1. Noise sources in MOSFET transistors.

The noise sources in a MOS transistor are:
thermal noise in the channel,
1/f noise,
Noise in the resistive poly gate,
noise due to the distributed substrate resistance,
shotnoise associated with the leakage current of the drain source reverse diodes.
For normal use, only the first two items are important. The other noise sources must be taken into account for very low noise applications.

1.1. Channel thermal noise.

A MOSFET has in normal working order an inverse resistive channel between the drain and the source. The gate voltage forms with minority carriers the channel. In the extreme case when the drain source voltage $V_{DS} = 0$, the channel can be treated as a homogeneous resistor. The noise in the channel is:

$$i_d^2 = 4 \cdot k \cdot T \cdot g_0$$ **Equation 1.**

k Boltzman constant,
T Absolute temperature
g_0 channel conductance at zero drain-source voltage.

Normally however the voltage $V_{DS} \neq 0$. The V_{GS} is higher as the V_{GD}. Therefor the channel is more conductive nearby the source, compared to the drain. Conclusion is the channel is not a homogeneous resistor anymore. To calculate the noise it must be split into small parts (Δx). The noise must be calculated in every one of those parts and then integrated along the whole channel.
The formula is:

$$i_d^2 = 4 \cdot k \cdot T \cdot \frac{m^2 \cdot W^2}{L^2 \cdot I_{DS}} \cdot \int_0^{V_{DS}} Q_n^2(V) dV$$ **Equation 2.**

W Channel width,
L Channel length,
μ Effective channel mobility,
I_{DS} Drain source current.

The formula for $Q_n(x)$ is:

$$Q_n(x) = C_{ox} \cdot (V_{GS} - V_T(x) - V(x))$$ **Equation 3.**

C_{ox} gate oxide capacitance per unit area,
$V_T(x)$ Threshold voltage at position x,
$V(x)$ Channel potential.

In general is the Voltage $V_T(x)$ position or channel potential depending.
By neglecting this dependency the integral in equation 1 becomes:

$$i_d^2 = 4 \cdot k \cdot T \cdot m \cdot C_{ox} \cdot \frac{W}{L} \cdot \frac{2}{3} \left[\frac{3 \cdot (V_{GS} - V_T) \cdot V_{DS} - 3 \cdot (V_{GS} - V_T)^2 - V_{DS}^2}{2 \cdot (V_{GS} - V_T) - V_{DS}} \right]$$

Equation 4.

With the MOSFET we have 3 operation areas: linear region ($V_{DS}<V_{GS}-V_T$), saturation point ($V_{DS}=V_{GS}-V_T$) and saturation region ($V_{DS}>V_{GS}-V_T$).
In the linear region is equation 4 the same as equation 1.
At the saturation point we can simplify the equation to:

$$i_d^2 = 4 \cdot k \cdot T \cdot \frac{2}{3} \cdot m \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_T) = 4 \cdot k \cdot T \cdot \frac{2}{3} \cdot g_m$$

Equation 5.

g_m The transconductance of the MOSFET.

In the saturation region theoretically its not allowed to use equation 5, however experimentally has been found out that it's a good approximation, as long as the device shows a good saturation. The reason for this is the fact that the cut-off region nearby the drain is much smaller then the resistive reverse channel which is responsible for the noise.
The expression in equation 5 predicts the thermal noise in the channel without the substrate effect. In practice the thermal noise is higher. This is due to the fact, that V_T is depending on the channel potential V(x).
The integral in equation 2 is now hard to solve. In short the result is:

$$i_d^2 = 4 \cdot k \cdot T \cdot g \cdot g_m$$

Equation 6.

The factor g is a complex function of the basic transistor parameters and bias conditions. To give g a value a numerical approach is required.
For modern CMOS processes with oxide thickness t_{ox} in the order of 50 nm and with a lower substrate doping N_b of about 10^{15} - 10^{16} cm^{-3} the factor g is between 0.67 and 1.

The current noise in the channel also generates noise in the gate through the gate-channel capacitance $C_{ox}WL$. The gate noise is due to the capacitative coupling frequency depending. (f^2).The gate noise is about:

$$i_g^2 \approx 4 \cdot k \cdot T \frac{1}{5 \cdot g_m} \cdot (2 \cdot p \cdot f)^2 \approx 4 \cdot k \cdot T \cdot \frac{g_m}{5} \cdot \left(\frac{f}{f_T} \right)^2$$

C_{GS} gate-source capacitance.

$f_T \approx \frac{g_m}{2 \cdot p \cdot C_{GS}}$ The cut off frequency of the MOSFET.

When we compare the gate noise with the drain current noise the only time the gate current noise is important is when the frequency $f>f_T$. Therefore in most practical situations this noise can be neglected.

1.2. 1/f Noise in MOS Transistors.

1/f noise has been observed in all kind of devices, from homogeneous metal film resistors and different kind of resistors to semiconductor devices and even in chemical concentrated cells. Because 1/f noise is well spread over the components, people think that there a fundamental physical mechanism is behind it. Till now such a mechanism is not yet found.
There is a lot of experimental evidence that several mechanisms are involved in generating 1/f noise.
The MOS transistor has the highest 1/f noise of all active semiconductors, due to their surface conduction mechanism.
The result is: several theory's and physical models competing together to explain the 1/f noise in a MOSFET. These theory's and models differ in detail but are all based on the mobility fluctuation model expressed by the Hooge empirical relation, and the carrier density or number fluctuation model first introduced by McWhorter.

1.2.1. The Mobility Fluctuation Model.

When we use the Mobility Fluctuation model we refer to the 1/f noise as Δμ-1/f noise. The model is described by the Hooge empirical equation:

$$\frac{i_f^2}{I^2} = \frac{a_l}{N \cdot f}$$ **Equation 7.**

The parameter a_l is called the Hooge 1/f noise parameter. *N* is the total number of free carriers in the device and *I* the short circuit current through the device.
The equation has been checked experimentally with several homogeneous metals and semiconductors (Si and GaAs) and proved to be OK. With all the samples a value of about $2*10^{-3}$ for a_l is found. The fact that a_l almost constant indicates that the Δμ-1/f noise is a fundamental phenomenon.
Theoretical you can derive from Equation 7 the assumption that 1/f noise is caused by *N* independent free carriers, each generating 1/f noise due to mobility fluctuation.
Experiments proved that only photon scattering rises the 1/f noise.
If in one particular device, for example a MOSFET more than one scattering mechanism exist, impurity scattering, surface scattering, the Hooge 1/f noise parameter a_l should be reduced to:

$$a = a_l \cdot \left(\frac{m_{eff}}{m_l} \right)^2$$ **Equation 8.**

m_l is the mobility when only lattice scattering is present and m_{eff} is the effective mobility of the device according to Matthiessen's theorem:

$$\frac{1}{m_{eff}} = \frac{i}{m_l} + \sum_{i}^{n} \frac{1}{m_i}$$ **Equation 9.**

In equation 9 n is the total number of different scattering processes.

Important is the fact, that the Hooge equation is only valid for homogeneous devices. A not homogeneous device the differential form of the equation must be used.

When a MOSFET is used in the linear region, $V_{DS} << V_{SAT} = V_{GS} - V_T$, the inverse channel is almost a homogeneous resistor.

Under strong inversion condition the total number N if the free charge in the inversion channel is given by:

$$q \cdot N = C_{ox} \cdot W \cdot L \cdot (V_{GS} - V_T)$$ **Equation 10.**

The drain source current I_{DS} for very small drain source voltages U_{DS} is:

$$I_{DS} = \frac{q \cdot m_{eff} \cdot N}{L^2} \cdot V_{DS}$$ **Equation 11.**

When we fill these two equations into the Hooge equation (7) the result is:

$$i_f^2 = a_l \left(\frac{m_{eff}}{m_l} \right)^2 \cdot \frac{q \cdot m_{eff} \cdot I_{DS} \cdot V_{DS}}{L^2 \cdot f}$$ **Equation 12.**

When we check equation 12 with the reality we find a deviation of an order of 10^3 - 10^4. Therefore the conclusion is that the Hooge equation is <u>not valid</u> for the MOS inversion channel.

For a MOSFET in the saturation region, with the drain source voltage V_{DS} is larger than the saturation voltage, the channel is, even in the x direction, not a homogeneous layer. To solve this we divide the channel into a number of small sections Δx. For each section the Hooge equation can be used. By integrating these over the whole channel the total 1/f noise power spectrum can be calculated:

$$i_f^2 = a_l \cdot \frac{q \cdot m_f (V_{GS} - V_T) \cdot I_{DS}}{L^2 \cdot f}$$ **Equation 13.**

The transconductance of a MOS transistor is related to I_{DS} by:

$$g_m = \sqrt{2 \cdot m_{eff} \cdot C_{ox} \cdot \frac{W}{L} \cdot I_{DS}}$$ **Equation 14.**

The equivalent input 1/f noise voltage spectrum density is then:

$$v_f^2 = a_l \cdot \frac{q \cdot m_f \cdot (V_{GS} - V_T)}{2 \cdot m_{eff} \cdot C_{ox} \cdot W \cdot L \cdot f} \qquad \textbf{Equation 15.}$$

According to equation 15 is the 1/f noise proportional to V_{GS} - V_T, and inversely proportional to the gate oxide capacitance per unit area C_{ox} and the gate area *WL*, provided that m_{eff} and m_f do not change with to V_{GS} - V_T. These proportionalities have been seen in some experiments, the majority of experiments however show a different outcome. These experiments can be declared with the number fluctuation model.

1.2.2. The number fluctuation model.

In the number fluctuation model we call the 1/f noise Δn-1/f noise.
It is believed to be caused by the random trapping and detrapping of the mobile carriers in the trap located at Si-SiO_2 interface and within the gate oxide. This causes a signal with a Lorentzian or generation-recombination spectrum. Superposition or a large number of these signals with the proper time constant result in a 1/f-noise spectrum.
According to this model, the Δn-1/f noise is proportional with the effective trap density near the quasi-Fermi level of the inverse carriers. This has been verified by a large number of experiments.
Also explained by this model is the 1/f noise in the weak inversion region. It is observed that the relative 1/f noise current i_f^2 / I^2 has a plateau in this region, which can be explained in the model.

In the interface between Si-SiO_2 and in the gate oxide (oxide traps) additional energy states exist. These states and traps communicate randomly with the free charges in the channel. This mechanism obeys the Schokley Read Hall statistics. By using this statistics the mean square fluctuation dn_t in the number of trapped carriers in a volume DV = ($Wdxdy$) at a specific position is given by:

$$dn_t = \frac{4 \cdot t}{1 + w^2 \cdot t^2} \cdot N_t \cdot f_t \cdot f_{pt} \cdot \Delta V \qquad \textbf{Equation 16}$$

t = trapping time constant [s].
N_t= trap density [cm^{-3}].
$f_t = \frac{1}{1 + e^{(E_t - F_t)/k \cdot T}}$ = Fraction of filled traps under steady state condition.
E_t = trap energy level [eV].
F_t = trap quasi-Fermi level [eV].

The fluctuation dn_t causes fluctuations in the channel free carriers dN that in its turn causes fluctuation in the channel current dI_{DS}.
In strong inversion it is shown that $d_N = dn_t$.

The number of free carriers N is:

$$q \cdot N = \int_0^L q \cdot n(x) dx = \int_0^L W \cdot C_{ox} \cdot (V_{GS} - V_T - V(x)) dx \qquad \textbf{Equation 17.}$$

The drain source current is given by:

$$I_{DS} = q \cdot n(x) \cdot m \cdot \frac{dV(x)}{dx} \qquad \textbf{Equation 18.}$$

For the fluctuation of the drain current due to fluctuation in trapped carriers in an elementary volume DV is:

$$di_d^2(f) = \left(\frac{I_{DS}}{L \cdot n(x)} \right)^2 \cdot dN^2 = \frac{m \cdot q^2 I_{DS}}{L^2 \cdot W \cdot C_{ox} \cdot (V_{SAT} - V(x))} \cdot \frac{dV(x)}{dx} dn_t^2$$

Equation 19.

To calculate the total 1/f noise we have to integrate equation 19 along the channel, over the energy band gap and into the oxide. The general expression is given in equation 20

$$i_d^2(f) = \int_0^{VDS} \int_{E_v}^{E_c} \int_0^d \frac{m \cdot q^2 \cdot I_{DS}}{L^2 \cdot C_{OX} \cdot (V_{SAT} - V(x))} \cdot \frac{4 \cdot t(y,E,V)}{1 + w^2 \cdot t(y,E,V)} \cdot N_t(E,y) \cdot f_t(E,V) \cdot f_{pt}(E,V) dydEdv$$

Equation 20

To evaluate this integral we must first determinate the distribution of $N_t(E,y)$, $t(y,E,V)$ and the function $f_t(E,V)$.
When we have a single trap with an uniform distribution into the oxide, $N_t(E_t,y) = N_t(E_t)$. The distribution of trapping time constant $t(y,E,V)$ based on the SRH statistics and the tunnelling model of McWorter:

$$t(y) = t_0 \cdot e^{a \cdot y} \text{ with } t_0 = \frac{1}{c \cdot (n_s + n_1)} \qquad \textbf{Equation 21}$$

α	$= 10^8$ cm^{-1},	McWorter tunnelling constant.
$n_s = n_i \cdot e^{(F_n - E_i)/k \cdot T}$		The surface carrier concentration.
$n_1 = n_i \cdot e^{(F_t - E_i)/k \cdot T}$		
c	$= 10^{-8}$ cm^3/s	The electron capture coefficient.

In the practical case $t_0 = 10^{-10}$ s. The traps are distributed into the oxide d = 50 Å, the maximum trapping time constant is $t_{max} = 5\ 10^{11}$ s.
With this information we can work out the integral into the oxide:

$$\int_0^d \frac{4 \cdot t(y,E,V)}{1 + w^2 \cdot t^2(y,E,V)} \cdot N_t(E_t) \cdot \frac{1}{a \cdot w} \cdot [\arctan w \cdot t_{\max} - \arctan w \cdot t_0] =$$

$$N_t(E_t) \cdot \frac{1}{4 \cdot a \cdot f}$$

$$\text{for } \frac{1}{2 \cdot p \cdot t_{\max}} < f < \frac{1}{2 \cdot p \cdot t_0}$$ **Equation 22.**

Under the condition of a constant space trap distribution a pure 1/f-noise spectrum is obtained in the frequency range of $10^{-13} < f < 10^{9}$ Hz. Normally the space trap distribution is not constant, the spectrum shows an $1/f^{\alpha}$ spectrum with α <> 1.

The next integral to solve is the integral over the trap distribution in the band gap. The exact trap distribution $N_t(E)$ is of minor importance. The only traps that are effective in noise generation, are the traps around the electron quasi-Fermi level F_n. Therefor a good approximation for the integral over the energy band is given by:

$$\int_{E_v}^{E_c} N_t(E) \cdot f_t(E,V) f_{pt}(E,V) dE = 4 \cdot k \cdot T \cdot N_t(F_n) \cdot f_t(F_n) \cdot f_{pt}(F_n)$$

Equation 23.

After solving these integrals equation 20 is reduced to:

$$i_d^2(f) = \frac{m \cdot q^2 \cdot I_{DS}}{L^2 \cdot C_{OX}} \cdot \frac{k \cdot T}{a \cdot f} \int_0^{V_{DS}} \frac{1}{(V_{SAT} - V(x))} \cdot N_t(F_n) \cdot f_t(F_n) \cdot f_{pt}(F_n) dV$$

Equation 24.

The quasi-Fermi level is not a constant along the channel. Due to this the function $f_t f_{pt}$ varies as well. The expression using SRH statistics is:

$$f_t(F_n) \cdot f_{pt}(F_n) = \frac{n_s^4}{\left(2 \cdot n_s^2 + n_i^2\right)^2}$$ **Equation 25**

$n_s(x)$ is the surface density of the inversion electrons in the channel which is according to the elementary MOS theory related to the channel potential:

$$n_s = n_{s0} \cdot \frac{V_{SAT} - V(x)}{V_{SAT}}$$ **Equation 26**

With the equations 25 and 26 the integral in equation 24 can be solved, provided that $N_t(F_n)$ is known. For the simplest case where $N_t(F_n)$ is uniformly distributed in the energy band or at least around F_n the integral is given by:

$$i_d^2(f) = \frac{m \cdot q^2 \cdot I_{DS}}{L^2 \cdot C_{ox}} \cdot \frac{k \cdot T \cdot N_t}{a \cdot f} \cdot \frac{1}{16} \cdot \ln\left[\frac{2}{2 \cdot \left(\frac{V_{SAT} - V_{DS}}{V_{SAT}}\right)^2 + \left(\frac{n_i}{n_{so}}\right)^2}\right]$$

Equation 27

The solution in equation 27 is valid under the conditions that:

The traps are uniformly distributed in the space and energy band, or at least around the electron quasi-Fermi level F_n.

The working area for this formula is the linear region till the saturation point $V_{DS}>V_{SAT}=V_{GS}-V_T$.

In the saturation ($V_{DS}>V_{SAT}$) area the channel thermal noise saturates as well. If in equation 27 $V_{DS}<<V_{SAT}$, the second term in the denominator in the *ln*-function can be ignored. The drain current noise increases with the increase of the drain source voltage. The equation reduces to:

$$i_d^2(f)=\frac{m\cdot q^2\cdot I_{DS}}{L^2\cdot C_{ox}}\cdot\frac{k\cdot T\cdot N_t}{a\cdot f}\cdot\frac{1}{16}\cdot\ln\left[\frac{\sqrt{2}\cdot n_{so}}{n_i}\right]=\frac{K_F\cdot I_{DS}}{C_{ox}\cdot L^2\cdot f}\qquad\textbf{Equation 28.}$$

The value of K_F is constant if $N_t(F_n)$ and n_{so} do not change that much with the bias condition.

Combining equation 28 with equation 14 gives:

$$v_f^2(f)=\frac{K_F}{2\cdot m\cdot C_{ox}^2\cdot W\cdot L\cdot f}=\frac{K_f}{C_{ox}^2\cdot W\cdot L\cdot f}\qquad\text{Equation 29}$$

1.2.3. Comparing equation 15 and equation 29, we see two discrepancies between the two models (Δμ-1/f and Δn-1/f).

1) In equation 29 the noise is independent of any dc bias condition, while in equation 15 it is directly proportional to the effective gate voltage ($V_{gs}-V_T$).

2) In equation 29 the noise is inversely proportional to C_{ox}^2, while according to equation 15 it is proportional to C_{ox}.

The two discrepancies between the two models don't mean that one of the models is incorrect. Other parameters like the effective mobility μ may affect the noise dependency on the dc bias condition and the oxide thickness.

Many experiments however show a strong dependence of the 1/f noise in a MOSFET on the state density. They can be fitted in equation 29.

1.3. Additional Noise Sources.

In very low noise applications some other noise sources can get important. These sources are:

1) the thermal noise associated with the resistive poly-gate R_g,
2) Thermal noise due to the resistive substrate R_b,
3) the shot noise associated with the leakage current of the drain-source inverse diodes.

Because the leakage current is always much smaller then the drain current I_{DS}, it's effect is always negligible. On the other hand, if you don't take special precautions in the layout, the noise due to R_g and R_b can prevail over the channel thermal noise.

1.3.1. Resistive Poly-gate Noise.

To calculate the noise due to the poly-gate we use the same procedures as for calculating the MOS channel thermal noise.

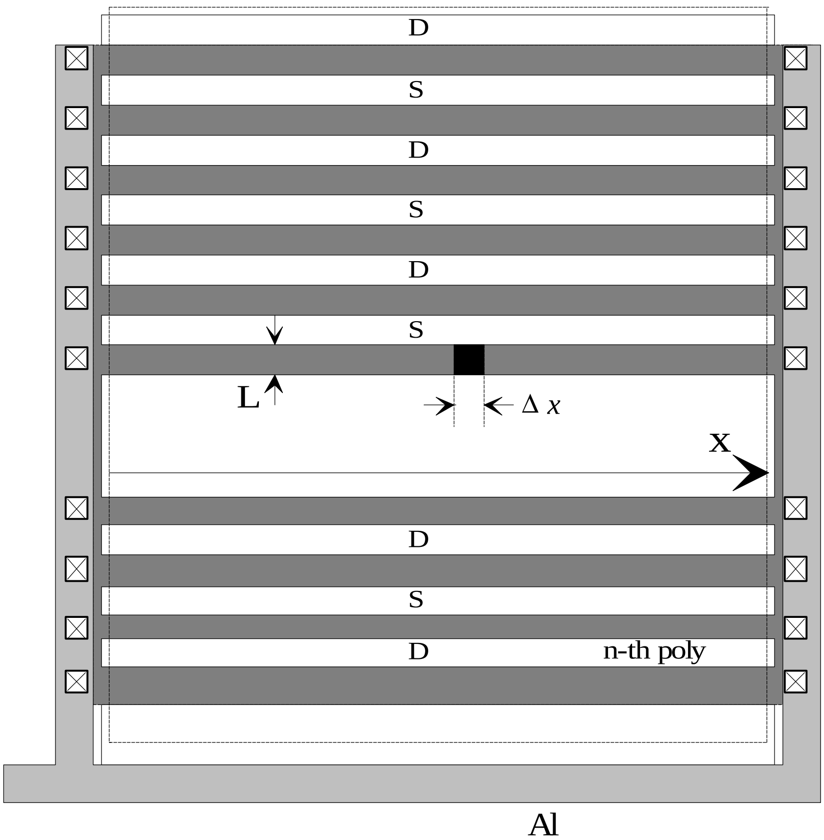

Figure 1: The finger structure of a MOS transistor.

In practice the finger structure is the most used for a MOS-FET with a large W/L ratio. The advantages of this structure are:

1) Lower C_{BS} and C_{BD} capacitance.
2) More useful in layout to realise.

The total channel width W in this structure is divided into n poly stripes with an equal width of $W_I = W / n$. Both ends of the stripes are connected to aluminium (Al) strips for low noise reasons. The sheet resistance of Al is about 0.05 Ω/sq. and of poly the value is about 25 Ω/sq. The noise contribution of the Al is negligible compared with the contribution of the poly.

The thermal noise of the section *Dx* is, according to the Nyquist theorem:

$$\Delta V_x = 4 \cdot k \cdot T \cdot r_{sq} \cdot \frac{\Delta x}{L}$$ **Equation 30**

r_{sq} is the sheet resistance of the poly gate. The voltage fluctuation *dV (x)* in the I-th stripe is given by:

$$dV(x) = dV(0) - \frac{x}{W_i} \cdot \Delta V_x \text{ for } 0 < x < x_0.$$ **Equation 31**

$$dV(x) = dV(W_i) - \frac{W_1 - x}{W_i} \cdot \Delta V_x \text{ for } x_0 < x < w_i.$$ **Equation 32**

Both ends of a stripe are short-circuited via Al, so $dV(0) = dV(W_i)$. The drain current fluctuations due to the noise voltage are given by:

$$\Delta di_d = \int_0^{W_i} dV(x) \cdot dg_{mi}(x) dx = \frac{(W_i - 2x_0)}{2} \cdot \Delta V_x \cdot dg_{mi}$$ **Equation 33**

The term $dg_{mi}(x)$ is the transconductance per unit length at position x. We assume that it's constant over the whole stripe W_i.
The drain noise current per stripe is given by:

$$\Delta i_d^2 = \int_0^{W_i} \Delta di_d^2 = 4 \cdot k \cdot T \cdot \frac{R_i}{12} \cdot g_{mi}^2$$ **Equation 34**

In this formula $R_i = r_{sq} \cdot \frac{W_i}{L}$ is the resistance and $g_{mi} = dg_{mi} \cdot W_i$ is the transconductance of the i-th stripe. The total noise current is given by simply multiply the expression by n.

$$\Delta i_d^2 = 4 \cdot k \cdot T \cdot n \cdot \frac{R_i}{12} \cdot g_{mi}^2 = 4 \cdot k \cdot T \cdot \frac{R_g}{12 \cdot n^2} \cdot g_m^2$$ **Equation 35**

$R_g = nR_I$ is the total gate resistance and $g_m = ng_{mi}$ is the total transconductance of the MOSFET.

When we observe equation 35 it's clear that in order to minimise the noise due to the resistive gate the number *n* should be as large as possible.
A second remark is that if in the finger structure only one side of the gate is connected together with Al, the noise increases with a factor 4.

1.3.2. Substrate resistance noise.

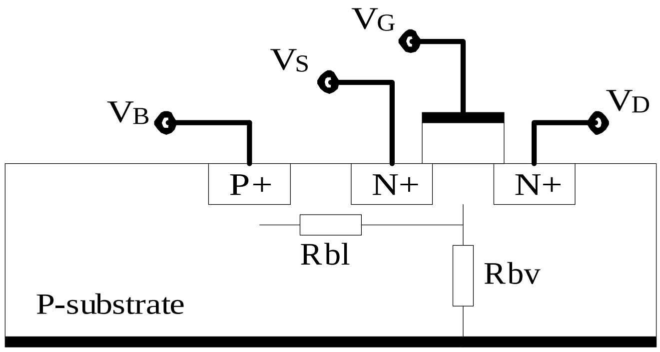

Figure 2: The substrate resistors in a nMOS transistor.

In figure 2 the substrate resistors in a nMOS transistor are drawn. The calculation of the noise due to these resistors is more complicated as from the poly gate, because of the distributed nature of the resistors. When we compare both resistors we see that the value of R_{bv} is much larger as R_{bl}. Due to this for noise calculations we can neglect R_{bv}.

The resistor R_{bv} is proportional to the distance between the bulk contact and the channel and with $1/W_i$. The result of the noise in the resistor is the modulation of the channel current due to the bulk transconductance.

$$g_{mb} = g_m \cdot \frac{K_2}{2 \cdot \sqrt{2 \cdot \Phi_f + V_{BS}}}$$ **Equation 36.**

In this formula K_2 is defined before. F_F is the difference between the quasi-Fermi level and the intrinsic level.
The total current noise due to the bulk resistor is there for:|

$$i_{dB}^2 = 4 \cdot k \cdot T \cdot n \cdot B \cdot \frac{b}{W_i} \cdot g_{mbi}^2 = 4 \cdot k \cdot T \cdot B \cdot \frac{b}{W} \cdot g_{mbi}^2$$ **Equation 37.**

The only way to reduce the bulk noise is by biasing the bulk to minimise the noise.

Reference: Low-Noise Wide-Band Amplifiers in Bipolar and CMOS Technologies, Zhong Yuan Chang and Willy M.C. Sansen.