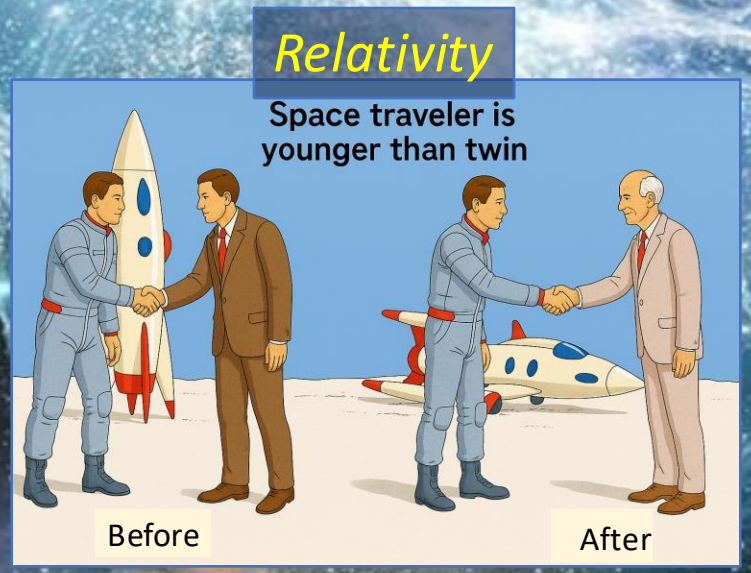
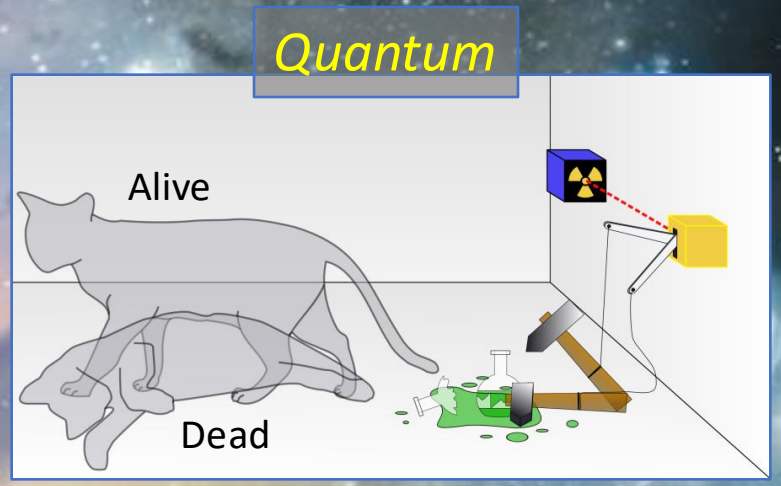
The Relativistic Quantum World

A lecture series on
Relativity Theory and Quantum Mechanics





Marcel Merk
Studium Generale Maastricht
Sep 10 – Oct 8, 2025

The Relativistic Quantum World

Relativity	Sep. 10:	Lecture 1: The Principle of Relativity and the Speed of Light Lecture 2: Time Dilation and Lorentz Contraction
	Sep. 17:	Lecture 3: The Lorentz Transformation and Paradoxes Lecture 4: General Relativity and Gravitational Waves
Quantum Mechanics	Sep. 24:	Lecture 5: The Early Quantum Theory Lecture 6: Feynman's Double Slit Experiment
	Oct. 1 :	Lecture 7: Wheeler's Delayed Choice and Schrodinger's Cat Lecture 8: Quantum Reality and the EPR Paradox
Standard Model	Oct. 8:	Lecture 9: The Standard Model and Antimatter Lecture 10: Why is there something rather than nothing?

Lecture notes, written for this course, are available: www.nikhef.nl/~i93/Teaching/
Prerequisite for the course: High school level physics & mathematics.

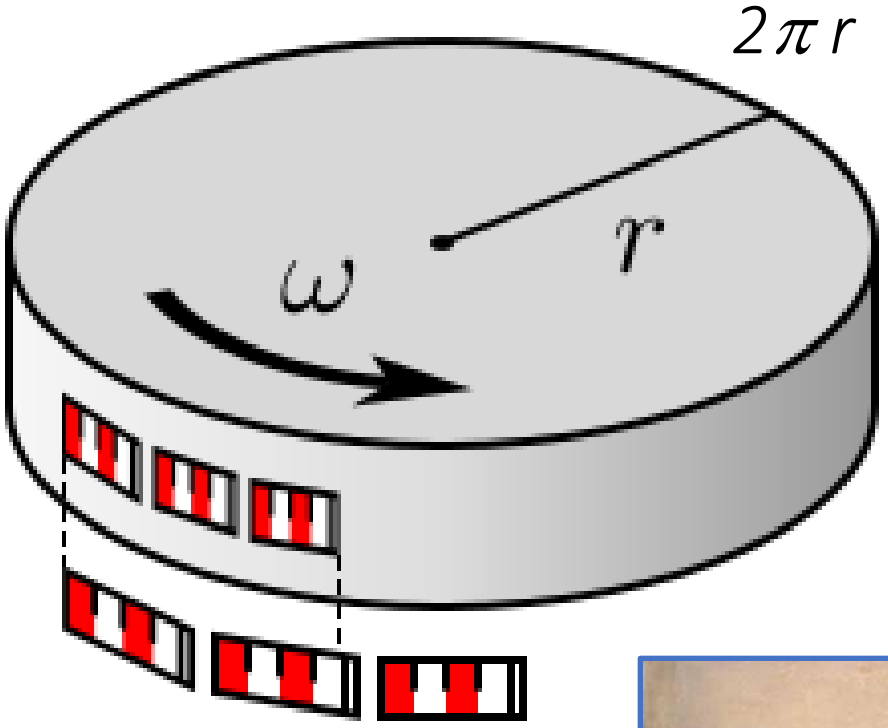
Lecture 4

General Relativity and Gravitational Waves

“Do not worry about your difficulties in mathematics.
I can assure you mine are still greater.”
- Albert Einstein

Ehrenfest Paradox - 1909

Rotating disk with ruler on the edge:
Circumference: $C = 2 \pi r$

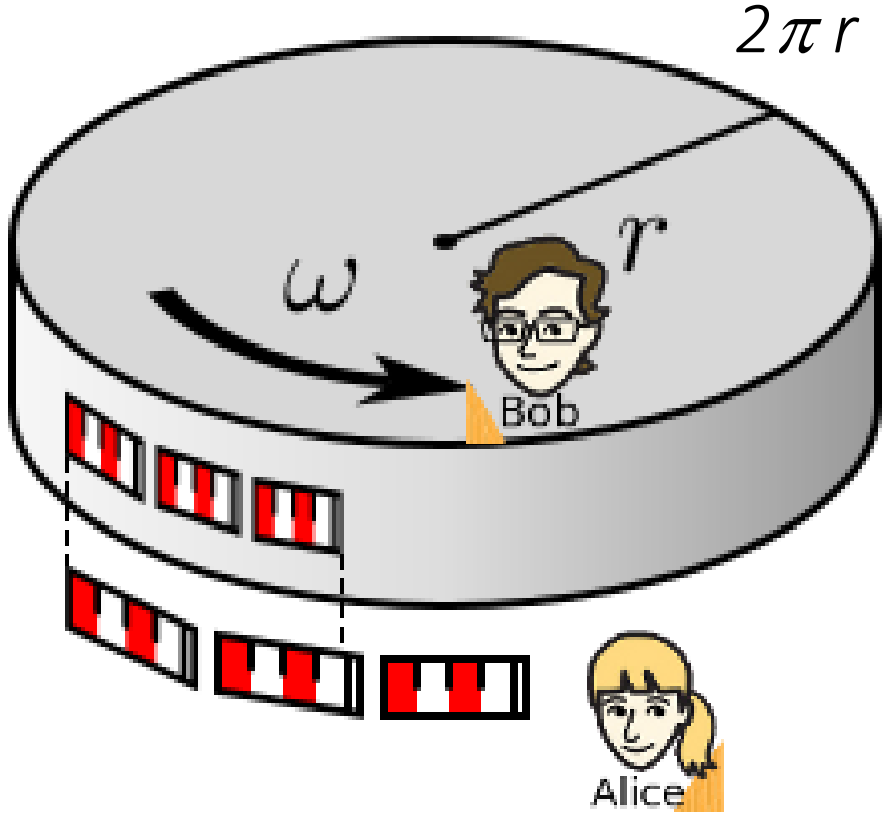


Ehrenfest Paradox - 1909

Rotating disk with ruler on the edge:
Circumference: $C = 2 \pi r$

Alice stands next to the disk and sees *rulers on disk* Lorentz contracted:
$C = 2 \pi r / \gamma$
➔ Circumference is smaller!

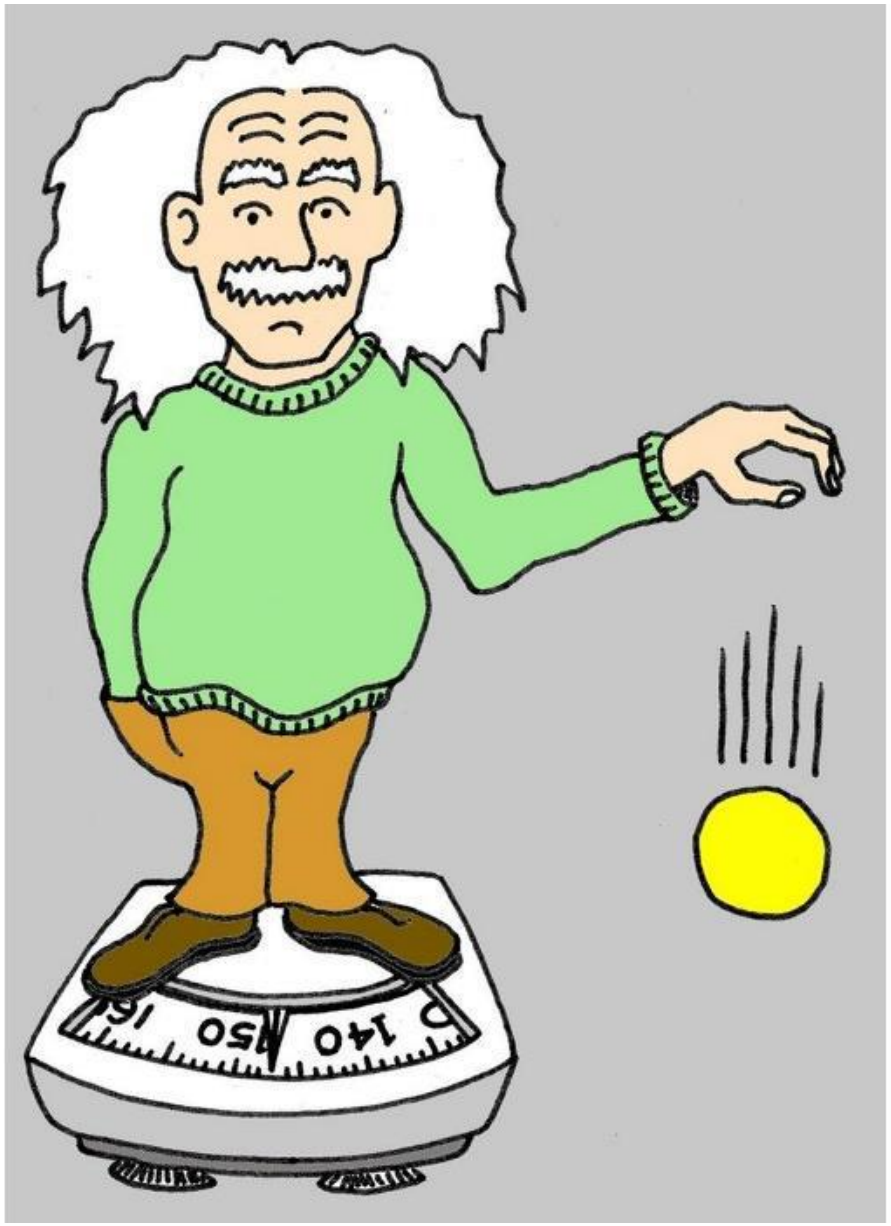
Bob moves on the disk and sees *rulers next to disk* contracted:
$C = 2 \pi r \cdot \gamma$
➔ Circumference is larger!

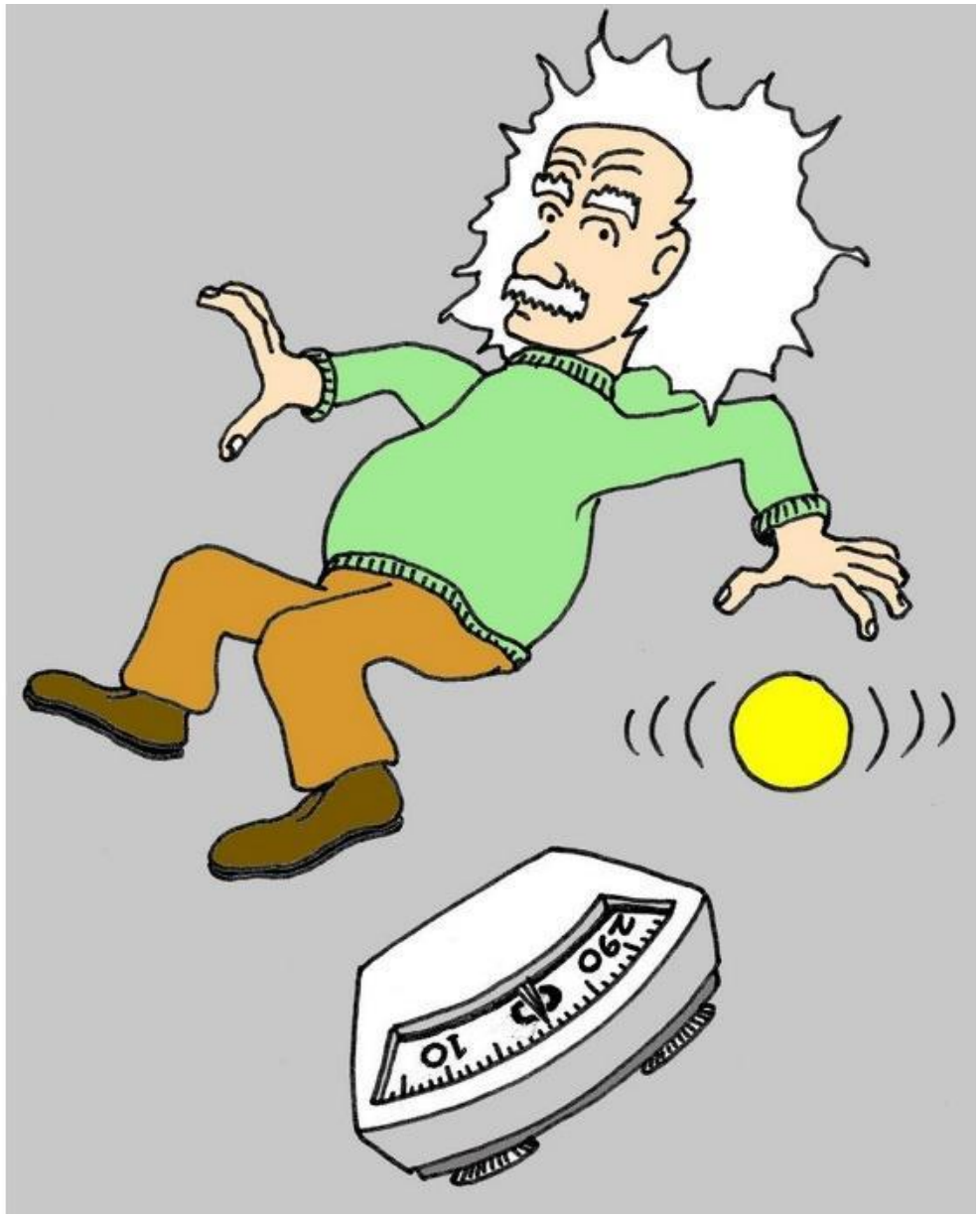


Also called:
Rigidity paradox
Einstein took it seriously!

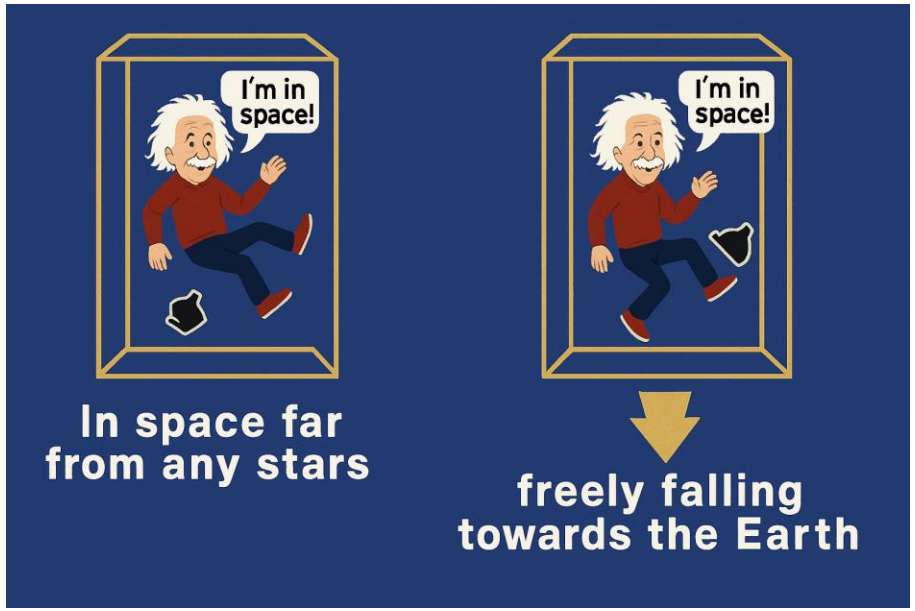
A rotating object is ***not an inertial*** frame:
- Postulate of relativity only worked for ***inertial frames***
- Need to adapt the postulates: special relativity → ***general relativity***

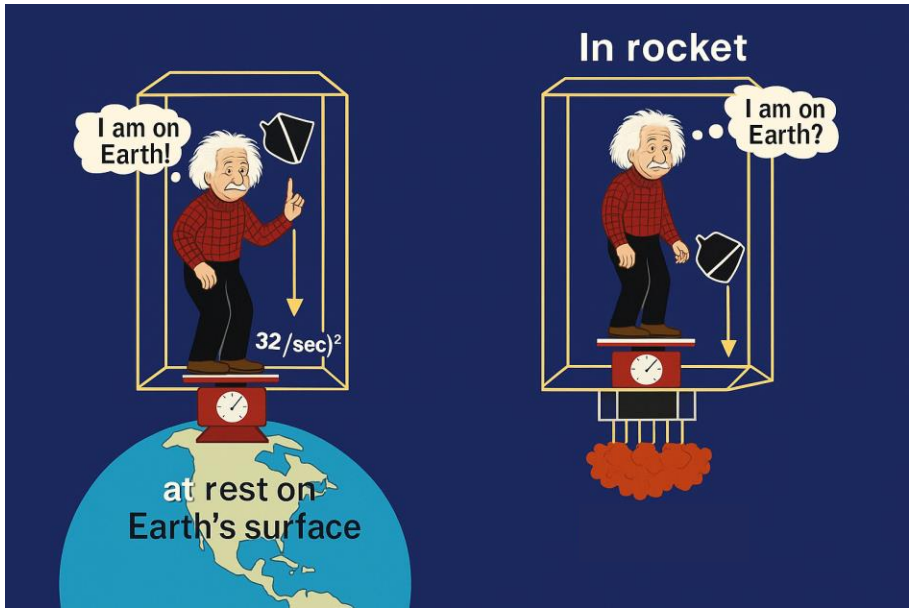
General Relativity: Einstein's "happiest thought"



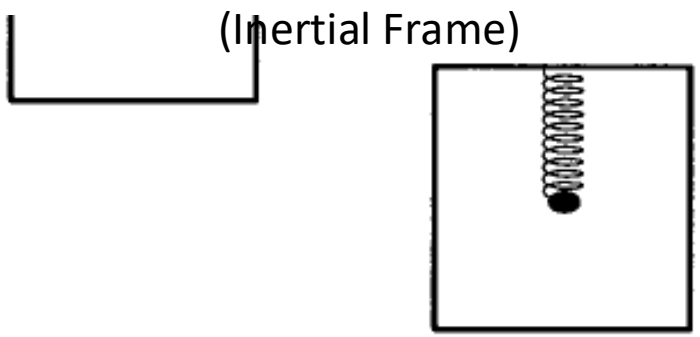


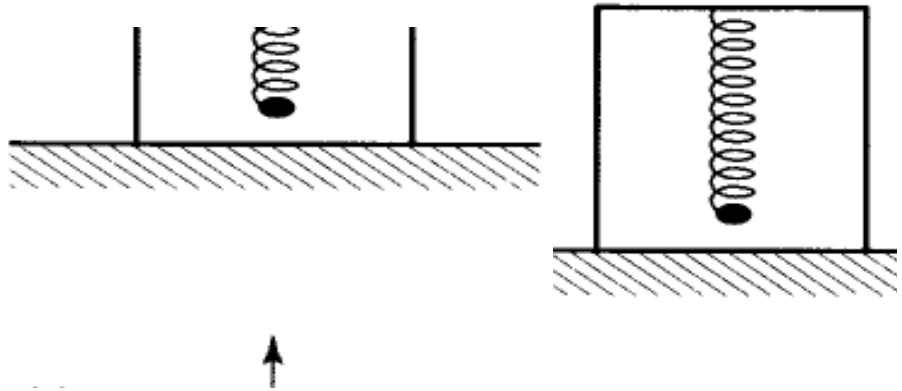
The Equivalence Principle





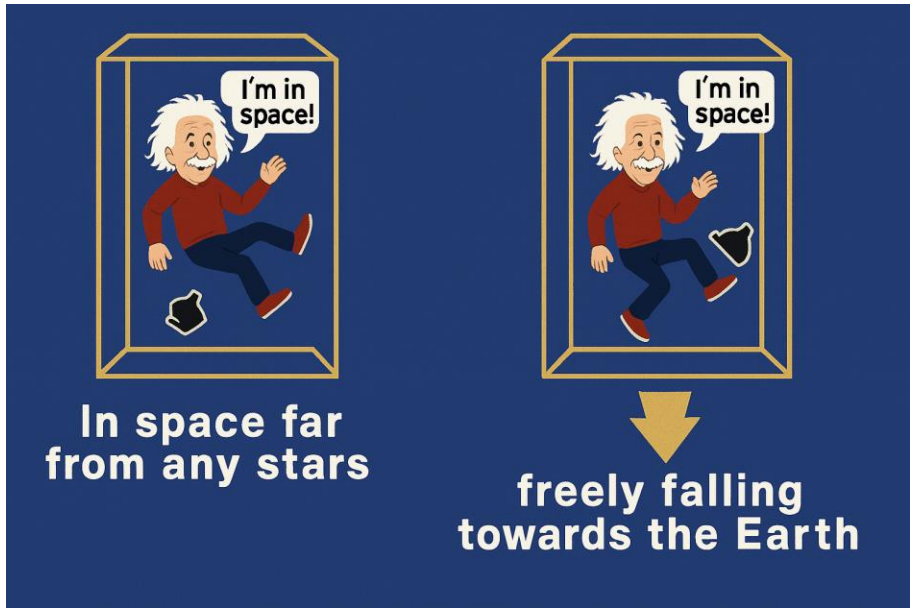
(Inertial Frame)

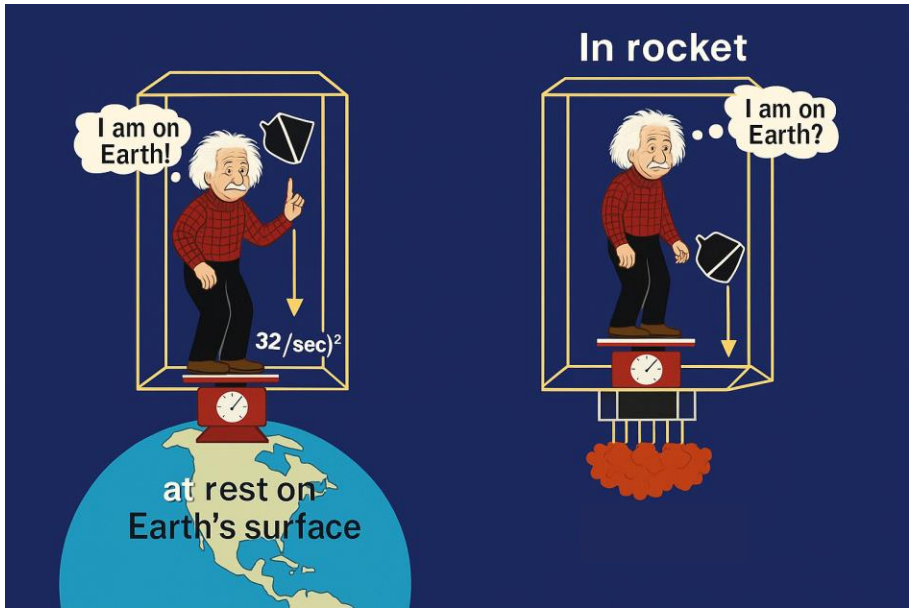




Einstein's "happiest thought": there is ***no way*** to determine whether you are standing on the earth or accelerating upwards in a rocket in space!

The Equivalence Principle





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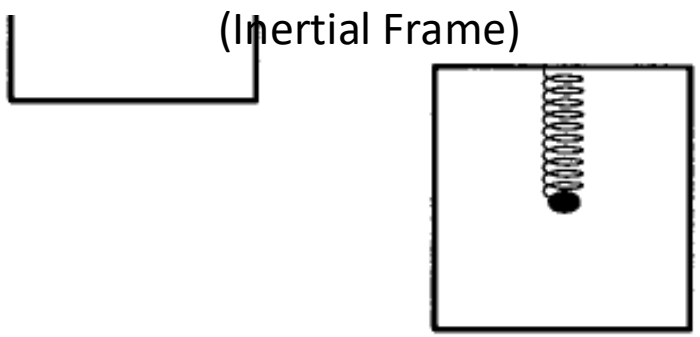
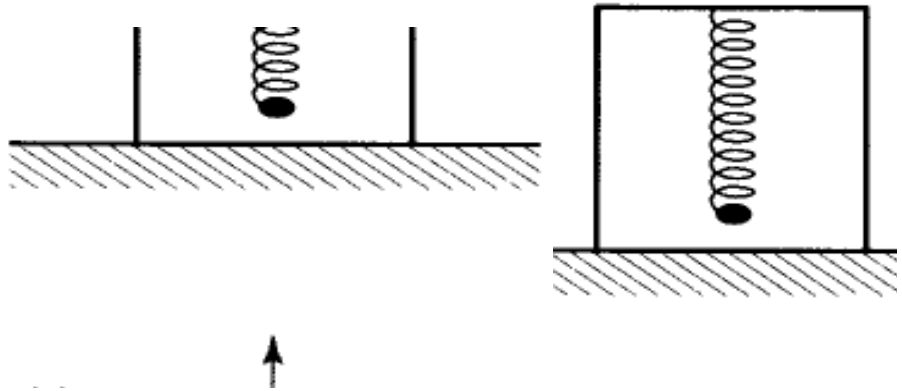


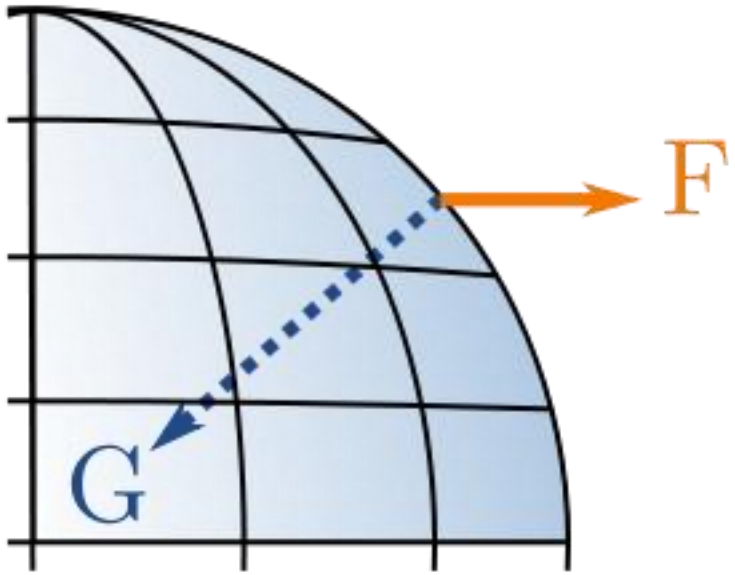


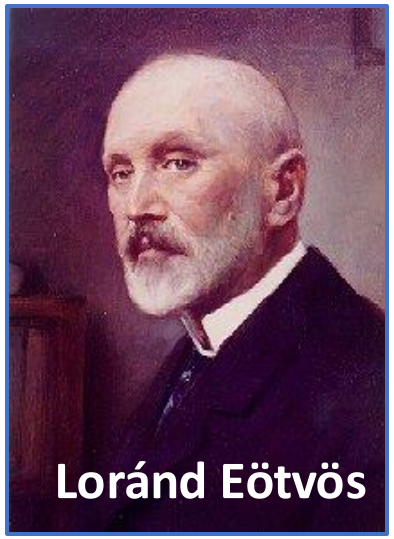
There is no difference between acceleration force and gravitation.

"Gravitational mass" = "Inertial mass"

Experimental confirmation: the Eötvös Experiment

Direction of gravity and centrifugal force on earth



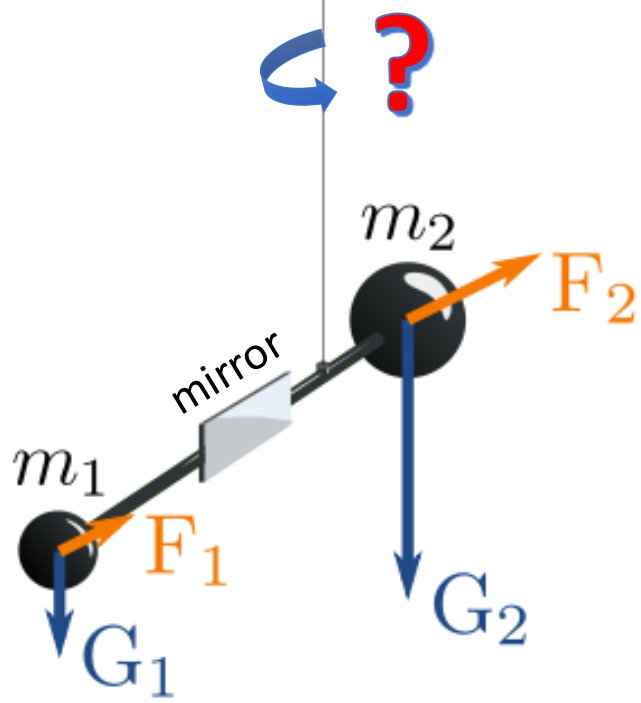


$F = m_i a$ with $a = \frac{v^2}{R}$

$G = m_g g$ with $g = \frac{GM_\oplus}{R^2}$

➔ $m_i = m_g$

Small (m_1) and big (m_2) mass on a rod suspended by a thin fiber



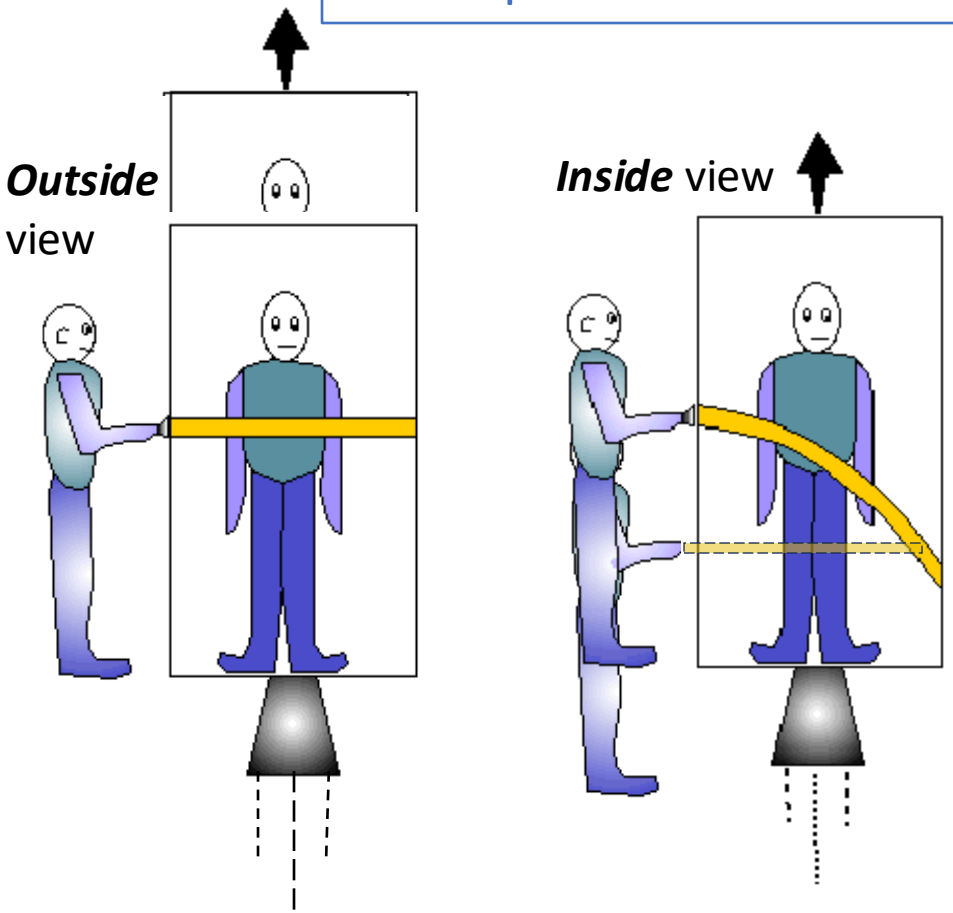
Gravity force **G** depends on Newton's law of gravity: ***gravitational mass***
Centrifugal force **F** depends on Newton's law of motion inertial mass: ***inertial mass***

The system did **not** rotate. ➔ $F_1/F_2 = G_1/G_2$

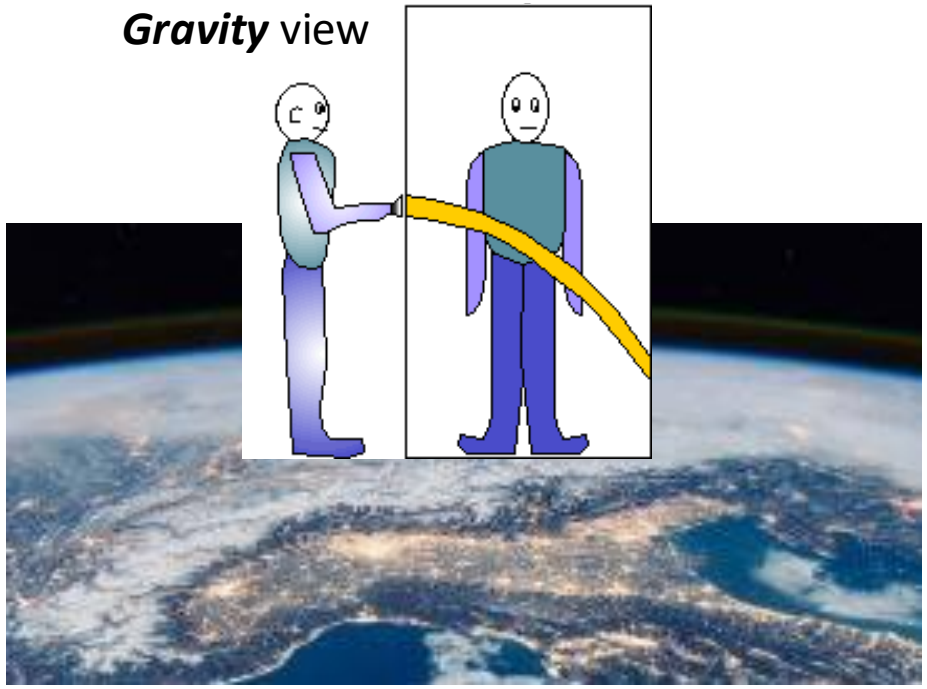
➔Experimental proof that indeed gravitational mass is equivalent to inertial mass.

Bending of Light

Consequence of acceleration and gravity being equivalent



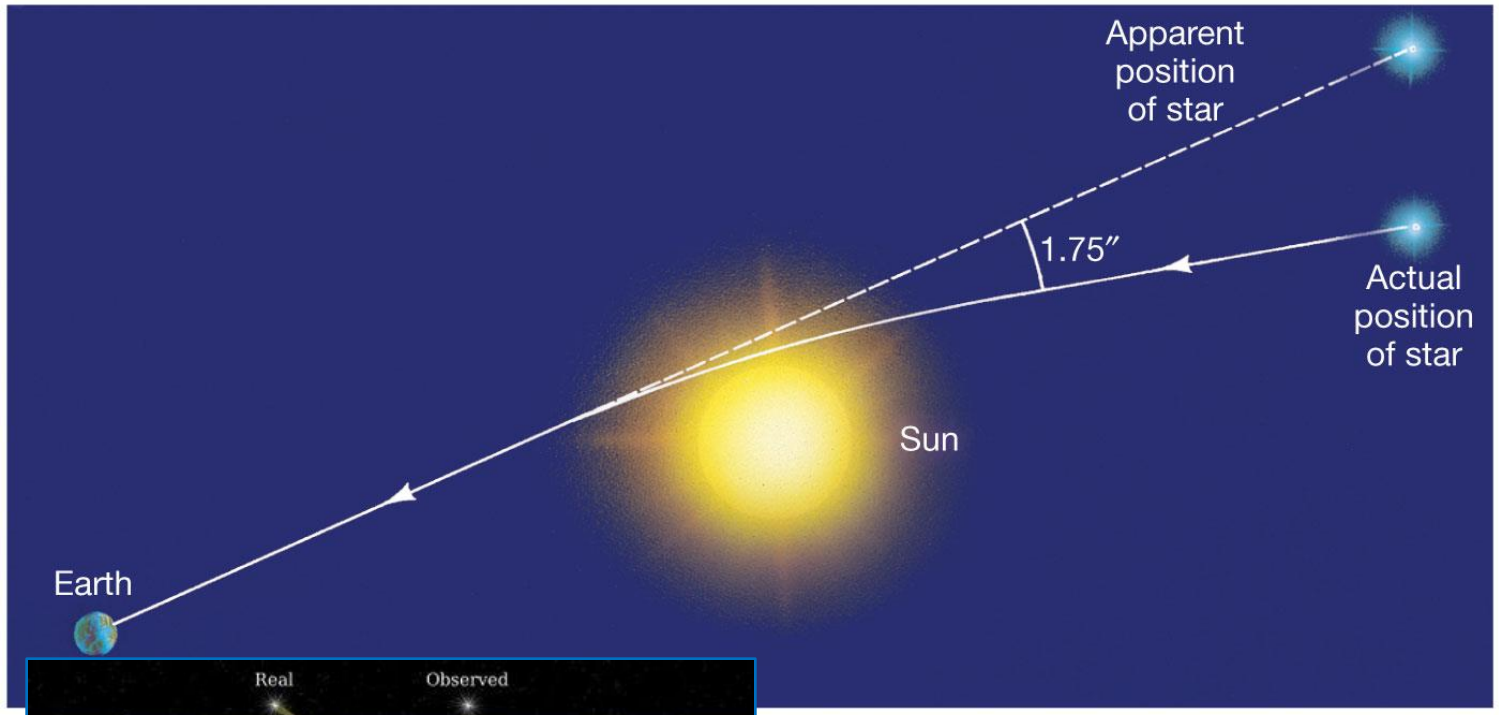
A: Lightbeam in accelerating rocket

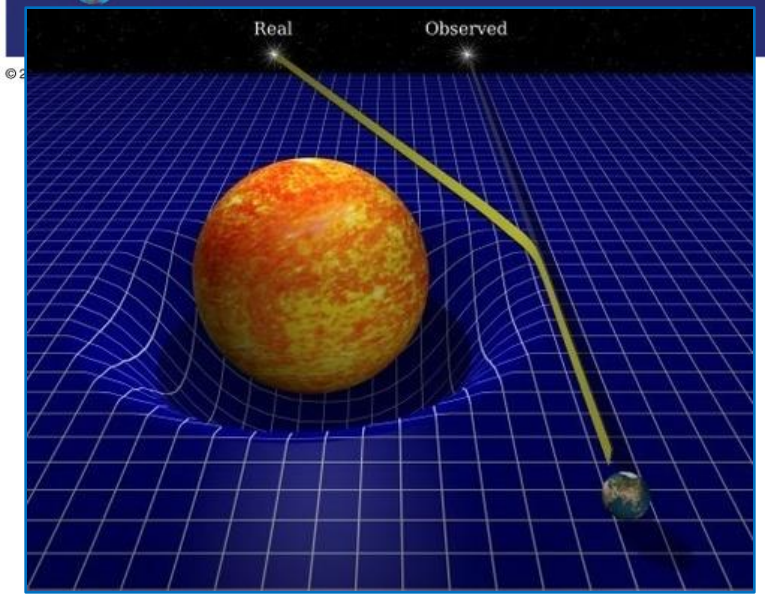


B: Lightbeam in gravitational field

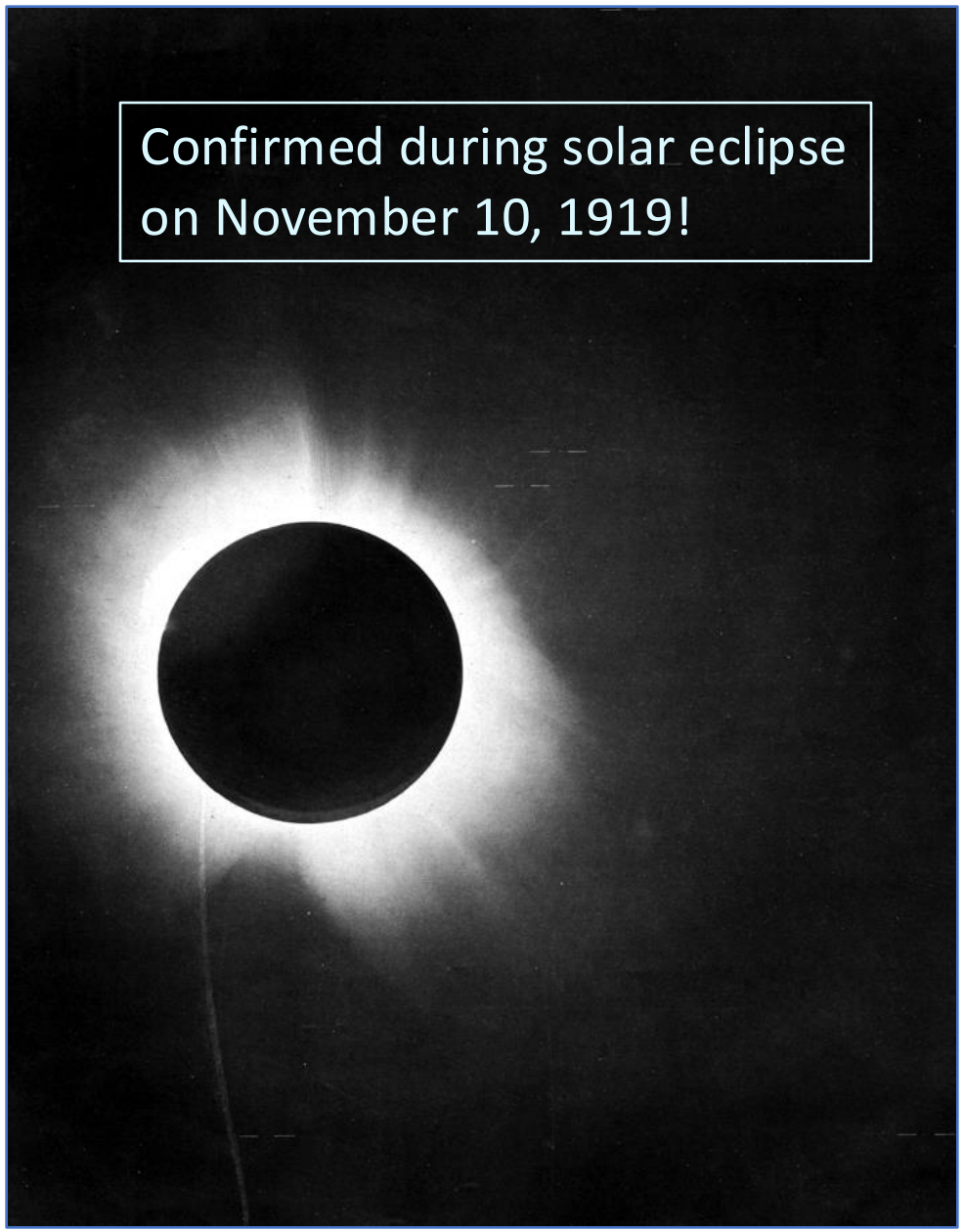
Prediction of Einstein: light beam bends under gravity!

Bending of light in gravitation field of the Sun



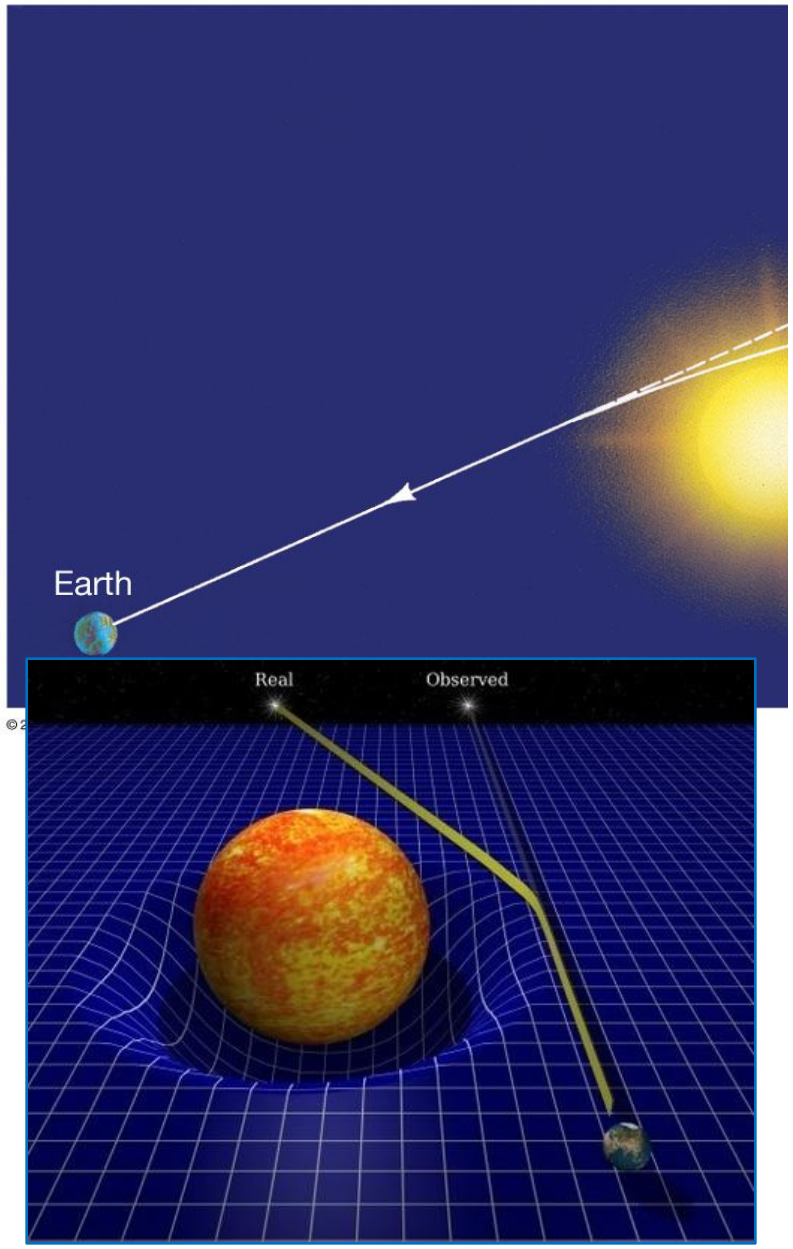


Light wants to go straight but space is curved!



Confirmed during solar eclipse on November 10, 1919!

Bending of light in gravitation field of the Sun



LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

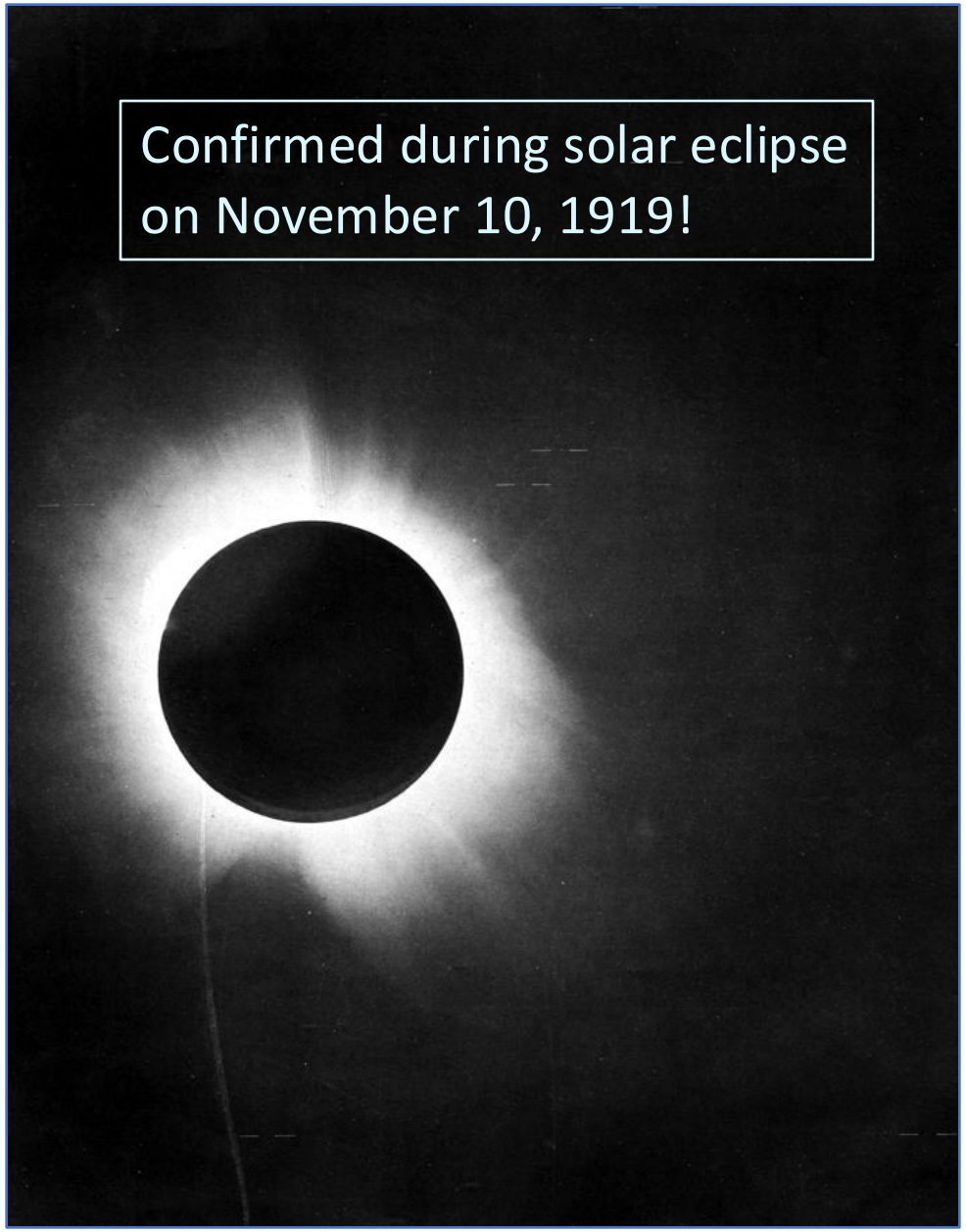
EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

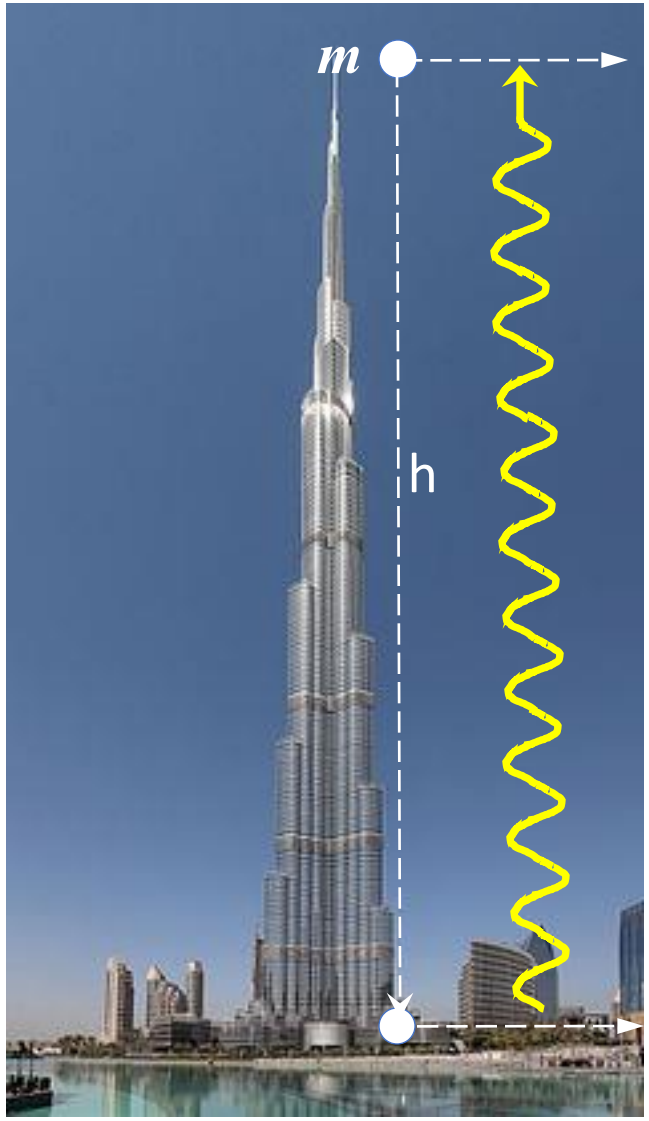
New York Times 1919



Confirmed during solar eclipse on November 10, 1919!

Einstein's next thought experiment on light

Particle with mass ***m*** falling from tower:



$$E = mc^2 \quad \Rightarrow \quad E' > E$$

From quantum mechanics we know that the energy of light is related to frequency (and wavelength): $E = hf = hc/\lambda$

$E' > E$??

Perpetuum mobile? ➔ No!

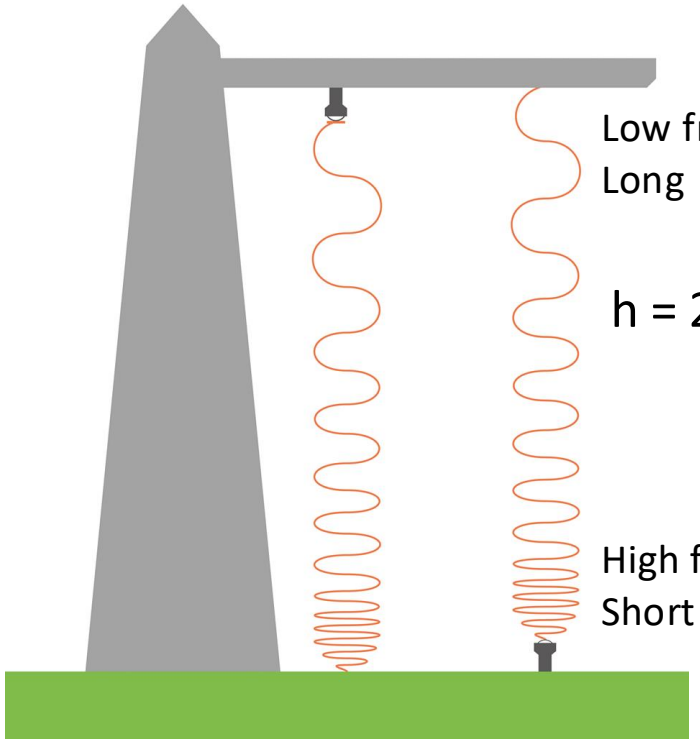
Photon **loses energy** gh/c^2 as it travels up the gravitational field!
→ Wavelength red-shift

$$E' = mc^2 + \frac{1}{2}mv^2 = mc^2 + mgh$$
$$= mc^2\,(1 + gh/c^2) \quad \Rightarrow \quad E' = hf'$$

$(E_{kin} = E_{pot})$

The Harvard Tower Experiment

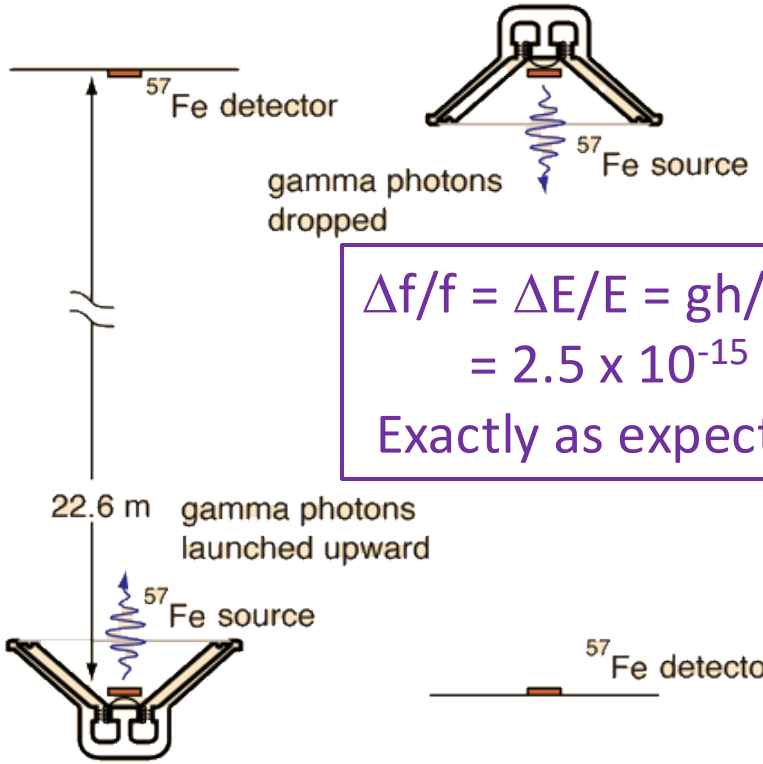
Harvard Tower Experiment (Pound-Rebka) at Jefferson lab in Harvard (1960):
Measure red-shift of photons in earth gravitational field.



Low frequency =
Long wavelength

h = 22.5 m

High frequency =
Short wavelength



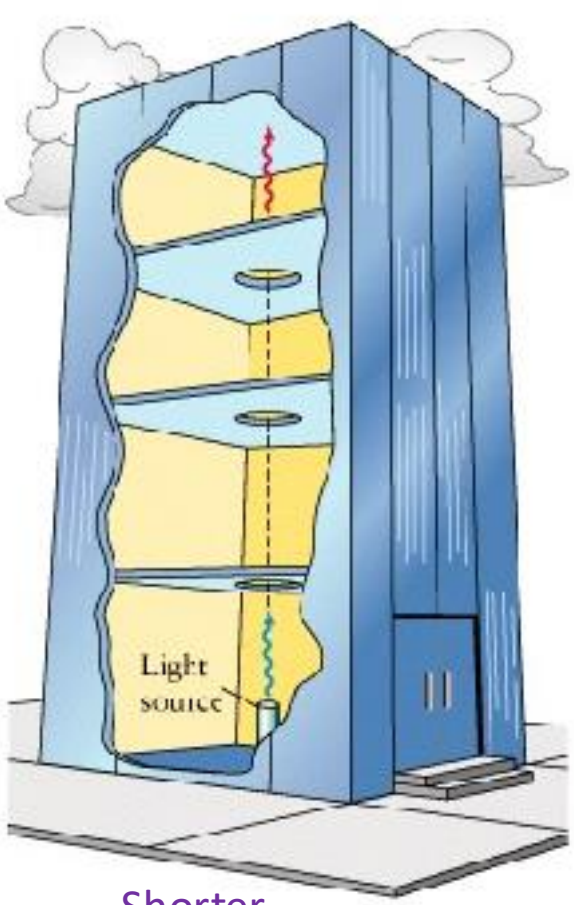
$\Delta f/f = \Delta E/E = gh/c^2$
$= 2.5 \times 10^{-15}$
Exactly as expected!

Gravitational Time Dilation

The photon loses energy as it climbs the gravitational field.

$$c = \lambda f$$

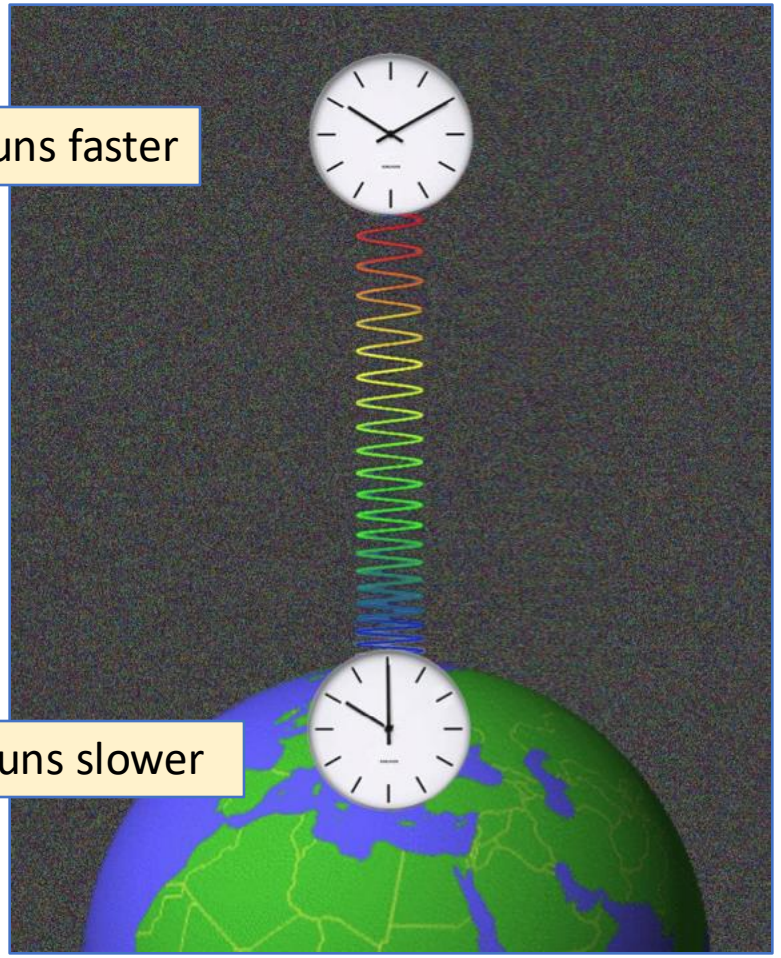
Longer wavelength



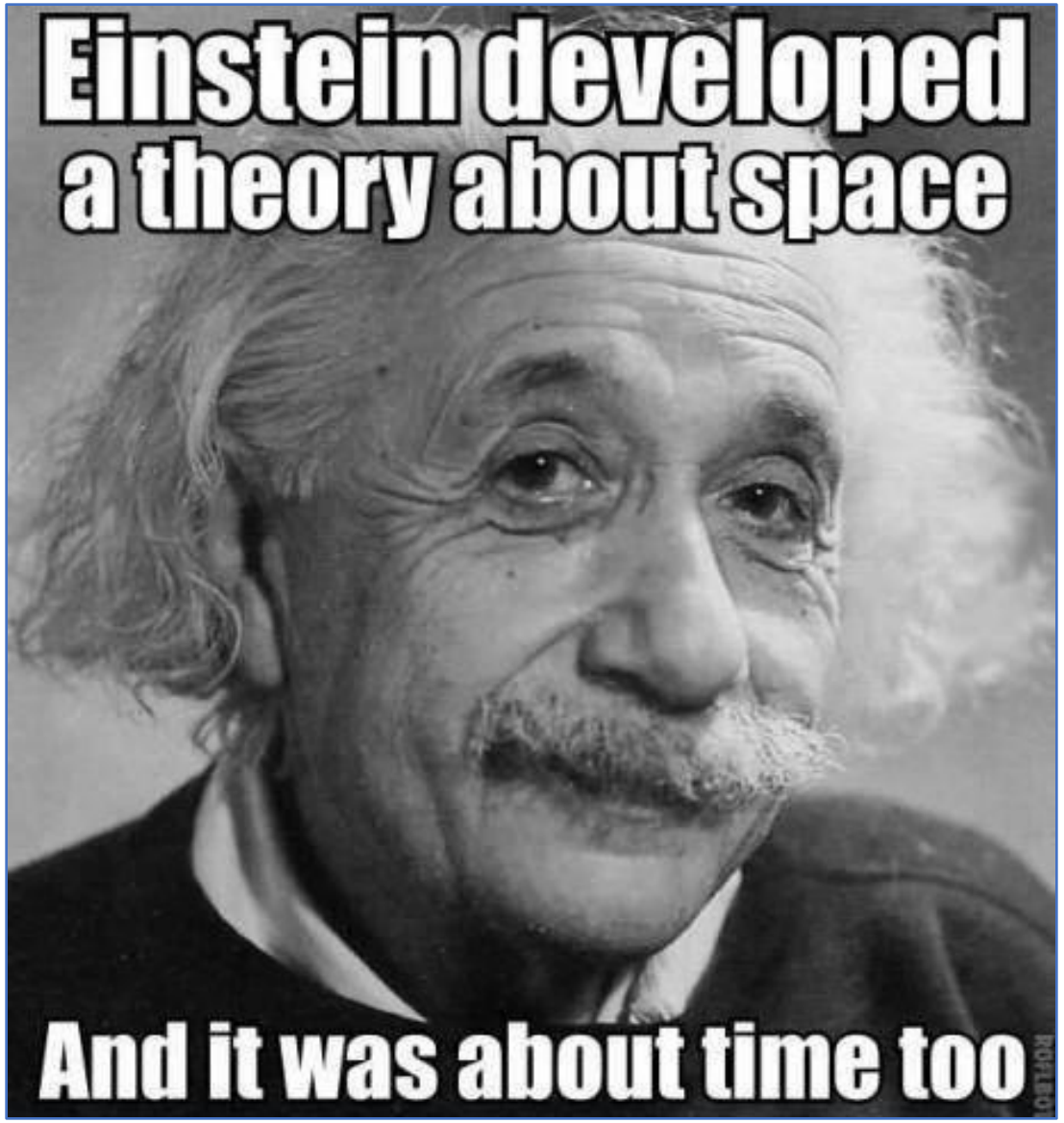
Shorter wavelength

Lower frequency

Higher frequency



➔Time ticks faster at higher altitude.



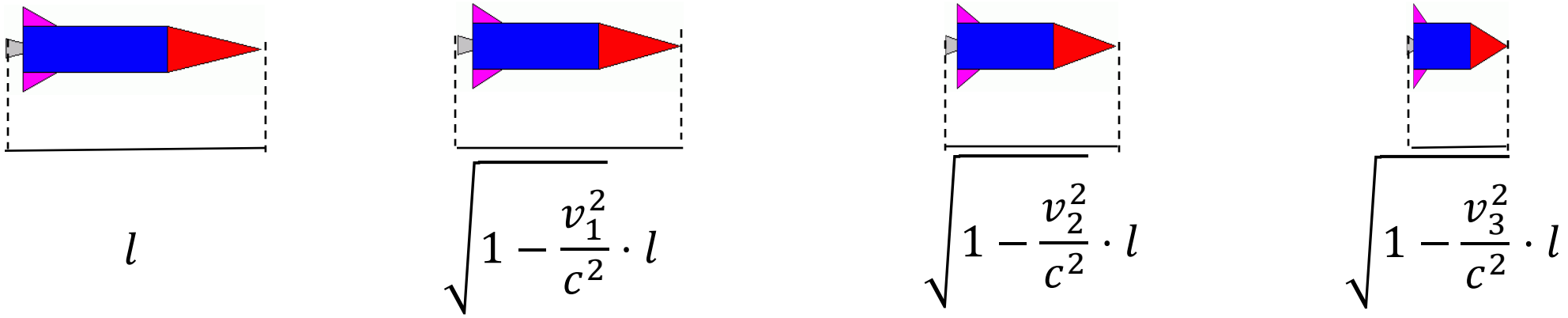
DEPARTMENT OF MATHEMATICS

"Space is curved and time is relative, eh? — Is this your way of asking for a day off?"

Accelerating Rocket

From special relativity we know that space contracts at high velocity

Velocity : $0 < v_1 < v_2 < v_3$



Space is seen to shrink further and further with increasing velocity!

$$1/\gamma = \sqrt{1-\frac{v^2}{c^2}}$$

Free falling object and Einstein's Equivalence Principle

$$E_{pot} = G\frac{mM_\oplus}{R}$$

↓

$$E_{kin} = E_{pot}$$

$$\frac{1}{2}mv^2 = G\frac{mM_\oplus}{R}$$

↓

$$v^2 = 2\frac{GM_\oplus}{R}$$

↓

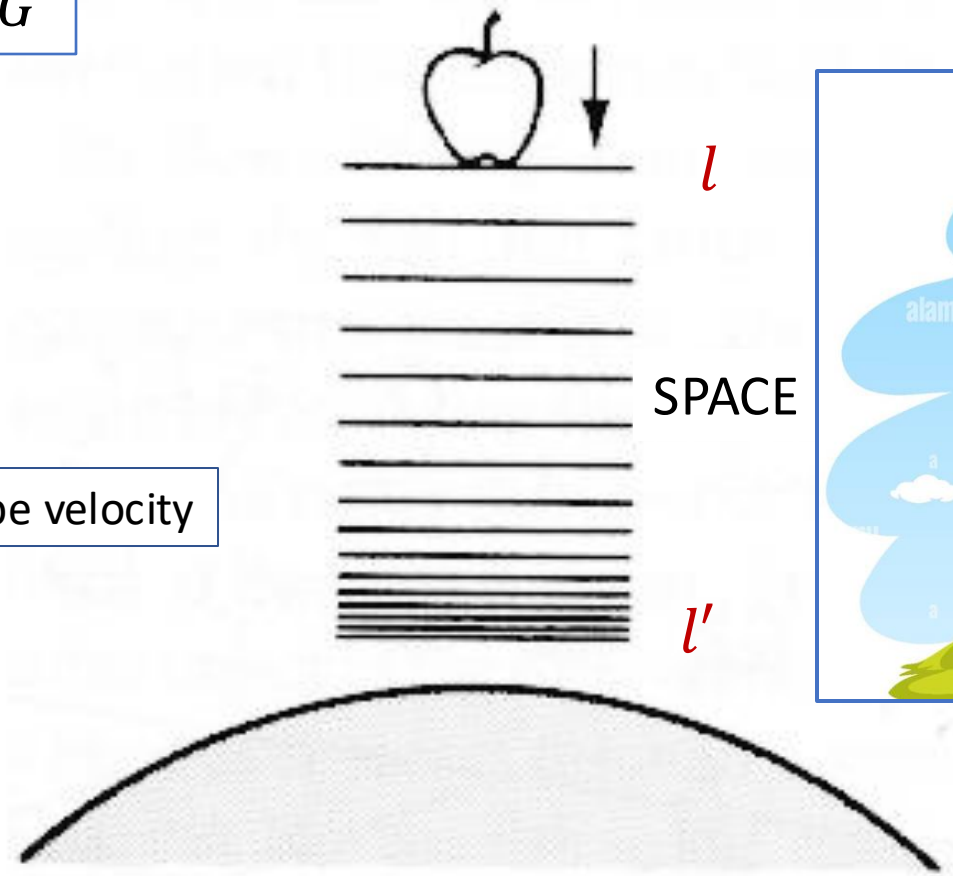
$$\sqrt{1-\frac{v^2}{c^2}} = \sqrt{1-2\frac{GM_\oplus}{Rc^2}}$$

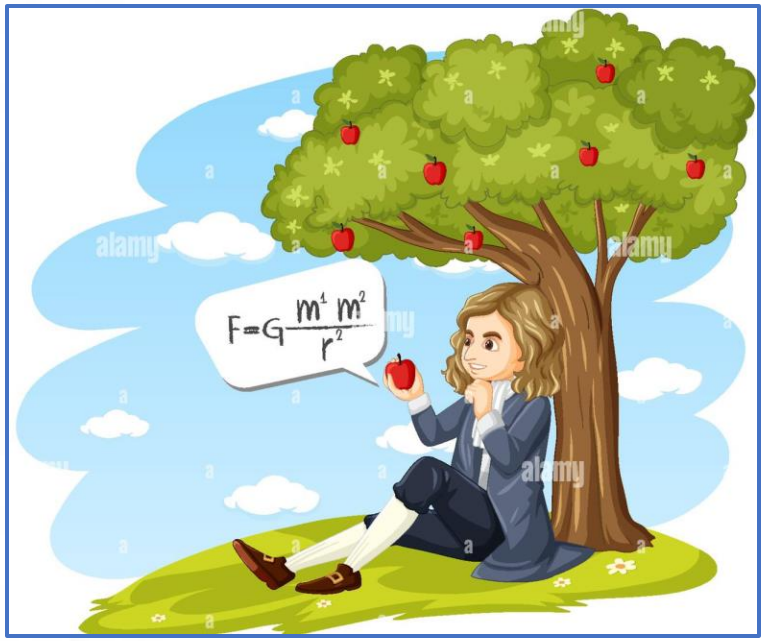
Newton's Constant G

v =Escape velocity

Lorentz factor

Falling apple:





Space shrinkage ("curvature"):

$$l' = \sqrt{1-2\frac{GM_\oplus}{Rc^2}} \cdot l$$

Free falling object and Einstein's Equivalence Principle

$$E_{pot} = G\frac{mM_\oplus}{R}$$

↓

$$E_{kin} = E_{pot}$$

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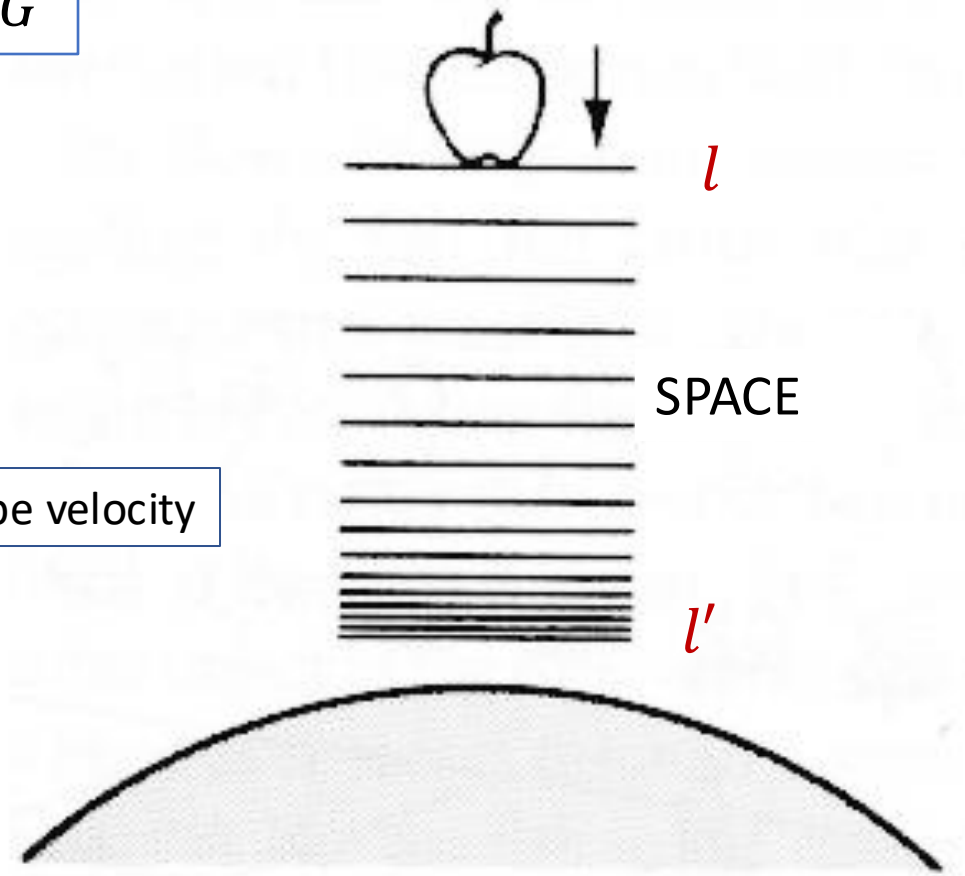
Falling apple:



Compare to accelerating rocket:

Space shrinkage ("curvature"):

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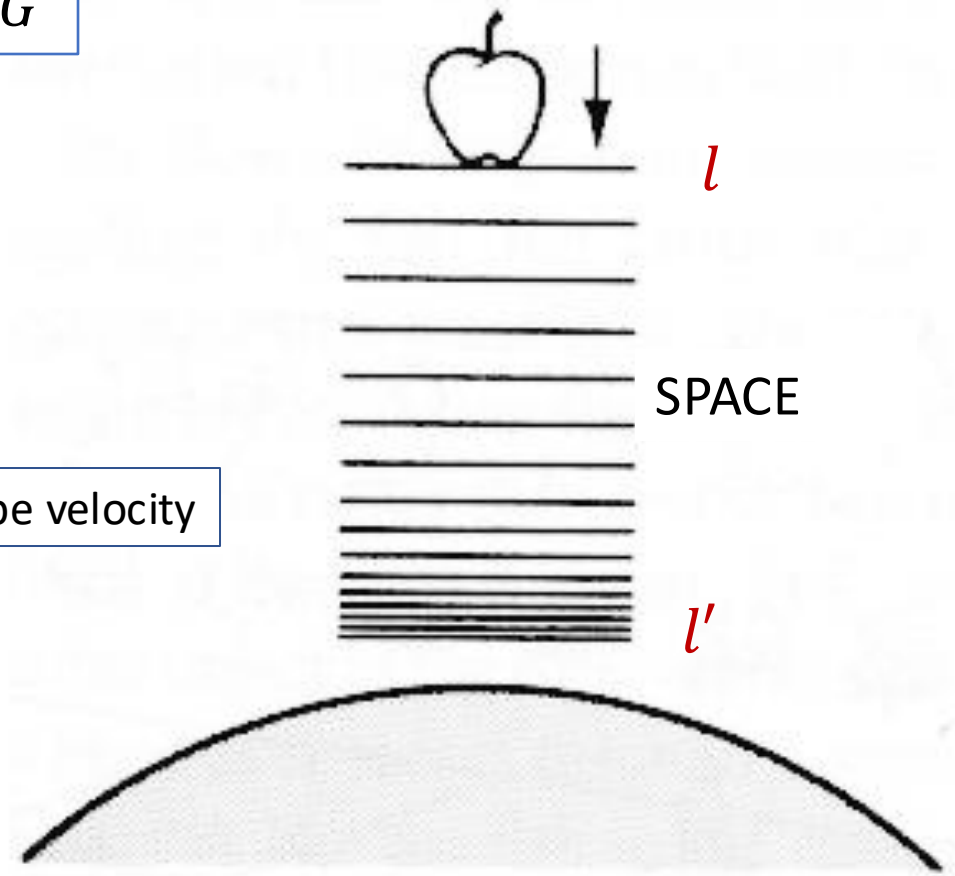


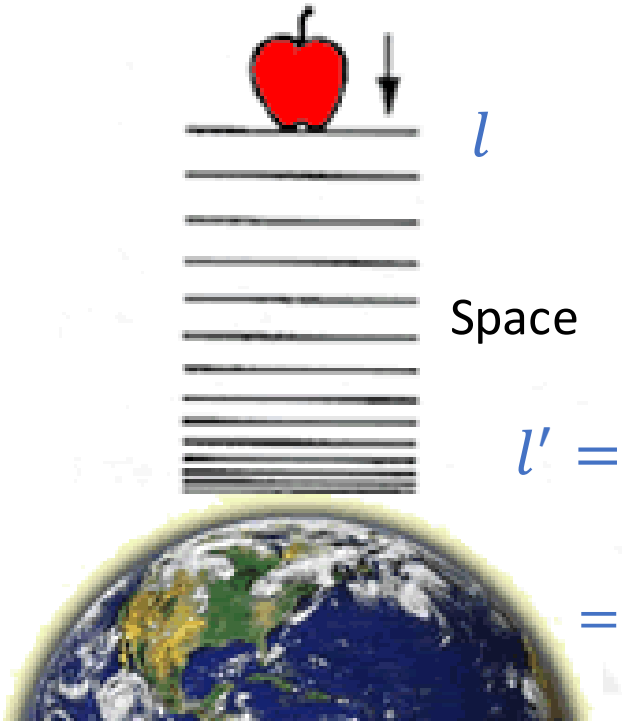
Compare to accelerating rocket:

Space shrinkage ("curvature"):

$$l' = \sqrt{1-2\frac{GM_\oplus}{Rc^2}} \cdot l = \frac{1}{\gamma}L$$

Space-Time curvature

A falling apple accelerates and units of space get more and more contracted:

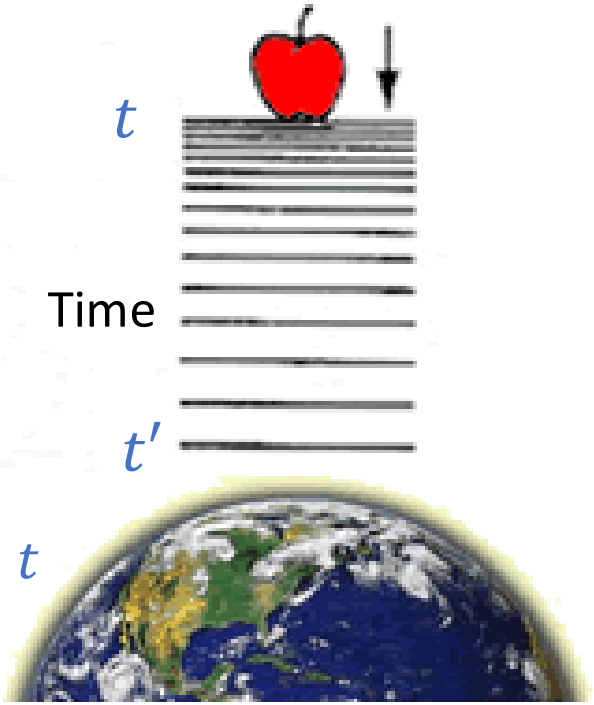


$$l' = 1/\gamma \cdot l$$

$$= \sqrt{1 - 2\frac{GM_{\oplus}}{Rc^2}} \cdot l$$

Space contracts near mass and dilates away from it.

An apple falls into the gravitational field and time runs slower and slower:



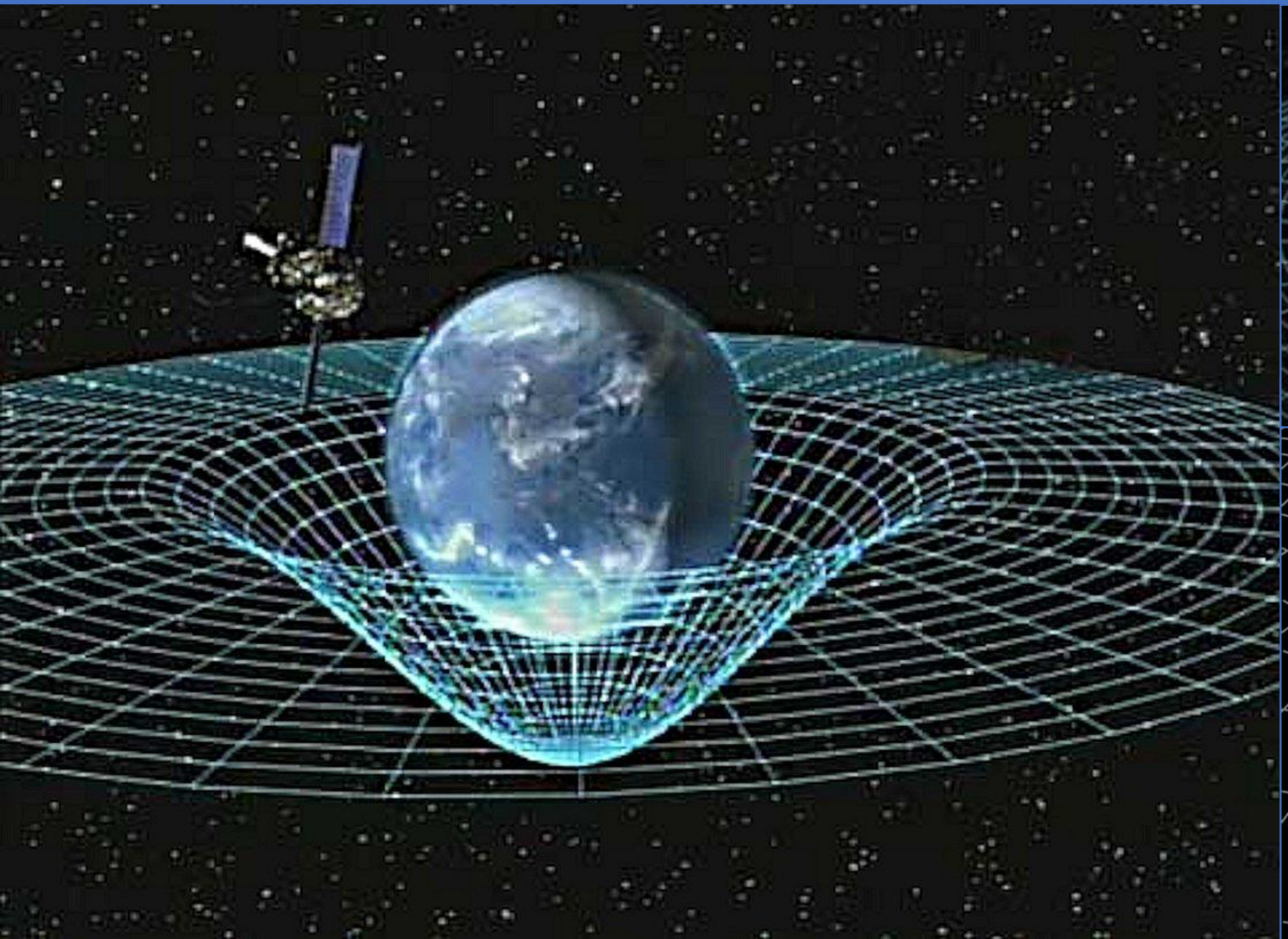
$$t' = \gamma \cdot t$$

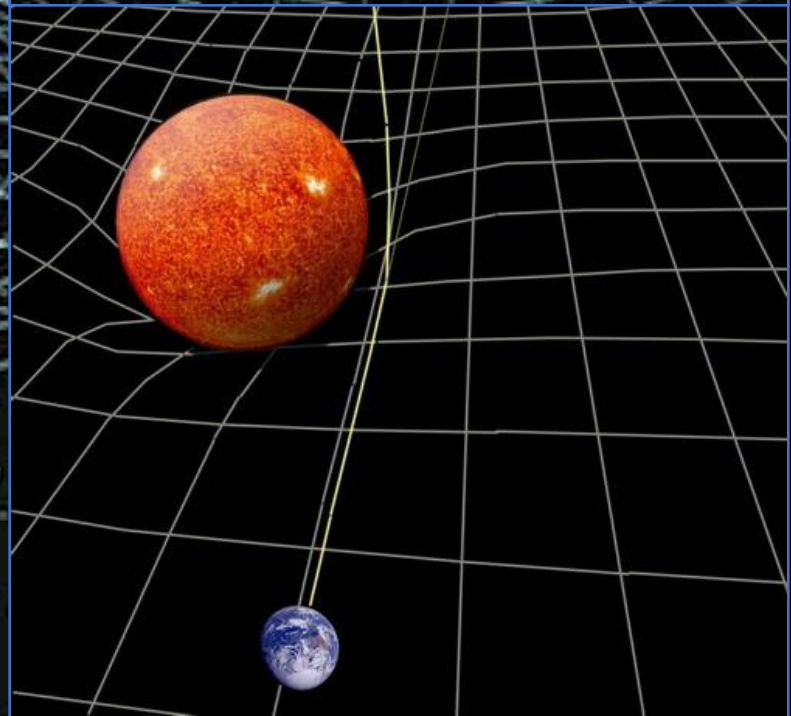
$$= \frac{1}{\sqrt{1 - 2\frac{GM_{\oplus}}{Rc^2}}} \cdot t$$

Time slows near mass and speeds up away from it.

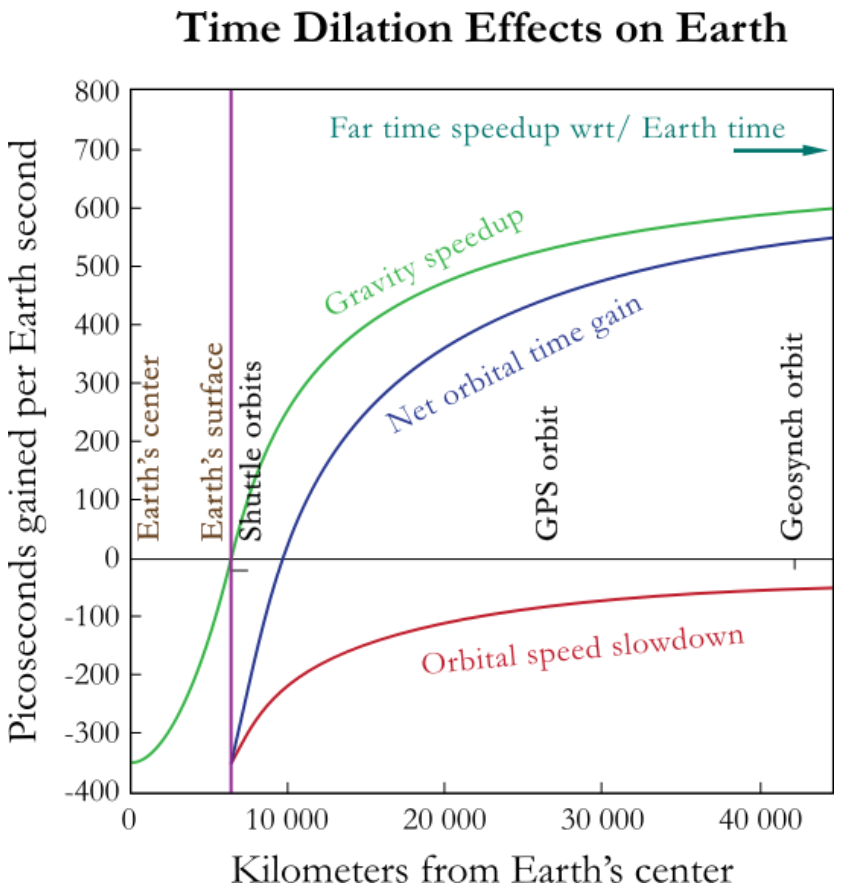
Space-time is curved in the presence of mass

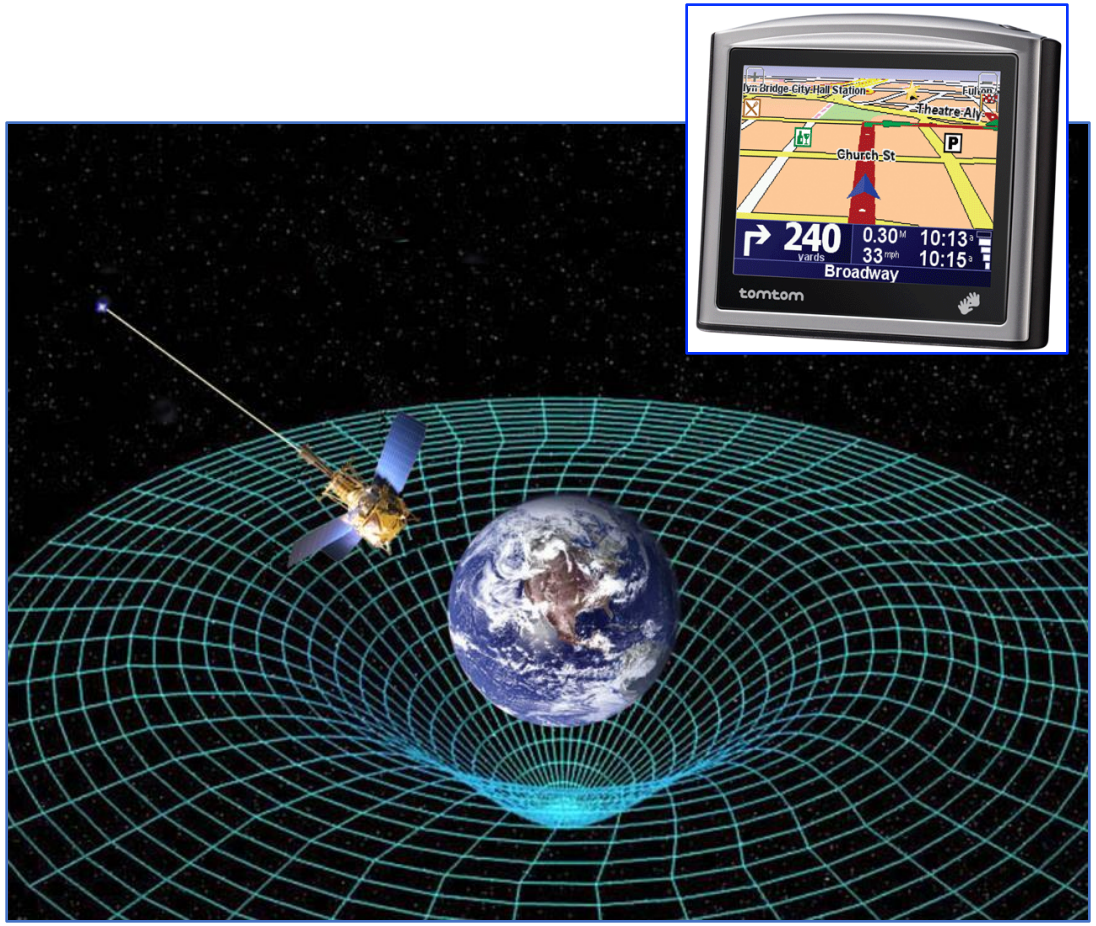
Mass causes curvature in space-time





Relativity and GPS



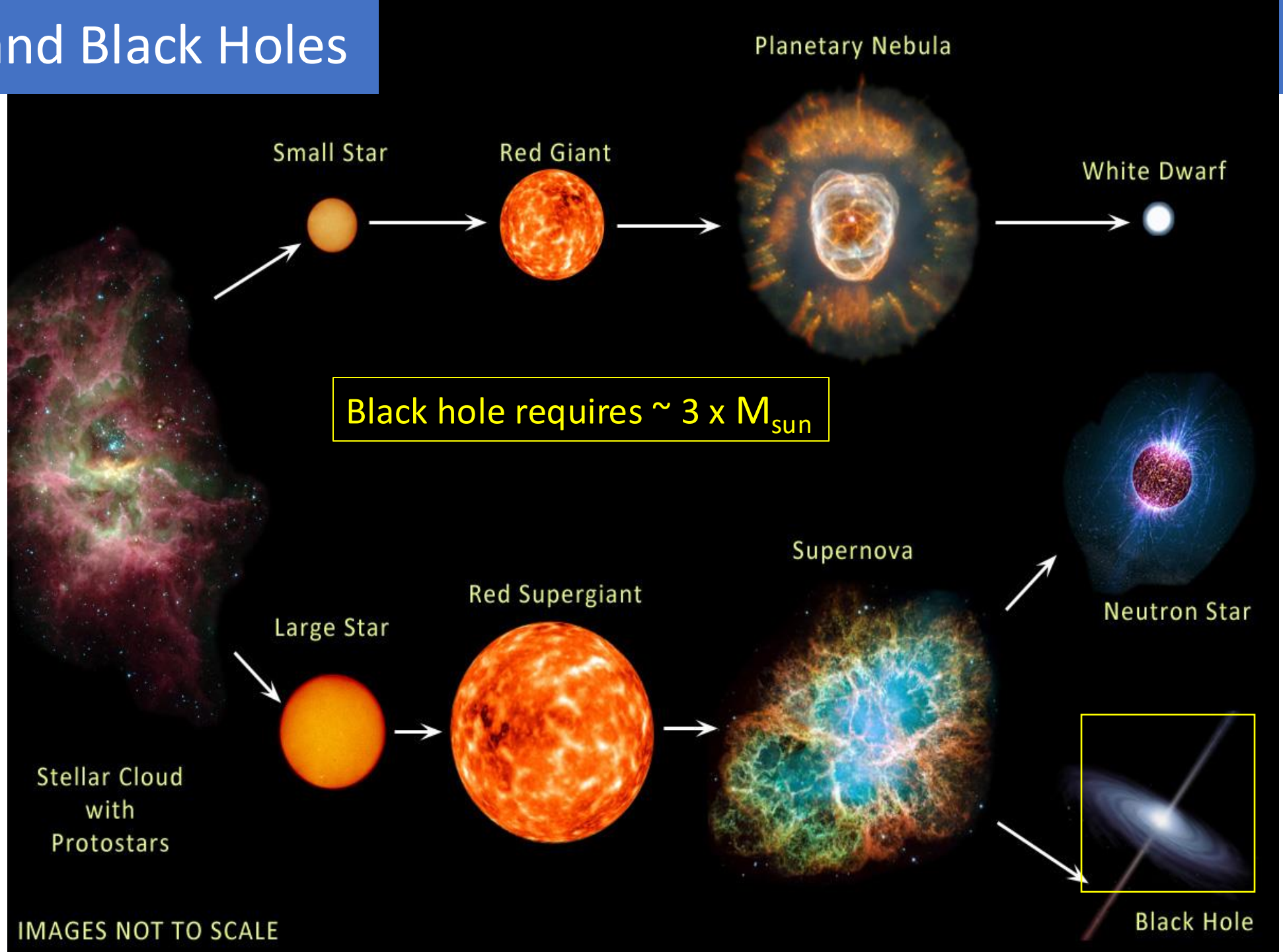


Two effects:

- **Time speeds up** at the satellite in comparison to earth surface **due to gravity**
- **Time slows down** at the satellite **due to high velocity** compared to person on earth

➔ Clocks in satellite and on earth de-synchronize with ~ 40 msec per day!

Stars and Black Holes



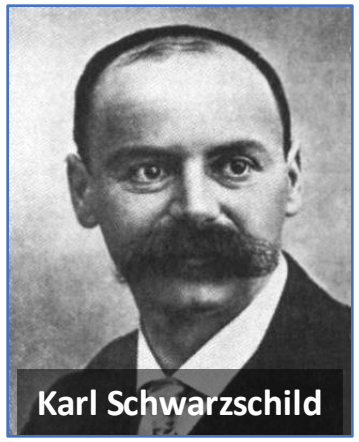
Stars and Black Holes

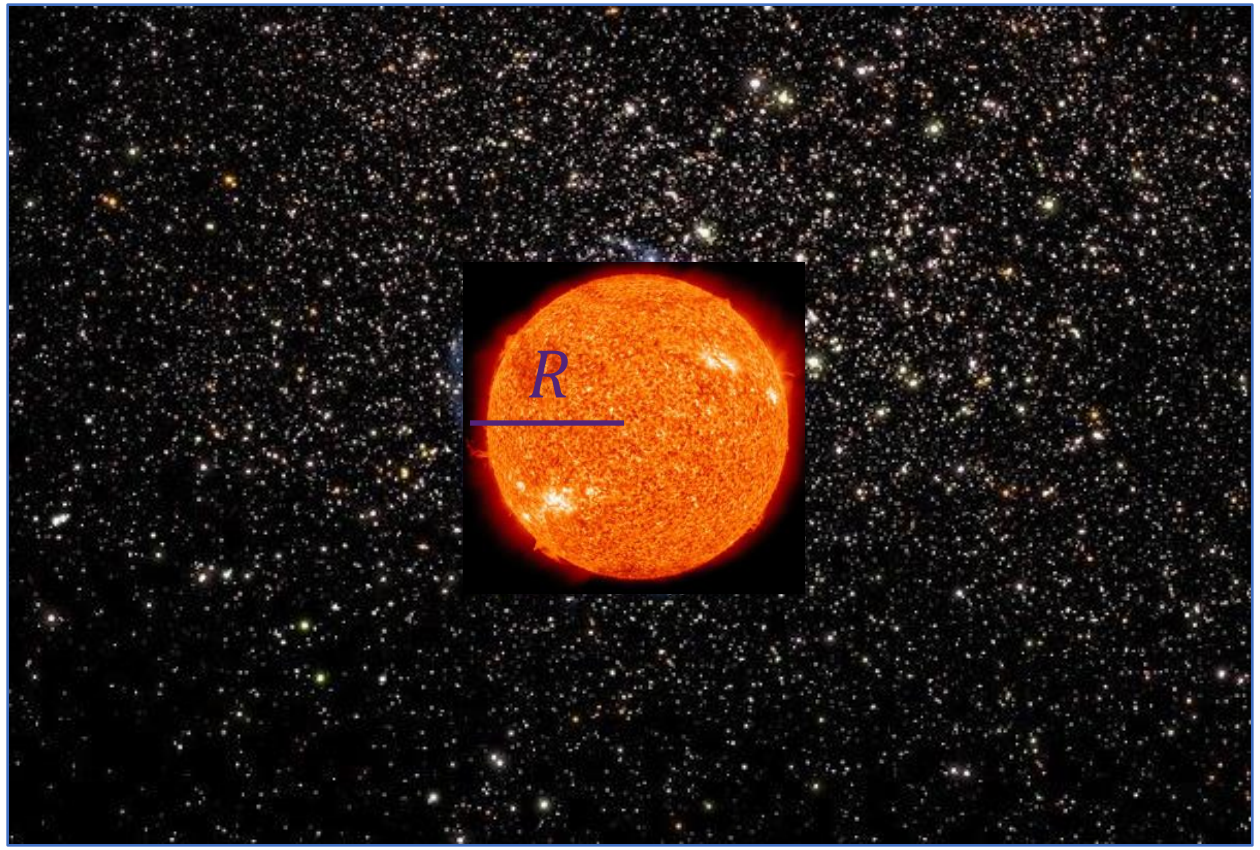
Gravitational time slowdown near a star with mass M:

$$\Delta t' = \Delta t \sqrt{1 - \frac{2GM}{Rc^2}}$$

Schwartzschild radius: $R_s = \frac{2GM}{c^2}$

$$\Delta t' = \Delta t \sqrt{1 - \frac{R_s}{R}}$$





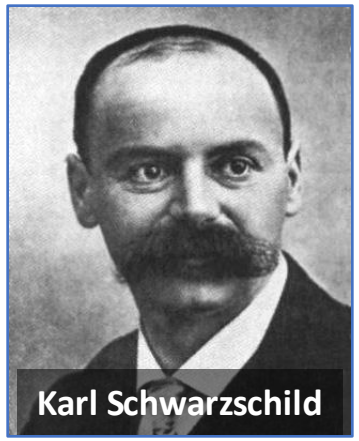
Stars and Black Holes

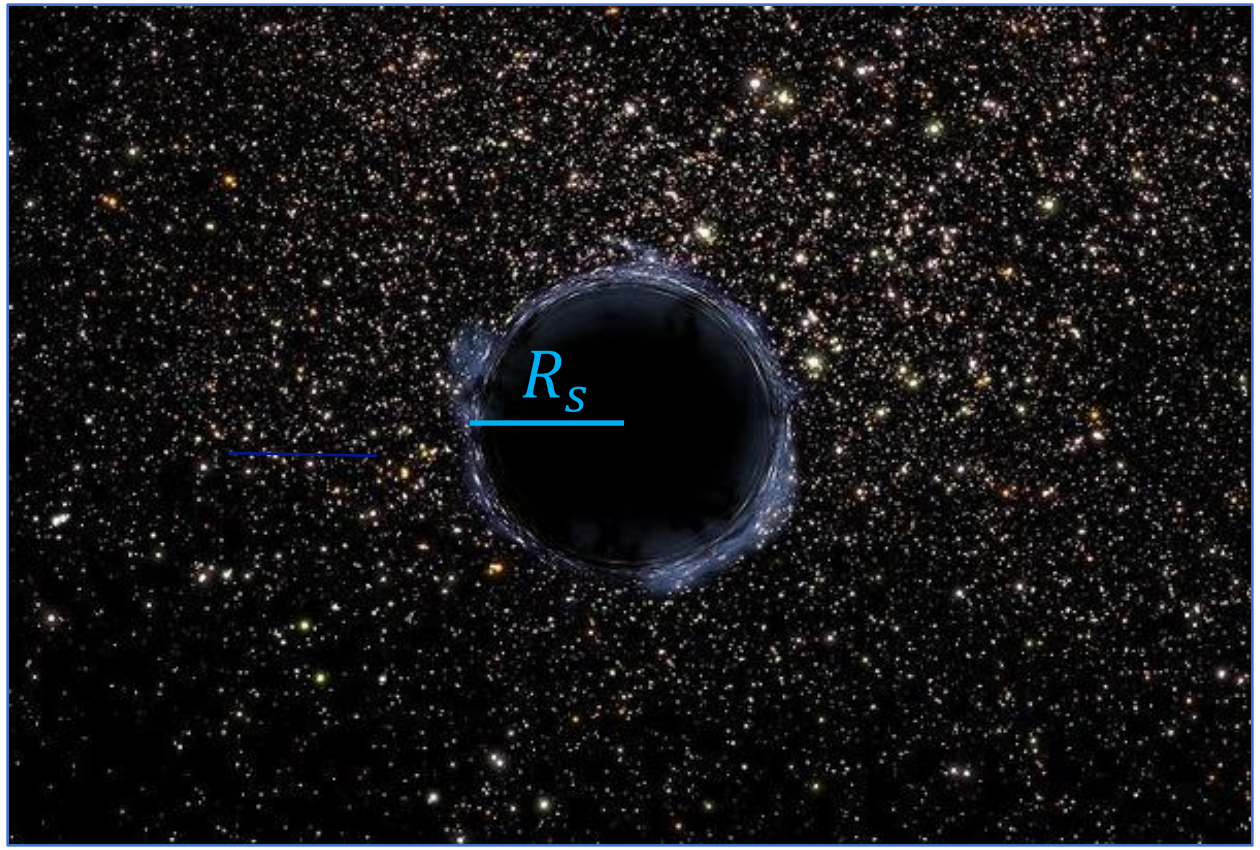
Gravitational time slowdown near a star with mass M:

$$\Delta t' = \Delta t \sqrt{1 - \frac{2GM}{Rc^2}}$$

Schwartzschild radius: $R_s = \frac{2GM}{c^2}$

$$\Delta t' = \Delta t \sqrt{1 - \frac{R_s}{R}}$$


Karl Schwarzschild



Time stand-still:

If $R = R_s$ then $\Delta t = 0$

(Time stands still at the horizon of a black-hole)

Example our sun: $G = 6.67 \times 10^{-11}\ m^3/kg\ s^2$ (Newton's gravitation constant)

$M_{sun} = 2 \times 10^{30}\ kg$

➔ $R_S = 3\ km$ for a black hole

Stars and Black Holes

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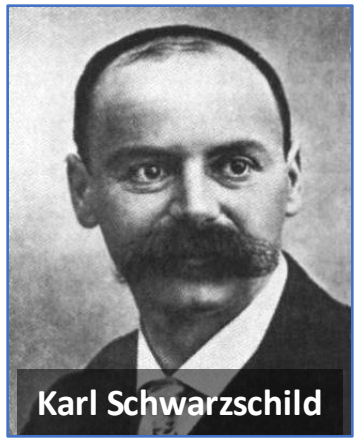
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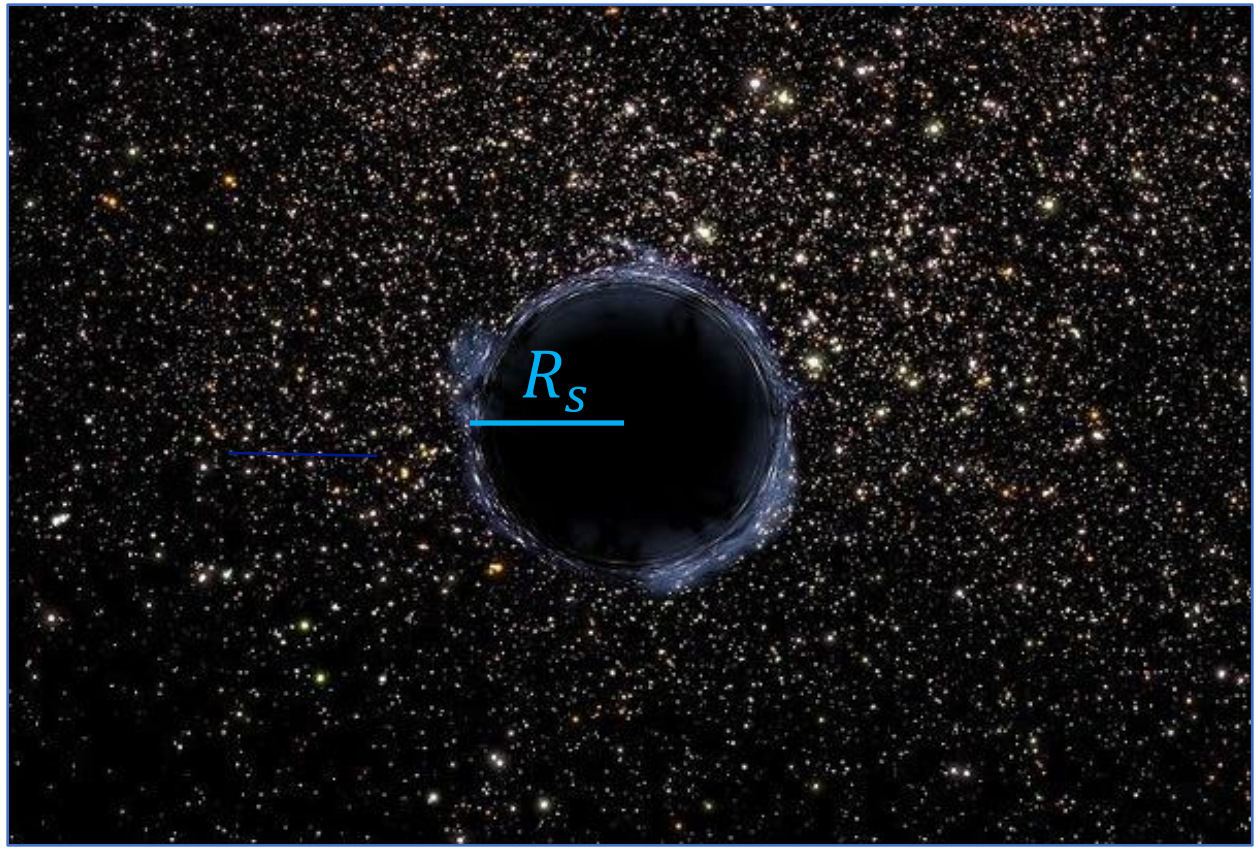

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$M_{earth} = 6 \times 10^{24}\ kg$

➔ $R_S = 9\ mm$ for a black hole

Stars and Black Holes

Gravitational time slowdown near a star with mass M:

$$\Delta t' = \Delta t \sqrt{1 - \frac{2GM}{Rc^2}}$$

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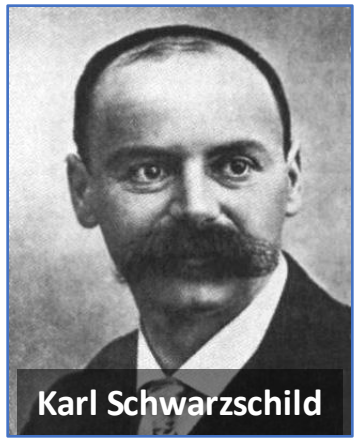
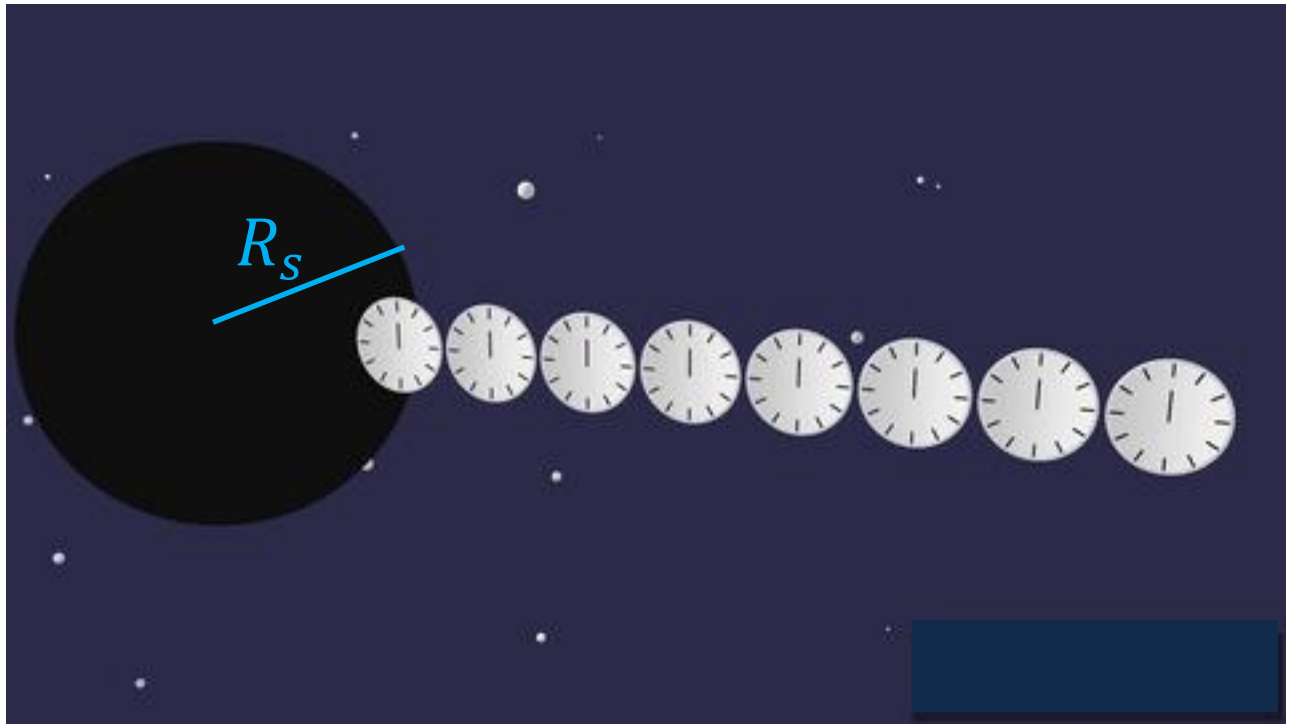

Karl Schwarzschild



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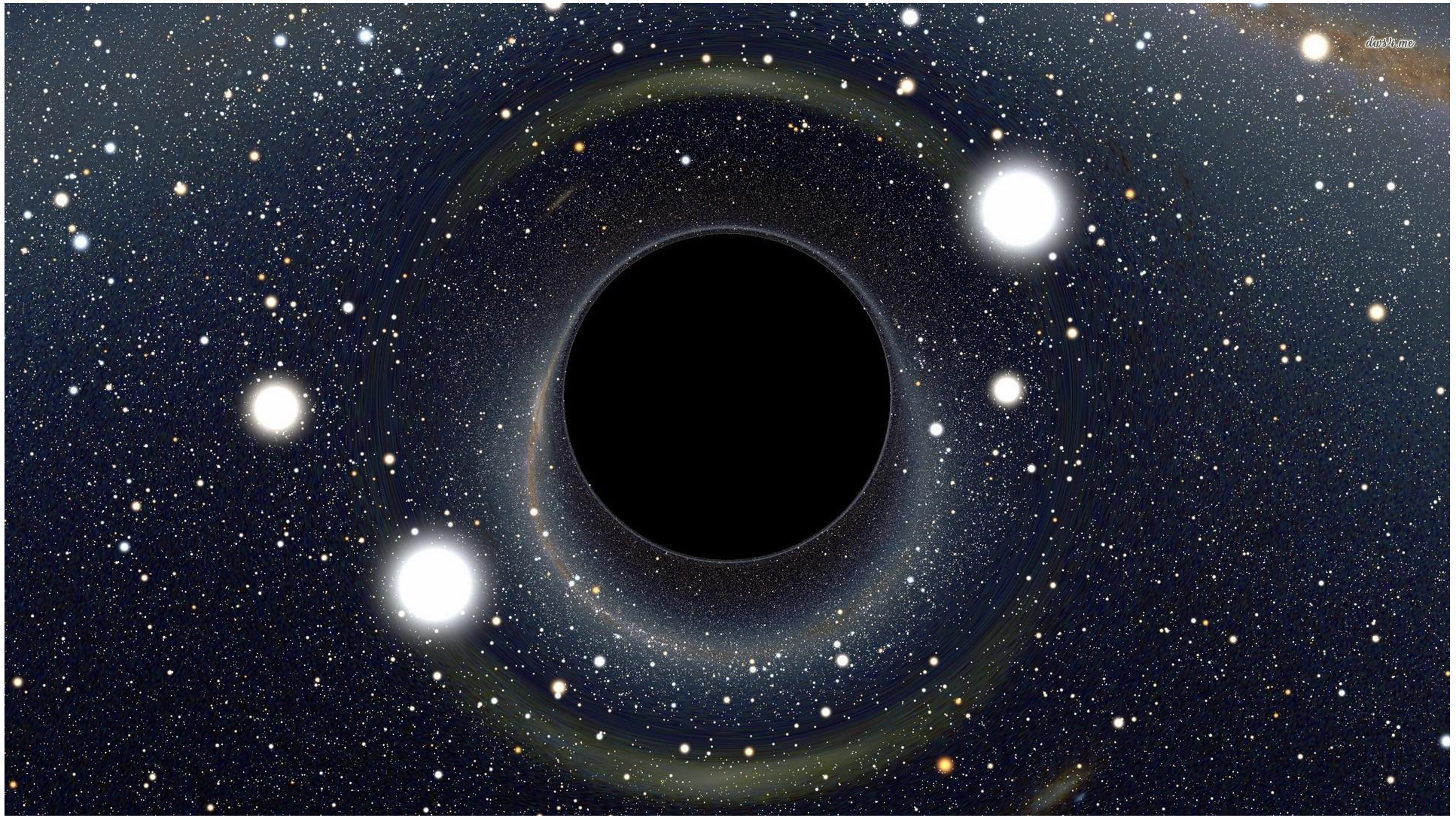
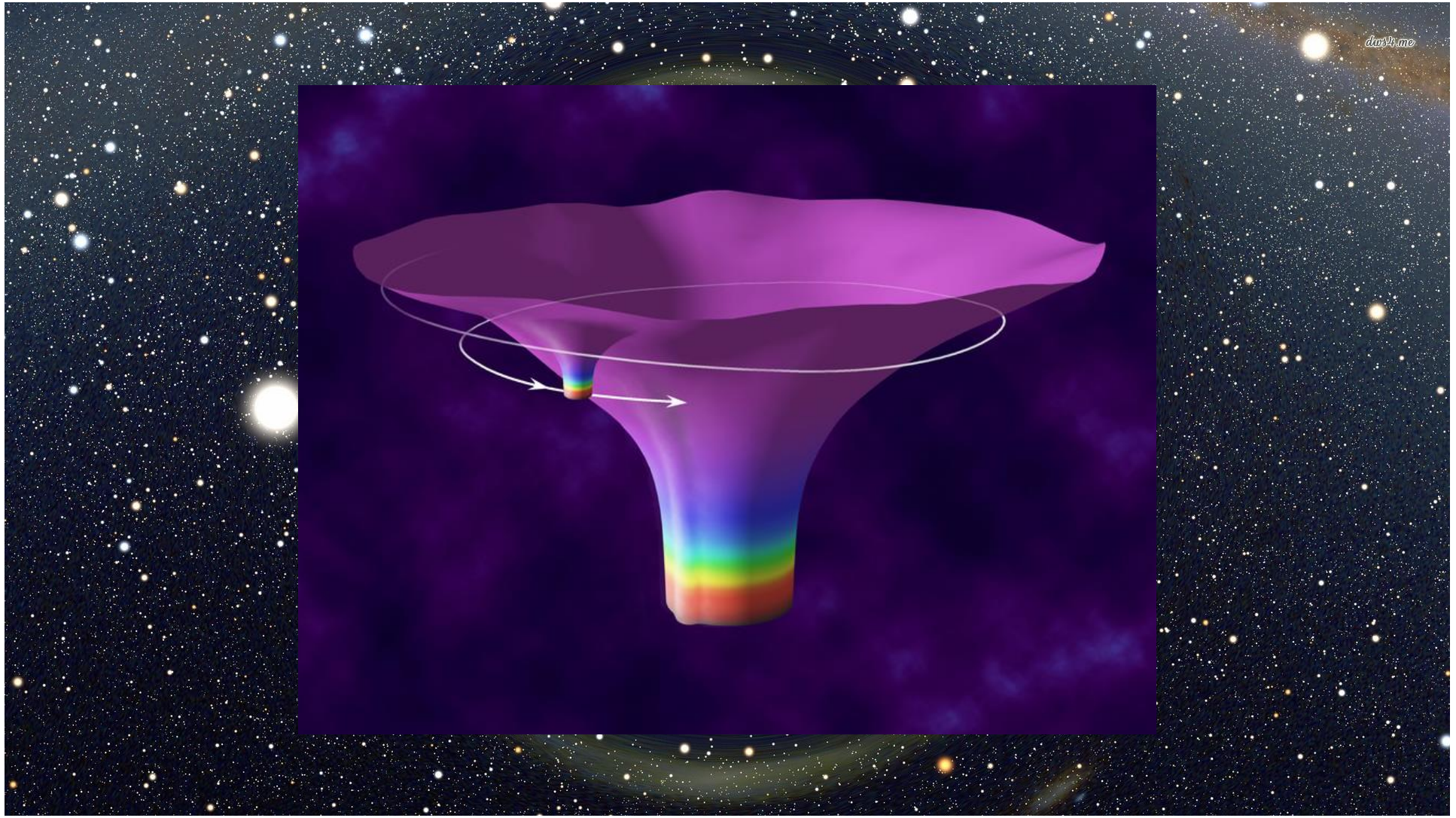
➔ $R_S = 9\ mm$ for a black hole

What *is* a black hole?



Purely curved space-time!

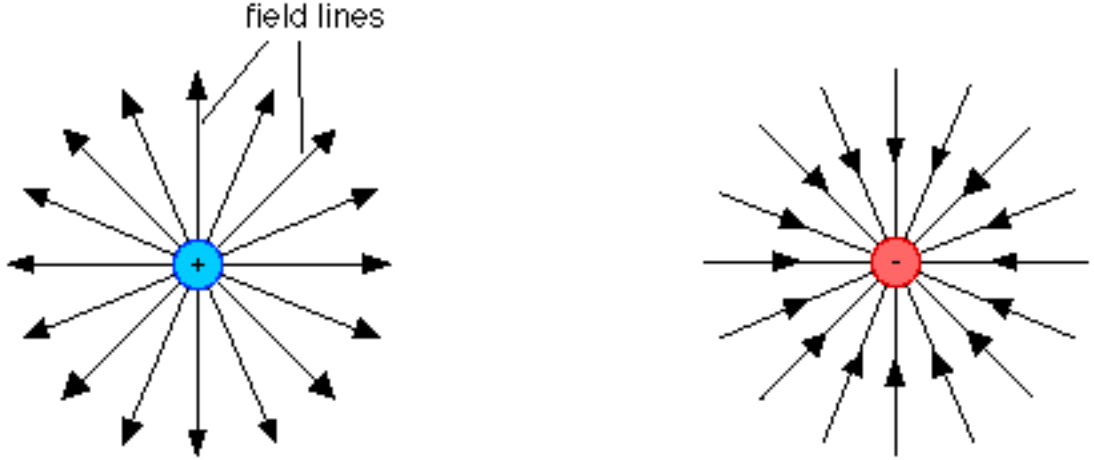
What *is* a black hole?



What happens when two black holes meet?

Intermezzo: Electric vs Gravitational Fields

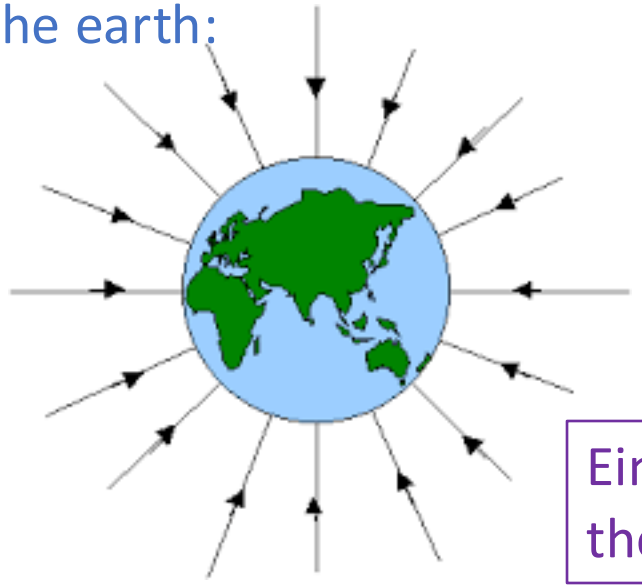
Electric field of positive and negative charged particle:



$$F_e = q\,E = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$$

Field equations have similar form

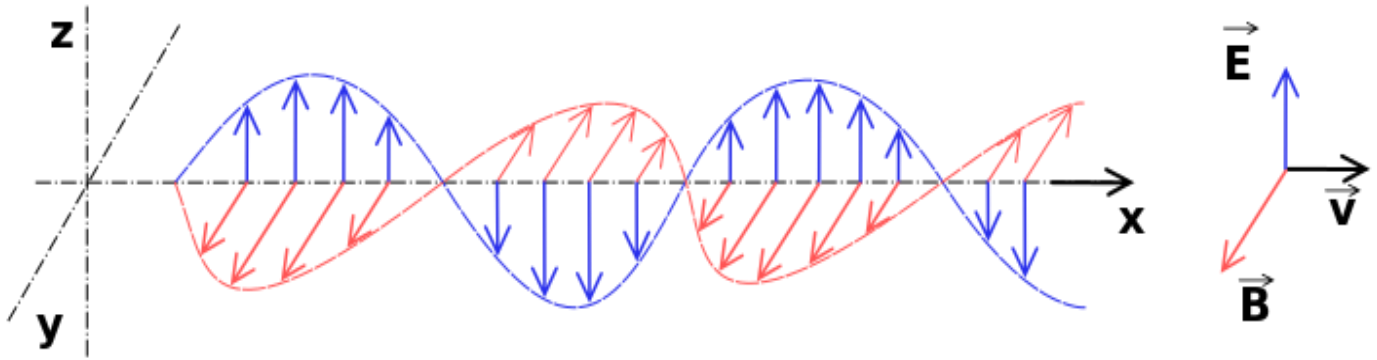
Gravitational field of the earth:

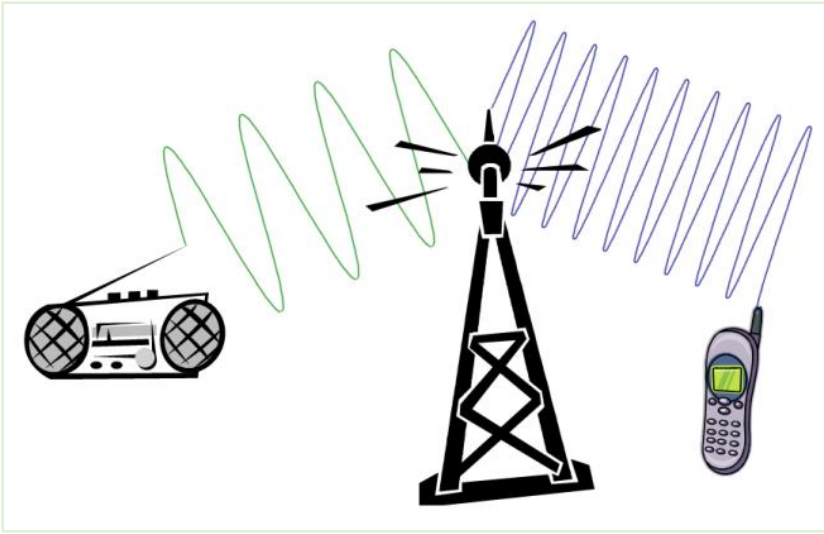


$$F_g = m\,g = -G\,\frac{mM}{R^2}$$

Einstein spent most of his life looking for a unified theory of electromagnetism and general relativity.

Electromagnetic and Gravitational Waves





Electromagnetic wave:

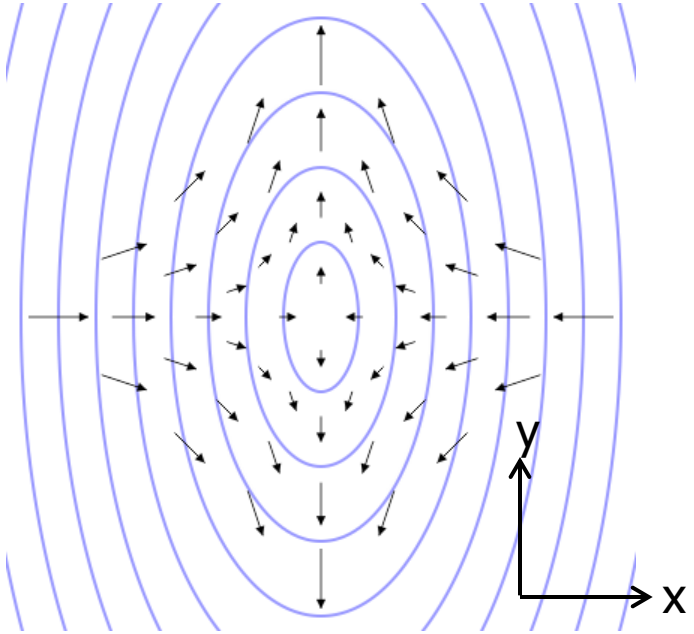
Changing electric and magnetic field propagating through space.

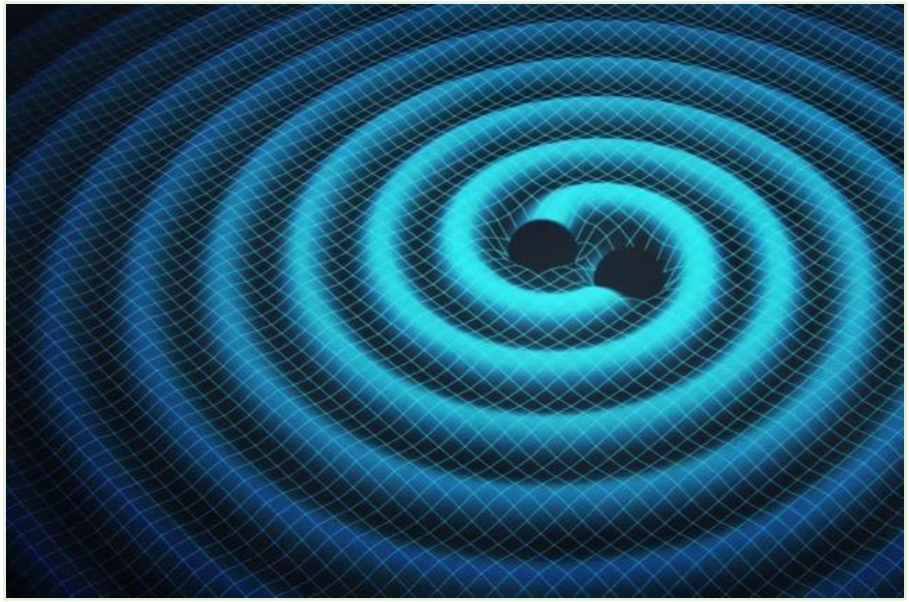
Caused by moving (accelerating!) electric charges.

Gravitational wave:

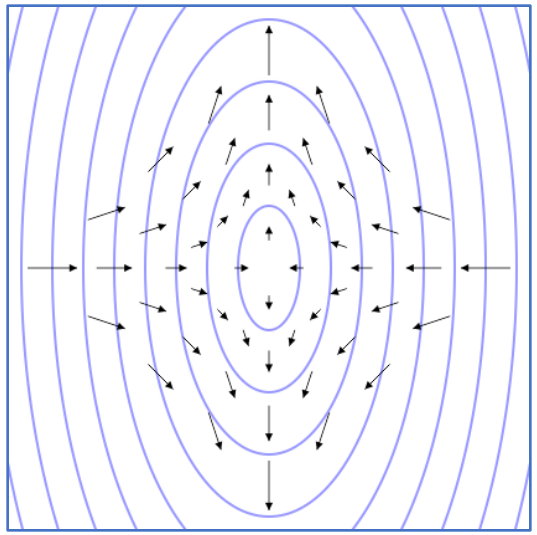
Changing space-time field.

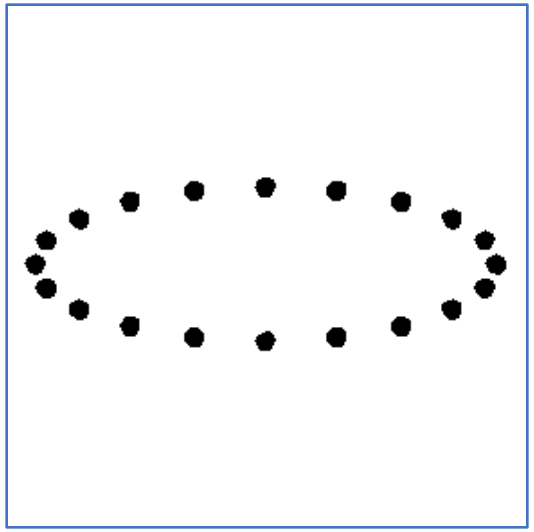
Caused by moving (accelerating!) masses

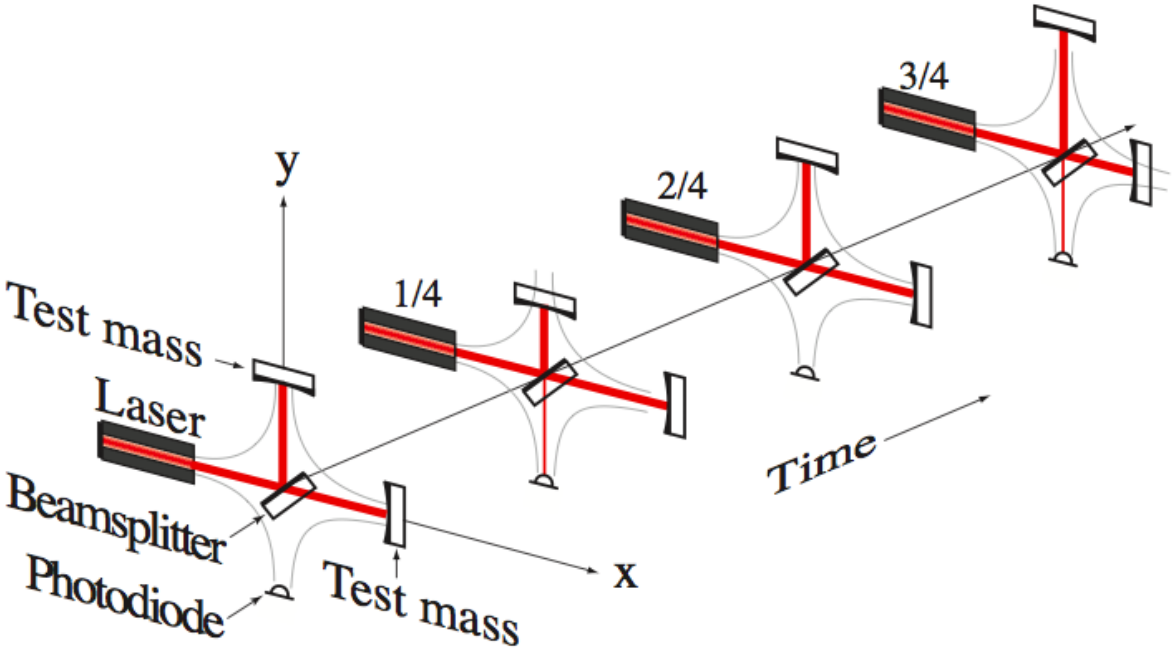


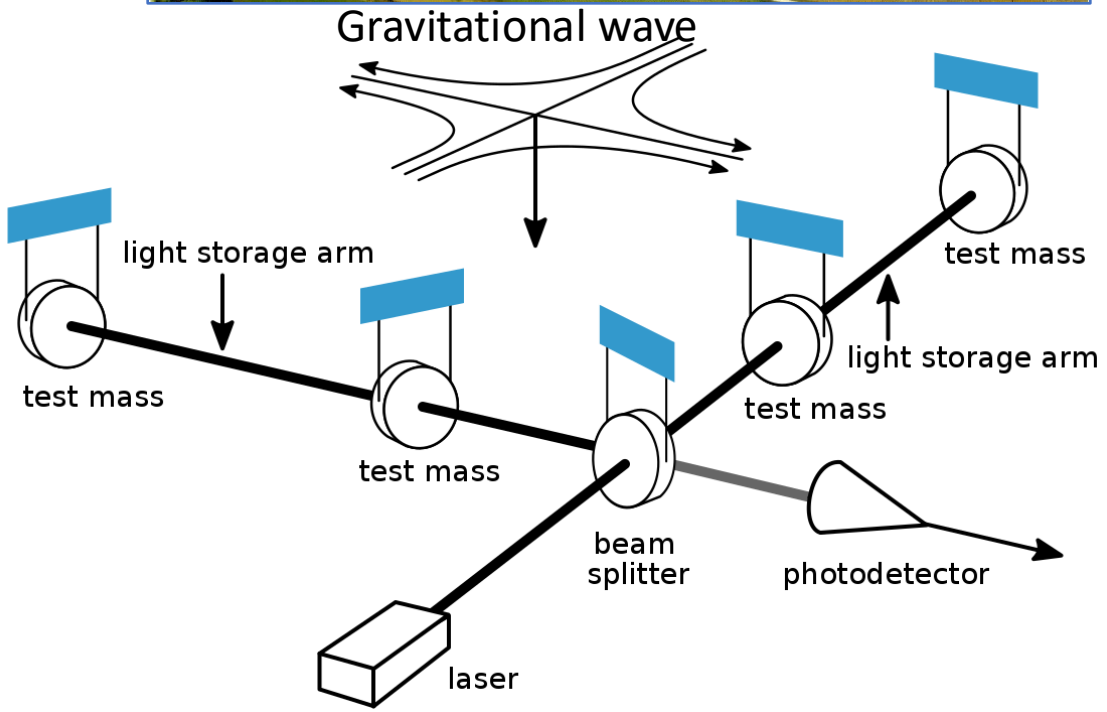


Remember the interferometer!



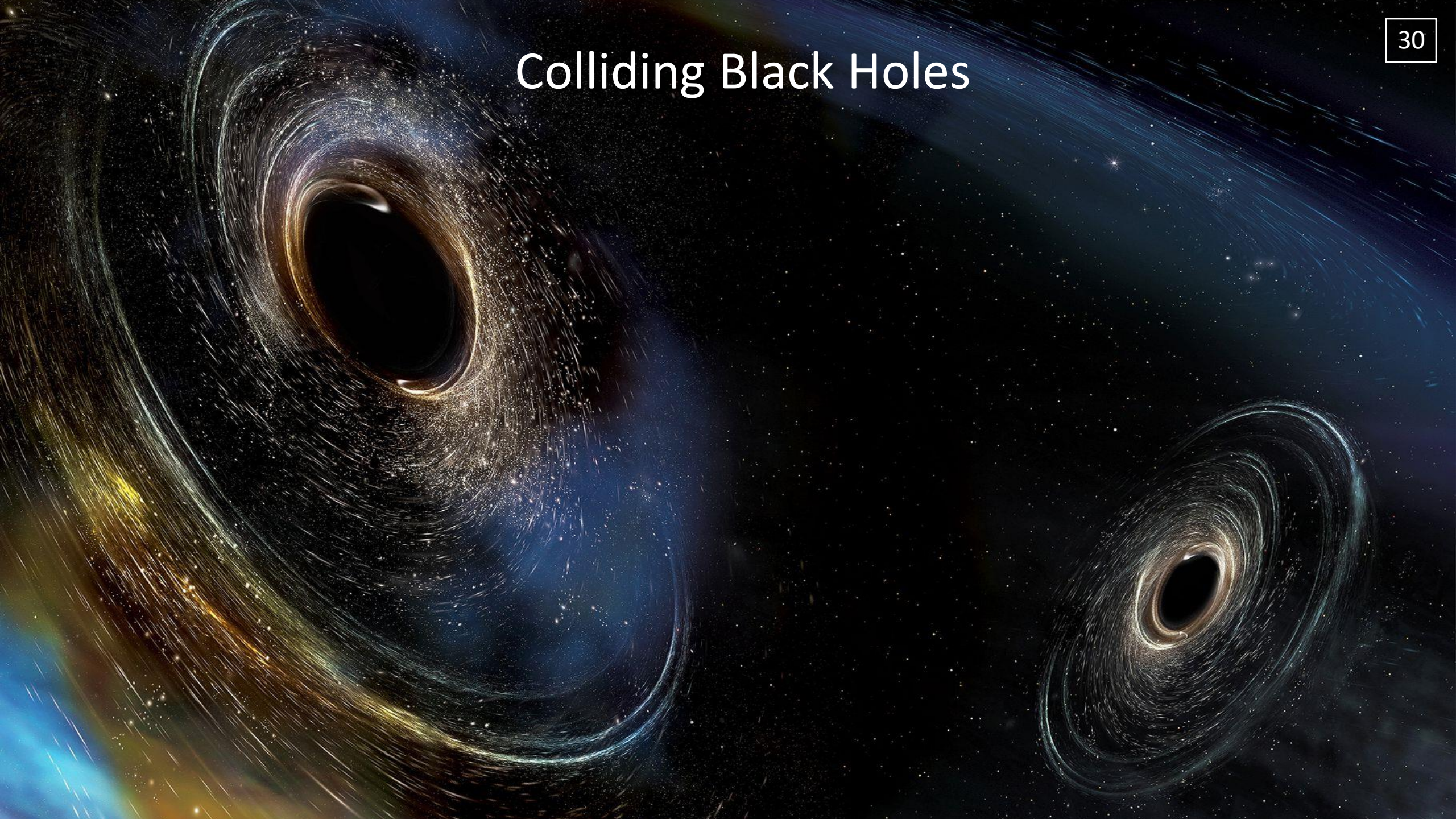




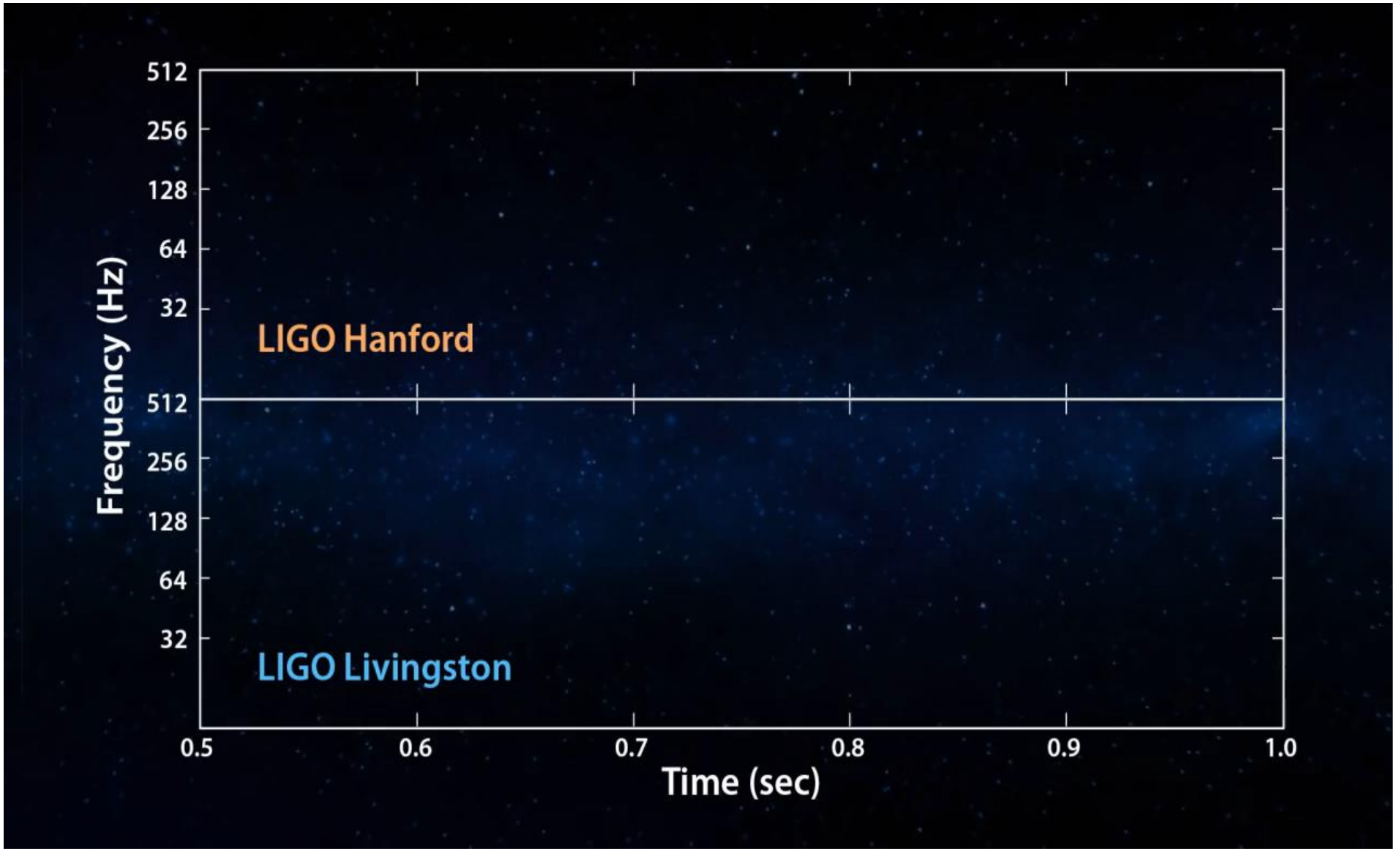


Current Facilities in the World

Colliding Black Holes



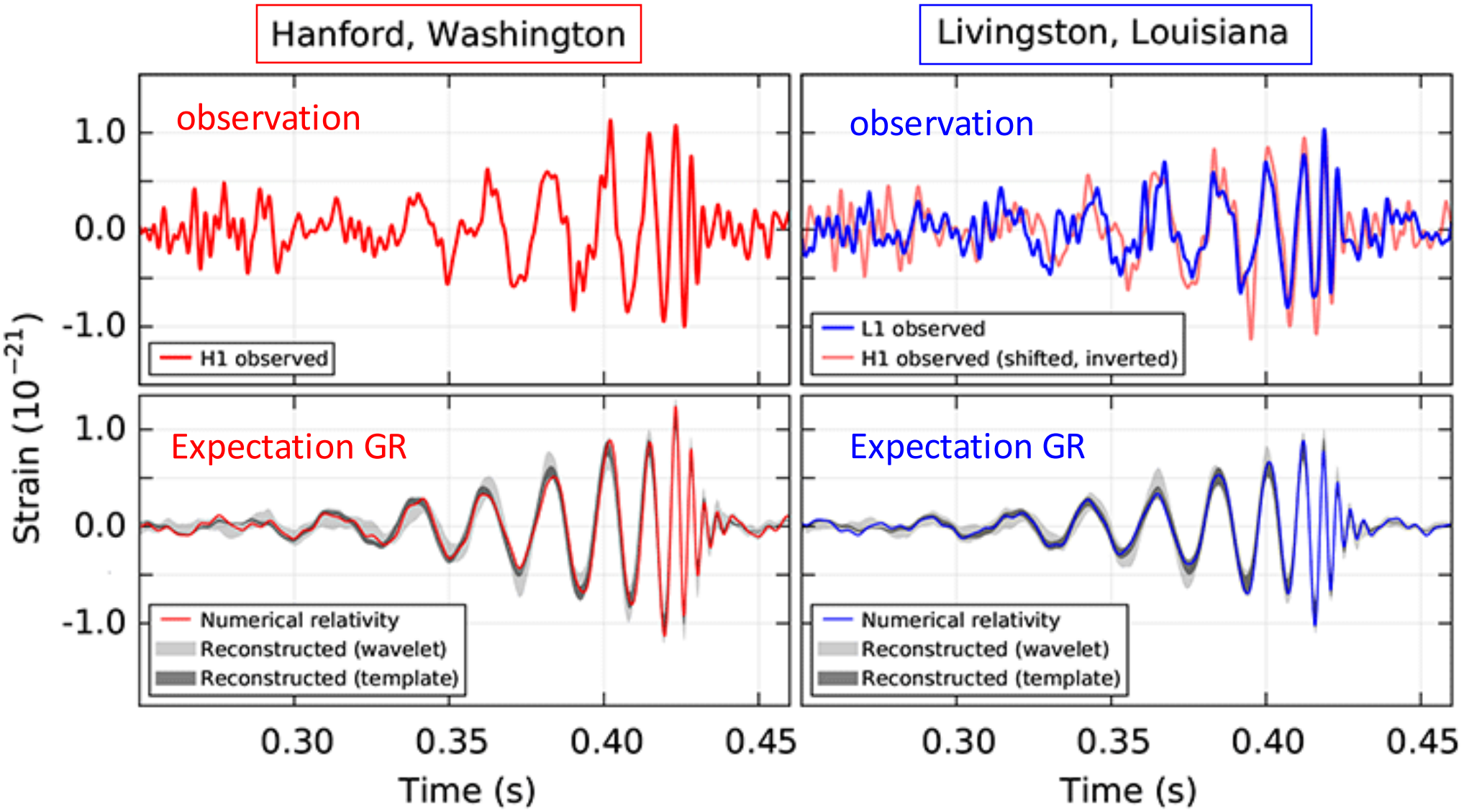
First detection of gravitational waves: GW150914



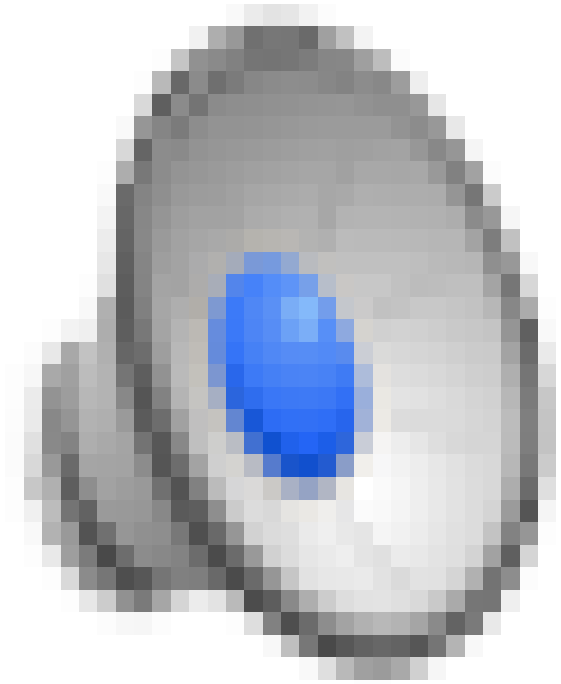
"Chirp" of colliding black holes at 1.3 billion lightyears distance

Consistent signals seen in Washington and Louisiana

(GW150914)

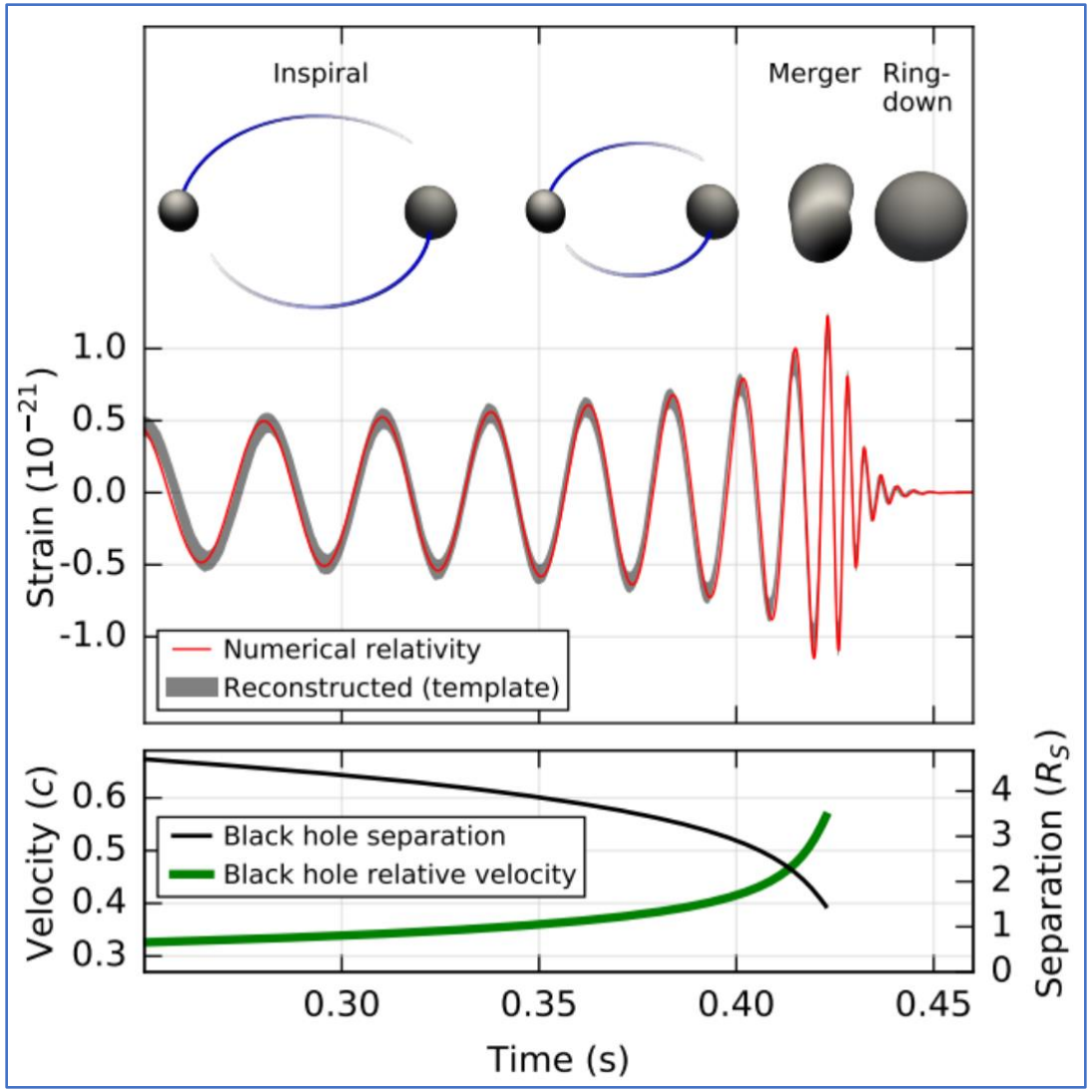


Detection of Gravitational Waves



Two Merging Black Holes

Two massive colliding/merging black holes:



Distance: 1.3 billion lightyears

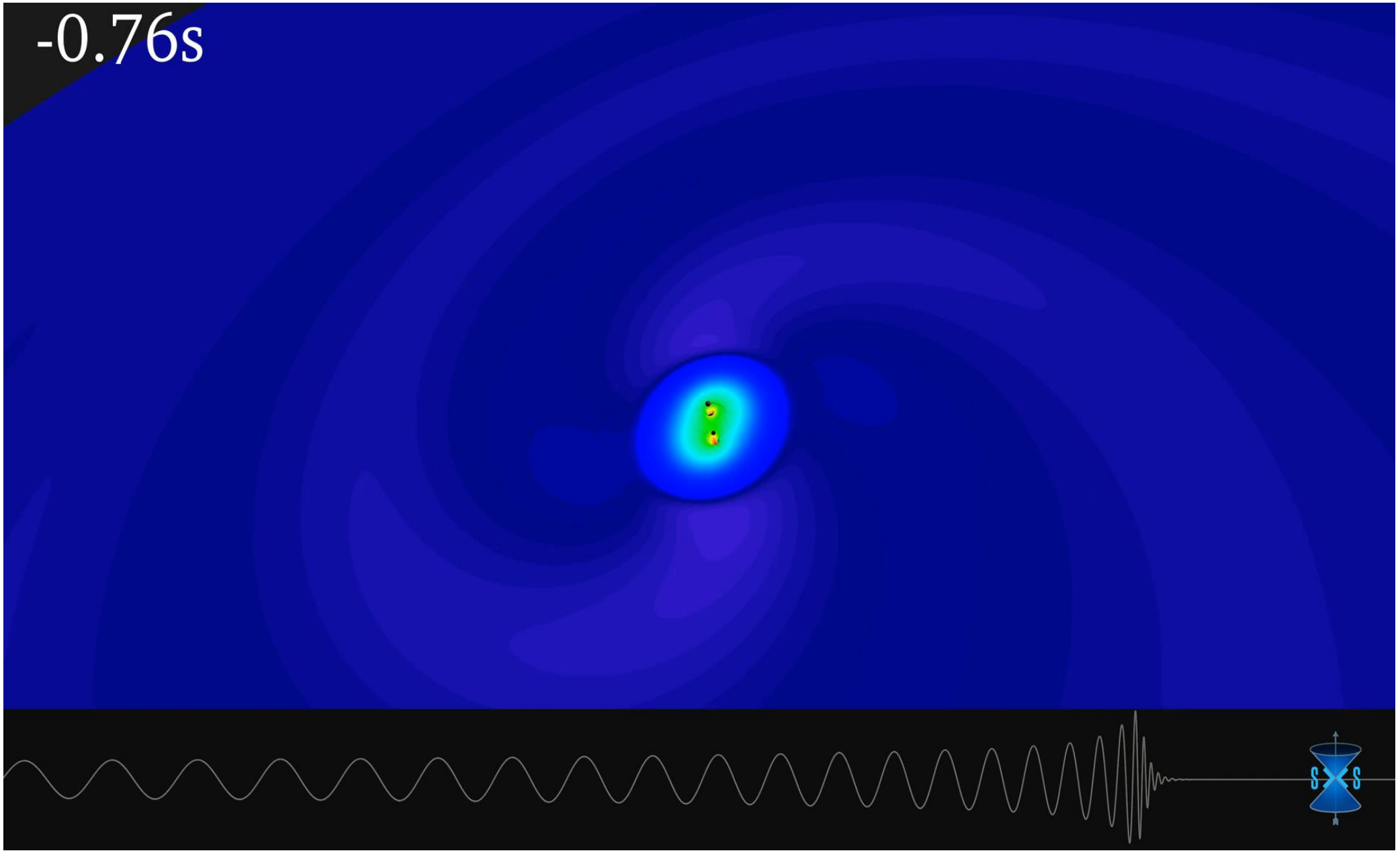
B.H.1 = 36 x mass of the sun
B.H.2 = 29 x mass of the sun
New BH: 62 solar masses
➔3 solar masses of energy ($E=mc^2$) radiated into space

Relative change of space (strain)
0.00000000000000000001%

Rotation speed increasing to half the light speed!

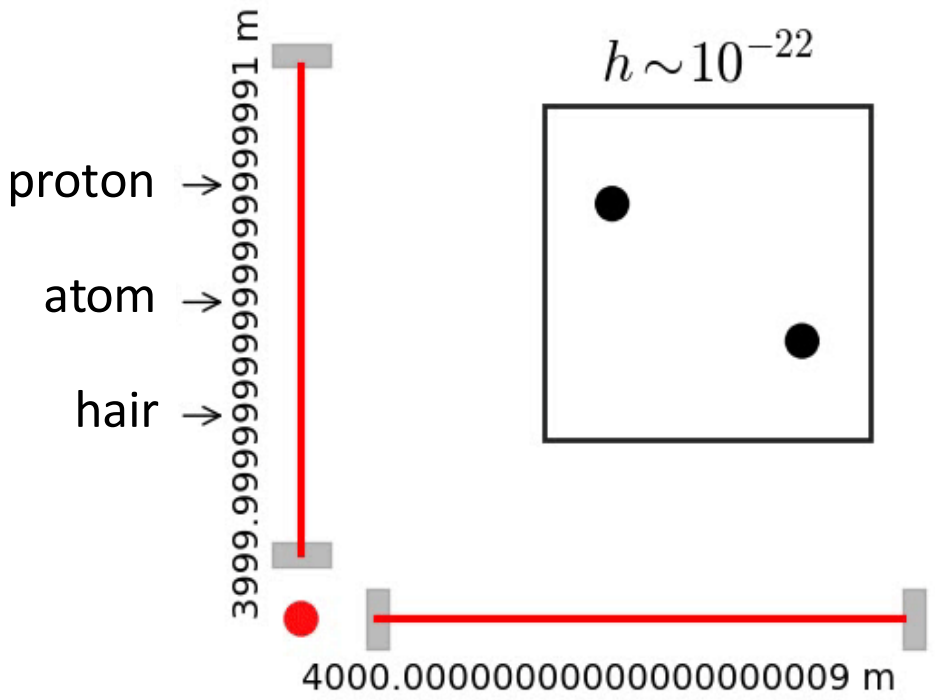
More energy was emitted in gravitational waves than all the visible (EM) energy of all stars in the universe!

Numerical Relativity Simulation for GW150914



Which precision is needed?

Laser
(f,



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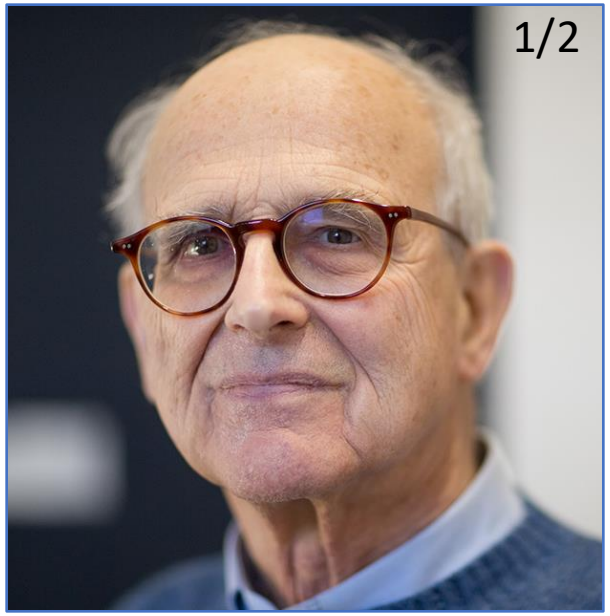
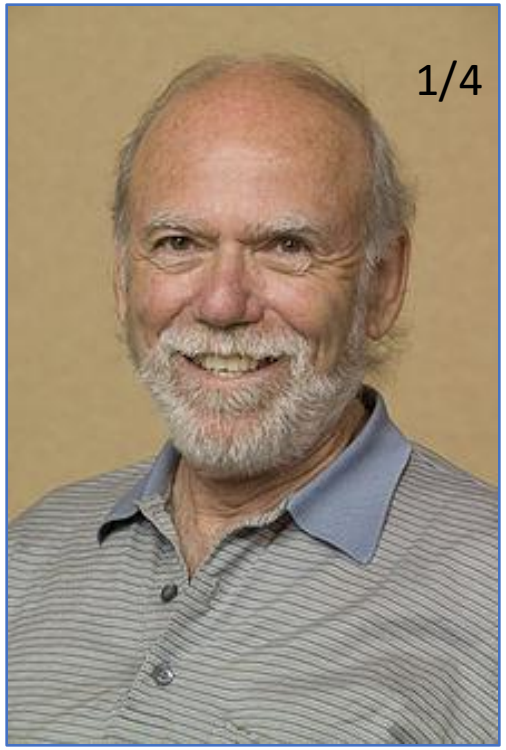
Nobel Prize in Physics 2017



Rainer Weiss

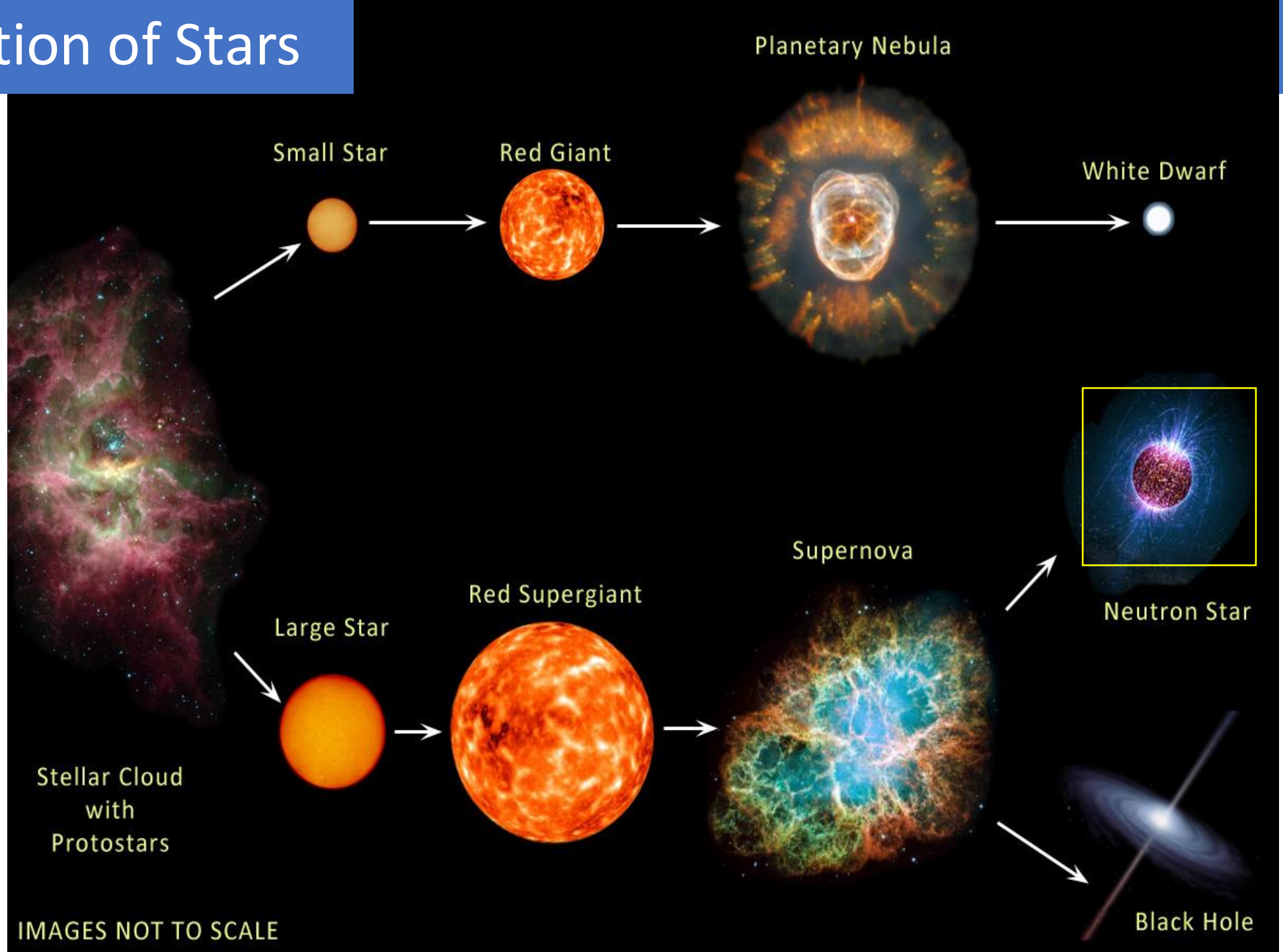


Barry C. Barish

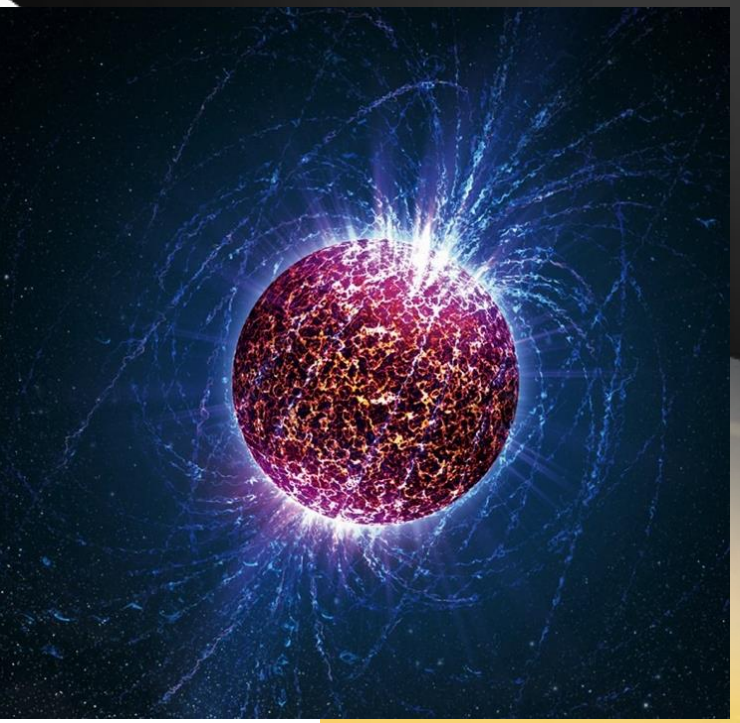
Kip S. Thorne

"For decisive contributions to the LIGO detector and the observation of gravitational waves"

Evolution of Stars

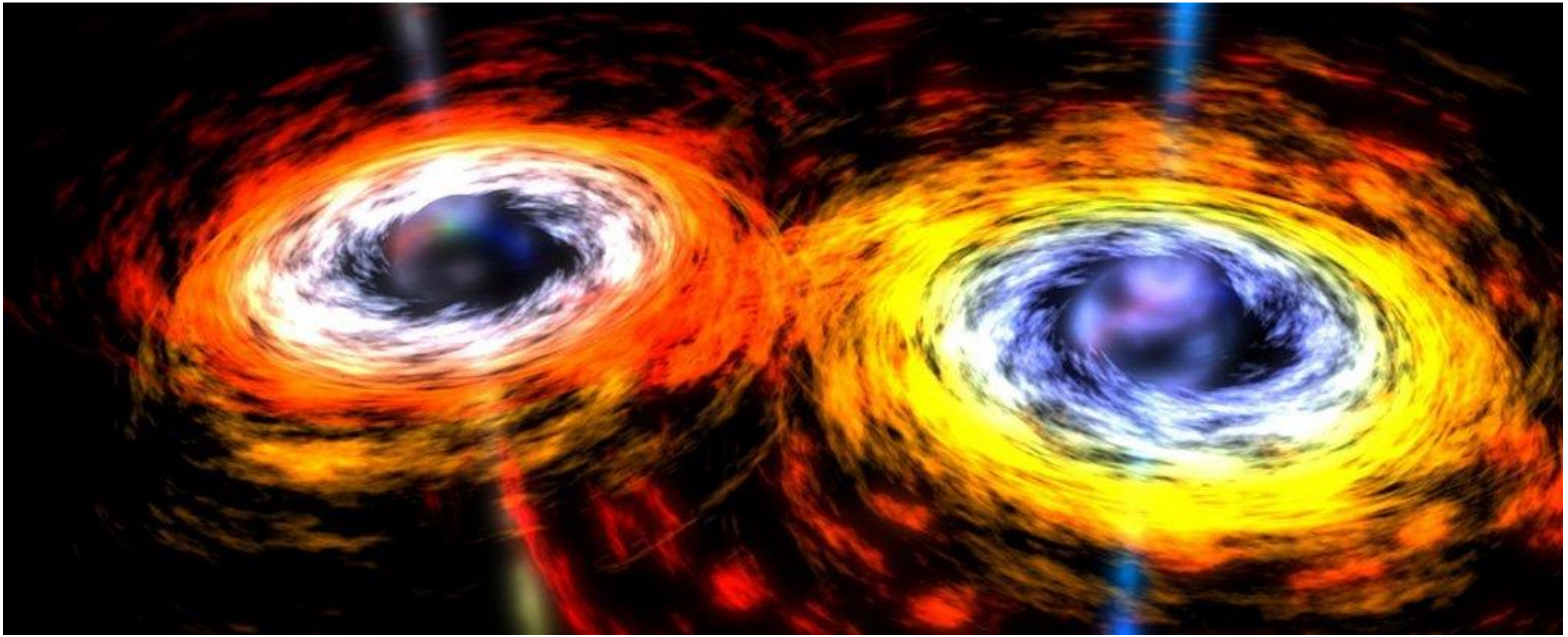


Neutron Star



Coalescing Neutron Stars: “Multimessenger” Astronomy

Gravitational Waves and …

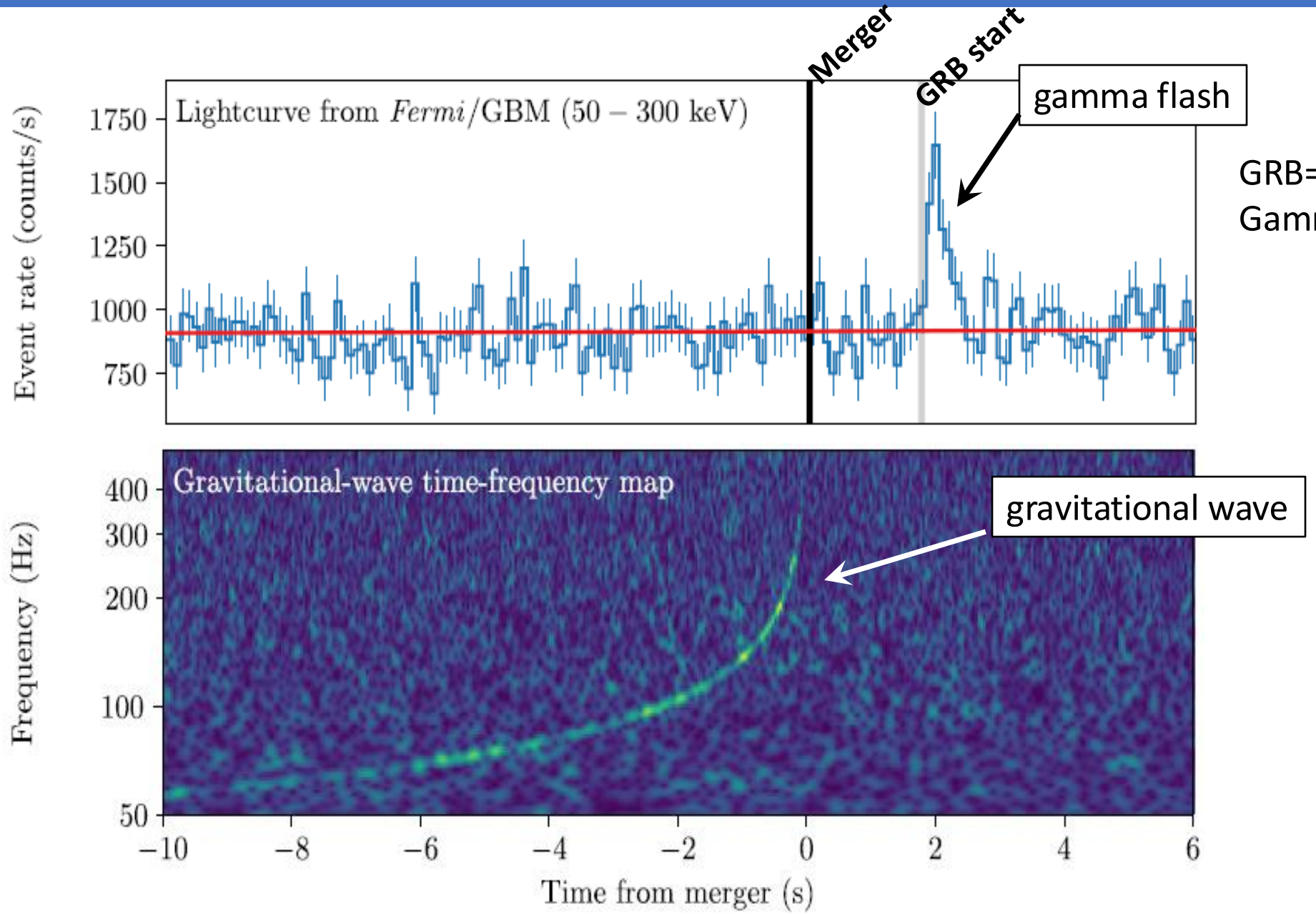


Coalescing Neutron Stars: “Multimessenger” Astronomy

Production of elements more massive then Iron (Fe) are produced: gold, platinum, uranium, …

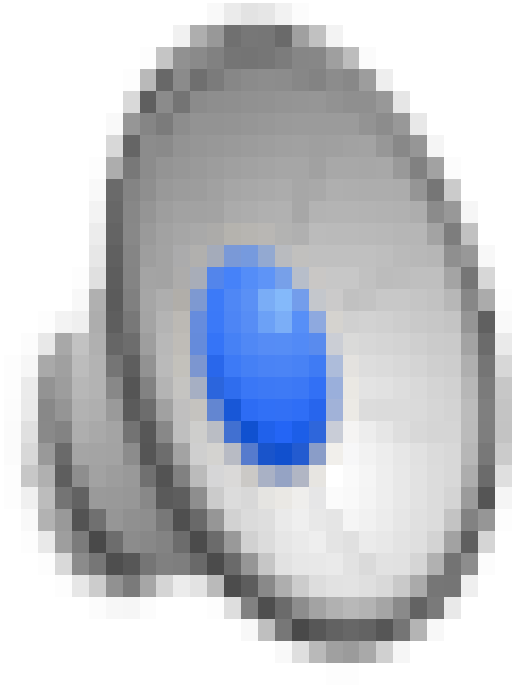

Gamma flash 1.7 sec later…
INTEGRAL
Fermi Space Telescope

“Multimessenger” Astronomy



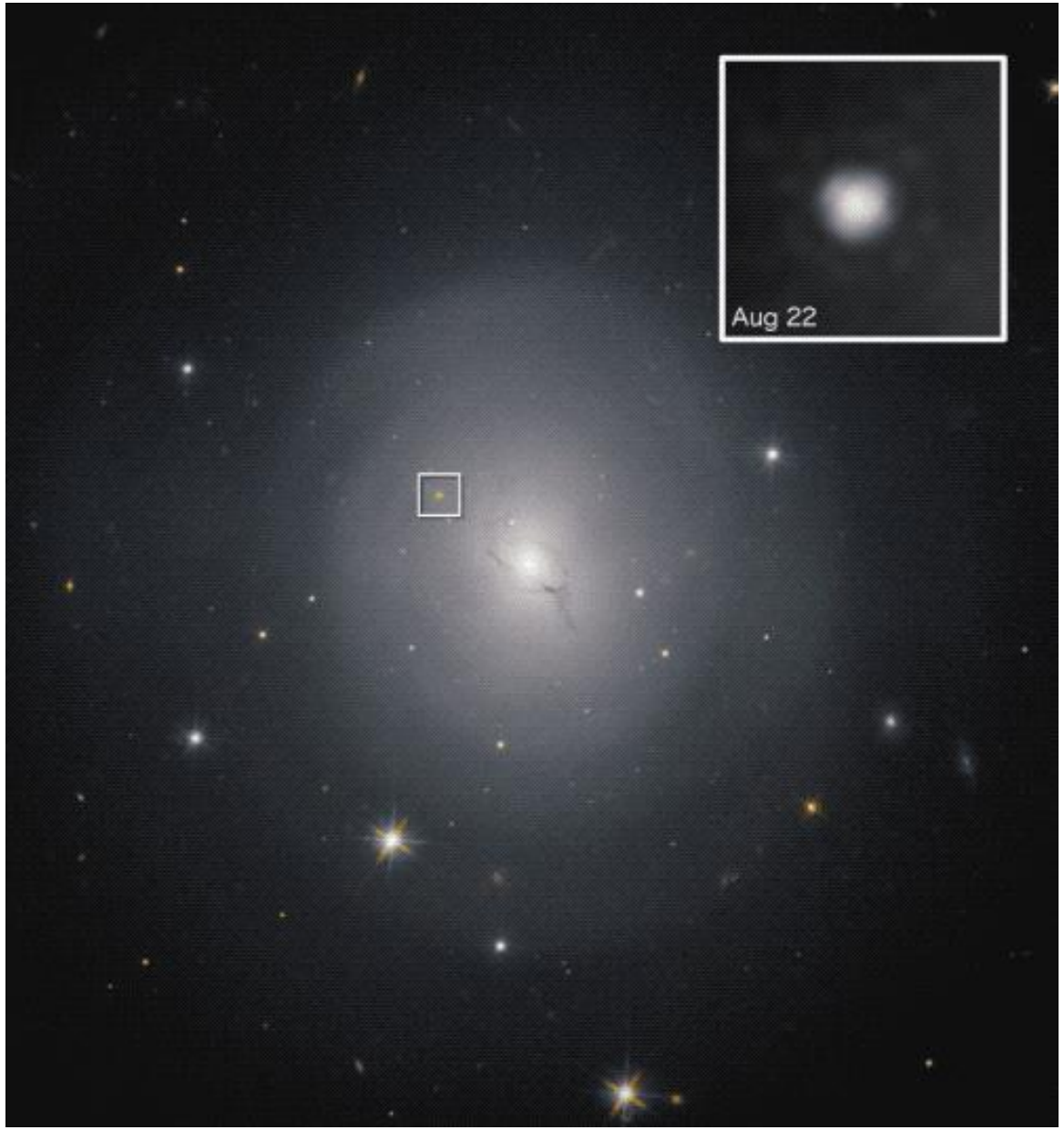
GRB=
Gamma Ray Burst

Neutron star merger: multimessenger



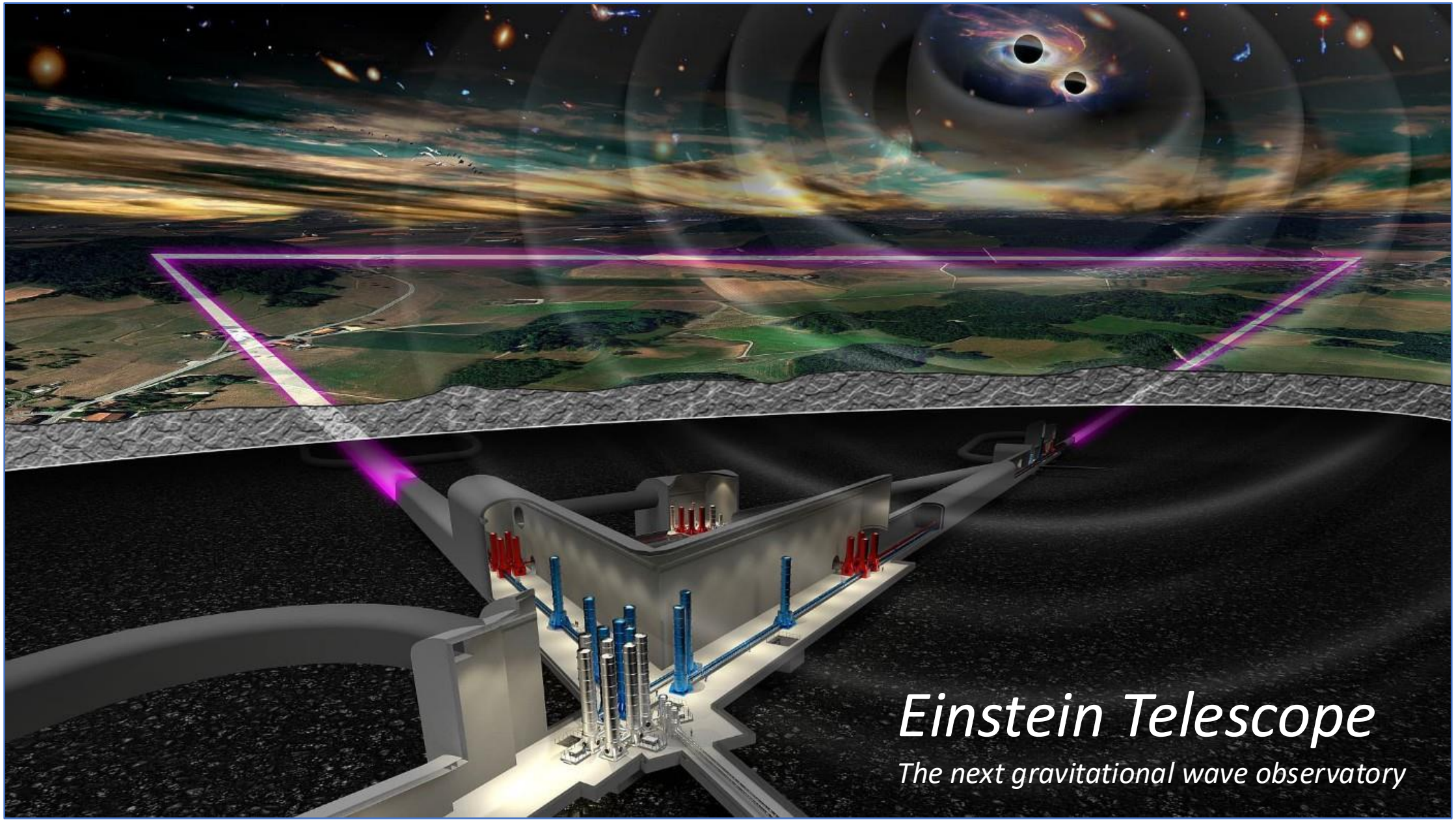
Kilonova: production heavy elements: gold, platinum etc.

Optisch Signaal



In a kilonova explosion elements more massive then Iron (Fe) are produced: Gold, platinum, uranium, …

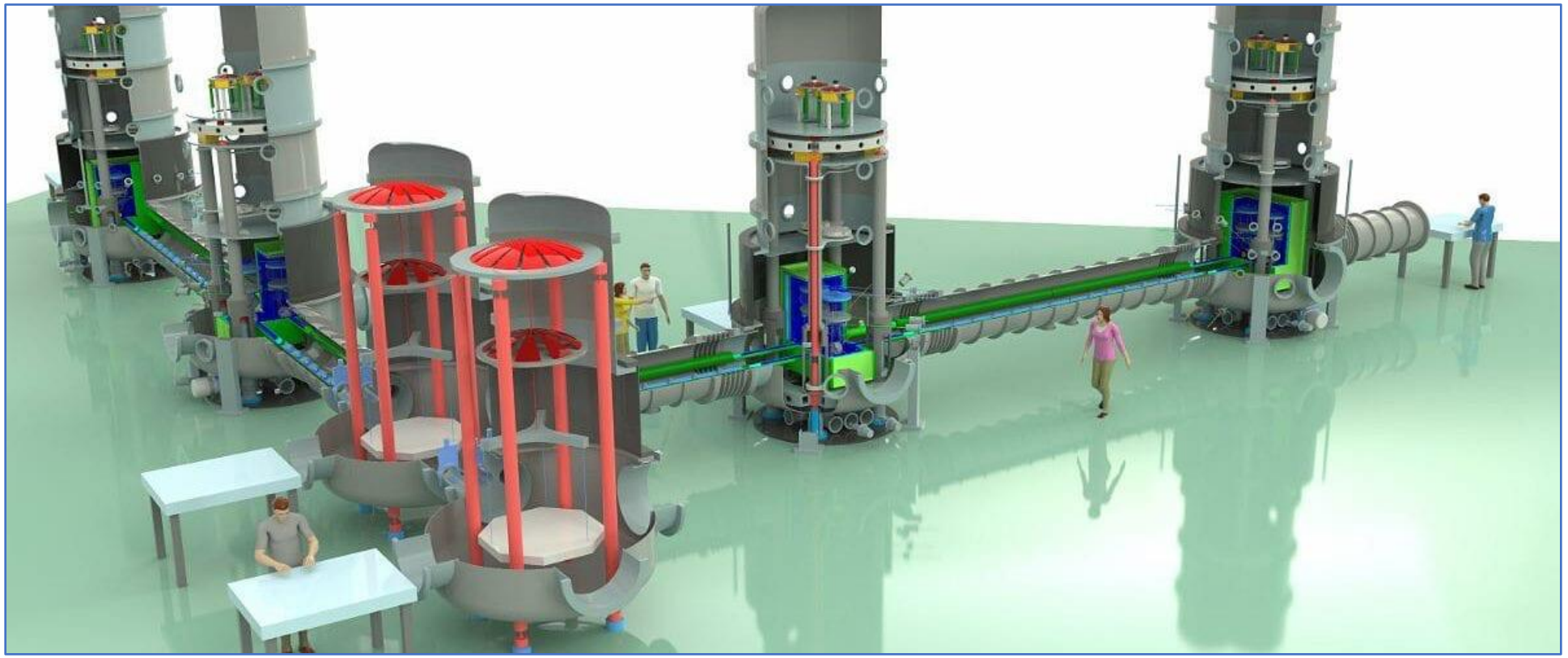
Possible Future Facility…



Underground Triangle: 10x10x10 km

ET Pathfinder in Maastricht

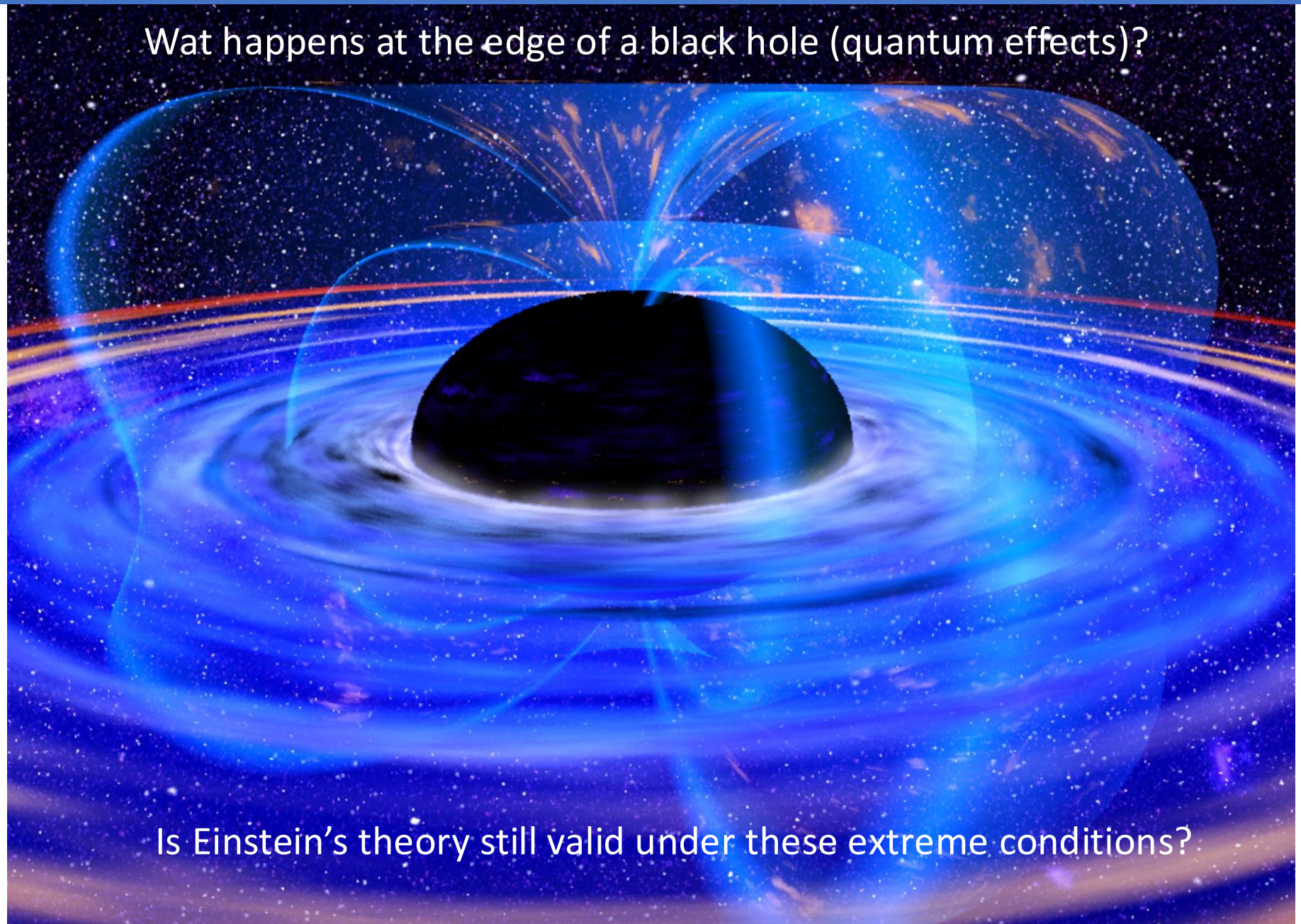
ET Pathfinder



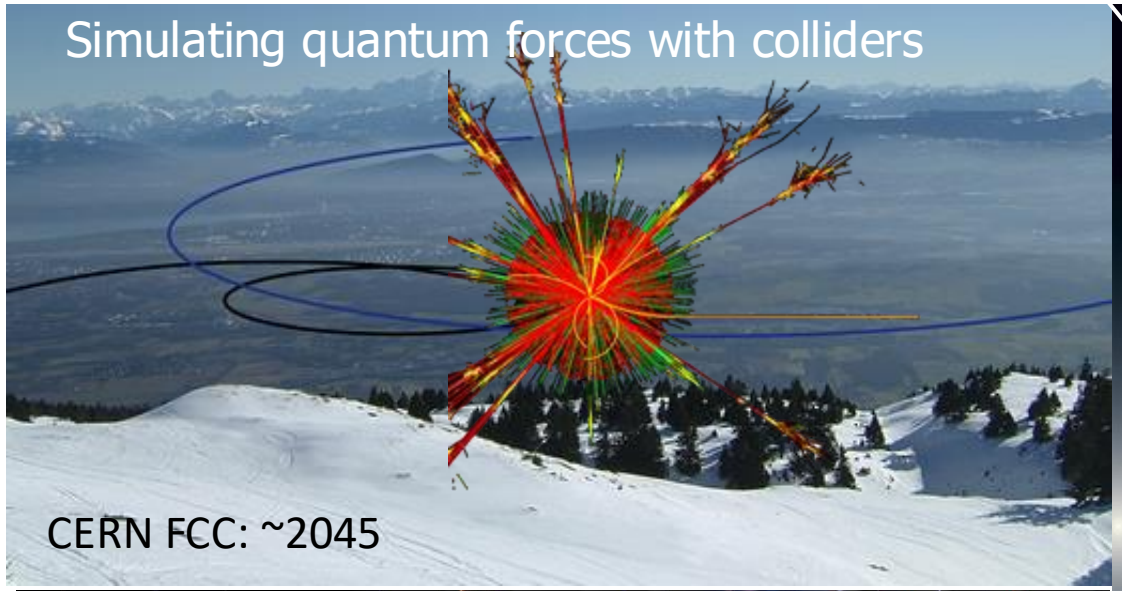
Nikhef, RWTH Aachen, UCL Louvain, Hasselt, Ghent, Antwerp, VUB Brussels, ULB Brussels, Liege, Radboud University Nijmegen, TU Eindhoven and Hamburg

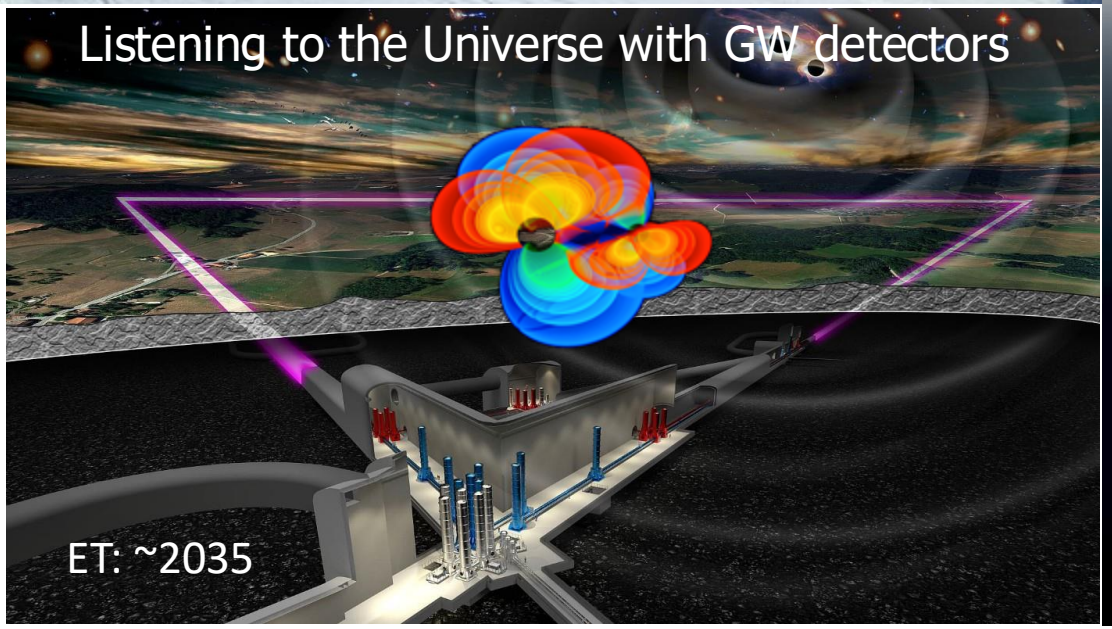
ET Pathfinder in Dub30

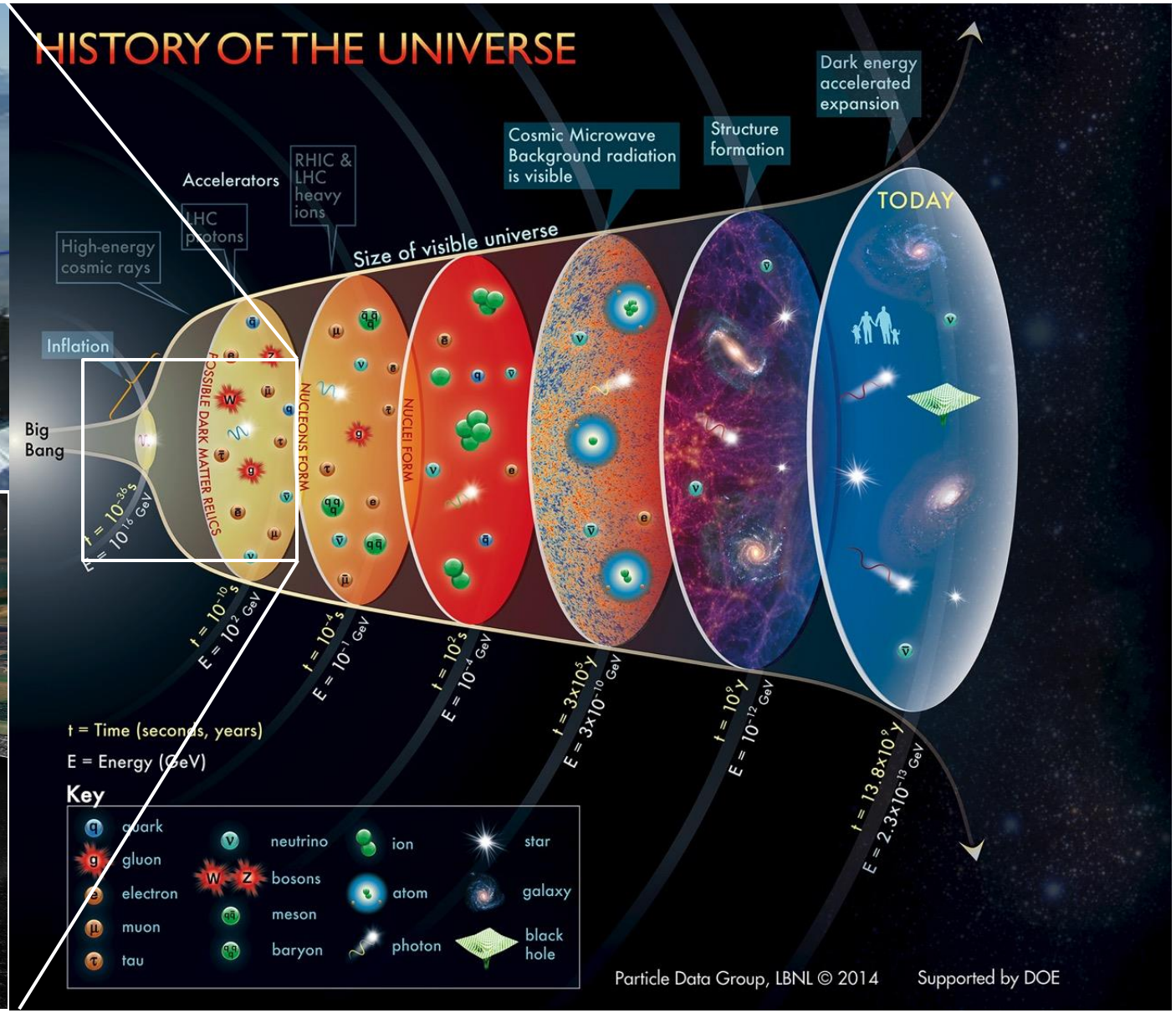
Fundamental Black hole physics



Looking into the Big Bang

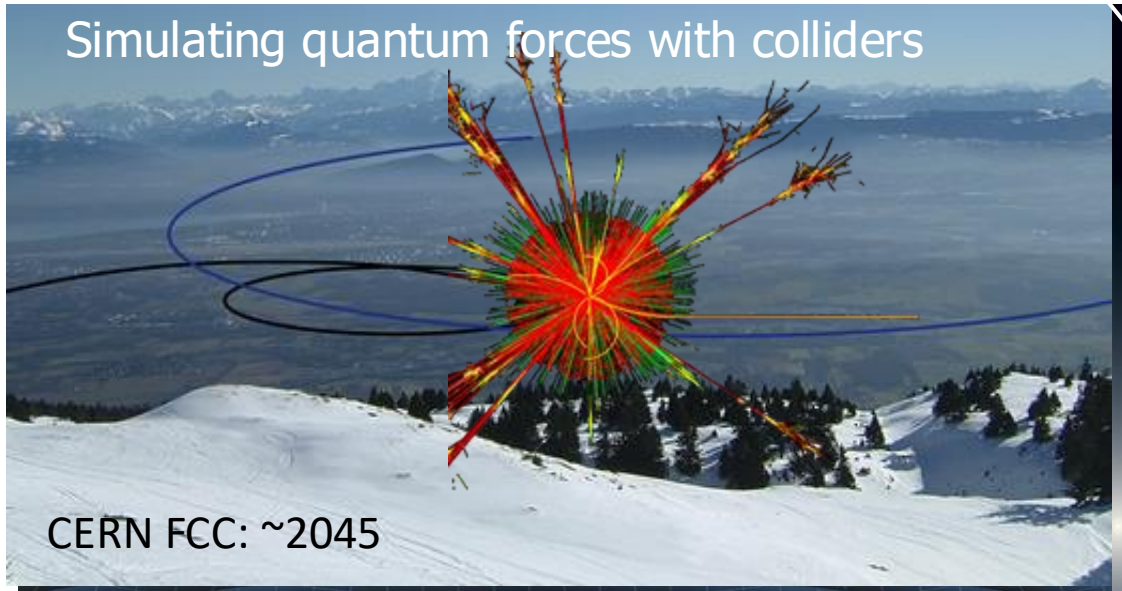






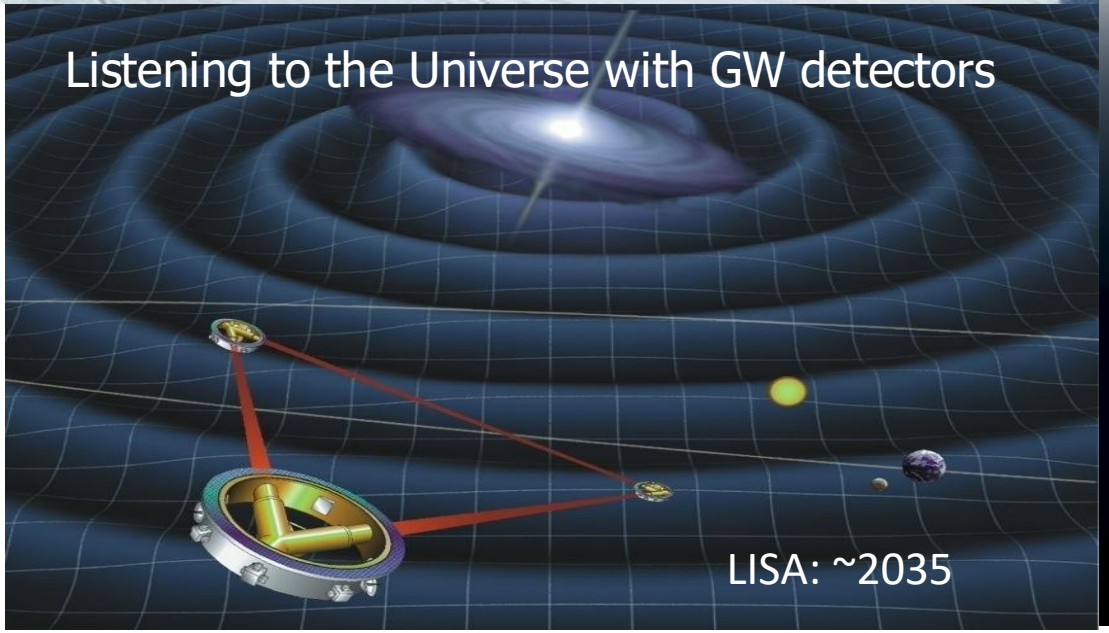
Looking into the Big Bang

Simulating quantum forces with colliders

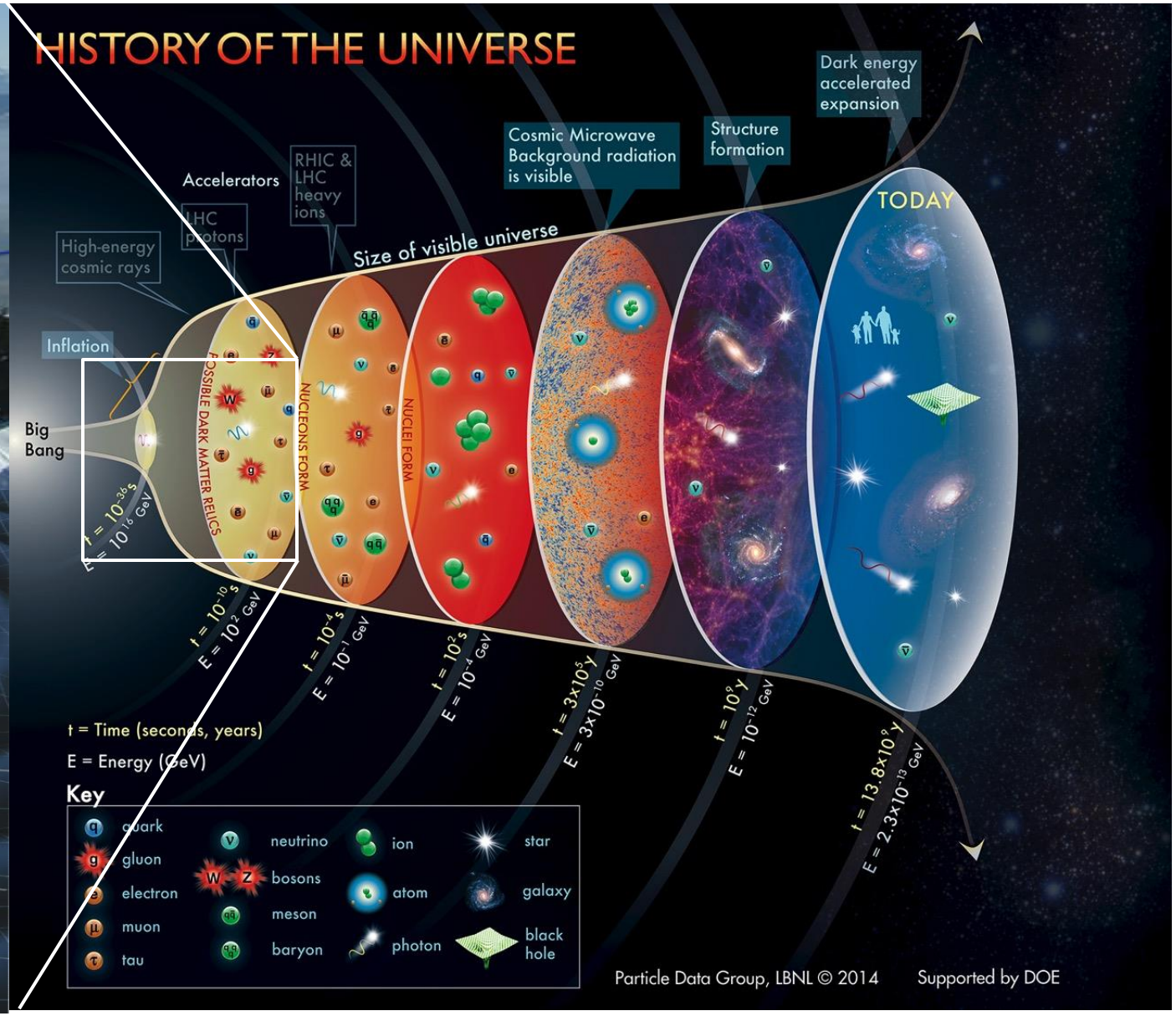


CERN FCC: ~2045

Listening to the Universe with GW detectors



LISA: ~2035

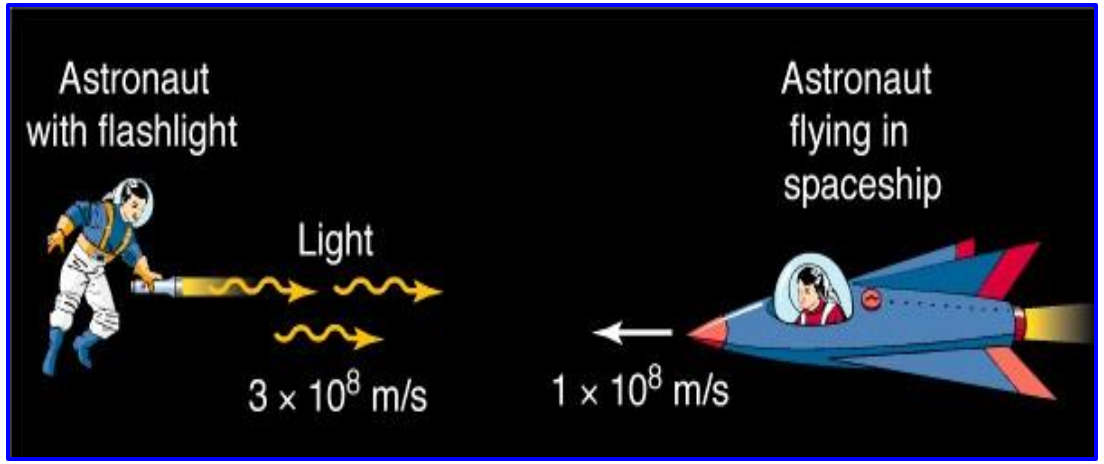


Relativity Theory in Summary

Special Relativity

All observers moving in inertial frames:

- Have identical laws of physics,
- Observe the same speed of light: c.

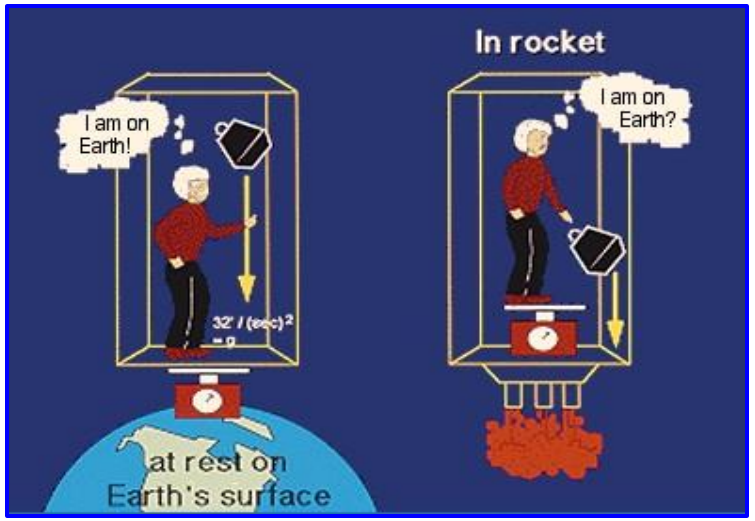


Consequences:

- Simultaneity is not the same for everyone,
- Distances shrink, time slows down at high speed,
- Velocities do not add-up as expected.

General Relativity

- A free falling person is also inertial frame,
- Acceleration and gravitation are equivalent: Inertial mass = gravitational mass

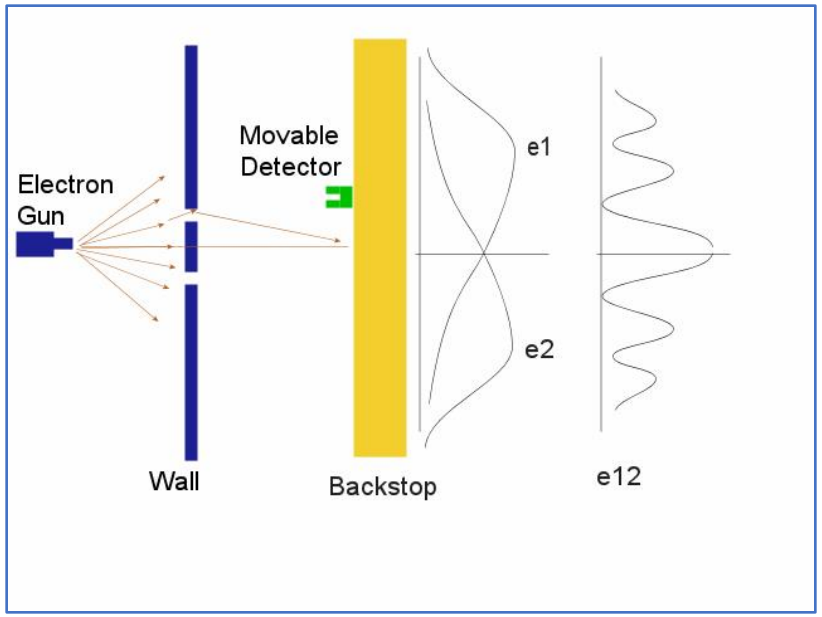


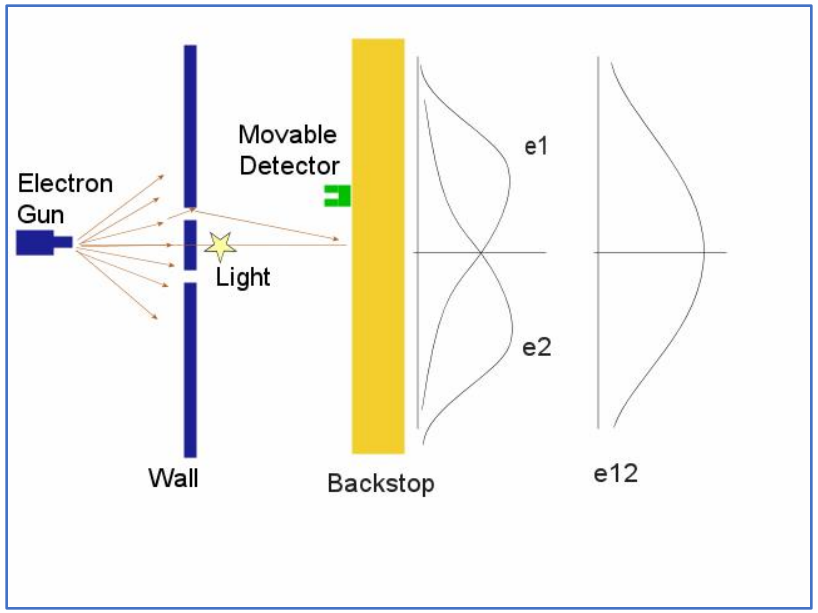
Consequences:

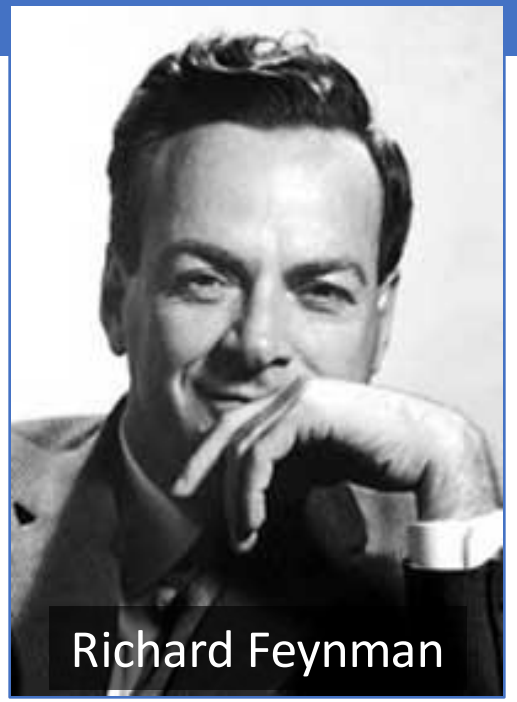
Space-time is curved:

- Light bends around a massive object,
- Time slows down and space shrinks in gravitational fields,
- Gravitational radiation exists.

Next week: Quantum Mechanics

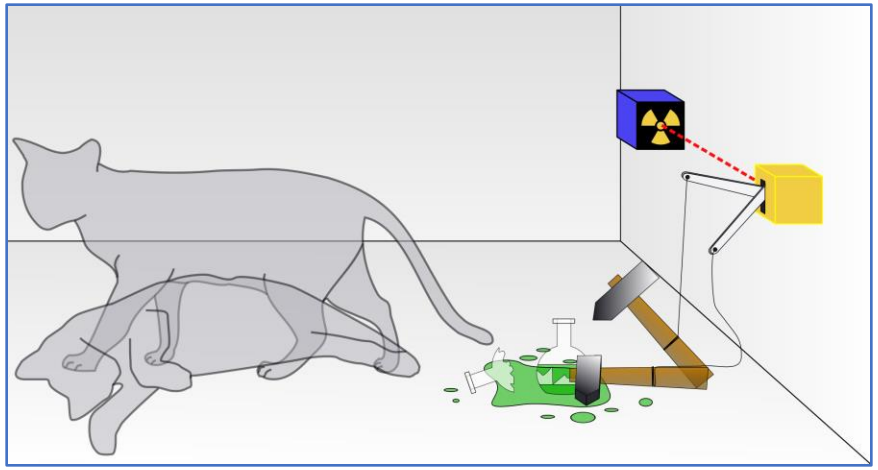






Quantum mechanics developed by Bohr and Heisenberg leads to ”absurd” thought experiments of Feynman and Wheeler. Einstein and Schrödinger did not like it.

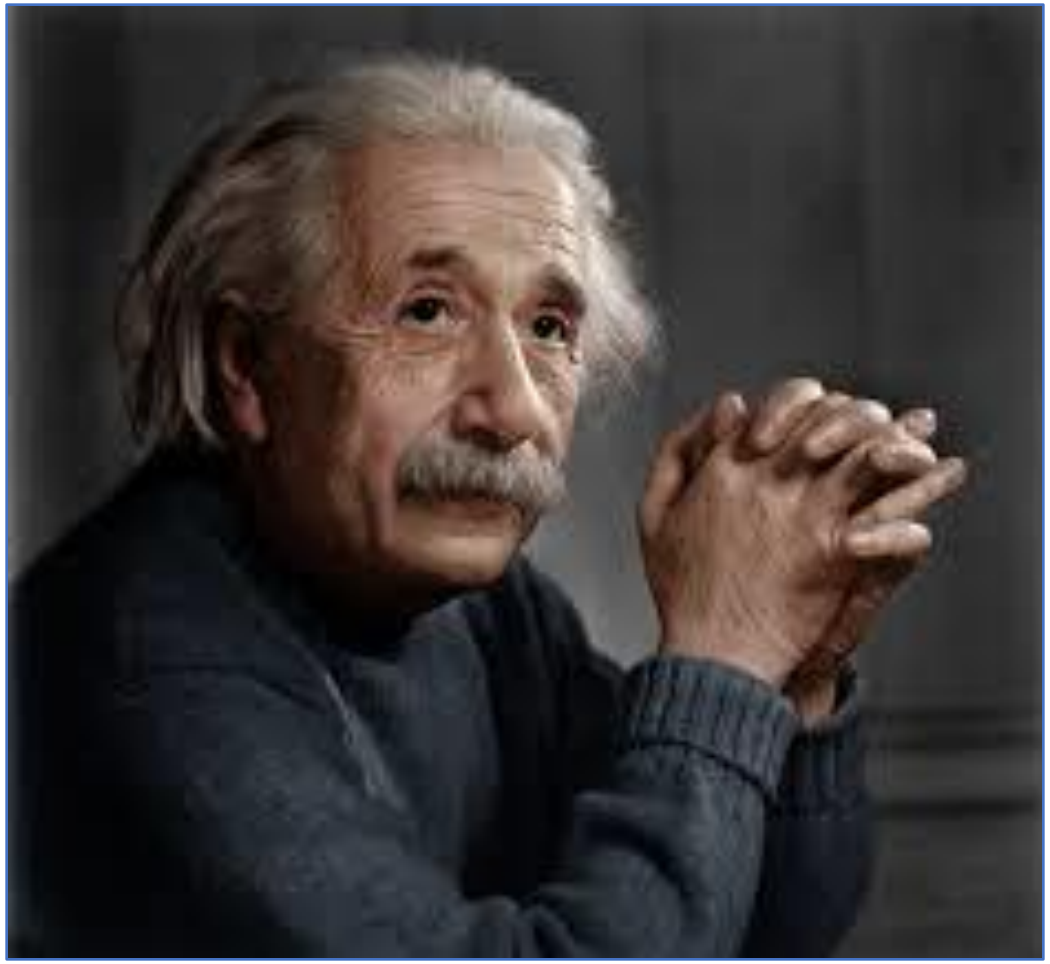
Even today people are debating its interpretation….



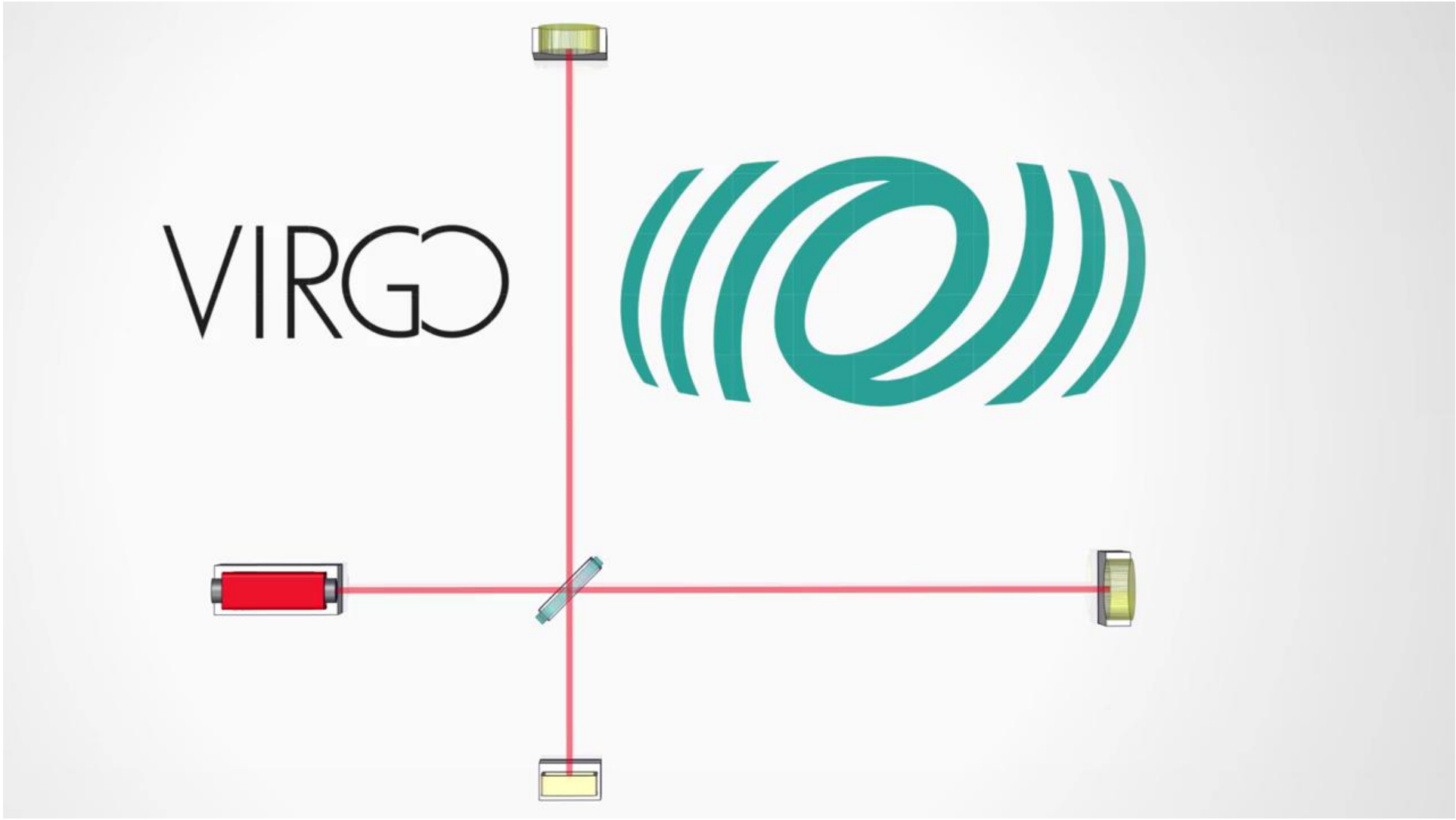
Extra Slides

Einstein Quotes

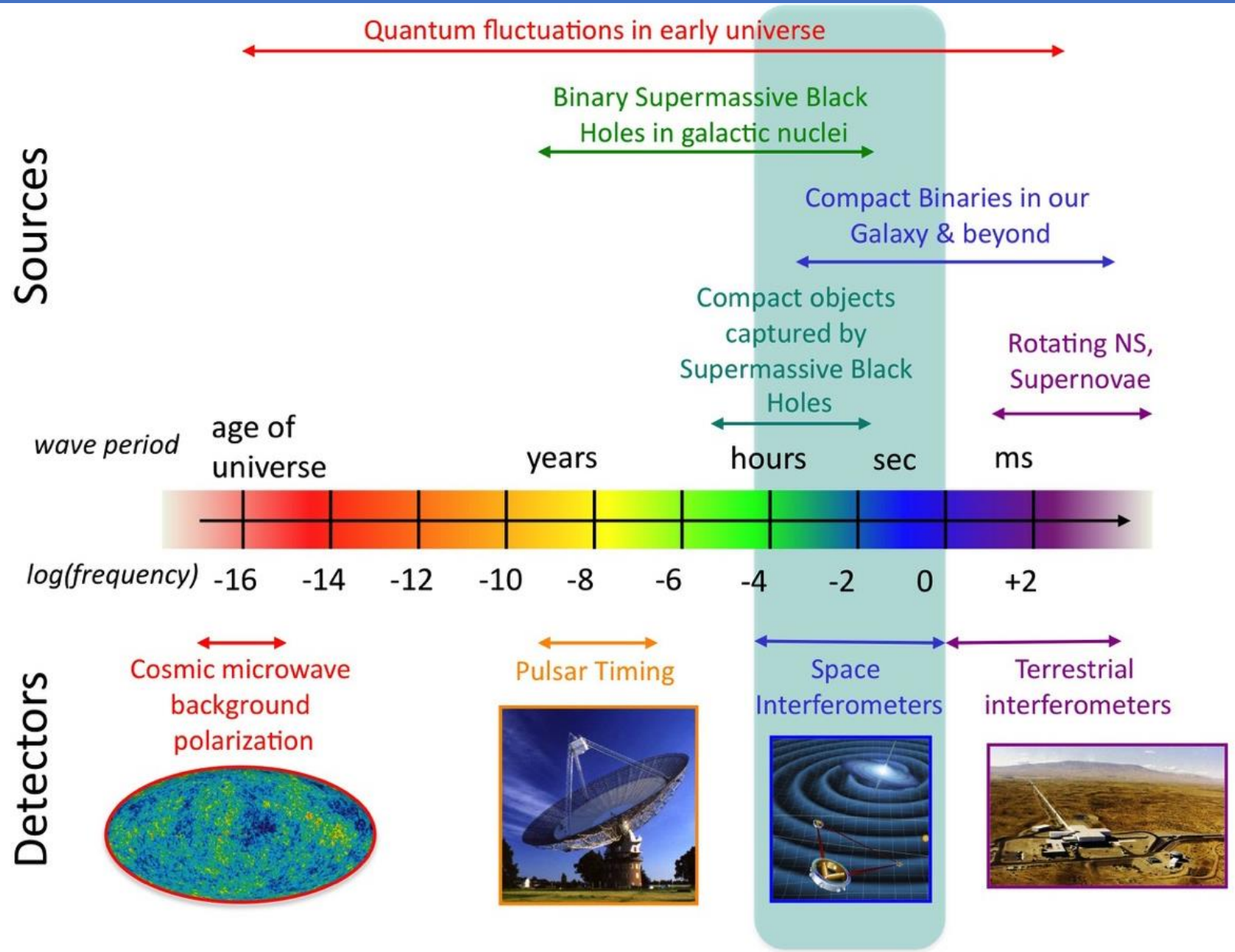
- “Imagination is more important than knowledge”
- “Education is what remains after one has forgotten what one has learned at school.”
- “I fear the day that technology will surpass our human interaction. The world will have a generation of idiots.”
- “A person who never made a mistake never tried anything new.”



Gravitation Wave Detection Method (in Dutch)

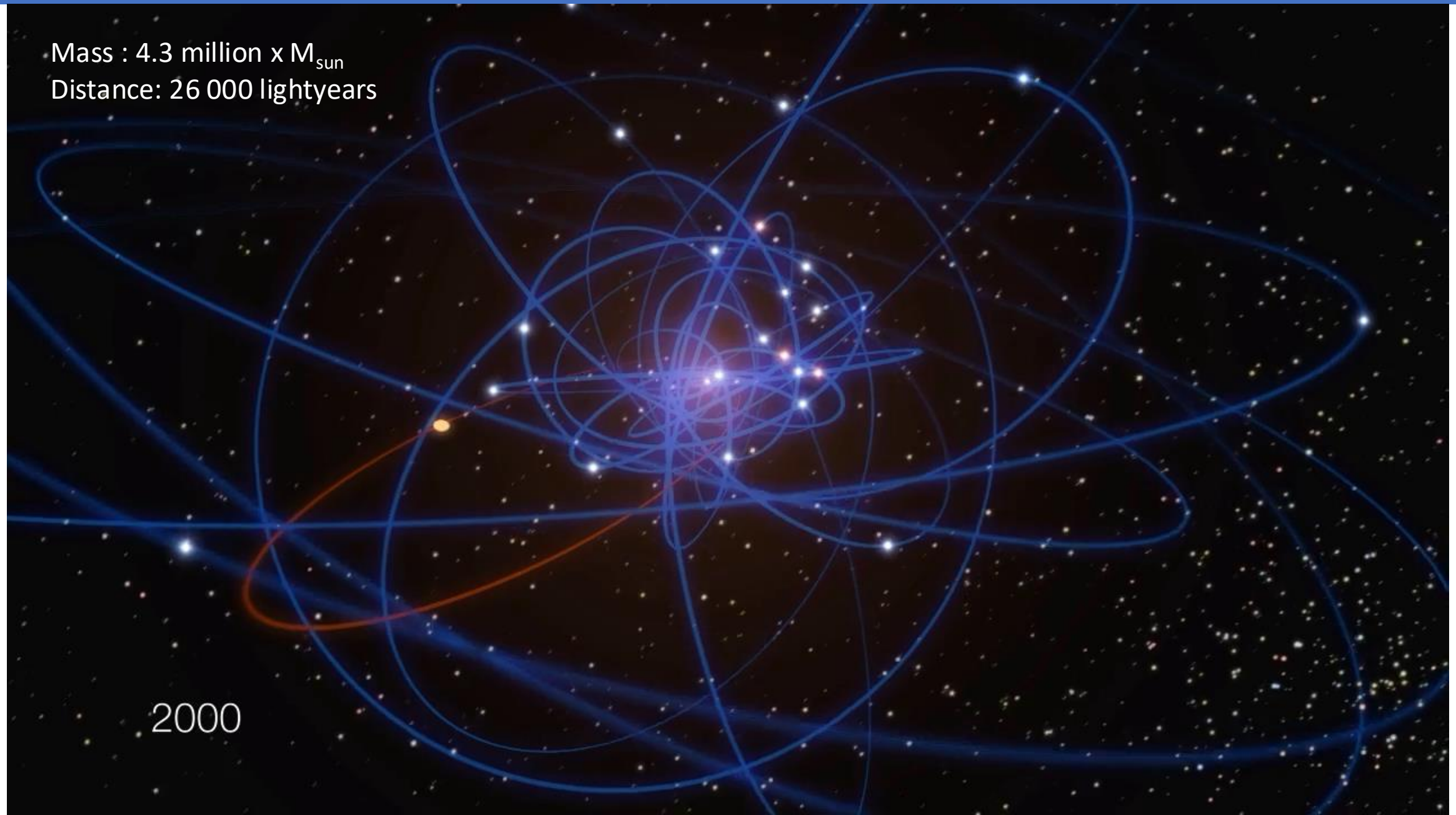


The Gravitational Wave Spectrum



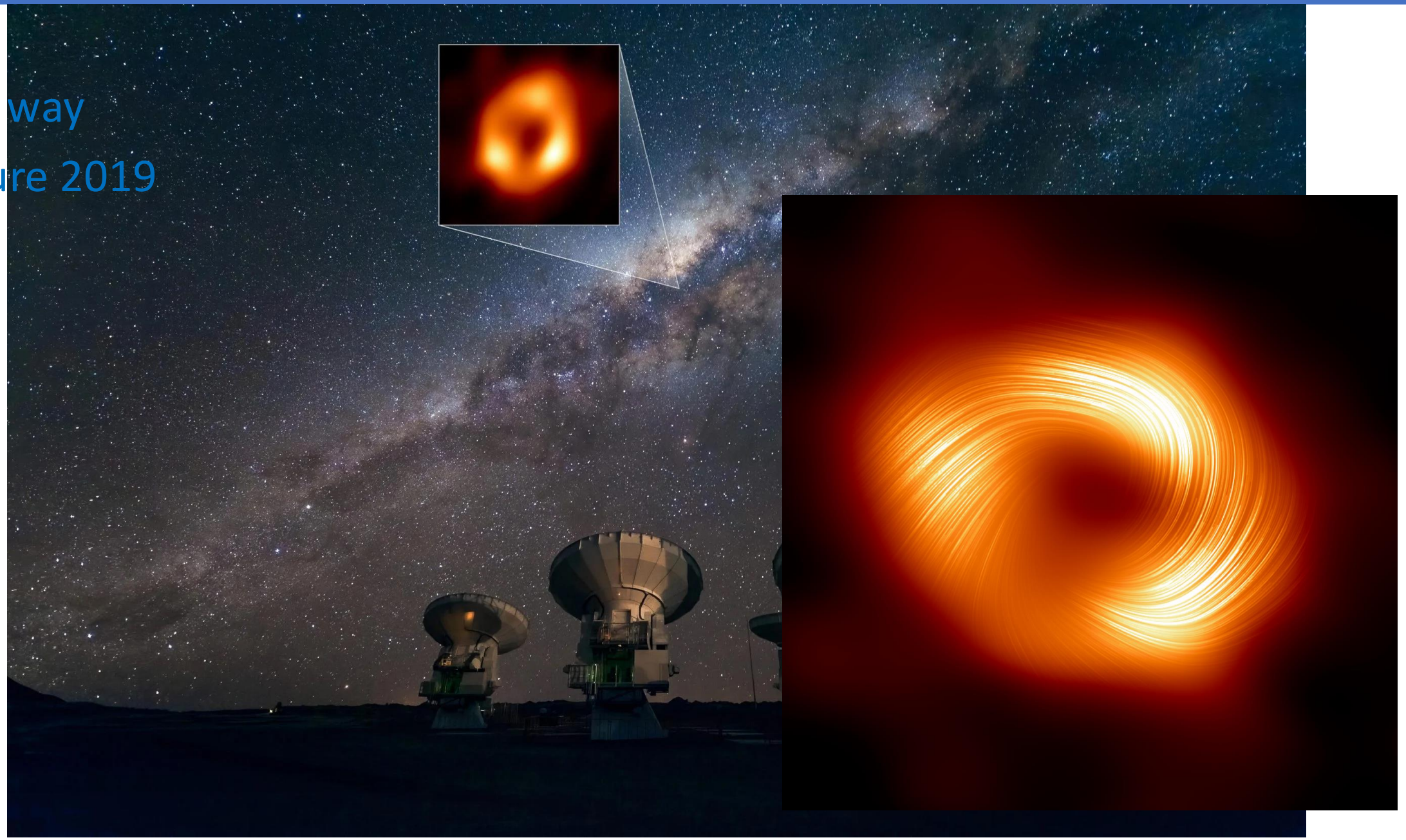
Center of our Milky Way Galaxy

Supermassive Black Hole in the center of our Galaxy



Black Hole in the Center of the Milky Way

- Mily way
- Picture 2019



Black Hole in center galaxy seen in polarized light

- EHT
- Strong Magnetic fields
- Here in polarized light

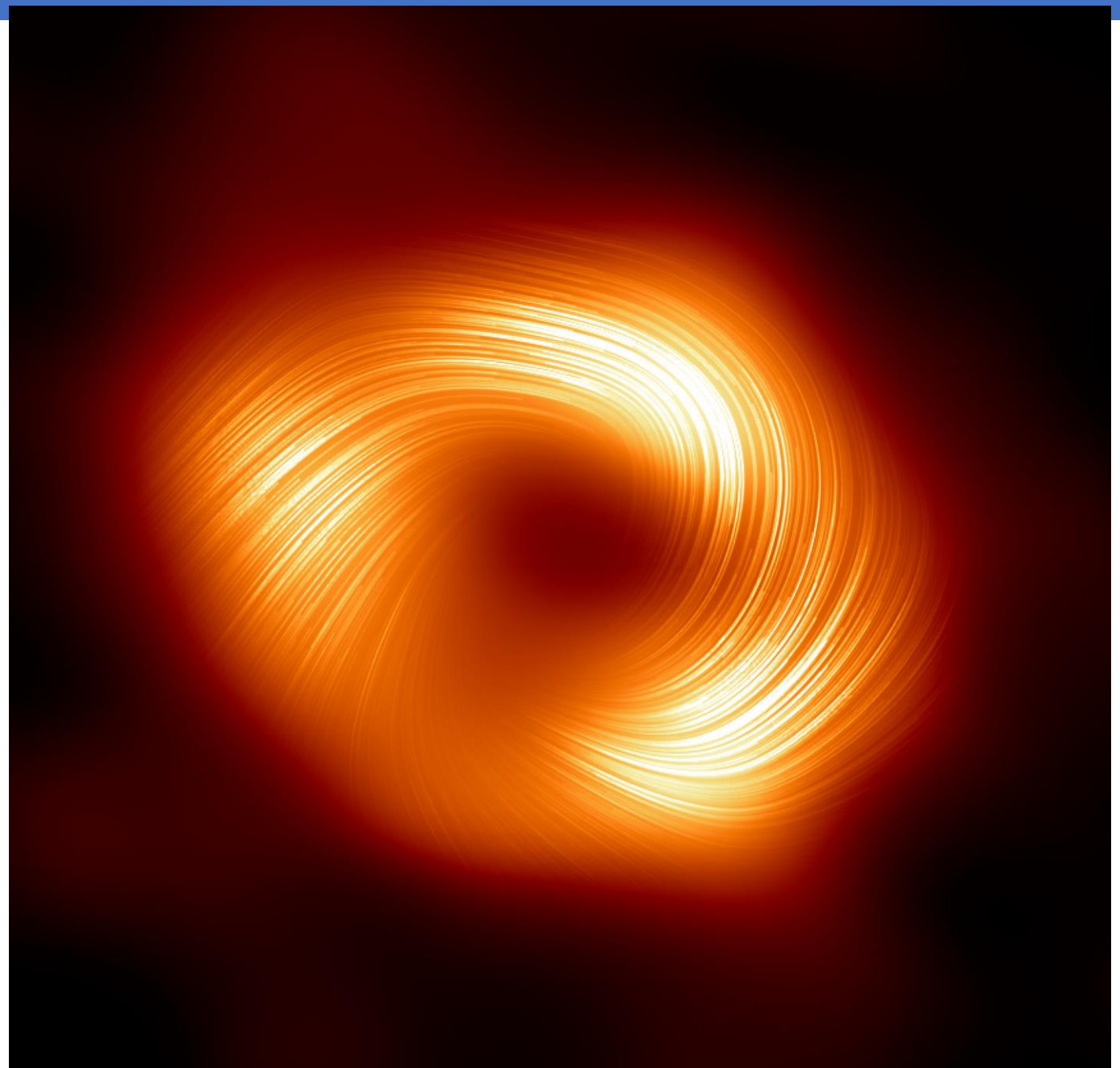
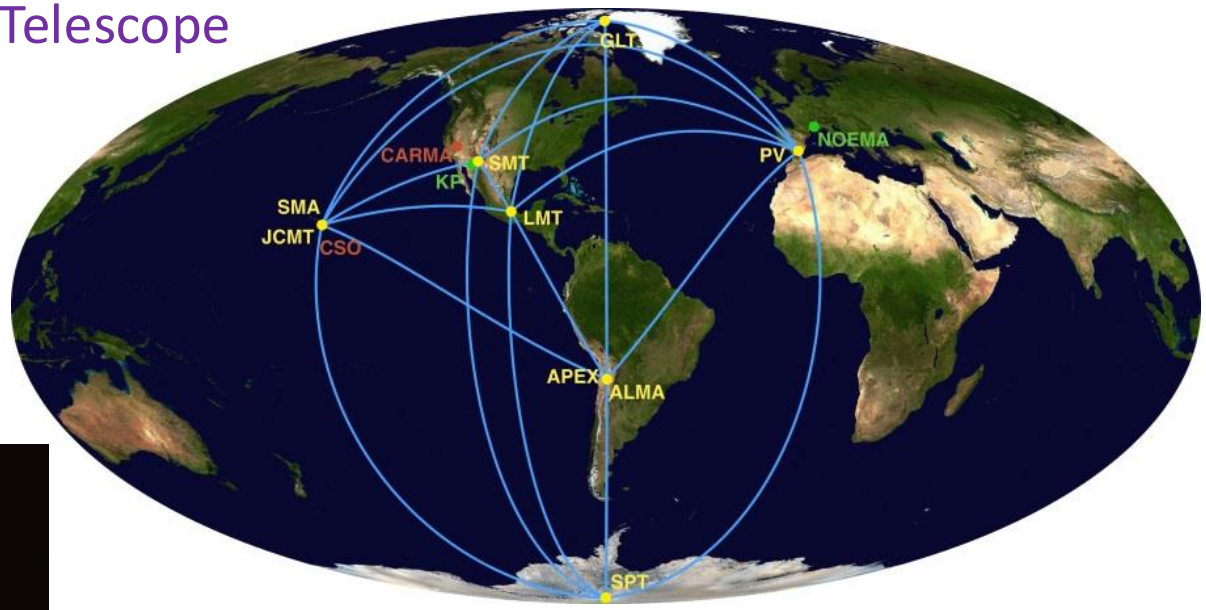


Image of the first (supermassive) black hole – 10 April 2019

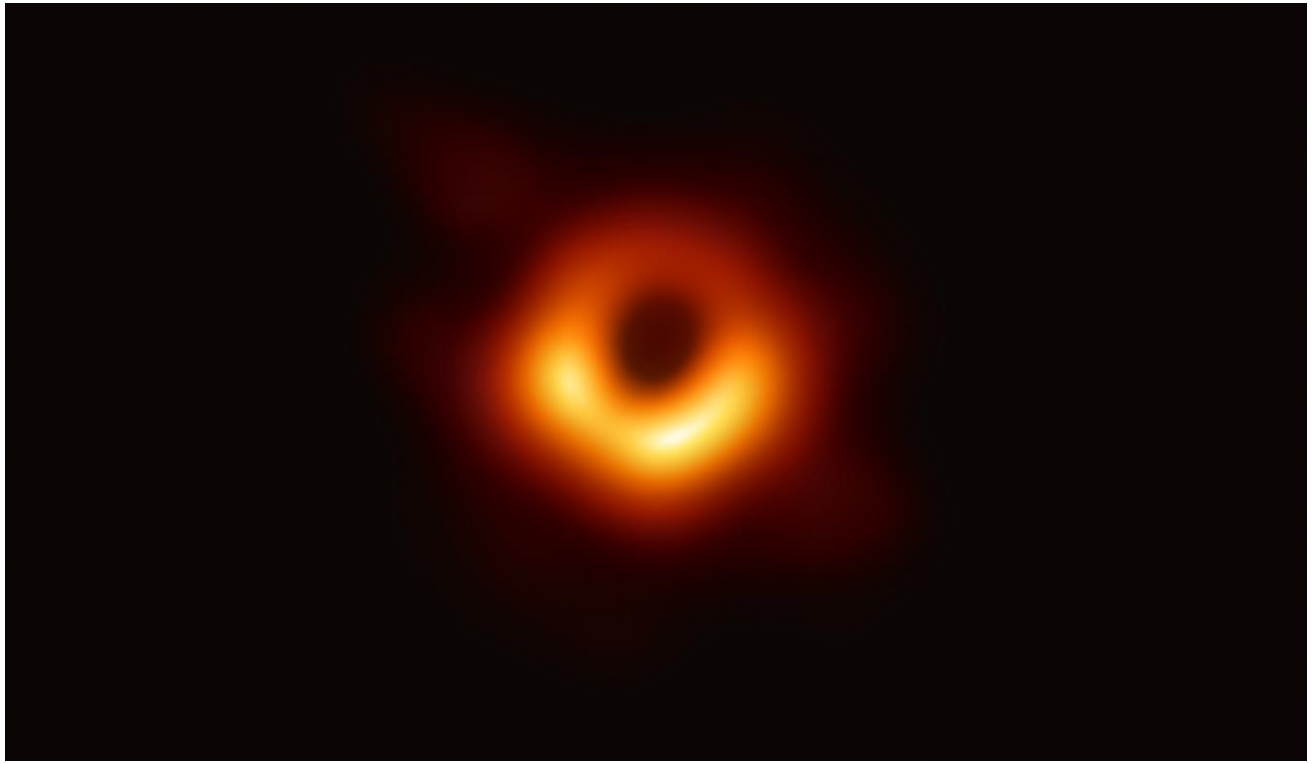
Event Horizon Telescope



Supermassive black hole in the center M87 galaxy

Distance: 55 million light years

Mass: 6.5 billion x the sun



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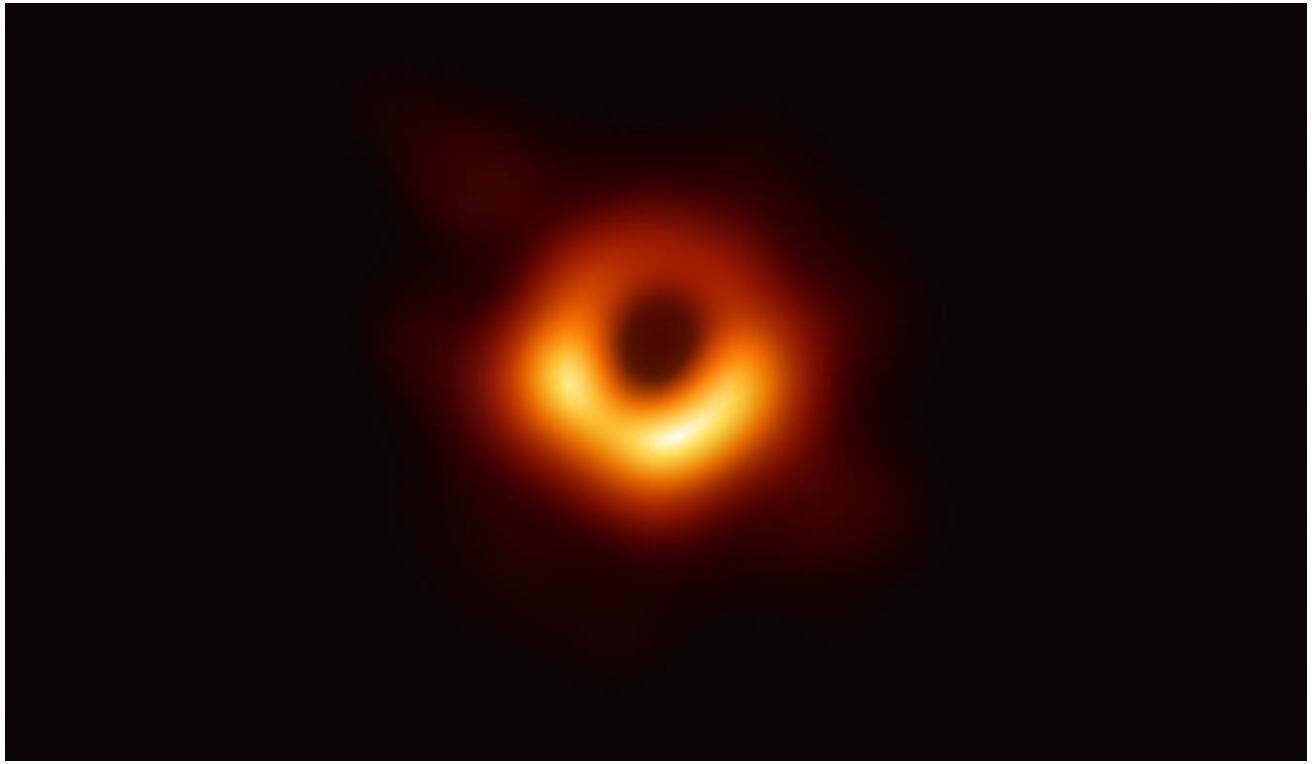
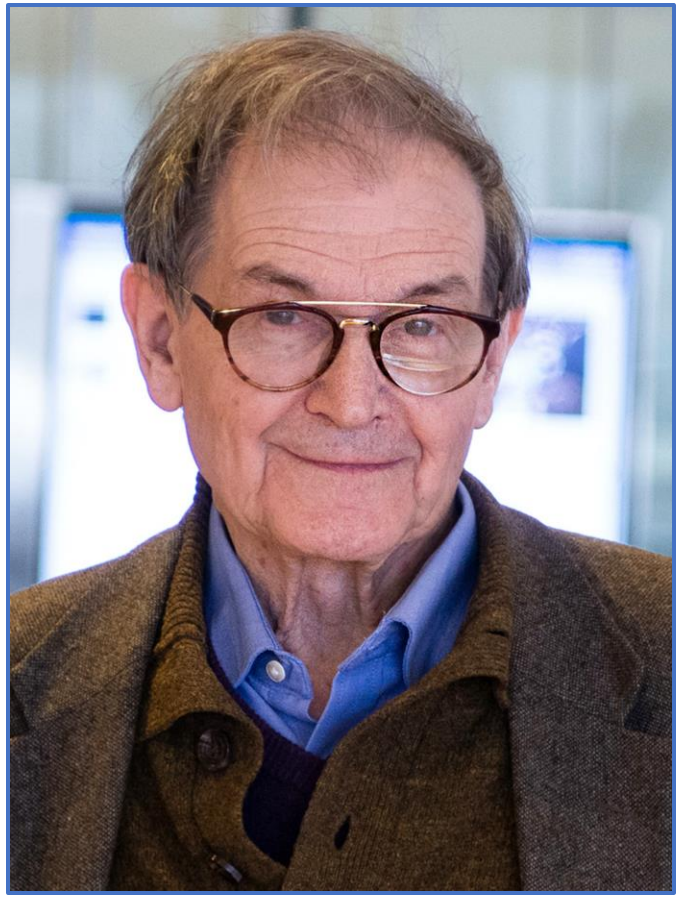


Nobel Prize in Physics (Oct 6, 2020) – Black Holes

½ : Roger Penrose



For the discovery that black hole formation is a robust prediction of the general theory of relativity.

¼ : Reinhard Genzel

¼ : Andrea Ghez

For the discovery of a supermassive compact object at the centre of our galaxy.


Virgo


LIGO Livingston


LIGO Hanford

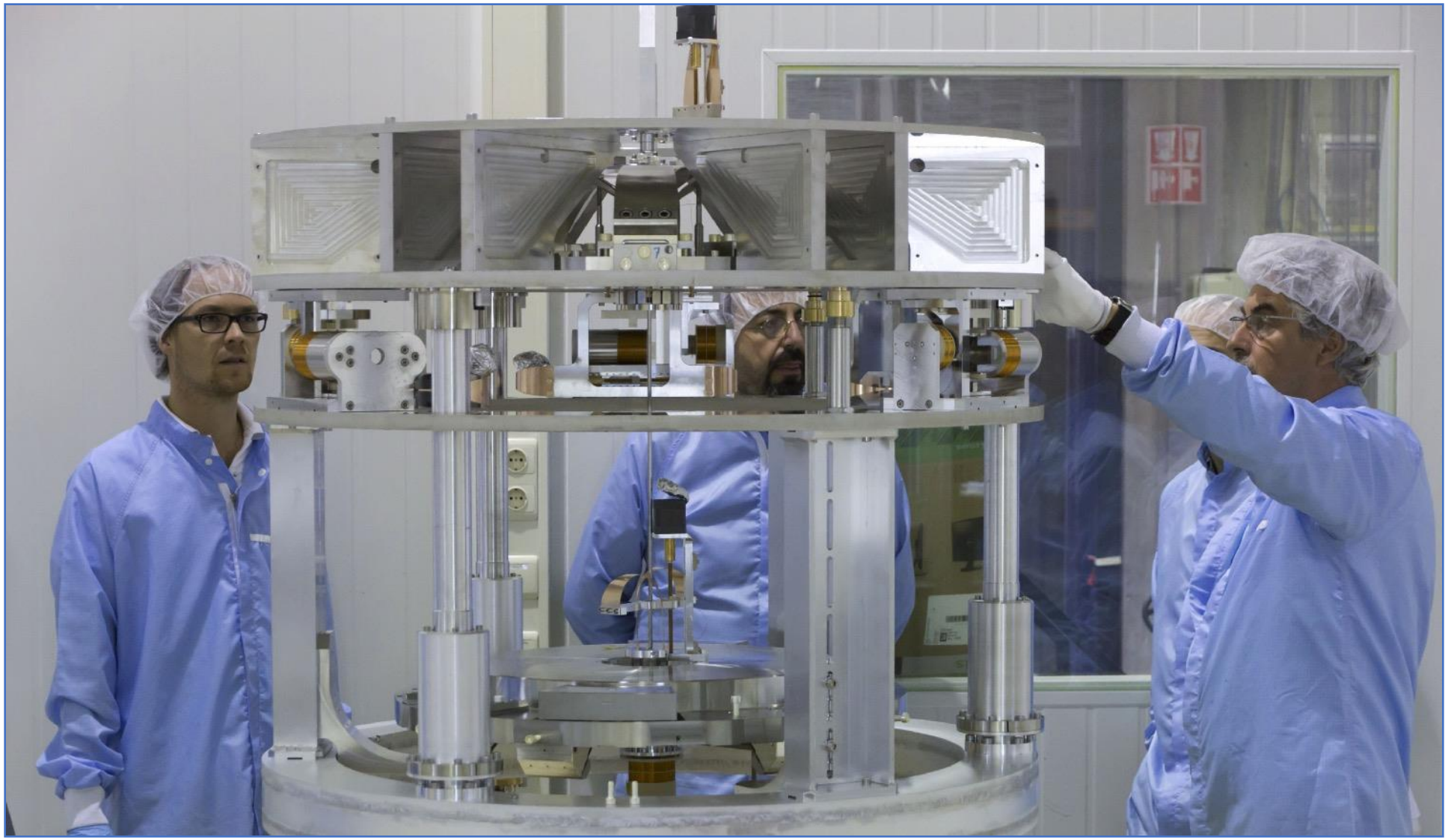

GEO 600
GEO 600

The Virgo Experiment in Pisa

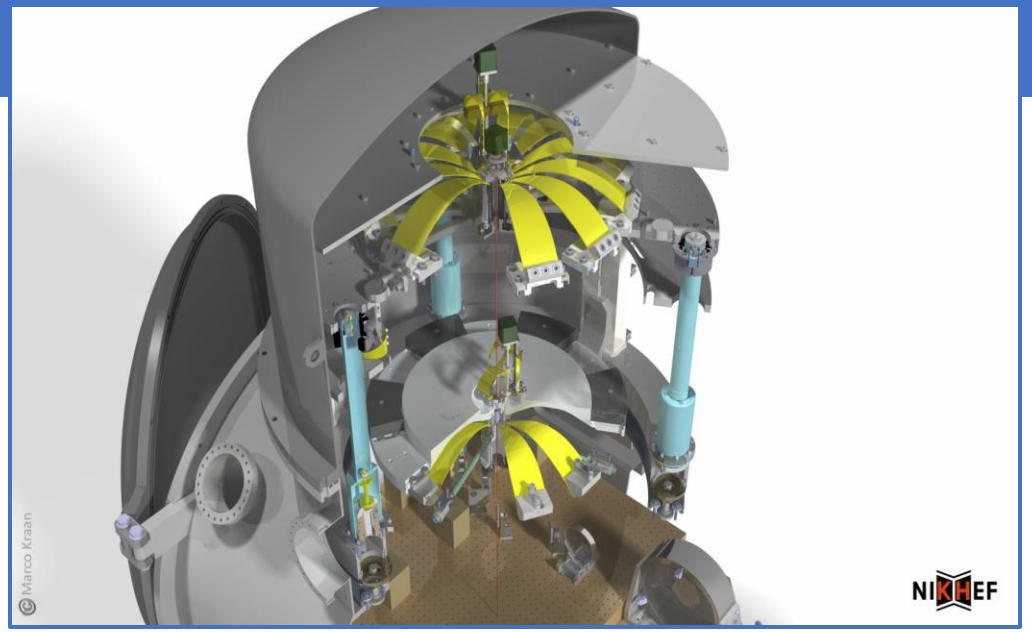
Ultrahigh Vacuüm

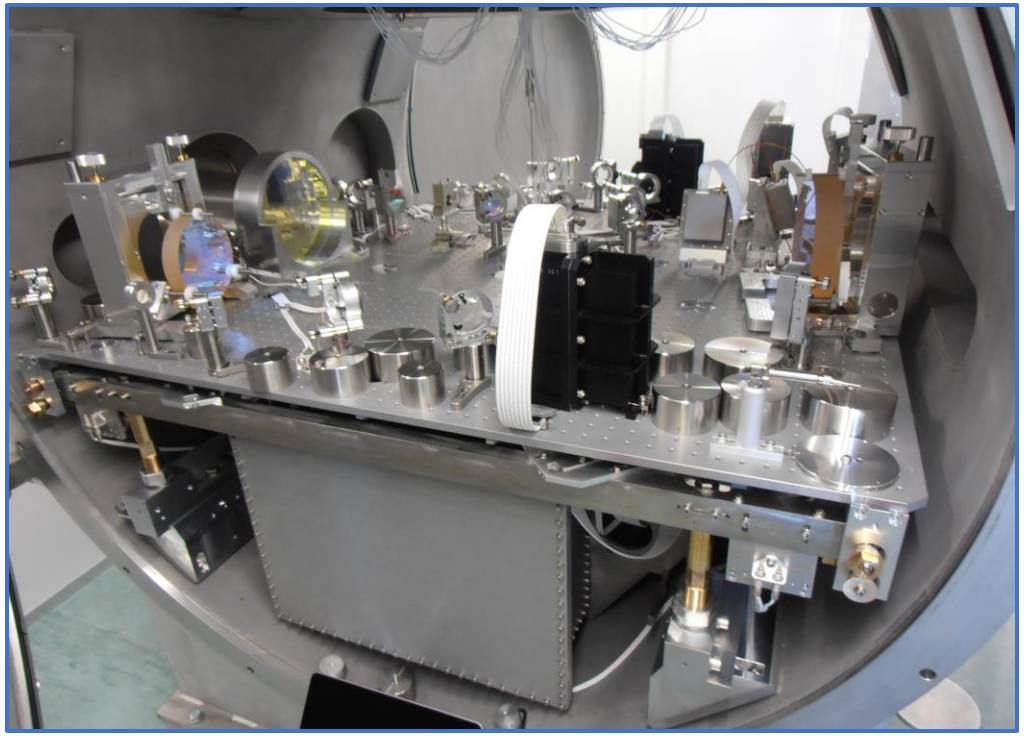
Largest vacuüm vessel in Europe: Pressure ~ 10^{-10} mbar

Seismic dampingsystem – made @Nikhef

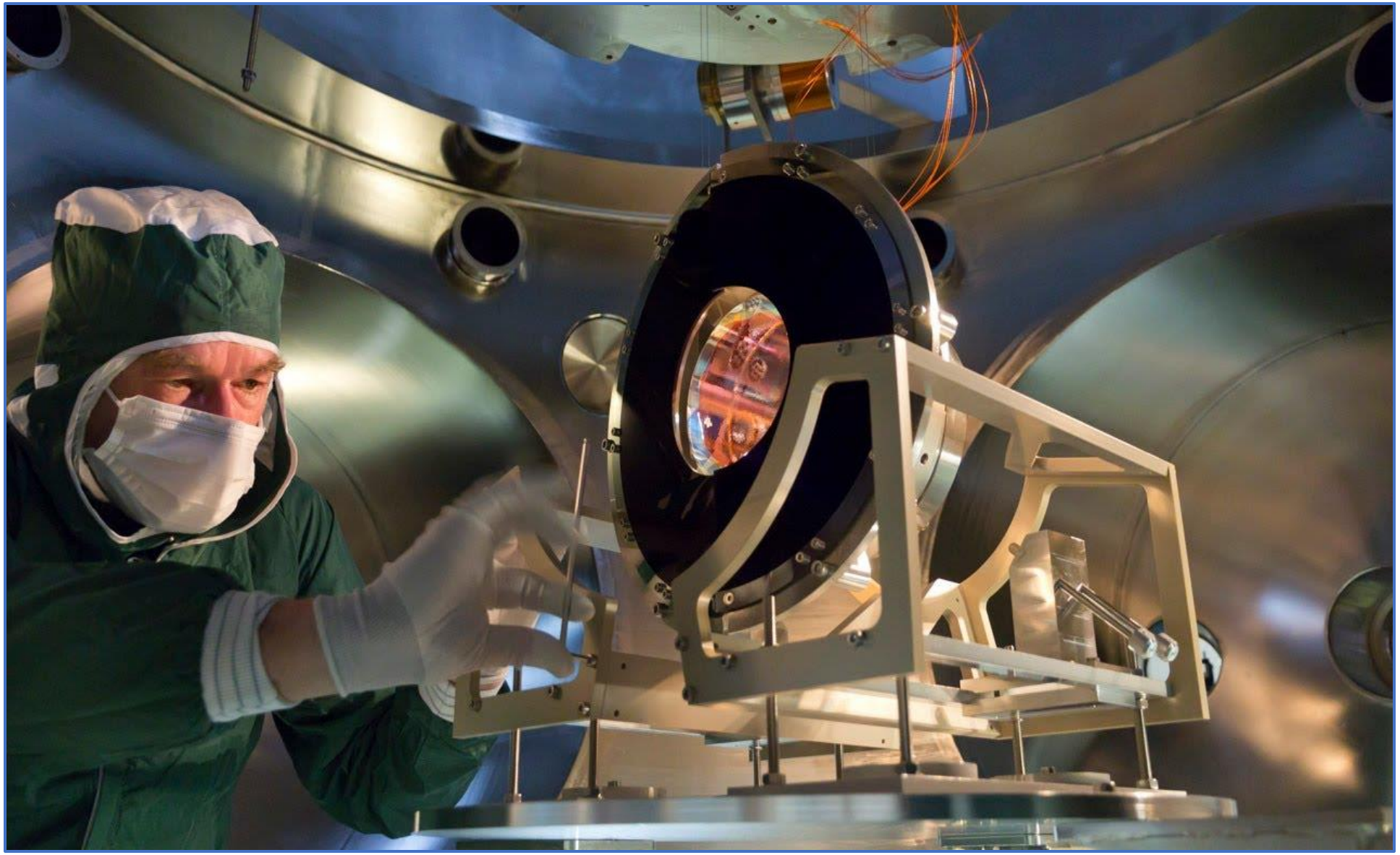


Seismic Damping Table

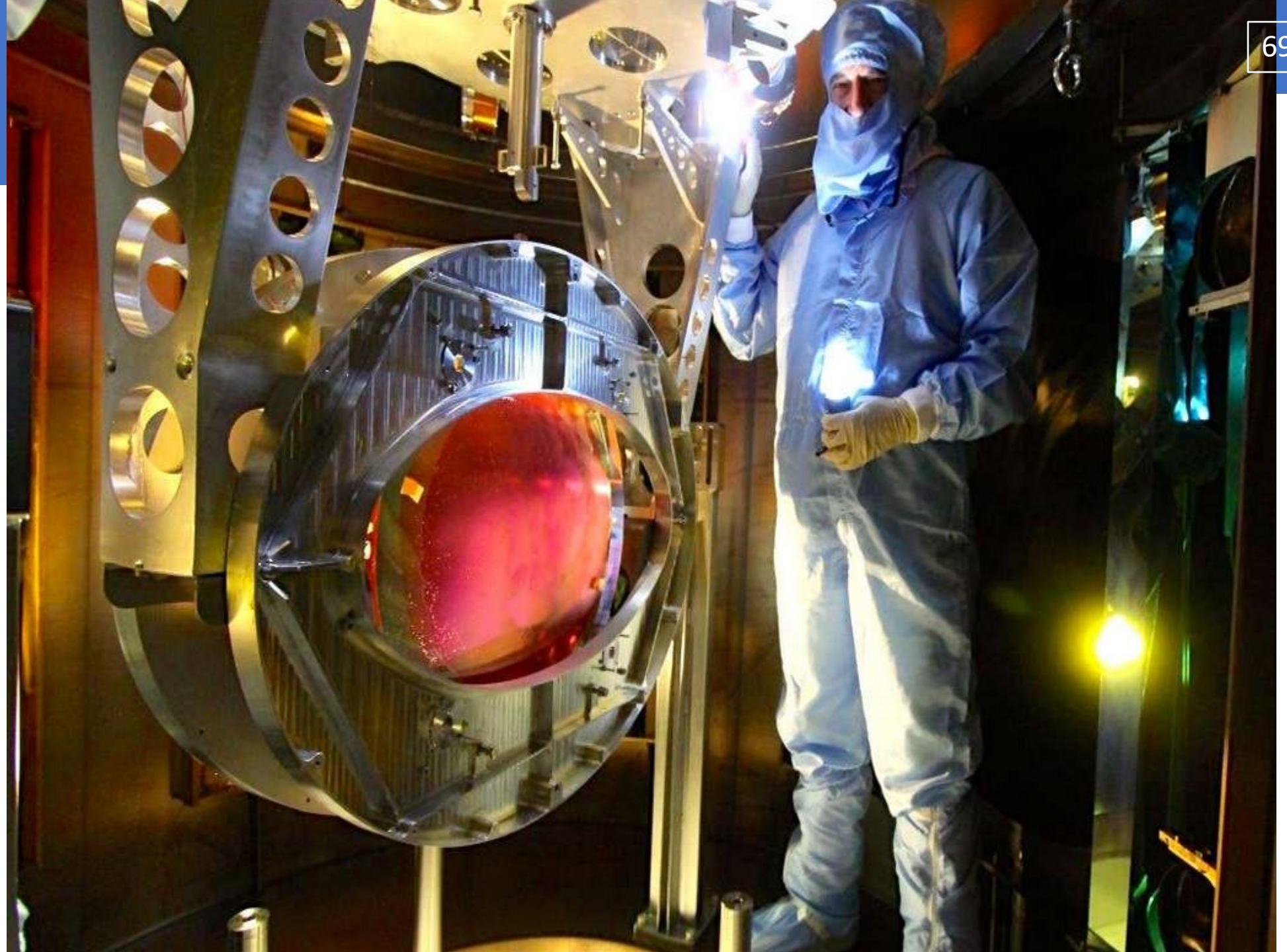


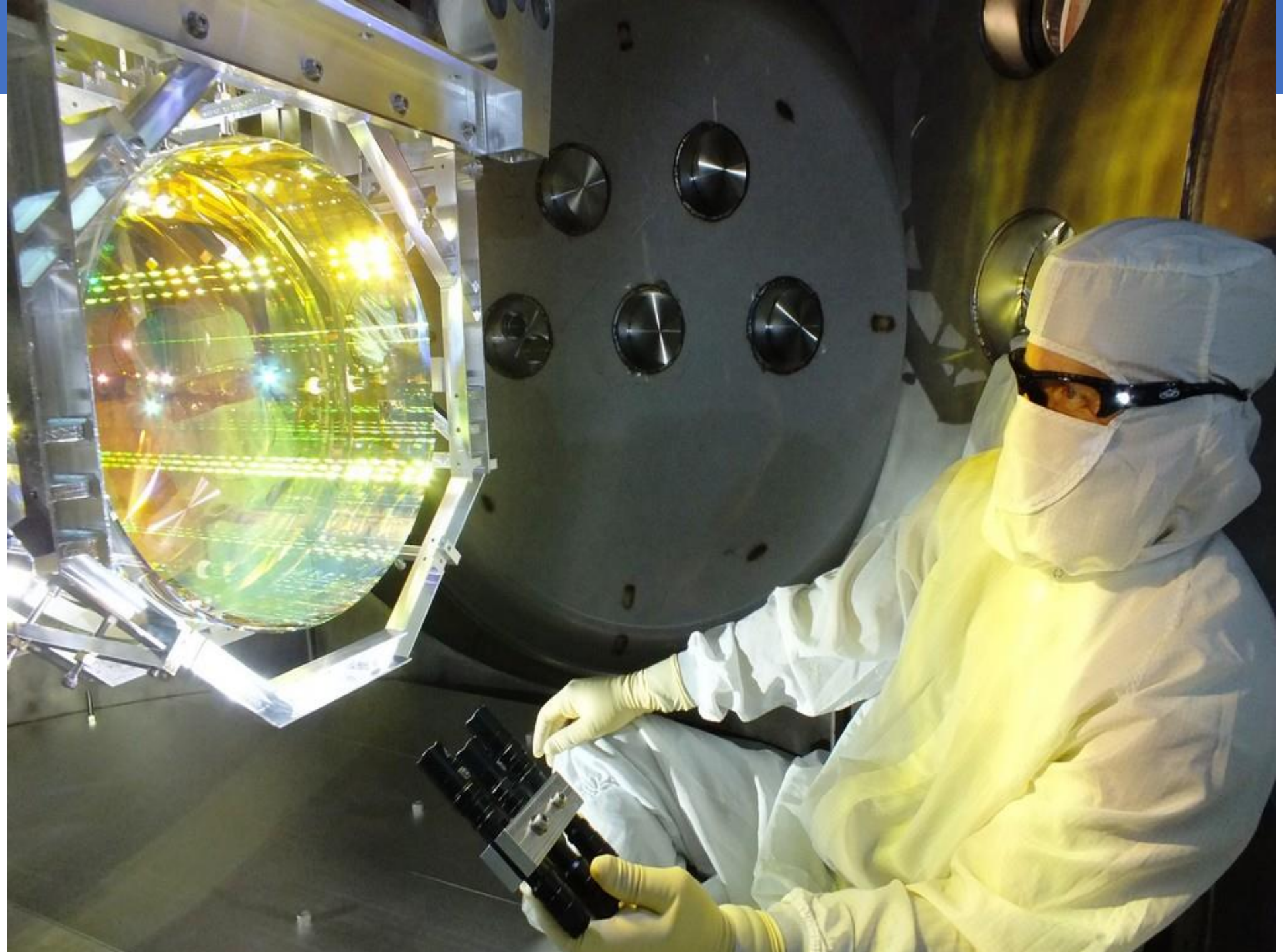


Input mode cleaner end mirror



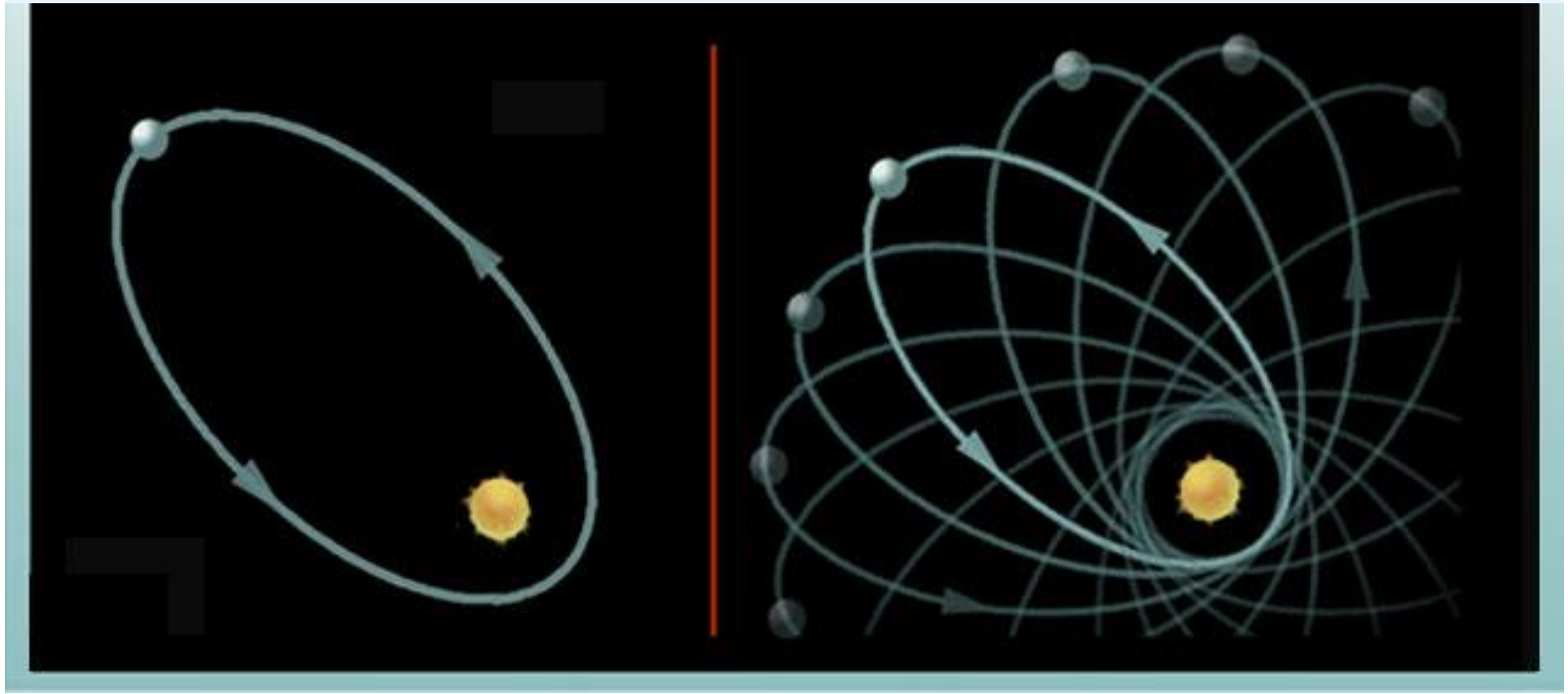
Beam Splitter



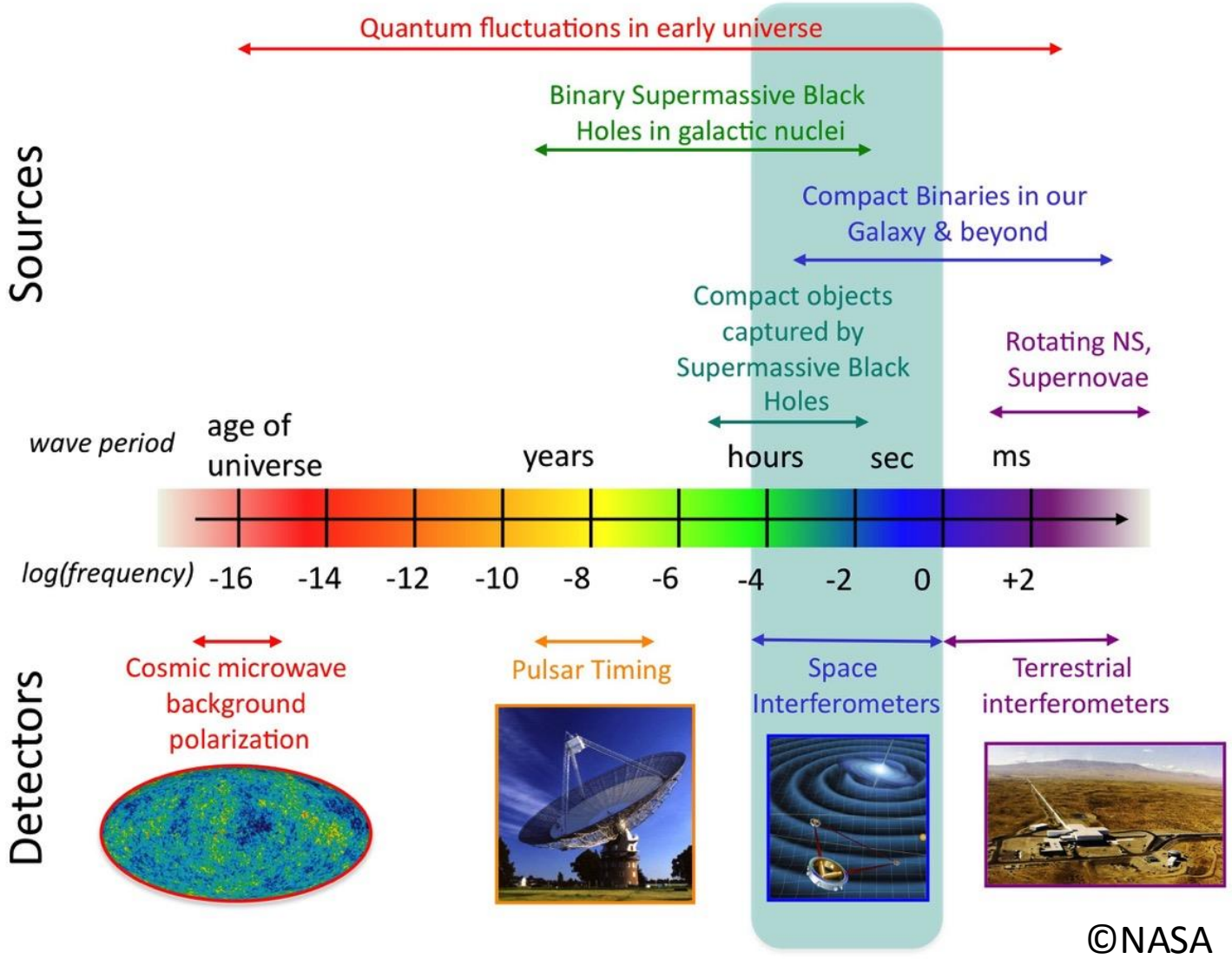


Precessie van de Mercuriusbaan

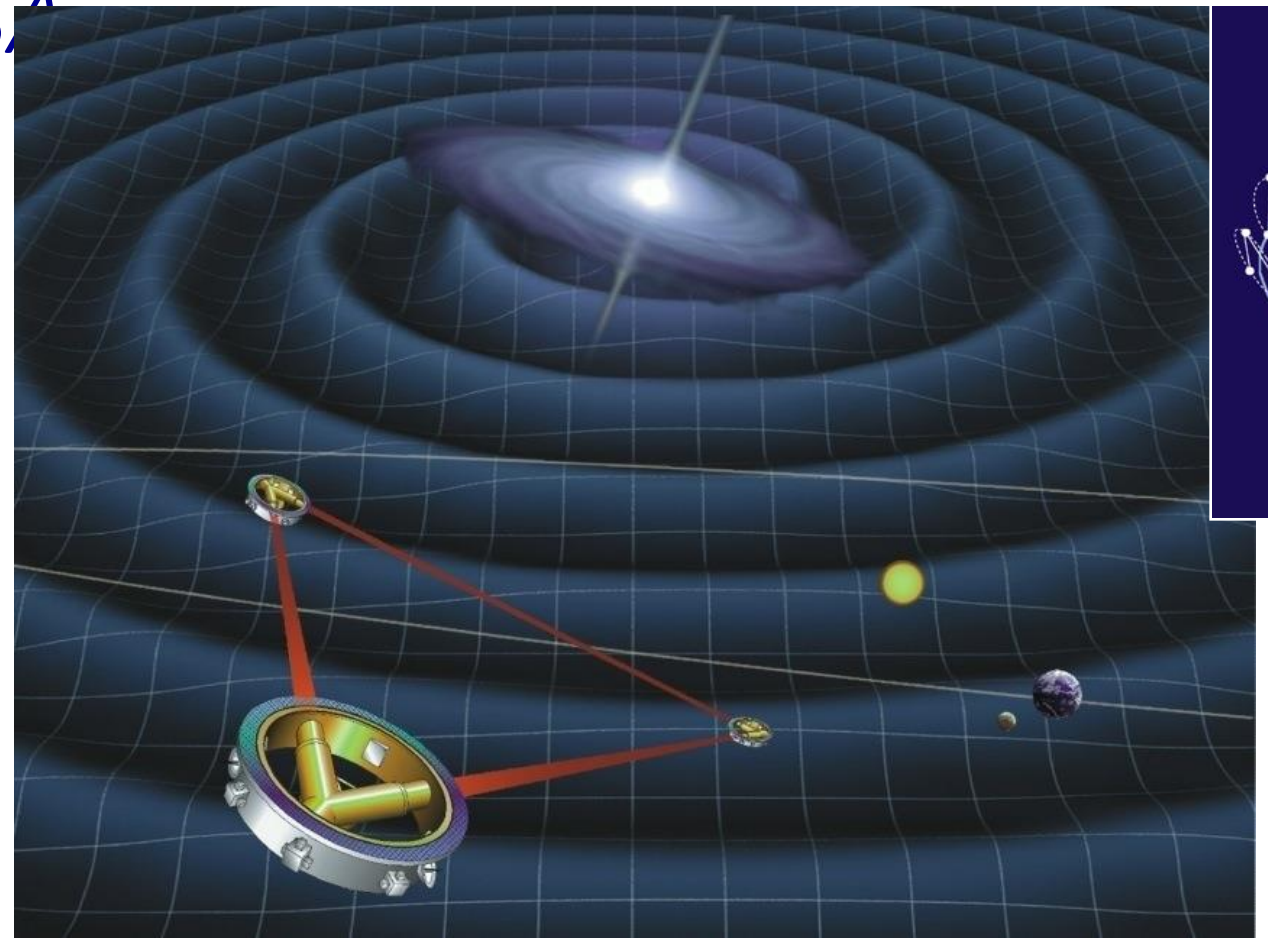
Einstein voorspelde wat Newton niet kon

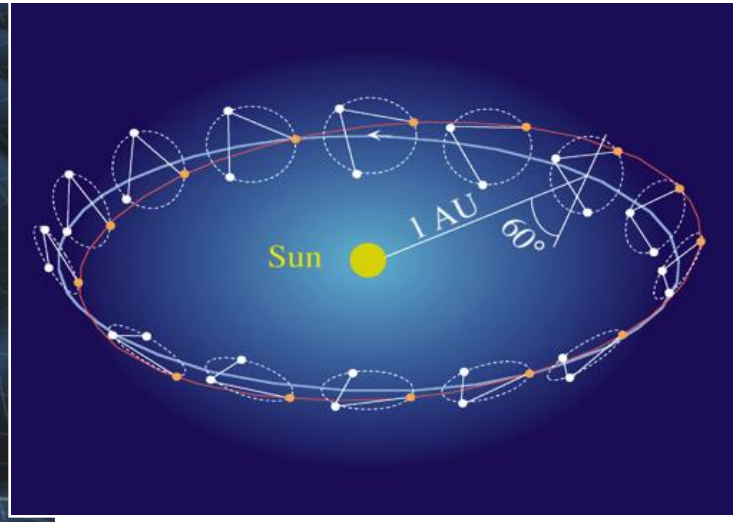


The Gravitational Wave Spectrum



LISA





Armen van 10^9 m