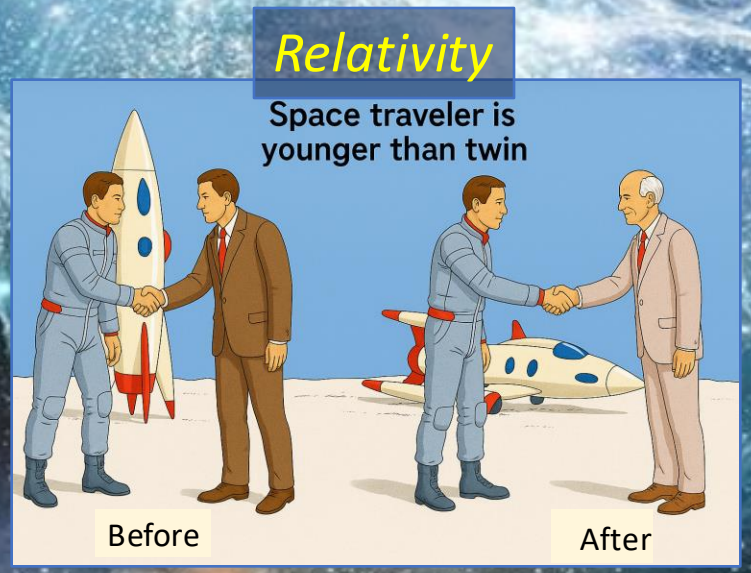
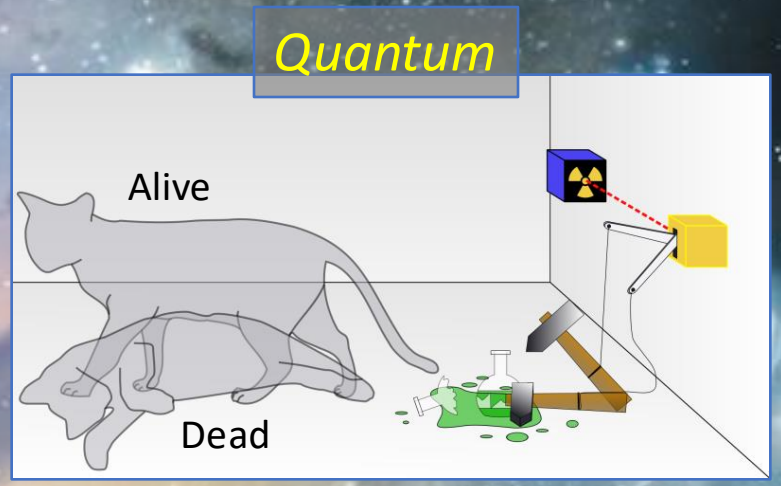
The Relativistic Quantum World

A lecture series on
Relativity Theory and Quantum Mechanics





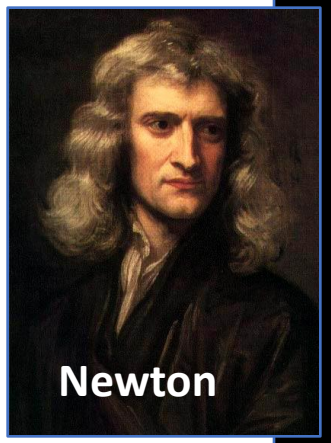
Marcel Merk
Studium Generale Maastricht
Sep 10 – Oct 8, 2025

The Relativistic Quantum World

Relativity	Sep. 10:	Lecture 1: The Principle of Relativity and the Speed of Light Lecture 2: Time Dilation and Lorentz Contraction
	Sep. 17:	Lecture 3: The Lorentz Transformation and Paradoxes Lecture 4: General Relativity and Gravitational Waves
Quantum Mechanics	Sep. 24:	Lecture 5: The Early Quantum Theory Lecture 6: Feynman's Double Slit Experiment
	Oct. 1 :	Lecture 7: Wheeler's Delayed Choice and Schrodinger's Cat Lecture 8: Quantum Reality and the EPR Paradox
Standard Model	Oct. 8:	Lecture 9: The Standard Model and Antimatter Lecture 10: Why is there something rather than nothing?

Lecture notes, written for this course, are available: www.nikhef.nl/~i93/Teaching/
Prerequisite for the course: High school level physics & mathematics.

Relativity and Quantum Mechanics

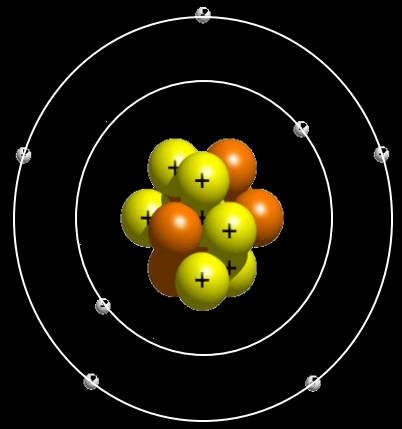


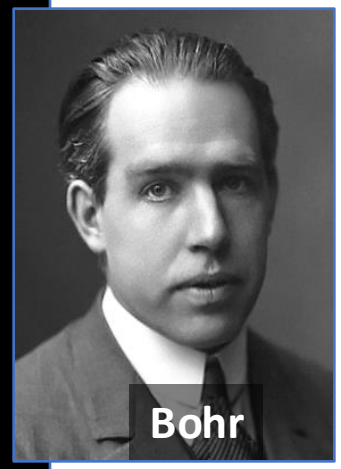
Classical Mechanics

Quantum Mechanics

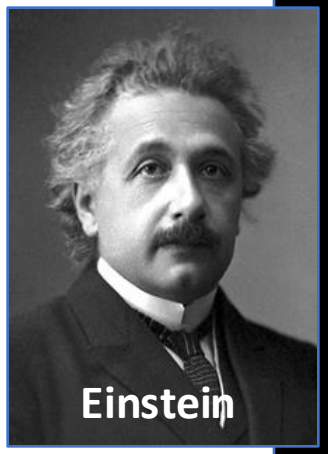
Smaller Sizes (ħ)

Higher Speed (c)

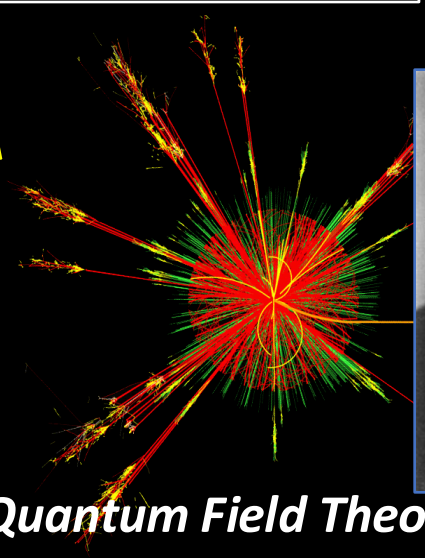




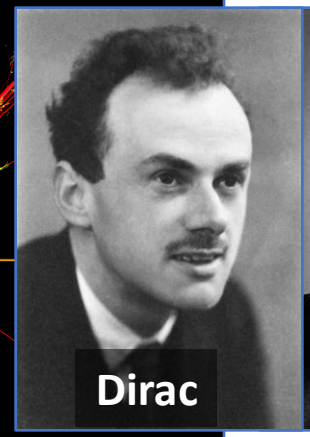
From our experience we are not used to relativistic and quantum mechanical phenomena. That's why we find it counterintuitive.

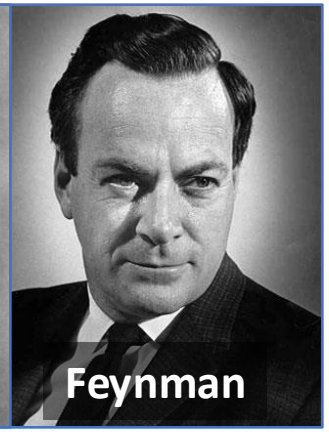


Relativity Theory



Quantum Field Theory





Lecture 5

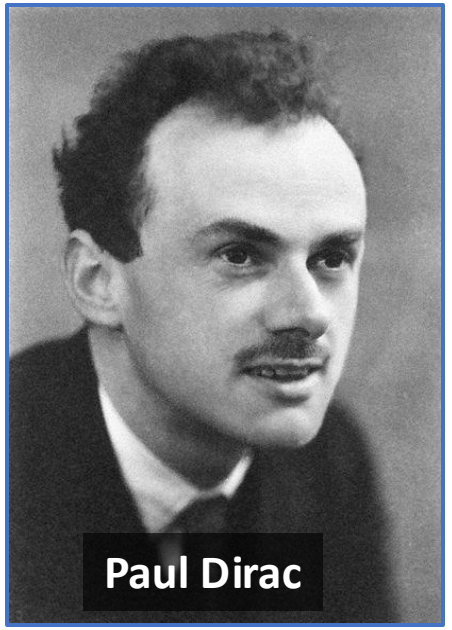
The Early Quantum Theory

“If Quantum Mechanics hasn’t profoundly shocked you, you haven’t understood it yet.”
- Niels Bohr

“Gott würfelt nicht (God does not play dice).”
- Albert Einstein

”Einstein, stop telling God what to do!”
- Niels Bohr

Key Persons of Quantum Mechanics



Niels Bohr:	Nestor of the ”Copenhagen Interpretation”
Erwin Schrödinger:	Inventor of the quantum mechanical wave equation
Werner Heisenberg:	Inventor of the uncertainty relation and “matrix mechanics”
Paul Dirac:	Inventor of relativistic wave equation: Antimatter!
Max Born:	Inventor of the probability interpretation of the wave function

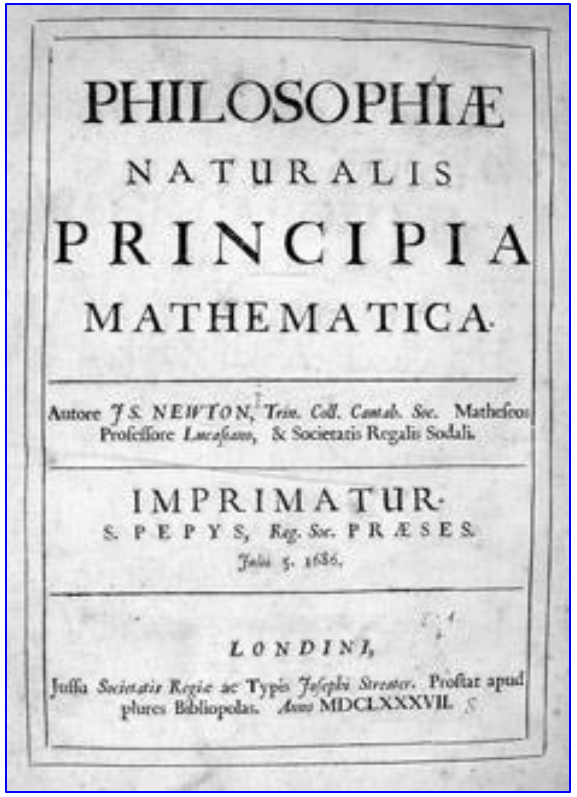
We will focus of the Copenhagen Interpretation and work with the concept of Schrödinger’s wave-function: ψ

1927 Solvay Conference Brussels

Deterministic Universe

Mechanics Laws of Newton:

1. The law of inertia: a body in rest moves with a constant speed
2. The law of force and acceleration: $F = ma$
3. The law: Action = - Reaction



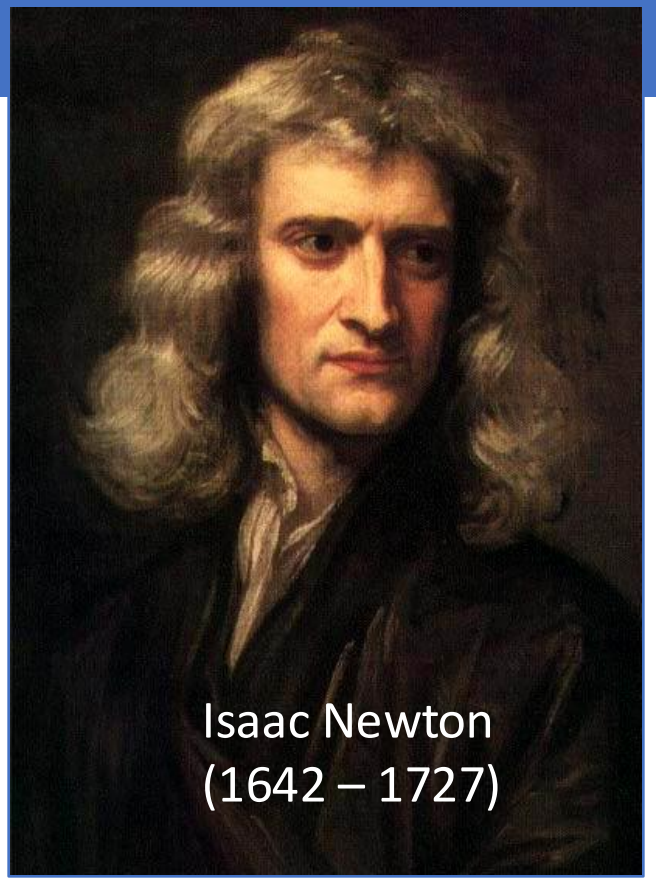
PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA

Autore *J.S. NEWTON*, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Juſſu Societatis Regiæ ac Typis Joſephi Streater. Proſtat apud plures Bibliopolas. Anno MDCLXXXVII.

"Principia" (1687)



Isaac Newton
(1642 – 1727)

- Classical Mechanics leads to a deterministic universe.
  - From ***exact initial conditions future can be predicted***.
- Quantum mechanics introduces a fundamental element of chance in the laws of nature: Planck's constant: h.
  - ***Quantum mechanics only makes statistical predictions***.

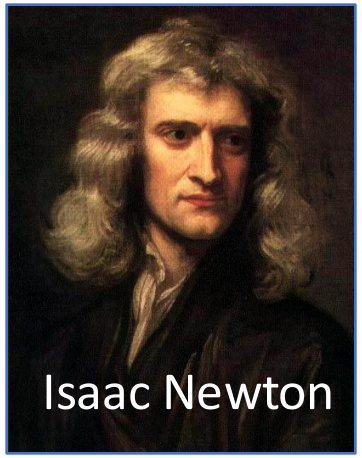
The Nature of Light

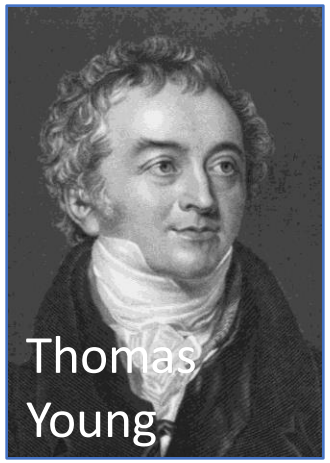
Isaac Newton (1642 – 1727):
Light is a stream of particles.

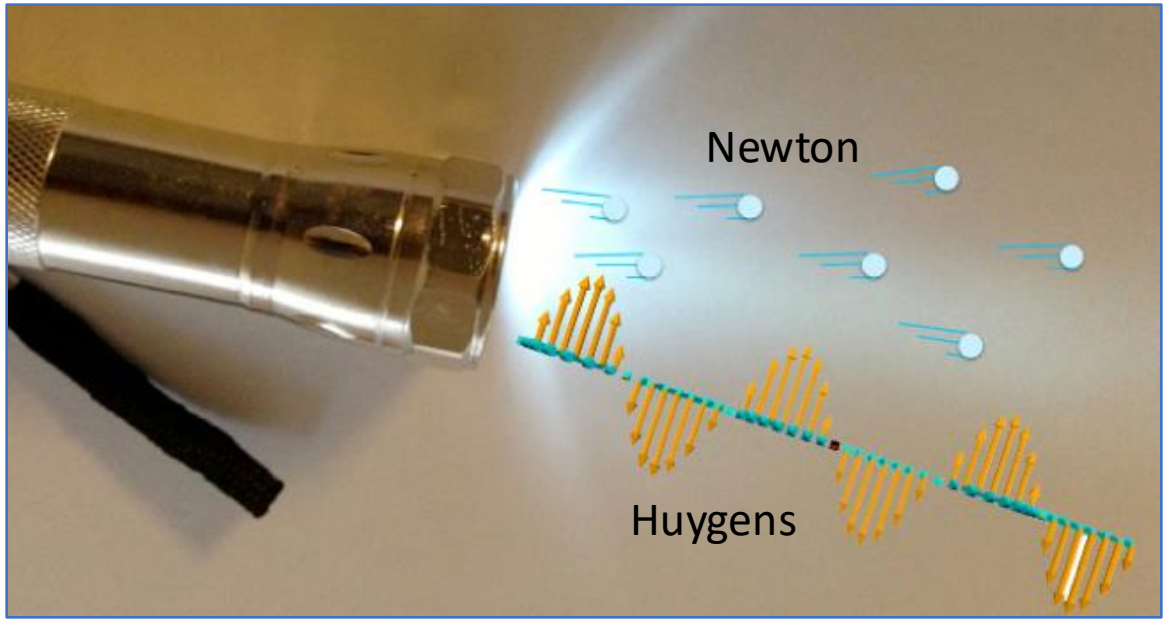
Christiaan Huygens (1629 – 1695):
Light consists of waves.

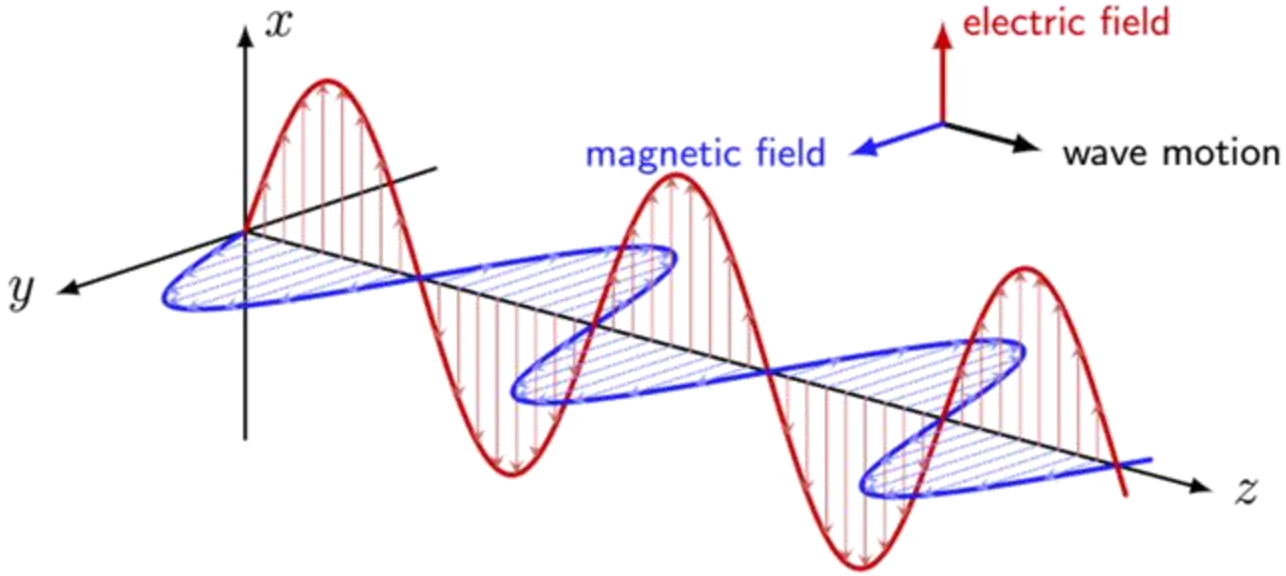
Thomas Young (1773 – 1829):
Interference observed: Light is waves!

James Clerk Maxwell (1831 – 1879):
Light is a wave of electromagnetic fields.



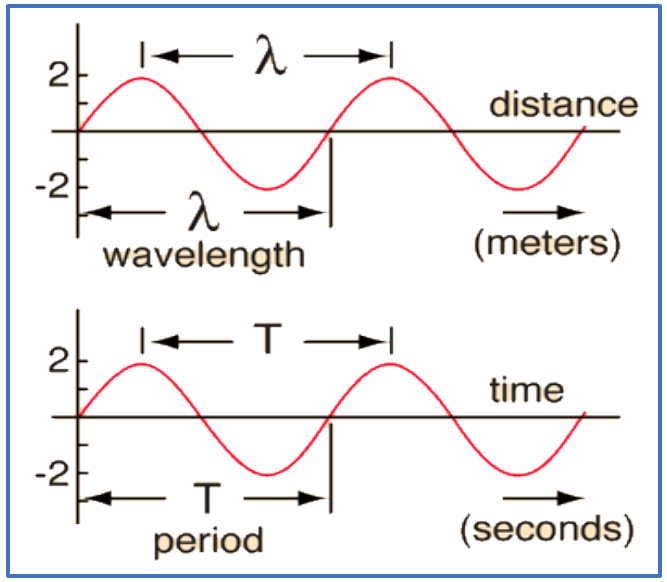






Waves & Interference : water, sound, light

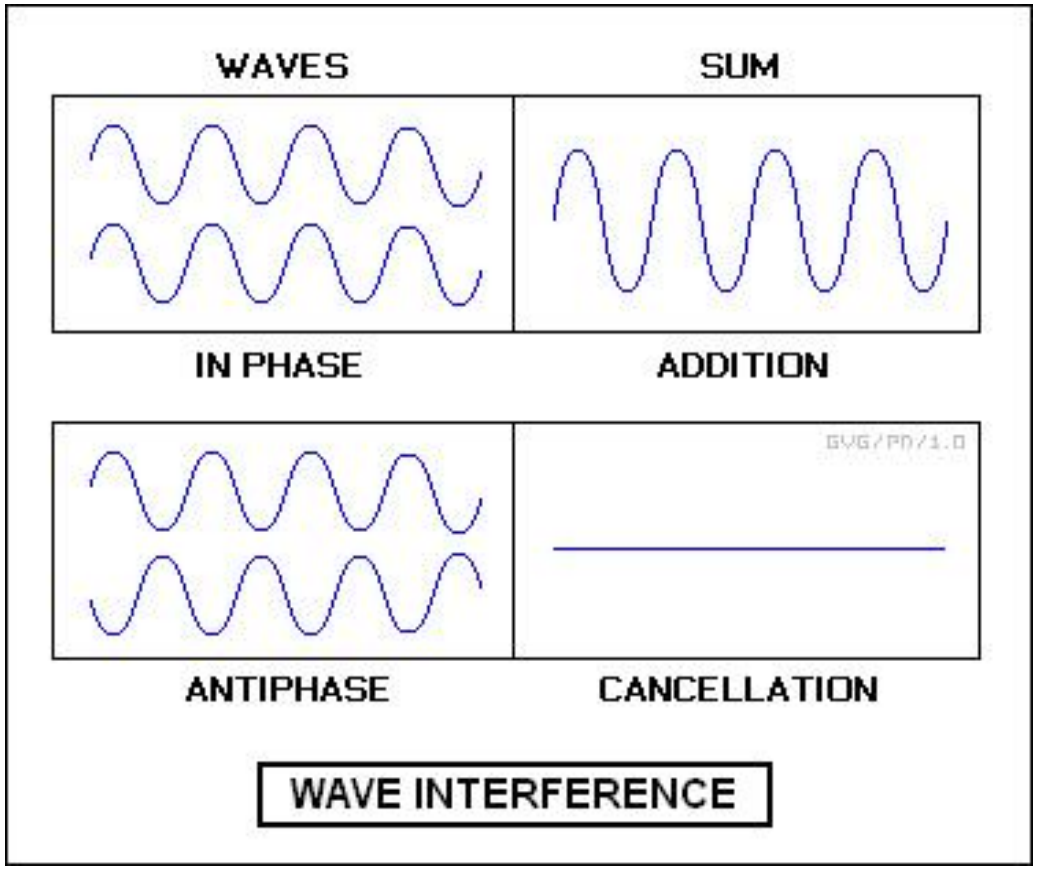
Principle of a wave



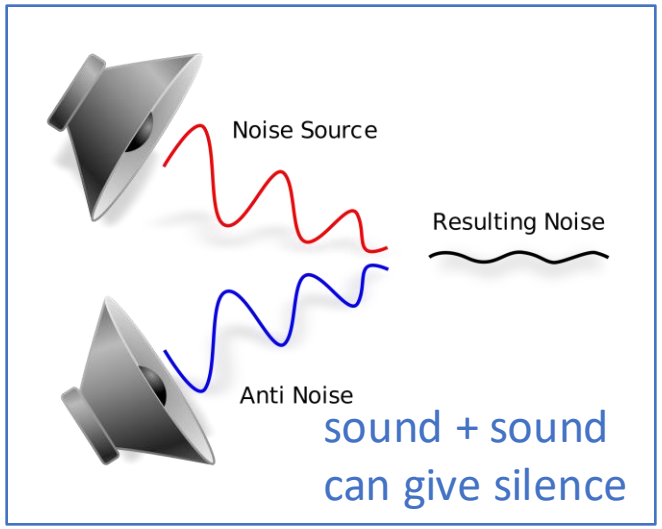
Water: Interference pattern:

$$\lambda = v/f$$

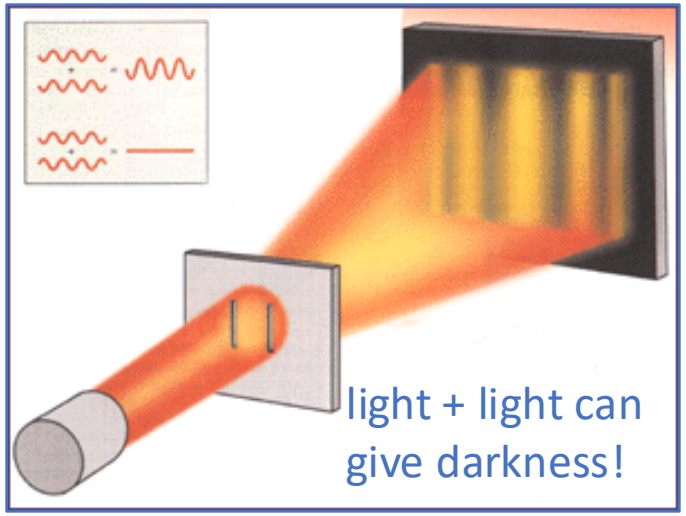
$$f = 1/T$$



Sound: Active noise cancellation:

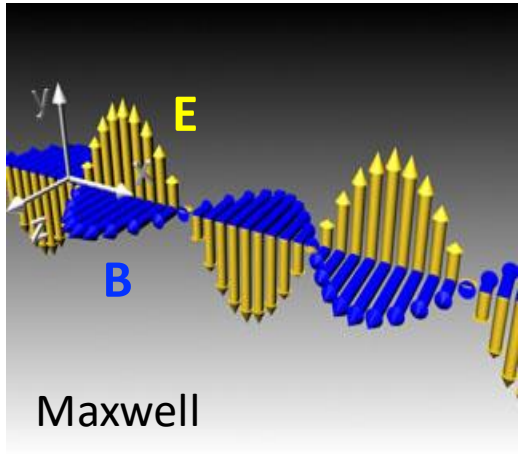


Light: Thomas Young experiment:



Interference with Water Waves

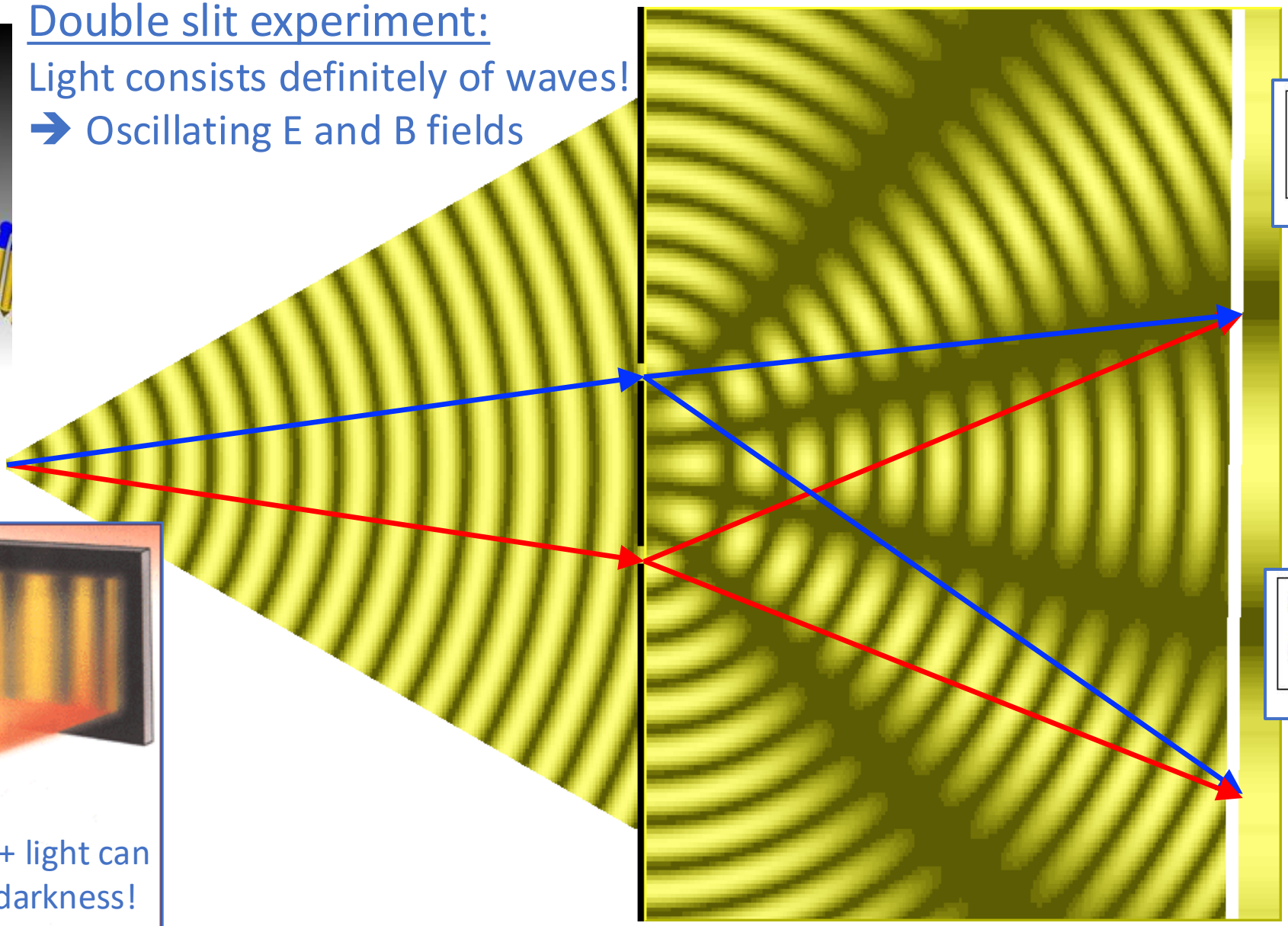
Interfering Light Waves

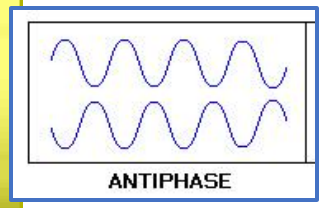


Double slit experiment:

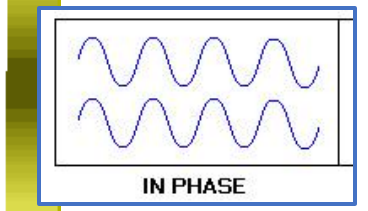
Light consists definitely of waves!

➔ Oscillating E and B fields

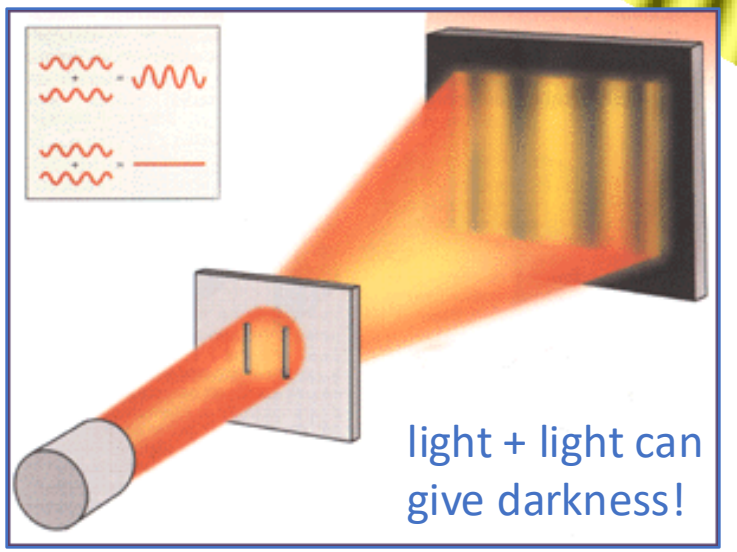




Cancellation:
light + light
= dark

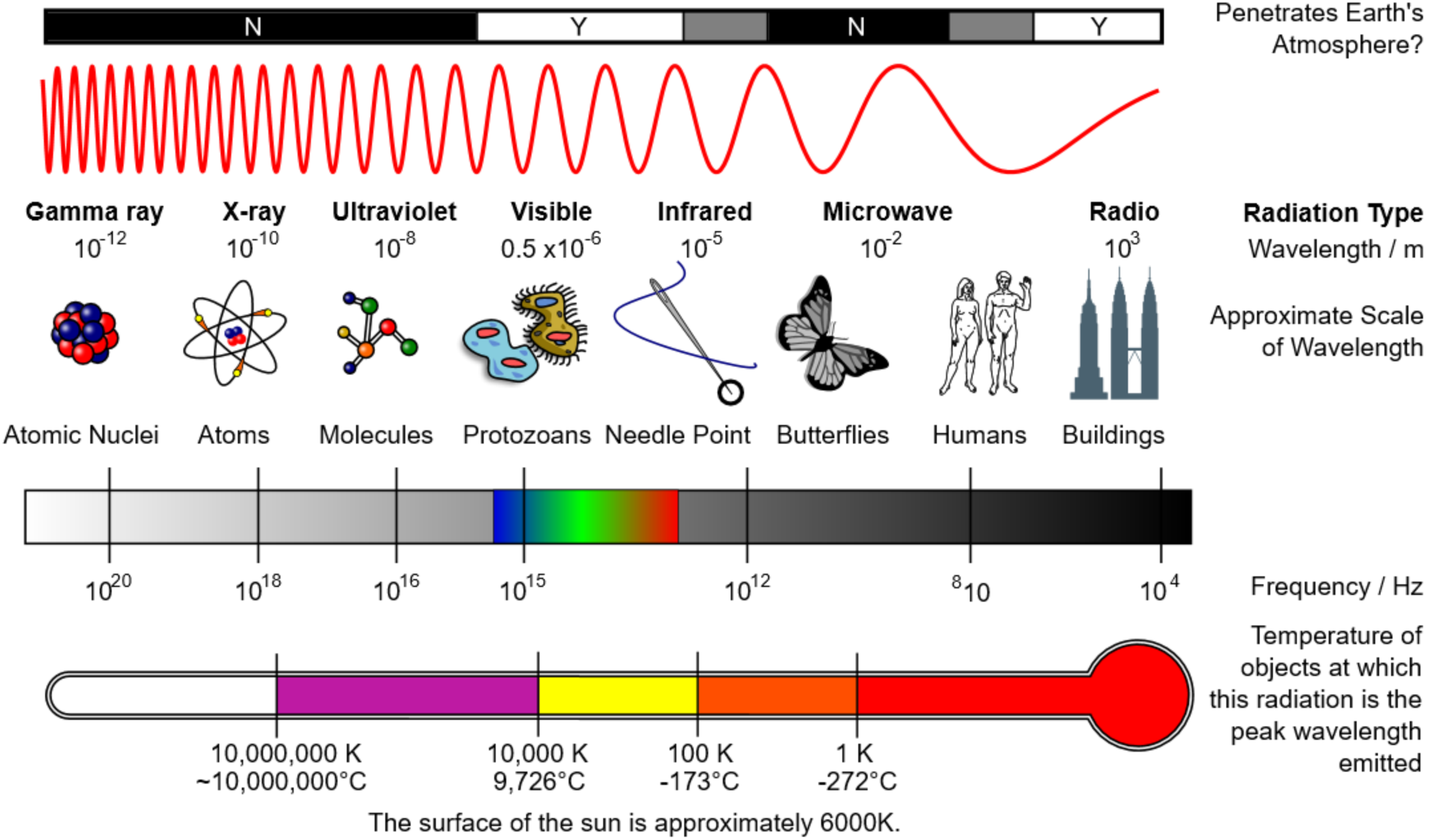


Amplification:
light + light
= more light



light + light can give darkness!

What can you see with light waves?



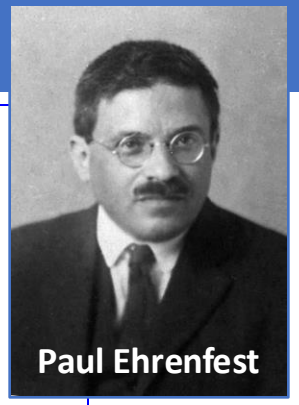
"Look, I'm a big-picture guy. I say 'Let there be light,' you guys figure out if it's a wave or a particle."

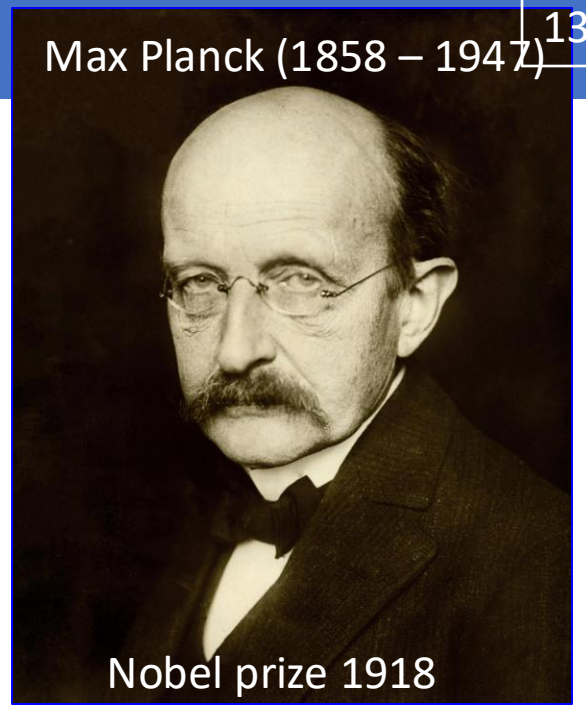
Particle nature: Quantized Light

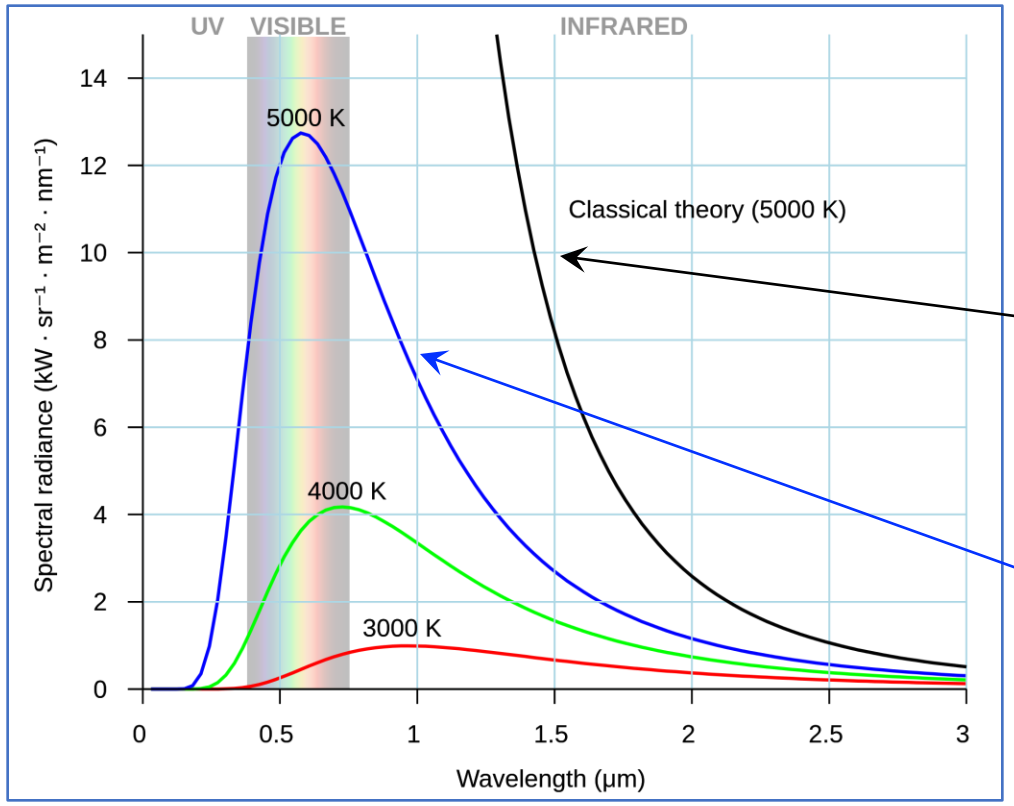
"UV catastrophe" in Black Body radiation spectrum:

If you heat a body it emits radiation. Classical thermodynamics predicts the amount of light at very short wavelength to be infinite!

Planck invented an ad-hoc solution:
For some reason material emitted light in "packages".


Paul Ehrenfest


Max Planck (1858 – 1947)
Nobel prize 1918

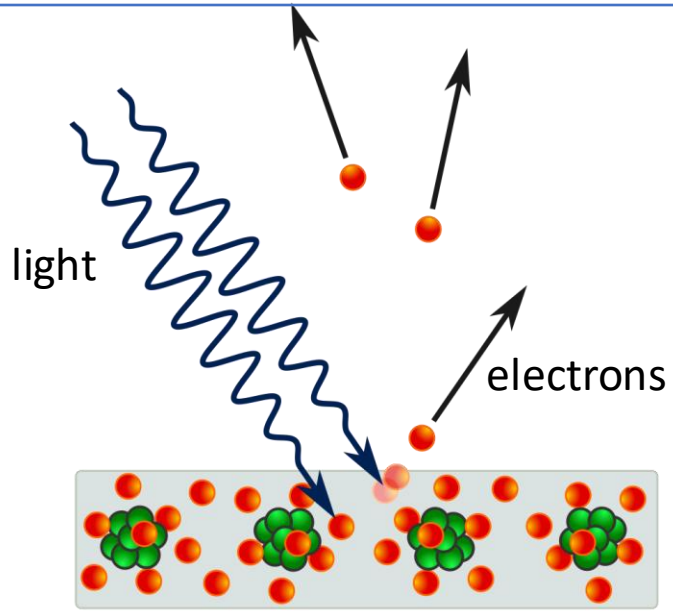


Planck's constant:
$h = 6.62 \times 10^{-34}$ Js

Classical theory:
There are more short wavelength "oscillations" of atoms than large wavelength "oscillations".

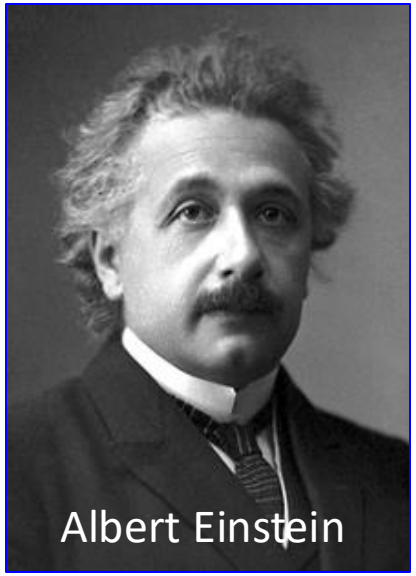
Quantum theory:
Light of high frequency (small wavelength) requires more energy: $E = hf$ (h = Planck's constant)

Photoelectric Effect



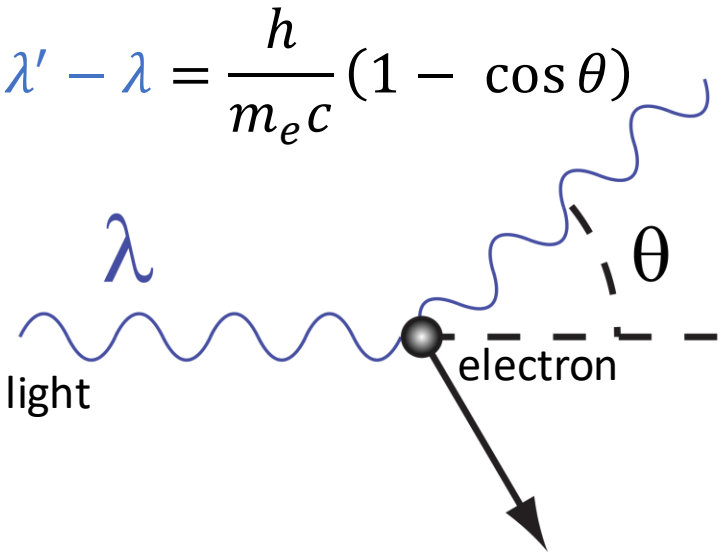
Photoelectric effect:
Light kicks out electron with ***E = h f***
(Independent on light intensity!)

Light ***consists*** of quanta.
(Nobelprize 1921)


Albert Einstein

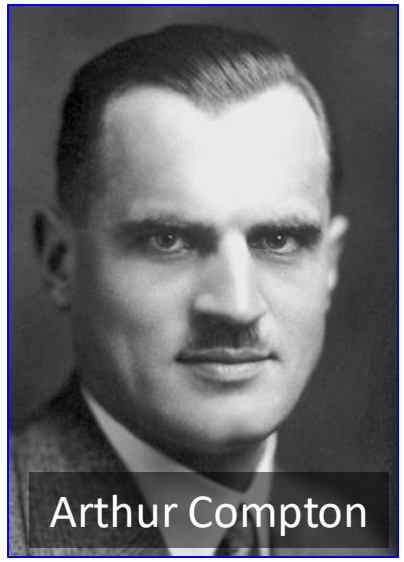
Wave: $E = hf = hc/\lambda$ → $\lambda = hc/E$
Momentum: $p = E/c$ → $E = pc$
It follows that: $\lambda = h/p$

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

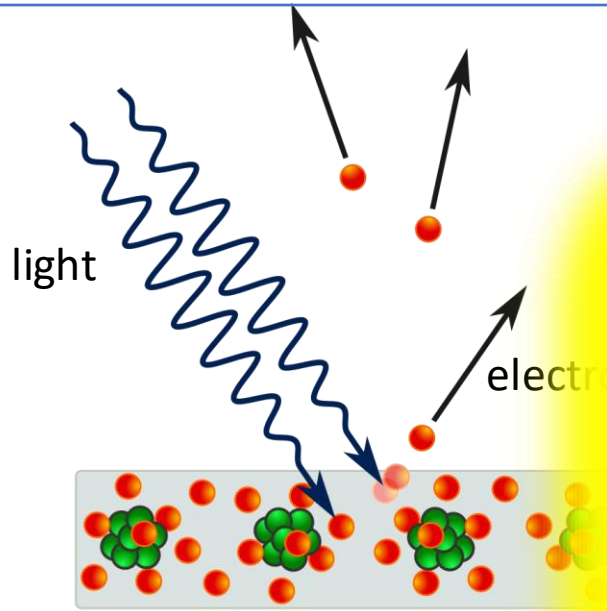


Compton Scattering:
"Playing billiards" with light quanta and electrons.

Light behaves as a particle with: ***λ = h / p***
(Nobelprize 1927)


Arthur Compton

Photoelectric Effect



Photoelectric effect:

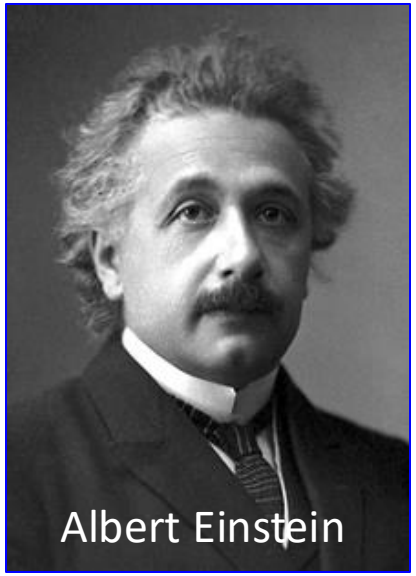
Light kicks out electron with ***E = h f***

(Independent on light intensity!)

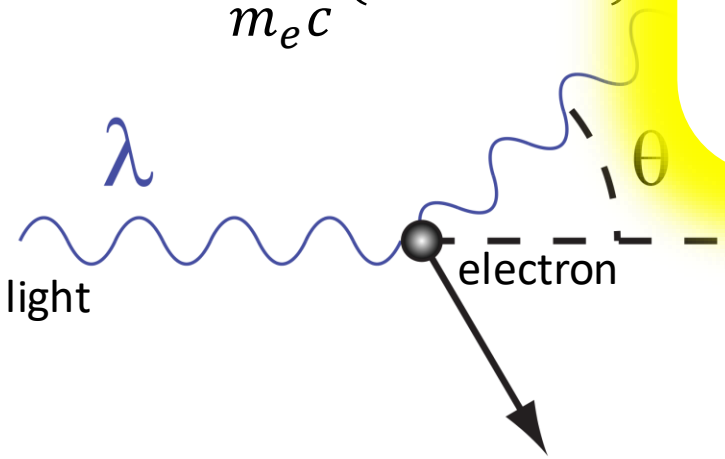
LIGHT IS A WAVE!

$\lambda = hc/E$

$E = pc$



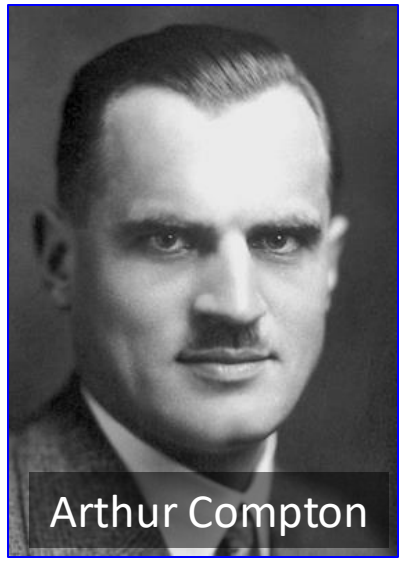
$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$



"Playing billiards" with light quanta and electrons.

Light behaves as a particle with: ***λ = h / p***

(Nobelprize 1927)

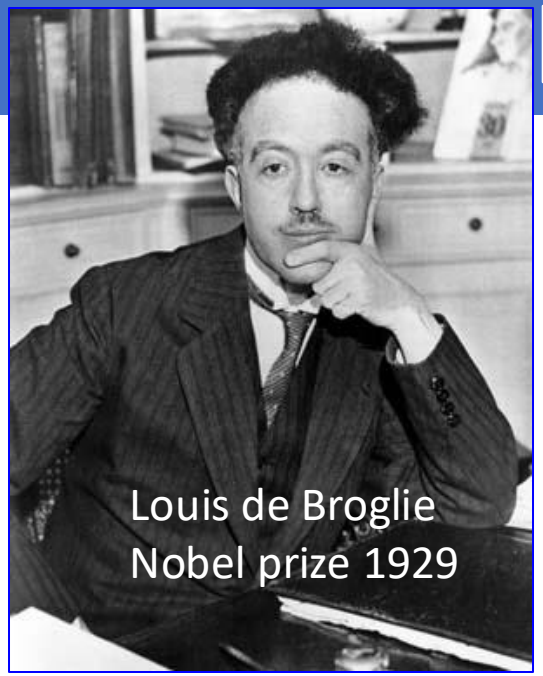


"Once and for all I want to know what I am paying for. When the electric company tells me whether light is a wave or a particle I'll write my check."

Matter Waves

Louis de Broglie - PhD Thesis(!) 1924 (Nobel prize 1929):
If light are particles incorporated in a wave, it suggests that particles (electrons) “are carried” by waves.

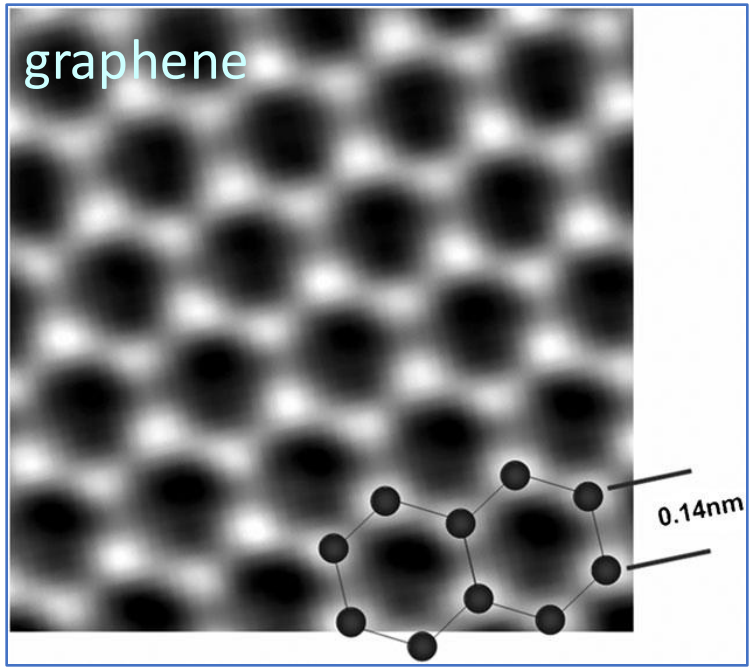
Original idea: a physical wave ➔ Quantum mechanics: probability wave!


Louis de Broglie
Nobel prize 1929

Particle wavelength: $\lambda = h/p \rightarrow \lambda = h/(mv)$

Wavelength visible light:
400 – 700 nm
Use $h = 6.62 \times 10^{-34}$ Js to calculate:
- Wavelength electron with v = 0.1 c:
  0.024 nm
- Wavelength of a fly (m = 0.01 gram, v = 10 m/s):
  0.000000000000000000062 nm


electron microscope



Matter Waves

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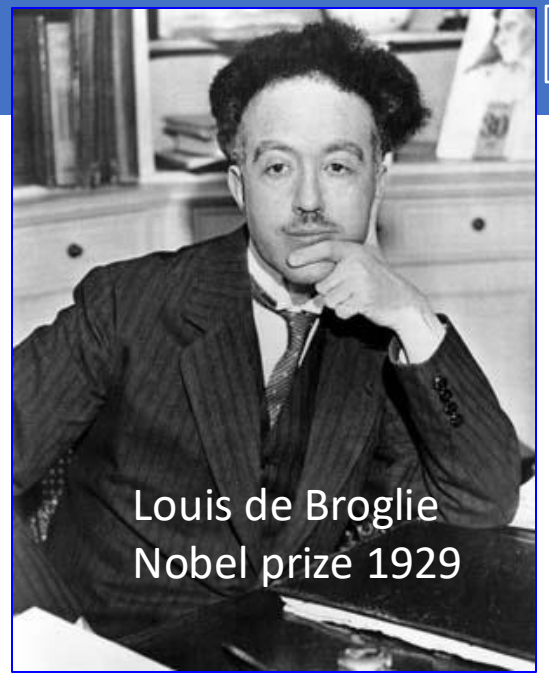
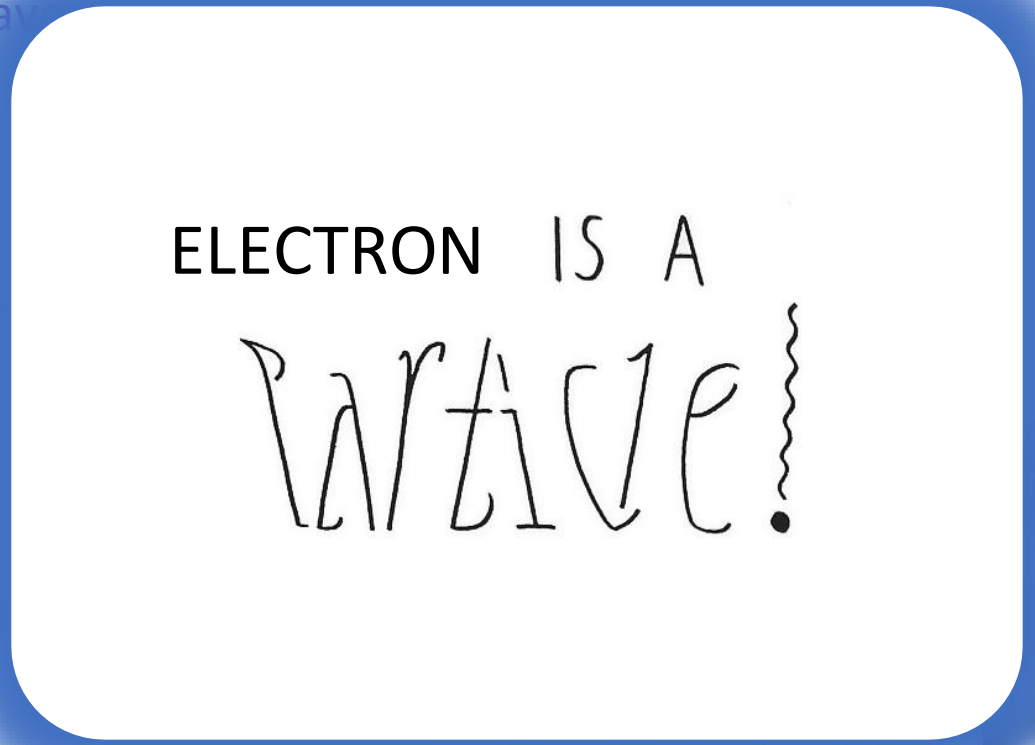
Original idea: a physical wave

Particle wave





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- Wavelength electron with
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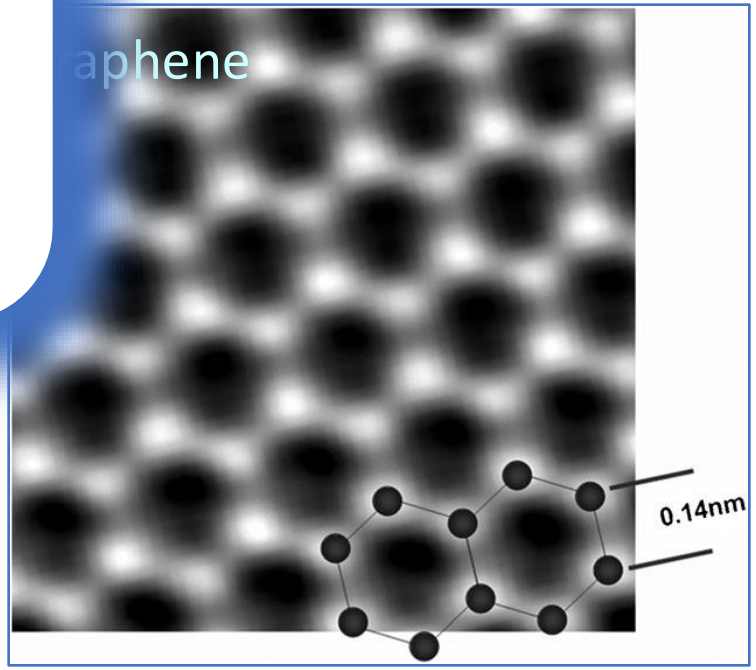
aphene



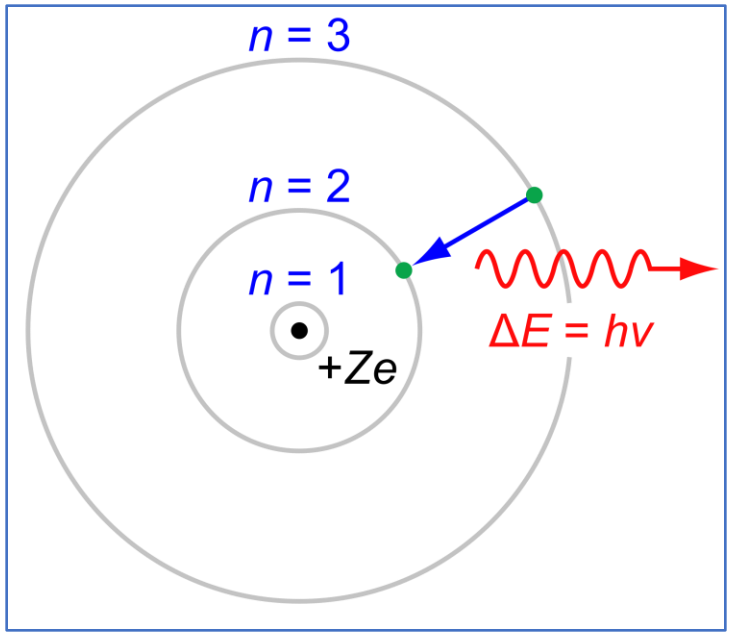
Speaking about Microscopes…

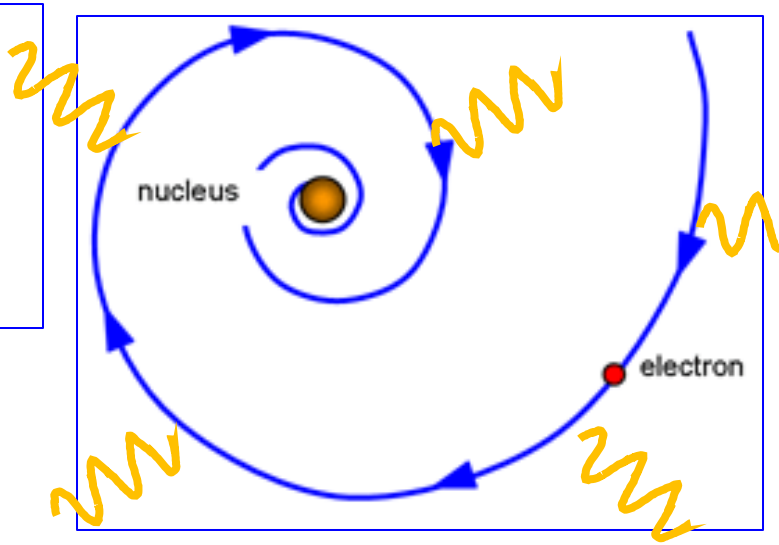
The Quantum Atom of Niels Bohr

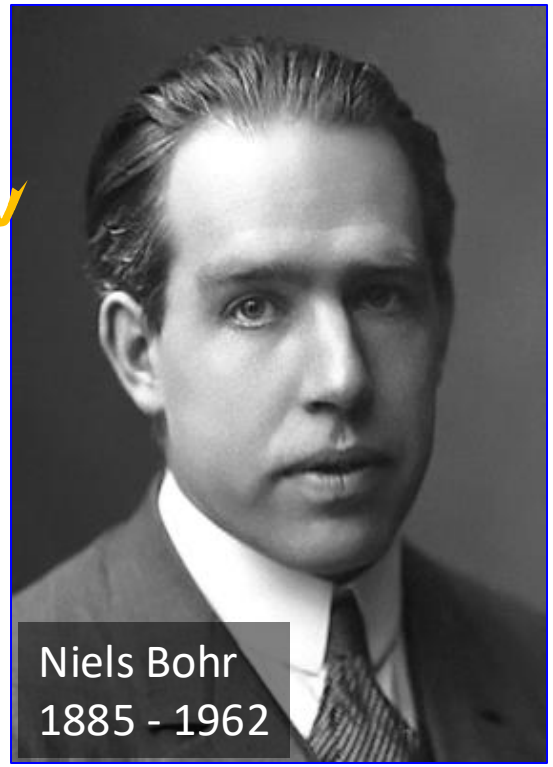
The classical Atom is unstable!

Expect: $t < 10^{-10}$ s

Niels Bohr: Atom is only stable for specific orbits: “energy levels”.







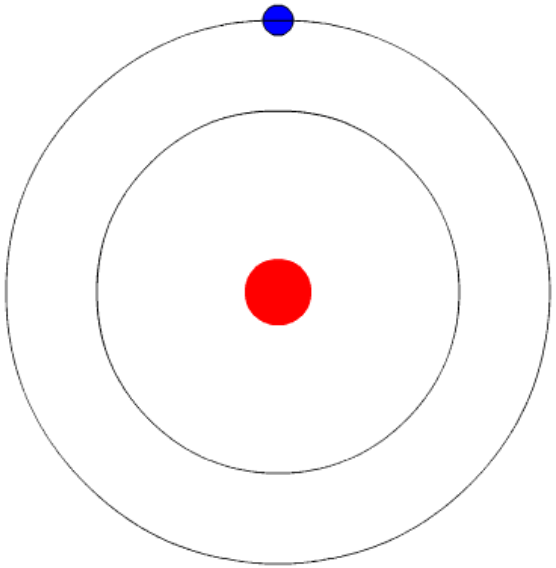
An electron can jump from a high to lower level by emitting a light quantum with corresponding energy difference.

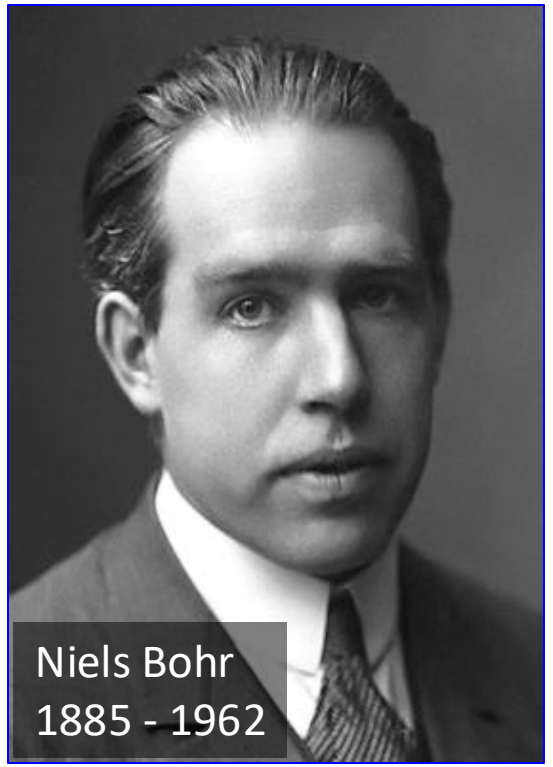
The Quantum Atom of Niels Bohr

The classical Atom is unstable!

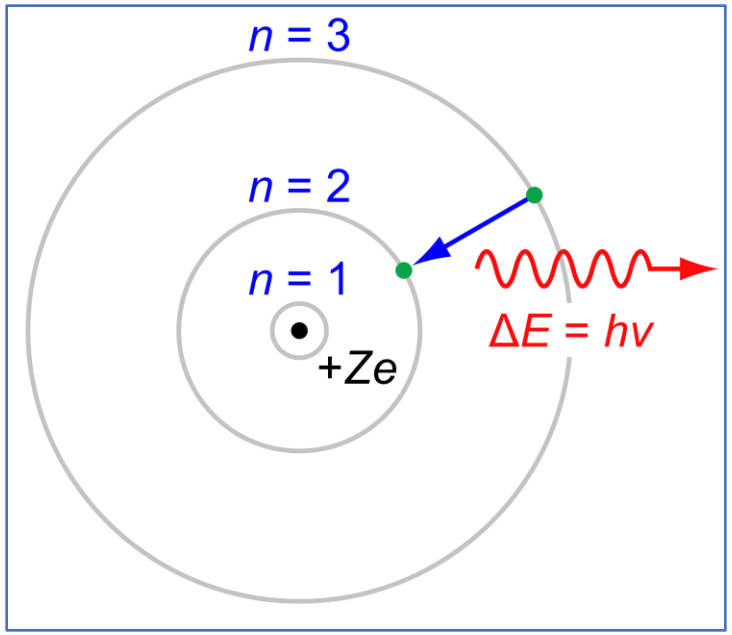
Expect: $t < 10^{-10}$ s

Niels Bohr: Atom is only stable for specific orbits: “energy levels”.





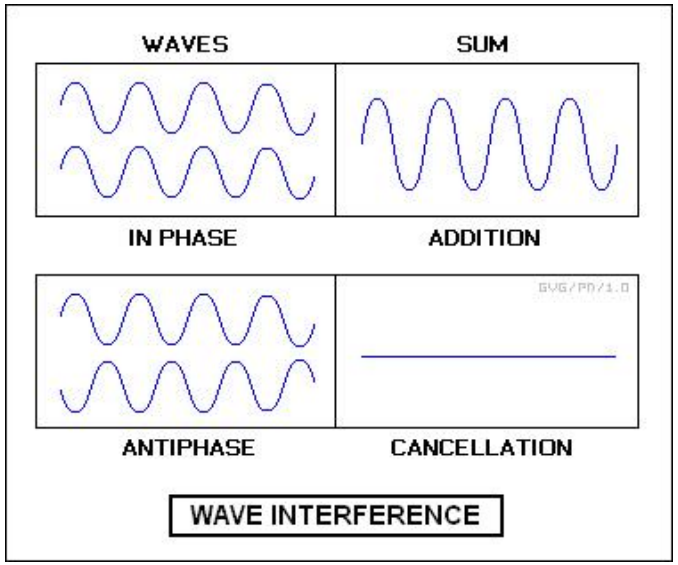
Niels Bohr
1885 - 1962

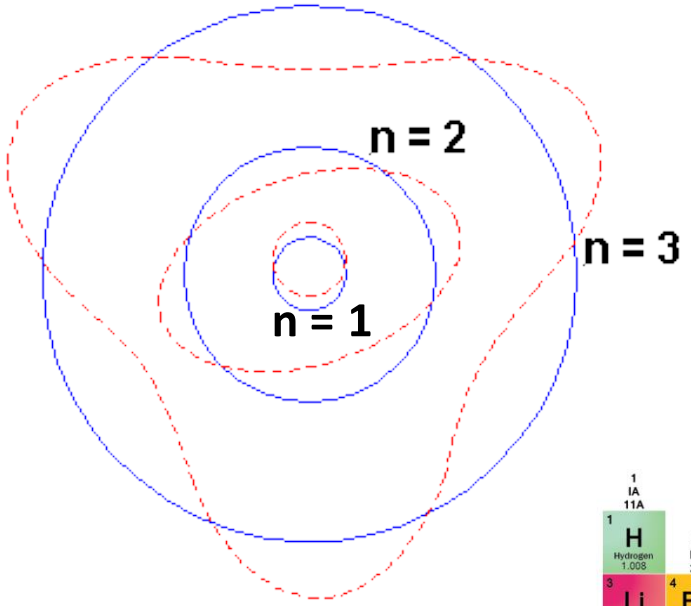


An electron can jump from a high to lower level by emitting a light quantum with corresponding energy difference.

Hydrogen “Balmer” spectrum of wavelengths: $E_n = -13.6\ \mathrm{eV}/n^2$

Schrödinger: Bohr atom and de Broglie waves



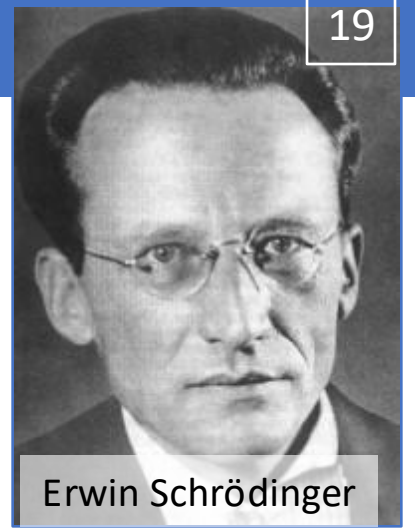


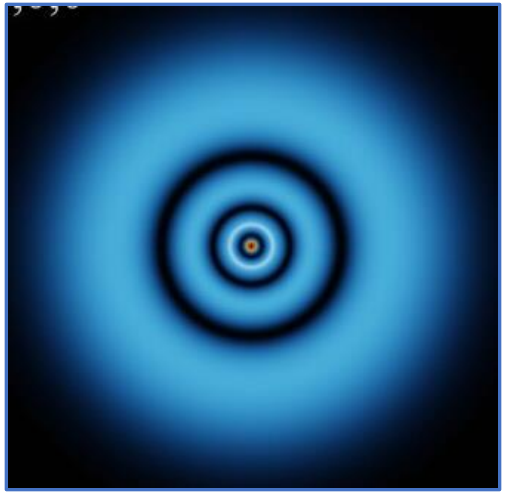
If orbit length "fits":

$2\pi r = n\lambda$ with $n = 1, 2, 3, \ldots$

The wave positively interferes with itself!

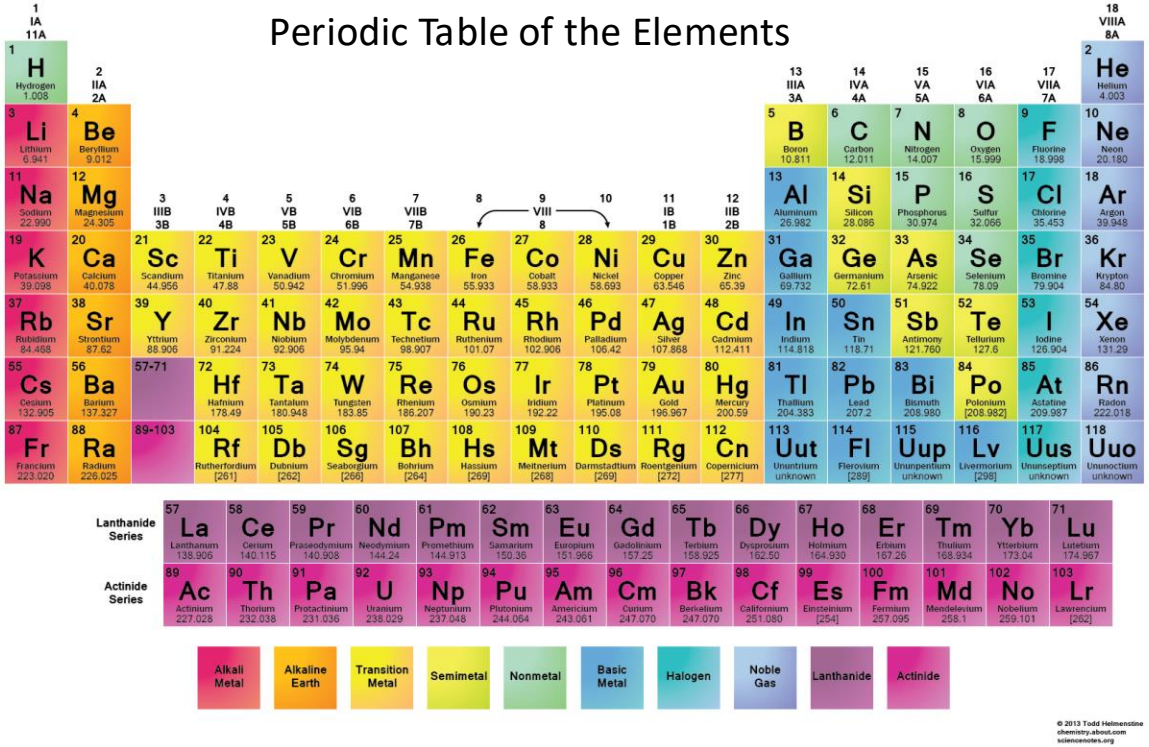
➔ Stable orbits!


Erwin Schrödinger

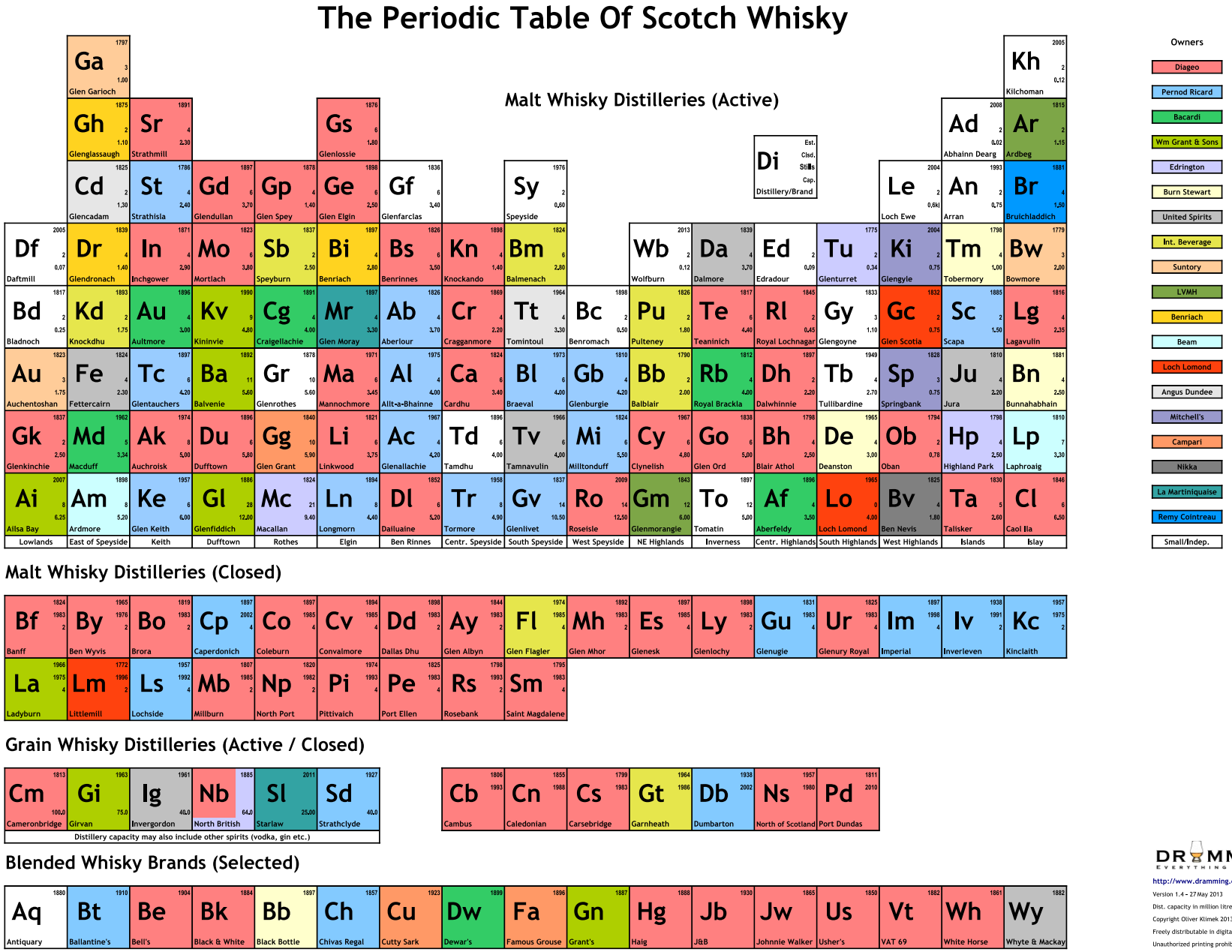

More realistic atom: probability waves

Energy levels explained
→ atom explained
Outer shell electrons
→ "chemistry explained"

Periodic Table of the Elements



Not yet explained: “Dramming”



DRAMMING
EVERYTHING WHISKY

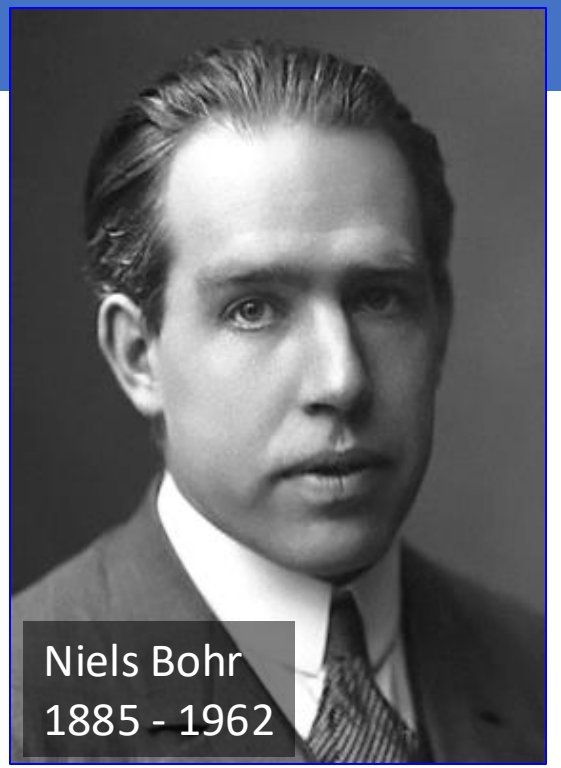
http://www.dramming.com

Version 1.4 – 27 May 2013

Dist. capacity in million litres p.a. except Le

Complementarity

Subatomic matter is not just waves and it is not just particles.
It is nothing we know from macroscopic world.


Niels Bohr
1885 - 1962

Copenhagen Interpretation (Niels Bohr):

One can observe wave characteristics or particle characteristics of quantum objects, ***never both*** at the same time.

Particle and Wave aspects of a physical object are ***complementary***.

Similarly one can never determine from a quantum object at the same time: ***energy and time, position and momentum*** (and more).

Heisenberg Uncertainty Relation

A measurement of a characteristic of quantum matter affects the object.
Heisenberg's "non-commuting" observables:

You cannot know ***position*** and ***momentum*** at the same time:

$$xp - px = i\hbar$$

usually written in the form:

$$\Delta x \ \Delta p \ \geq \ \hbar/2$$

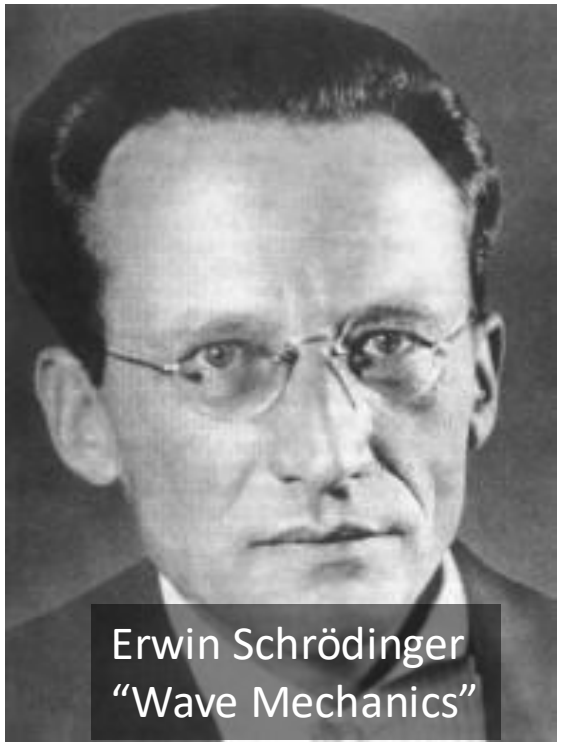
You cannot know both **energy** and **time**:

$$Et - tE = i\hbar$$

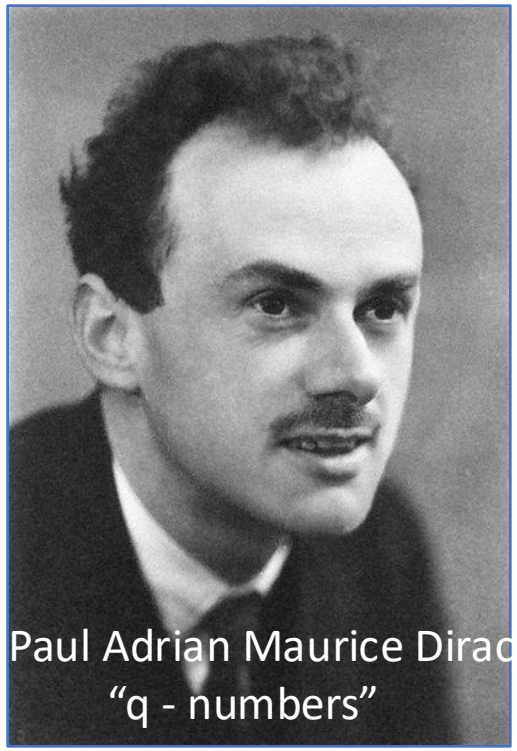
usually written in the form

$$\Delta E \ \Delta t \ \geq \ \hbar/2$$

Werner Heisenberg
"Matrix mechanics"



Erwin Schrödinger
"Wave Mechanics"



Paul Adrian Maurice Dirac
"q - numbers"

It is a fundamental aspect of nature.

Not related to limited technology!

Heisenberg Uncertainty Relation

It is a fundamental aspect of nature.

Not related to limited technology!

Heisenberg Uncertainty Relation: for experts

Energy and time:

In quantum mechanics energy is the "rate of change in time of a wavefunction": $E\psi \rightarrow i\hbar \frac{\partial}{\partial t}\psi$

It fundamentally requires time to measure energy, ie. change in time.

Consequence: $\Delta E\ \Delta t \geq\ \hbar/2$ Measuring energy means locality in time is lost.

Position and momentum:

In quantum mechanics momentum is "rate of change in space of a wavefunction": $p\psi \rightarrow -i\hbar \frac{\partial}{\partial x}\psi$

It fundamentally requires space to measure momentum, ie. change in space.

Consequence: $\Delta x\ \Delta p \leq\ \hbar/2$ Measuring momentum means locality in space is lost.

It is a fundamental aspect of nature.

Not related to limited technology!

…and if you REALLY want to see where it comes from….

1. Replace E by the operator $i\hbar\frac{\partial}{\partial t}$ operating on wave function ψ:

$$Et(\psi) = i\hbar\frac{\partial}{\partial t}(t\psi) = i\hbar\psi + i\hbar t\frac{\partial\psi}{\partial t}$$

$$tE(\psi) = ti\hbar\frac{\partial\psi}{\partial t} = i\hbar t\frac{\partial\psi}{\partial t}$$

$$\Rightarrow \quad \boxed{(Et - tE)\,\psi = i\hbar\,\psi}$$

2. Replace p by the operator $-i\hbar\frac{\partial}{\partial x}$ operating on wave function ψ:

$$px(\psi) = -i\hbar\frac{\partial}{\partial x}(x\psi) = -i\hbar\psi - i\hbar x\frac{\partial\psi}{\partial x}$$

$$xp(\psi) = -xi\hbar\frac{\partial\psi}{\partial x} = -i\hbar x\frac{\partial\psi}{\partial tx}$$

$$\Rightarrow \quad \boxed{(xp - px)\,\psi = i\hbar\,\psi}$$

"Non commuting observables"

It is a fundamental aspect of nature.

Not related to limited technology!

…and if you REALLY want to see where it comes from….

1. Replace E by the operator $i\hbar\frac{\partial}{\partial t}$ operating on wave function ψ:

$$Et(\psi) \quad \text{…} \quad i\hbar t\frac{\partial\psi}{\partial t}$$

$$tE(\psi)$$

$$\rightarrow \quad \boxed{(Et - tE)\,\psi = i\hbar\,\psi}$$

2. Re… $\frac{\partial}{\partial x}$ operating on wave function ψ:

$$px(\psi) \quad \text{…} \quad -i\hbar x\frac{\partial\psi}{\partial x}$$

$$xp(\psi)$$

$$\rightarrow \quad \boxed{(xp - px)\,\psi = i\hbar\,\psi}$$

"Non commuting observables"

It is a fundamental aspect of nature.

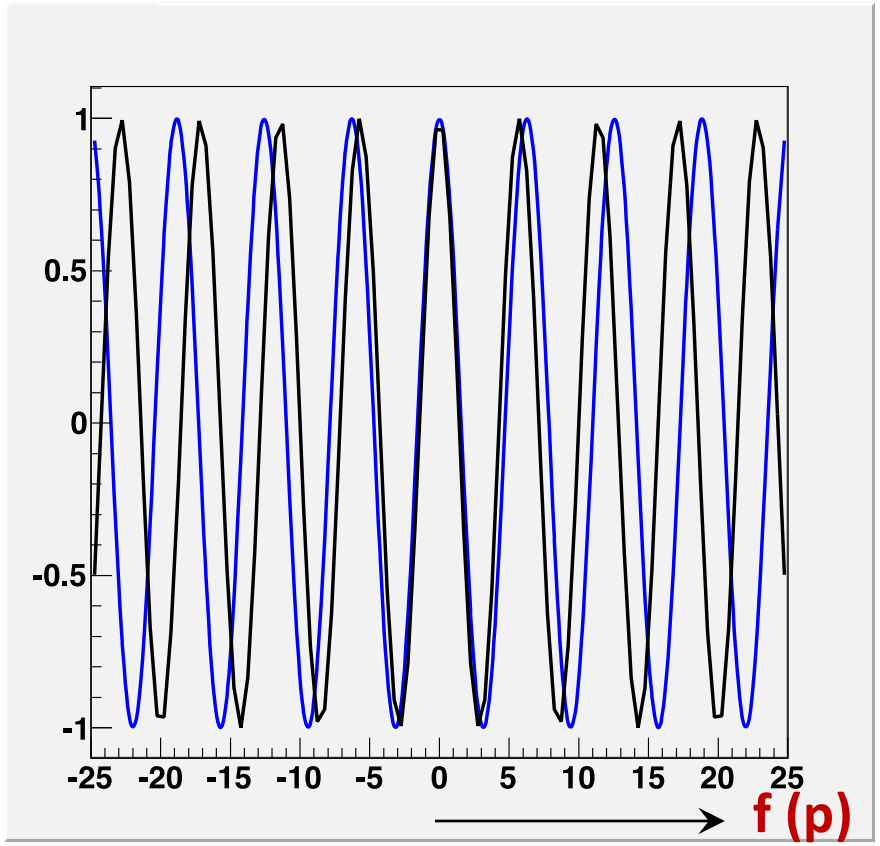
Not related to limited technology!

Waves and Uncertainty

Use the "wave-mechanics" picture of Schrödinger

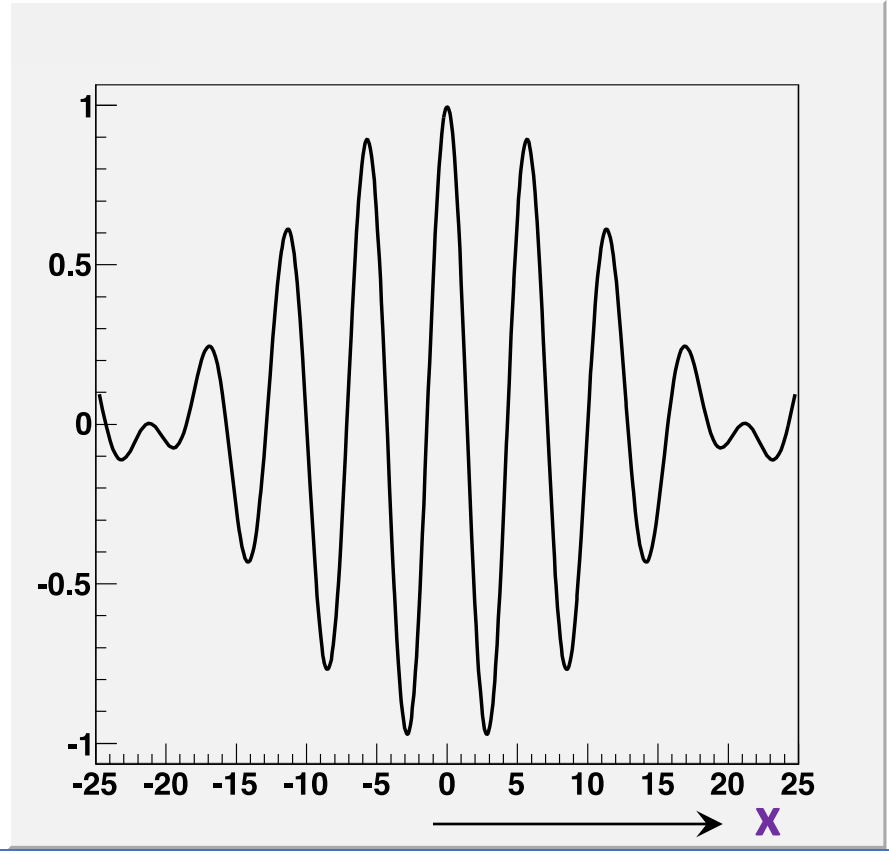
A wave has an exactly defined frequency.

Two waves: $p_1 = hf_1/c$, $p_2 = hf_2/c$



A particle has an exactly defined position.

Wave Packet: sum of black and blue wave



The more waves are added, the more the wave packet looks like a particle, or,
If we try to determine the position **x**, we destroy the frequency/momentum **p** and vice versa.

Waves and Uncertainty

Use the “wave-mechanics” picture of Schrödinger

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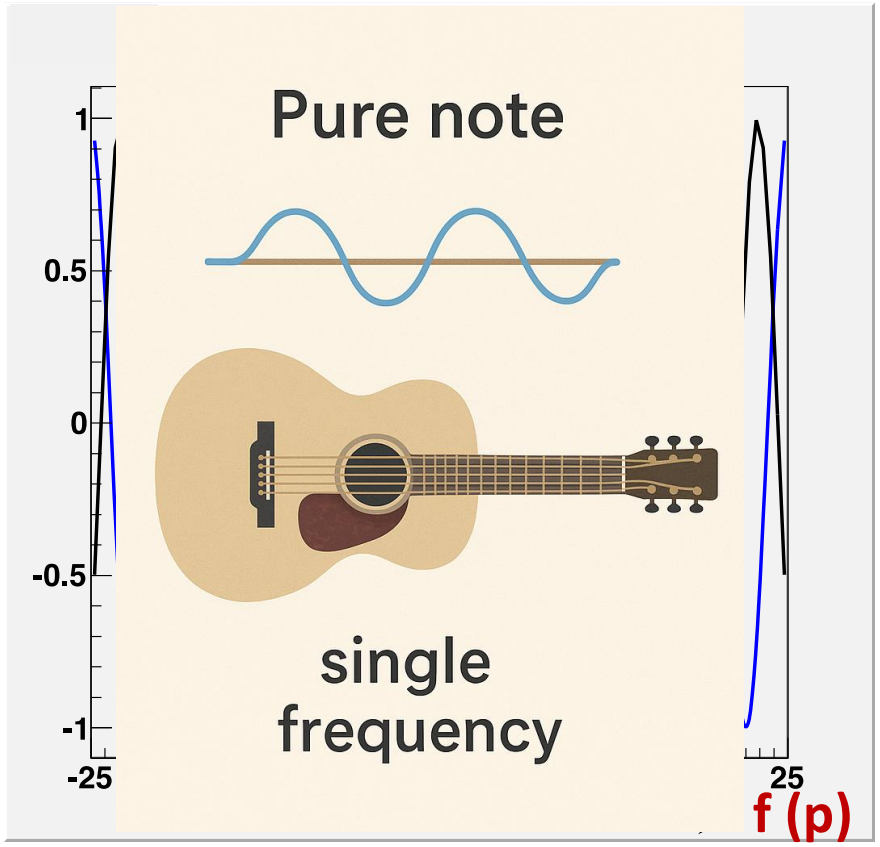
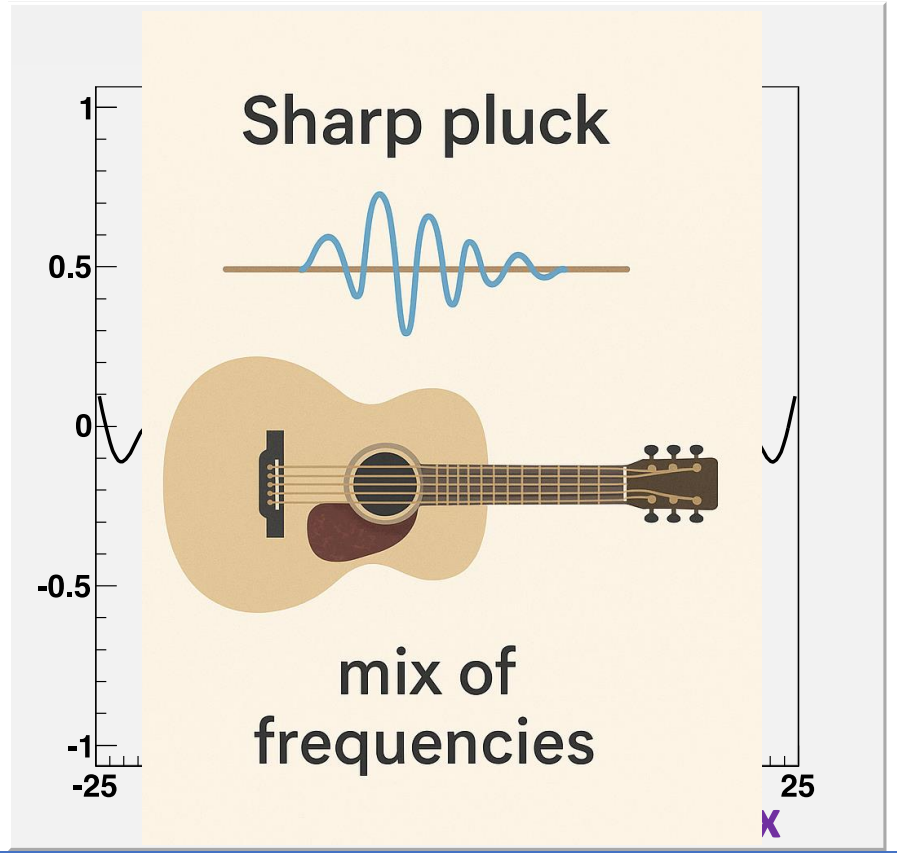


A particle has an exactly defined position.

Wave Packet: sum of black and blue wave



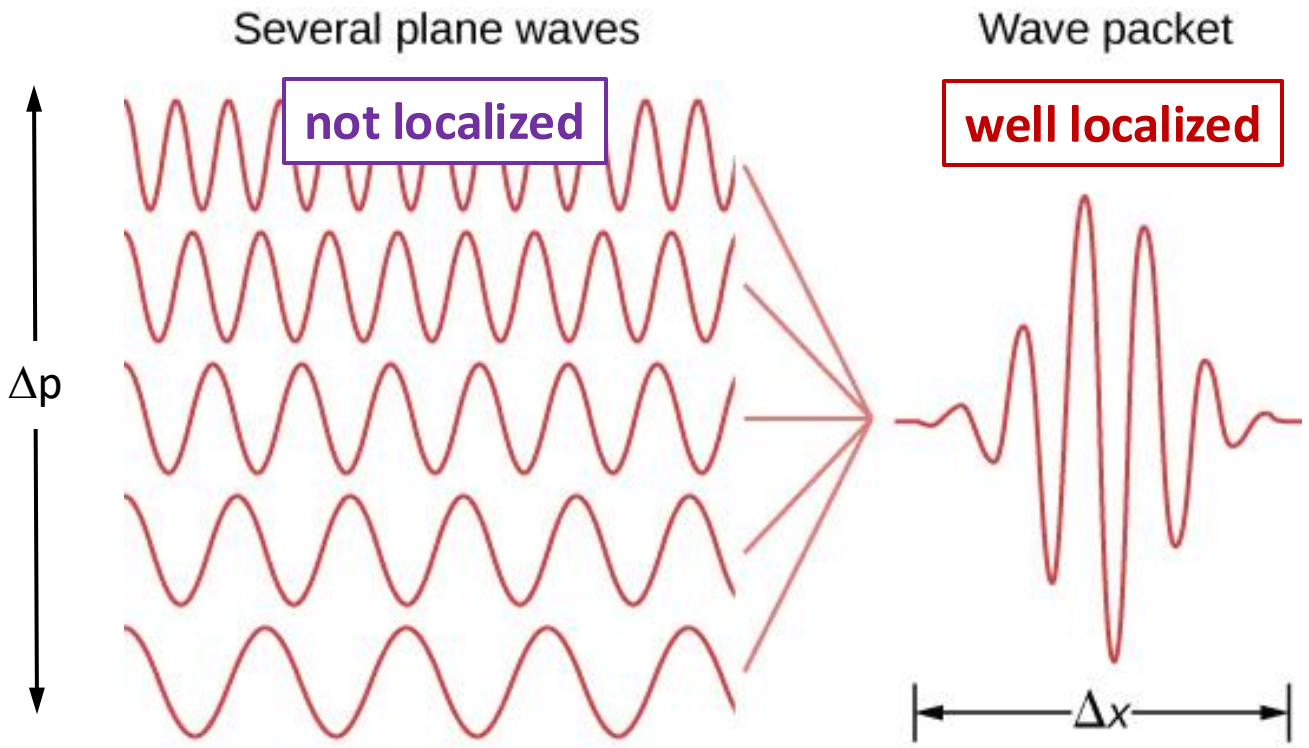
The more waves are added, the more the wave packet looks like a particle, or,
If we try to determine the position **x**, we destroy the frequency/momentum **p** and vice versa.

Waves and Particles

"Particle:"

Pure waves of different frequency, i.e. different momentum $p = hf/c$

Wave package, i.e. "particle"



Uncertainty Revisited

Measure precise momentum
→ You force the quantum to ***have*** a specific frequency
→ You lose the locality of the quantum
→ No position information.

Measure precise position
→ You force the quantum to ***have*** a specific position
→ You lose the momentum (frequency) information of the quantum

The uncertainty relation at work

Shine a beam of light through a narrow slit which has a opening size Δx.
The light comes out over an undefined angle that corresponds to Δp_x.

$$\Delta x \, \Delta p_x \sim \hbar/2$$

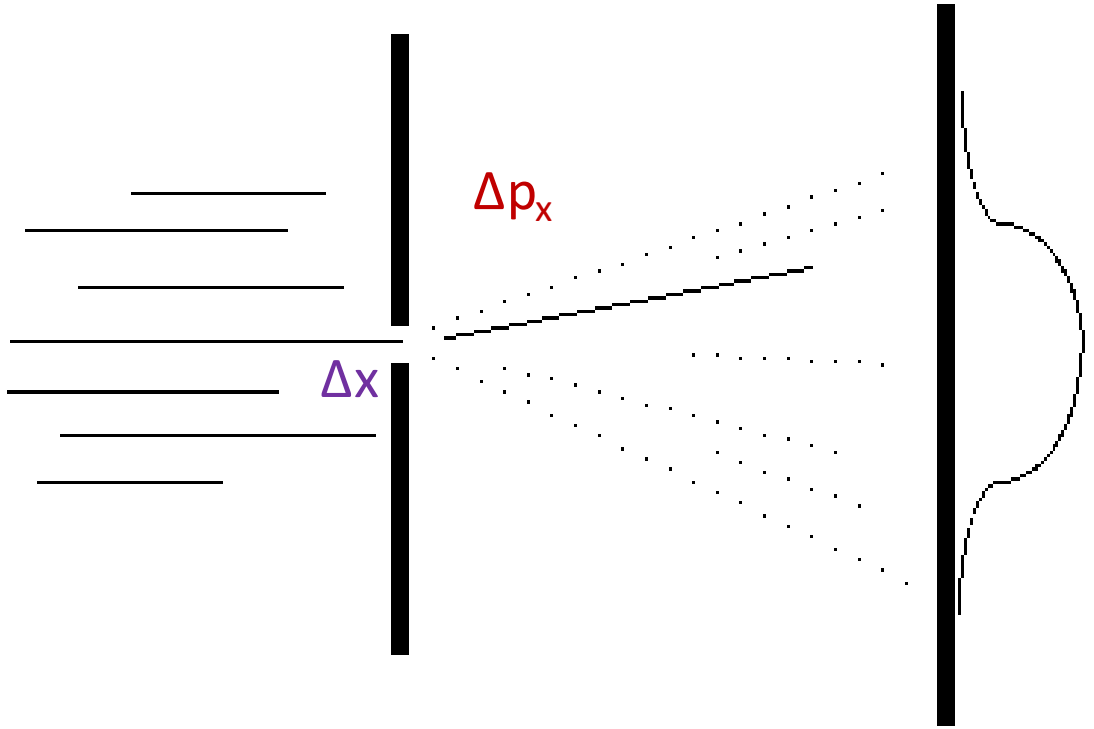
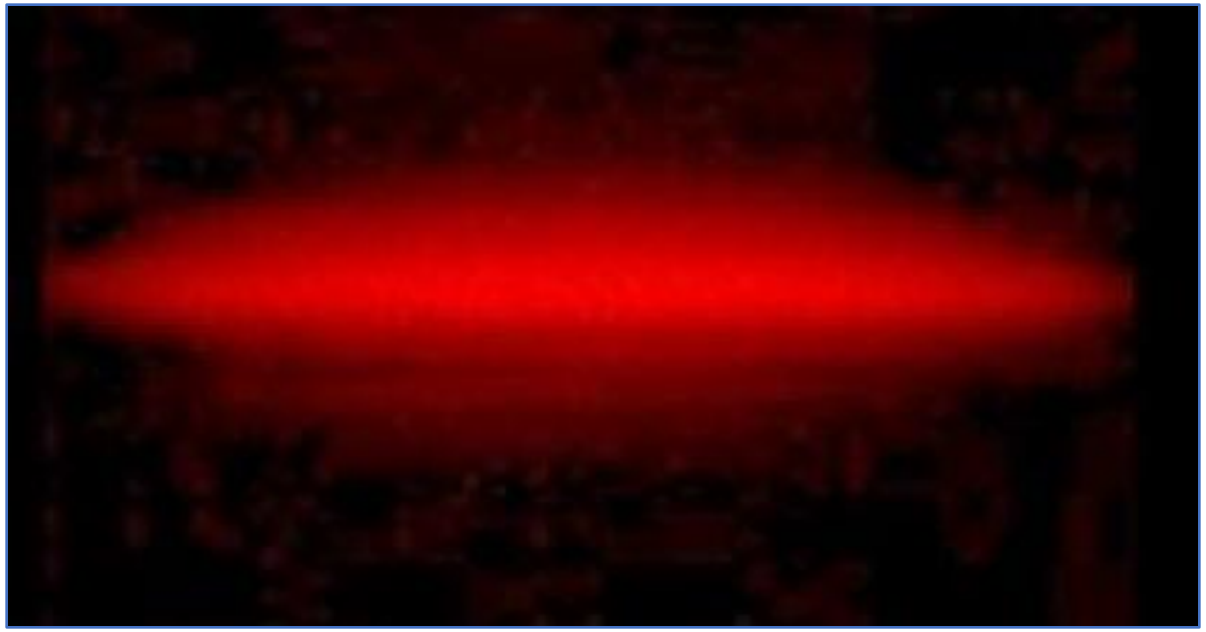
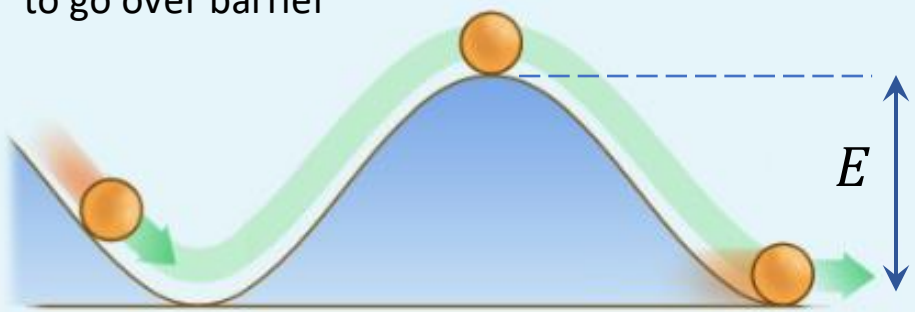


Image of a laser pointer after passing through a slit:

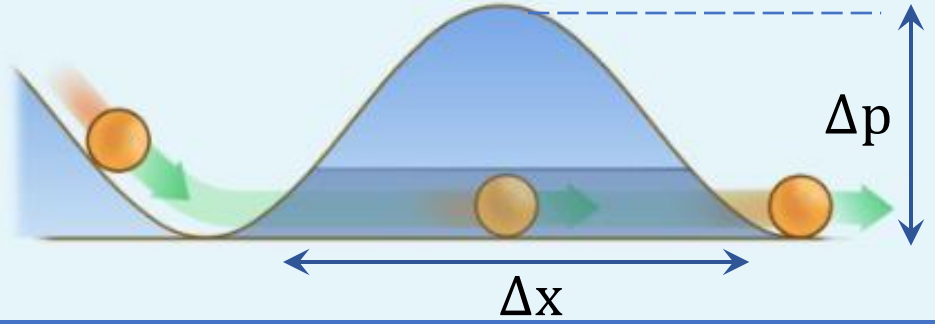


Consequence: Particles can quantum tunnel through a wall!

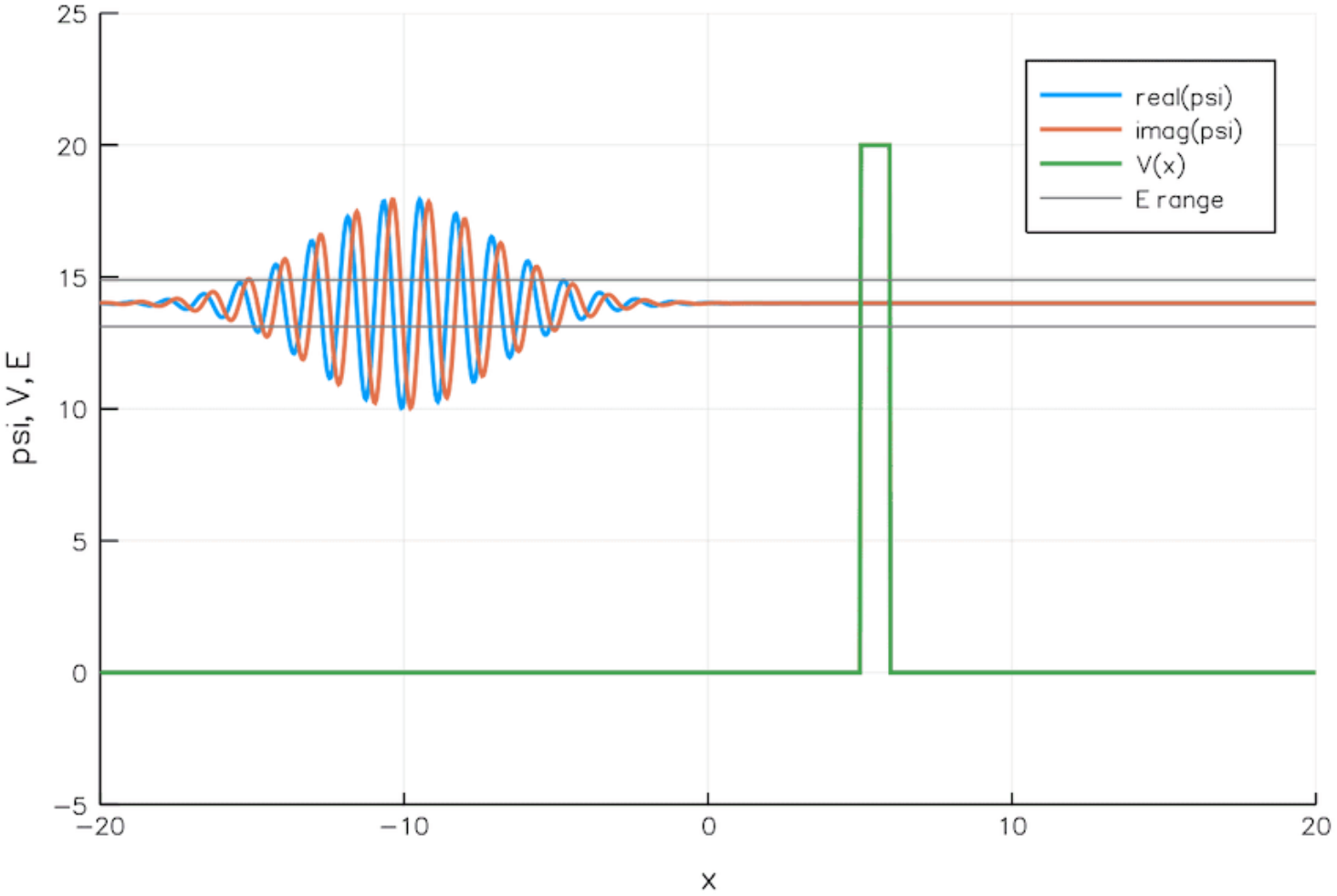
Classic: particle needs enough energy $E = \frac{1}{2}mv^2$ to go over barrier



Quantum: particle can tunnel through the barrier. Even if its energy is not enough.

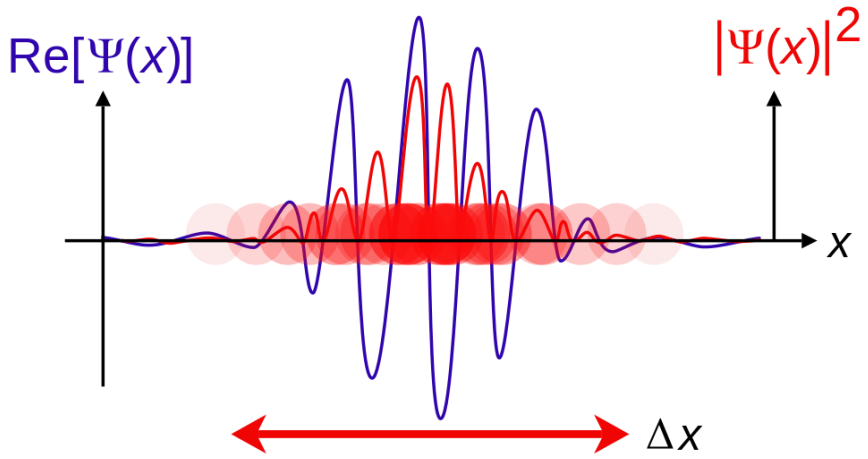


Quantum explanation: particles are waves packets:
"The particles energy (or momentum) is uncertain enough to pass through if the time (or space) is short enough."

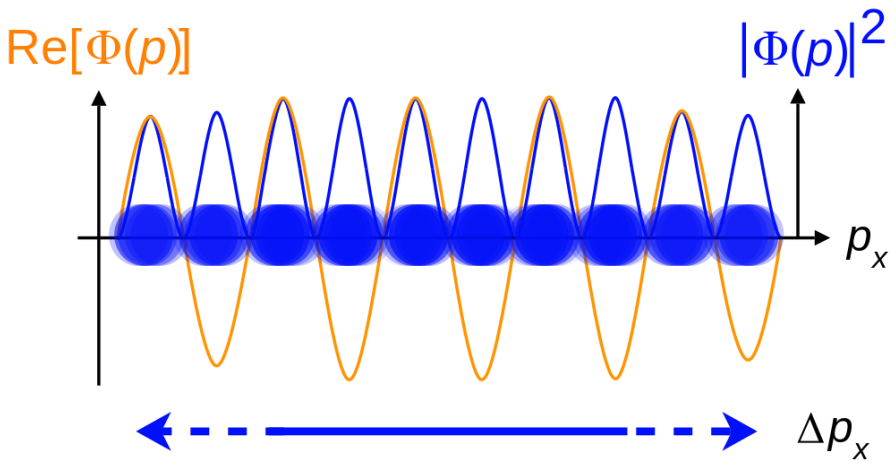


The wave function ψ

Position fairly known

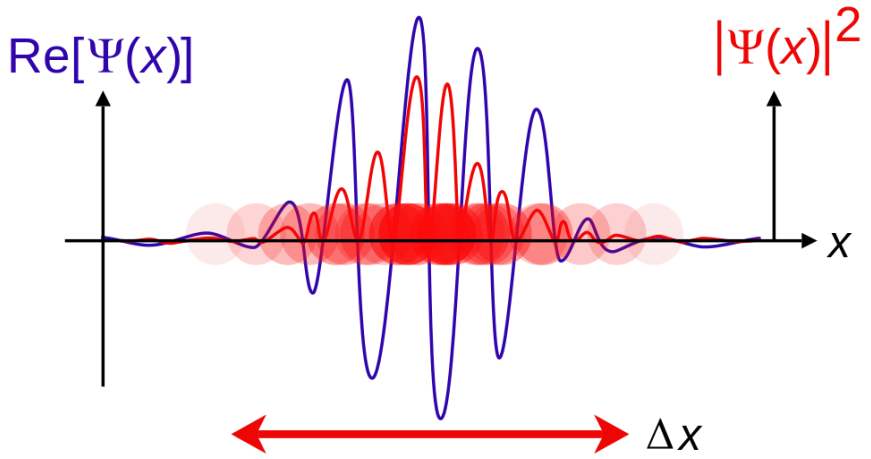


Momentum badly known

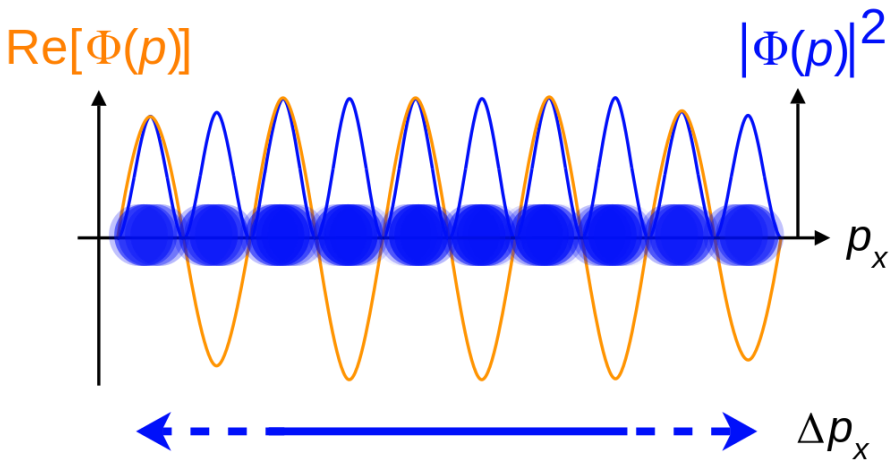


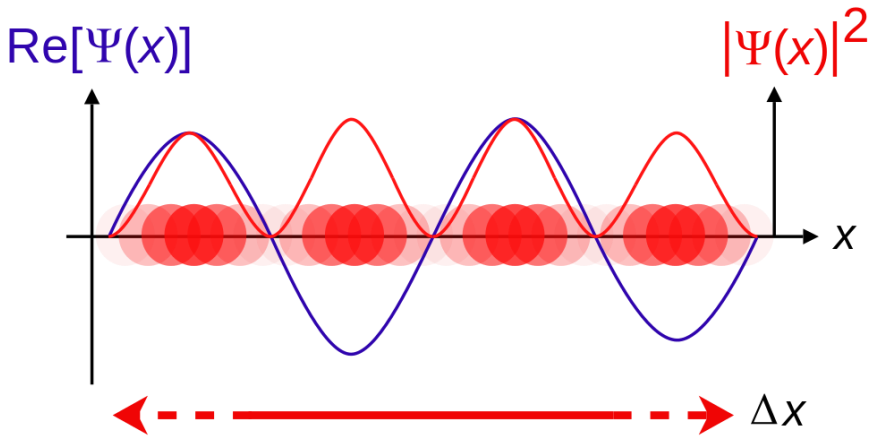
The wave function ψ

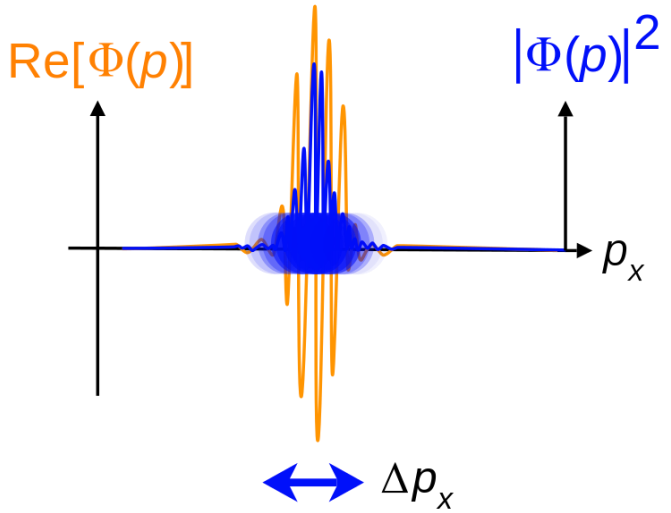
Position fairly known



Momentum badly known



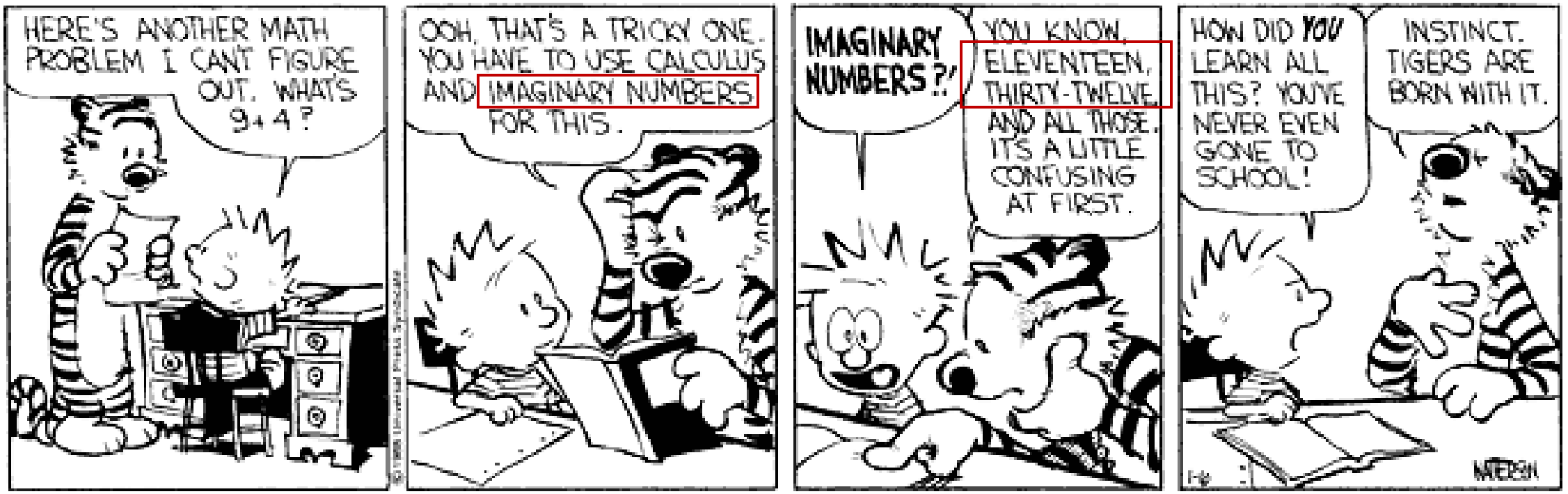




Position badly known

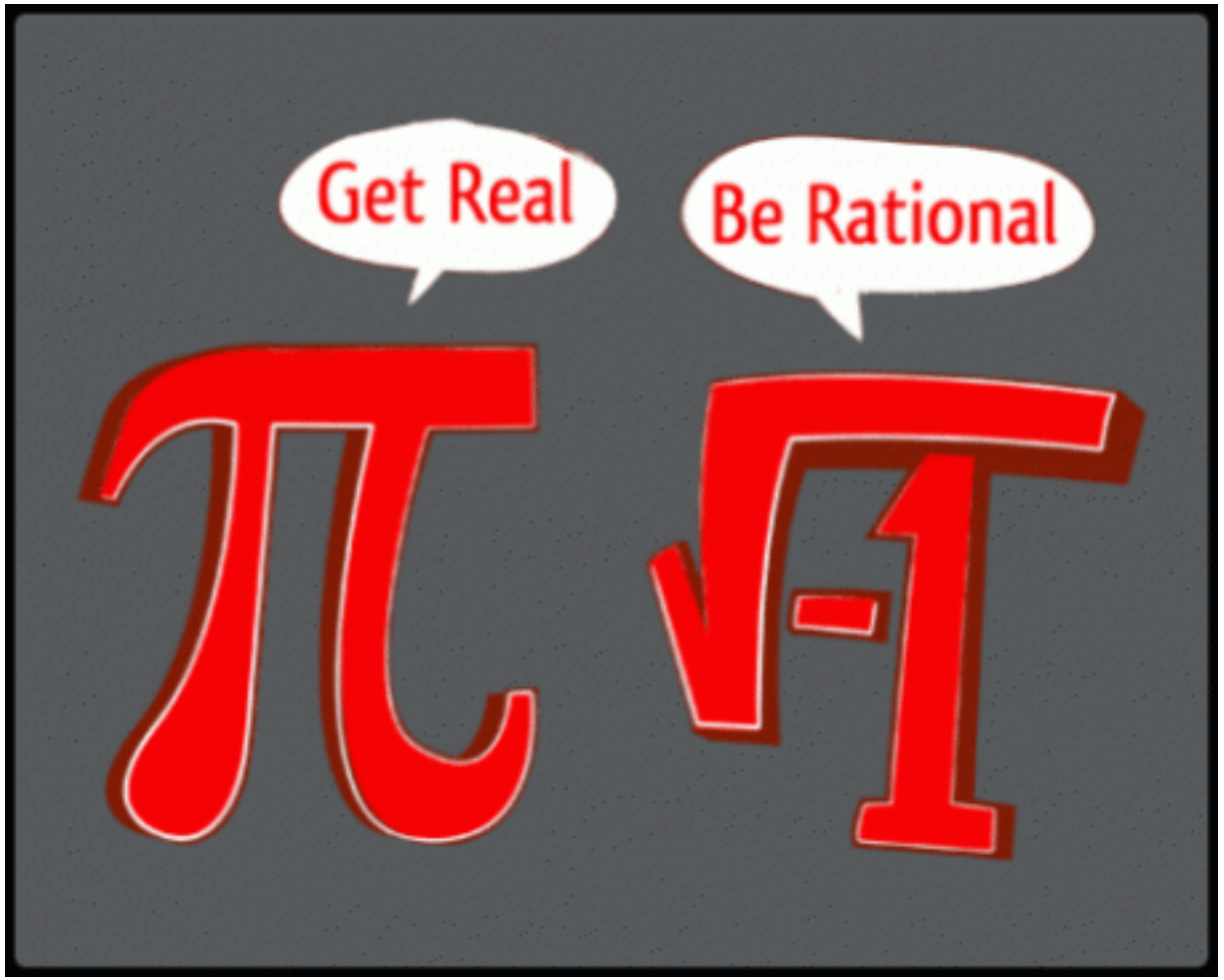
Momentum fairly known

Imaginary Numbers



"Life is complex, it has both real and imaginary parts"

- Anonymous


Get Real
Be Rational
π
√-1

The Copenhagen Interpretation: what **is** ψ ?

The wave function ψ is not a real object.
The only physical meaning is that its square gives the probability to find a particle at a position x and time t.

$$\text{Prob}(x,t) = |\psi(x,t)|^2 = \psi\,\psi^*$$

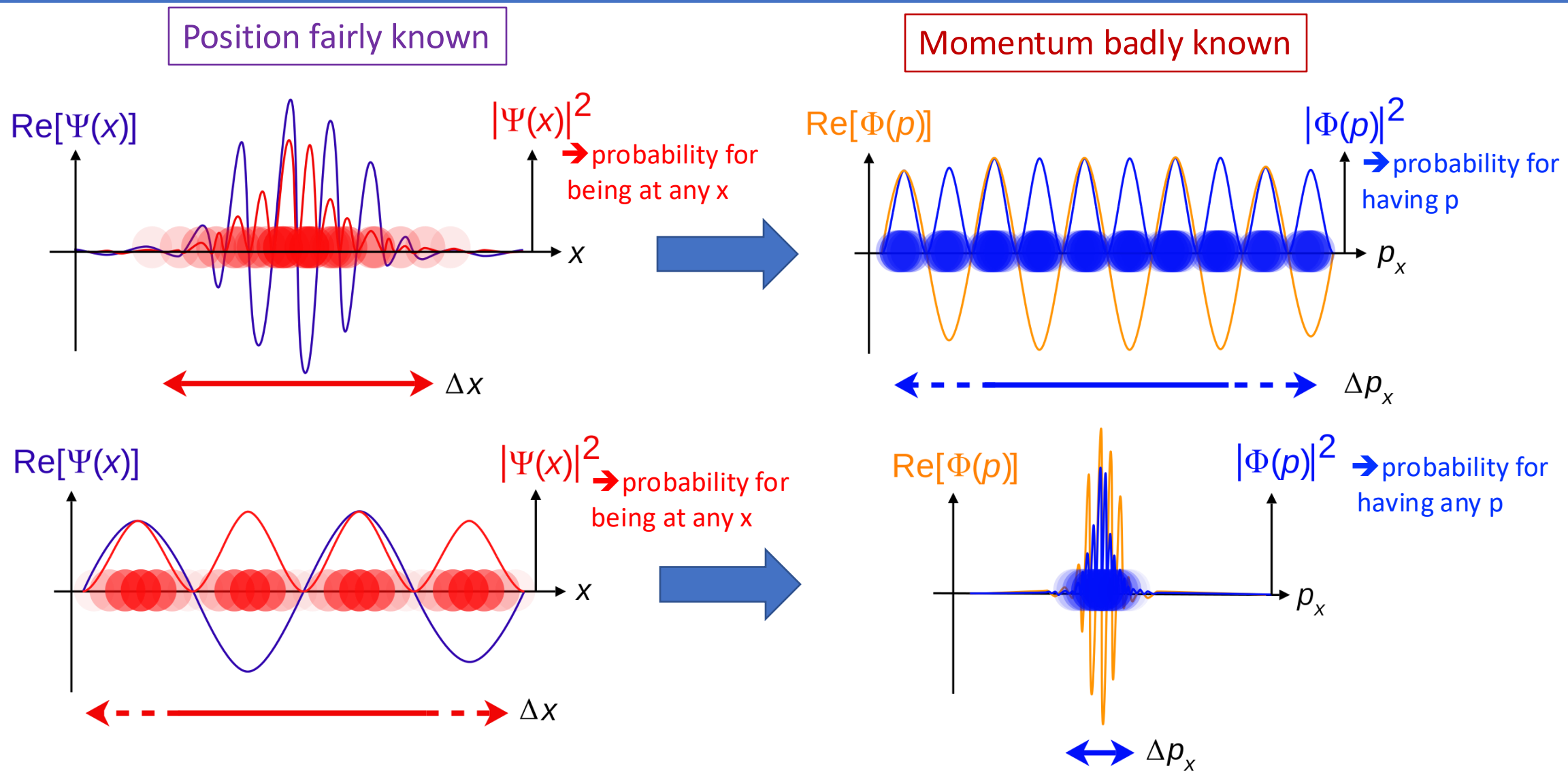

Niels Bohr


Max Born

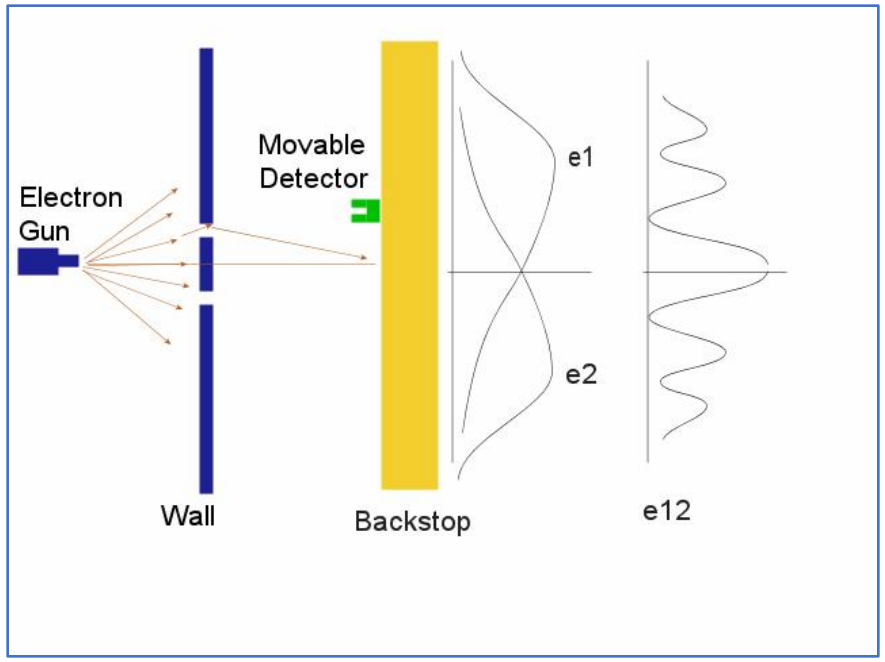
Quantum mechanics allows only to calculate ***probabilities*** for possible outcomes of an experiment and is non-deterministic, contrary to classical theory.
Einstein: "Gott würfelt nicht."

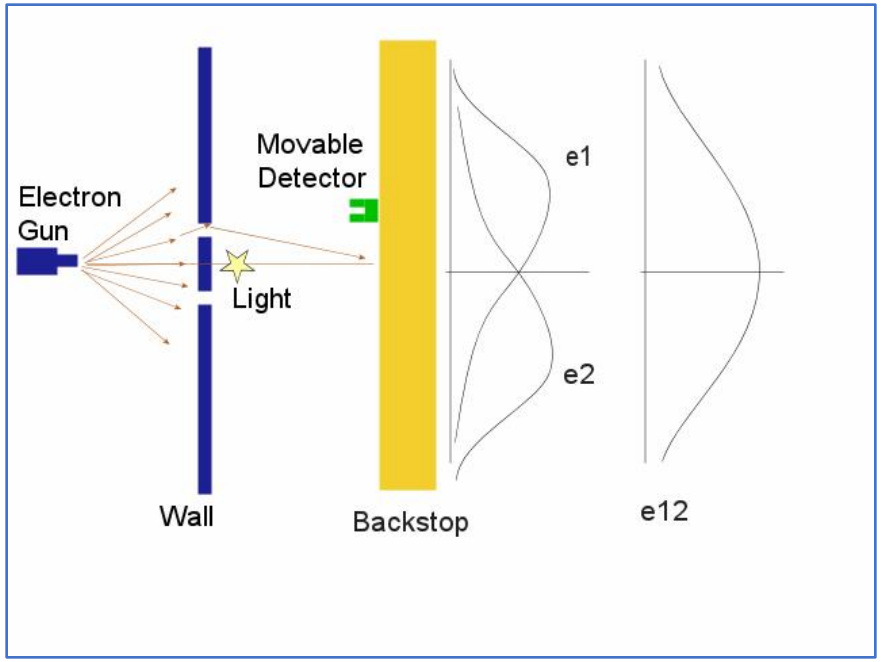
The mathematics for the ***probability*** of the quantum wave-function is the same as the mathematics of the ***intensity*** of a classical wave function.

The wave function ψ



Next Lecture: double slit experiment





The core of quantum mechanics illustrated by Feynman.
Einstein and Schrödinger did not like it.
Wheeler later took it to the extreme.
Even today people are debating its interpretation.