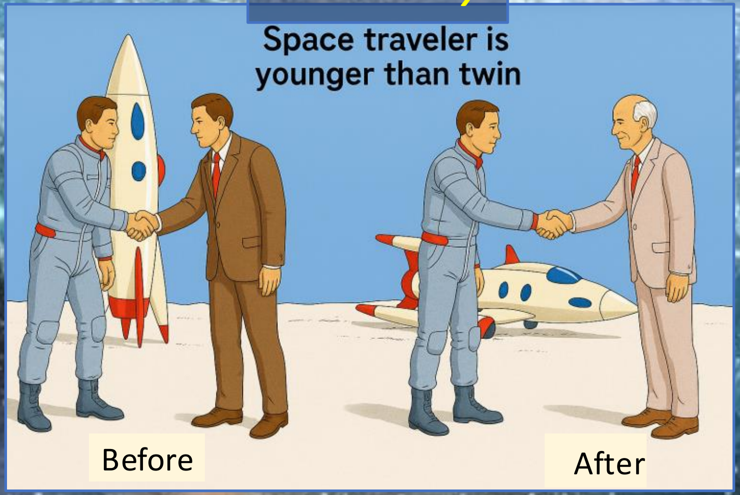
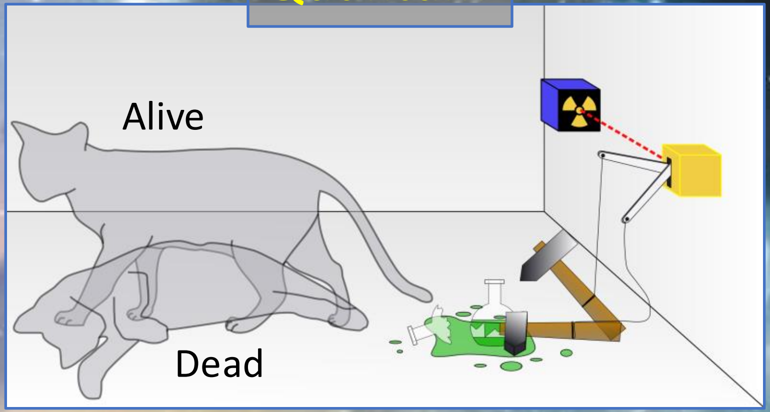
The Relativistic Quantum World

A lecture series on
Relativity Theory and Quantum Mechanics

Relativity



Quantum



Marcel Merk
Studium Generale Maastricht
Sep 10 – Oct 8, 2025

The Relativistic Quantum World

Relativity

Sep. 10: Lecture 1: The Principle of Relativity and the Speed of Light
Lecture 2: Time Dilation and Lorentz Contraction

Sep. 17: Lecture 3: The Lorentz Transformation and Paradoxes
Lecture 4: General Relativity and Gravitational Waves

Quantum Mechanics

Sep. 24: Lecture 5: The Early Quantum Theory
Lecture 6: Feynman's Double Slit Experiment

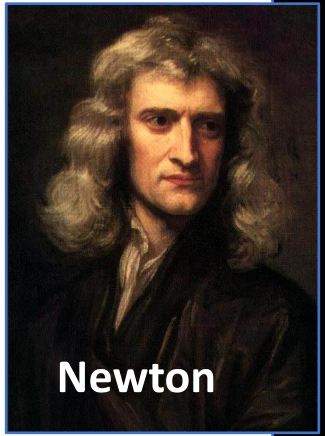
Oct. 1 : Lecture 7: Wheeler's Delayed Choice and Schrodinger's Cat
Lecture 8: Quantum Reality and the EPR Paradox

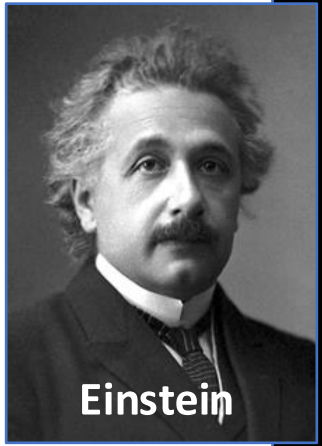
Standard Model

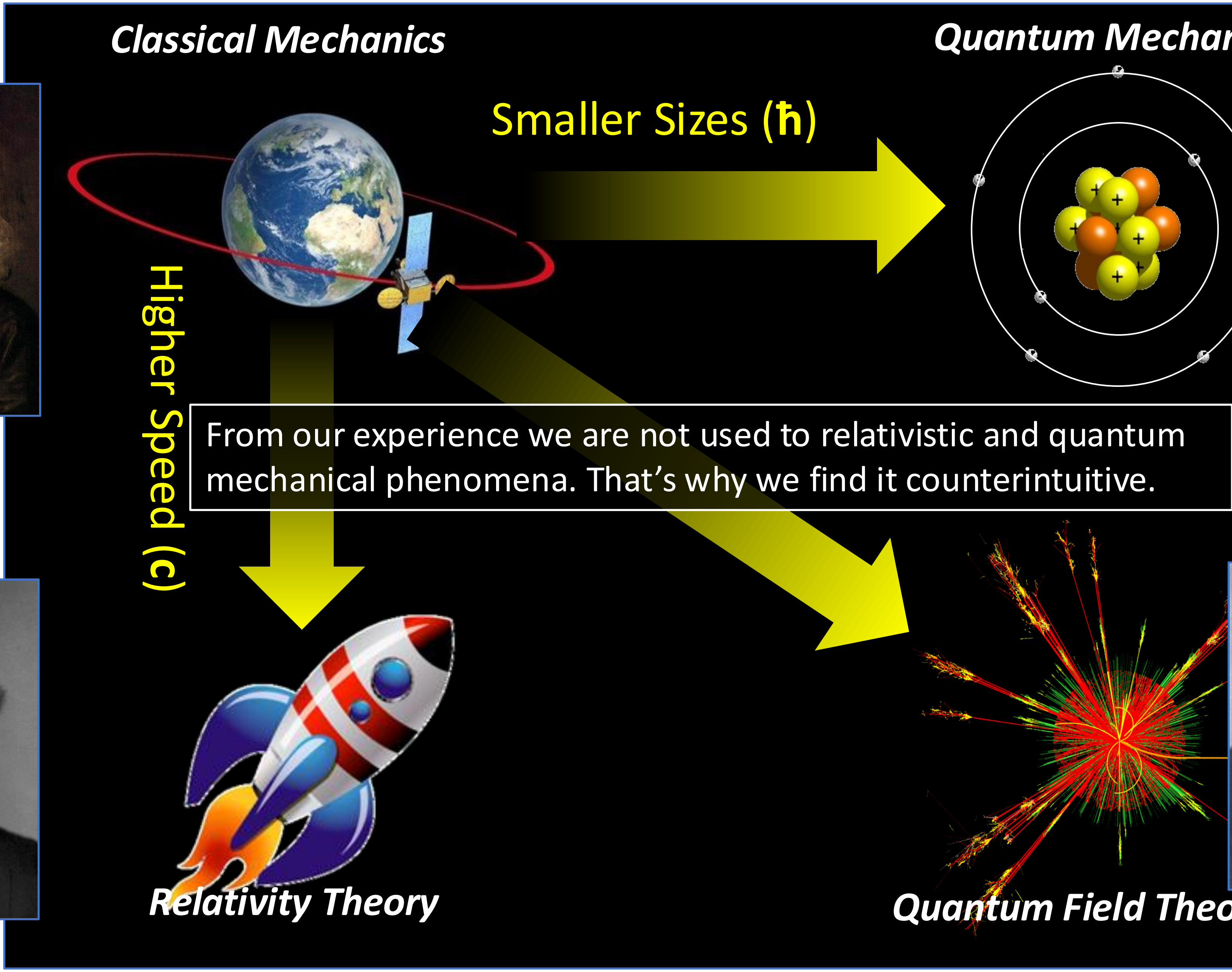
Oct. 8: Lecture 9: The Standard Model and Antimatter
Lecture 10: Why is there something rather than nothing?

Lecture notes, written for this course, are available: www.nikhef.nl/~i93/Teaching/
Prerequisite for the course: High school level physics & mathematics.

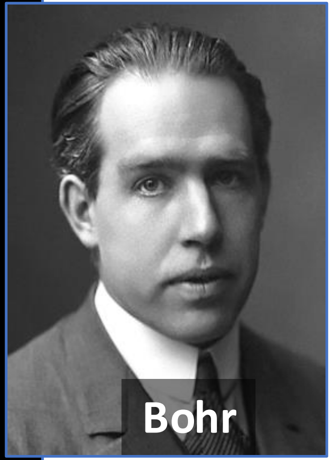
Relativity and Quantum Mechanics

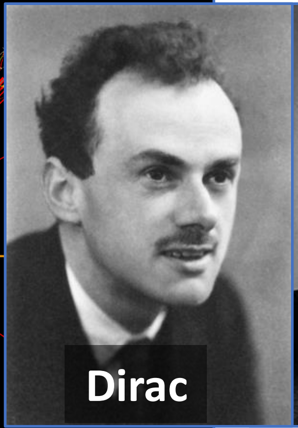


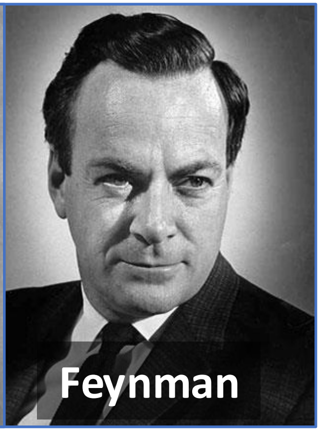




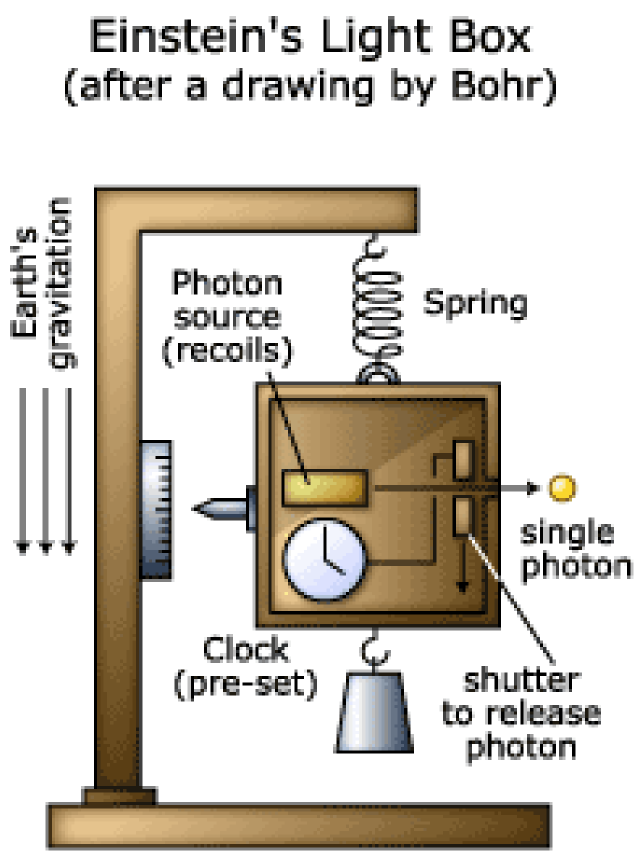
From our experience we are not used to relativistic and quantum mechanical phenomena. That’s why we find it counterintuitive.

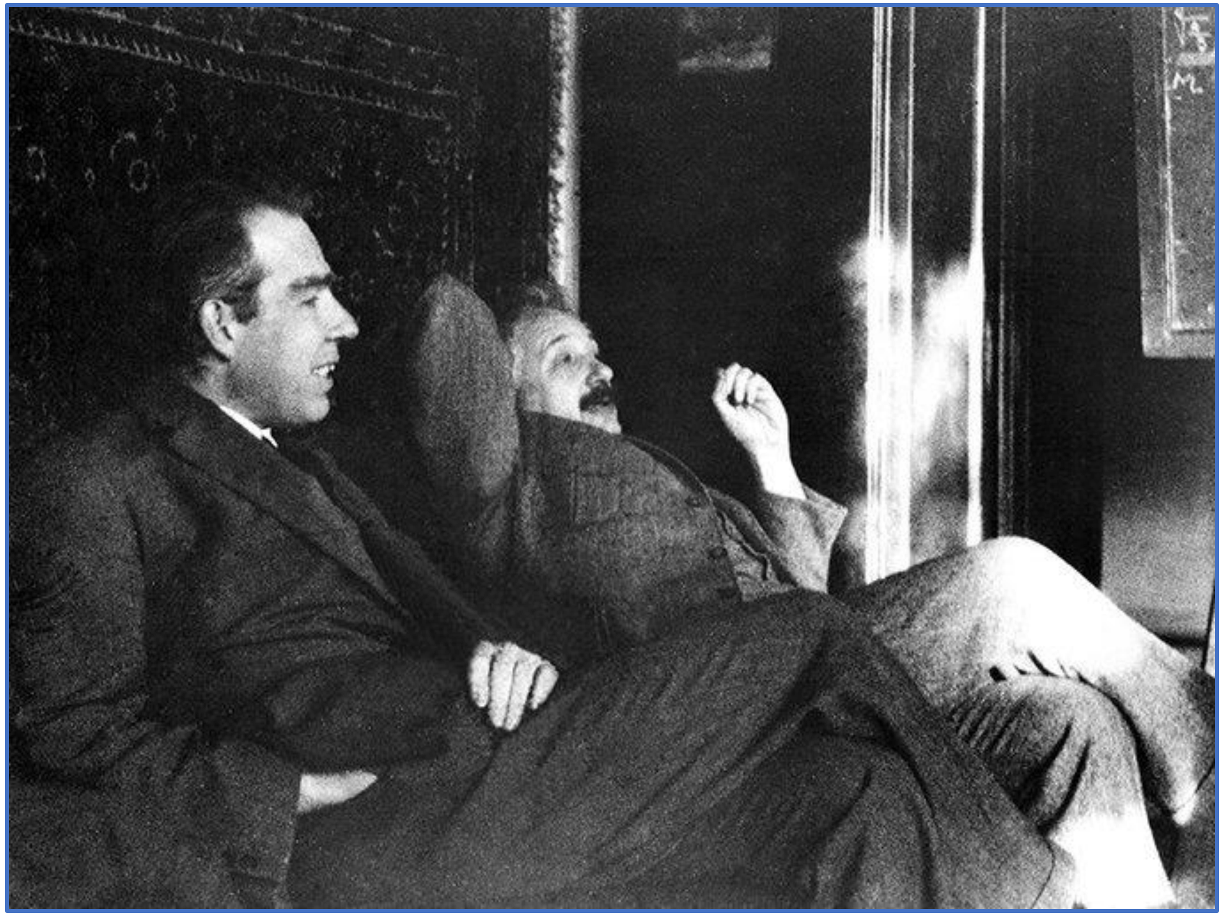






A “Gedanken” Experiment





Bohr and Einstein at Ehrenfest’s home in Leiden

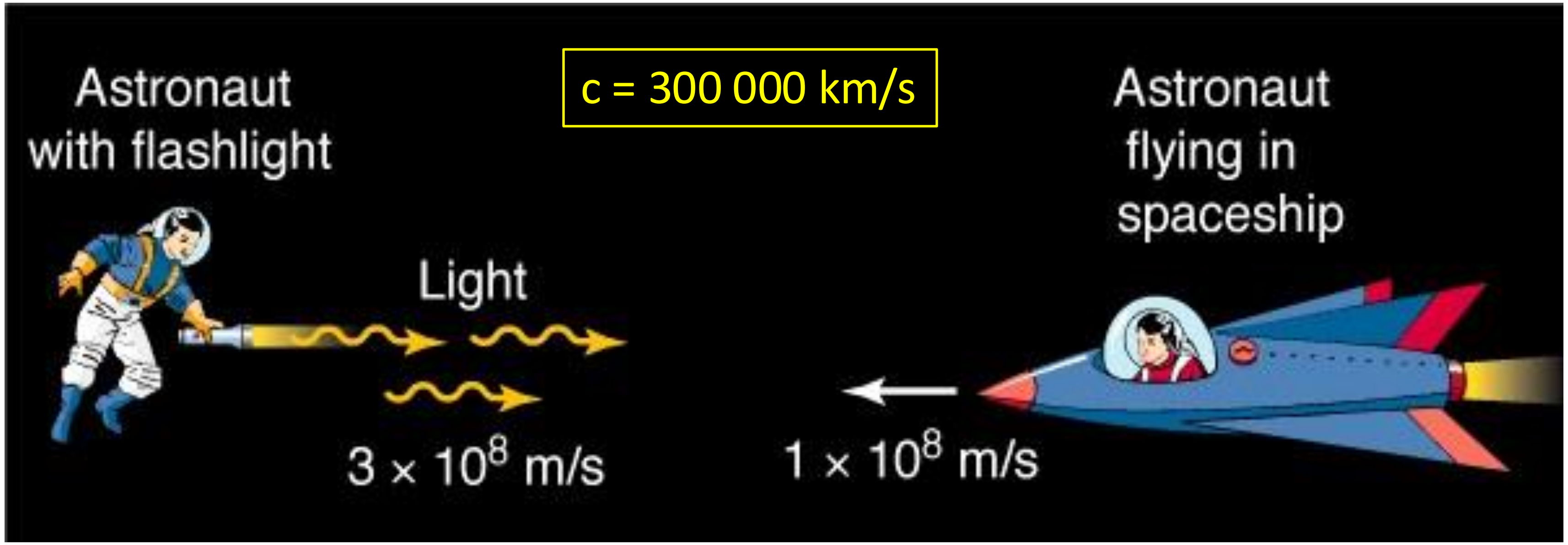
A useful tool: <u>Thought experiments</u>:
Consider an experiment that is not limited by our level of technology.
Assume the apparatus works so perfectly that we only test the limits of the laws of nature!

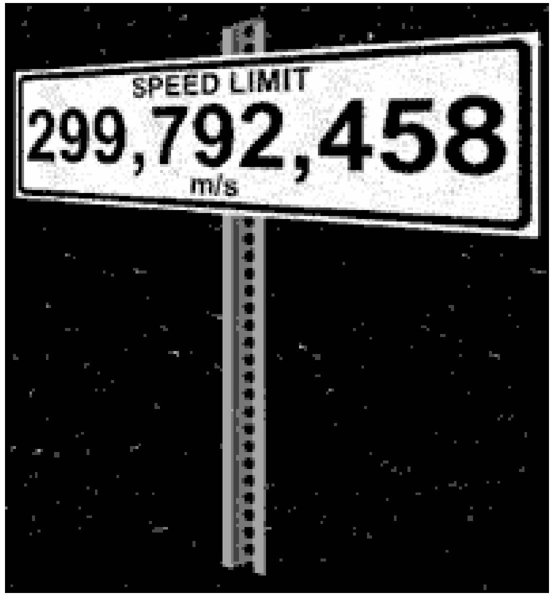
The Story Sofar

Postulates of Special Relativity

Two observers in so-called inertial frames, i.e. they move with a constant relative speed to each other, observe that:

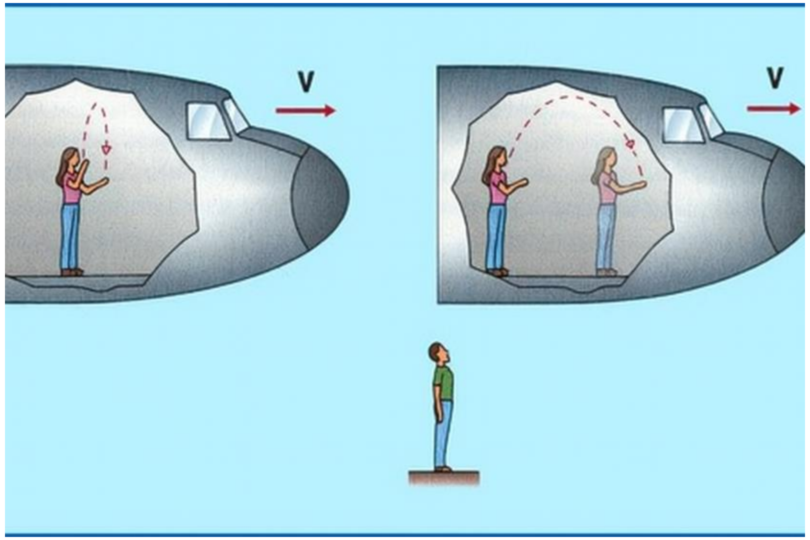
1) The laws of physics for each observer are the same,
2) The speed of light in vacuum for each observer is the same.



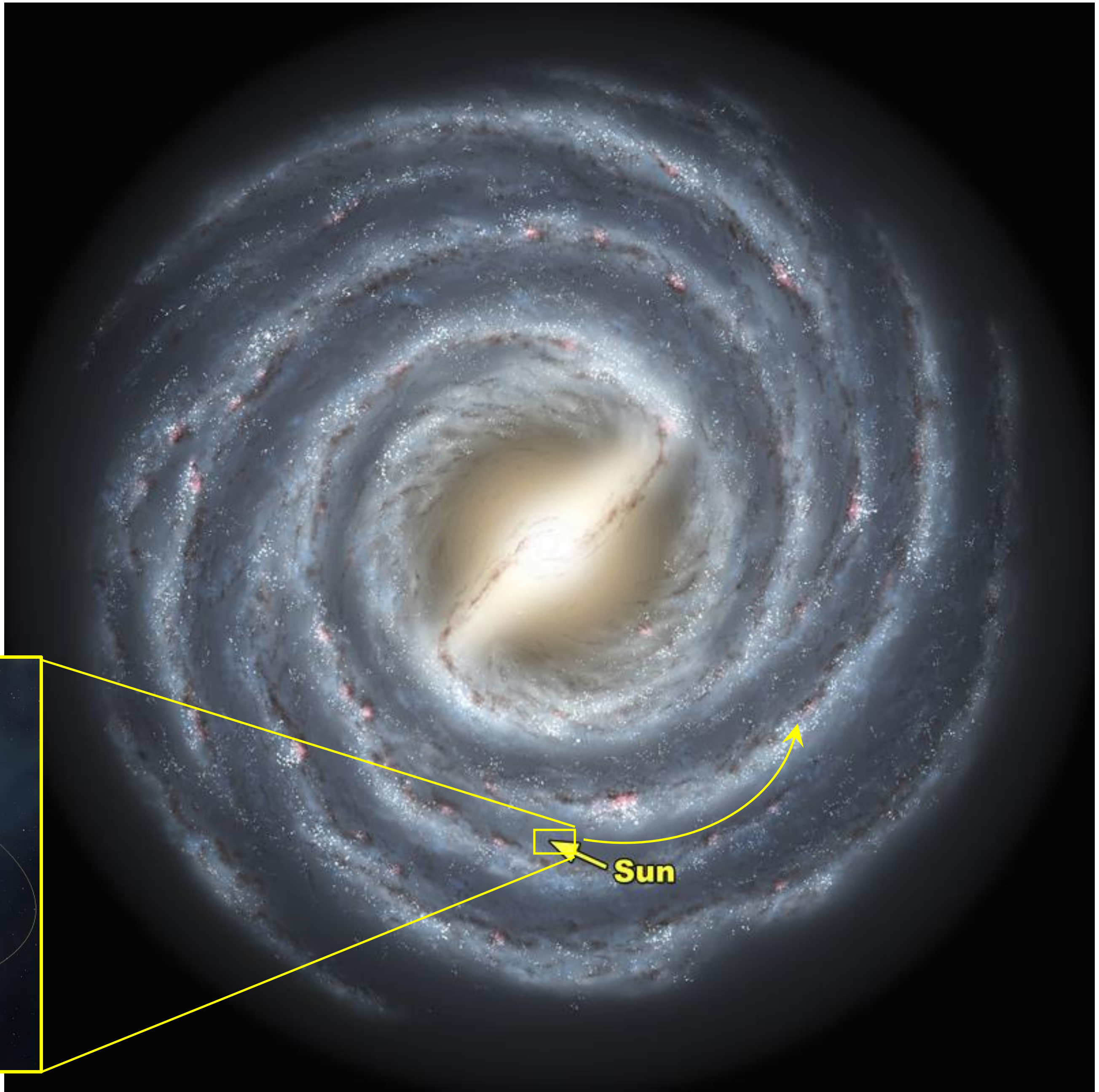


“Absolute velocity” is meaningless.

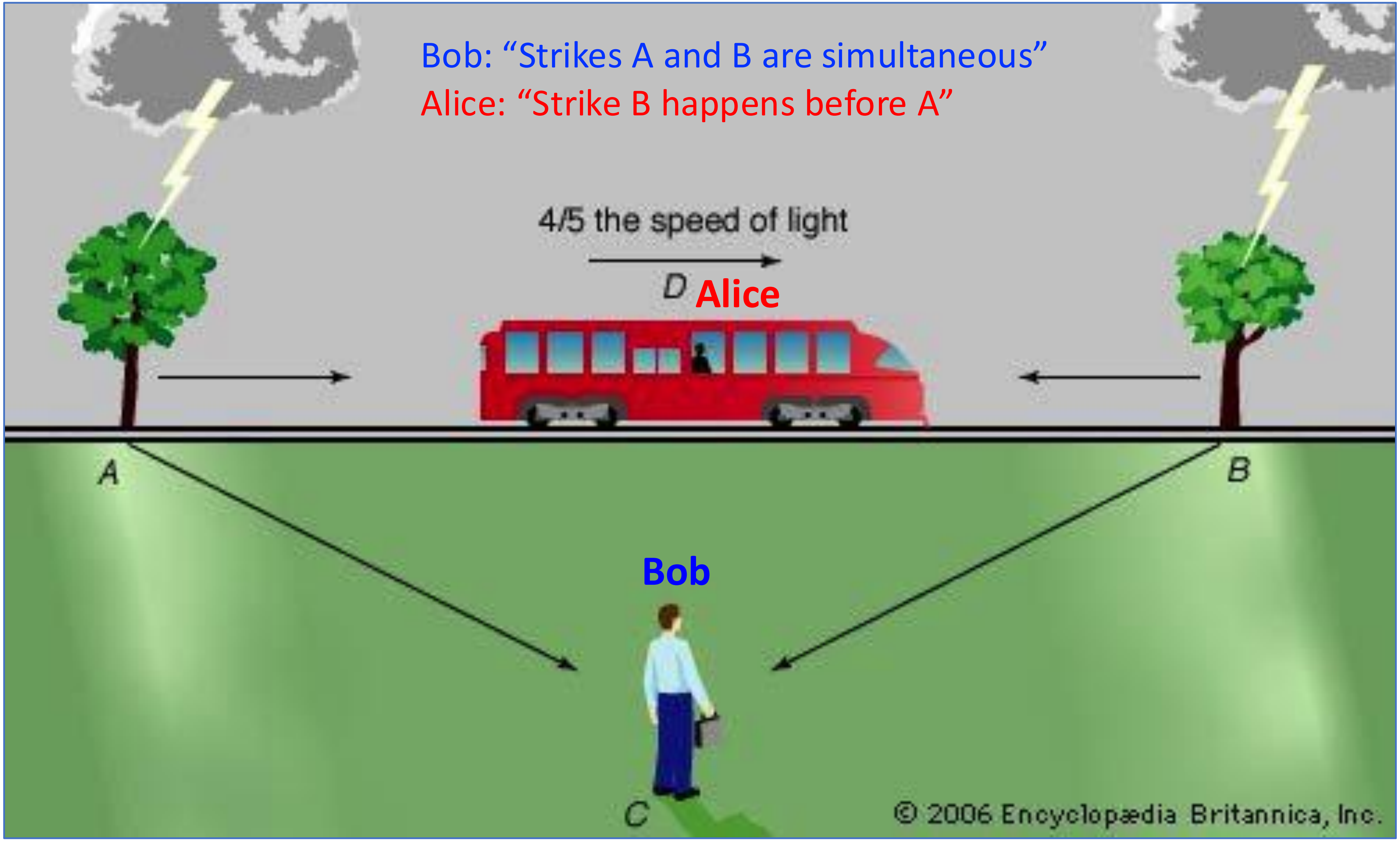
The Story Sofar: Principle of Relativity



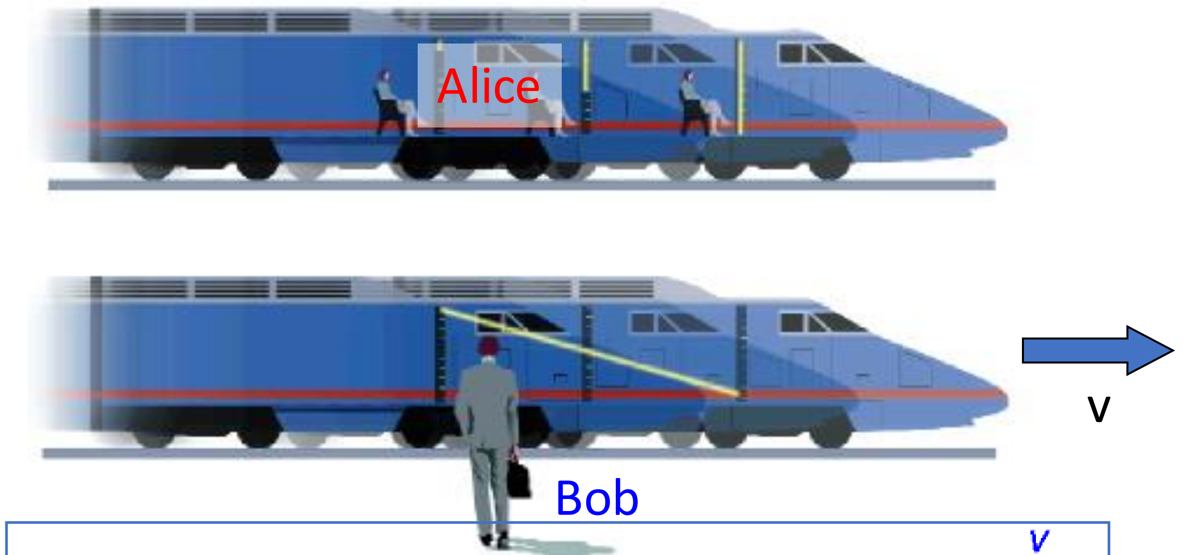
The Sun and earth move in space
with a speed of 828000 km/h.
We do not notice it!

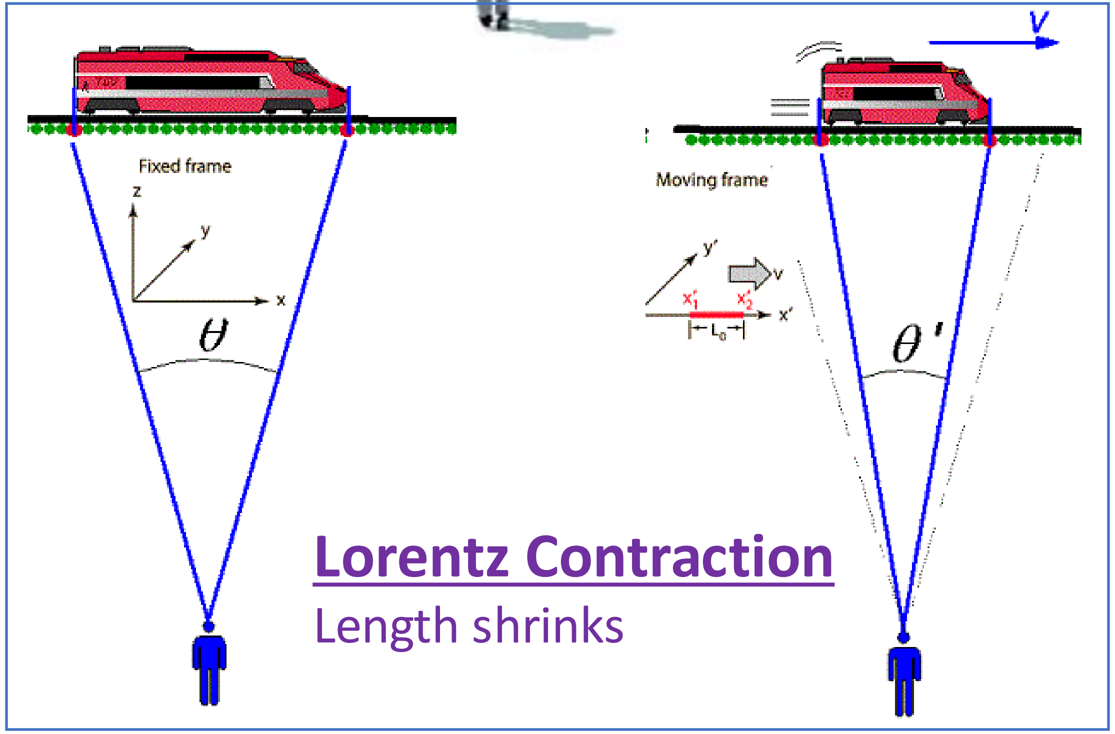


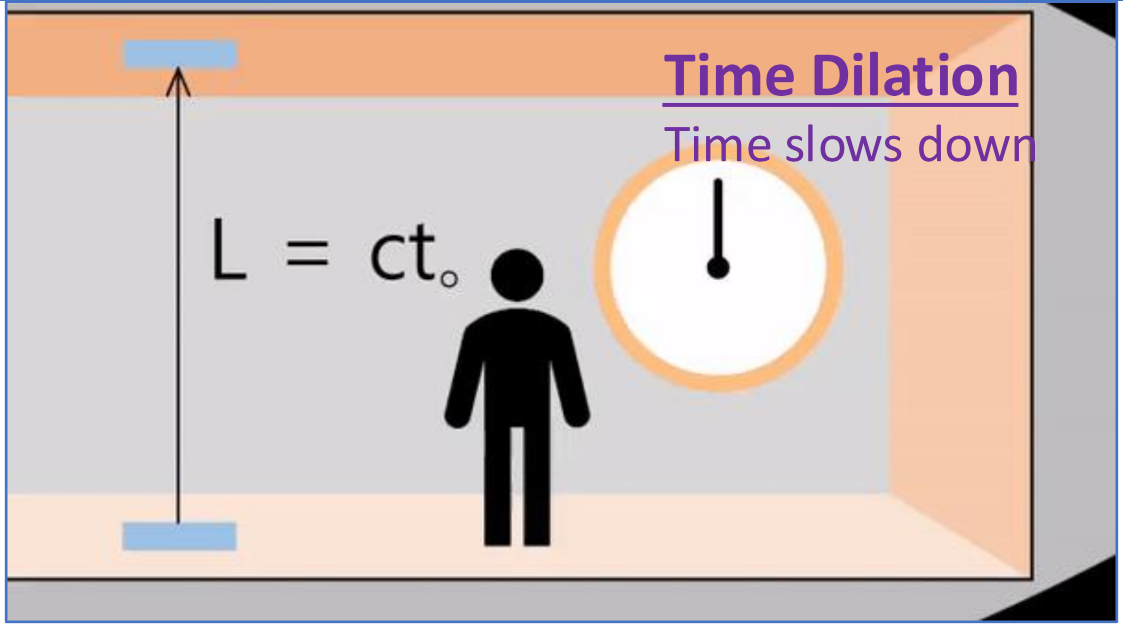
The Story Sofar: Simultaneity

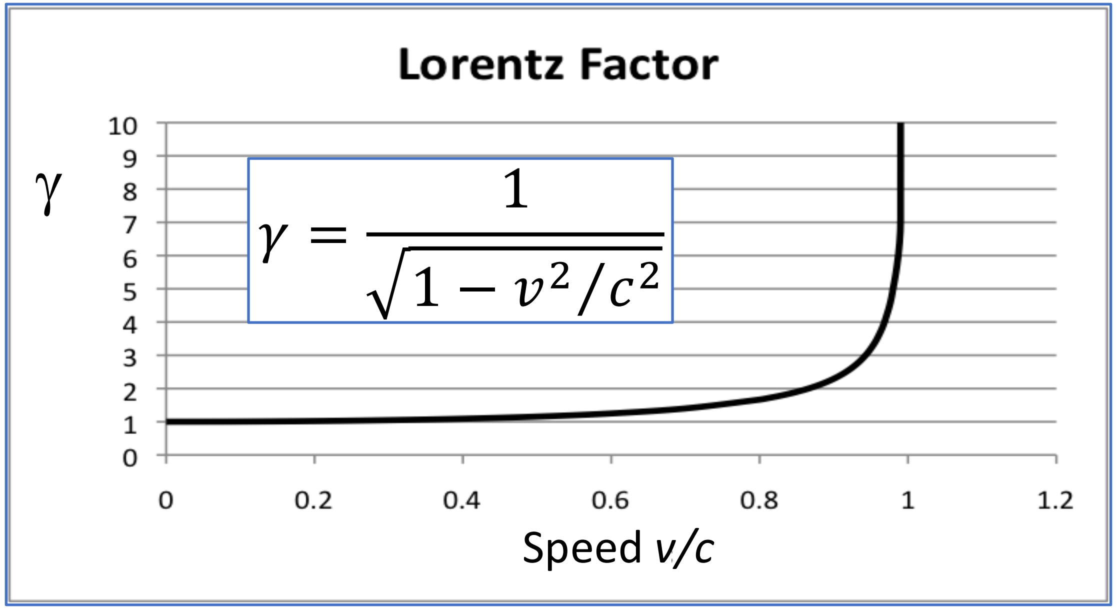


The Story Sofar: Time Dilation and Lorentz Contraction



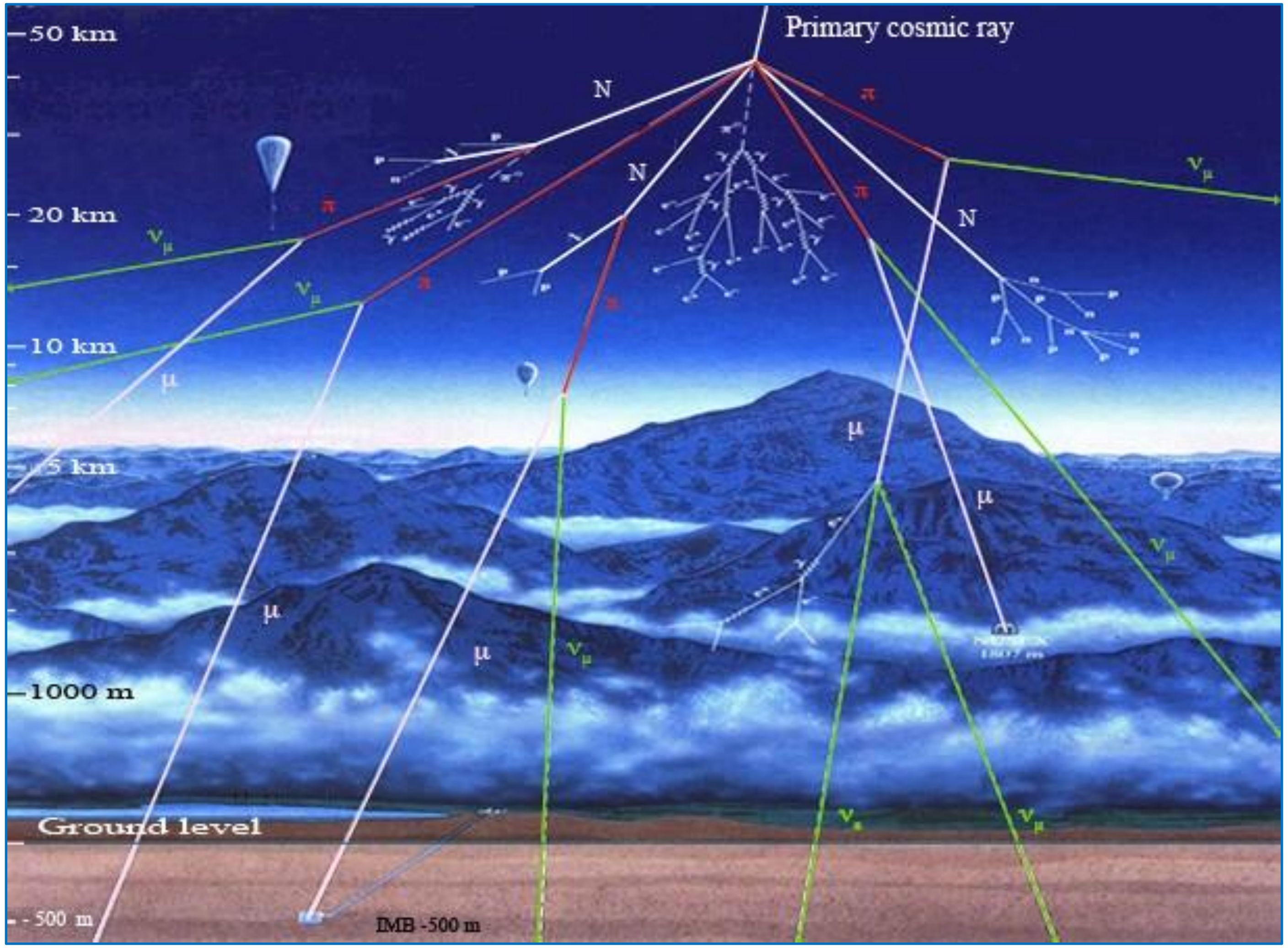


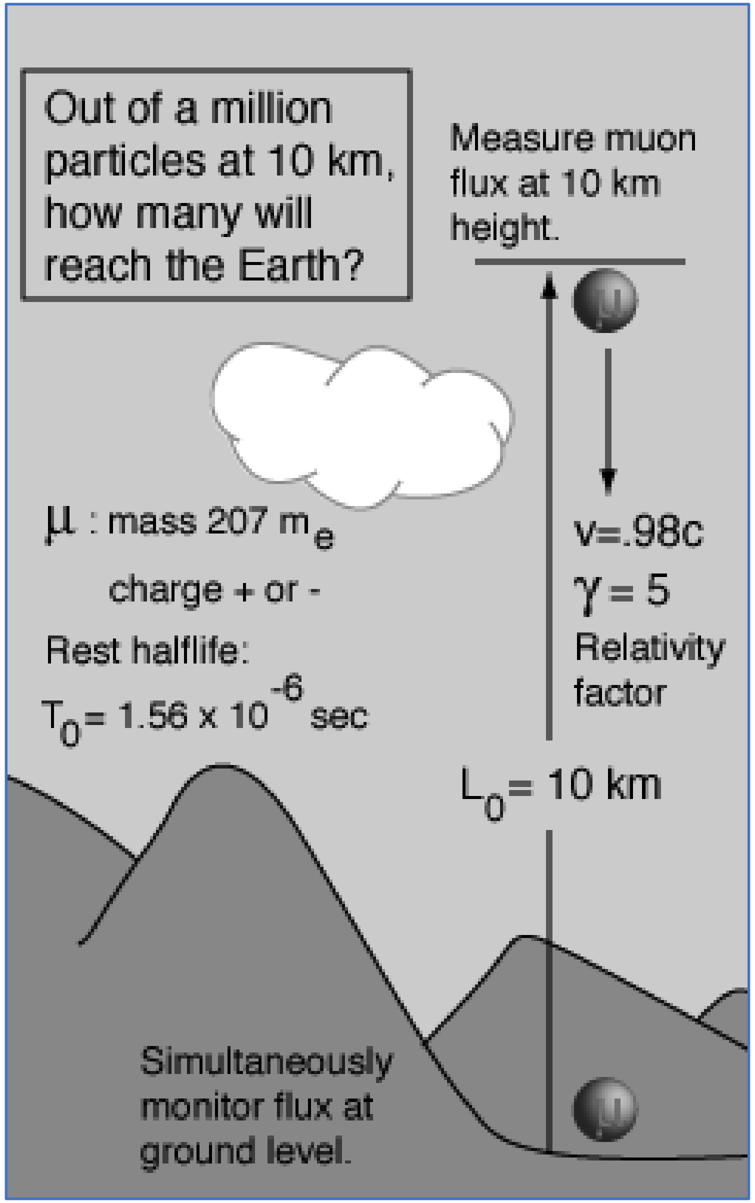




The Story Sofar: A Real Experiment

Muon particles are created at 10 km height. They have a half-lifetime of 1.56 μs , too short to reach the ground, but:...

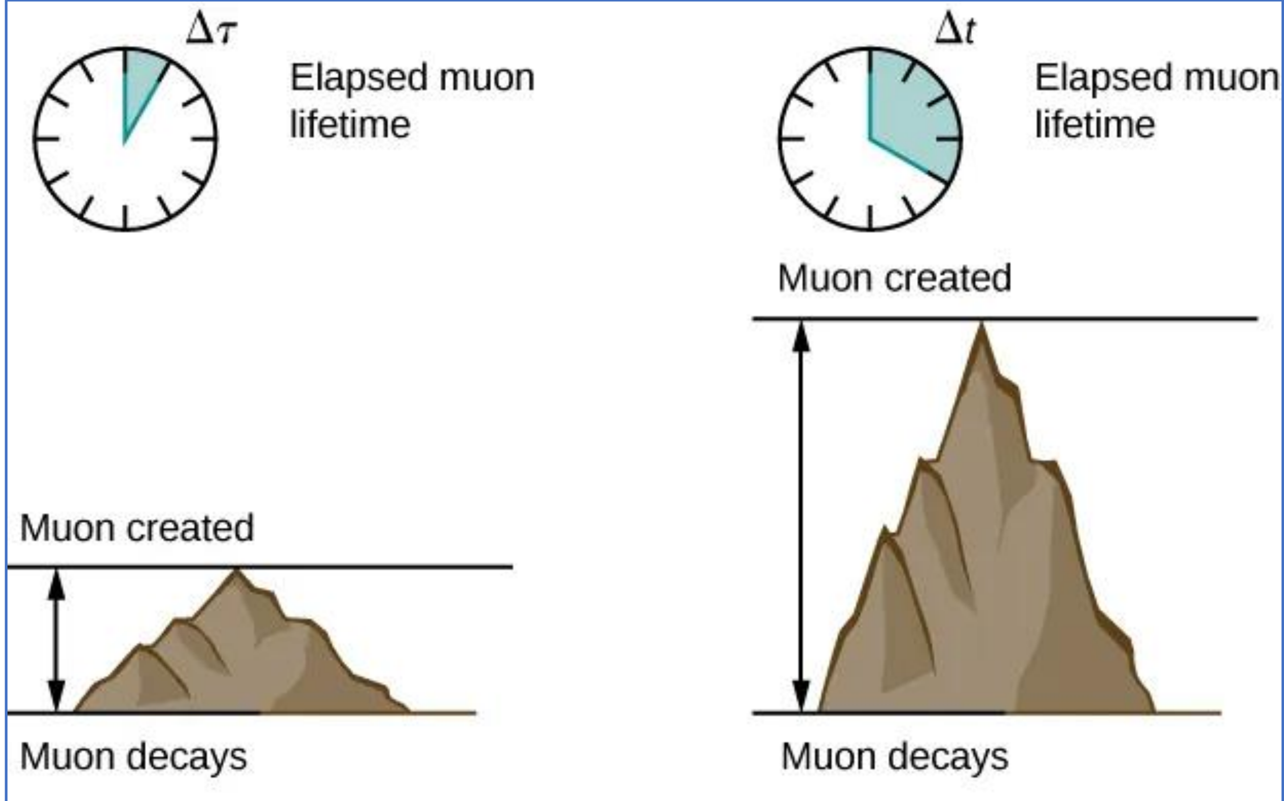




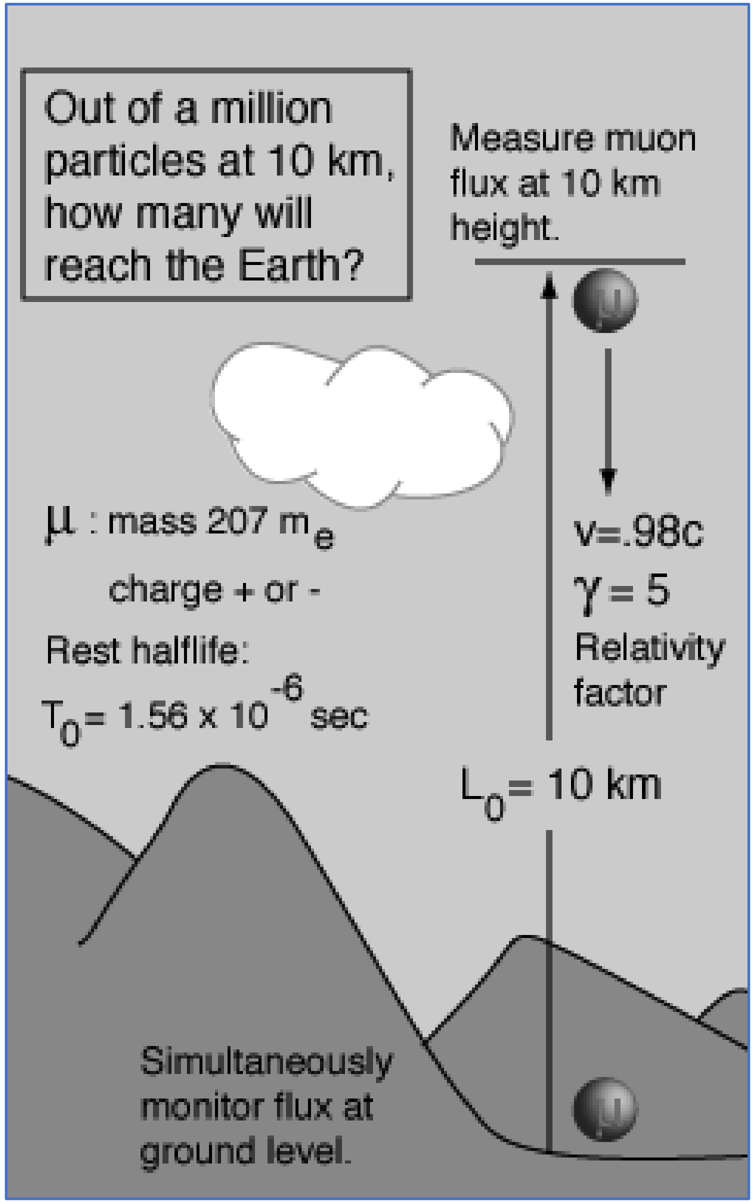
The Story Sofar: A Real Experiment

Muon particles are created at 10 km height. They have a half-lifetime of 1.56 μs , too short to reach the ground, but:

- As seen from an observer on earth they live a factor 5 longer
- As seen from the muon particle the distance is a factor 5 shorter



(a) Muon's reference frame (b) Earth's reference frame



Lecture 3

The Lorentz Transformation and Paradoxes

"Imagination is more important than knowledge."

- Albert Einstein

Coordinate Systems

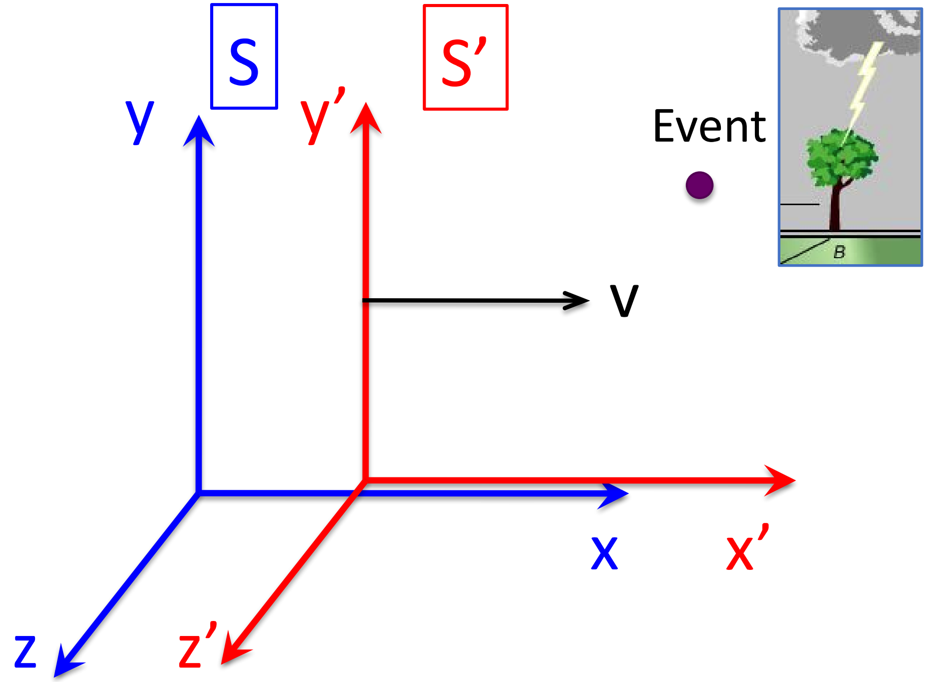
A reference system or coordinate system is used to determine the time and position of an event.

Reference system S is linked to observer Bob at position (x,y,z) = (0,0,0)
An event is fully specified by giving its coordinates and time: (t, x, y, z)

Reference system S’ is linked to observer Alice who is moving with velocity v with respect to Bob. The event has: (t’, x’, y’, z’)

How are the coordinates of an event, say a lightning strike in a tree, expressed in coordinates for Bob and for Alice?

(t, x, y, z) → (t’, x’, y’, z’)



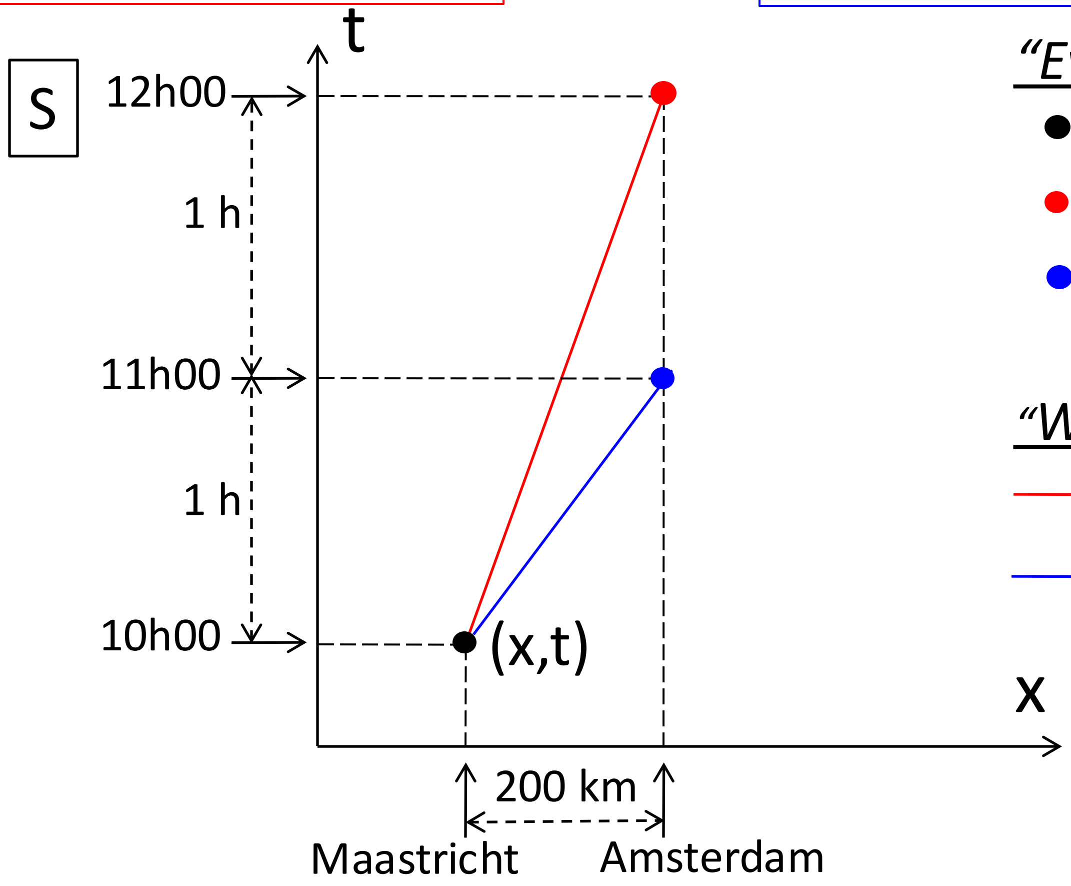
S: “Bob”

S’: “Alice”

Space-time diagram

Bob drives from Maastricht to Amsterdam with 100 km/h.

Alice drives from Maastricht to Amsterdam with 200 km/h.



"Events":

- Departure Alice & Bob
- Arrival Bob
- Arrival Alice

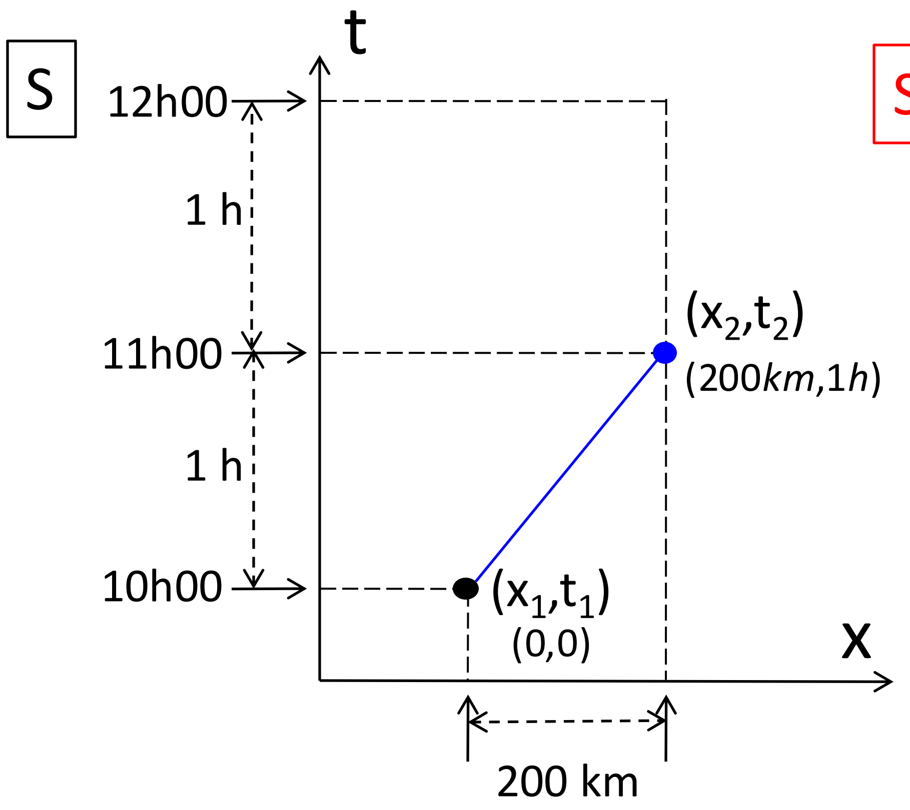
"World lines":

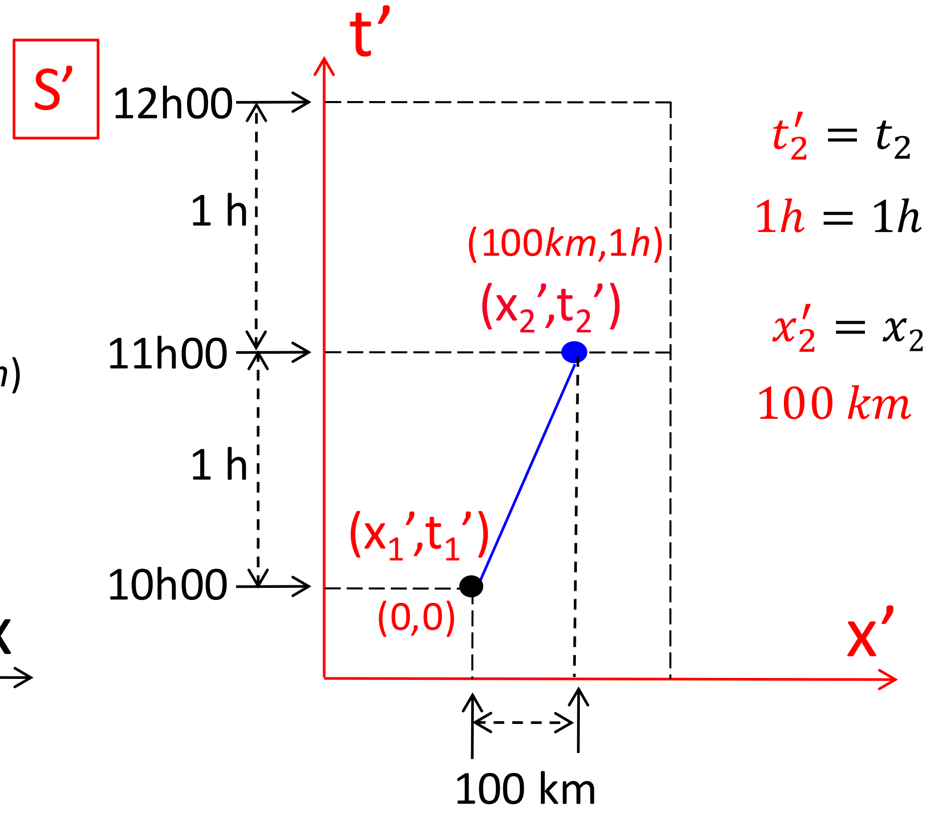
- Bob's world-line
- Alice's world-line

Events with space-time coordinates: (x,t)
More general: it is a 4-dimensional coordinate system: (x,y,z,t)

Coordinate transformation

How does Alice's trip look like in the coordinates of the reference system of Bob?





$t_2' = t_2$

$1h = 1h$

$x_2' = x_2 - vt$

$100\ km = 200\ km\ -$
$100\ km/h \times\ 1h$

Alice as seen from Maastricht
S = fixed reference system in Maastricht

Alice as seen from Bob
S' = fixed reference to Bob

Bob's reference frame S' moves with velocity v (100 km/h) with respect to Maastricht S

Coordinate transformation

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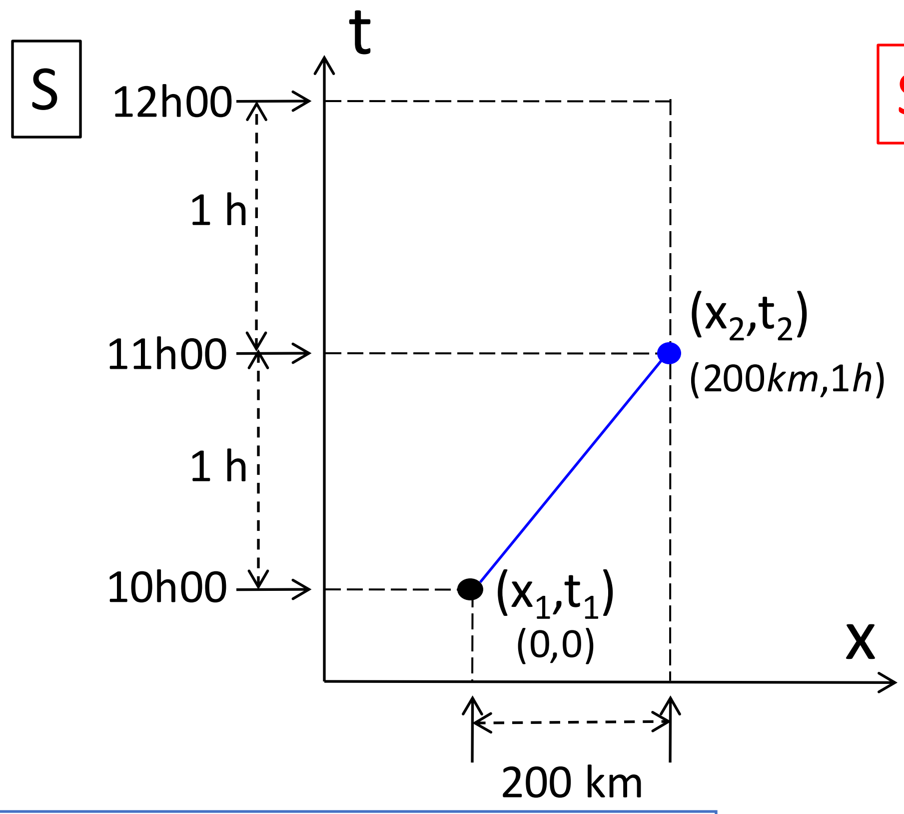

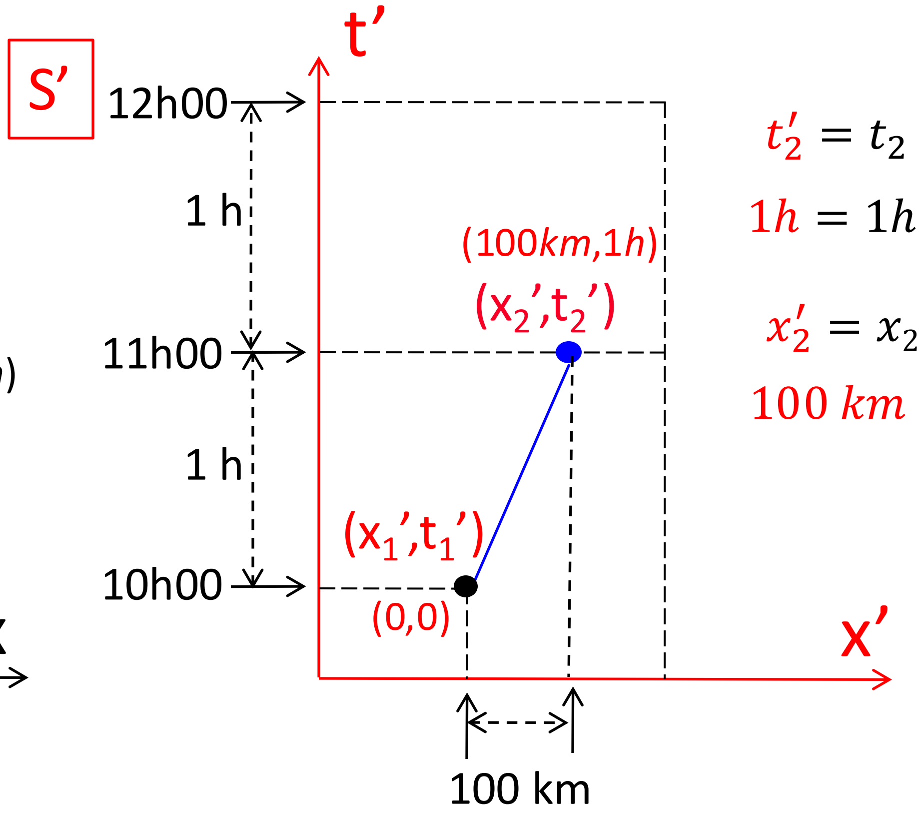

$t_2' = t_2$

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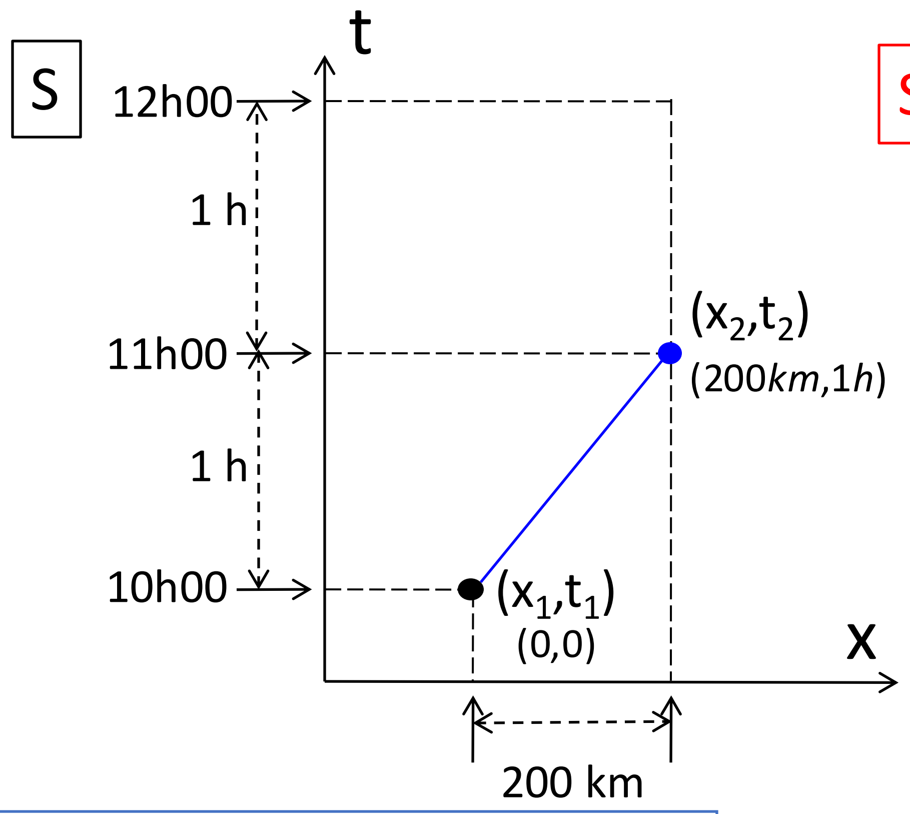
$100\ km = 200\ km - 100\ km/h \times 1h$

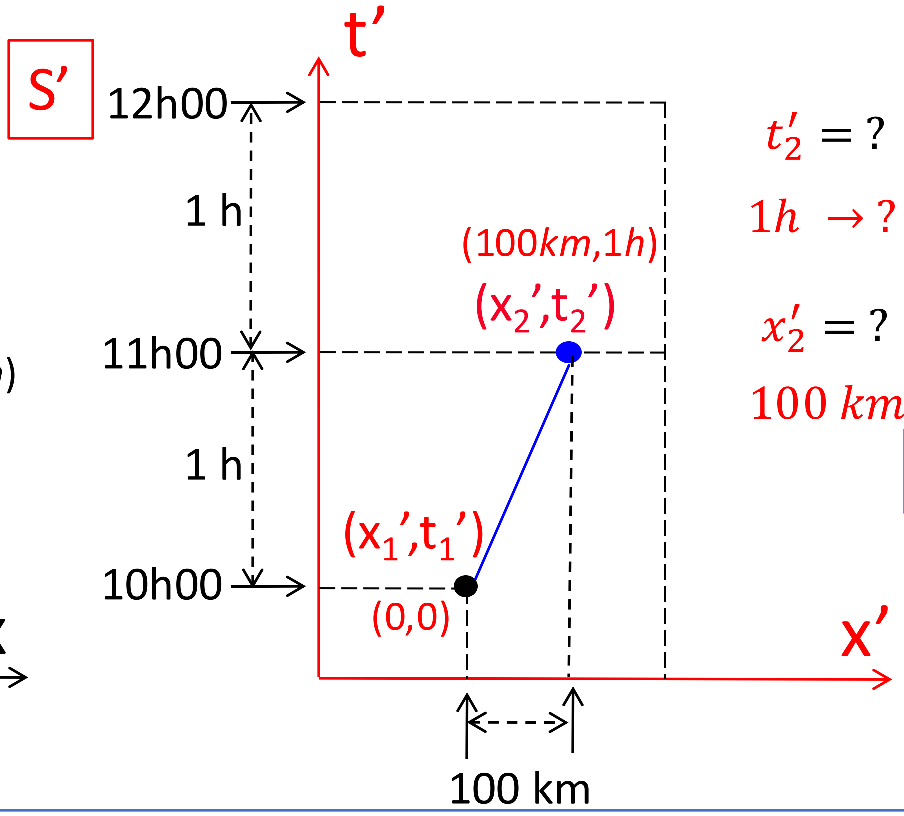
Classical (Gallilei Transformation):

$$t' = t$$
$$x' = x - v\,t$$

Coordinate transformation

How does Alice's trip look like in the coordinates of the reference system of Bob?





$t_2' = ?$

$1h \rightarrow ?$ $1h - 1.5 \times 10^{-11}s$

$x_2' = ?$

$100\ km \rightarrow ?$

$100km + 0.4\ nm$

Classical (Gallilei Transformation):

$$t' = t$$
$$x' = x - v\,t$$

Relativistic (Lorentz Transformation):

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$
$$x' = \gamma\,(x - v\,t)$$

with: $\gamma = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

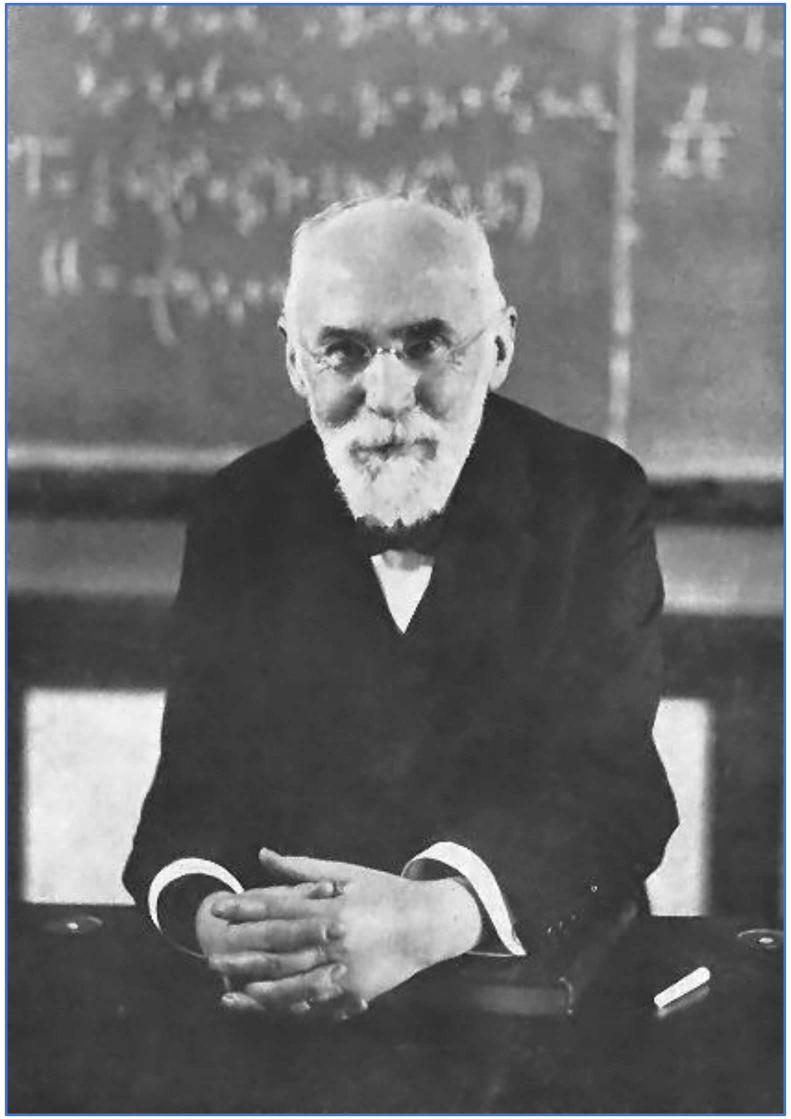
Lorentz Transformations

Hendrik Anton Lorentz (1853 – 1928)

Dutch Physicist in Leiden
(Nobelprize 1902 with Pieter Zeeman)

To explain the Michelson-Morley experiment he assumed that bodies contracted due to intermolecular forces as they were moving through the ether.
(He believed in the existence of the ether)

Einstein derived it from the relativity principle and also saw that time has to be modified.



Let’s go crazy and derive the Lorentz Transformation…

Start with classical Galilei Transformation:

$$x' = x - vt$$

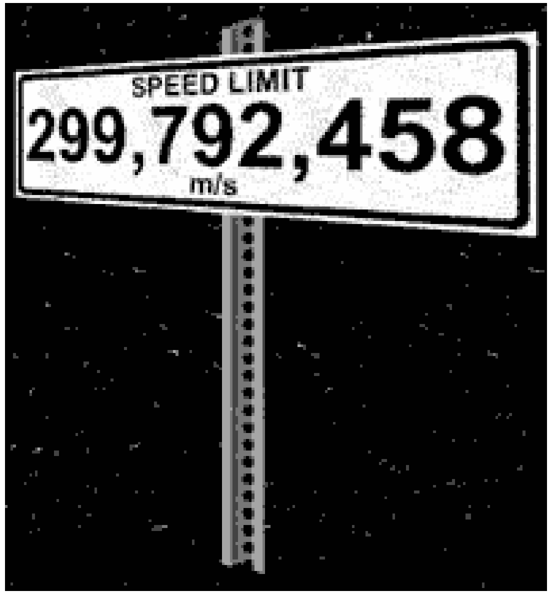
$$x = x' + vt'$$

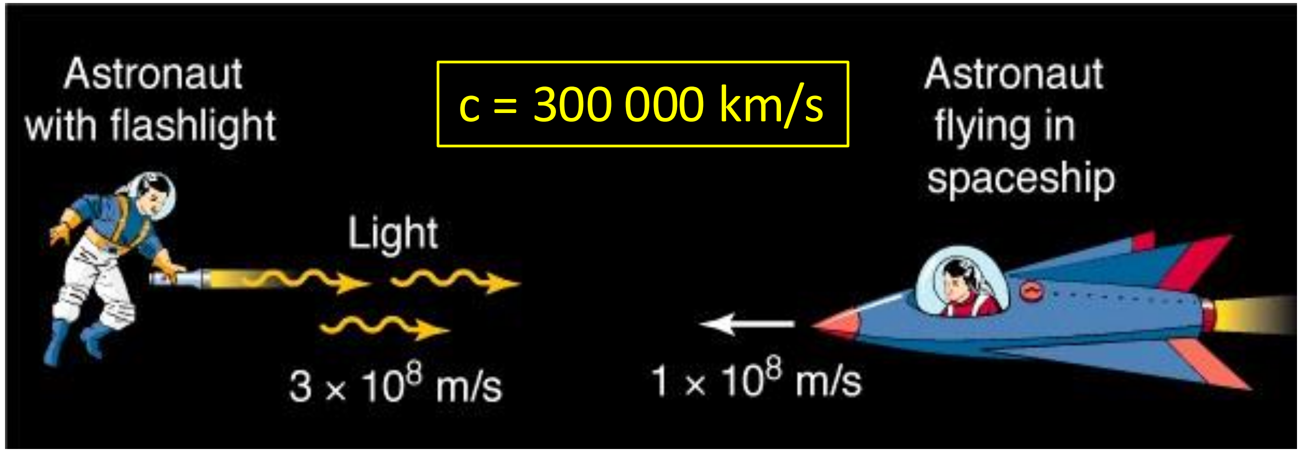
Let’s try relativity by including a factor f:

$$x' = f(x - vt)$$

$$x = f(x' + vt')$$

For light: $x = ct$ and $x' = ct'$





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$$x = f(x' + vt')$$

For light: $x = ct$ and $x' = ct'$, so:

$$ct' = f(ct - vt)$$

$$ct = f(ct' + vt')$$

Then: $$t' = f\left(\frac{c-v}{c}\right)t$$

$$t = f\left(\frac{c+v}{c}\right)t'$$

Substitute first into second:

$$t = f\left(\frac{c+v}{c}\right) f\left(\frac{c-v}{c}\right) t$$

Divide by t: $$1 = \left(\frac{c+v}{c}\right)\left(\frac{c-v}{c}\right) f^2 = \left(\frac{c^2-v^2}{c^2}\right) f^2$$

It follows then that: $$f^2 = \frac{c^2}{c^2 - v^2} = \frac{1}{1 - v^2/c^2}$$

So that we find: $$f = \frac{1}{\sqrt{1 - v^2/c^2}} = \gamma$$

Therefor we have derived the Lorentz transformation: $$x' = \gamma(x - vt)$$

Similarly we find the Lorentz transformation for time: (see lecture notes)

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

whereas the Galilei translation was:

$$t' = t$$

Let's go crazy and derive the Lorentz Transformation…

Start with classical Galilei Transfo

$$x' = x - vt$$

$$x = x' + vt'$$

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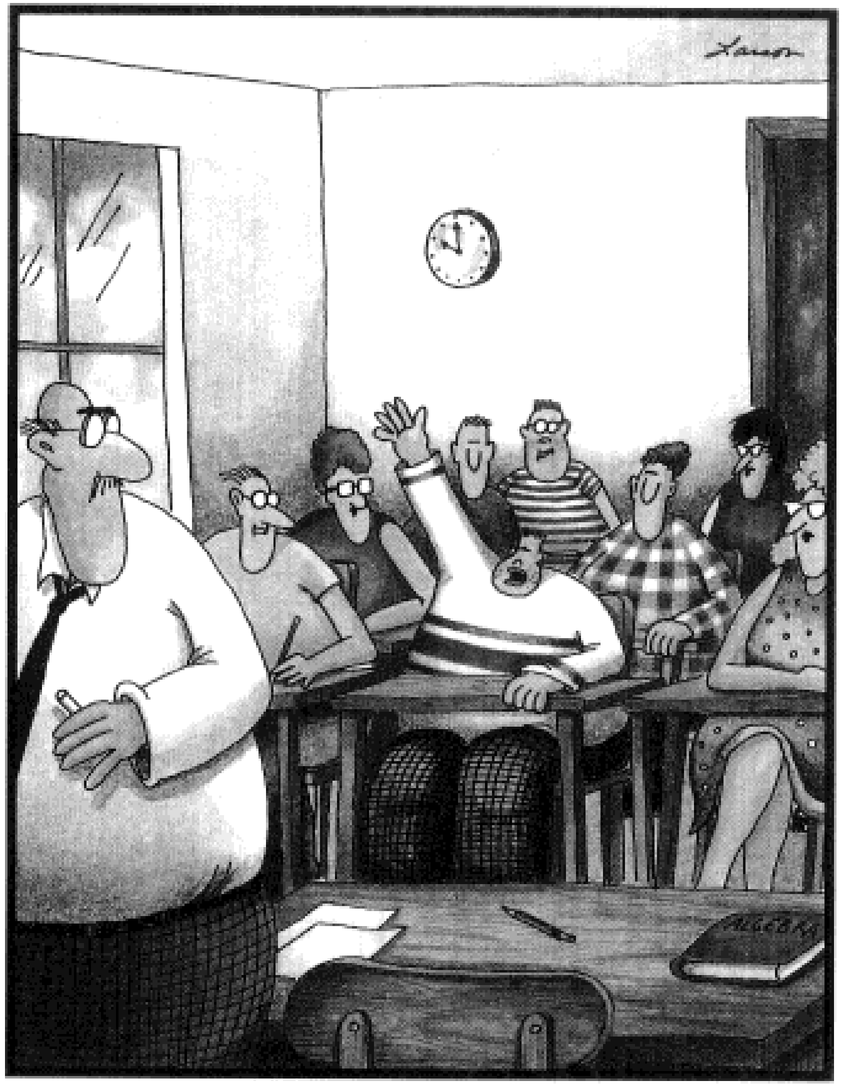
Substitute first into second:

$$t = f\left(\frac{c+v}{c}\right) f\left(\frac{c-v}{c}\right)$$



"Mr. Osborne, may I be excused? My brain is full."

$$\left(\frac{\cdot+v}{c}\right)\left(\frac{c-v}{c}\right) f^2 = \left(\frac{c^2-v^2}{c^2}\right) f^2$$

$$f^2 = \frac{c^2}{c^2 - v^2} = \frac{1}{1 - v^2/c^2}$$

$$f = \frac{1}{\sqrt{1 - v^2/c^2}} = \gamma$$

ived the

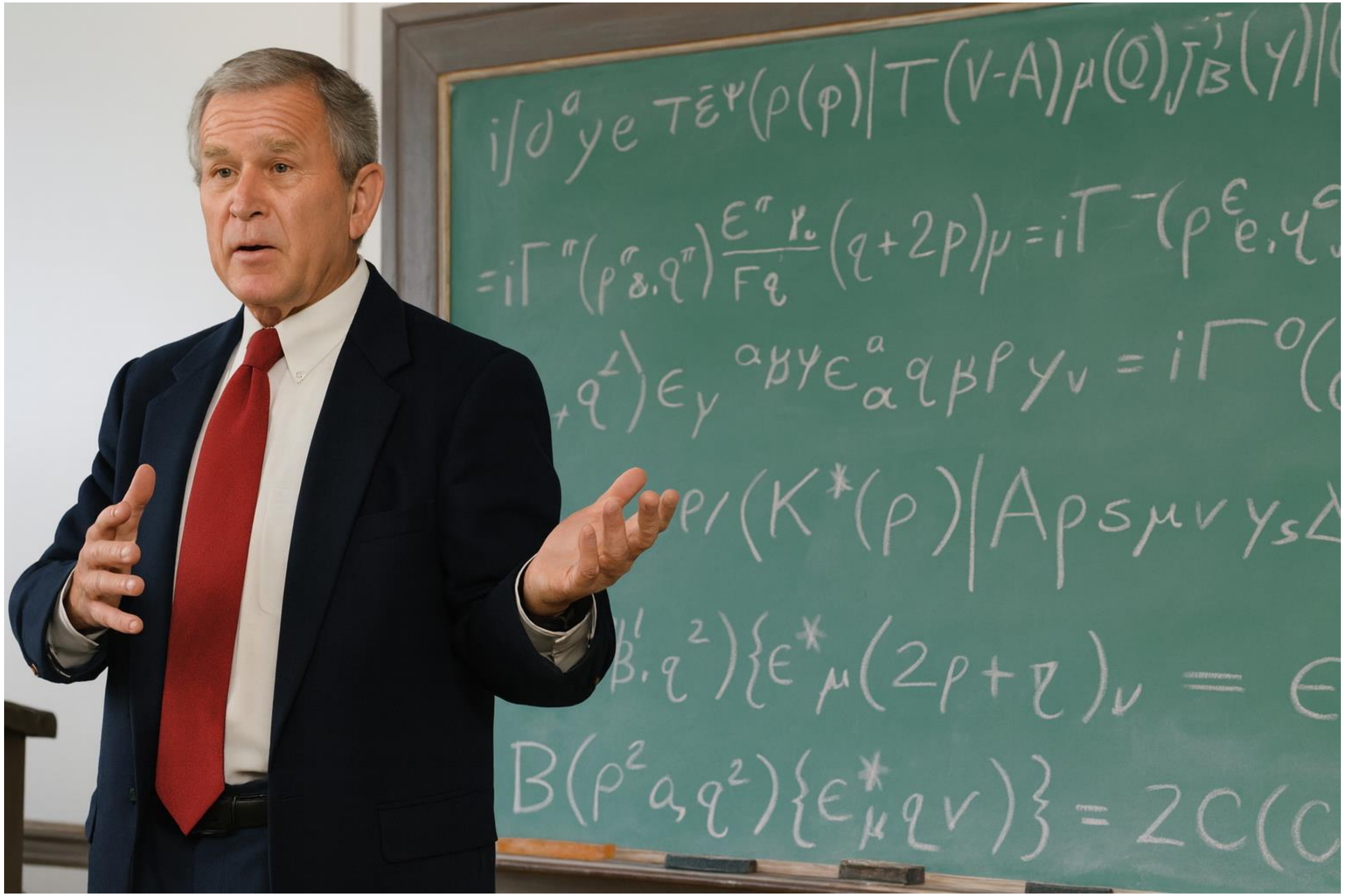
on: $$x' = \gamma(x - vt)$$

orentz

e: $$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$t' = t$$

Mathematics isn't too hard…



The classical limit (everyday life experience)

Lorentz transformation:

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

With new variables:

$$\beta = \frac{v}{c}$$

Fraction of lightspeed

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Relativistic factor

Daily life experience: speed ***much lower*** than lightspeed:

$$v \ll c \ , \ \beta \ll 1$$

$$\gamma \approx 1$$

$$ct' = \gamma(ct - \beta x)$$

$$x' = \gamma(x - \beta ct)$$

(Einstein)

$$t' \approx t$$

$$x' \approx x - vt$$

(Galilei)

The classical limit (everyday life experience)

Lorentz transformation:

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

In everyday life we ***do not see*** the difference between the classical and relativity theory!

With new variables:

$$\beta = \frac{v}{c}$$

Fraction of lightspeed

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Relativistic factor

$$ct' = \gamma(ct - \beta x)$$

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$$\gamma \approx 1$$

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$$x' \approx x - vt$$

(Galilei)

Paradoxes

Paradoxes Case 1: Sherlock & Watson

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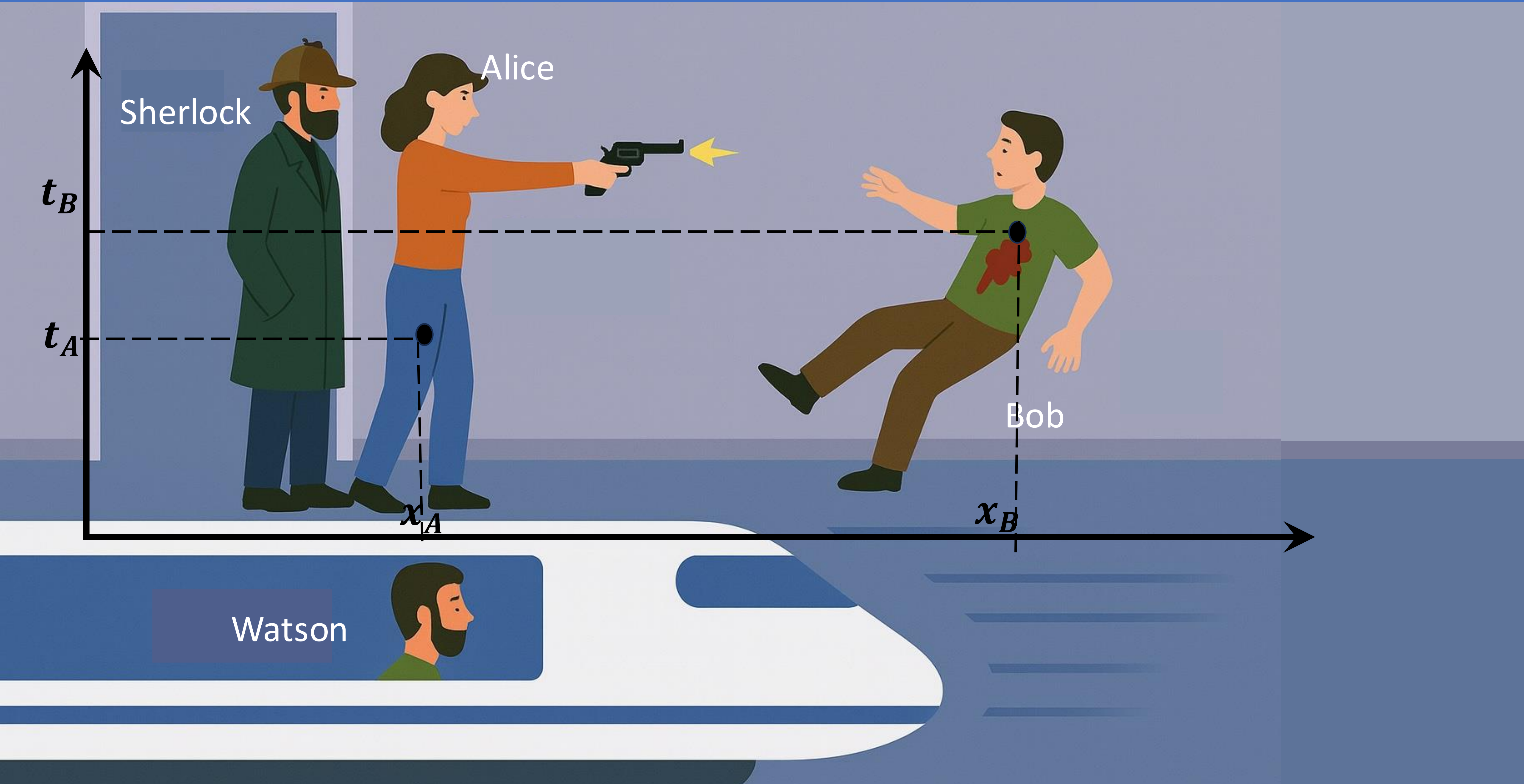
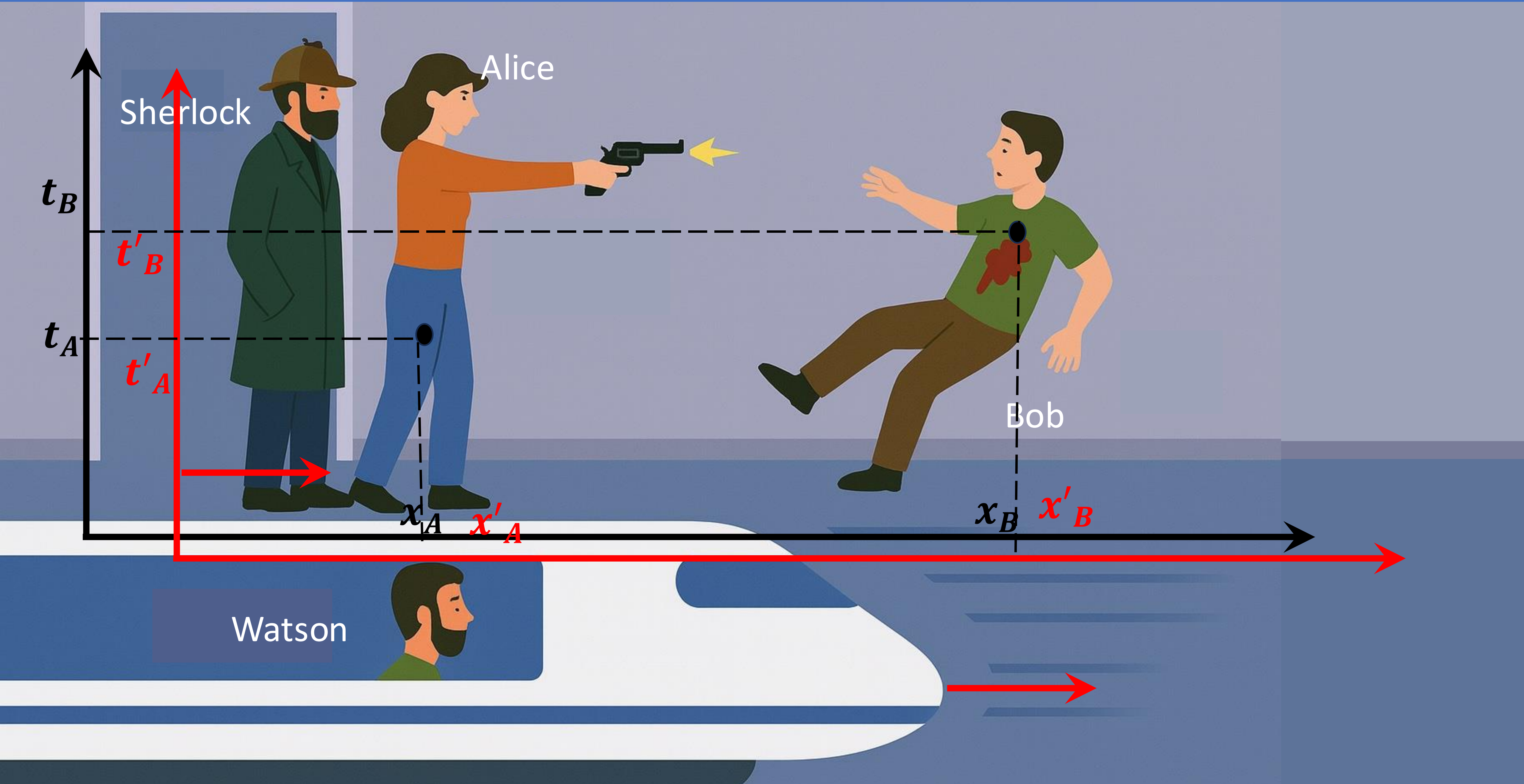


Paradoxes Case 1: Sherlock & Watson



Paradoxes Case 1: Sherlock & Watson

A murder scene is being investigated.

Alice enters a room and from the doorstep shoots Bob, who dies. (*Thought experiment!*)

Sherlock (S) stands at the doorstep (next to Alice) and observes the events.
Alice shoots at $t = t_A$ from position $x = x_A$
Bob dies at $t = t_B$ at position $x = x_B$

Watson (S') passes by on a fast train and sees the same scene. He sees:

- Alice shoots at $t' = t_A'$ from position $x' = x_A'$
- Bob dies at $t' = t_B'$ at position $x' = x_B'$

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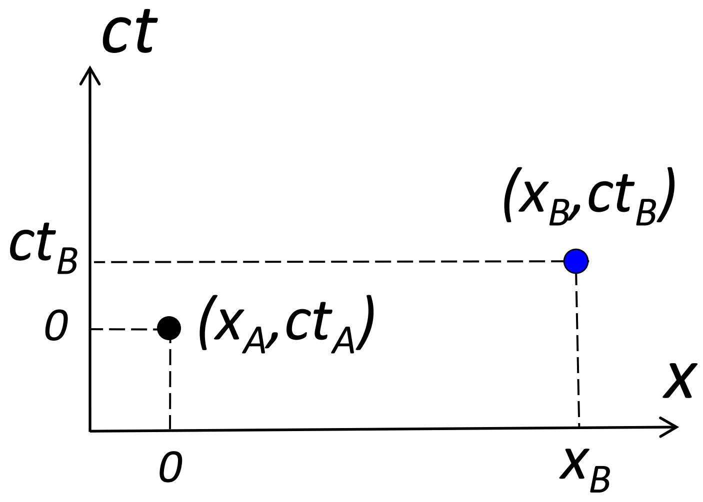
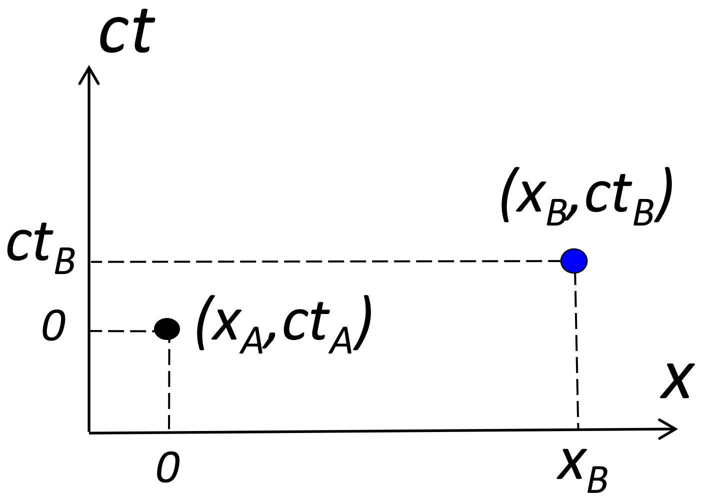
Sherlock: Alice shoots at Bob from x_A at time t_A
Bob dies on position x_B and time t_B

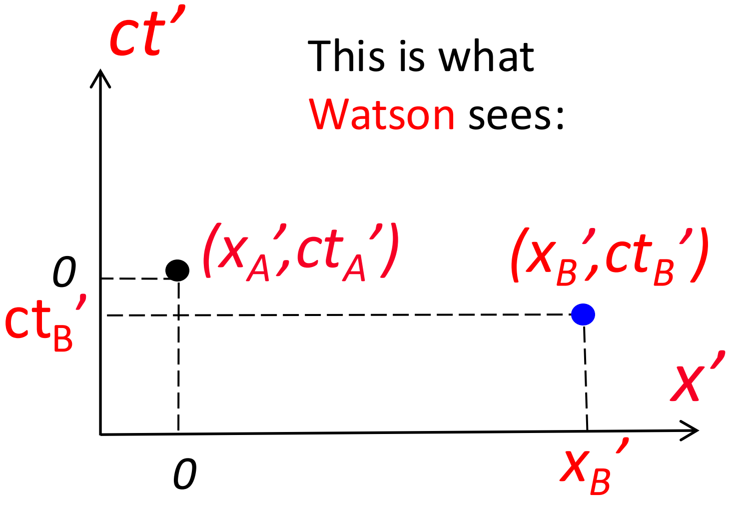


Causality

Sherlock: Alice shoots at Bob from x_A at time t_A

Bob dies on position x_B and time t_B





This is what Watson sees:

?

What does Watson see at ***v=0.6c***?

$$\beta = 0.6 \quad , \quad \gamma = \frac{1}{\sqrt{1-0.6^2}} = 1.25$$

$$\begin{aligned} ct' &= \gamma\,(ct - \beta\, x) \\ x' &= \gamma\,(x - \beta\, ct) \end{aligned}$$

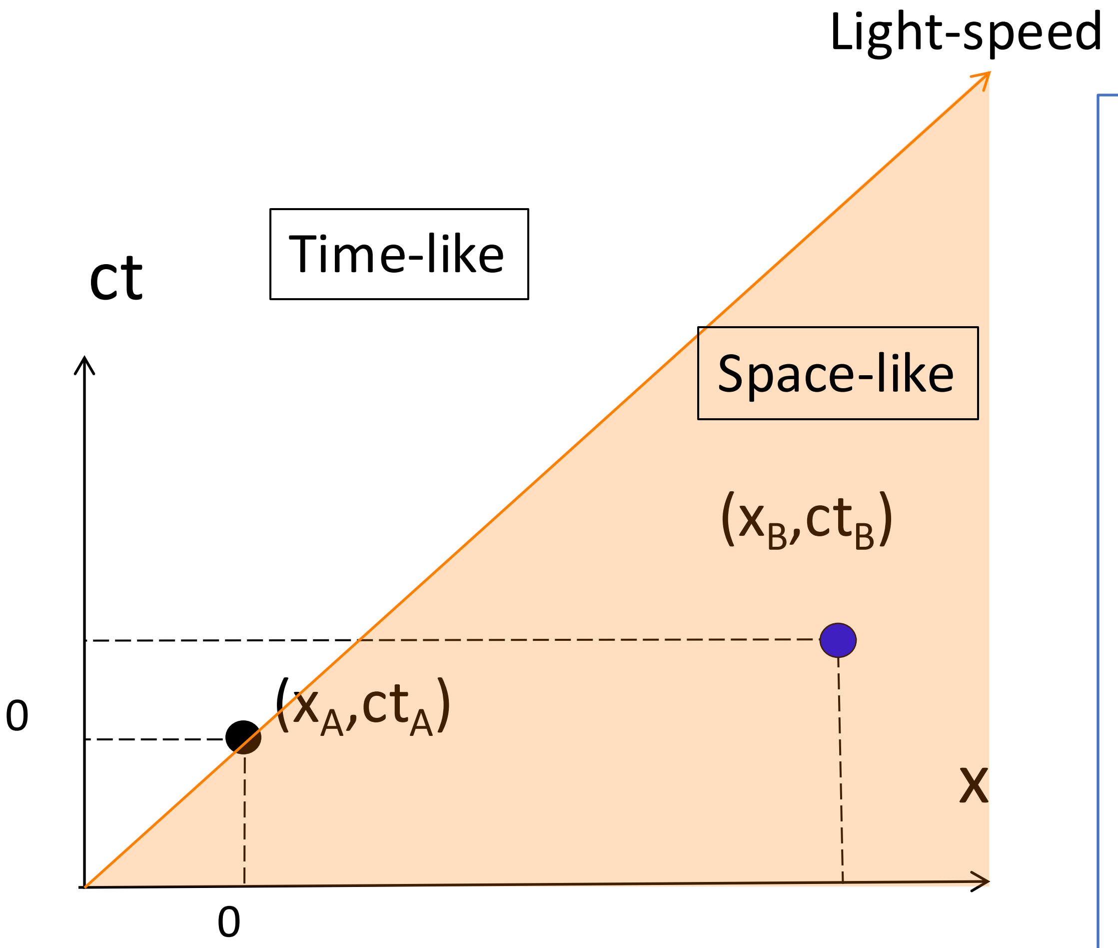
To make the calculation easy let's take $x_A = 0$ and $t_A = 0$:

$$\begin{aligned} ct'_A &= 0 \\ x'_A &= 0 \\ ct'_B &= 1.25\,(ct_B - 0.6\, x_B) \\ x'_B &= 1.25\,(x_B - 0.6\, ct_B) \end{aligned}$$

If distance $\boldsymbol{x_B} > \boldsymbol{ct_B}/0.6$ then $\boldsymbol{ct_B' < 0}$:
Bob dies **before** Alice shoots the gun!

What is wrong?

The situation was not possible to begin with!



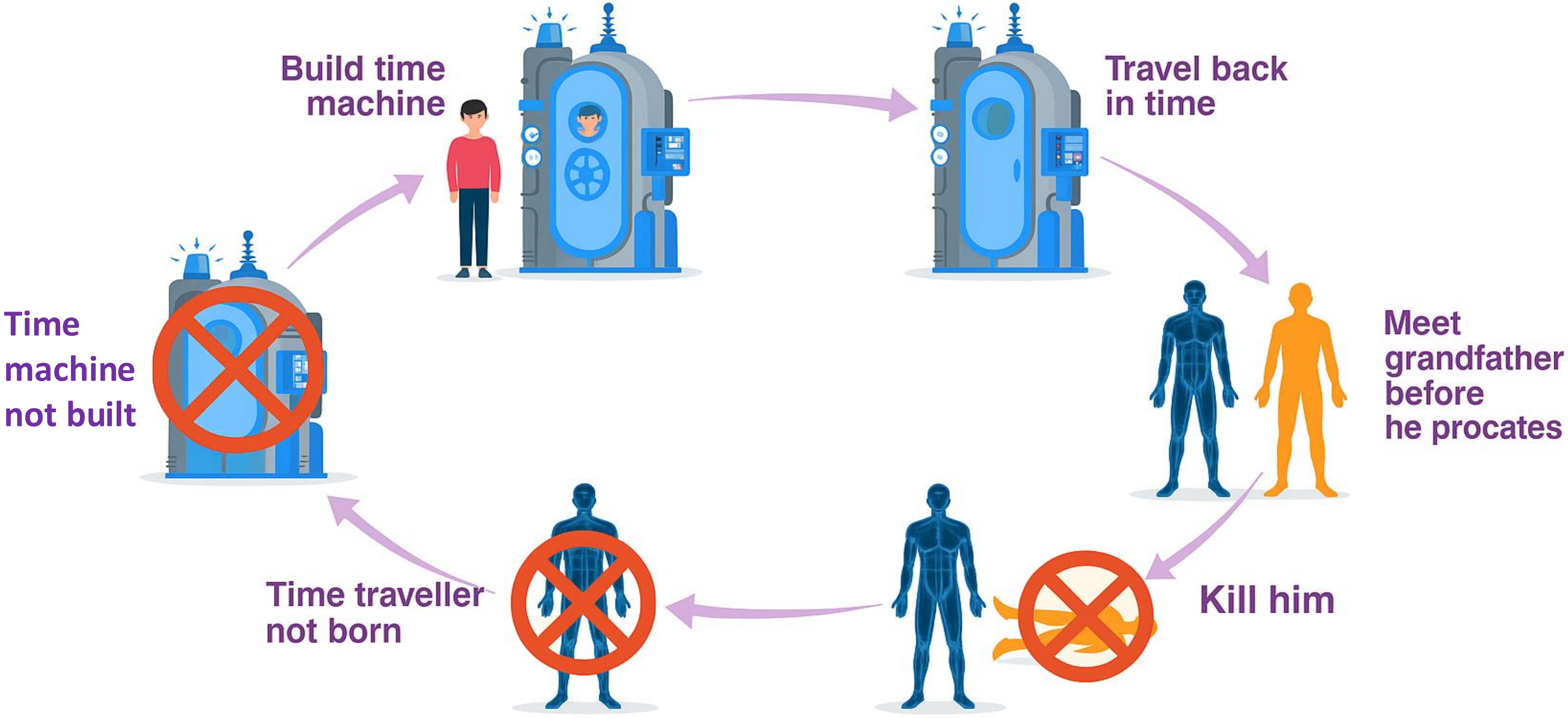
Nothing can travel faster than the speed of light, also not the bullet of gun!

The requirement: $x_B > c\, t_B / 0.6$ implies a bullet speed of :
$v = x_B/t_B > c / 0.6 = 1.67\, c$!
Faster than speed of light!

Travelling faster than light would imply you go back in time for some observers.

Causality is not affected by the relativity theory!

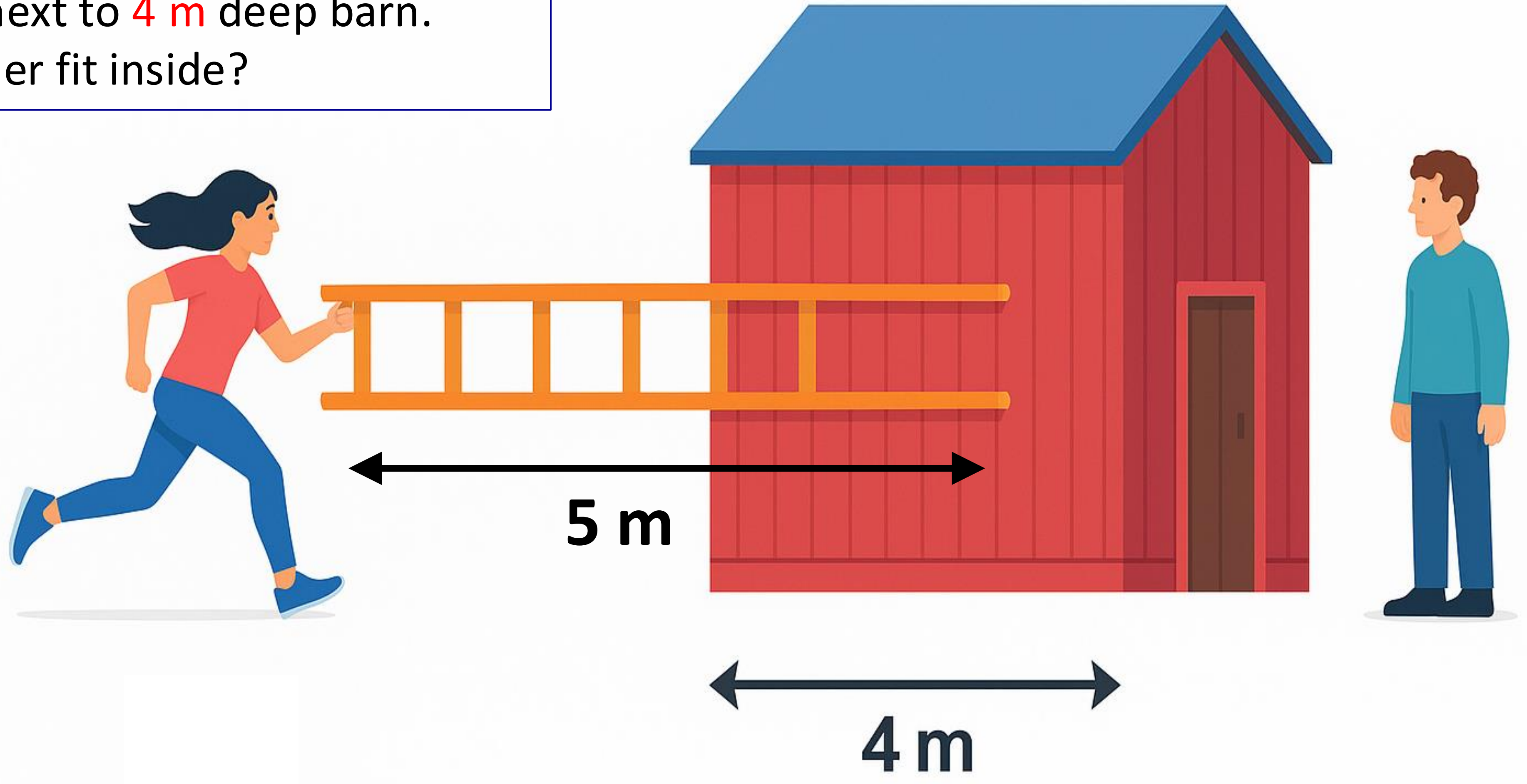
“Grandfather Paradox”



Travelling **backwards** in time is not possible!

Paradox 2: A ladder in a barn?

Alice runs towards a barn with
v = 0.8 c (γ =1.66).
She carries a 5 m long ladder.
Bob stands next to 4 m deep barn.
Will the ladder fit inside?



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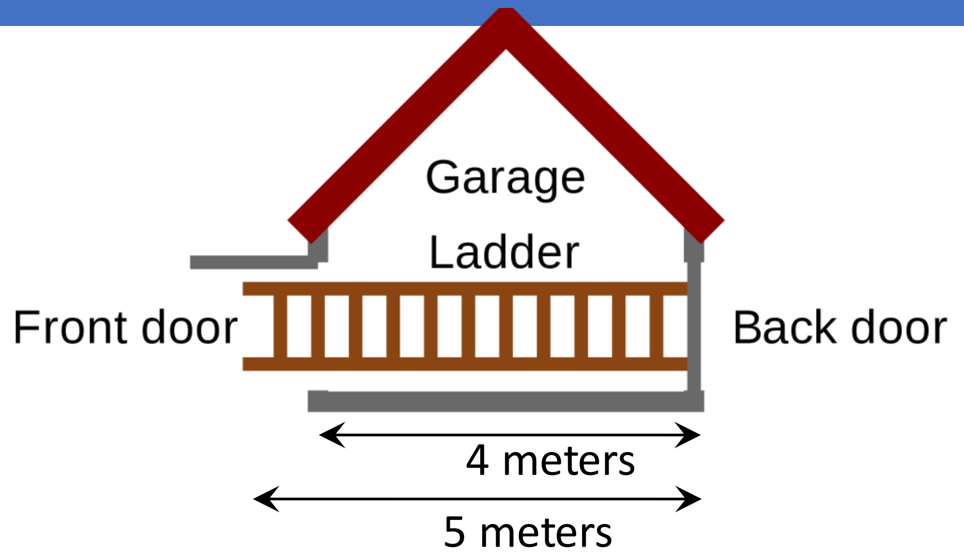
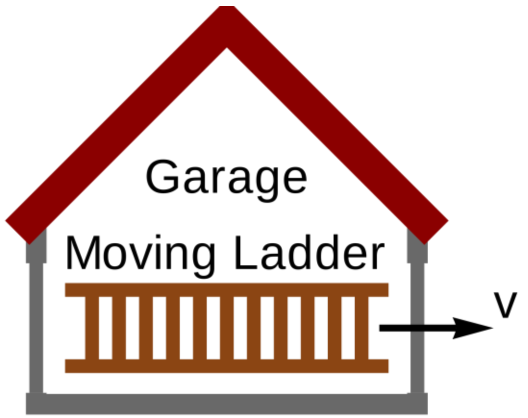
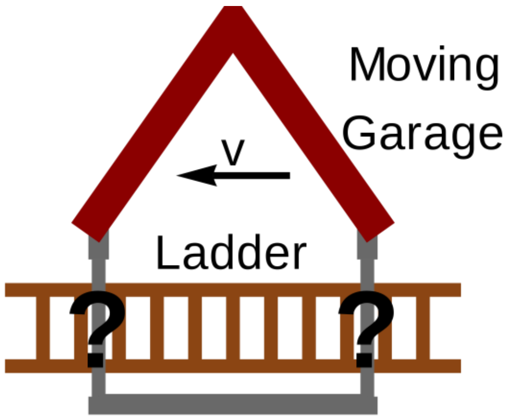


Bob: sure, no problem!
He sees a L/γ = 3 m long ladder



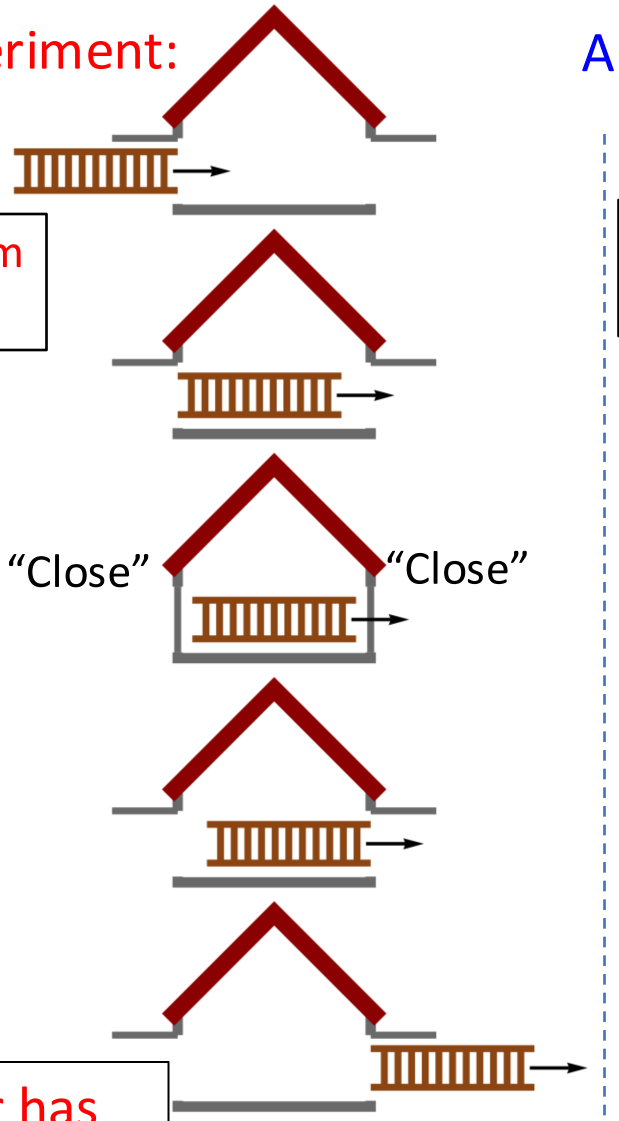
Alice: no way!
She sees a L/γ = 2.4 m deep barn



Paradox 2: A ladder in a barn?

Bob's experiment:

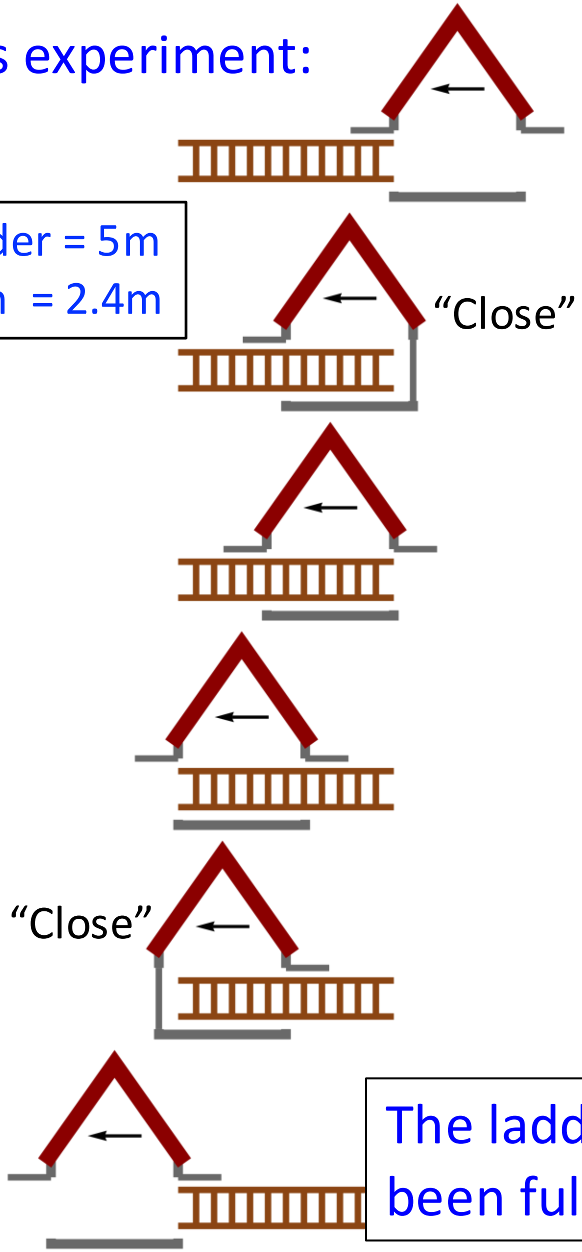
Ladder = 3m
Barn = 4m



The ladder has been fully inside

Alice's experiment:

Ladder = 5m
Barn = 2.4m



The ladder has **not** been fully inside

Bob and Alice don't agree on simultaneity of events.

Alice claims the back door is opened before the front door is closed.

Bob claims both doors are closed at the same time.

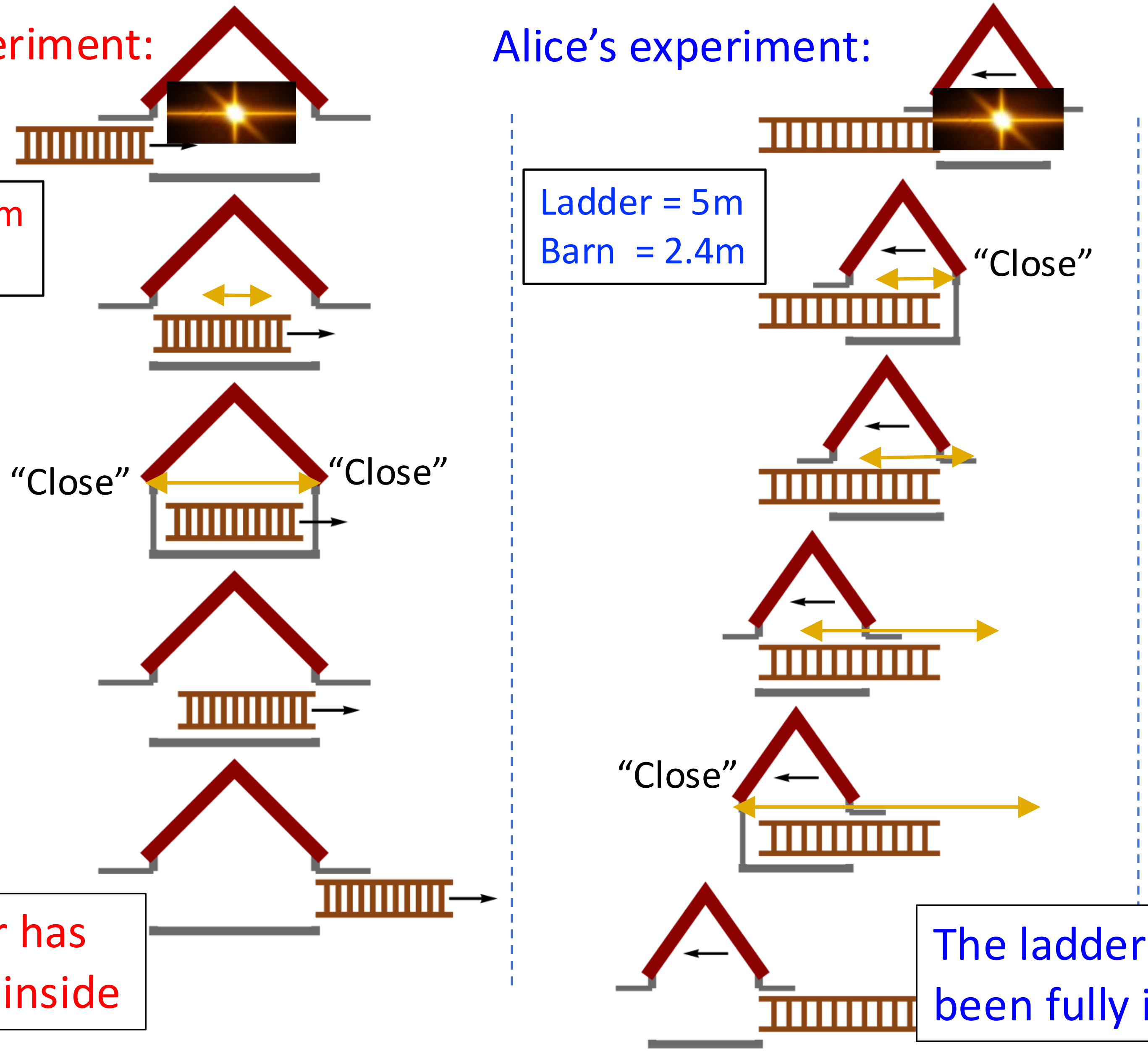
Paradox 2: A ladder in a barn?

Bob's experiment:

Ladder = 3m
Barn = 4m

Alice's experiment:

Ladder = 5m
Barn = 2.4m



The ladder has been fully inside

The ladder has **not** been fully inside

Bob and Alice don't agree on simultaneity of events.

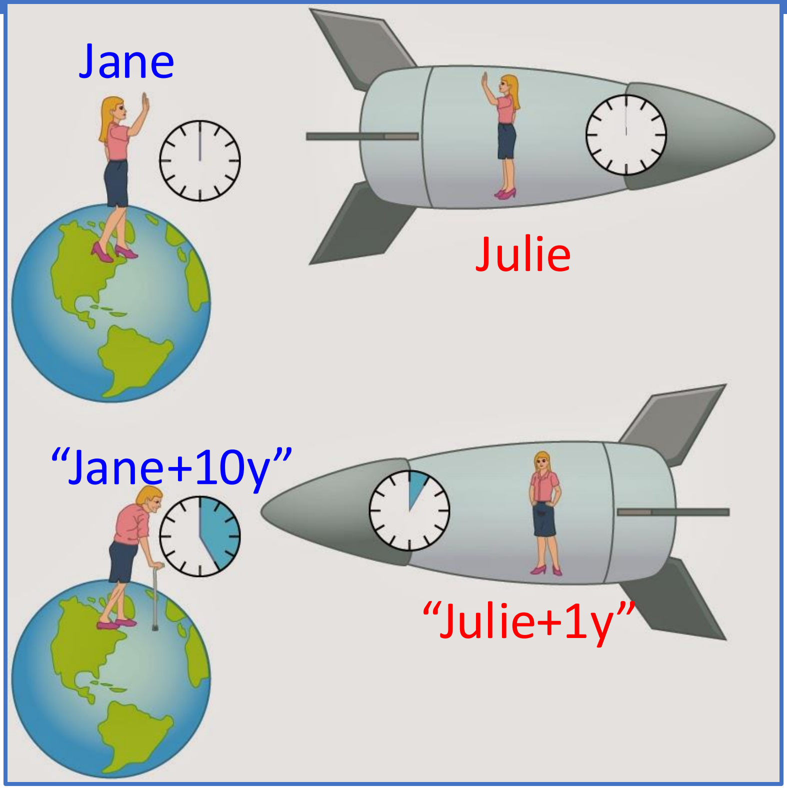
Alice claims the back door is opened before the front door is closed.

Bob claims both doors are closed at the same time.

*But will it stay inside?!
To stay inside the ladder must stop (negative accelleration)
Compressibility → lightspeed

Paradox 3: The Twin Paradox

Meet identical twins: Jane and Julie



Julie travels to a star with v = 99.5% of c (γ = 10) and returns to Jane on earth after one year travel.
Jane has aged 10 years, Julie only 1 year.

Jane understands this. Due to Julie’s high speed time went slower by a factor of 10 and therefore ***Jane has aged more than Julie***.

But Julie argues: the only thing that matters is our *relative speed*!
From her spaceship time goes *slower* on earth! She claims ***Jane should be younger***!

Who is right?

Answer: special relativity holds for ***constant*** relative velocities.
When Julie turns around she slows down, turns and accelerates back.
At that point time on earth progresses fast for her, so that ***Jane is right*** in the end.

Paradox 3: The Twin Paradox

1971: A *real* experiment!

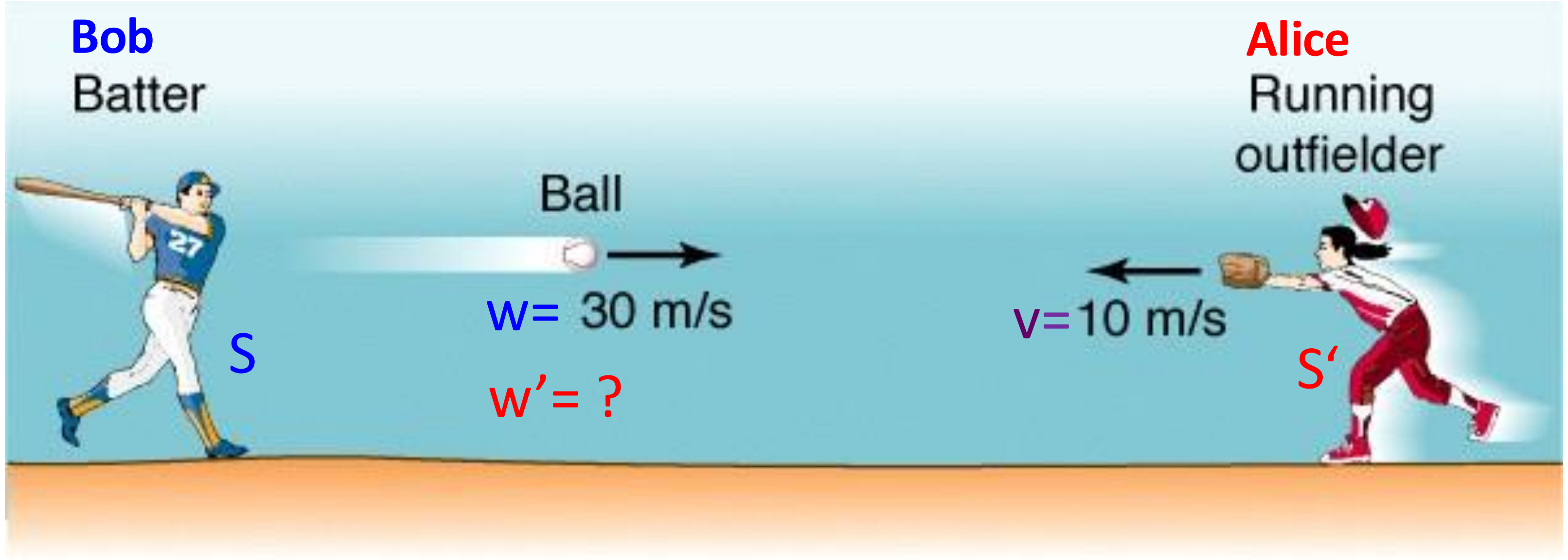
Joseph Hafele & Richard Keating tested it with 3 atomic cesium clocks.

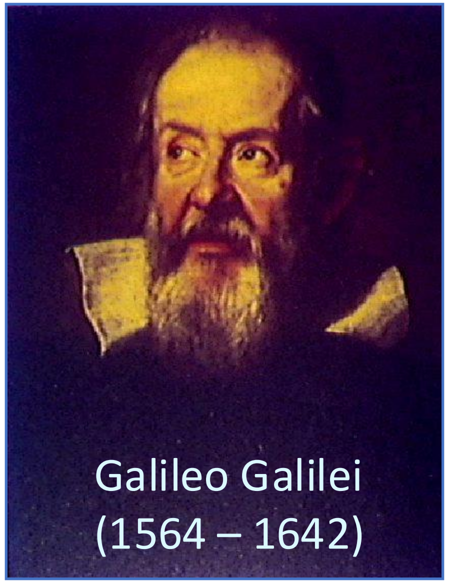
- One clock in a plane ***westward*** around the earth (against earth rotation)
- One clock in a plane ***eastward*** around the earth (with earth rotation)
- One clock stayed behind in the lab.

The clock that went ***eastward*** was 300 nsec behind, in agreement with relativity.

Remember Calvin's father

Galilei and Einstein Transformation law




Galileo Galilei
(1564 – 1642)

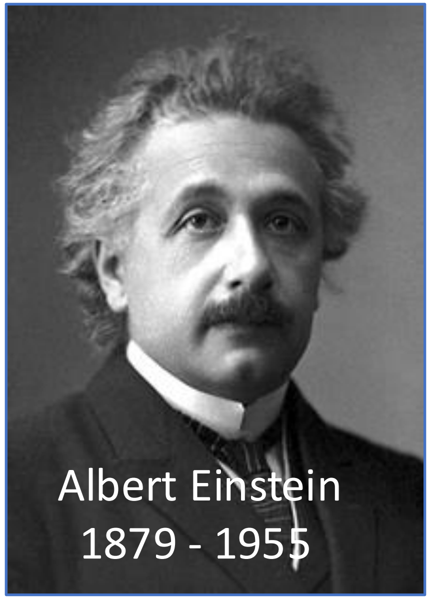
With which speed do Alice and the ball hit by Bob approach each other?
Intuitive law (daily experience): 30 m/s + 10 m/s = 40 m/s

Galilei formula: $w' = w + v = 30 + 10 = 40\ m/s$

Einstein formula: $w' = \dfrac{w + v}{1 + \dfrac{vw}{c^2}} = \dfrac{30 + 10}{1 + \dfrac{30 \times 10}{9 \times 10^{16}}} =$

(see lecture 3)

$= 39.99999999999997\ m/s$


Albert Einstein
1879 - 1955

Derive the laws for adding speed

Galilei Transformation:

$$x' = x + vt$$
$$t' = t$$

In the frame of S we have:

$$x = w\,t$$

Then it follows:

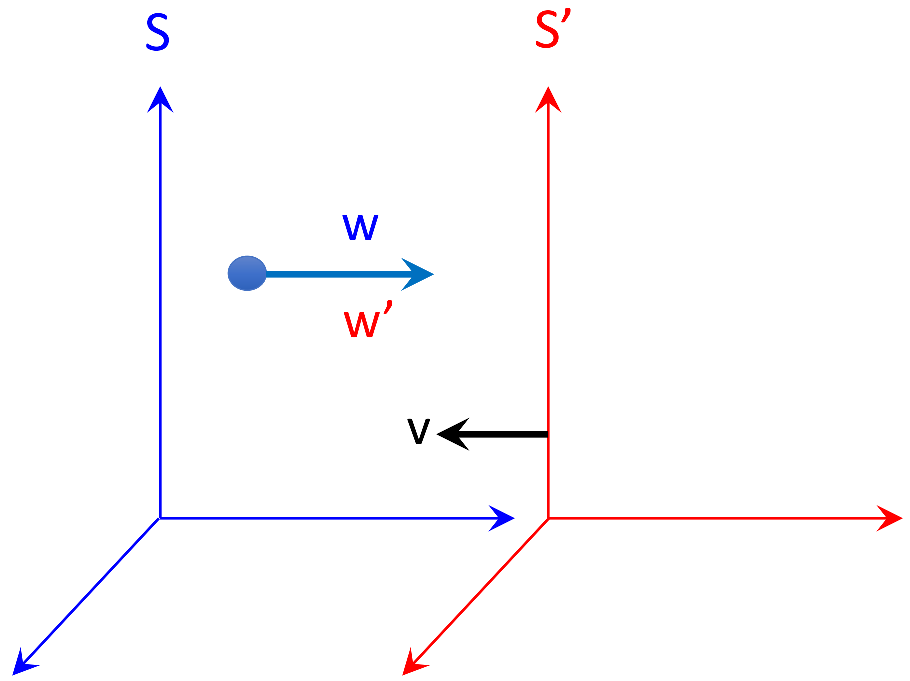
$$x = w\,t'$$
$$x' - v\,t = w\,t'$$
$$x' - v\,t' = w\,t'$$
$$x' = (v + w)\,t'$$

Therefore in S':

$$w' = w + v$$

Lorentz Transformation:

$$x' = \gamma(x + vt)$$
$$t' = \gamma\left(t + \frac{v}{c^2}x\right)$$



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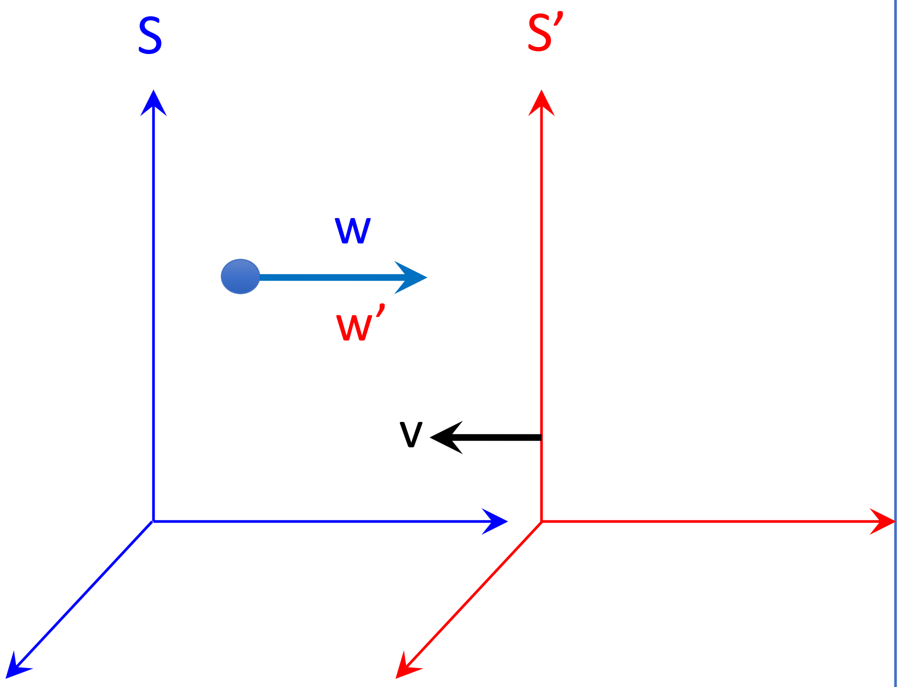

Re-write the laws: $x' = \gamma x + \gamma v\, t$

$$t' = \gamma t + \gamma \frac{v}{c^2} x$$

Substitute in frame S: $x = wt$ to find: $x' = \gamma w t + \gamma v\, t$

$$t' = \gamma t + \gamma \frac{vw}{c^2} t$$

Invert the equation for t': $t = \frac{1}{\gamma}\left(\frac{1}{1 + \frac{vw}{c^2}}\right) t'$

Put into the expression for x': $x' = \gamma(v + w)\frac{1}{\gamma}\left(\frac{1}{1 + \frac{vw}{c^2}}\right) t'$

Which should be: $x' = w't'$, therefor: $w' = \frac{w + v}{1 + \frac{vw}{c^2}}$

Derive the laws for adding speed

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Therefore in S′:

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$$x' = \gamma(x + vt)$$
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Re-write the laws: $x' = \gamma x + \gamma v\,t$, $t' = \gamma t + \gamma\frac{v}{c^2}x$

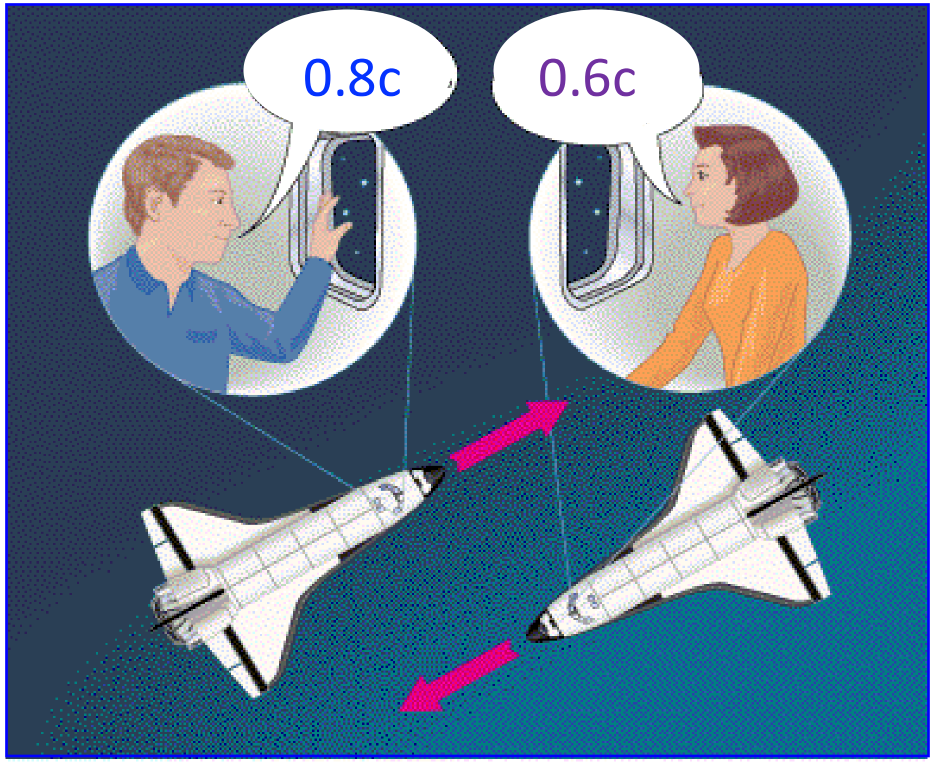
Substitute in frame S: $x = wt$ to find: $x' = \gamma wt + \gamma v\,t$, $t' = \gamma t + \gamma\frac{vw}{c^2}t$

Invert the equation for t': $t = \frac{1}{\gamma}\left(\frac{1}{1 + \frac{vw}{c^2}}\right)t'$

Put into the expression for x': $x' = \gamma(v + w)\frac{1}{\gamma}\left(\frac{1}{1 + \frac{vw}{c^2}}\right)t'$

Which should be: $x' = w't'$, therefor: $w' = \frac{w + v}{1 + \frac{vw}{c^2}}$

Large effects at relativistic speeds



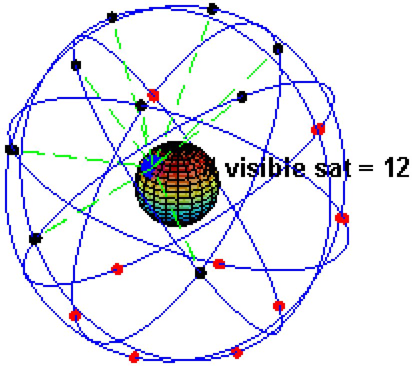
Bob in a rocket passes a star with 0.8 c
Alice in a rocket passes a star with 0.6 c
In opposite directions.

What is their relative speed?

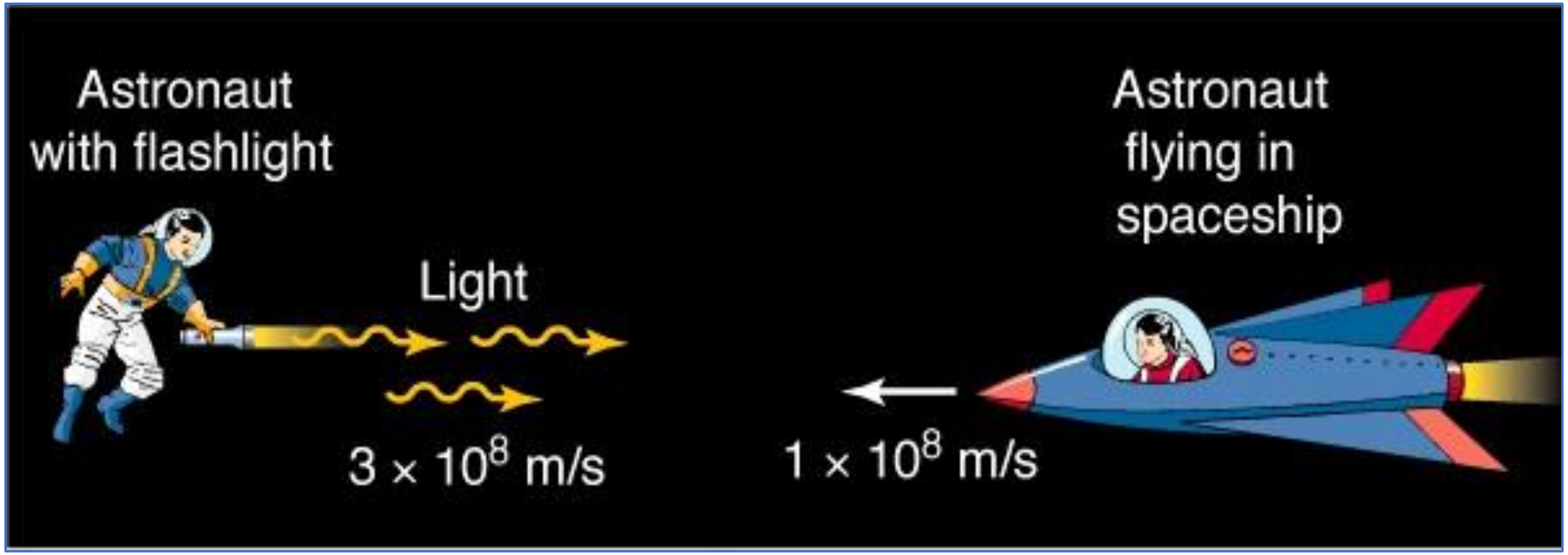
$$w' = \frac{0.8c + 0.6c}{1 + (0.8 \times 0.6)} = 0.95c$$

Very different than w' = 0.8 c + 0.6 c = 1.4 c !!

Without relativity theory GPS technology makes mistakes of about 10 km/day !



How about Alice seeing light coming from Bob?



How fast does the light go for Alice?

➔ Just put $w = c$ into Einstein's formula:

$$w' = \frac{c+v}{1+\frac{cv}{c^2}} = \frac{c+v}{1+\frac{v}{c}} = \frac{c+v}{\frac{1}{c}(c+v)} = \frac{1}{\frac{1}{c}} = c$$

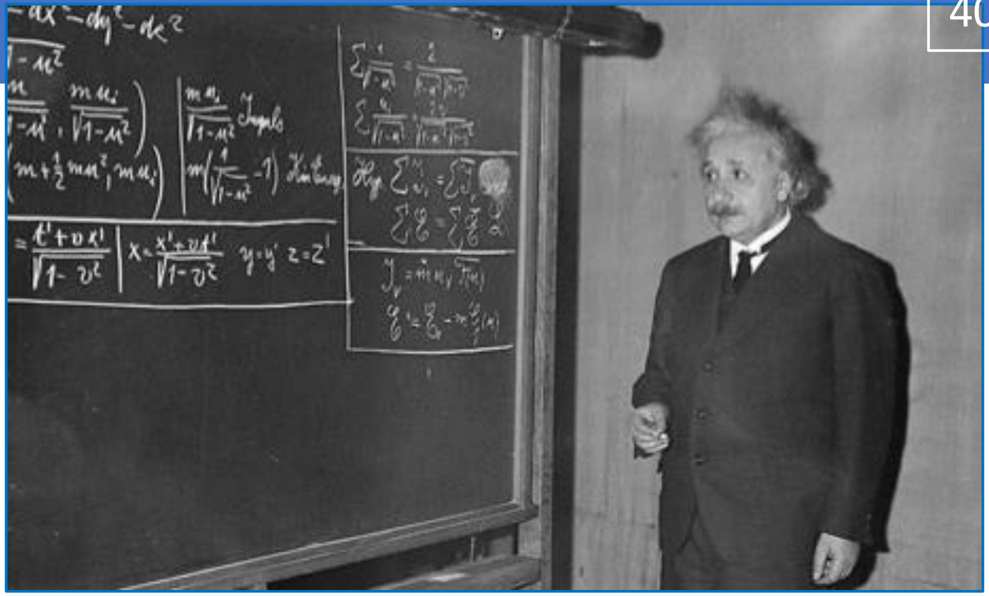
The speed of light is always the same for each observer!

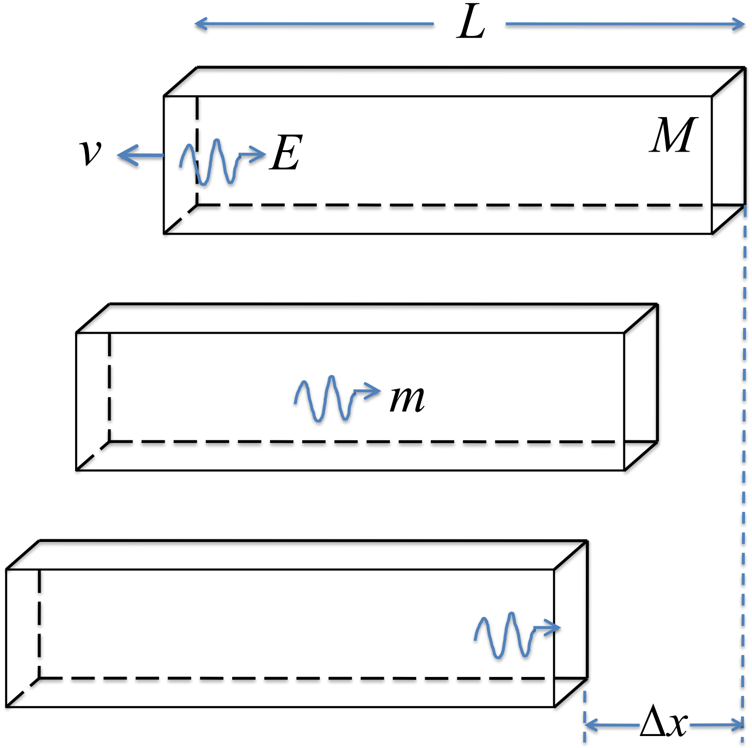
$E=mc^2$: see lecture notes

Consider a box with length *L* and mass *M* floating in deep space.

A photon is emitted from the left wall and a bit later absorbed in the right wall.

Center of Mass of box + photon must stay unchanged.





$Mv = E/c$ — Action = - Reaction: photon momentum is balanced with box momentum

$t = L/c$ — Time it takes the photon

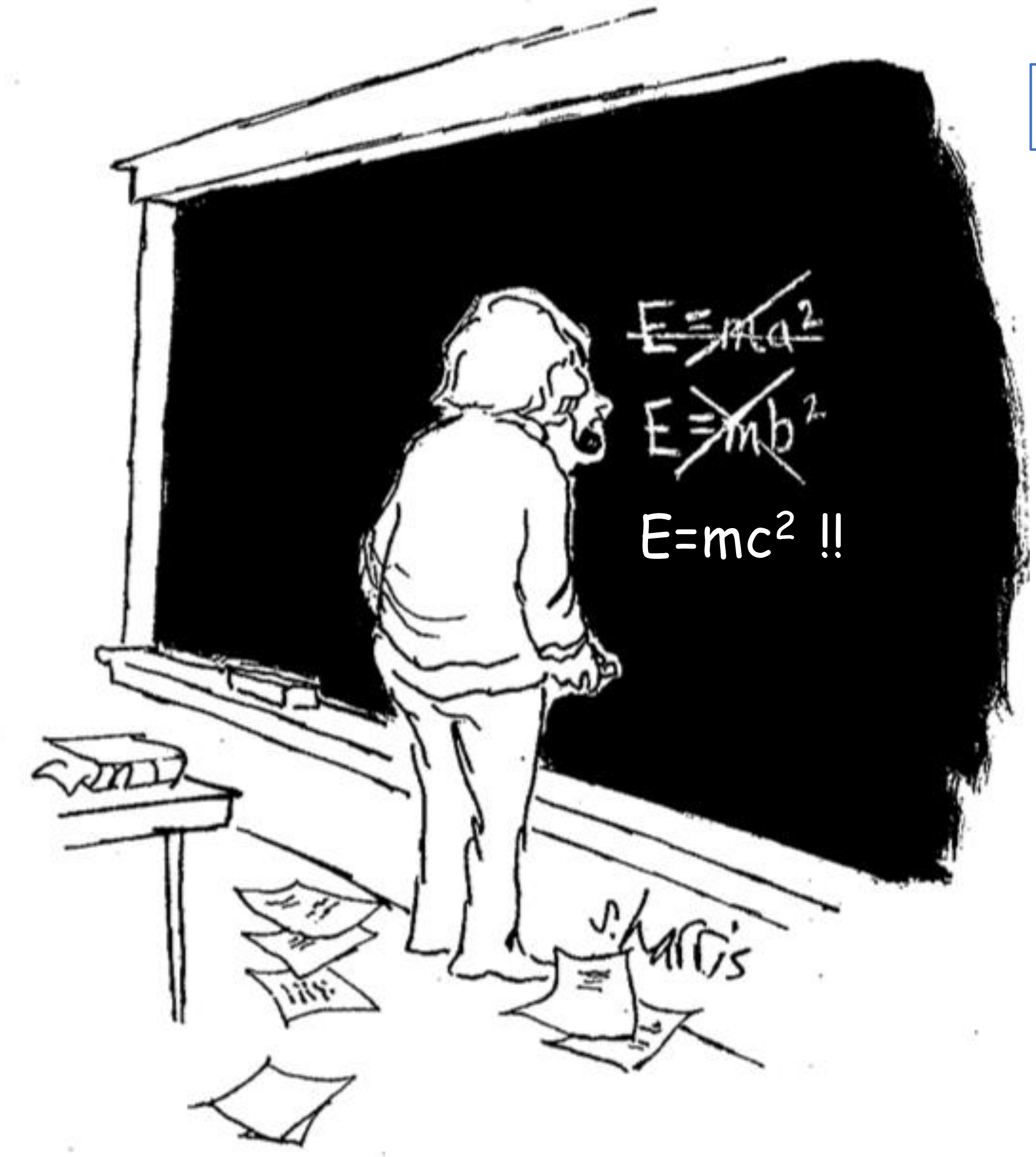
$\Delta x = vt$ — Distance that the box has moved

$M\Delta x = mL$ — C.O.M. does not move: box compensated by photon C.O.M.

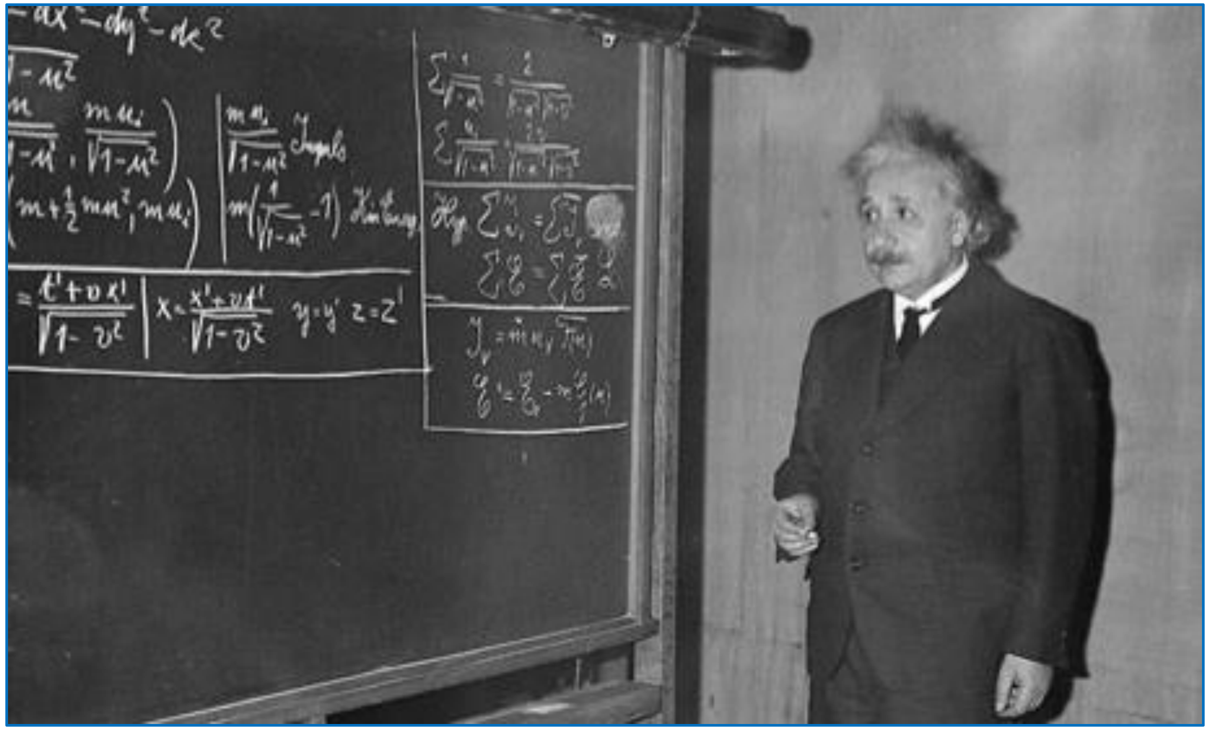
$EL/c^2 = mL$ — Substitute the above equations

$E = mc^2$ — Equivalence of mass and energy!

$E=mc^2$: see lecture notes



An easier way to derive it

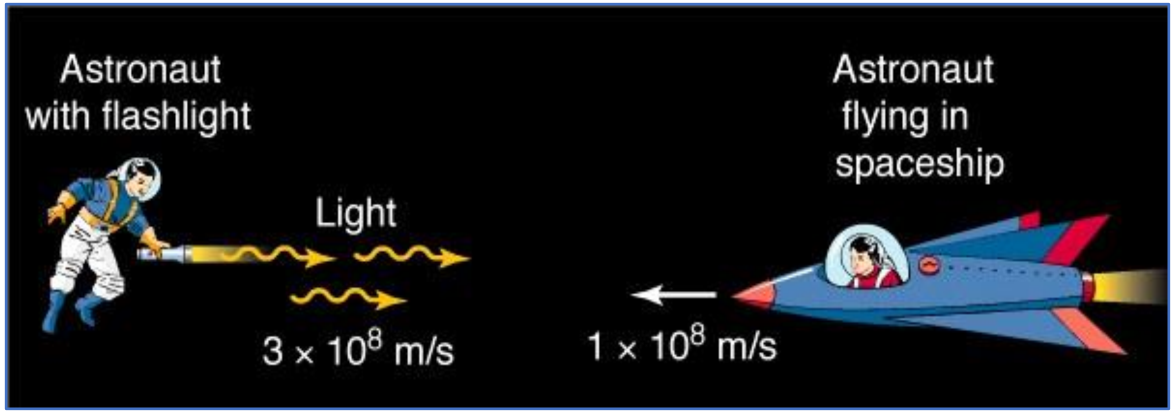


Equivalence of mass and energy!

Conclusions Special Relativity

Simple principle:

- Laws of physics of inertial frames are the same.
- Speed of light is the same for all observers.



Big Consequences:

Space and time are seen differently for different observers.

- Alice's time is a mixture of Bob's time and space and vice versa.
- Alice's space is a mixture of Bob's time and space and vice versa.

$$ct' = \gamma(ct - \beta x)$$
$$x' = \gamma(x - \beta ct)$$

- Time dilation and Lorentz contraction
- Energy and mass are equivalent

Next Lecture: General Relativity

General Relativity: inertial mass = gravitational mass