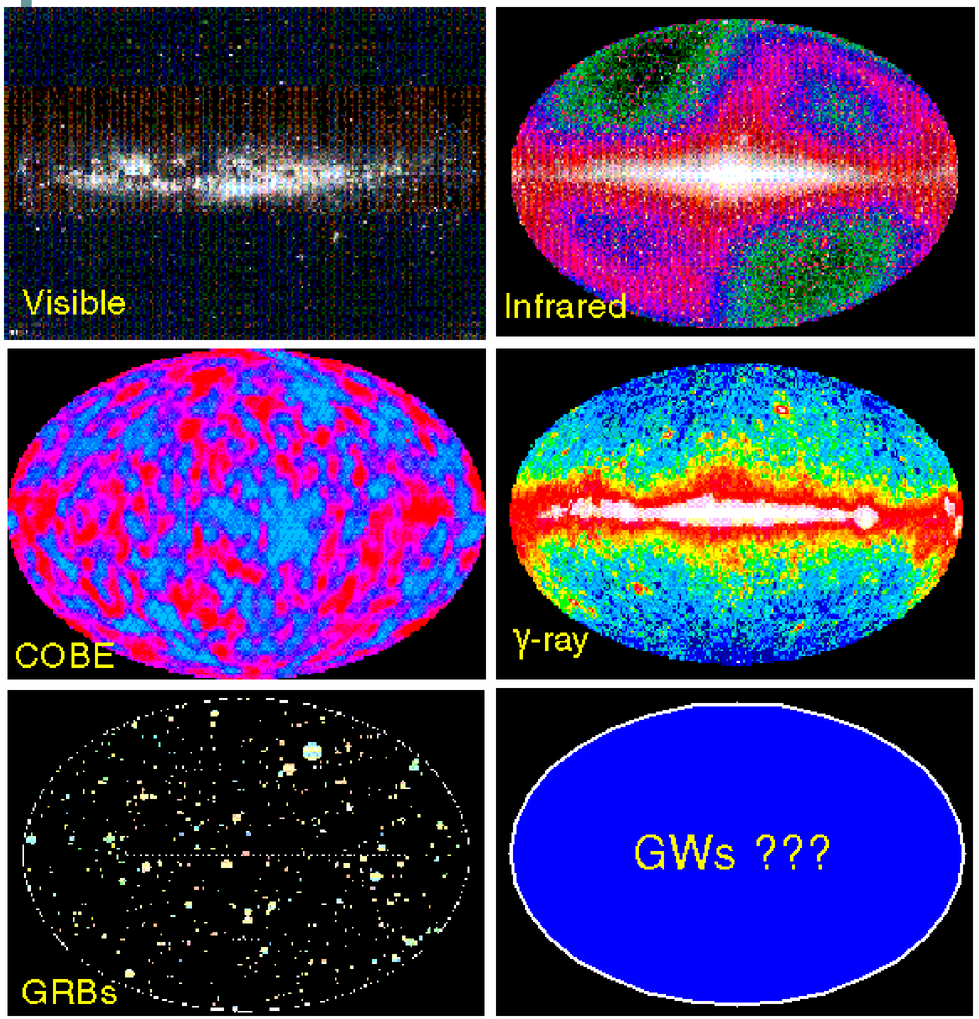
Visible: 600 nm

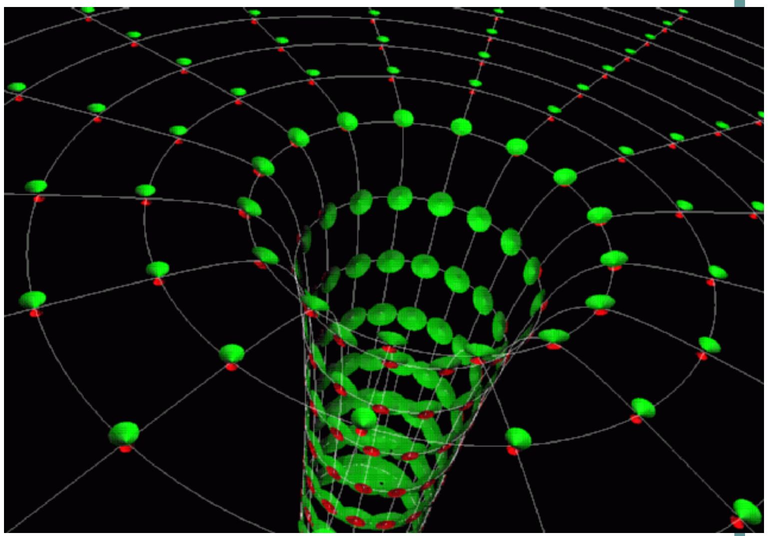

Infrared: 100 mm


Radio: 21cm

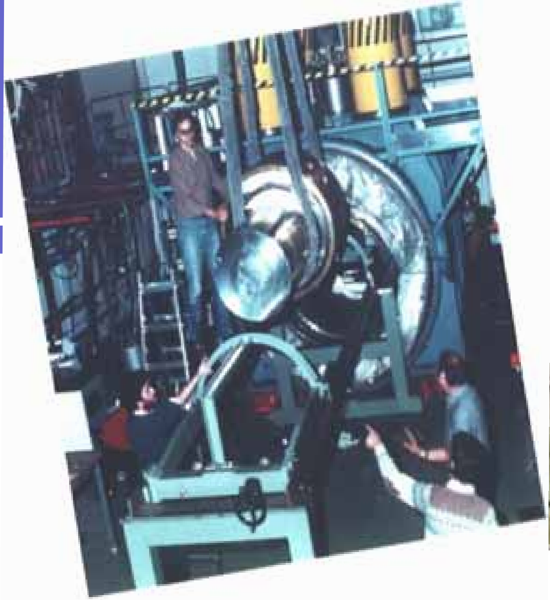

Radio – HI filter

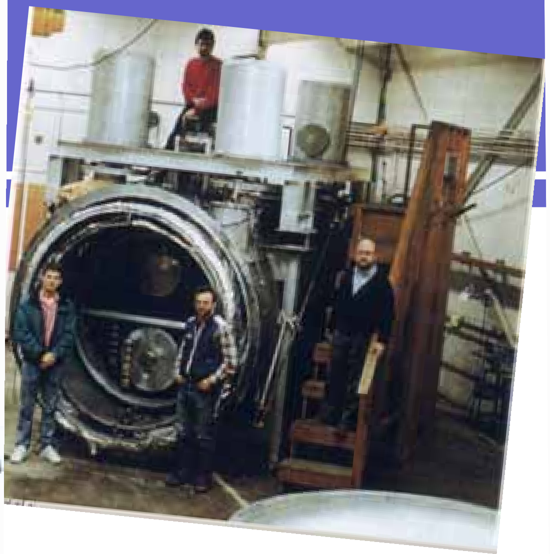
A New Window on the Universe



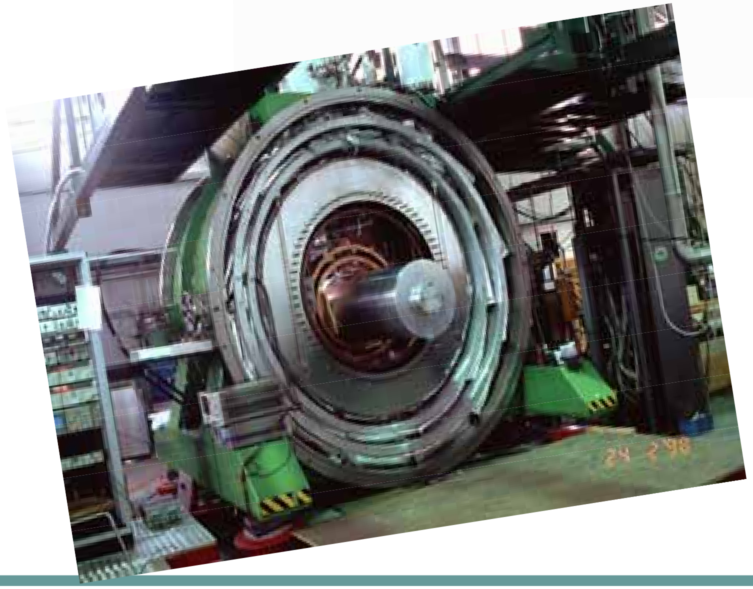


Gravitational Waves will provide a new way to view the dynamics of the Universe





ALLEGRO AURIGA EXPLORER NAUTILUS NIOBE



The GW detector network

GEO600 (German-British)
Hannover, Germany

LIGO (USA)
Hanford, WA & Livingston, LA

TAMA (Japan)
Mitaka

AIGO (Australia),
Wallingup Plain, Perth

VIRGO (French-Italian)
Cascina, Italy

About the lectures…

- Theory of Gravitational Waves
- Gravitational Wave Detectors
  - Signal Analysis
- Sources of Gravitational Waves

Gravitational Waves

- **Why Gravitational Waves?**
  - Fundamental aspect of General Relativity
  - Originate in the **most violent events** in the Universe
  - Major **challenge to present technology**
- **Why we have not seen them yet?**
  - They **carry enormous amount of energy** **but**
  - They **couple very weakly** to detectors.
- **How we will detect them?**
  - **Resonant Detectors** (Bars & Spheres)
  - **Interferometric Detectors** on Earth
  - **Interferometers** in Space

Gravitational vs E-M Waves

- **EM waves are radiated by individual particles**, **GWs are due to non-spherical bulk motion of matter**. I.e. the information carried by EM waves is stochastic in nature, while the GWs provide insights into coherent mass currents.
- **The EM will have been scattered many times**. **In contrast, GWs interact weakly with matter and arrive at the Earth in pristine condition**. Therefore, GWs can be used to probe regions of space that are opaque to EM waves. Stiil, the weak interaction with matter also makes the GWs fiendishly hard to detect.
- Standard astronomy is based on **deep imaging of small fields of view**, while **gravitational-wave detectors cover virtually the entire sky**.
- **EM radiation has a wavelength smaller than the size of the emitter**, **while the wavelength of a GW is usually larger than the size of the source**. Therefore, we cannot use GW data to create an image of the source. GW observations are more like audio than visual.

Morale: GWs carry information which would be difficult to get by other means.

Uncertainties and Benefits

- **Uncertainties**
  - The strength of the sources (may be orders of magnitude)
  - The rate of occurrence of the various events
  - The existence of the sources

- **Benefits**
  - **Information about the Universe that we are unlikely ever to obtain in any other way**
  - Experimental tests of fundamental laws of physics which cannot be tested in any other way
  - **The first detection of GWs will directly verify their existence**
  - By comparing the arrival times of EM and GW bursts we can measure their speed with a fractional accuracy **$\sim 10^{-11}$**
  - **From their polarization properties of the GWs we can verify GR prediction that the waves are transverse and traceless**
  - **From the waveforms we can directly identify the existence of black-holes.**

Information carried by GWs

- **Frequency**

$$f \sim 10^4\,\mathrm{Hz} \rightarrow \rho \sim 10^{16}\,\mathrm{gr/cm}^3$$
$$f \sim 10^{-4}\,\mathrm{Hz} \rightarrow \rho \sim 1\mathrm{gr/cm}^3$$

$$f_{dyn} \sim \left(\frac{GM}{R^3}\right)^{1/2} \sim (G\rho)^{1/2}$$

- **Rate of frequency change**

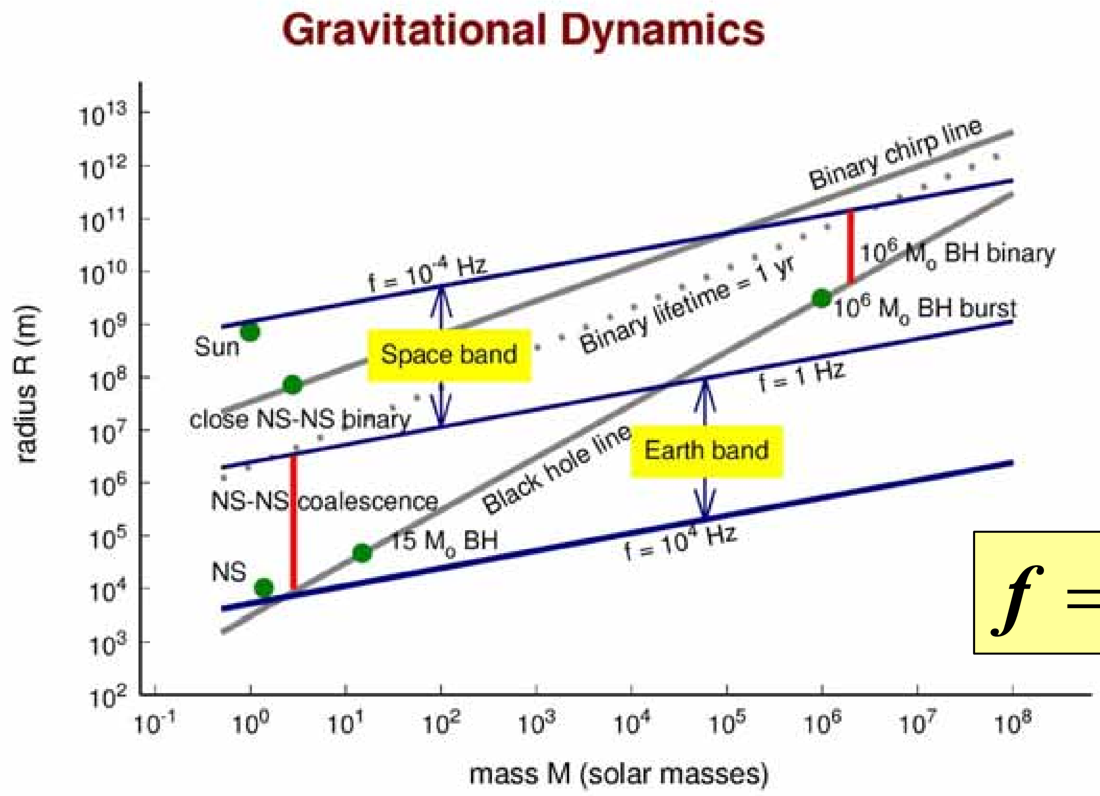
$$\dot{f}/f \sim (M_1, M_2)$$

- **Damping**

$$\tau \sim M^3/R^4$$

- **Polarization**
  - Inclination of the symmetry plane of the source
  - Test of general relativity
- **Amplitude**
  - Information about the strength and the distance of the source ($h \sim 1/r$).
- **Phase**
  - Especially useful for detection of binary systems.

Gravitational Dynamics



$$f = (GM / R^3)^{1/2}$$

GW Frequency Bands

- High-Frequency: 1 Hz - 10 kHz
  - (Earth Detectors)
- Low-Frequency: 10^{-4} - 1 Hz
  - (Space Detectors)
- Very-Low-Frequency: 10^{-7} - 10^{-9} Hz
  - (Pulsar Timing)
- Extremely-Low-Frequency: 10^{-15}-10^{-18} Hz
  - (COBE, WMAP, Planck)

Gravitation & Spacetime Curvature

Newton

$$\nabla^2 U = 4\pi G\rho$$

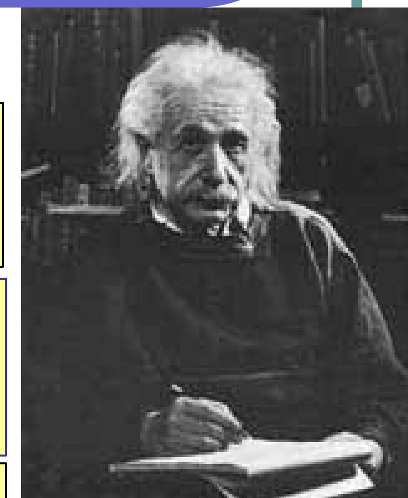
$$\frac{d^2\vec{x}}{dt^2} = \nabla U$$

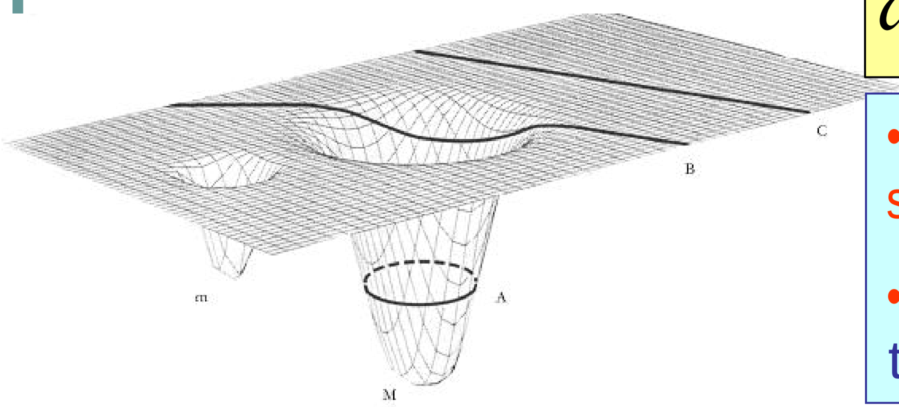
Einstein

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

$$\frac{d^2\vec{x}}{ds^2} \sim f(g^{\mu\nu})$$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$





- Matter dictates the degree of spacetime deformation.
- Spacetime curvature dictates the motion of matter.

GWs fundamental part of Einstein's theory

Linearized Gravity

- Assume a **small perturbation** on the background metric:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$$

- The **perturbed Einstein's equations** are:

$$h_{\alpha\beta;\mu}{}^{;\mu} + g_{\alpha\beta}h^{\mu\nu}{}_{;\nu\mu} - 2h_{\mu(\alpha}{}^{;\mu}{}_{;\beta)} + 2R_{\mu\alpha\nu\beta}h^{\mu\nu} - 2R_{\mu(\alpha}h_{\beta)}{}^{\mu} = kT_{\alpha\beta}$$

- **limit**)…
- And by **choosing a gauge**:

$$\tilde{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2} n_{\mu\nu} h_{\alpha}{}^{\alpha}$$

$$\tilde{h}^{\mu\nu}{}_{;\mu} = 0$$

- **Simple wave equation**:

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right)\tilde{h}^{\mu\nu} \equiv \partial_\lambda \partial^\lambda \tilde{h}^{\mu\nu} = kT^{\mu\nu}$$

Lorentz or Hilbert or De Donder gauge

Transverse-Traceless (TT)-gauge

- **Plane wave solution**

$$\tilde{h}^{\mu\nu} = A^{\mu\nu} e^{ik_a x^a}$$

$$A^{\mu\nu} k_\mu = 0$$

$$k^\mu k_\mu = 0$$

- TT-gauge (wave propagating in the z-direction)

$$A^{\mu\nu} = h_+ \varepsilon_+^{\mu\nu} + h_\times \varepsilon_\times^{\mu\nu}$$

$$\varepsilon_+^{\mu\nu} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \varepsilon_\times^{\mu\nu} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- **Riemann tensor**
- **Geodesic deviation**

$$R_{j0k0}{}^{TT} = -\frac{1}{2}\frac{\partial^2}{\partial t^2} h_{jk}{}^{TT}$$

$$\frac{d^2\xi_k}{dt^2} \approx -R_{k0j0}{}^{TT}\xi^j = \frac{1}{2}\frac{\partial^2 h_{jk}{}^{TT}}{\partial t^2}\xi^j$$

- **…and the tidal force**

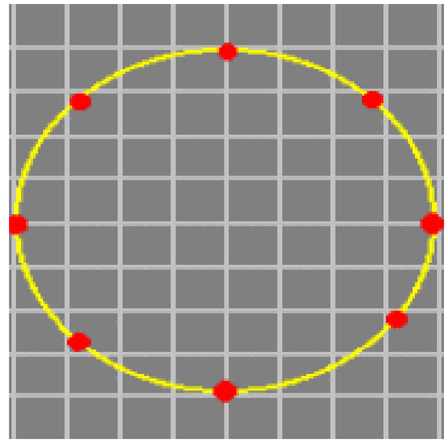
$$f^k \simeq m \cdot R^k_{0j0} \cdot \xi^j$$

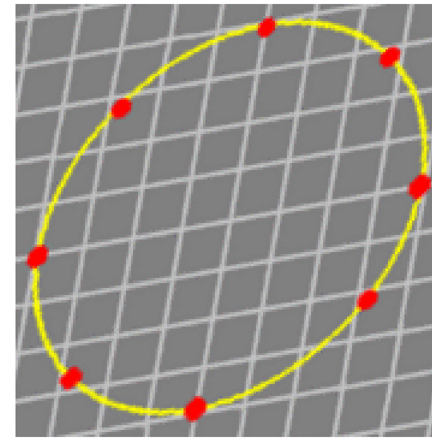
Gravitational Waves II

$$h^{\mu\nu} = h_+ \varepsilon_+^{\mu\nu} \cos[\omega(t-z)]$$

$$\Delta x = -\frac{1}{2} h_+ \cos\left[\omega(t-z)\right] x$$

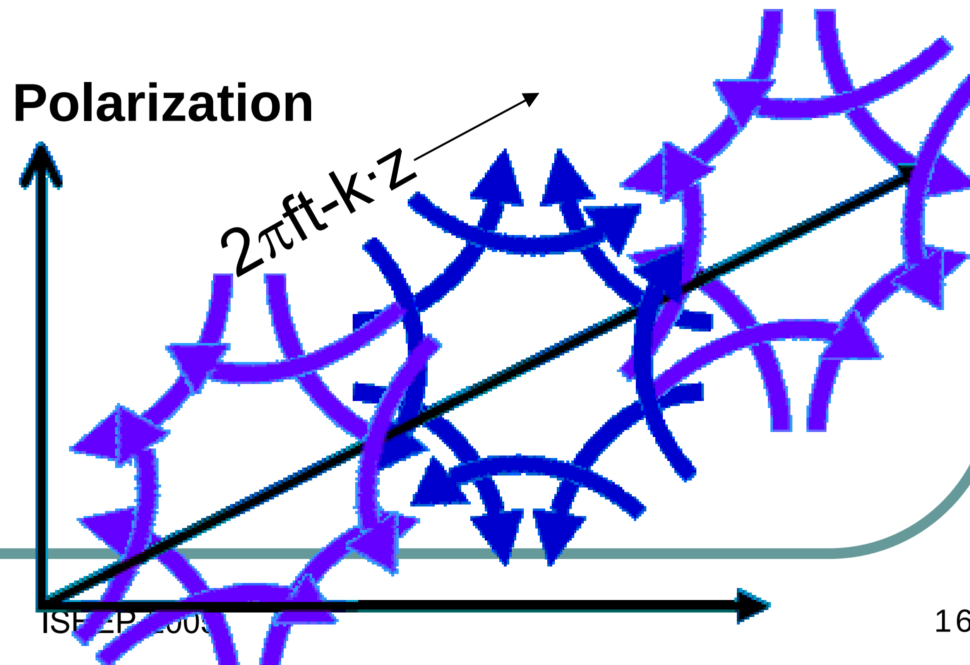
$$\Delta y = \frac{1}{2} h_+ \cos\left[\omega(t-z)\right] y$$



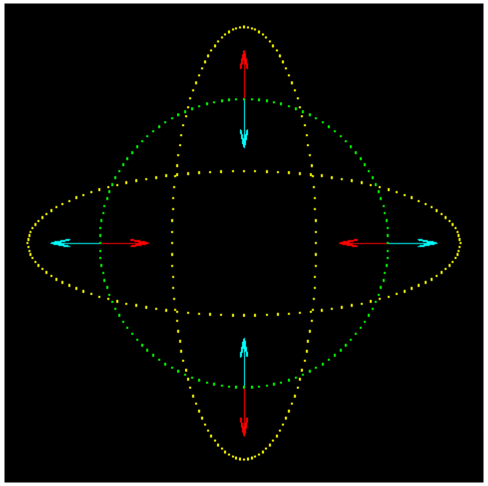


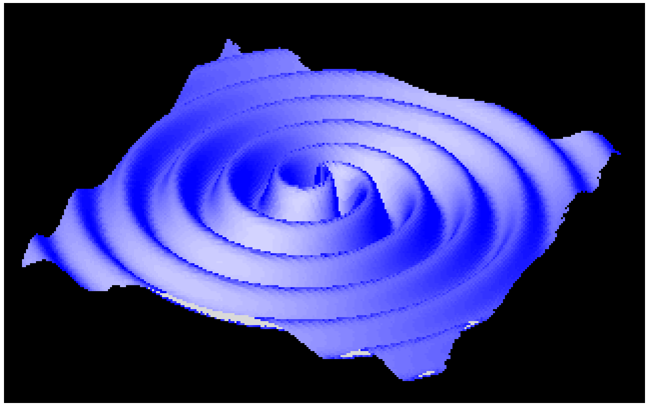
…in other words

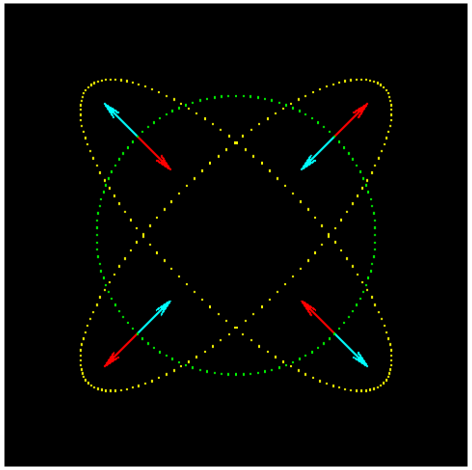
$$\frac{\Delta \ell}{\ell} = h$$

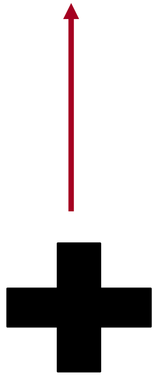


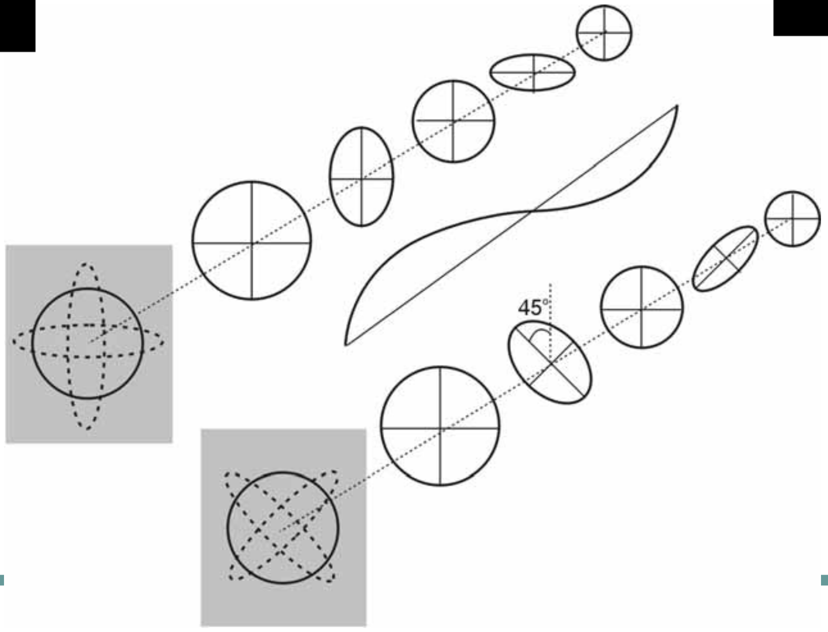
GW Polarizations











Stress-Energy carried by GWs

GWs exert forces and do work, they must carry energy and momentum

- The energy-momentum tensor in an arbitrary gauge

$$t_{\mu\nu}^{(GW)} = \frac{1}{32\pi}\left\langle \tilde{h}_{\alpha\beta;\mu}\tilde{h}^{\alpha\beta}{}_{;\nu} - \frac{1}{2}\tilde{h}_{;\mu}\tilde{h}_{;\nu} - \tilde{h}^{\alpha\beta}{}_{;\beta}\tilde{h}_{\alpha\mu;\nu} - \tilde{h}^{\alpha\beta}{}_{;\beta}\tilde{h}_{\alpha\nu;\mu} \right\rangle$$

- …in the TT-gauge:

$$t_{\mu\nu}^{(GW)} = \frac{1}{32\pi}\left\langle \tilde{h}^{jk\ TT}_{;\mu} \cdot \tilde{h}_{jk;\nu}^{TT} \right\rangle$$

- …it is divergence free

$$t_{\mu;\nu}^{\nu\ (GW)} = 0$$

- For waves propagating in the z-direction

$$t_{00}^{(GW)} = -\frac{1}{c}t_{0z}^{(GW)} = \frac{1}{c^2}t_{zz}^{(GW)} = \frac{1}{16\pi}\frac{c^2}{G}\left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle$$

- for a SN exploding in Virgo cluster the energy flux on Earth

$$t_{0z}^{(GW)} \approx \frac{\pi}{4}\frac{c^3}{G} f^2 \left\langle h_+^2 + h_\times^2 \right\rangle = 320 \times \left(\frac{f}{1kHz}\right)^2 \left(\frac{h}{10^{-21}}\right)^2 \frac{\text{ergs}}{\text{cm}^2\text{sec}}$$

- The corresponding EM energy flux is:

$$\sim 10^{-9}\,\text{erg}\cdot\text{cm}^{-2}\cdot\text{sec}^{-1}$$

Wave-Propagation Effects

GWs affected by the large scale structure of the spacetime exactly as the EM waves

- The magnitude of h_{jk}^{TT} falls of as $1/r$
- The polarization, like that of light in vacuum, is parallel transported radially from source to earth
- The time dependence of the waveform is unchanged by propagation except for a frequency-independent **redshift**

$$\frac{f_{\text{received}}}{f_{\text{emitted}}} = \frac{1}{1+z}$$

We expect

- Absorption, scattering and dispersion
- Scattering by the background curvature and tails
- Gravitational focusing
- Diffraction
- Parametric amplification
- Non-linear coupling of the GWs (frequency doubling)
- Generation of background curvature by the waves

The emission of grav. radiation

If the energy-momentum tensor is varying with time, GWs will be emitted

- The retarded solution for the linear field equation

$$\left(-\frac{\partial^2}{\partial t^2}+\nabla^2\right)\tilde{h}^{\mu\nu}=kT^{\mu\nu}_{(\text{matter})}$$

$$h^{\mu\nu}=2\int\frac{T^{\mu\nu}(t-|\vec{x}-\vec{x}'|,\vec{x}')}{|\vec{x}-\vec{x}'|}d^3x'$$

- For a point in the radiation zone in the slow-motion approximation

$$h^{\mu\nu}\approx\frac{2}{r}\int T^{\mu\nu}(t-r,\vec{x}')d^3x'\sim\frac{2}{r}\frac{\partial^2}{\partial t^2}\left[Q^{jk}(t-r)\right]^{TT}$$

- Where Q_{kl} is the quardupole moment tensor

$$Q^{kl}\equiv\int\rho(t,x^k)\left(x^kx^l-\frac{1}{3}r^2\delta^{kl}\right)d^3x$$

- **Power emitted in GWs**

$$L_{GW}=-\frac{dE}{dt}=\frac{1}{5}\frac{G}{c^5}\sum_{ij}\left\langle\dddot{Q}_{ij}\cdot\dddot{Q}_{ij}\right\rangle$$

Angular and Linear momentum emission

- **Angular momentum** emission

$$\frac{dJ_i^{GW}}{dt} = \frac{2}{5}\sum_{jkl} \varepsilon_{ijk} \left\langle \ddot{Q}_{jl} \cdot \dddot{Q}_{lk} \right\rangle$$

- **Linear momentum** emission

$$\frac{dP_i^{GW}}{dt} = \frac{2}{63}\sum_{jk} \left\langle \dddot{Q}_{jk} \cdot \ddddot{Q}_{jki} \right\rangle + \frac{16}{45}\sum_{jkl} \varepsilon_{ijk} \left\langle \dddot{Q}_{jl} \cdot \dddot{P}_{lk} \right\rangle$$

Q_{ijk} : mass octupole moment

P_{ij} : current quadrupole moment

Linearized GR vs Maxwell

	Einstein	Maxwell
Potentials	$h_{\alpha\beta}(x)$	$\left(\Phi(x), \vec{A}(x)\right)$
Sources	$T_{\alpha\beta}$	$\left(\rho_{\text{elect}}, \vec{J}\right)$
Lorentz gauge	$\tilde{h}^{\alpha\beta}{}_{;\alpha}=0$	$\frac{\partial\Phi}{\partial t} + \vec{\nabla}\cdot\vec{A} = 0$
Wave equation	$\Box\tilde{h}_{ij} = -8\pi T_{ij}$	$\Box\vec{A} = -4\pi\vec{J}$
Solution	$\tilde{h}^{ij} = 2\int d^3x' \frac{[T^{ij}]_{ret}}{\lvert\vec{x}-\vec{x}'\rvert}$	$\vec{A} = \int d^3x' \frac{[\vec{J}]_{ret}}{\lvert\vec{x}-\vec{x}'\rvert}$
Solution (asymp)	$\tilde{h}^{ij} = 2\frac{\left[\ddot{Q}^{ij}\right]_{ret}}{r}$	$\vec{A} = \frac{\left[\dot{\vec{p}}\right]_{ret}}{r}$
Radiated Power	$\frac{dE}{dt} = \frac{1}{5}\left\langle \dddot{Q}_{ij}\cdot\dddot{Q}^{ij}\right\rangle$	$\frac{dE}{dt} = \frac{2}{3}\left\langle \ddot{\vec{p}}^2\right\rangle$

Back of the envelope calculations!

- **Characteristic time-scale** for a mass element to move from one side of the system to another is:

$$T \sim \frac{R}{\upsilon} \sim \frac{R}{(M/R)^{1/2}} = \left(\frac{R^3}{M}\right)^{1/2}$$

- The **quadrupole moment** is approximately:

$$\dddot{Q}_{ij} \sim \frac{MR^2}{T^3} \sim \frac{M\upsilon^2}{T} \sim \frac{E_{ns}}{T} \sim \left(\frac{M}{R}\right)^{5/2}$$

- **Luminosity**

$$L_{GW} \sim \frac{G}{c^5}\left(\frac{M}{R}\right)^5 \sim \frac{G}{c^5}\left(\frac{M}{R}\right)^2 \upsilon^6 \sim \frac{c^5}{G}\left(\frac{R_{Sch}}{R}\right)^2\left(\frac{\upsilon}{c}\right)^6$$

$$\frac{c^5}{G} = 3.63\times10^{59}\, erg/s = 2.03\times10^5 M_{\odot} c^2/s$$

- The **amplitude** of GWs at a distance ***r*** (R~R_{Schw}~10Km and r~10Mpc~3x10^{19}km):

$$h \sim \frac{\ddot{Q}}{r} \sim \frac{1}{r}\left(\frac{MR^2}{T^2}\right) \sim \frac{1}{r}\frac{M^2}{R} \sim \ldots \sim 10^{-19}$$

- **Radiation damping**

$$\tau_{react} = \frac{E_{kin}}{L_{GW}} \sim \left(\frac{R}{M}\right)^{5/2} T \sim \left(\frac{\upsilon}{c}\right)\left(\frac{R}{R_{Schw}}\right)^3 T$$

What we should remember…

- **Length variation**

$$\frac{\Delta \ell}{\ell} = h$$

- **Amplitude**

$$h^{jk} \approx \frac{2}{r} \ddot{Q}^{jk}$$

- **Power emitted**

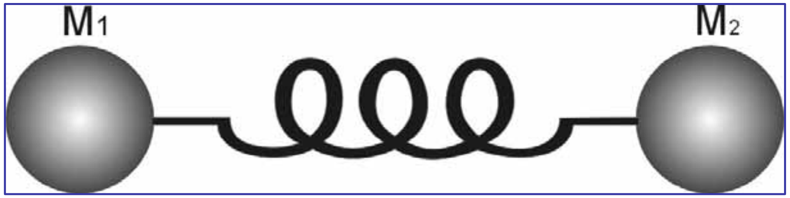
$$L_{GW} = -\frac{dE}{dt} = \frac{1}{5} \frac{G}{c^5} \sum_{ij} \left\langle \dddot{Q}_{ij} \dddot{Q}_{ij} \right\rangle$$

Vibrating Quadrupole

- The position of the two masses

$$x = \pm[x_0 + \xi \sin(\omega t)] \quad , \quad x_0 \ll \xi$$

- The quadrupole moment of the system is



$$Q^{kl}(t-r) \simeq \left[1 + \frac{2\xi}{x_0}\sin\omega(t-r)\right] Q_0^{kl}$$

$$Q_0^{kl} = \begin{pmatrix} -2mx_0^2 & 0 & 0 \\ 0 & -2mx_0^2 & 0 \\ 0 & 0 & 4mx_0^2 \end{pmatrix}$$

- The radiated gravitational field is

$$h^{kl} = \frac{2}{3}\left(\frac{\xi}{x_0}\right)\frac{\omega^2}{r}\sin[\omega(t-r)]Q_0^{kl}$$

- The emitted power

$$-\frac{dE}{dt} = \frac{G}{45c^5}\left\langle \dddot{Q}_{kl}\dddot{Q}_{kl} \right\rangle = \frac{16}{15}\frac{G}{c^5}(mx_0\xi)^2\omega^6$$

- And the damping rate of the oscillator is

$$\gamma_{rad} = -\frac{1}{E}\left\langle \frac{dE}{dt} \right\rangle = \frac{16}{15}\frac{G}{c^5} m x_0^2 \omega^4$$

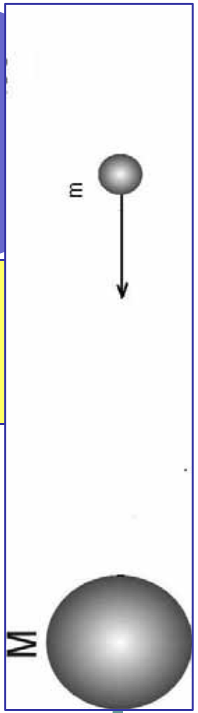
Two-body collision

- The radiated power
- The energy radiated during the plunge from $z=\infty$ to $z=-R$
- If $R=R_{Schw}$ ($M=10M_\odot$ & $m=1M_\odot$)

$$-\frac{dE}{dt}=\frac{8}{15}\frac{G}{c^5}m^2(3\dot{z}\,\ddot{z}+z\,\dddot{z})^2$$

$$-\Delta E=\frac{4}{105}\frac{1}{R^{7/2}}\frac{G}{c^5}m^2(2GM)^{5/2}$$

$$\frac{1}{2}m\dot{z}^2=\frac{GmM}{|z|}$$

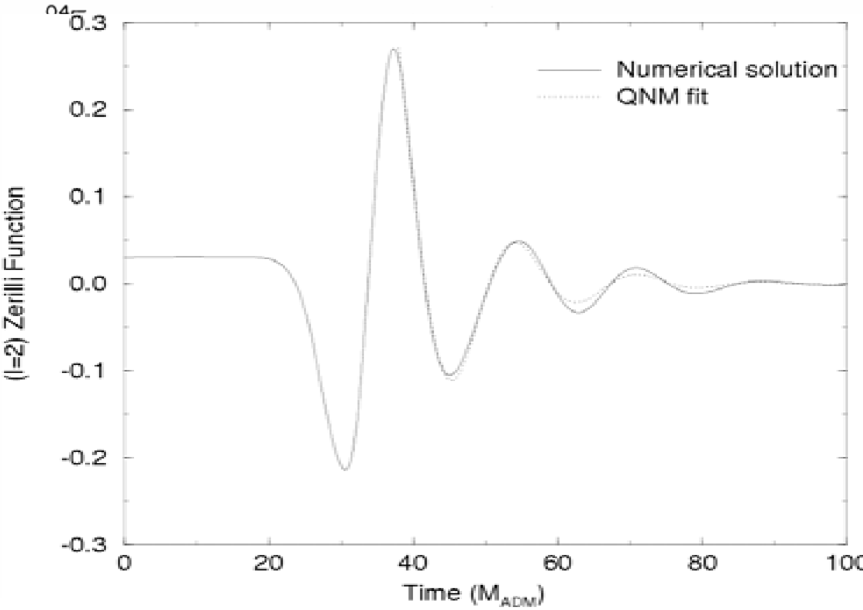


$$-\Delta E\quad=0.0190mc^2\frac{m}{M}$$

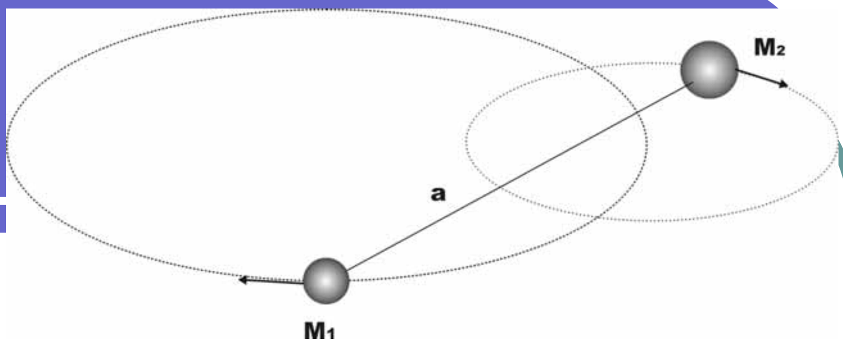
$$-\Delta E_{true}=0.0104mc^2\frac{m}{M}\rightarrow 2\times10^{51}erg$$

$$\Delta t\simeq R/\upsilon\simeq R/c\simeq 30km/c\approx10^{-4}s$$

$$f\simeq10^4\,Hz$$



Rotating Quadrupole (a binary system)



THE BEST SOURCE FOR GWs

- **Radiated power**

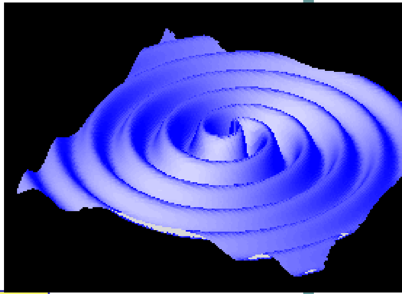
$$-\frac{dE}{dt}=\frac{32}{5}\frac{G}{c^5}\mu^2 a^4\omega^6=\frac{32}{5}\frac{G}{c^5}\frac{M^3\mu^2}{a^5}$$

- Energy loss leads to **shrinking of their orbital separation**

$$\frac{da}{dt}=-\frac{64}{5}\frac{G^3}{c^5}\frac{\mu M^2}{a^3}$$

- **Period changes** with rate

$$\frac{\dot{P}}{P}=-\frac{96}{5}\frac{G^3}{c^5}\frac{\mu M^2}{a^4}$$



- ...and the system **will coalesce** after

$$T_{inspiral}=\frac{5}{256}\frac{c^5}{G^3}\frac{a_0^4}{\mu M^2}$$

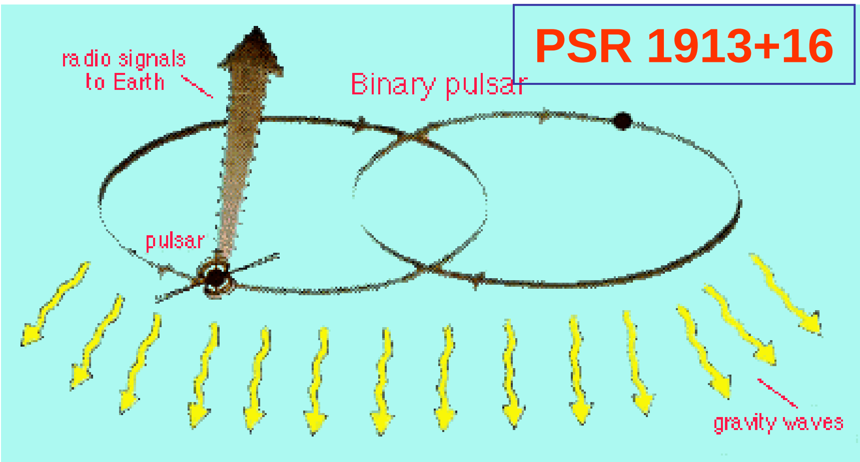
- The **total energy loss** is

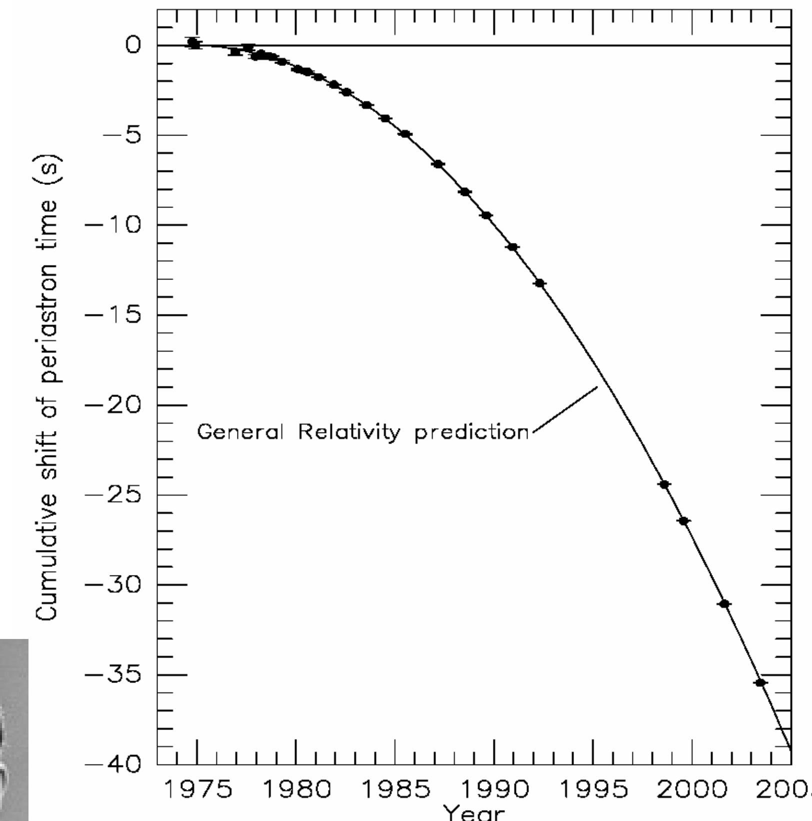
$$\Delta E_{rad}=\frac{G}{2}\mu M\left(\frac{1}{a_0}-\frac{1}{a}\right)$$

- Typical **amplitude** of GWs

$$h\approx 5\times10^{-22}\left(\frac{M}{2.8M_\odot}\right)^{2/3}\left(\frac{\mu}{0.7M_\odot}\right)\left(\frac{f}{100Hz}\right)^{2/3}\left(\frac{15Mpc}{r}\right)$$

First verification of GWs



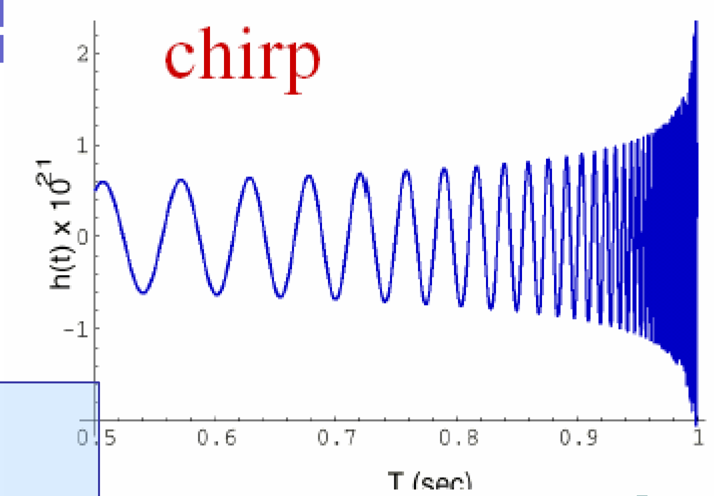


Nobel 1993

Hulse & Taylor

$$\frac{\dot{P}_{b,corrected}}{\dot{P}_{b,GR}} = 1.0013 \pm 0.0021$$

Binary systems (examples)



PSR 1913+16

$M_1 = M_2 \sim 1.4 M_\odot$, P=7h 45m 7s, r=5kpc,

$h_{earth} \sim 10^{-20}$, $f \sim 10^{-4}$Hz, $T_{insp} \sim 3\times10^{8}$yr

$dP_{theo}/dt = -7.2\times10^{-12}$s/yr $dP_{obs}/dt = -(6.9\pm06)\times10^{-12}$s/yr

The LIGO/VIRGO binary (10-1000Hz)

$M_1 = M_2 \sim 1.4 M_\odot$, $f_0 = 10$Hz, $f_{final} = 1000$Hz,

$T_{insp} \sim 15$min, after ~15000 cycles (inspiral/merging 300Mpc)

$M_1 = 50 M_\odot$, $M_2 \sim 50 M_\odot$, $f_0 = 10$Hz,

$f_{final} = 100$Hz, (inspiral/merging 400Mpc)

$$f_{BH} \sim 12kHz\left(\frac{1M_\odot}{M}\right)$$

The LISA binary (10^{-5}-10^{-2}Hz)

$M_1 = M_2 \sim 10^6 M_\odot$, $f_0 = 10^{-4}$Hz, $f_{final} = 0.01$Hz, (inspiral/merging at r~3Gpc)

$M_1 = M_2 \sim 10^5 M_\odot$, $f_0 = 10^{-4}$Hz, $f_{final} = 0.1$Hz, (inspiral/merging at r~3Gpc)

$M_1 = M_2 \sim 10^4 M_\odot$, $f_0 = 10^{-3}$Hz, $f_{final} = 1$Hz, (inspiral at r~3Gpc)

Smaller Stars/BHs plunging into super-massive ones

An interesting observation

- **The observed frequency change will be:**

$$\dot{f} \sim f^{11/3} M_{chirp}^{5/3}$$

$$M_{chirp}^{5/3} = \mu M^{2/3}$$

- **The corresponding amplitude will be :**

$$h \sim \frac{M_{chirp}^{5/3} f^{2/3}}{r} = \frac{\dot{f}}{f^{3} r}$$

•Since both frequency and its rate of change are measurable quantities, we can immediately compute the chirp mass.

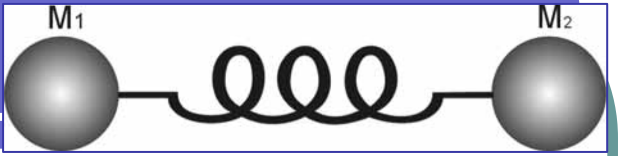
•The third relation provides us with a direct estimate of the distance of the source

•Post-Newtonian relations can provide the individual masses

2^{nd} Part

DETECTION of GWs

A Quadrupole Detector



Tidal force is the driving force of the oscillator

- Plane wave

$$h^{\mu\nu} = h_+ \varepsilon^{\mu\nu} e^{i(\omega t - kz)}$$

- Displacement & Tidal force

$$\xi_x \approx L h_+ e^{i\omega t} \qquad f_x \approx m L h_+ \omega^2 e^{i\omega t}$$

- Equation of motion

$$\ddot{\xi} + \dot{\xi}/\tau + \omega_0{}^2 \xi = -\frac{1}{2}\omega^2 L h_+ e^{i\omega t}$$

- Solution

$$\xi = \frac{\omega^2 L h_+ e^{i\omega t}/2}{\omega_0^2 - \omega^2 + i\omega/\tau}$$

$$\xi_{\max} = \omega_0 \tau \cdot L \cdot h_+ / 2 = Q \cdot L \cdot h / 2$$

- **Cross section**

$$\sigma = \frac{32\pi}{15}\frac{G}{c^3}\omega_0 \cdot Q \cdot M \cdot L^2$$

- Weber's detector:
  $M = 1410 kg$, $L = 1.5 m$,
  $d = 66 cm$, $\omega_0 = 1660 Hz$,
  $Q = 2x10^5$.

$$\sigma_{Weber} \simeq 3 \times 10^{-19} cm^2$$

Quadrupole Detector Limitations

Problems

- Very small cross section $\sim 3x10^{-19} cm^2$.
- Sensitive to periodic GWs **tuned** in the right frequency of the detector
- Sensitive to bursts **only** if the pulse has a substantial component at the resonant frequency
- The **width of the resonance** is:

$$\Delta\nu \sim \gamma / 2\pi \sim 10^{-2}\, Hz$$

•Thermal noise limits our ability to detect the energy of GWs.

•The excitation energy has to be greater than the thermal fluctuations $E \gtrsim kT$

$$h_{\min} \geq \frac{1}{\omega_0 L Q}\sqrt{\frac{15kT}{M}} \sim 10^{-20}$$

BURSTS

•Periodic signals which match the resonant frequency of the detector are extremely rare.

•A great number of events produces short pulses which spread radiation over a wide range of frequencies.

•The minimum detectable amplitude is

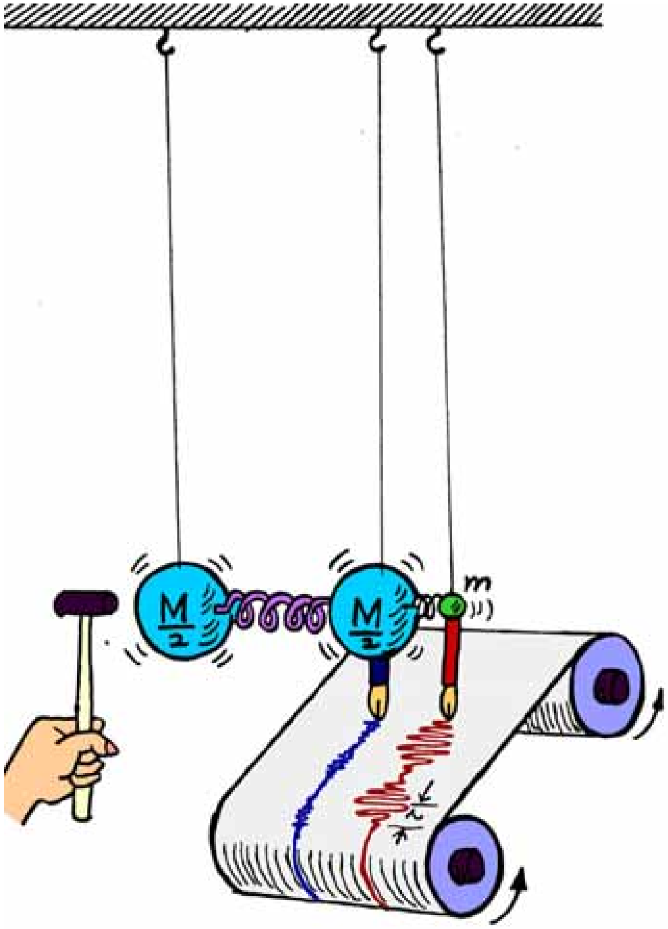
$$h_{\min} \geq \frac{1}{\omega_0 L}\sqrt{\frac{30kT_{eff}}{\pi M}} \sim 10^{-16}$$

The total energy of a pulse from the Galactic center (r=10kpc) which will provide an amplitude of $h \sim 10^{-16}$ or energy flux $\sim 10^9$ erg/cm^2.

$$4\pi r^2 \times 10^9\, erg / cm^2 = 10^{55}\, erg$$
$$\approx 10 M_\odot c^2 \;!!!$$

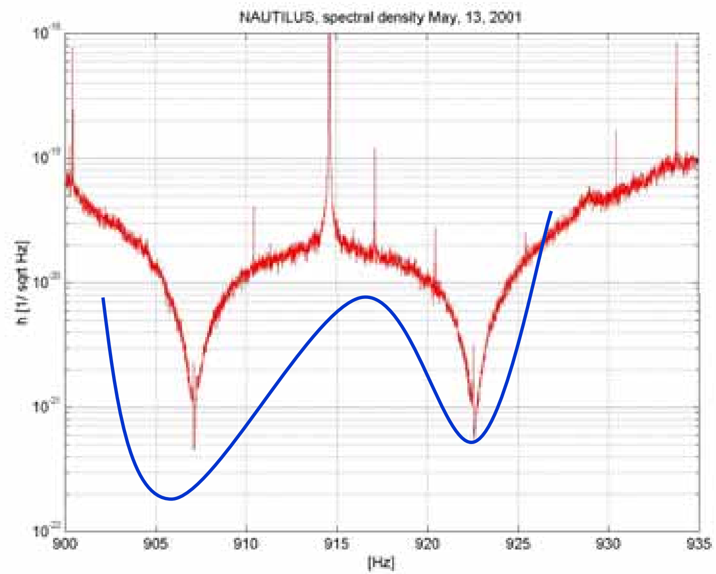
Improvements to resonant detectors

- Have higher Q
- Operate in extremely low temperatures (mK)
- Larger masses
- Different geometry
- Better electronic sensors



Modern Bar Detectors

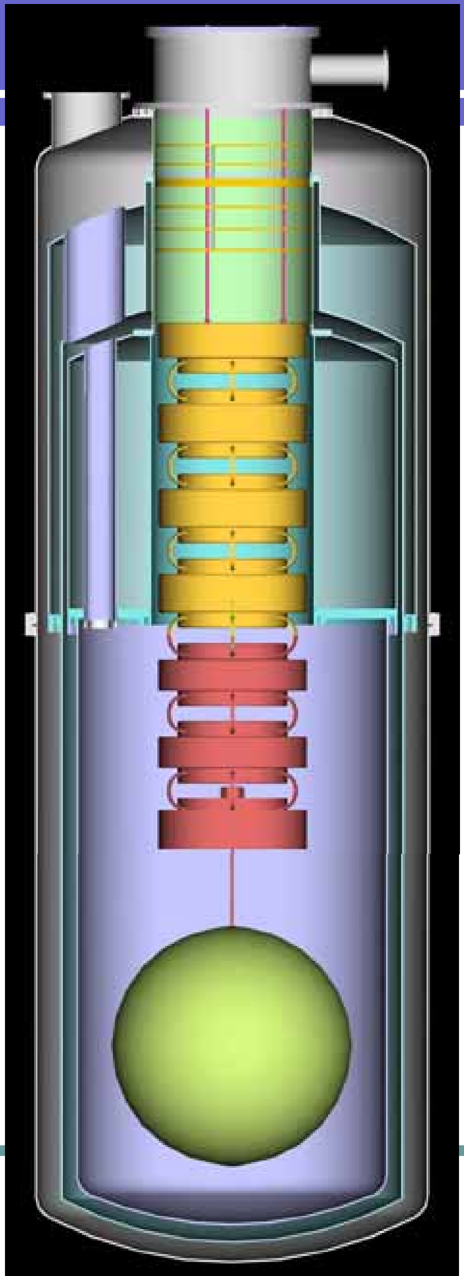
	WEBER	NAUTILUS
mass(kg)	1410	2270
Length(m)	1.53	2.97
ω_0(Hz)	1660	910
$Q = \omega/\gamma$	2×10^5	2.3×10^6
$\sigma\ (\omega_0)_{\text{abs}}\ (\text{cm}^2)$	2×10^{-19}	70×10^{-19}
Typical pulse sensitivity h	10^{-16}	9×10^{-19}



Exploiting the resonant-mass detector technique: the spherical detector

M = 1-100 tons
Sensitivity:
10^{-23} - 10^{-24} $Hz^{-1/2}$;
h ~ 10^{-21} - 10^{-22}

- **Omnidirectional**
- Capable of **detecting source position**
- Capable of **measuring polarization**



MINIGRAIL
Leiden (Netherlands)

MARIO SHENBERG
Sao Paulo (Brasil)

SFERA
Frascati (Italy)

CuAl(6%) sphere
Diameter= 65 cm
Frequency = 3 kHz
Mass = 1 ton

Laser Interferometers

- The output of the detector is

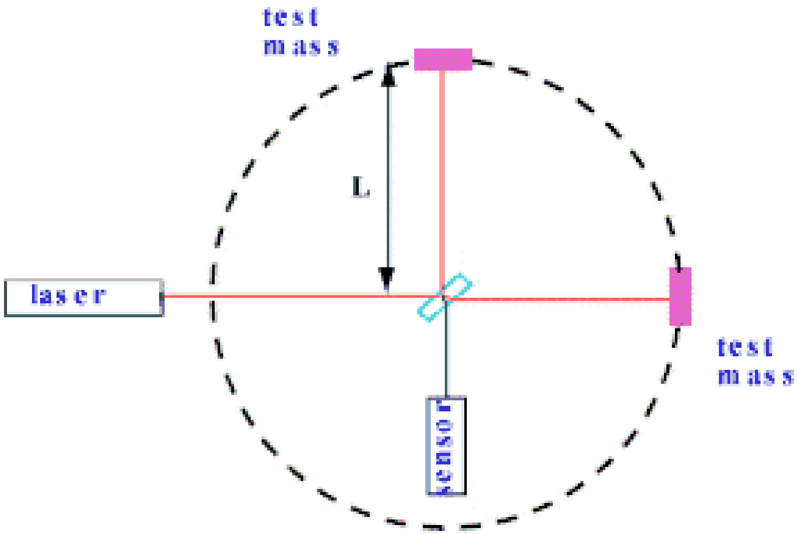
$$\frac{\Delta L}{L} = F_+ h_+(t) + F_\times h_\times(t) = h(t)$$

- Technology allows measurements $\Delta L \sim 10^{-16} cm$.
- For signals with $h \sim 10^{-21}$-10^{-22} we need arm lengths $L \sim 1$-$10 km$.
- Change in the arm length by ΔL corresponds to a phase change

$$\Delta\varphi = \frac{4\pi b \Delta L}{\lambda} \sim 10^{-9}\,\text{rad}$$

- The number of photons reaching the photo-detector is proportional to laser-beam's intensity $[\sim \sin^2(\Delta\varphi/2)]$

$$N_{\text{out}} = N_{\text{input}} \sin^2(\Delta\varphi/2)$$



OPTIMAL CONFIGURATION

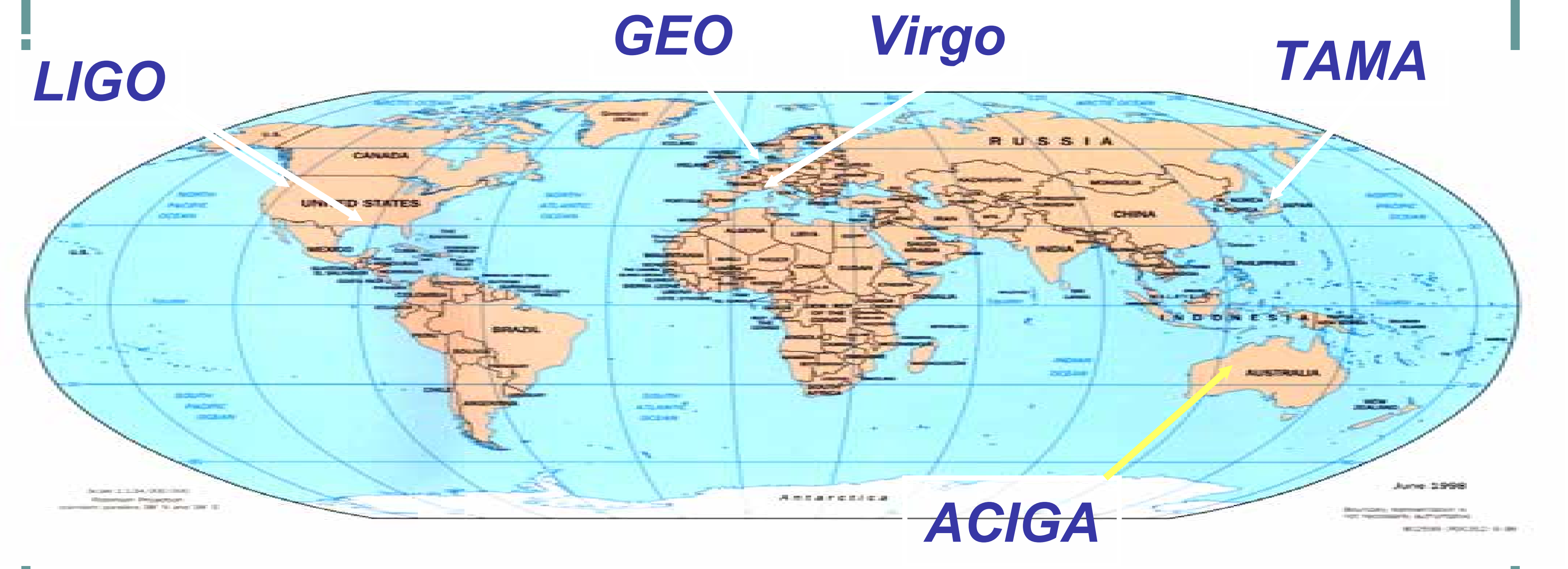
•Long arm length L

•Large number of reflections b

•Large number of photons (but be aware of radiation pressure)

•Operate at interface minimum $\cos(2\pi b \Delta L/\lambda)=1$.

International Network *interferometers*



Simultaneously detect signal (within msec)

detection confidence

- **locate the sources**
- **decompose polarization of gravitational waves**

Noise Sources I

- Photon Shot Noise
  - The number of emerging photons is subject to statistical fluctuations
  - Implies an uncertainty in the measurement of *ΔL*.
- Radiation Pressure Noise
  - Lasers produce radiation pressure on the mirrors
  - Uncertainty in the measurement of the deposited momentum leads to an uncertainty in the position of the mirrors
- Quantum Limit
  - If we try to minimize PSN and RPN with respect to laser power we get a minimum detectable stain
  - Heisenberg's principle sets an additional uncertainty in the measurement of ΔL ($\Delta L\ \Delta p \gtrsim \hbar$, if $\Delta p \sim m\ \Delta L / \tau$ …) and the minimum detectable strain is

$$\delta N_{out} \sim \sqrt{N_{out}}$$

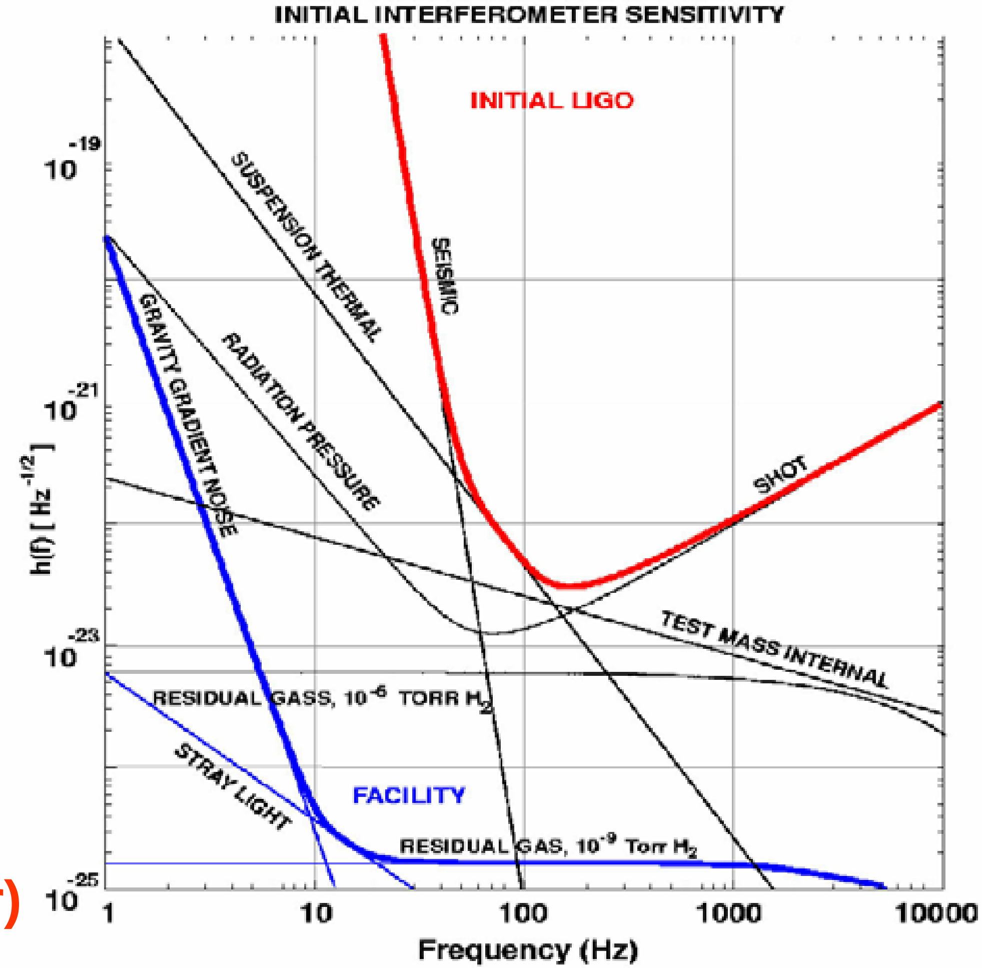
$$h_{\min} = \frac{\delta(\Delta L)}{L} = \frac{\Delta L}{L} \sim \frac{1}{bL}\sqrt{\frac{\hbar c \lambda}{2\pi\tau I_0}}$$

$$h_{\min} = \frac{\tau}{m}\frac{b}{L}\sqrt{\frac{\tau \hbar I_0}{c\lambda}}$$

$$h_{\min} = \frac{1}{L}\sqrt{\frac{\tau\hbar}{m}}$$

LIGO/Virgo Sensitivity

- Seismic Noise
  - Important below 60Hz. Dominates over all other types of noise.
- Residual gas-phase noise
  - Statistical fluctuations in the residual gas density induce fluctuations of the refraction index and as a result on the monitored phase shift.
  - Vacuum pipes **(~10^{-9} torr)**



About the noise in the detectors

The sensitivity of gravitational wave detectors is limited by noise

- The non-Gaussian noise may occur several times per day (e.g. strain releases in the suspension systems) and can be removed via comparisons of the data streams from various detectors
- The Gaussian noise obeys Gaussian probability distribution and characterized by a *power spectral density* $S_n(f)$ (dimensions of time :Hz^{-1})
- The noise amplitude is the square root of $S_n(f)$
- The dimensionless quantity $h_n^2(f) \equiv f S_n(f)$ is called effective noise
- Displacement or strain noise $h_L(f) \equiv L h_n(f)$ and the corresponding noise spectrum $S_L(f) \equiv L^2 S_n(f)$
- The observed signal $o(t)$ at the output of a detector consists of the true gravitational wave $h(t)$ strain and of Gaussian noise $n(t)$.
- If a signal is present in the data
- Long lived signals are easier to recognize than short bursts of the same amplitude

$$\langle n^2(t) \rangle = 2 \int_0^\infty S_n(f) df$$

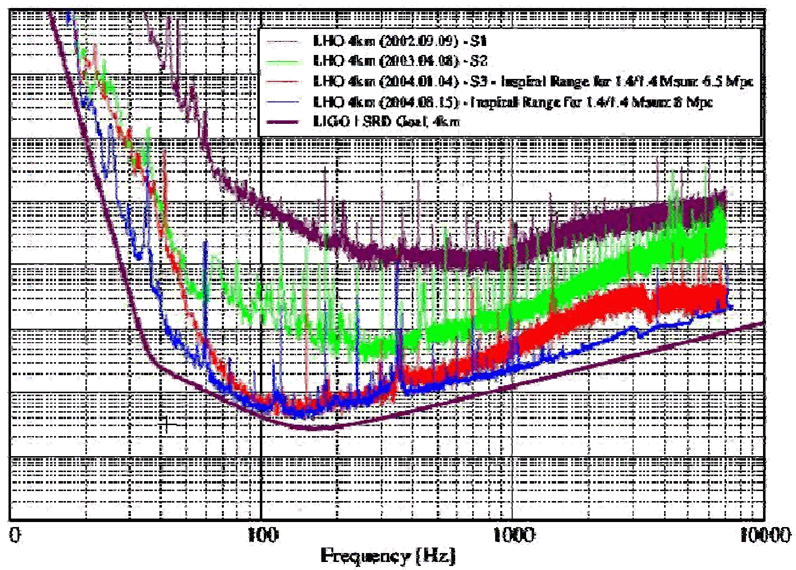
$$o(t) = n(t) + h(t)$$

$$\left(\frac{S}{N}\right)^2_{opt} = 2 \int_0^\infty \frac{|\tilde{h}(f)|^2}{S_n(f)} df$$

$$h_{\text{eff}} = h\sqrt{n}$$

Current Sensitivity of Virgo and LIGO

- **Very good progress over the past 3 years** and we are fast approaching the designed sensitivity goals
- TAMA reached the designed sensitivity (2003)
- LIGO (and GEO600) have reached within a factor of ~2 of their design sensitivity
- Currently focussing on:
  - upper limit studies
  - testing data analysis pipelines
  - detector characterization
- VIRGO is following very fast



Current sensitivities *12 Sep 05*

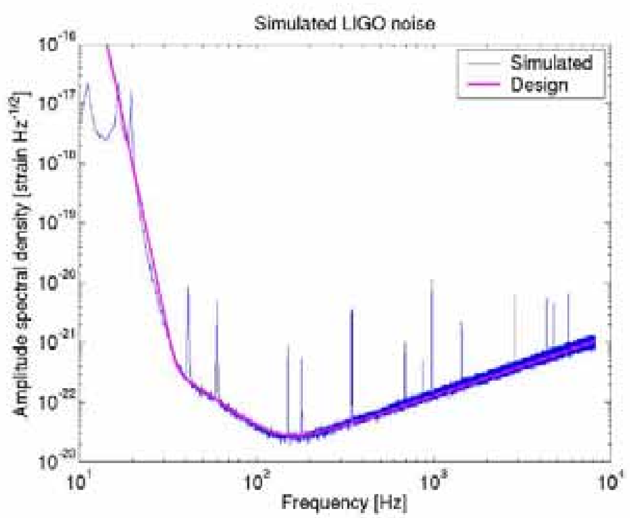


Figure 1. The LIGO design sensitivity curve and the spectrum of the simulated data.

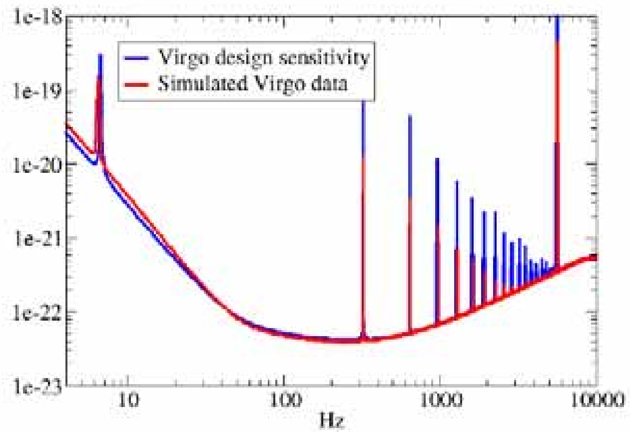
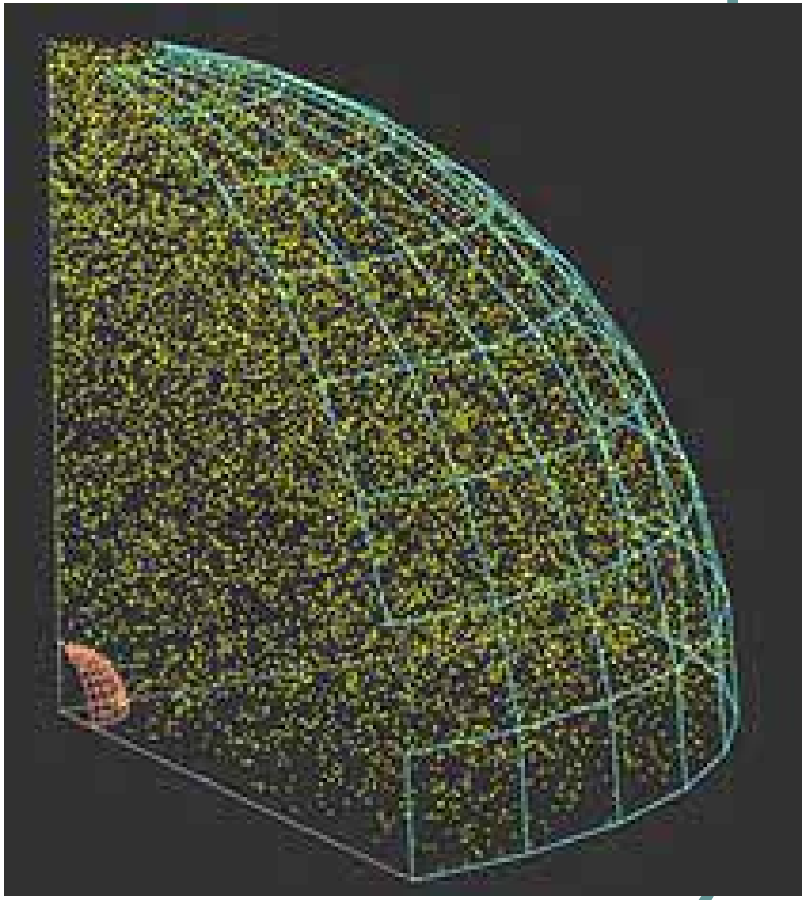


Figure 2. The Virgo design sensitivity curve and the spectrum of the simulated data.

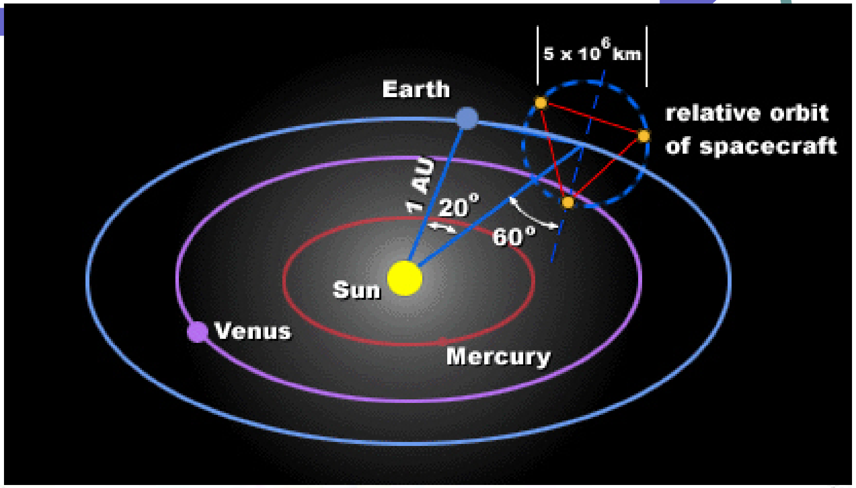
Future Gravitational Wave Antennas

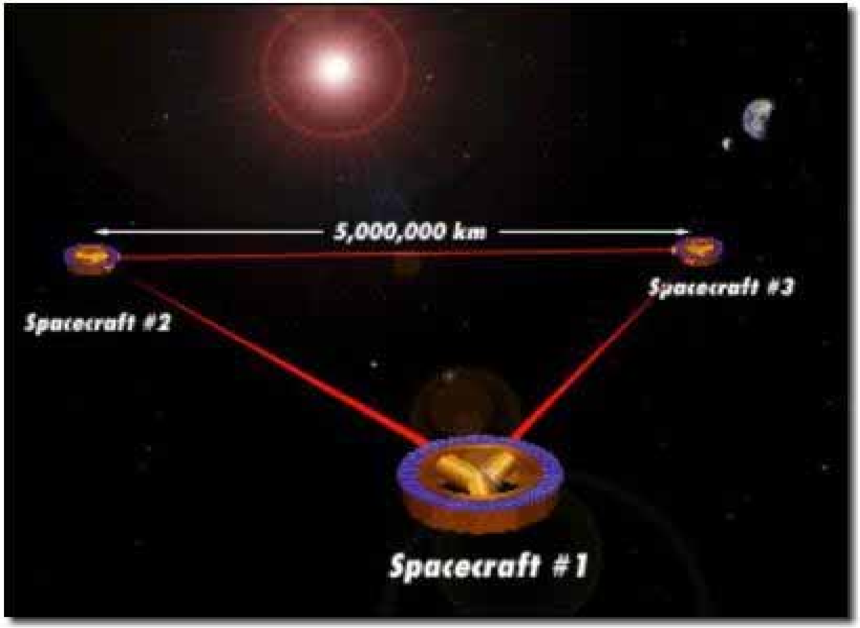
- **Advanced LIGO**
  - 2006-2008; planning under way
  - 10-15 times more sensitive than initial LIGO
- **High Frequency GEO**
  - 2008? Neutron star physics, BH quasi-normal modes
- **EGO: European Gravitational Wave Observatory**
  - 2012? Cosmology



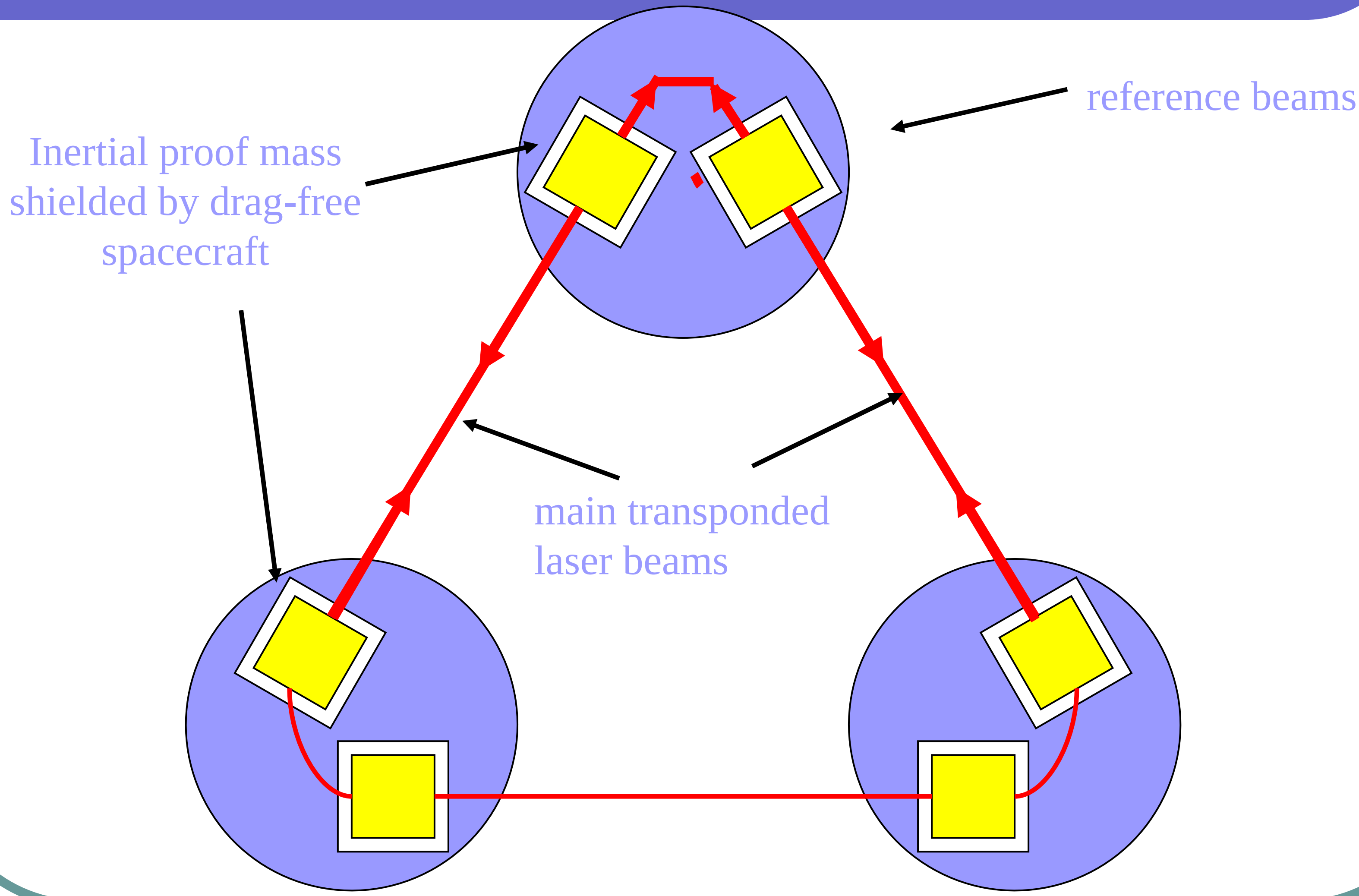
LISA the space interferometer

- LISA is low frequency detector.
- With arm lengths 5,000,000 km targets at 0.1mHz – 0.1Hz.
- Some sources are very well known (close binary systems in our galaxy).
- Some other sources are extremely strong (SM-BH binaries)
- LISA's sensitivity is roughly the same as that of LIGO, but at 10^5 times lower frequency.
- Since the gravitational waves energy flux scales as $F \sim f^2 \cdot h^2$, it has **10 orders** better energy sensitivity than LIGO.





LISA Outline Lay-Out

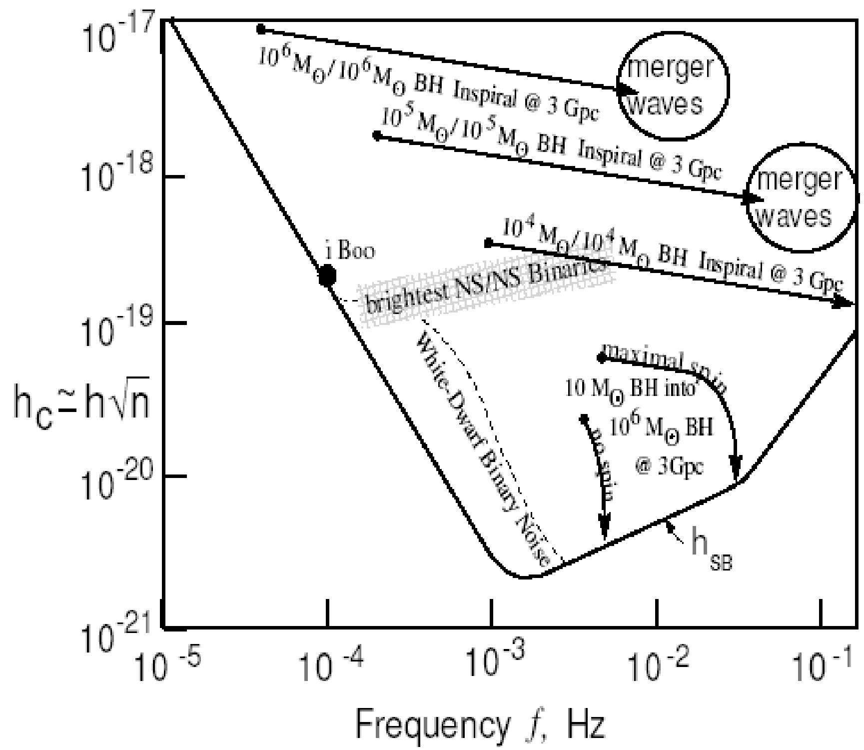


LISA Highlights

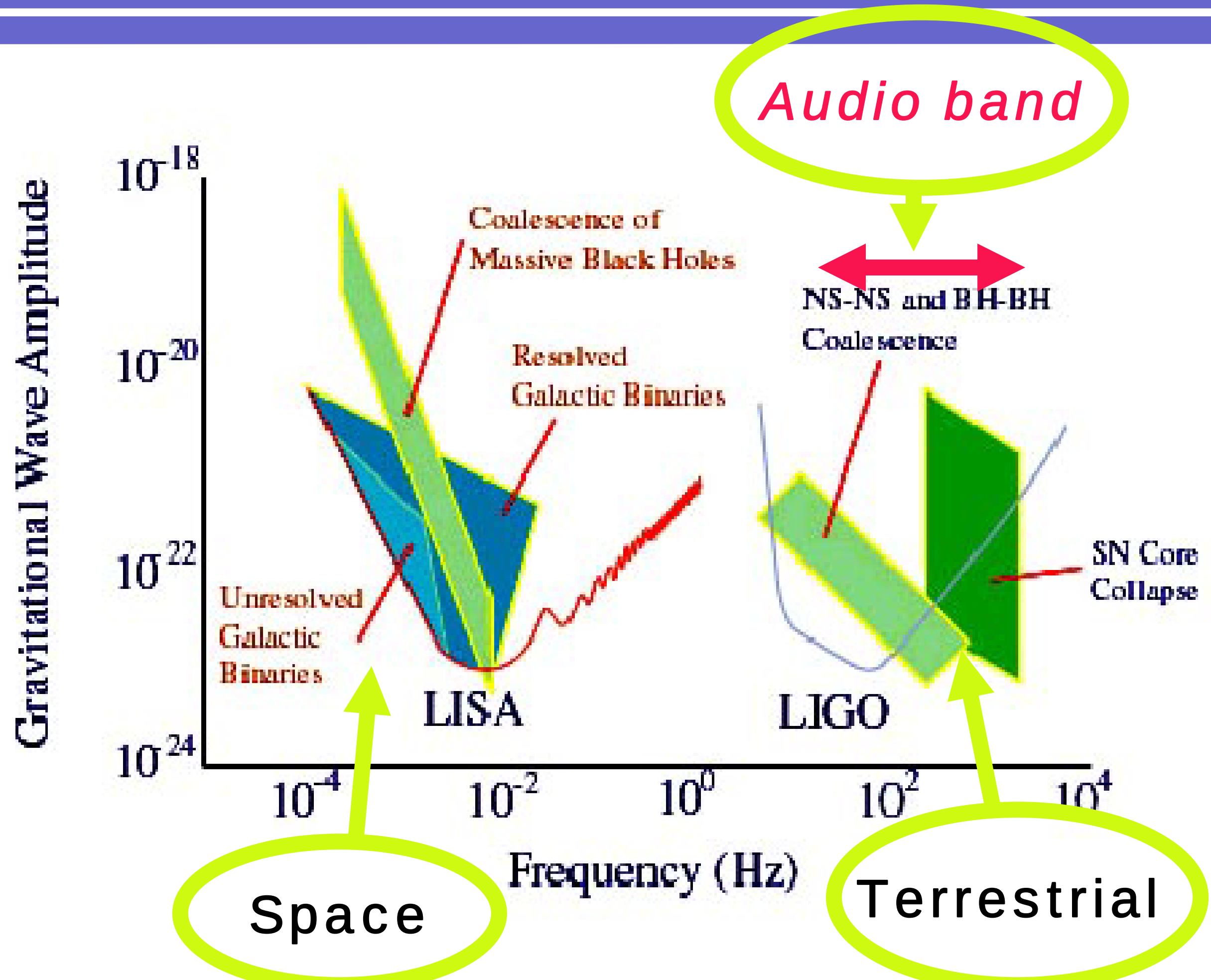
- Mapping the location of ***all*** binary compact objects in the galaxy (up to 1yr detection threshold).
- Detecting massive BH mergers anywhere in the universe
- Testing theories of MBH creation
- Compact objects around massive black holes (highly unequal mass systems)
  - allows probing of GR near the event horizon
  - easier to model and understand theoretically (perturbative approach)
  - as the compact object spirals in, it’s gravity radiation gives us a “map” of spacetime near the MBH horizon

LISA's Sensitivity

- At frequencies $f \gtrsim 10^{-3}$ Hz, LISA's noise is due to photon counting noise (shot noise).
- The sensitivity curve steepens at $f \sim 3 \times 10^{-2}$ Hz because at larger f the waves' period is shorter than the round-trip light travel time in each arm.
- For $f \lesssim 10^{-3}$ Hz, the noise is due to buffeting-induced random motions of the spacecraft, that cannot removed by the drag-compensation system.



Frequency range of astrophysics sources

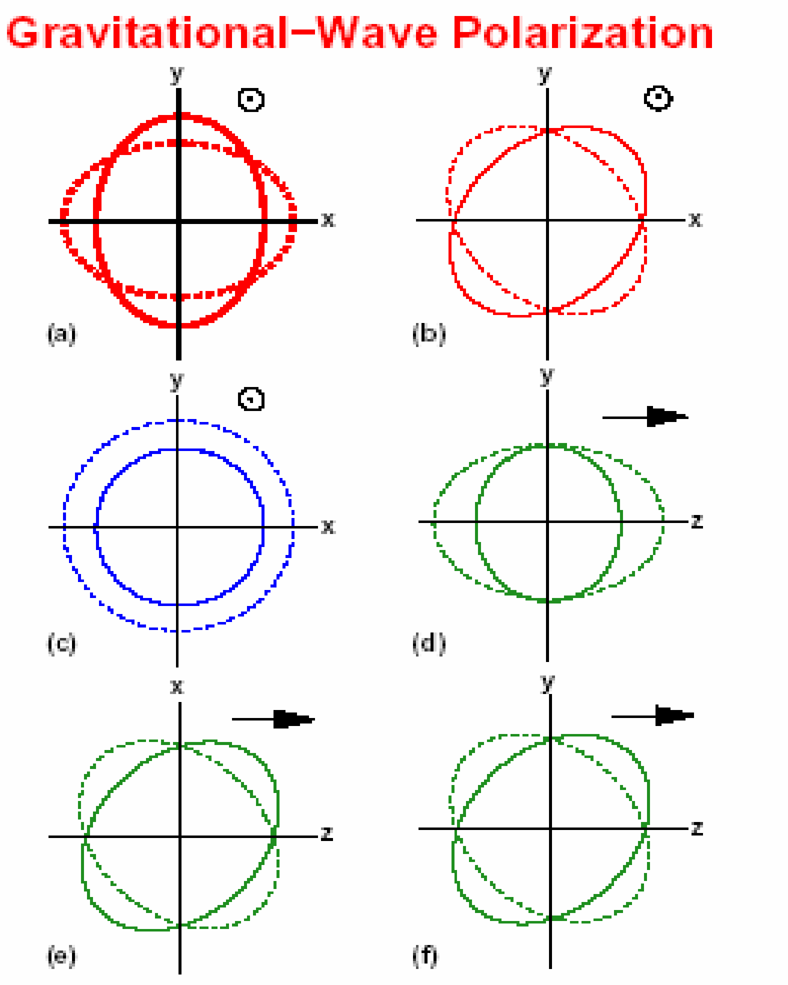


Gravitational Waves over ~8 orders of magnitude

Grav. Wave Polarization I

- A GW detector measures the local components of a symmetric 3x3 tensor, (the "electric" components of the Riemann tensor, R_{0i0j}.
- The 6 independent components can be expressed in terms of polarizations (modes with specific transformation properties under null rotations).
- **2** representing quadrupolar deformations
- **3** are transverse to the direction of propagation
- 1 representing a monopole ``breathing'' deformation.

- GR predicts only the first 2 transverse quadrupolar modes, independently of the source



The END of the 1st day