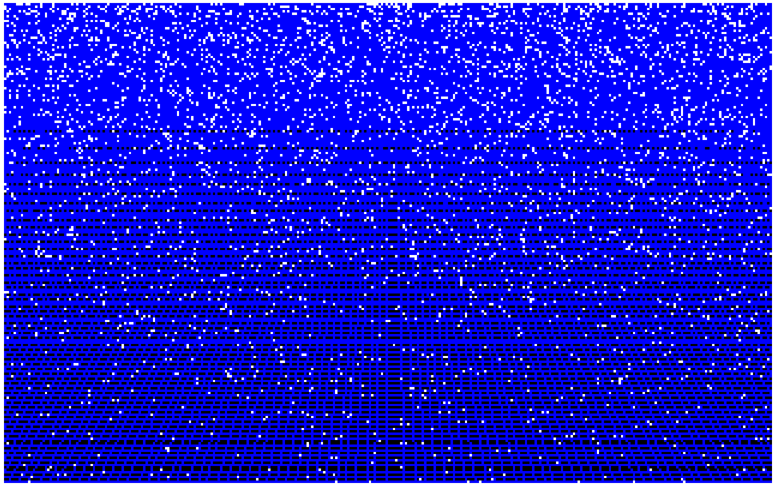
Deeltjes en velden

HOVO Cursus



Jo van den Brand

31 oktober 2013

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Overzicht

- **Docent informatie**
  - Jo van den Brand & Gideon Koekoek
  - Email: jo@nikhef.nl en gkoekoek@gmail.com
  - 0620 539 484 / 020 592 2000
- **Rooster informatie**
  - Donderdag 10:00 – 13:00, HG 08A-05 (totaal 10 keer)
  - Collegevrije week: 24 oktober 2013
- **Boek en website**
  - David Griffiths, *Introduction to Elementary Particles*, Wiley and Sons, ISBN 978-3-527-40601-2 (2008)
  - Zie website URL: www.nikhef.nl/~jo
- **Beoordeling**
  - Huiswerkopgaven 20%, tentamen 80%

Inhoud

- Inleiding
  - Deeltjes
  - Interacties
- Relativistische kinematica
  - Speciale relativiteitstheorie
  - Viervectoren
  - Energie en impuls
- Quantumfysica
  - Formalisme
  - Spin van deeltjes
- Structuur van hadronen
- Symmetrieën
  - Behoudwetten, quarkmodel
  - Symmetriebreking
- Veldentheorie
  - Lagrange formalisme
  - Feynman regels
- Quantumelektrodynamica
  - Diracvergelijking
- Quarks en hadronen
  - Quantumchromodynamica
- Elektrozwakke wisselwerking
- Higgs formalisme

Symmetrieën

Resume quantummechanica

- Systeem wordt beschreven door een golffunctie, Ψ
- Fysische observabele correspondeert met hermitische operator, *O*
- Verwachtingswaarden worden gegeven door eigenwaarden
- Tijdsafhankelijkheid wordt gegeven door de Schrödingervergelijking

$$< O >= \int \psi_a^* \mathbf{O} \psi_a dV.$$

$$\int (\mathbf{O}\psi)^* \phi dV = \int \psi^* \mathbf{O}^\dagger \phi dV,$$

$$i\hbar \frac{\partial \psi}{\partial t} = \mathbf{H}\psi$$

$$-i\hbar \frac{\partial \psi^*}{\partial t} = (\mathbf{H}\psi)^* = \psi^* \mathbf{H}$$

$$\begin{aligned} \frac{\partial}{\partial t} < \ O \ > &= \frac{\partial}{\partial t} \int \psi^* \mathbf{O} \psi dV \\ &= \int \left(\frac{\partial \psi^*}{\partial t} \mathbf{O} \psi + \psi^* \mathbf{O} \frac{\partial \psi}{\partial t} \right) dV \\ &= \frac{i}{\hbar} \int \quad [\mathbf{H}, \mathbf{O}] \equiv \mathbf{HO} - \mathbf{OH} = 0 \end{aligned}$$

< O > bewegingsconstante, als [**H**,**O**] = 0

$$\mathbf{H}\psi = E\psi \quad \text{en} \quad \mathbf{O}\psi = o\psi$$

Symmetrieën

Symmetrie transformatie U

- **tijdsonafhankelijk**
- **Unitair**
- **Commuteert met H**

$$\psi'(\vec{r},t) = \mathrm{U}\psi(\vec{r},t)$$

Invullen in SV

$$\mathrm{H} = \mathrm{U}^{-1}\mathrm{H}\mathrm{U} = \mathrm{U}^{\dagger}\mathrm{H}\mathrm{U}$$

$$[\mathrm{H},\ \mathrm{U}] = 0$$

Continue transformatie met generator **G**

$$\mathrm{U} = e^{i\epsilon \mathrm{G}} = 1 + i\epsilon\mathrm{G} + \frac{1}{2!}(i\epsilon\mathrm{G})^2 + ..$$

Er geldt

$$\mathrm{U}^{\dagger}\mathrm{U} = e^{-i\epsilon\mathrm{G}^{\dagger}}e^{i\epsilon\mathrm{G}} = 1 \quad \rightarrow \quad \boxed{\mathrm{G}^{\dagger} = \mathrm{G}}$$

G hermitisch ⇔ observabele

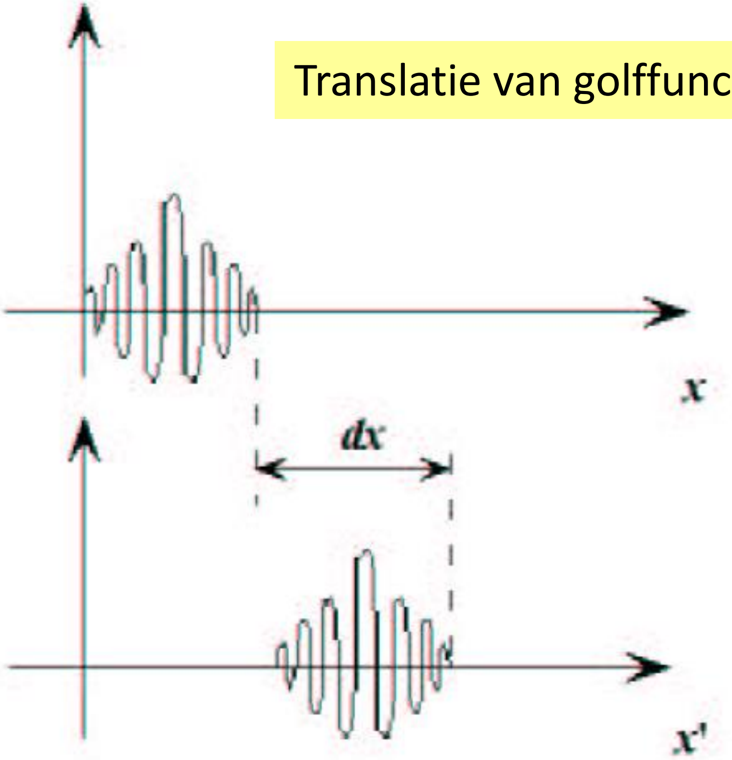
$$[\mathrm{H},\ \mathrm{U}] = 0 \quad \longrightarrow \quad \mathrm{H}(1 + i\epsilon\mathrm{G}) - (1 + i\epsilon\mathrm{G})\mathrm{H} = 0$$

$$[\mathrm{H},\ \mathrm{G}] = 0$$

Symmetrie ⇔ behoudswet

Behoud van impuls

Translatie van golffunctie van een deeltje



$$\begin{aligned}\psi'(x') = \psi(x-\epsilon) &= \psi(x) - \epsilon\frac{d\psi(x)}{dx} + .. \\ &= (1 + i\epsilon\mathbf{G} + ..)\psi(x)\end{aligned}$$

$$\mathbf{G} = i\frac{d}{dx} = -\frac{1}{\hbar}\mathrm{p_x}$$

impulsoperator

p_x is behouden

Algemeen translatieoperator

$$\mathrm{U}(\vec{a}) = \exp(-\frac{i}{\hbar}\vec{a}\cdot\vec{\mathbf{P}})$$

$$\begin{array}{llll} \vec{r}' & = \mathrm{U}\ \vec{r}\ \mathrm{U}^{-1} & = \vec{r} - \vec{a} & : \ \text{ruimtecordinaten} \\ \vec{p}' & = \mathrm{U}\ \vec{p}\ \mathrm{U}^{-1} & = \vec{p} & : \ \text{impulsen} \\ \vec{s}' & = \mathrm{U}\ \vec{s}\ \mathrm{U}^{-1} & = \vec{s} & : \ \text{spins} \end{array}$$

Behoud van lading

Verval van elektron:
$\tau > 4.6 \times 10^{26}$ jaar

$$\cancel{e \to \gamma + \nu_e}$$

Lading is veelvoud van *e*

$$q = Qe$$

$$a + b + .. + i \to c + d + .. + f$$

Additief quantumgetal

$$\sum Q_i = \sum Q_f$$

Ladingsbehoud ⇔ corresponderende symmetrie?

Golffunctie van deeltje met lading *q*

$$i\hbar \frac{\partial \psi_q}{\partial t} = \mathbf{H}\psi_q$$

Ladingsoperator **Q**

$$[\mathbf{H},\ \mathbf{Q}] = 0 \longrightarrow \mathbf{Q}\psi = q\psi$$

Bijbehorende symmetrie

$$\boxed{\psi'_q = e^{i\epsilon\mathbf{Q}}\psi_q} \longrightarrow i\hbar \frac{\partial \psi'_q}{\partial t} = \mathbf{H}\psi'_q$$

Ijktransformaties en ijkinvariantie

Globaal want ε = constant

Lokale ijksymmetrieën

Lading is in *elk* ruimtetijd punt behouden → lokale symmetrie

Unitaire groep U(1)

$$\psi_q' = e^{i\epsilon(\vec{r},t)\mathbf{Q}}\psi_q = e^{i\epsilon(x)\mathbf{Q}}\psi_q$$

$$i\hbar\frac{\partial\psi_q'}{\partial t} = \mathbf{H}\psi_q'$$

vrij deeltje

Schrödingervergelijking niet invariant, want

$$\frac{\partial\psi_q'}{\partial t} = e^{i\epsilon(x)\mathbf{Q}}\frac{\partial\psi_q}{\partial t} + \underline{e^{i\epsilon(x)\mathbf{Q}}\frac{\partial\epsilon}{\partial t}}\psi \neq e^{i\epsilon(x)\mathbf{Q}}\frac{\partial\psi_q}{\partial t}$$

Voer nieuw dynamische veld in en laat deeltje hiermee wisselwerken

$$\boxed{\begin{aligned} &\mathbf{B} = \nabla\times\mathbf{A} \quad \text{en} \quad \mathbf{E} = -\nabla\phi - \frac{\partial\mathbf{A}}{\partial t} \\ &\mathbf{A}' = \mathbf{A} + \nabla\epsilon, \quad \text{en} \quad \phi' = \phi + \frac{\partial\epsilon}{\partial t} \end{aligned}}$$

$$i\hbar\frac{\partial}{\partial t} \to i\hbar\frac{\partial}{\partial t} - q\phi, \qquad \text{en} \qquad \frac{\hbar}{i}\nabla \to \frac{\hbar}{i}\nabla - q\mathbf{A}$$

$$\left(i\hbar\frac{\partial}{\partial t} - q\phi\right)\psi_q = \frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\mathbf{A}\right)^2\psi_q$$

Minimale substitutie

$$\boxed{\begin{aligned} \left(i\hbar\tfrac{\partial}{\partial t} - q\phi'\right)\psi_q' &= e^{i\epsilon\mathbf{Q}}\left(i\hbar\tfrac{\partial}{\partial t} - q\phi\right)\psi_q, \\ \tfrac{1}{2m}\left(\tfrac{\hbar}{i}\nabla - q\mathbf{A}'\right)^2\psi_q' &= e^{i\epsilon\mathbf{Q}}\tfrac{1}{2m}\left(\tfrac{\hbar}{i}\nabla - q\mathbf{A}\right)^2\psi_q \end{aligned}}$$

$$i\hbar\frac{\partial\psi_q}{\partial t} = \frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\mathbf{A}\right)^2\psi_q + \underline{q\phi\psi_q}$$

Behoud van baryongetal

Proton is ook stabiel
$\tau > 2.1 \times 10^{29}$ jaar

Baryongetal $B = 1$
voor p en n

quarks $B = 1/3$
antiquarks $B = -1/3$

Leptonen, mesonen
hebben $B = 0$

$$\begin{array}{lll}
p & \rightarrow e^+\pi^0 & \tau > 5.5 \times 10^{32} \text{ jaar} \\
p & \rightarrow \mu^+\pi^0 & \tau > 2.7 \times 10^{32} \text{ jaar} \\
p & \rightarrow e^+\gamma & \tau > 4.6 \times 10^{32} \text{ jaar} \\
p & \rightarrow e^+ \text{ wat dan ook} & \tau > 0.6 \times 10^{30} \text{ jaar}
\end{array}$$

$$\boxed{\begin{array}{ccccccccc}
 & & n & \rightarrow & p & + & e^- & + & \bar{\nu}_e \\
 & & & & & & & & \\
Q & : & 0 & = & 1 & - & 1 & + & 0 \\
B & : & 1 & = & 1 & + & 0 & + & 0 \\
\hline
L_e & : & 0 & = & 0 & + & 1 & - & 1
\end{array}}$$

Behoud van leptongetal

Observatie: verval treedt in paren op: e^+e^-, $\mu^+\mu^-$, $\tau^+\tau^-$

Voer leptongetallen in

L_e, L_μ, L_τ

$$
\begin{array}{lcccccccc}
 & n & \rightarrow & p & + & e^- & + & \bar{\nu}_e \\
 & {}^3\mathrm{H} & \rightarrow & {}^3\mathrm{He} & + & e^- & + & \bar{\nu}_e \\
L_e & : & 0 & = & 0 & + & 1 & - & 1.
\end{array}
$$

$$
\begin{array}{lccccccccl}
 & p & \rightarrow & n & + & e^+ & + & \nu_e & & \text{in kernen} \\
\text{bijvoorbeeld} & {}^{35}\mathrm{Ar} & \rightarrow & {}^{35}\mathrm{Cl} & + & e^+ & + & \nu_e & & \text{typisch } \beta^+ - \text{verval} \\
L_e \quad : & 0 & = & 0 & - & 1 & + & 1 & &
\end{array}
$$

$$
\begin{array}{lcccccccc}
\text{of} & {}^{37}\mathrm{Ar} & + & e^- & \rightarrow & {}^{37}\mathrm{Cl} & + & \nu_e & \\
L_e \quad : & 0 & + & 1 & = & 0 & + & 1 & .
\end{array}
$$

spin neutrino: 1/2

Verschil tussen neutrino's en antineutrino's?

Cowan en Reines: detectie neutrino

Basisidee van experiment

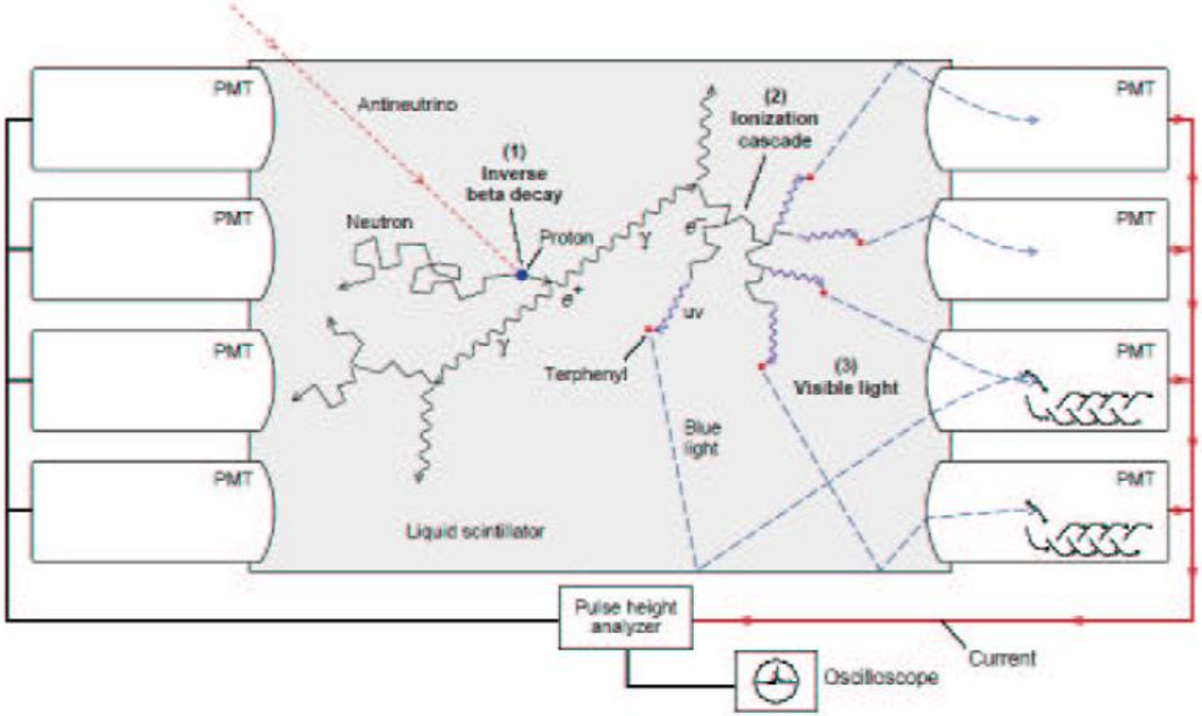
- Gebruik antineutrino’s
- Kernreactor als bron (700 MW)
- Detector: 200 liter water + $CdCl_2$
- Vloeistofscintillatoren (3 - 1400 liter elk)

$$\begin{array}{lccccccccc} & \bar{\nu}_e & + & p & \rightarrow & n & + & e^+ \\ L_e \ : & -1 & + & 0 & \rightarrow & 0 & - & 1. \end{array}$$

$$\boxed{e^+ + e^- \rightarrow 2\gamma}$$

Resultaat:

$$< \ \sigma \ >= (12^{+7}_{-4}) \times 10^{-44} \ \ \mathrm{cm}^2$$



Davis: detectie neutrino

Bij dezelfde reactor gemeten: reactie verloopt niet!

$$\begin{array}{lccccccccc} & & \bar{\nu}_e & + & {}^{37}\mathrm{Cl} & \rightarrow & {}^{37}\mathrm{Ar} & + & e^- \\ L_e & : & -1 & + & 0 & \rightarrow & 0 & + & 1! \end{array}$$

Neutrino's van de zon induceren de reactie:

$$\begin{array}{lccccccccc} & & \nu_e & + & {}^{37}\mathrm{Cl} & \rightarrow & {}^{37}\mathrm{Ar} & + & e^- \\ L_e & : & 1 & + & 0 & \rightarrow & 0 & + & 1! \end{array}$$

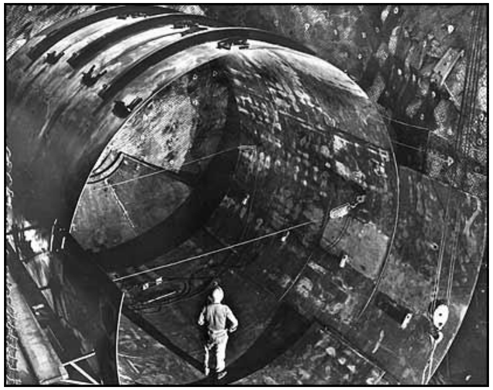
Neutrino's en antineutrino's zijn verschillende deeltjes

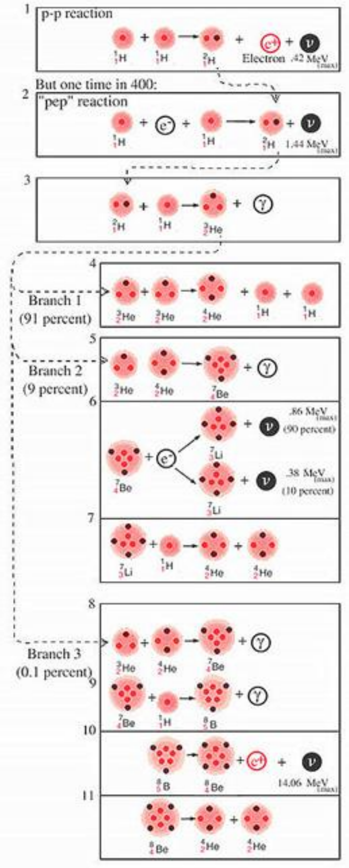
Echter: flux factor 2 – 3 te laag!
Opgelost: neutrino oscillaties!
Neutrino's hebben massa!!!

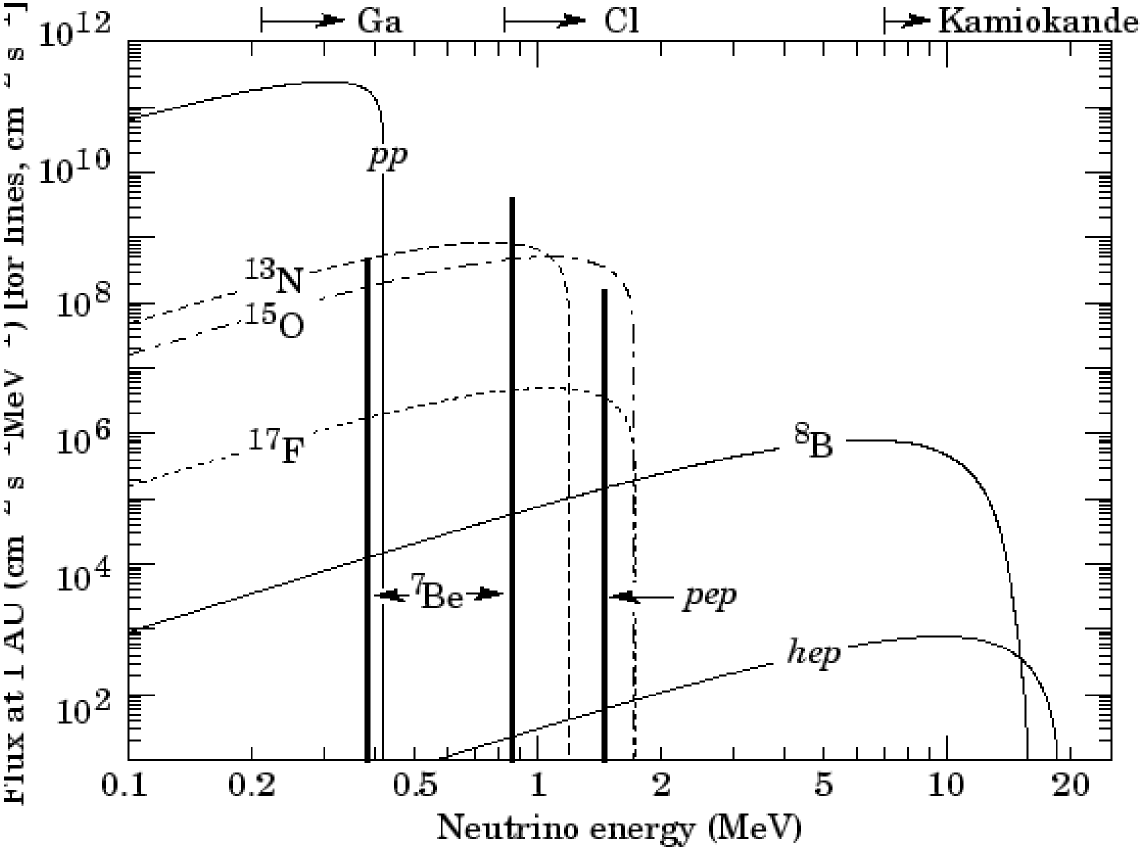
Neutrino’s van de zon

Specifiek energiespectrum van neutrino’s
Drempelenergie verschilt per detectiemedium

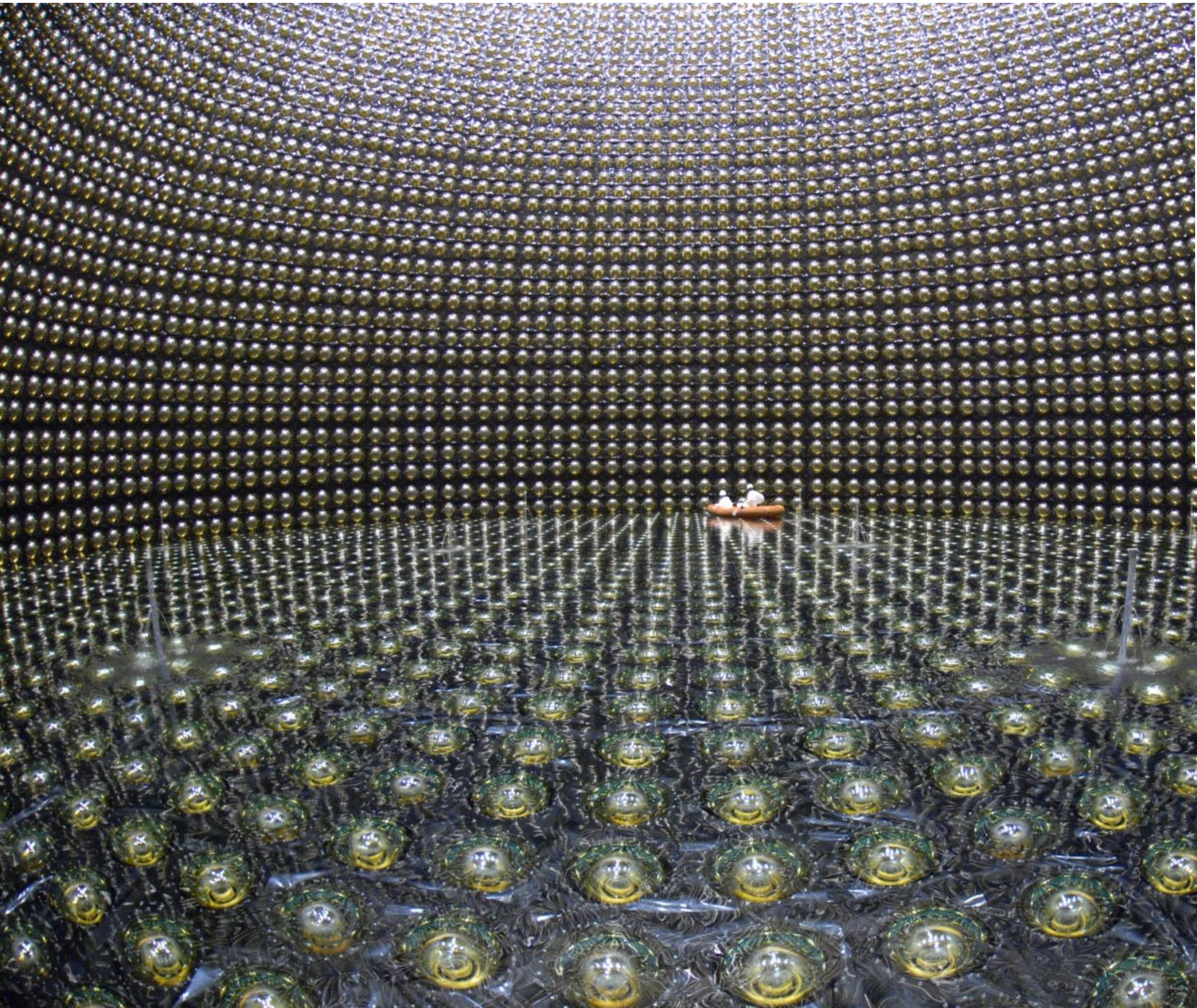
Eerste experimenten gebruikten $\mathrm{Cl}(\nu_e, e^-)\mathrm{Ar}$
Ray Davis, Homestake, South Dakota

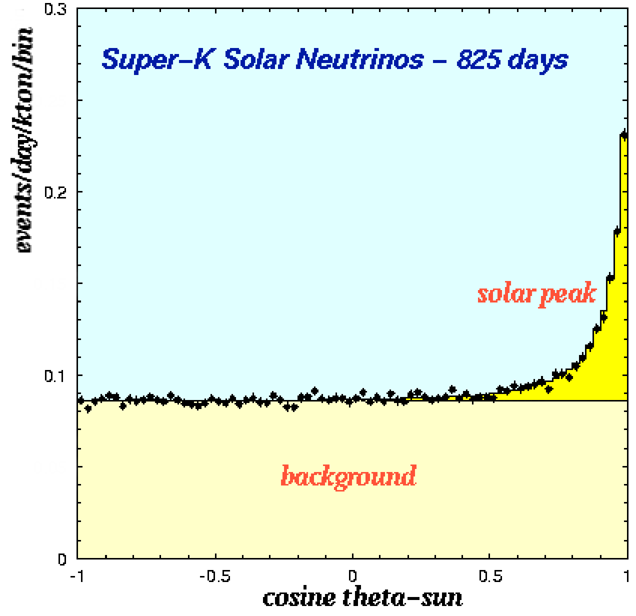




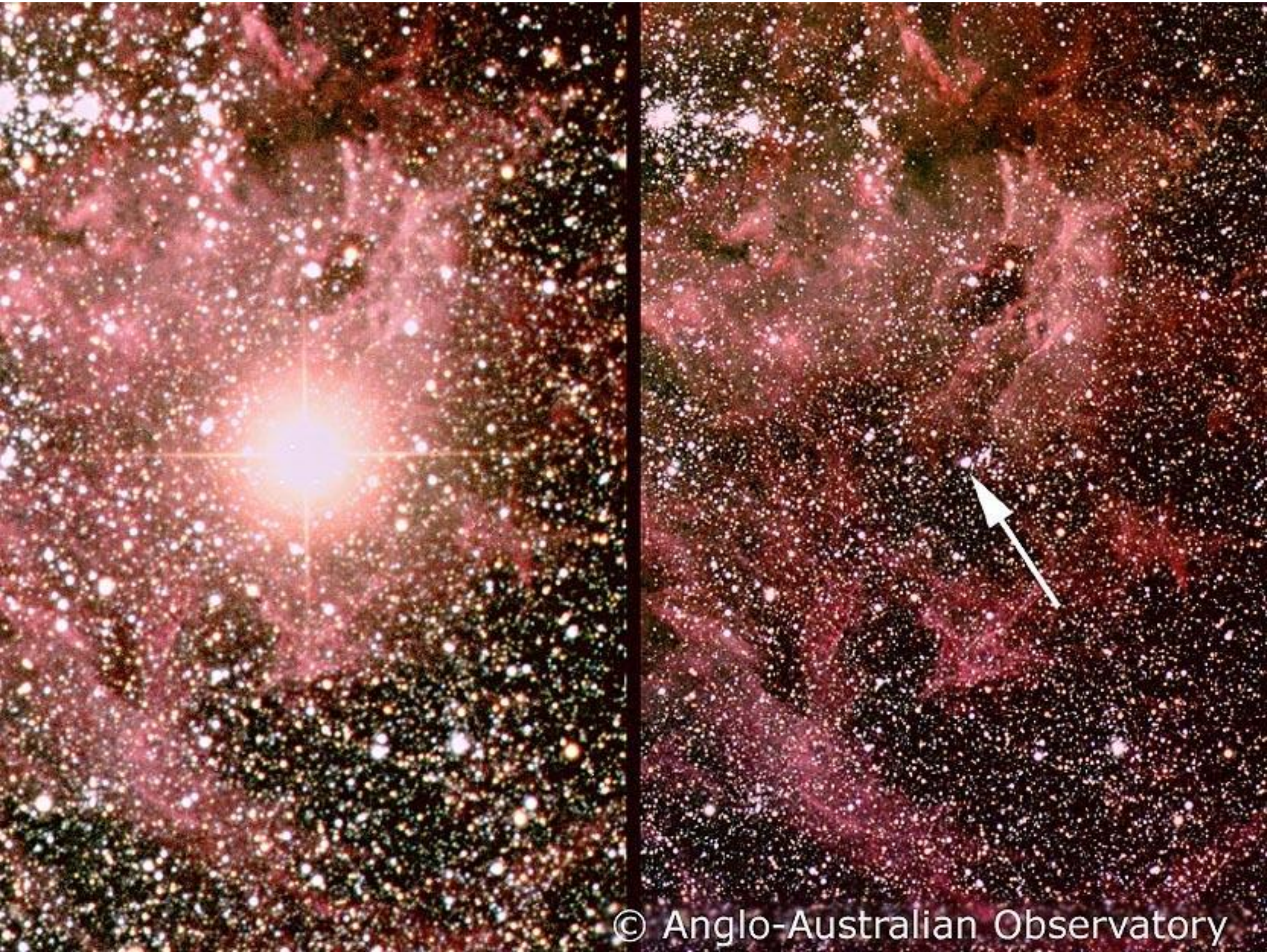


Superkamiokande





SN1987A



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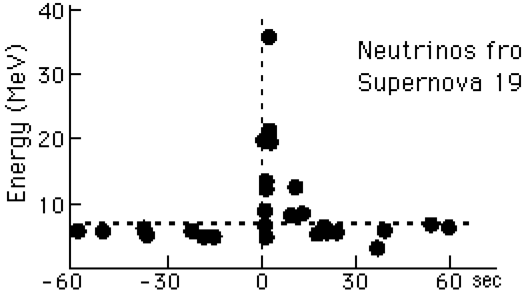
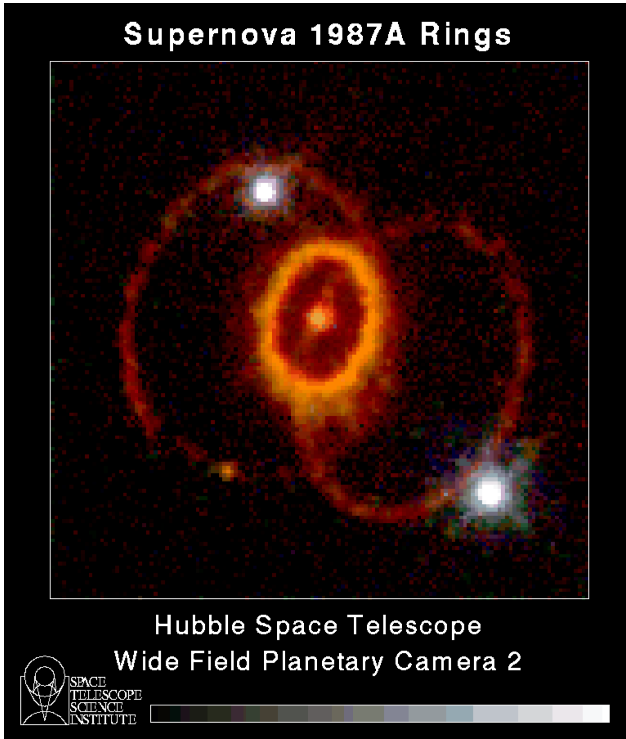
Superkamiokande

Detection of neutrino’s from SN1987A

Spectrum in agreement with supernova models

Limit on mass of neutrino





Borexino in Gran Sasso

Detect low energy (< 1 MeV) neutrino’s from decay of ^{7}Be

Check MSW effect: neutrino oscillations are affected by matter due to the presence of electrons

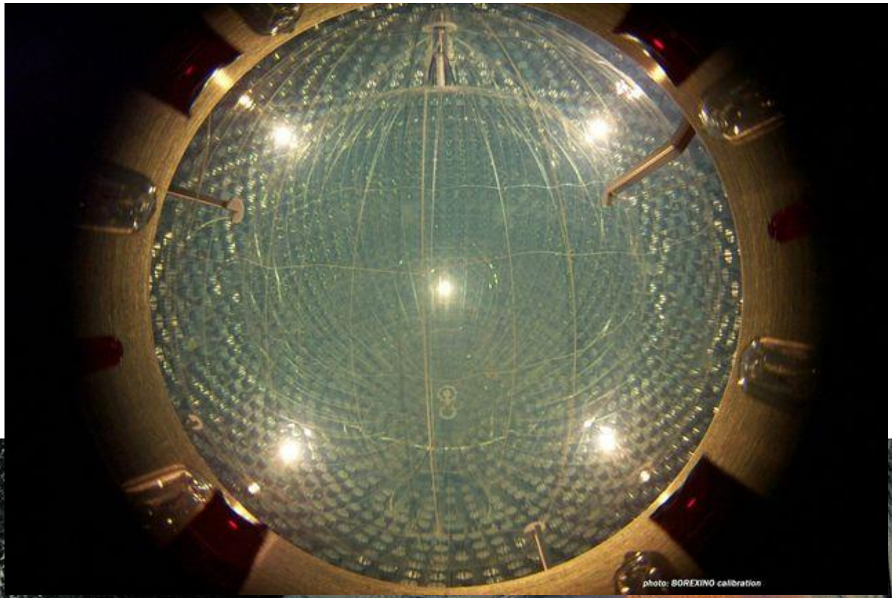


photo: BOREXINO calibration

OPERA in Gran Sasso

Fire muon neutrino's from SPS at CERN to OPERA in Gran Sasso

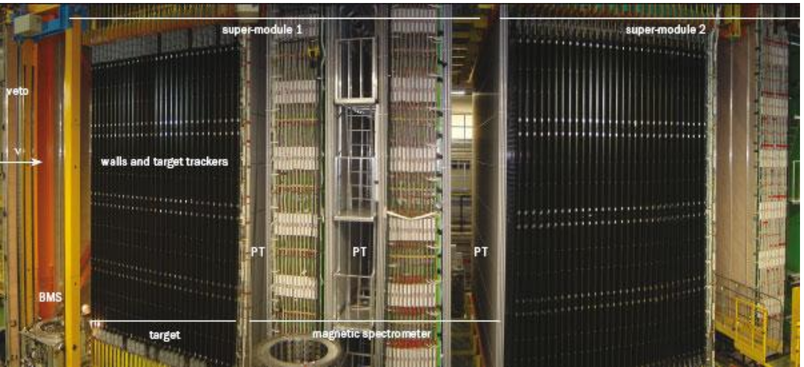
Detect appearance of tau neutrino's

- May 31, 2010
- June 6, 2012
- March 26, 2013

Emulsion in lead sheets and scintillator trigger planes

Neutrino's travel faster than speed of light

- Fiber optic cable problem
- Incorrect clock
- Claim withdrawn in July 2012

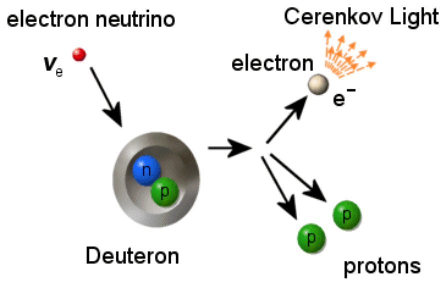


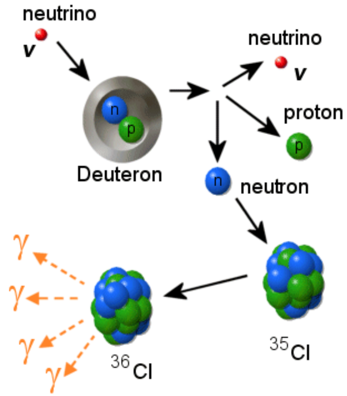
Sudbury Neutrino Observatory

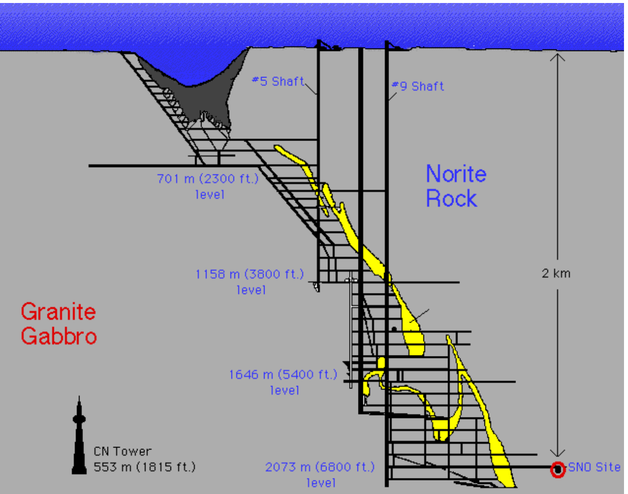
Heavy water – 1000 tons

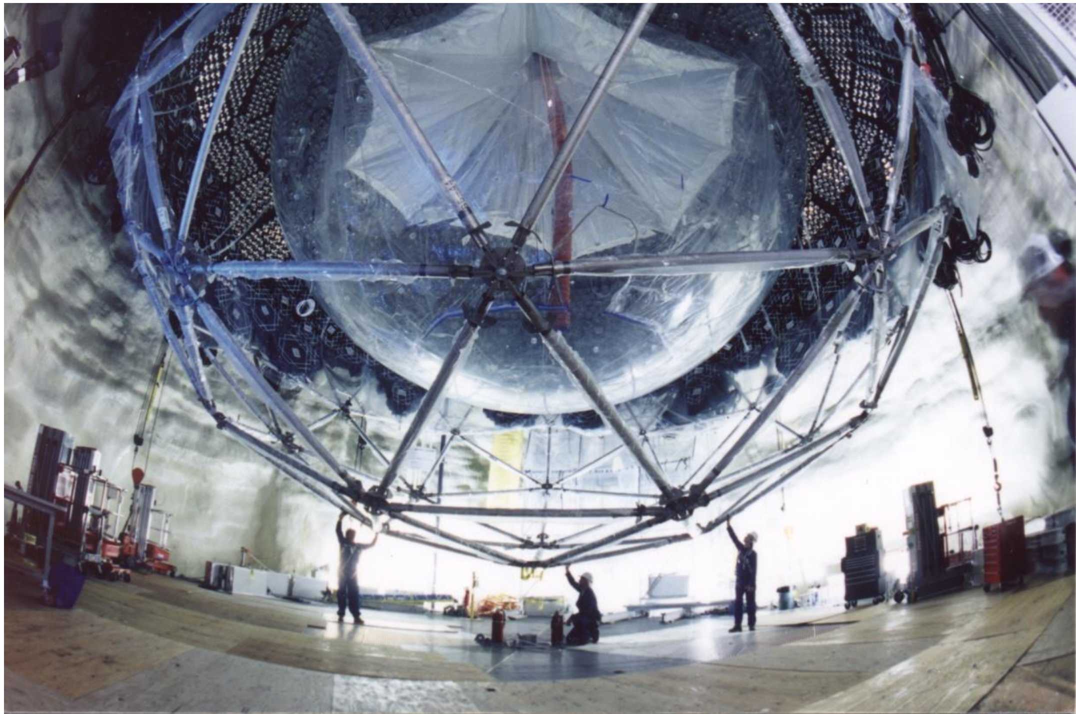
Electron-neutrino converts neutron into proton and electron. Cherenkov radiation from electron is detected

All neutrino species can break-up the deuterium nucleus. The neutron is captured forming tritium and a 6 MeV gamma







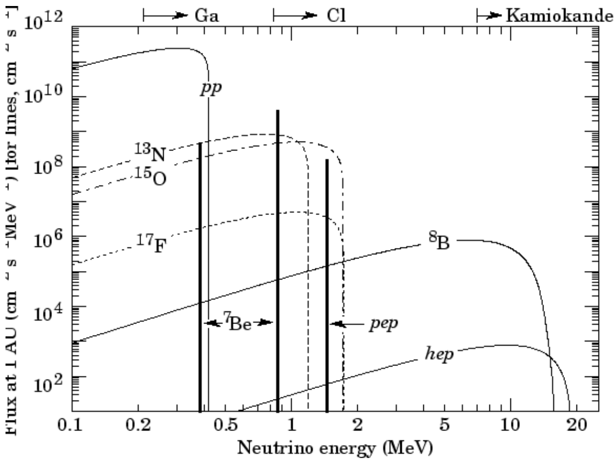


Solar neutrino problem

Early (Cl) experiments (1968) showed that the sun did not produce enough neutrino's (by about a factor 3). SNU units are used…

Kamiokande (water; designed for proton decay) sees relatively more neutrino's than the Cl detectors. Problems with expected energy distribution?

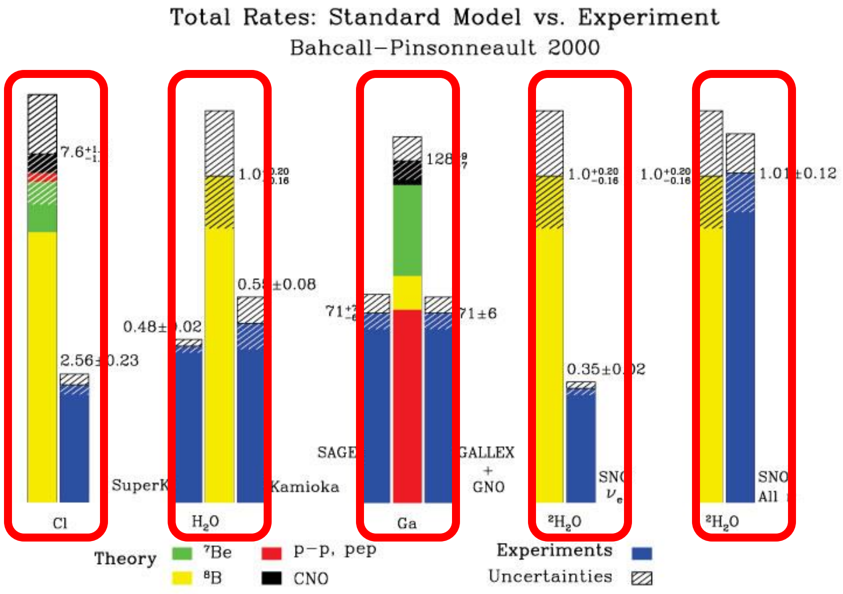
Gallium data accounted for by pep and hep



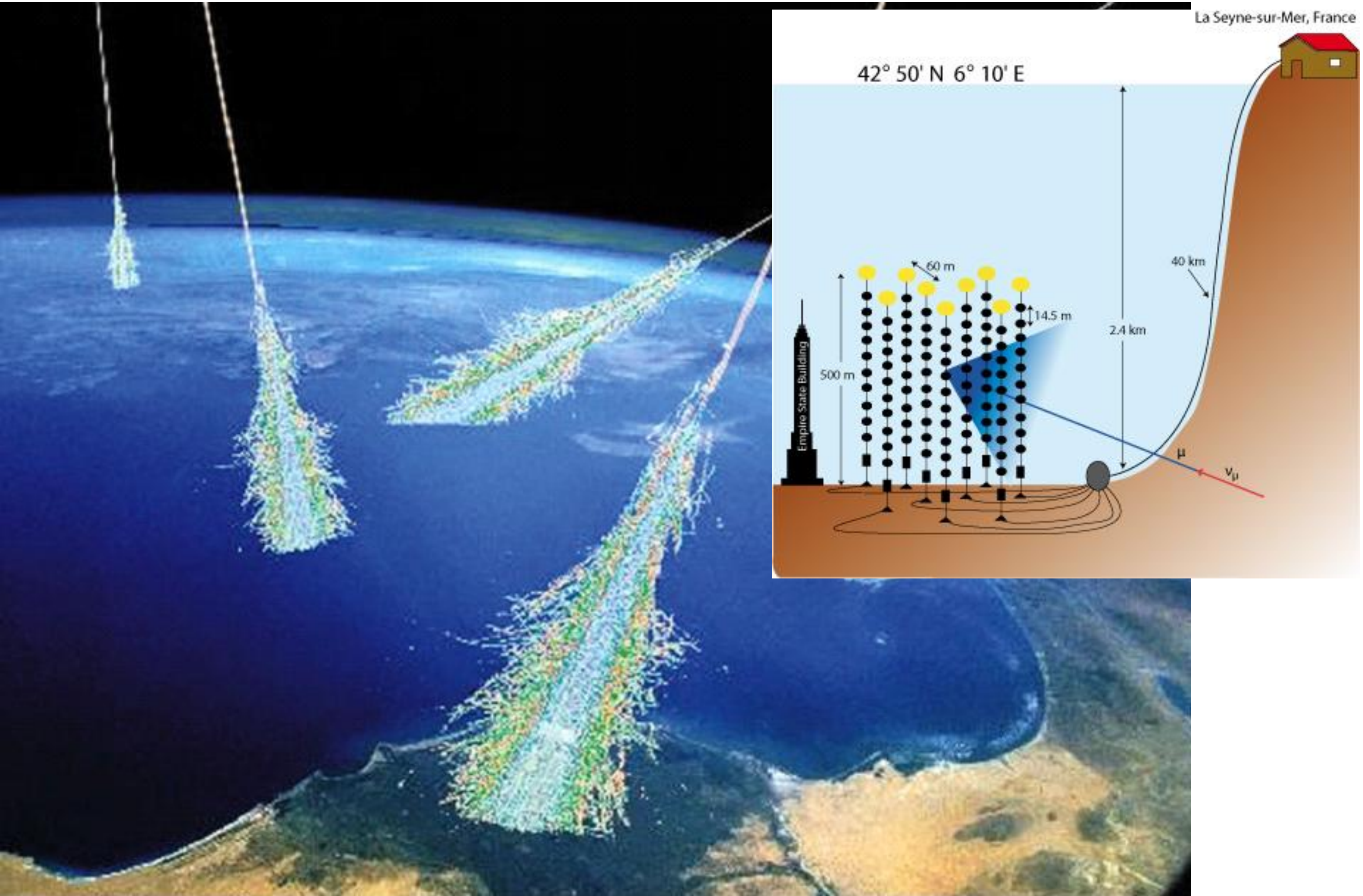
SNO electron neutrino's account for one third of events

SNO is sensitive to all neutrino flavors (through NC interactions) and with all neutrino's in agreement with solar model

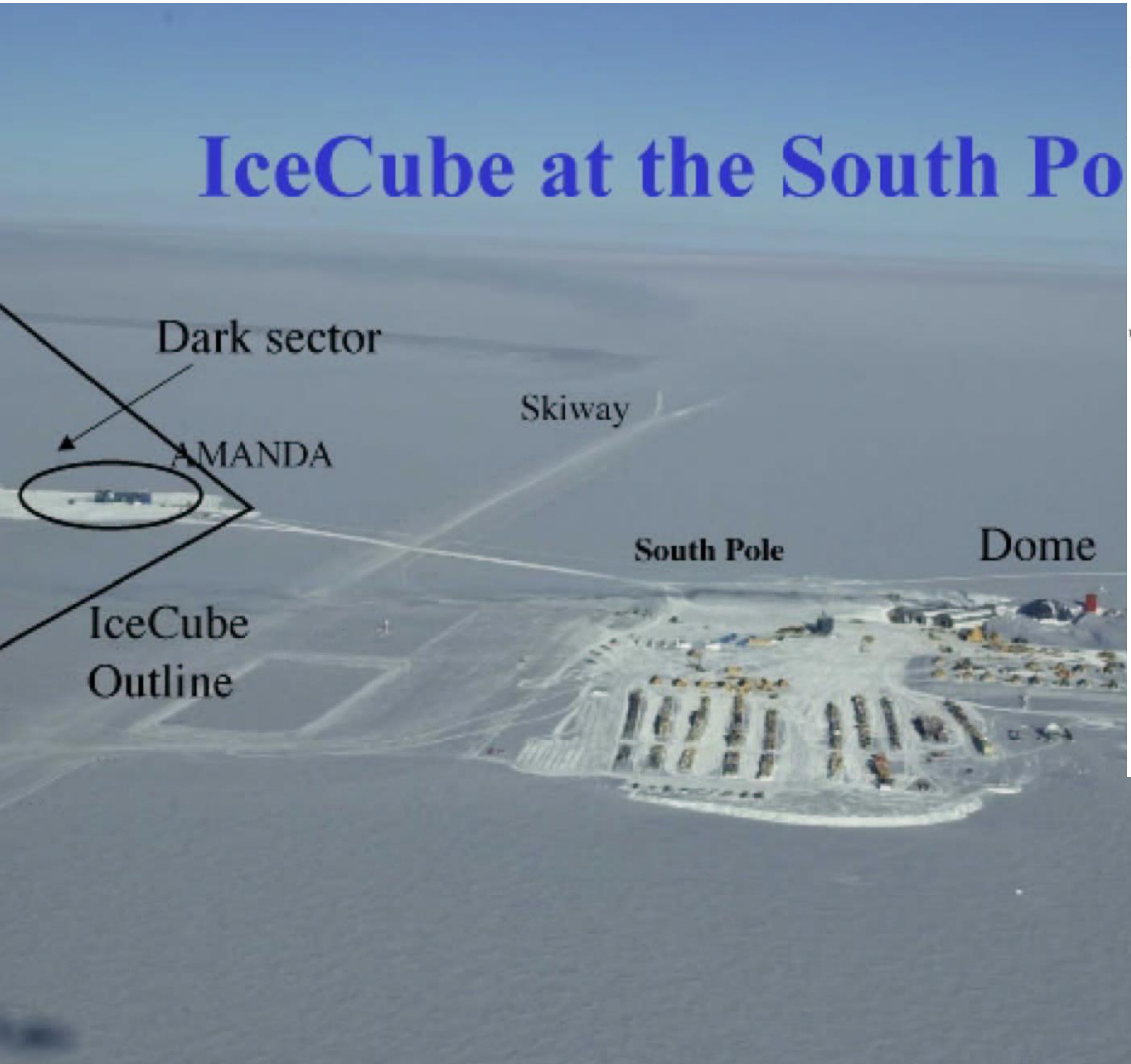
Neutrino's oscillate! (thus have mass)

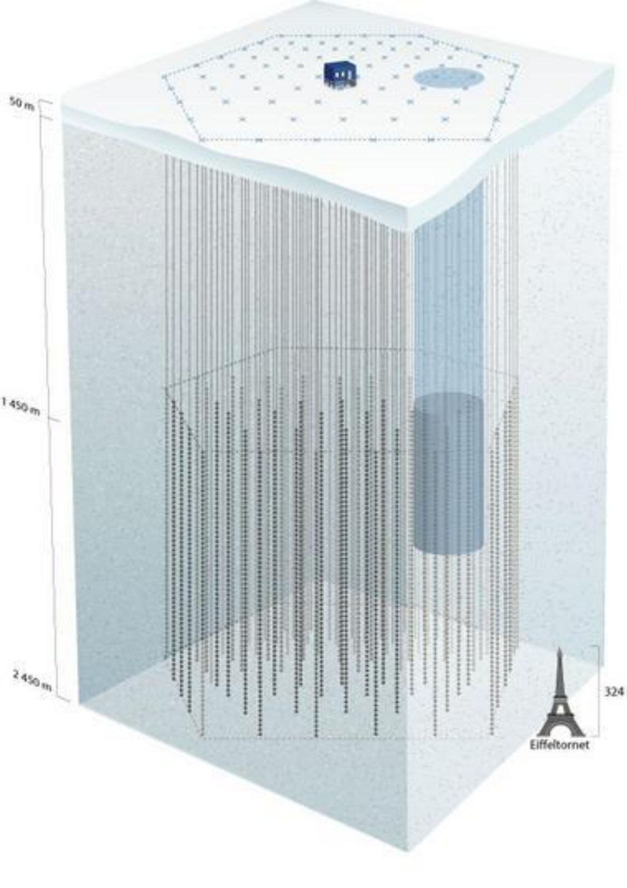


Kosmische neutrino's – Antares en KM3NET



Kosmische neutrino's – Amanda en Icecube





Zware neutrino's

Pion verval

$$\begin{aligned} \pi^+ &\to \mu^+ + \nu_\mu \qquad \text{B.R.} &= 0.999878 \\ \pi^+ &\to e^+ + \nu_e \qquad \text{B.R.} &= 1.2 \times 10^{-4} \end{aligned}$$

Antideeltjes hebben dezelfde levensduur en B.R.

$$\begin{aligned} \pi^- &\to \mu^- + \bar{\nu}_\mu \\ \pi^- &\to e^- + \bar{\nu}_e . \end{aligned}$$

Reacties

$$\begin{array}{llccccccccl} & & \nu_e & + & n & \to & p & + & e^- & & \\ L_e & : & 1 & + & 0 & = & 0 & + & 1 & & \underline{\text{treedt op}} \\ L_\mu & : & 0 & + & 0 & = & 0 & + & 0 & & \\ & & & & & & & & & & \\ & & \nu_\mu & + & n & \to & p & + & e^- & & \\ L_e & : & 0 & + & 0 & \neq & 0 & + & 1 & & \text{treedt niet op} \\ L_\mu & : & 1 & + & 0 & \neq & 0 & + & 0 & & \\ & & & & & & & & & & \\ & & \nu_\mu & + & n & \to & p & + & \mu^- & & \\ L_e & : & 0 & + & 0 & = & 0 & + & 0 & & \underline{\text{treedt op}} \\ L_\mu & : & 1 & + & 0 & = & 0 & + & 1 & & \end{array}$$

B.R. < 10^{-12}

$$\mu^- \to e^- + e^+ + e^-$$

$$\begin{aligned} \mu^- + {}^{32}\text{S} &\to e^- + {}^{32}\text{S}, \quad \sigma/\sigma_{\nu,\ \text{capture}} < 7 \times 10^{-11} \\ \mu^- + {}^{32}\text{S} &\to e^+ + {}^{32}\text{Si}, \quad \sigma/\sigma_{\nu,\ \text{capture}} < 9 \times 10^{-10} \end{aligned}$$

treedt niet op

Spiegeling in de ruimte en pariteit

Unitaire pariteitsoperatie

$$\begin{aligned} \vec{\mathbf{r}}' &= \mathcal{P}\,\vec{\mathbf{r}}\,\mathcal{P}^{-1} &&= -\vec{\mathbf{r}} \qquad & \vec{L}' &= (-\vec{r}\times(-\vec{p})) &&= \vec{L} \\ \vec{\mathbf{p}}' &= \mathcal{P}\,\vec{\mathbf{p}}\,\mathcal{P}^{-1} &&= -\vec{\mathbf{p}} \qquad & \vec{s}' &= \vec{s}. \end{aligned}$$

Andere quantumgetallen, *Q, B, …* blijven gelijk

Indien spiegelinvariant

$$[\mathbf{H},\ \mathcal{P}] = 0 \longrightarrow \begin{aligned} \mathbf{H}\psi &= E\psi \\ \mathcal{P}\psi &= \pi\psi. \\ \pi = \pm 1 \end{aligned}$$

Gezamenlijke eigentoestanden

Vb. waterstofatoom potentiaal sferisch symmetrisch

$$\mathbf{H}(\vec{r}) = \mathbf{H}(-\vec{r}) = \mathbf{H}(r) \longrightarrow [\mathbf{H}, \mathcal{P}] = 0$$

$$\psi(r,\vartheta,\varphi) = \chi(r)Y_l^m(\vartheta,\varphi)$$

Pariteit golffuncties $(-1)^l$

niet-ontaarde systemen

Pariteitschending in β-verval

C.S. Wu *et al.* *(1957)*

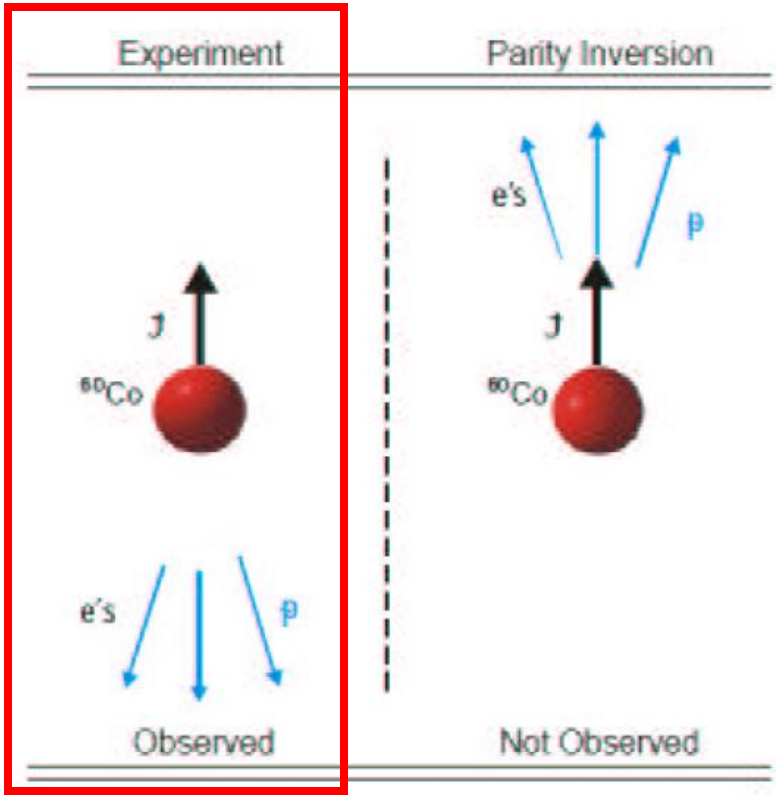
Gepolariseerde kern – ^{60}C

$J^\pi = 5^+ \longrightarrow J^\pi = 4^+$

$E(M) = E_0 - g\mu_N BM$

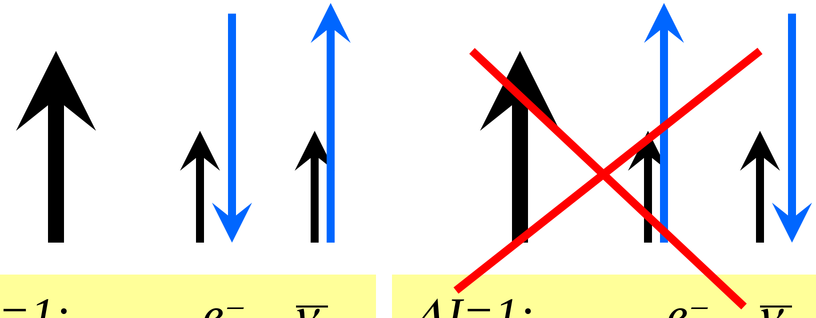
$$\frac{n(M')}{n(M)} = \frac{e^{-E(M')/kT}}{e^{-E(M)/kT}} = e^{\frac{(M'-M)g\mu_N B}{kT}}$$

Detecteer $\boldsymbol{p}_e$



⇒

Pseudoscalaire grootheid $< \underline{\boldsymbol{p}_e \cdot J\ >}$

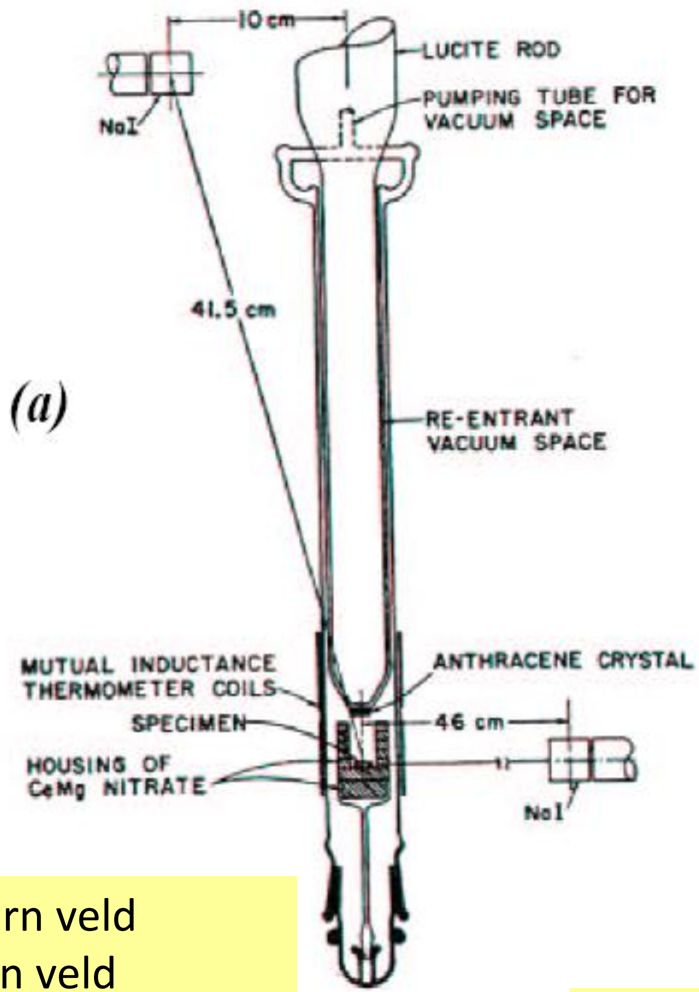


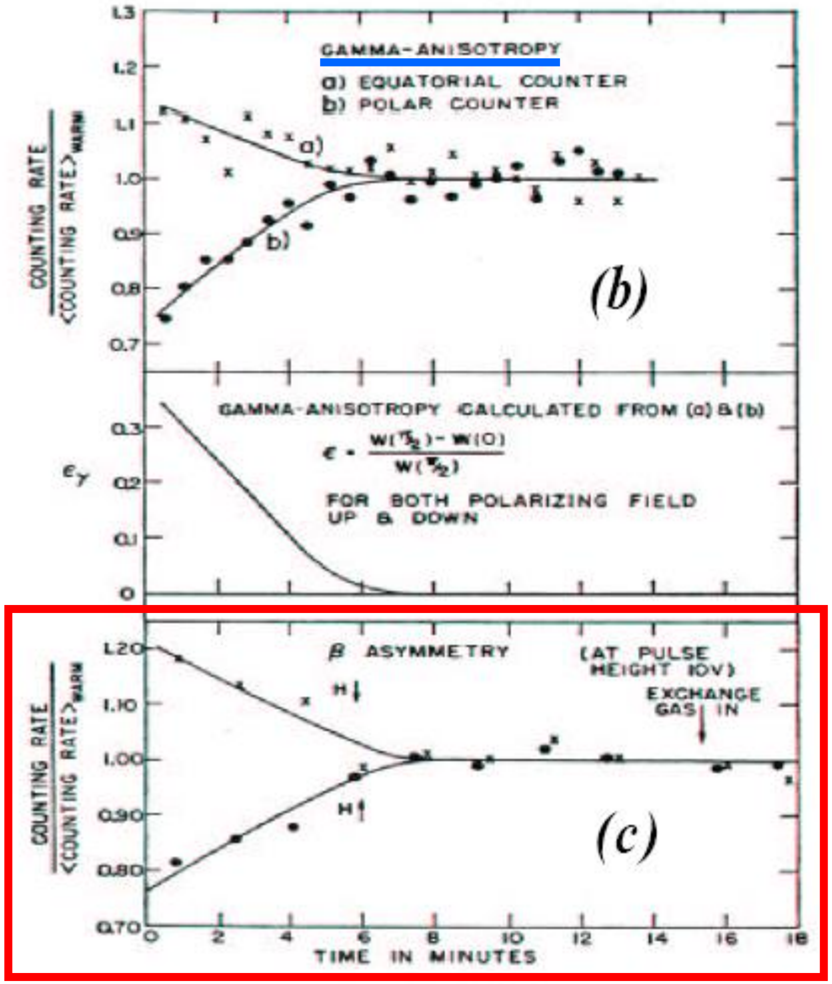
$\Delta J=1$: e^- $\bar{\nu}_e$

$\Delta J=1$: e^- $\bar{\nu}_e$

Pariteitschending in β-verval

C.S. Wu *et al.*





0.05 T extern veld
100 T intern veld

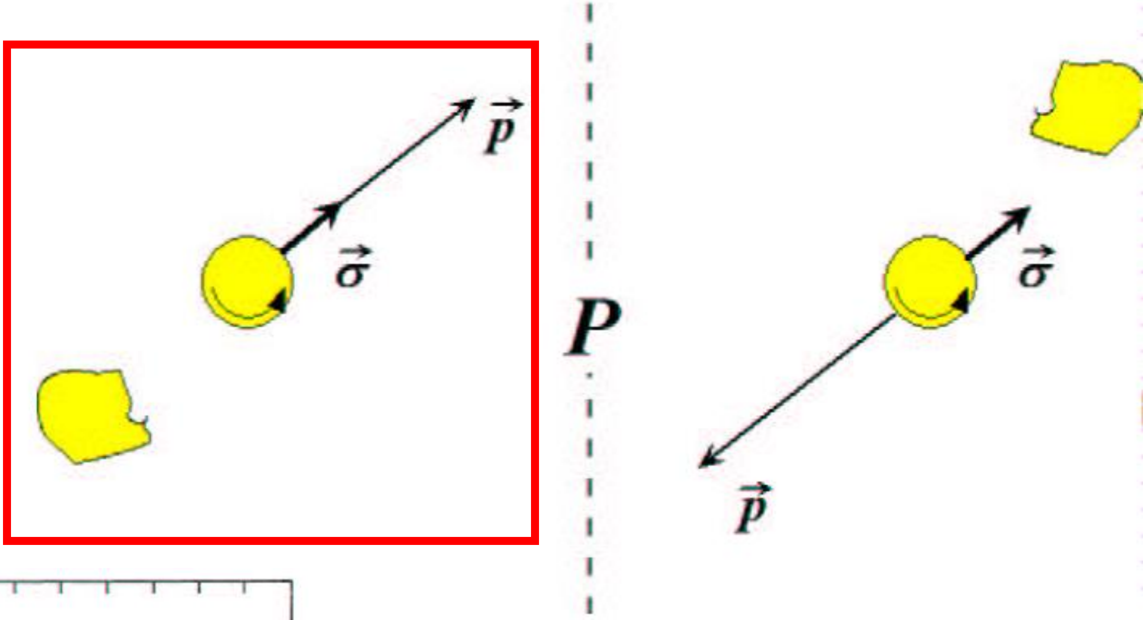
Adiabatische demagnetisatie: 10 mK

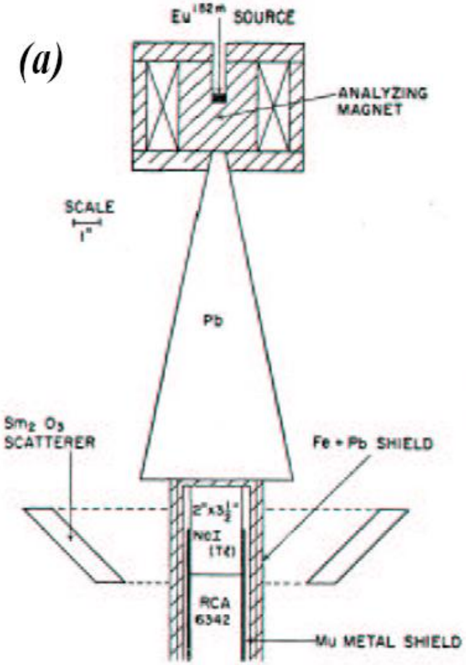
Heliciteit van leptonen

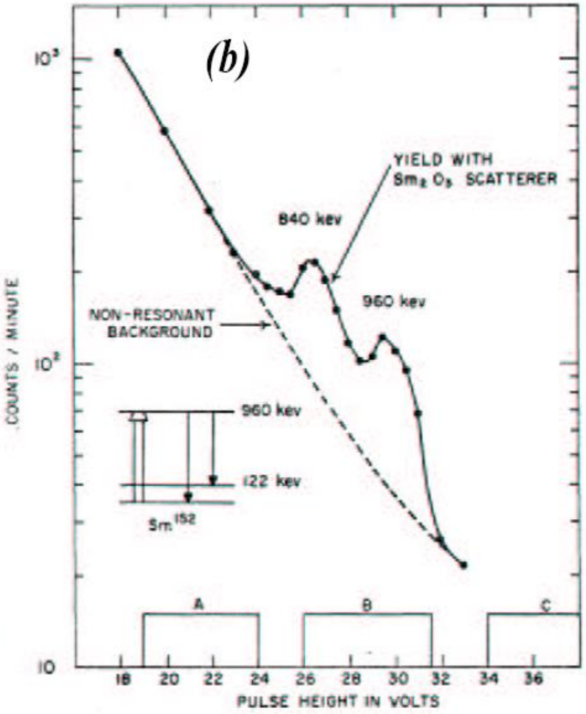
C.S. Wu $A = < \boldsymbol{p}_e \cdot J >$

Goldhaber, Grodzins en Sunyar
$h = < \boldsymbol{p}_e \cdot \sigma >$

1958: heliciteit neutrino







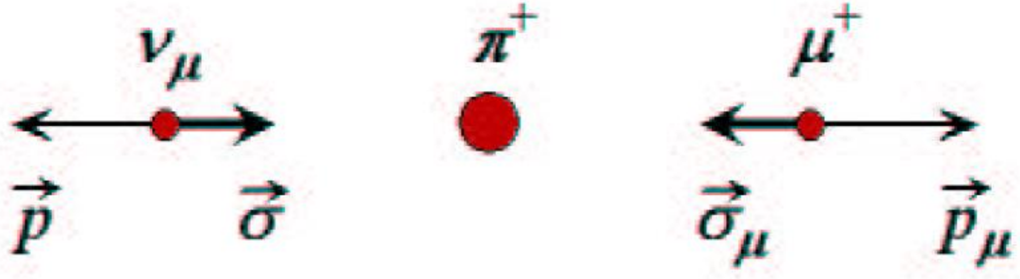
$^{152}\mathrm{Eu} + e^- \rightarrow {}^{152}\mathrm{Sm}^* + \nu_e$

$^{152}\mathrm{Sm}^* \rightarrow {}^{152}\mathrm{Sm} + \gamma$

Resultaat:
$< h_{\nu e} = -1.0 \pm 0.3 >$

Pionverval: heliciteitsonderdrukking

Waarneming:



$$\begin{aligned} \pi^+ &\rightarrow \mu^+ + \nu_\mu, \qquad \text{B.R.} = 0.999878 \\ \pi^+ &\rightarrow e^+ + \nu_e, \qquad \text{B.R.} = 1.2 \times 10^{-4} \end{aligned}$$

Incorrect:

$$\frac{\lambda_e}{\lambda_\mu} = \frac{1 + (m_e/m_\pi)^2}{1 + (m_\mu/m_\pi)^2} \frac{1 - (m_e/m_\pi)^2}{1 - (m_\mu/m_\pi)^2} \simeq 3.5$$

Correctiefactor:

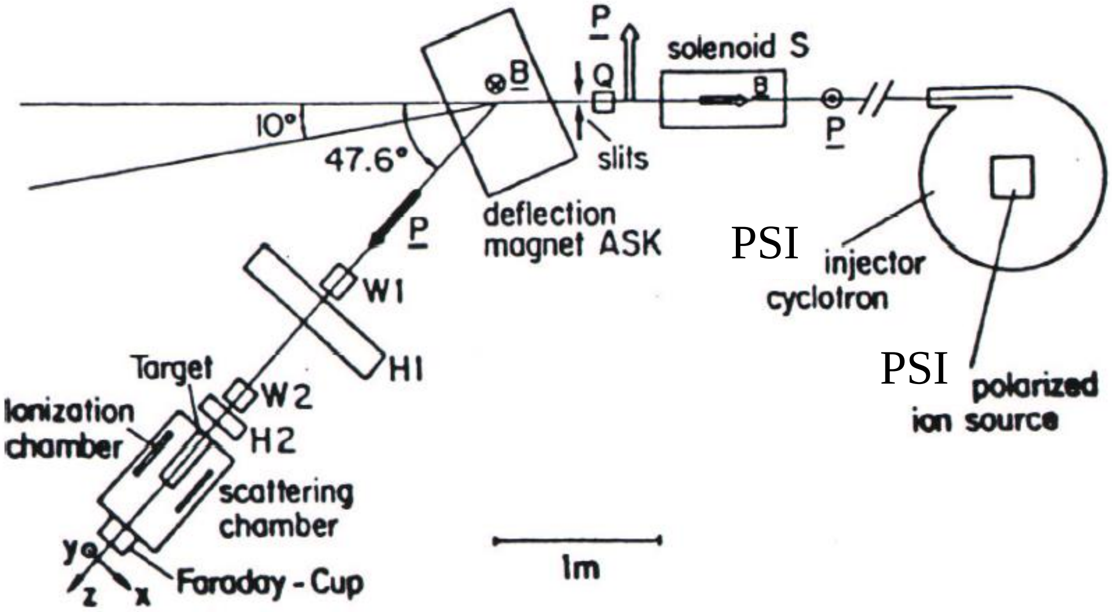
$$f = \frac{1 - v_e/c}{1 - v_\mu/c} = \frac{m_e^2}{m_\mu^2} \frac{1 + (m_\mu/m_\pi)^2}{1 + (m_e/m_\pi)^2} = 3.7 \times 10^{-5}$$

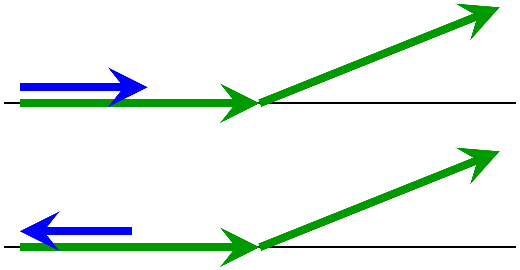
Conclusie:

- alle neutrino's (ν_e, maar ook ν_μ en ν_τ) hebben een heliciteit -1, en alle antineutrino's ($\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$) hebben een heliciteit $+1$.
- De in β-verval uitgezonden geladen leptonen (e^-) hebben een heliciteit $-v/c$, terwijl de antideeltjes (e^+) een heliciteit $+v/c$ hebben.

Sterke wisselwerking: behoud van pariteit

pp verstrooiing
T_p = 50 MeV
Longitudinale *p*
polarisatie





Resultaat:

$$\frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} = (-1.5 \pm 0.2) \times 10^{-7}$$

Intrinsieke pariteit (eigenpariteit)

We kennen intrinsieke pariteit toe aan deeltjes, $P_N = +1$

Gebruik behoud van pariteit in sterke wisselwerking om P_π te bepalen

Reactie:

$$\pi^- + d \rightarrow n + n$$

Neem aan:

$$J_\pi = 0, \quad J_d = 1, \quad J_n = \frac{1}{2}$$

$$\mathcal{P}_d = +1$$

Selecteer pionvangst

$$l_{\pi d} = 0 \quad \Rightarrow \quad |\vec{J}_{\text{tot}}| = |\vec{l}_{\pi d} + \vec{J}_\pi + \vec{J}_d| = 1 = |\vec{l}_{nn} + \vec{J}_n + \vec{J}_n|$$

$$\mathcal{P}_{\text{tot}} = \mathcal{P}_\pi \cdot \mathcal{P}_d \cdot (-1)^{l_{\pi d}} = \mathcal{P}_\pi = (\mathcal{P}_n)^2 \, (-1)^{l_{nn}}$$

golffunctie van de beide neutronen antisymmetrisch

$\Rightarrow$ ${}^3P_1$-toestand met $l_{nn} = 1$

${}^{2S+1}L_J$

$P_\pi = -1$